

Hopfield Networks

Introduction

The goal of this project was to investigate the associative memory capacity of a Hopfield network by modifying different variables in the construction of networks and analyzing the outcomes. The base network used in this experiment had 100 neurons, 50 imprinted patterns, and no self-coupling. To examine the effects of these variables, the number of neurons and imprinted patterns were changed in separate runs.

For accurate basal results, 2000 runs were performed of the 100 neuron/50 imprinted pattern network. For the sake of time, 100 runs were performed for the other three experiments.

Analysis

First we'll look at the base neural network with 100 neurons and 50 imprinted patterns.

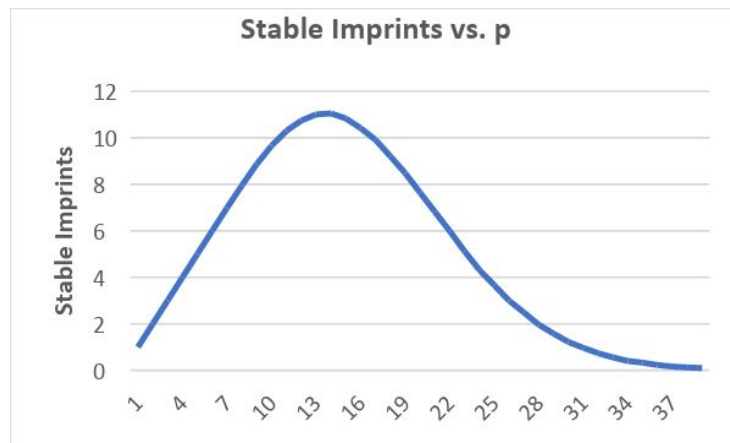


Figure 1) Number of stable imprints against number of imprinted patterns; neurons = 100, $p = 50$

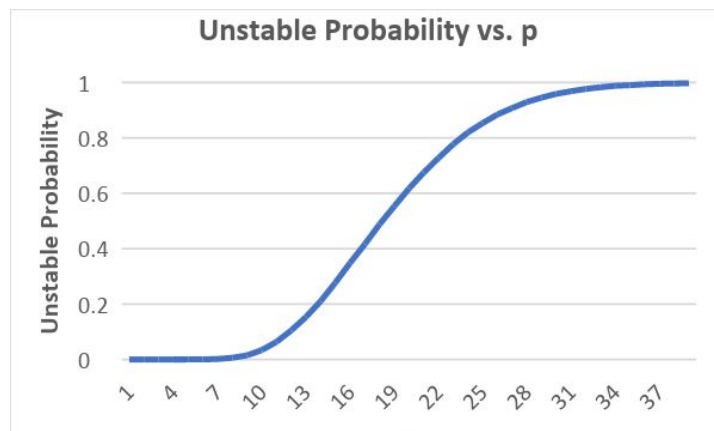


Figure 2) Fraction of unstable imprints against number of imprinted patterns; neurons = 100, $p = 50$

As shown in Figure 2, the probability of the imprinted pattern increases as the number of imprinted patterns increases. In fact, once approximately 30 patterns are imprinted, the probability of instability is >0.95 . There is also a probability of 0 to have an unstable pattern below a p of ~ 6 . Looking at Figure 1, we can see the number of stable imprints is very close to reaching its global minimum around 30 runs. The maximum stable imprints we find happens at $p = 14$, with 11.05 imprints. After $p = 14$, the number of stable patterns constantly decreases to zero, reaching 0.09 at $p = 39$. This data shows us that given a Hopfield network with 100 neurons, you can ensure a memory is stable for at most 6 imprinted patterns.

Now we will analyze neural networks with different amounts of neurons: one with 50 and another with 200; first let's look at a network with 50 neurons.

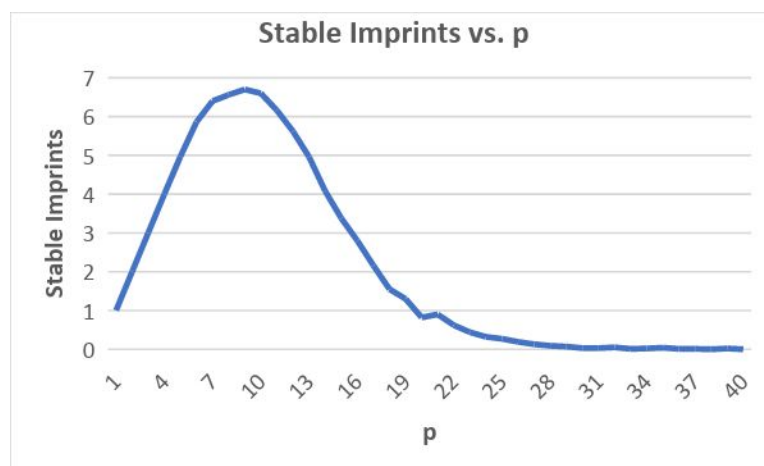


Figure 3) Number of stable imprints against number of imprinted patterns; neurons = 50, $p = 50$

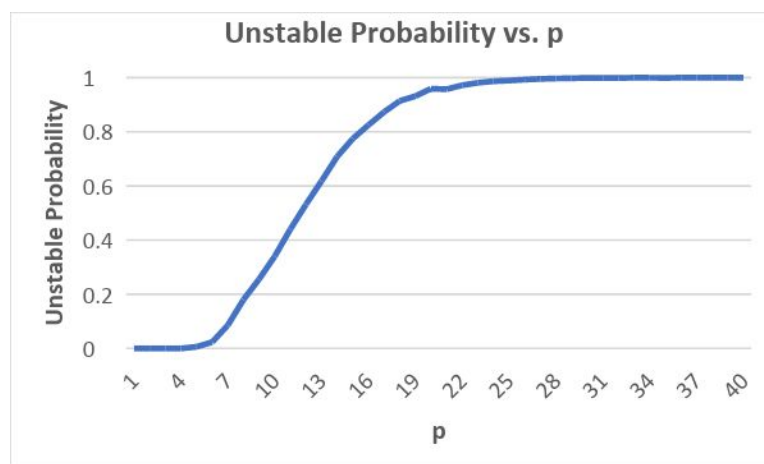


Figure 4) Fraction of unstable imprints against number of imprinted patterns; neurons = 50, $p = 50$

Figures 3 and 4 look very similar to Figures 1 and 2, but their values are “shrunk” in a sense. This makes sense, as the fewer neurons a network has, the fewer ways it has to ensure stability. Looking at Figure 3, the max number of stable imprints is 6.7 at $p = 9$ in

contrast to 100-neuron net's 11.05 imprints. Similarly, in Figure 4, having >0.95 probability of instability occurs earlier at $p = 20$.

We are about to look at a network with 200 neurons—we should be able to assume behavior opposite that of the transition from 100 to 50, where we have more stable imprints and a lower probability of instability.

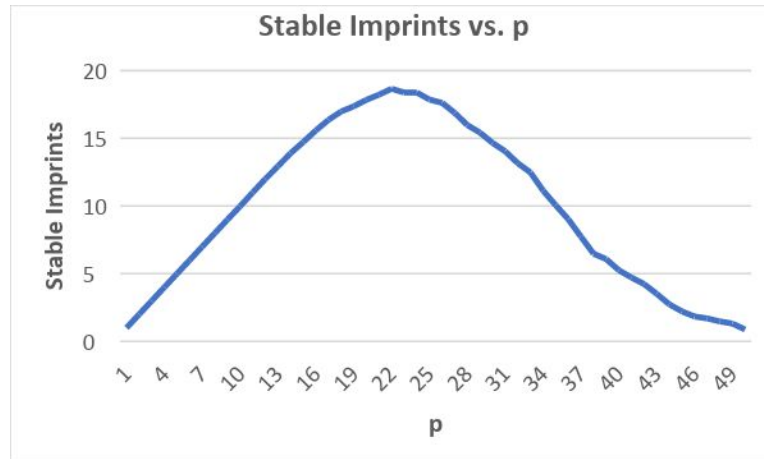


Figure 5) Number of stable imprints against number of imprinted patterns; neurons = 200, $p = 50$

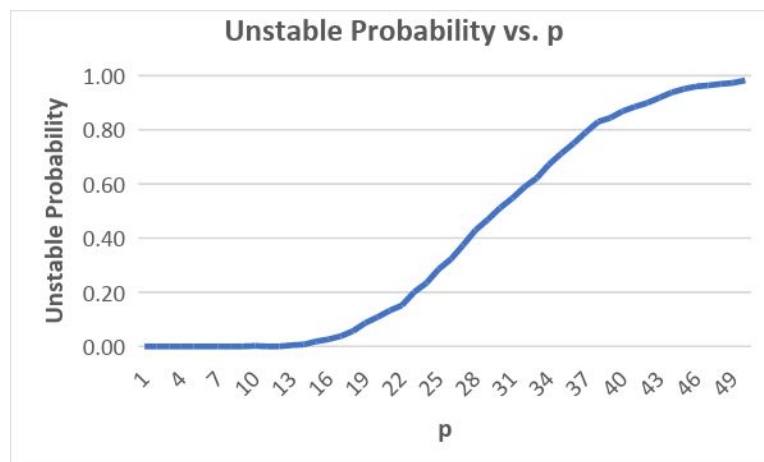


Figure 6) Fraction of unstable imprints against number of imprinted patterns; neurons = 200, $p = 50$

As predicted, Figure 5 shows that the maximum number of stable imprints increased to 18.6 at $p = 23$. Additionally, the number of stable imprints does not reach a minimum at 0, even at $p = 50$. This confirms that as a network's size increases, so does its stability. Therefore one can guarantee a more stable memory by simply increasing the number of neurons in their network.

Finally, we will analyze how changing the number of imprinted patterns affects the networks. Presumably, nothing of import will change as all this will do is extend the ends of each graph, which are already approaching their local respective maximums/minimums.

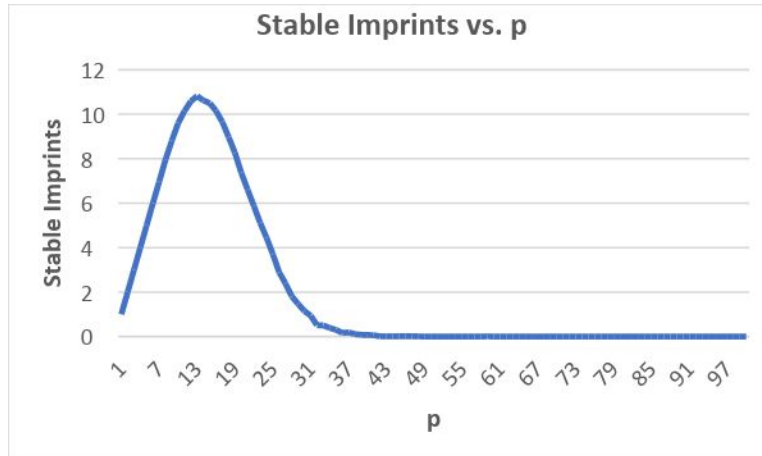


Figure 7) Number of stable imprints against number of imprinted patterns; neurons = 100, $p = 100$

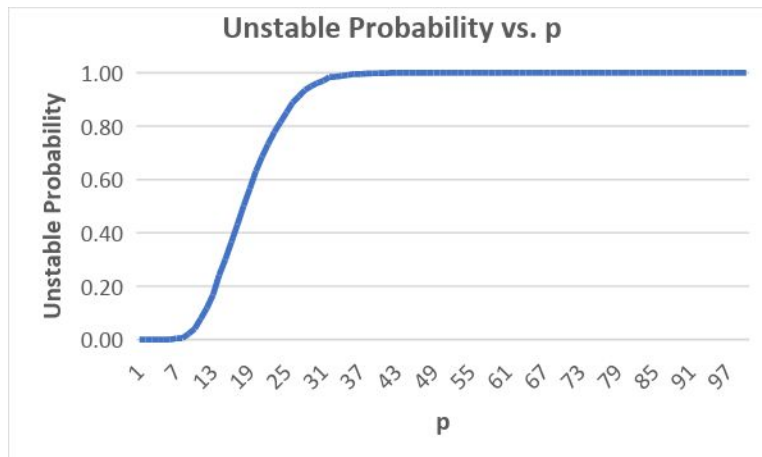


Figure 8) Fraction of unstable imprints against number of imprinted patterns; neurons = 100, $p = 100$

As predicted, Figures 7 and 8 show simply an extension of their 100-neuron counterparts, Figures 1 and 2. Increasing the number of imprinted patterns does not increase stability. This is because each network has an upper limit of imprinted patterns, after which no more stable patterns can be imprinted. Increasing the number of imprinted patterns if you've already hit that limit then, obviously, does not affect the results in any way.

In conclusion, increasing the number of neurons in a Hopfield network directly correlates to an increase in stable memory.