



TP5 : logique classique

\neg et \vee et \rightarrow exm:

Exercice 1 :

1.

$$\frac{\frac{\frac{\neg(P \vee \neg P) \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P) \rightarrow \neg(P \vee \neg P)} \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P)} \quad \vee_e(P \vee \neg P)$$

2.

$$\frac{\frac{\frac{\neg(P \vee \neg P) \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P) \rightarrow \neg(P \vee \neg P)} \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P)} \quad \vee_e(P \vee \neg P)$$

$$\frac{\frac{\frac{\neg(P \vee \neg P) \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P) \rightarrow \neg(P \vee \neg P)} \quad \neg(P \vee \neg P)}{\neg(P \vee \neg P)} \quad \vee_e(P \vee \neg P)$$

3.

$$\frac{\frac{\frac{\neg(P \rightarrow Q) \quad \neg(P \rightarrow Q)}{\neg(P \rightarrow Q) \rightarrow \neg(P \rightarrow Q)} \quad \neg(P \rightarrow Q)}{\neg(P \rightarrow Q)} \quad \vee_e(P \rightarrow Q)$$

$$\begin{array}{c} \overline{p \vdash p \vee a}^h \quad \overline{p, p \vee a \vdash a}^h \\ \hline \overline{p \vdash a} \quad \lambda_c(p \vee a) \\ \hline p \vee a, p \rightarrow a \vdash \perp \\ \hline \vdash p \vee a \rightarrow a | p \rightarrow a | \rightarrow i \end{array}$$

[illegible]

[illegible]

Exercise 2:

$\neg(A \wedge B)$ absurde:

Exercice 3

$$\frac{P_1, B \vdash A \quad P_1, A \vdash B}{P_1 \vdash A} \text{ ex } \neg$$

Exercise 4:

$$\frac{\Gamma, A \vee B}{\Gamma \vdash A \vee B}$$

$$\frac{\Pi, \neg A \vdash B}{\Pi \vdash A \vee B}$$

$$\frac{\frac{\frac{\Pi, A \vdash A}{\Pi, A \vdash A \vee B} \vee_i | 1}{\Pi \vdash A \vee A} \vee_e | 1}{\Pi \vdash A \vee B} \vee_e | 1$$

Exercise 5 :

$$\neg P \rightarrow P \vdash_K P$$

$$\frac{\frac{\frac{\frac{\Pi, \neg P \vdash \neg P}{\Pi, \neg P \vdash \neg P} \neg_i}{\Pi, \neg P \vdash P} \neg_e}{\Pi \vdash \neg P} \neg_e}{\neg P \rightarrow P \vdash P} \rightarrow_e$$

$$\vdash_K (P \rightarrow Q) \vee (Q \rightarrow P)$$

$$\frac{\frac{\frac{\frac{\Pi, P \vdash P}{\Pi, P \vdash P} \neg_i}{\Pi, P \vdash Q} \neg_e}{\Pi \vdash P \rightarrow Q} \rightarrow_e}{\vdash (P \rightarrow Q) \vee (Q \rightarrow P)} \vee_e$$

Exercise 6

$$P \rightarrow (Q \vee R) \equiv_K (P \rightarrow Q) \vee (P \rightarrow R)$$

$$\frac{\frac{\frac{\frac{\Pi, P \vdash P}{\Pi, P \vdash P} \neg_i}{\Pi, P \vdash Q \vee R} \neg_e}{\Pi \vdash P \rightarrow (Q \vee R)} \rightarrow_e}{\frac{\frac{\frac{\frac{\Pi, P \vdash P}{\Pi, P \vdash P} \neg_i}{\Pi, P \vdash Q} \neg_e}{\Pi \vdash P \rightarrow Q} \rightarrow_e}{\frac{\frac{\frac{\Pi, P \vdash P}{\Pi, P \vdash P} \neg_i}{\Pi, P \vdash R} \neg_e}{\Pi \vdash P \rightarrow R} \rightarrow_e}{\Pi \vdash (P \rightarrow Q) \vee (P \rightarrow R)} \vee_i | 1}{P \rightarrow (Q \vee R) \vdash (P \rightarrow Q) \vee (P \rightarrow R)} \vee_e$$

