Definition. $\Gamma_{\mathcal{U}} :=$ *construct it s.t.* $\Gamma_{\mathcal{U}} \subset \mathcal{U}$ *and, for a function* g, $g(\Gamma_{\mathcal{U}}) = \mathcal{U}$.

Definition. $\gamma : \mathcal{P}(\mathcal{P}(\mathbb{N})) \to \mathcal{P}(\mathcal{P}(\mathbb{N}))$, *such that it generates the ultrafilter from its generator.*

Lemma. $g(\Gamma_{\mathcal{U}}) = \mathcal{U}$

Proof. Prove it.

Definition. $\overline{\mathcal{U}} := g(\overline{\Gamma_{\mathcal{U}}})$

Lemma. $\overline{\mathcal{U}}$ *is an ultrafilter on* $\mathbb{N}$.

Proof. Properties to check are: (a): $\emptyset \notin \overline{\mathcal{U}}$, (b): $\forall_{X\in\overline{\mathcal{U}}}\forall_{Y\subseteq\mathbb{N}} X \subseteq Y \to Y \in \overline{\mathcal{U}}$, (c): $\forall_{X,Y\in\overline{\mathcal{U}}} X \cap Y \in \overline{\mathcal{U}}$, (d): $\forall_{X\subseteq\mathbb{N}} X \in \overline{\mathcal{U}} \vee \overline{X} \in \overline{\mathcal{U}}$.
Prove it.

Definition. $\overline{\mathfrak{U}} := \prod_{i\in\mathbb{N}} \mathfrak{A}_i / \overline{\mathcal{U}}$

Corollary. *For* $\varphi \in \mathcal{L}_\epsilon$, $\exists_{j\in\mathbb{N}} \mathfrak{A}_j \models \neg\varphi \Rightarrow (\mathfrak{U} \models \varphi \Rightarrow \overline{\mathfrak{U}} \models \neg\varphi)$

Proof. Change the claim according to intuition and then prove it.