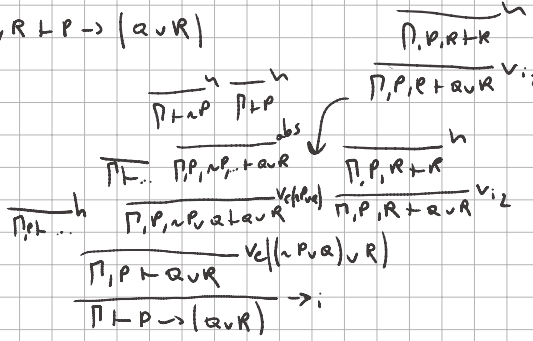
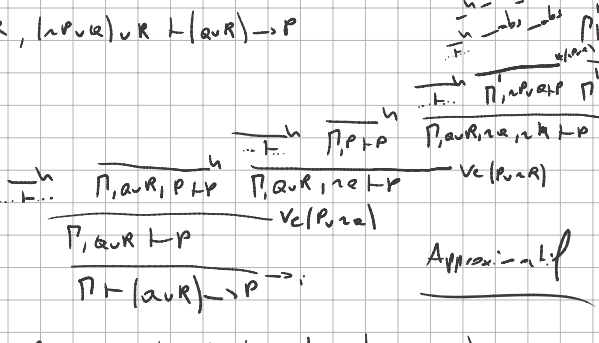
DST 2020

I - Preuves de séquents propositionnels

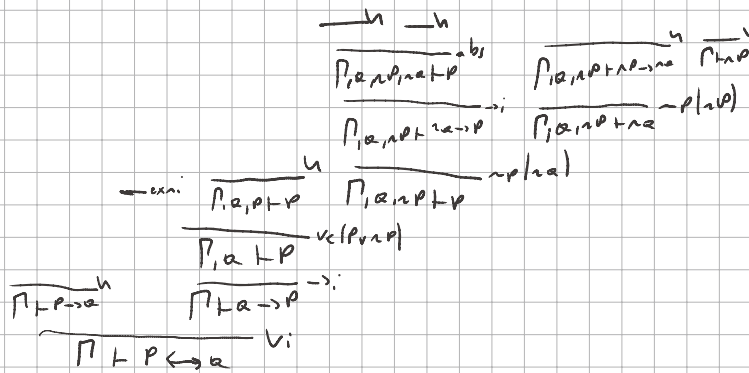
1. $(\neg P \vee Q) \vee R \vdash P \to (Q \vee R)$



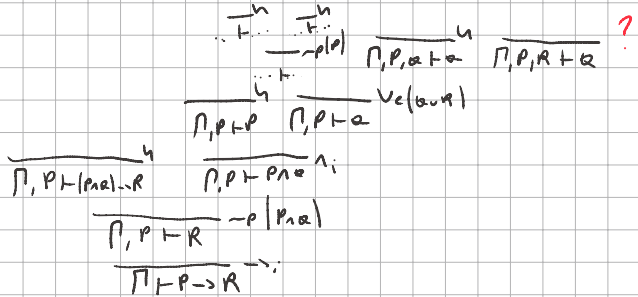
2. $P \vee \neg Q,\ P \vee \neg R,\ (\neg P \vee Q) \vee R \vdash (Q \vee R) \to P$



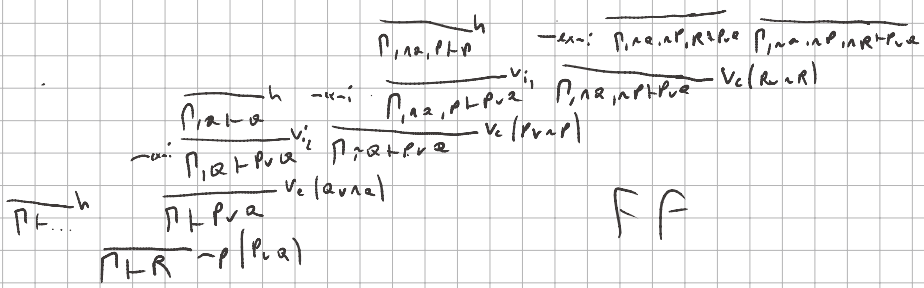
3. $(LK)\ P \to Q,\ \neg P \to \neg Q \vdash (P \to Q) \wedge (Q \to P) \quad \models P \leftrightarrow Q$



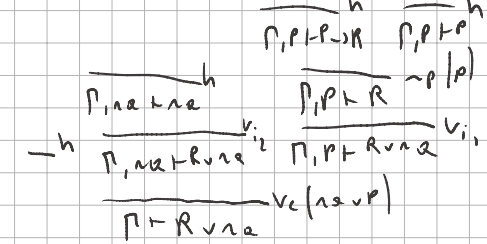
4. $P \to (Q \vee R),\ (P \wedge Q) \to R \vdash P \to R$



5. $(\mathrm{LK})\ (P \vee Q) \to R,\ \neg P \to \neg Q \to R \vdash R$



6. $\neg Q \vee P,\ P \to R \vdash R \vee \neg Q$



II. Règles dérivées

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash B}{\Gamma \vdash A}\ ?$$

Soit $\Gamma = \{A \vee B, B\}$

On a alors $\dfrac{\dfrac{}{\Gamma \vdash A \vee B}\,h \quad \dfrac{}{\Gamma, A \vdash B}\,h}{\Gamma \vdash A}\ ?$

Le séquent est prouvable

Soit v la valuation suivante

$v(A) = F$; $v(B) = V$; $v(A \vee B) = V$

Cette valuation rend vraies les hypothèses et fausse la conclusion, donc ce séquent est non valide $\Rightarrow$ pas prouvable d'après le méta-théorème de correction.

$\sim P \to P \vdash_K P$

$\vdash_K (P \to Q) \vee (Q \to P)$

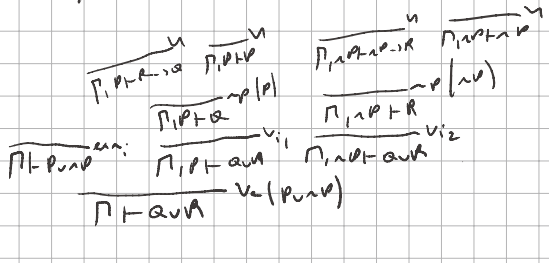
$$\dfrac{\dfrac{}{\sim P \to P, P \vdash P}\,h \quad \dfrac{}{\sim P \to P, \sim P \vdash P}\,h}{\sim P \to P \vdash_K P}\ \text{exm}$$

$$\dfrac{\dfrac{}{\Gamma \vdash \sim P \to P}\,h \quad \dfrac{}{\Gamma \vdash P}\,h}{v_c(P \vee \sim P)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P, \alpha \vdash P}\,h}{P \vdash \alpha \to P}\,\to i}{P \vdash (P \to \alpha) \vee (\alpha \to P)}\,v_{i2} \quad \dfrac{\dfrac{\dfrac{}{\neg P, P \vdash \alpha}\,h\ \text{abs}}{\neg P \vdash P \to \alpha}\,\to i}{\neg P \vdash (P \to \alpha) \vee (\alpha \to P)}\,v_{i1}}{\vdash (P \to \alpha) \vee (\alpha \to P)}\,v_c(P \vee \neg P)$$

Correction :

$P \to Q,\ \neg P \to R \vdash Q \vee R$



$(Q \wedge \neg R) \vee (Q \wedge P) \vdash Q \wedge (R \to P)$