Номер 1.1

Вариант 4

In[]:=
```
A = Table[If[i > j, 1, If[i == j, i + 1, If[i < j, 2]]], {i, 7}, {j, 7}]
MatrixForm[A]
```

Out[]=

$$\{\{2, 2, 2, 2, 2, 2, 2\}, \{1, 3, 2, 2, 2, 2, 2\}, \{1, 1, 4, 2, 2, 2, 2\}, \{1, 1, 1, 5, 2, 2, 2\}, \{1, 1, 1, 1, 6, 2, 2\}, \{1, 1, 1, 1, 1, 7, 2\}, \{1, 1, 1, 1, 1, 1, 8\}\}$$

Out[]//MatrixForm=

$$\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 3 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 4 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 5 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 6 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 7 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 8 \end{pmatrix}$$

In[]:=
```
B = Table[8 * i - i^2, {i, 7}]
MatrixForm[B]
```

Out[]=

$$\{7, 12, 15, 16, 15, 12, 7\}$$

Out[]//MatrixForm=

$$\begin{pmatrix} 7 \\ 12 \\ 15 \\ 16 \\ 15 \\ 12 \\ 7 \end{pmatrix}$$

a)

In[]:=
```
n = Norm[A, ∞]
```

Out[]=

$$14$$

In[]:=
```
inv = Norm[Inverse[A], ∞]
```

Out[]=

$$\frac{25}{14}$$

In[]:=
```
num = N[n * inv] (*-число обусловленности*)
```

Out[]=

$$25.$$

б)

In[•]:=
```
X = LinearSolve[A, B]
```

Out[•]=

$$\left\{-\frac{93}{20}, \frac{7}{20}, \frac{37}{20}, \frac{131}{60}, \frac{29}{15}, \frac{4}{3}, \frac{1}{2}\right\}$$

в)

```
(*1я система*)
```

In[•]:=
```
B1 = Table[If[i == 7, 0.01 * 0.01 * B[[7]], 0], {i, 7}, {j, 1}]
MatrixForm[B1]
```

Out[•]=
```
{{0}, {0}, {0}, {0}, {0}, {0}, {0.0007}}
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.0007 \end{pmatrix}$$

In[•]:=
```
X1 = LinearSolve[A, B + B1]
```

Out[•]=
```
{{-4.65002}, {0.349983}, {1.84998}, {2.18332}, {1.93332}, {1.33332}, {0.5001}}
```

```
(*2я система*)
```

In[•]:=
```
B2 = Table[If[i == 7, 0.01 * 0.1 * B[[7]], 0], {i, 7}, {j, 1}]
MatrixForm[B2]
```

Out[•]=
```
{{0}, {0}, {0}, {0}, {0}, {0}, {0.007}}
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.007 \end{pmatrix}$$

In[•]:=
```
X2 = LinearSolve[A, B + B2]
```

Out[•]=
```
{{-4.65017}, {0.349833}, {1.84983}, {2.18317}, {1.93317}, {1.33317}, {0.501}}
```

```
(*3я система*)
```

```
In[•]:= B3 = Table[If[i == 7, 0.01 * 1 * B[[7]], 0], {i, 7}, {j, 1}]
         MatrixForm[B3]
```

Out[•]=

```
{{0}, {0}, {0}, {0}, {0}, {0}, {0.07}}
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.07 \end{pmatrix}$$

```
In[•]:= X3 = LinearSolve[A, B + B3]
```

Out[•]=

```
{{-4.65167}, {0.348333}, {1.84833}, {2.18167}, {1.93167}, {1.33167}, {0.51}}
```

г)

```
In[•]:= limpr1 = num * Norm[B1, ∞] / Norm[B + B1, ∞]
```

Out[•]=

```
0.00109375
```

```
In[•]:= 0.00109375` // PercentForm
```

Out[•]//PercentForm=

```
0.1094%
```

```
In[•]:= limpr2 = num * Norm[B2, ∞] / Norm[B + B2, ∞]
```

Out[•]=

```
0.0109375
```

```
In[•]:= 0.0109375` // PercentForm
```

Out[•]//PercentForm=

```
1.094%
```

```
In[•]:= limpr3 = num * Norm[B3, ∞] / Norm[B + B3, ∞]
```

Out[•]=

```
0.109375
```

```
In[•]:= 0.109375` // PercentForm
```

Out[•]//PercentForm=

```
10.94%
```

д)

In[•]:=
$$p1 = \frac{\text{Norm}[X - X1, \infty]}{\text{Norm}[X1, \infty]} \text{ // PercentForm}$$

Out[•]//PercentForm=
0.002151%

In[•]:=
$$p2 = \frac{\text{Norm}[X - X2, \infty]}{\text{Norm}[X2, \infty]} \text{ // PercentForm}$$

Out[•]//PercentForm=
0.0215%

$$p3 = \frac{\text{Norm}[X - X3, \infty]}{\text{Norm}[X3, \infty]} \text{ // PercentForm}$$

Out[•]//PercentForm=
0.215%

Номер 2.2

In[1]:=
$$A = \text{Table}\left[\frac{1}{i + j - 1}, \{i, 7\}, \{j, 7\}\right]$$

MatrixForm[A]

Out[1]=
$$\left\{\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\}, \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right\}, \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}\right\}, \left\{\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}, \left\{\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}\right\}, \left\{\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}\right\}, \left\{\frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}\right\}\right\}$$

Out[2]//MatrixForm=
$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \end{pmatrix}$$

In[3]:=
```
B = Table[3 * i - 8, {i, 7}]
MatrixForm[B]
```

Out[3]= $\{-5, -2, 1, 4, 7, 10, 13\}$

Out[4]//MatrixForm=

$$\begin{pmatrix} -5 \\ -2 \\ 1 \\ 4 \\ 7 \\ 10 \\ 13 \end{pmatrix}$$

а)

In[5]:=
```
n = Norm[A, ∞]
```

Out[5]= $\frac{363}{140}$

In[6]:=
```
inv = Norm[Inverse[A], ∞]
```

Out[6]= 379 964 970

In[7]:=
```
num = N[n * inv]
```

Out[7]= 9.85195×10^8

б)

In[8]:=
```
X = LinearSolve[A, B]
```

Out[8]= $\{973, -45\,696, 502\,740, -2\,184\,000, 4\,400\,550, -4\,124\,736, 1\,453\,452\}$

в)

In[9]:=
```
(*1я система*)
B1 = Table[If[i == 7, 0.01 * 0.01 * B[[7]], 0], {i, 7}, {j, 1}]
MatrixForm[B1]
```

Out[9]= $\{\{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0.0013\}\}$

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.0013 \end{pmatrix}$$

In[11]:=
```
X1 = LinearSolve[A, B + B1]
```

Out[11]=

$$\{\{988.616\}, \{-46\,351.9\}, \{509\,299.\}, \{-2.21023 \times 10^6\}, \{4.44974 \times 10^6\}, \{-4.16802 \times 10^6\}, \{1.46788 \times 10^6\}\}$$

In[12]:=
```
(*2я система*)
B2 = Table[If[i == 7, 0.01 * 0.1 * B[[7]], 0], {i, 7}, {j, 1}]
MatrixForm[B2]
```

Out[12]=
$$\{\{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0.013\}\}$$

Out[13]//MatrixForm=
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.013 \end{pmatrix}$$

In[14]:=
```
X2 = LinearSolve[A, B + B2]
```

Out[14]=
$$\{\{1129.16\}, \{-52\,254.6\}, \{568\,326.\}, \{-2.44634 \times 10^6\}, \{4.89244 \times 10^6\}, \{-4.5576 \times 10^6\}, \{1.59774 \times 10^6\}\}$$

In[15]:=
```
(*3я система*)
B3 = Table[If[i == 7, 0.01 * 1 * B[[7]], 0], {i, 7}, {j, 1}]
MatrixForm[B3]
```

Out[15]=
$$\{\{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0.13\}\}$$

Out[16]//MatrixForm=
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.13 \end{pmatrix}$$

In[17]:=
```
X3 = LinearSolve[A, B + B3]
```

Out[17]=
$$\{\{2534.56\}, \{-111\,282.\}, \{1.1586 \times 10^6\}, \{-4.80742 \times 10^6\}, \{9.31946 \times 10^6\}, \{-8.45338 \times 10^6\}, \{2.89633 \times 10^6\}\}$$

г)

In[18]:=
```
limpr1 = num * Norm[B1, ∞] / Norm[B + B1, ∞] // PercentForm
```

Out[18]//PercentForm=
$$9850964\%$$

In[19]:=
```
limpr2 = num * Norm[B2, ∞] / Norm[B + B2, ∞] // PercentForm
```

Out[19]//PercentForm=
$$98421068\%$$

In[20]:=
$$\text{limpr3} = \text{num} * \frac{\text{Norm}[B3, \infty]}{\text{Norm}[B + B3, \infty]} \text{ // PercentForm}$$

Out[20]//PercentForm=

975440482%

д)

In[21]:=
$$\text{p1} = \frac{\text{Norm}[X - X1, \infty]}{\text{Norm}[X1, \infty]} \text{ // PercentForm}$$

Out[21]//PercentForm=

1.105%

In[22]:=
$$\text{p2} = \frac{\text{Norm}[X - X2, \infty]}{\text{Norm}[X2, \infty]} \text{ // PercentForm}$$

Out[22]//PercentForm=

10.05%

In[23]:=
$$\text{p3} = \frac{\text{Norm}[X - X3, \infty]}{\text{Norm}[X3, \infty]} \text{ // PercentForm}$$

Out[23]//PercentForm=

52.78%

Номер 2.1

In[60]:=
$$A = \begin{pmatrix} 7 & 3 & 0 & 0 & 0 \\ 2 & 16 & -4 & 0 & 0 \\ 0 & 5 & -12 & 4 & 0 \\ 0 & 0 & 3 & 21 & -6 \\ 0 & 0 & 0 & 4 & 7 \end{pmatrix};$$

$$B = \begin{pmatrix} -11 \\ 0 \\ -31 \\ -21 \\ 35 \end{pmatrix};$$

```
a = {0, 2, 5, 3, 4};
b = {7, 16, -12, 21, 7};
c = {3, -4, 4, -6, 0};
d = {-11, 0, -31, -21, 35};
L = {0, 0, 0, 0, 0};
M = {0, 0, 0, 0, 0};
L[[1]] = -c[[1]]/b[[1]];
M[[1]] = d[[1]]/b[[1]];
For[i = 2, i ≤ 5, i++,
 L[[i]] = -c[[i]]/(b[[i]] + a[[i]] * L[[i - 1]])
]
L
For[i = 2, i ≤ 5, i++,
 M[[i]] = (d[[i]] - a[[i]] * M[[i - 1]])/(b[[i]] + a[[i]] * L[[i - 1]])
]
M
X = {0, 0, 0, 0, 0};

X[[5]] = M[[5]];

For [i = 4, i ≥ 1, i--,
 X[[i]] = L[[i]] * X[[i + 1]] + M[[i]]
]
X


(*Ответ:*)
```

Out[77]=

$$\{-2, 1, 3, 0, 5\}$$

Номер 2.2

In[78]:=
$$A = \begin{pmatrix} 7 & 3 & 0 & 0 & 0 \\ 2 & 16 & -4 & 0 & 0 \\ 0 & 5 & -12 & 4 & 0 \\ 0 & 0 & 3 & 21 & -6 \\ 0 & 0 & 0 & 4 & 7 \end{pmatrix};$$

$$B = \begin{pmatrix} -11 \\ 0 \\ -31 \\ -21 \\ 35 \end{pmatrix};$$

```
a = {0, 2, 5, 3, 4};
b = {7, 16, -12, 21, 7};
c = {3, -4, 4, -6, 0};
d = {-11, 0, -31, -21, 35};
L = {0, 0, 0, 0, 0};
M = {0, 0, 0, 0, 0};
L[[1]] = -c[[1]]/b[[1]];
M[[1]] = d[[1]]/b[[1]];
For[i = 2, i ≤ 5, i++,
 L[[i]] = -c[[i]]/(b[[i]] + a[[i]] * L[[i - 1]])
]
L
For[i = 2, i ≤ 5, i++,
 M[[i]] = (d[[i]] - a[[i]] * M[[i - 1]])/(b[[i]] + a[[i]] * L[[i - 1]])
]
M
X = {0, 0, 0, 0, 0};

X[[5]] = M[[5]];

For [i = 4, i ≥ 1, i--,
 X[[i]] = L[[i]] * X[[i + 1]] + M[[i]]
]
```

Out[89]=
$$\left\{-\frac{3}{7}, \frac{14}{53}, \frac{106}{283}, \frac{566}{2087}, 0\right\}$$

Out[91]=
$$\left\{-\frac{11}{7}, \frac{11}{53}, 3, -\frac{2830}{2087}, 5\right\}$$

In[95]:=
```
X = N[X];
NumberForm[X, 3]
```
Out[96]//NumberForm=
```
{-2., 1., 3., 0., 5.}
```

In[97]:=
```
(*Ответ:*)
{1, 0, 1, 0, 1}
```
Out[97]=
```
{1, 0, 1, 0, 1}
```

Номер 3

In[115]:=
```
n = 10;
A = Table[If[i == j, 2 * n, 1], {i, 1, n}, {j, 1, n}];

B = Table[(2 * n - 1) * i + n * (n + 1) / 2 + (3 * n - 1) * (4 - 1), {i, 1, n}];
```

Метод Якоби

In[118]:=
```
jacobi[X0_, maxIterations_, tolerance_] := Module[
 {X = X0, Xprev, iterations = 0, error = tolerance + 1, n = Length[X0]},
 While[iterations < maxIterations && error > tolerance,
  Xprev = X;
  X = Table[(B[[i]] - Sum[A[[i, j]] * Xprev[[j]], {j, 1, n}] + A[[i, i]] * Xprev[[i]]) /
       A[[i, i]], {i, 1, n}];
  error = Max[Abs[X - Xprev]];
  iterations++;
 ];
 {X, iterations}
]
```

Метод Зейделя

In[100]:=
```
gaussSeidel[X0_, maxIterations_, tolerance_] :=
 Module[{X = X0, Xprev, iterations = 0, error = tolerance + 1, n = Length[X0]},
  While[iterations < maxIterations && error > tolerance, Xprev = X;
   Do[X[[i]] = (B[[i]] - Sum[A[[i, j]] * X[[j]], {j, 1, i - 1}] -
         Sum[A[[i, j]] * Xprev[[j]], {j, i + 1, n}]) / A[[i, i]], {i, 1, n}];
   error = Max[Abs[X - Xprev]];
   iterations++;];
  {X, iterations}]
```

In[119]:=
```
X0 = ConstantArray[0, n];
maxIterations = 1000;
tolerance = 10^-3;
```

Решение

In[122]:=
```
{Xjacobi, iterationsJacobi} = jacobi[X0, maxIterations, tolerance];
```

In[123]:=
```
N[Xjacobi]
```

Out[123]=
```
{4.00026, 5.00026, 6.00026, 7.00026, 8.00026, 9.00026, 10.0003, 11.0003, 12.0003, 13.0003}
```

In[124]:=
```
N[iterationsJacobi]
```

Out[124]=
```
13.
```

In[125]:=
```
{Xzeidel, iterationsZeidel} = gaussSeidel[X0, maxIterations, tolerance];
```

In[126]:=
```
N[Xzeidel]
```

Out[126]=
```
{3.99997, 4.99997, 5.99997, 6.99998, 7.99999, 9., 10., 11., 12., 13.}
```

In[127]:=
```
N[iterationsZeidel]
```

Out[127]=
```
6.
```

In[153]:=
```
ClearAll
```

Out[153]=
```
ClearAll
```

n = 20

In[154]:=
```
n = 20;
A = Table[If[i == j, 2 * n, 1], {i, 1, n}, {j, 1, n}];

B = Table[(2 * n - 1) * i + n * (n + 1) / 2 + (3 * n - 1) * (4 - 1), {i, 1, n}];
```

Метод Якоби

In[157]:=
```
jacobi[X0_, maxIterations_, tolerance_] := Module[
 {X = X0, Xprev, iterations = 0, error = tolerance + 1, n = Length[X0]},
 While[iterations < maxIterations && error > tolerance,
  Xprev = X;
  X = Table[(B[[i]] - Sum[A[[i, j]] * Xprev[[j]], {j, 1, n}] + A[[i, i]] * Xprev[[i]]) /
       A[[i, i]], {i, 1, n}];
  error = Max[Abs[X - Xprev]];
  iterations++;
 ];
 {X, iterations}
]
```

Метод Зейделя

In[158]:=
```
gaussSeidel[X0_, maxIterations_, tolerance_] :=
 Module[{X = X0, Xprev, iterations = 0, error = tolerance + 1, n = Length[X0]},
  While[iterations < maxIterations && error > tolerance, Xprev = X;
   Do[X[[i]] = (B[[i]] - Sum[A[[i, j]] * X[[j]], {j, 1, i - 1}] -
        Sum[A[[i, j]] * Xprev[[j]], {j, i + 1, n}]) / A[[i, i]], {i, 1, n}];
   error = Max[Abs[X - Xprev]];
   iterations++;];
  {X, iterations}]
```

In[159]:=
```
X0 = ConstantArray[0, n];
```

In[160]:=
```
maxIterations = 1000;
tolerance = 10^-3;
```

Решение

In[162]:=
```
{Xjacobi, iterationsJacobi} = jacobi[X0, maxIterations, tolerance];
```

In[163]:=
```
N[Xjacobi]
```

Out[163]=
```
{4.00019, 5.00019, 6.00019, 7.00019, 8.00019, 9.00019, 10.0002, 11.0002, 12.0002, 13.0002,
 14.0002, 15.0002, 16.0002, 17.0002, 18.0002, 19.0002, 20.0002, 21.0002, 22.0002, 23.0002}
```

In[164]:=
```
N[iterationsJacobi]
```

Out[164]=
```
15.
```

In[165]:=
```
{Xzeidel, iterationsZeidel} = gaussSeidel[X0, maxIterations, tolerance];
```

In[166]:=
```
N[Xzeidel]
```
Out[166]=
```
{4.00001, 5.00001, 6.00001, 7.00001, 8.00001, 9., 10.,
 11., 12., 13., 14., 15., 16., 17., 18., 19., 20., 21., 22., 23.}
```

In[167]:=
```
N[iterationsZeidel]
```
Out[167]=
```
7.
```

Вывод : У метода Зейделя меньше итераций в обоих случаях