Proof for – Fairness-Driven Downlink Optimization in NOMA-MEC with UAV-IRS for 5G/6G Networks

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APPENDIX A
PROOF OF LEMMA 1

By applying Jensen's inequality [1], we obtain the following approximation for the expected rate:

$$\mathbb{E}\left\{R_i\right\} \approx \log_2\left(1+\frac{\mathbb{E}\left\{\left|\boldsymbol{H}_i\right|^2\right\} p_i}{\mathbb{E}\left\{\left|\boldsymbol{H}_i\right|^2\right\} \sum\limits_{j=1, j\neq i}^{M} p_j+\sigma^2}\right) \tag{1}$$

Let $\hat{\mathrm{s}}_i^H = \sqrt{\frac{\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}}\overline{\mathrm{s}}_i^H$, $\check{\mathrm{s}}_i^H = \sqrt{\frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}}\widetilde{\mathrm{s}}_i^H$, $\hat{\boldsymbol{h}}_i^H = \sqrt{\frac{\kappa_2}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}}}\bar{\boldsymbol{h}}_i^H$, $\check{\boldsymbol{h}}_i^H = \sqrt{\frac{\rho_0-\kappa_2}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}}}\widetilde{\boldsymbol{h}}_i^H$, $\kappa_1 = \frac{K_1\rho_0}{K_1+1}$, and $\kappa_2 = \frac{K_2\rho_0}{K_2+1}$.

Then, the expected value of $\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\}$ can be decomposed as:

$$\begin{aligned}\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} &= \mathbb{E}\left\{\left|\left(\hat{\mathrm{s}}_i^H+\check{\mathrm{s}}_i^H\right)+\left(\hat{\boldsymbol{h}}_i^H+\check{\boldsymbol{h}}_i^H\right)\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\}\\ &\overset{(a)}{=} \left|\hat{\mathrm{s}}_i^H+\hat{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2+\mathbb{E}\left\{\left|\check{\mathrm{s}}_i^H\right|^2\right\}+\mathbb{E}\left\{\left|\check{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\}\end{aligned} \tag{2}$$

where the equality $\overset{(a)}{=}$ follows from the fact that $\check{\mathrm{s}}^H$ and $\check{\boldsymbol{h}}^H$ are zero-mean and independent of each other. Thus, we have:

$$\mathbb{E}\left\{\left|\check{\mathrm{s}}_i^H\right|^2\right\} = \frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}, \tag{3}$$

$$\mathbb{E}\left\{\left|\check{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\} = \frac{L\rho_0(\rho_0-\kappa_2)}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}\|\boldsymbol{q}_u-\boldsymbol{w}_s\|^2} \tag{4}$$

Let $\gamma_i = \frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}$, $\tau_i = L\rho_0(\rho_0-\kappa_2)$. Substituting (3) and (4) into (2), we obtain the expected effective composite channel power gain from the AP to GMD$_i$:

$$\begin{aligned}&\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} \triangleq \Omega_i\\ &= \left|\hat{\mathrm{s}}_i^H+\hat{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2+\gamma_i+\frac{\tau_i}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}\|\boldsymbol{q}_u-\boldsymbol{w}_s\|^2}\end{aligned} \tag{5}$$

Substituting (5) into (1), the upper bound of the expected achievable rate $\mathbb{E}\{R_i\}$ for GMD can be derived as (11). From formula (11),it is evident that the communication rate $\overline{R}_i$ is primarily influenced by the deterministic component of the LoS path, the larger path loss, and the IRS reflection characteristics. This suggests that when calculating $\overline{R}_i$, the reliance is on the statistical data of Channel State Information (CSI), rather than instantaneous changes. This is particularly feasible in IRS-assisted systems, as the passive operation of the IRS makes it challenging to obtain its instantaneous CSI.

APPENDIX B
PROOF OF THEOREM 2

Let $f(z) = \log_2(1+z)$. By applying the first-order Taylor expansion of $f(z)$ around z_0, we obtain:

$$\begin{aligned}&\log_2(1+z) \geq \log_2(1+z_0)+\frac{1}{\ln(2)}\cdot\frac{1}{1+z_0}(z-z_0),\\ &\log_2(1+z) \geq \log_2(1+z_0)+\frac{z_0}{1+z_0}\cdot\frac{1}{\ln(2)}\cdot\frac{1}{z_0}(z-z_0),\\ &\log_2(1+z) \geq \log_2(1+z_0)+\frac{z_0}{1+z_0}(\log_2 z-\log_2 z_0),\\ &\log_2(1+z) \geq \varepsilon\log_2 z+\eta,\end{aligned} \tag{6}$$

where $\varepsilon = \frac{z_0}{1+z_0}, \eta = \log_2(1+z_0)-\frac{z_0}{1+z_0}\log_2 z_0$.

Thus, the proof of Theorem 2 is complete.

APPENDIX C
PROOF OF LEMMA 3

The expression $\boldsymbol{X}_i^H\boldsymbol{\Phi}\boldsymbol{X}_i$ is a quadratic form, where $\boldsymbol{X}_i$ is a complex matrix. Given that $\boldsymbol{\Phi} = \breve{\boldsymbol{\theta}}\breve{\boldsymbol{\theta}}^H = (\breve{\boldsymbol{\theta}}\breve{\boldsymbol{\theta}}^H)^H = \boldsymbol{\Phi}^H$, $\boldsymbol{\Phi}$ is a Hermitian matrix.

For any non-zero vector $\boldsymbol{v}$, since $\boldsymbol{\Phi} = \breve{\boldsymbol{\theta}}\breve{\boldsymbol{\theta}}^H$, we have:

$$\boldsymbol{v}^H\boldsymbol{\Phi}\boldsymbol{v} = \boldsymbol{v}^H\breve{\boldsymbol{\theta}}\breve{\boldsymbol{\theta}}^H\boldsymbol{v} = (\breve{\boldsymbol{\theta}}^H\boldsymbol{v})^H(\breve{\boldsymbol{\theta}}^H\boldsymbol{v}) = |\breve{\boldsymbol{\theta}}^H\boldsymbol{v}|^2 \geq 0 \tag{7}$$

Thus, $\boldsymbol{\Phi}$ is Hermitian and positive semidefinite in the complex domain. Consequently $\boldsymbol{X}_i^H\boldsymbol{\Phi}\boldsymbol{X}_i$ is a convex function with respect to $\boldsymbol{\Phi}$. Additionally, the trace operation $\mathrm{Tr}(P_iY_i)$ is linear. Therefore, both $f(\boldsymbol{\Phi})$ and $g(\boldsymbol{\Phi})$ are convex functions with respect to $\boldsymbol{\Phi}$.

This completes the proof of Lemma 3.

APPENDIX D
PROOF LEMMA 4

Assume that in the optimal solution of problem **(P5.1)**, there exists $\xi > 0$ such that at least one of the constraints (33a) to (33d) is satisfied as follows:

$$(\overline{u}_i-\xi)^2 \geq \|\boldsymbol{q}_u-\boldsymbol{w}_i\|^2, i\in\mathcal{M}, \tag{8a}$$

$$\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^2 \geq (\underline{u}_i+\xi)^2, i\in\mathcal{M}, \tag{8b}$$

$$(\overline{u}_s-\xi)^2 \geq |\boldsymbol{q}_u-\boldsymbol{w}_s|^2, \tag{8c}$$

$$|\boldsymbol{q}_u-\boldsymbol{w}_s|^2 \geq (\underline{u}_s+\xi)^2, \tag{8d}$$

To ensure that all constraints (33a) to (33d) are satisfied with equality, we can carefully adjust the parameter ξ to either reduce the values of $\overline{u}_i$ and $\overline{u}_s$ or increase the values of $\underline{u}_i$ and $\underline{u}_s$. This adjustment allows us to increase the value of $\underline{\Omega}_i$ or decrease the value of $\overline{\Omega}_i$ to meet the equality conditions of constraints (34) and (35). Consequently, the objective function value is improved.

Thus, the optimal solution of problem **(P5.1)** will satisfy all equality conditions of constraints (33a) to (33d) and (34), (35). Therefore, we conclude that problem **(P5)** and problem **(P5.1)** are mathematically equivalent.

This completes the proof of Lemma 4.

References

[1] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.