On Notation

I write the following as symbols of the metalanguage:

- "$\Rightarrow$", "$\Leftarrow$", "$\Leftrightarrow$" as the material conditional.
- "&" or "+" as logical "and" like in "$ZFC + CH \models \aleph_1 = 2^{\aleph_0}$". I use & when dealing with arithmetic.
- "$\exists_x \varphi(x)$" is as usual.
- "$=$", "$\neg$" are as usual.

If a symbol is preceded by "$\mathcal{M} \models$" or has $\cdot^{\mathcal{M}}$ is part of the object language. Consider that all named symbols are logical constants, i.e. for $\mathcal{M}$ and $\mathcal{N}$ models $\Rightarrow^{\mathcal{M}} = \Rightarrow^{\mathcal{N}}$ and similarly for the other symbols. For a model $\mathcal{M}$, I write:

- "$\Rightarrow^{\mathcal{M}}$" as "$\rightarrow$"
- "$\&^{\mathcal{M}}$" as "$\wedge$"
- "$\exists^{\mathcal{M}}$", "$=^{\mathcal{M}}$", "$\neg^{\mathcal{M}}$" remain the same

I see an advantage in changing the symbol of relations only, distinguishing meta- and object language will otherwise be clear. Order of metalinguistic relations is as usual and has precedence on object language, e.g. $\mathcal{M} \models \varphi \leftrightarrow \psi \Rightarrow \varphi \rightarrow \psi$ is the same as $\mathcal{M} \models (\varphi \leftrightarrow \psi) \Rightarrow (\varphi \rightarrow \psi)$.
The symbol "$\models$" denotes classical logical consequence is to be understood into two different ways: a relation between a model and a sentence in the object language or between a theory and a sentence in the metalanguage. Both interpretations are familiar, note the two following examples: $ZFC \models \pi > 1$ & $2 + 1 = 3$ and $\mathcal{M} \models \neg\aleph_1 = 2^{\aleph_0} \wedge \exists_x 2 + x = 3$.
I also write usual abbreviations like $\neg a = b$ as $a \neq b$ or $\exists_x \varphi(x) \wedge \psi(x)$ as $\exists_{\varphi(x)} \psi(x)$.
I write ":=" for a relation between a syntactical object and a semantical object (or a second syntactical object) and denotes syntactical overwrite e.g., "$\varphi := 1+1 = 2$" every time I type "φ" the reader should understand it just like "$2 + 1 = 2$". I also use "'" to refer to the syntax of symbols.
Letters in Fraktur Alphabet are structures, like "$\mathfrak{A}$", "$\mathfrak{B}$", "$\mathfrak{U}$" and caligraphic letters are models, like "$\mathcal{M}$", "$\mathcal{N}$". Models are structure, though usually they are related to a theory e.g. a model of ZFC is also a structure. I also denote with the respective capital letter the domain of the model or structure, like "M", "A".
I write "$\mathcal{L}$" to refer to language of classical logic $\langle \neg, \wedge, \exists \rangle$, "$\mathcal{L}_\epsilon$" to refer to language of set theory $\langle \neg, \wedge, \exists, =, \in \rangle$ and "$\mathcal{L}_{\mathfrak{A}}$" to the language of a structure $\mathfrak{A}$.