



I - Preuves de sequents propositionals

$$1. (\neg P \vee Q) \vee R \vdash P \rightarrow (Q \vee R)$$

$$\begin{array}{c}
 R \vdash P \rightarrow (Q \vee R) \\
 \\
 \frac{}{\Gamma \vdash P} h \quad \frac{}{\Gamma \vdash Q} h \quad \frac{}{\Gamma \vdash R} h \\
 \frac{}{\Gamma \vdash P \rightarrow (Q \vee R)} \rightarrow i
 \end{array}$$

2. $p \vee \neg a, p \vee \neg R, (\neg p \vee R) \vee R \vdash (q \vee R) \rightarrow p$

$$R, (\lambda v.R) \cdot R \vdash (QvR) \rightarrow P$$

Approximately

3. $(LK) \quad p \rightarrow q, \neg p \rightarrow \neg q \vdash (p \rightarrow q) \wedge (q \rightarrow p) \quad | = p \leftrightarrow q$

[illegible]

9. $P \rightarrow (Q \vee R), (P \wedge Q) \rightarrow R \vdash P \rightarrow R$

[illegible]

$$S, (LK) (p, q) \rightarrow R, \sim p \rightarrow \sim q \rightarrow R \vdash R$$

[illegible]

6. $\neg \alpha \vee \beta, P \rightarrow R \vdash R \vee \neg \alpha$

$$\begin{array}{r} \frac{P_1, P_2, \dots, P_n \vdash R}{P_1, P_2, \dots, P_n \vdash R} \text{ (trivial)} \\ \frac{P_1, P_2, \dots, P_n \vdash R}{P_1, P_2, \dots, P_n \vdash R} \text{ (trivial)} \\ \frac{P_1, P_2, \dots, P_n \vdash R}{P_1, P_2, \dots, P_n \vdash R} \text{ (trivial)} \end{array}$$

III. Règles dérivées

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash B}{\Gamma \vdash A} ?$$

Soit $\Gamma = \{A \vee B, B\}$

On a alors $\frac{\frac{}{\Gamma \vdash A \vee B}^h \quad \frac{}{\Gamma, A \vdash B}^h}{\Gamma \vdash A} ?$

Le suivant est prouvable

Soit v la valuation suivante

$$v(A) = F \quad ; \quad \begin{cases} v(B) = V \\ v(A \vee B) = V \end{cases}$$

Cette valuation rend vraies les hypothèses et fausse la conclusion, donc ce suivant est non valide \Rightarrow pas prouvable d'après le méta-théorème de correction.

$$\neg P \rightarrow P \vdash_K P$$

$$\vdash_K (P \rightarrow Q) \vee (Q \rightarrow P)$$

ex: $\frac{\frac{}{\neg P \rightarrow P, P \vdash P}^h \quad \frac{\frac{}{\neg P \rightarrow P}^h \quad \frac{}{P \vdash P}^h}{\neg P \rightarrow P, \neg P \vdash P} \rightarrow \quad \neg P \vee P}{\neg P \rightarrow P \vdash_K P} \vee_c (P \vee \neg P)$

ex: $\frac{\frac{\frac{}{P, P \vdash P}^h}{P \vdash P \rightarrow P} \rightarrow \quad \frac{\frac{}{P, P \vdash P}^h \quad \frac{}{P \vdash P}^h}{P \vdash P \rightarrow P} \rightarrow \quad \frac{\frac{}{P \vdash P \rightarrow P}^h \quad \frac{}{P \vdash P \rightarrow P}^h}{P \vdash (P \rightarrow P) \vee (P \rightarrow \neg P)} \vee_{\vee} \quad \frac{\frac{}{P \vdash P \rightarrow P}^h \quad \frac{}{P \vdash P \rightarrow P}^h}{P \vdash (P \rightarrow P) \vee (P \rightarrow \neg P)} \vee_{\vee} \quad \frac{}{P \vdash (P \rightarrow P) \vee (P \rightarrow \neg P)} \vee_c (P \vee \neg P)$

Correction :

$$P \rightarrow Q, \neg P \rightarrow R \vdash Q \vee R$$

[illegible]

$$(a_1 \wedge \kappa) \vee (a_1 \wedge p) \vdash a_1 \mid \kappa \rightarrow p$$

[illegible]