Proof for – Fairness-Driven Downlink Optimization in NOMA-MEC with UAV-IRS for 5G/6G Networks

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Appendix A
Proof of Lemma 1

In order to validate the legitimate application of Jensen's inequality in mixed LoS/NLoS channel conditions, our analysis proceeds as follows: First, by incorporating the UAV's flight constraints, it is established from inequality (42) that $\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} < \infty$. Second, given that $\mathrm{SINR}_i \geq 0$, it follows that $R_i = \log_2(1+\mathrm{SINR}_i)$ is a concave function. Based on these two points, we can apply Jensen's inequality to derive the upper bound on the expected rate. [1],

$$\mathbb{E}\left\{R_i\right\} \leq \log_2\left(1+\frac{\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} p_i}{\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} \sum\limits_{j=1, j\neq i}^{M} p_j + \sigma^2}\right) \tag{1}$$

Given the characteristics of Jensen's inequality, the upper bound outlined in (1) proves to be fairly tight when SINR is low. This makes it a good fit for complex urban scenarios with AIRS assistance. Drawing from the theories presented in [2], [3], we can establish an approximation method for the achievable ergodic rate $\mathbb{E}\left\{R_i\right\} \approx \overline{R}_i$, which is detailed as follows:

$$\mathbb{E}\left\{R_i\right\} \approx \log_2\left(1+\frac{\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} p_i}{\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} \sum\limits_{j=1, j\neq i}^{M} p_j + \sigma^2}\right) \triangleq \overline{R}_i \tag{2}$$

Let $\hat{\mathbf{s}}_i^H = \sqrt{\frac{\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}}\overline{\mathbf{s}}_i^H$, $\breve{\mathbf{s}}_i^H = \sqrt{\frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}}\widetilde{\mathbf{s}}_i^H$, $\hat{\boldsymbol{h}}_i^H = \sqrt{\frac{\kappa_2}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}}}\overline{\boldsymbol{h}}_i^H$, $\breve{\boldsymbol{h}}_i^H = \sqrt{\frac{\rho_0-\kappa_2}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}}}\widetilde{\boldsymbol{h}}_i^H$, $\kappa_1 = \frac{K_1\rho_0}{K_1+1}$, and $\kappa_2 = \frac{K_2\rho_0}{K_2+1}$.

Then, the expected value of $\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\}$ can be decomposed as:

$$\begin{aligned}\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} &= \mathbb{E}\left\{\left|\left(\hat{\mathbf{s}}_i^H+\breve{\mathbf{s}}_i^H\right)+\left(\hat{\boldsymbol{h}}_i^H+\breve{\boldsymbol{h}}_i^H\right)\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\}\\ &\overset{(a)}{=} \left|\hat{\mathbf{s}}_i^H+\hat{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2+\mathbb{E}\left\{\left|\breve{\mathbf{s}}_i^H\right|^2\right\}+\mathbb{E}\left\{\left|\breve{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\}\end{aligned} \tag{3}$$

where the equality $\overset{(a)}{=}$ follows from the fact that $\breve{\mathbf{s}}^H$ and $\breve{\boldsymbol{h}}^H$ are zero-mean and independent of each other. Thus, we have:

$$\mathbb{E}\left\{\left|\breve{\mathbf{s}}_i^H\right|^2\right\} = \frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}, \tag{4}$$

$$\mathbb{E}\left\{\left|\breve{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2\right\} = \frac{L\rho_0(\rho_0-\kappa_2)}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}\|\boldsymbol{q}_u-\boldsymbol{w}_s\|^2} \tag{5}$$

Let $\gamma_i = \frac{\rho_0-\kappa_1}{\|\boldsymbol{w}_s-\boldsymbol{w}_i\|^{\beta_1}}$, $\tau_i = L\rho_0(\rho_0-\kappa_2)$. Substituting (4) and (5) into (3), we obtain the expected effective composite channel power gain from the AP to GMD_i:

$$\begin{aligned}&\mathbb{E}\left\{|\boldsymbol{H}_i|^2\right\} \triangleq \Omega_i\\ &= \left|\hat{\mathbf{s}}_i^H+\hat{\boldsymbol{h}}_i^H\boldsymbol{\Theta}\boldsymbol{G}\right|^2+\gamma_i+\frac{\tau_i}{\|\boldsymbol{q}_u-\boldsymbol{w}_i\|^{\beta_2}\|\boldsymbol{q}_u-\boldsymbol{w}_s\|^2}\end{aligned} \tag{6}$$

Substituting (6) into (2), the upper bound of the expected achievable rate $\mathbb{E}\{R_i\}$ for GMD can be derived as (11). From formula (11),it is evident that the communication rate $\overline{R}_i$ is primarily influenced by the deterministic component of the LoS path, the larger path loss, and the IRS reflection characteristics. This suggests that when calculating $\overline{R}_i$, the reliance is on the statistical data of Channel State Information (CSI), rather than instantaneous changes. This is particularly feasible in IRS-assisted systems, as the passive operation of the IRS makes it challenging to obtain its instantaneous CSI.

Appendix B
PROOF OF THEOREM 2

Define a new function $\phi(x) = \log_2(1+2^x)$. Let $x = \log_2 z$, then $2^x = 2^{\log_2 z} = z$. Thus, $\phi(\log_2 z) = \log_2(1+z)$.

The first-order derivative of $\phi(x)$, denoted as $\phi'(x)$, and the second-order derivative of $\phi(x)$, denoted as $\phi''(x)$, are as follows:

$$\begin{aligned}\phi'(x) &= \frac{d}{dx}\left[\log_2(1+2^x)\right] = \frac{1}{(1+2^x)\ln 2}\cdot\frac{d}{dx}(1+2^x)\\ &= \frac{2^x}{1+2^x}\end{aligned} \tag{7}$$

$$\phi''(x) = \frac{d}{dx}\left(\frac{2^x}{1+2^x}\right) = \frac{2^x\ln 2}{(1+2^x)^2} \tag{8}$$

Since $2^x > 0$ and $\ln 2 > 0$, it follows that $\phi''(x) > 0$ for all real numbers x. Hence, $\phi(x) = \log_2(1+2^x)$ is a convex function. We have

$$\phi(x) \geq \phi(x_0)+\phi'(x_0)(x-x_0) \tag{9}$$

By substituting $x = \log_2 z$ and $x_0 = \log_2 z_0$ into the convex function inequality, , we can derive that:

$$\log_2(1+z) \geq \log_2(1+z_0) + \frac{z_0}{1+z_0}(\log_2 z - \log_2 z_0)$$
$$\log_2(1+z) \geq \frac{z_0}{1+z_0}\log_2 z + \left(\log_2(1+z_0) - \frac{z_0}{1+z_0}\log_2 z_0\right) \tag{10}$$

Thus, the proof of Theorem 2 is complete.

Appendix C
PROOF OF LEMMA 3

To determine whether $f(\mathbf{\Phi}), g(\mathbf{\Phi})$ is a concave function, we analyze the second-order derivative with respect to $\mathbf{\Phi}$ and verify its negative semi-definiteness.

Let $u(\mathbf{\Phi}) = \mathrm{Tr}(P_i Y_i) + A_i p_i = \mathrm{Tr}(P_i \boldsymbol{X}_i^H \mathbf{\Phi} \boldsymbol{X}_i) + A_i p_i$. Using the matrix trace derivative formula $\frac{\partial \mathrm{Tr}(AXB)}{\partial X} = A^T B^T$, the derivative of $u(\mathbf{\Phi})$ with respect to $\mathbf{\Phi}$ is:

$$\frac{\partial u(\mathbf{\Phi})}{\partial \mathbf{\Phi}} = \boldsymbol{X}_i P_i \boldsymbol{X}_i^H = P_i \boldsymbol{X}_i \boldsymbol{X}_i^H. \tag{11}$$

According to the chain - rule of derivatives for composite functions, the first-order derivative of $f(\mathbf{\Phi})$ is:

$$\frac{\partial f(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}} = \frac{1}{u(\boldsymbol{\Phi})\ln 2} \cdot \frac{\partial u(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}} = \frac{P_i \boldsymbol{X}_i \boldsymbol{X}_i^H}{(\mathrm{Tr}(P_i \boldsymbol{X}_i^H \boldsymbol{\Phi} \boldsymbol{X}_i) + A_i p_i)\ln 2} \tag{12}$$

Let $g(\boldsymbol{\Phi}) = \frac{\partial f(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}}$. By using the quotient - rule for derivatives $\frac{\partial}{\partial \boldsymbol{\Phi}}\left(\frac{\boldsymbol{C}}{u(\boldsymbol{\Phi})}\right) = -\frac{\boldsymbol{C} \otimes \frac{\partial u(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}}}{u(\boldsymbol{\Phi})^2}$, where $\boldsymbol{C} = \frac{P_i \boldsymbol{X}_i \boldsymbol{X}_i^H}{\ln 2}$, and $\otimes$ represents the Kronecker product. Thus, the second-order derivative (Hessian matrix) is:

$$\frac{\partial^2 f(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}^2} = -\frac{(P_i \boldsymbol{X}_i \boldsymbol{X}_i^H) \otimes (P_i \boldsymbol{X}_i \boldsymbol{X}_i^H)}{(\mathrm{Tr}(P_i \boldsymbol{X}_i^H \boldsymbol{\Phi} \boldsymbol{X}_i) + A_i p_i)^2 (\ln 2)^2} \tag{13}$$

For any non - zero vector $\boldsymbol{z}$, consider the quadratic form:

$$\boldsymbol{z}^H \frac{\partial^2 f(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}^2} \boldsymbol{z} = -\frac{\left|\boldsymbol{z}^H (P_i \boldsymbol{X}_i \boldsymbol{X}_i^H) \boldsymbol{z}\right|^2}{(\mathrm{Tr}(P_i \boldsymbol{X}_i^H \boldsymbol{\Phi} \boldsymbol{X}_i) + A_i p_i)^2 (\ln 2)^2} \tag{14}$$

The expression $\boldsymbol{X}_i^H \mathbf{\Phi} \boldsymbol{X}_i$ is a quadratic form, where $\boldsymbol{X}_i$ is a complex matrix. Since $\mathbf{\Phi} = \breve{\boldsymbol{\theta}} \breve{\boldsymbol{\theta}}^H = (\breve{\boldsymbol{\theta}} \breve{\boldsymbol{\theta}}^H)^H = \mathbf{\Phi}^H$, $\mathbf{\Phi}$ is a Hermitian matrix.

For any non-zero vector $\boldsymbol{v}$, because $\mathbf{\Phi} = \breve{\boldsymbol{\theta}} \breve{\boldsymbol{\theta}}^H$, we have:

$$\boldsymbol{v}^H \mathbf{\Phi} \boldsymbol{v} = \boldsymbol{v}^H \breve{\boldsymbol{\theta}} \breve{\boldsymbol{\theta}}^H \boldsymbol{v} = (\breve{\boldsymbol{\theta}}^H \boldsymbol{v})^H (\breve{\boldsymbol{\theta}}^H \boldsymbol{v}) = |\breve{\boldsymbol{\theta}}^H \boldsymbol{v}|^2 \geq 0 \tag{15}$$

From the constraint (13b), we can deduce that $p_i > 0$. Thus, in the complex domain, $(\mathrm{Tr}(P_i \boldsymbol{X}_i^H \boldsymbol{\Phi} \boldsymbol{X}_i) + A_i p_i)^2 (\ln 2)^2 > 0$. Accordingly, we have:

$$\boldsymbol{z}^H \frac{\partial^2 f(\boldsymbol{\Phi})}{\partial \boldsymbol{\Phi}^2} \boldsymbol{z} \leq 0 \tag{16}$$

Therefore, the Hessian matrix is negative semi-definite within its domain, and the function $f(\boldsymbol{\Phi})$ is a concave function. Similarly, $g(\boldsymbol{\Phi})$ is also concave.

This completes the proof of Lemma 3.

Appendix D
PROOF LEMMA 4

Assume that in the optimal solution of problem (P5.1), there exists $\xi > 0$ such that at least one of the constraints (35a) to (35d) is satisfied as follows:

$$(\overline{u}_i - \xi)^2 \geq \|\boldsymbol{q}_u - \boldsymbol{w}_i\|^2, \quad i \in \mathcal{M}, \tag{17a}$$
$$\|\boldsymbol{q}_u - \boldsymbol{w}_i\|^2 \geq (\underline{u}_i + \xi)^2, \quad i \in \mathcal{M}, \tag{17b}$$
$$(\overline{u}_s - \xi)^2 \geq |\boldsymbol{q}_u - \boldsymbol{w}_s|^2, \tag{17c}$$
$$|\boldsymbol{q}_u - \boldsymbol{w}_s|^2 \geq (\underline{u}_s + \xi)^2, \tag{17d}$$

To ensure that all constraints (35a) to (35d) are satisfied with equality, we can carefully adjust the parameter ξ to either reduce the values of $\overline{u}_i$ and $\overline{u}_s$ or increase the values of $\underline{u}_i$ and $\underline{u}_s$. This adjustment allows us to increase the value of $\underline{\Omega}_i$ or decrease the value of $\overline{\Omega}_i$ to meet the equality conditions of constraints (36) and (37). Consequently, the objective function value is improved.

Thus, the optimal solution of problem (P5.1) will satisfy all equality conditions of constraints (35a) to (35d) and (36), (37). Therefore, we conclude that problem (P5) and problem (P5.1) are mathematically equivalent.

This completes the proof of Lemma 4.

References

[1] S. Boyd and L. Vandenberghe, Convex optimization. Cambridge university press, 2004.

[2] R. Tang, W. Feng, Y. Chen, and N. Ge, “Noma-based uav communications for maritime coverage enhancement,” China Communications, vol. 18, no. 4, pp. 230–243, 2021.

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