Ambiguity and Parsing Algorithms

CSCI 3136: Principles of Programming Languages

Agenda

- Announcements
  - Assignment 3 is out and due June 7.
  - Extended office hours: WF 2:30 – 4pm (more coming if needed)
  - Midterm will be on June 19, 10am – 11:30am in **CHEB 170**
- Readings:
  - Today: 2.3.0, 2.3.1
  - Note: I recommend using alternative texts for this part of the course:
  - E..g, Hopcorft et al, “Introduction to Automata Theory”
- Lecture Contents
  - Ambiguous Grammars
  - Left and Right Parse Tree Derivations
  - LL(K) and LR(K) Parsing
  - S-Grammars
  - LL(1) Parsing
  - LR(1) Parsing

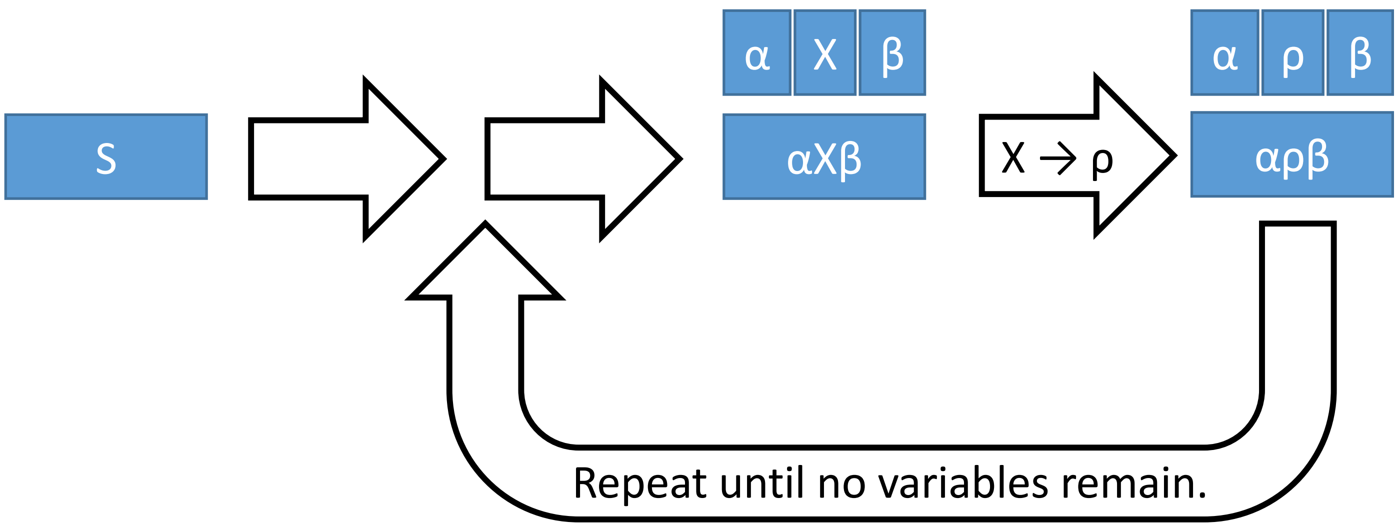
Recall: A CFG for Expressions

- $V = \{E, Op\}$
- $\Sigma = \{\text{identifier}, \text{number}, (,), +, -, *, /\}$
- P={

$$
\begin{aligned}
&E \rightarrow E\ Op\ E\\
&E \rightarrow -E\\
&E \rightarrow (\ E\ )\\
&E \rightarrow \text{number}\\
&E \rightarrow \text{identifier}\\
&Op \rightarrow +\\
&Op \rightarrow -\\
&Op \rightarrow *\\
&Op \rightarrow /
\end{aligned}
$$

  }
- $S = E$

Derivations in a Nutshell



Parse Tree of a Derivation (42+13)*11

$\sigma = \mathbf{E}$

$\Rightarrow \mathbf{E}$ Op E

$\Rightarrow$ ($\mathbf{E}$) Op E

$\Rightarrow$ ($\mathbf{E}$ Op E) Op E

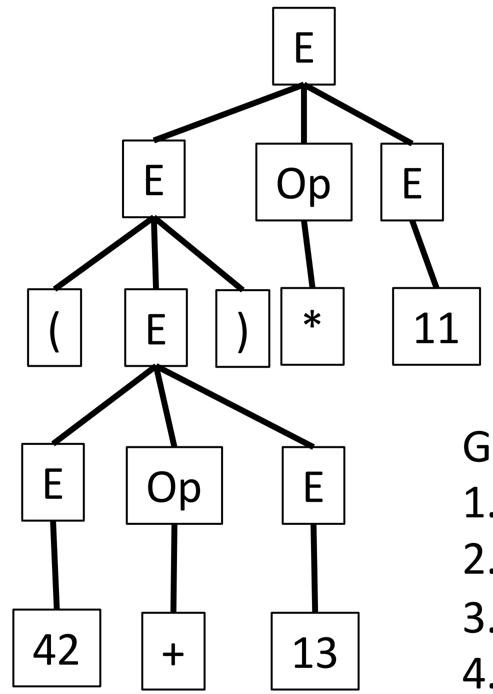
$\Rightarrow$ (42 **Op** E) Op E

$\Rightarrow$ (42 + $\mathbf{E}$) Op E

$\Rightarrow$ (42 + 13) **Op** E

$\Rightarrow$ (42 + 13) $*$ $\mathbf{E}$

$\Rightarrow$ (42 + 13) $*$ 11



Grammar
1. E → E Op E
2. E → −E
3. E → (E)
4. E → number
5. E → identifier
6. Op → +
7. Op → −
8. Op → $*$
9. Op → /

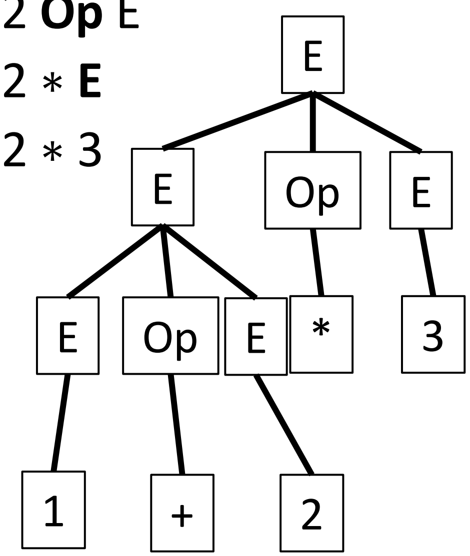
Another Example: 1 + 2 * 3

This is ambiguous!

σ = **E**

⇒ **E** Op E
⇒ **E** Op E Op E
⇒ 1 **Op** E Op E
⇒ 1 + **E** Op E
⇒ 1 + 2 **Op** E
⇒ 1 + 2 ∗ **E**
⇒ 1 + 2 ∗ 3

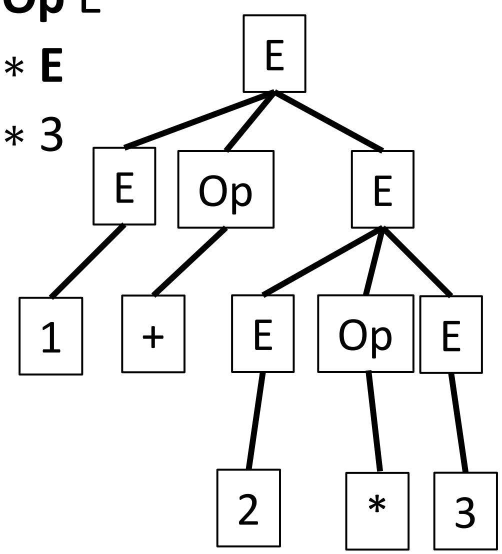
(1 + 2) * 3



σ = **E**

⇒ **E** Op E
⇒ 1 **Op** E
⇒ 1 + **E**
⇒ 1 + **E** Op E
⇒ 1 + 2 **Op** E
⇒ 1 + 2 ∗ **E**
⇒ 1 + 2 ∗ 3

1 + (2 * 3)



Grammar

1. E → E Op E
2. E → –E
3. E → (E)
4. E → number
5. E → identifier
6. Op → +
7. Op → –
8. Op → ∗
9. Op → /

Ambiguity

- Observations:
  - There are infinitely many grammars to specify the same language
  - There may be multiple parse trees for the same sentence!
- Definition: If multiple parse trees can be generated by G for the same sentence, then G is *ambiguous*.
- Definition: If L does not have an unambiguous grammar, then L is *inherently ambiguous*
  - Usually not the case for programming languages!

An Unambiguous Expression Grammar

Grammar

1. $E \rightarrow T$
2. $E \rightarrow E + T$
3. $E \rightarrow E - T$
4. $T \rightarrow F$
5. $T \rightarrow T * F$
6. $T \rightarrow T / F$
7. $F \rightarrow$ number
8. $F \rightarrow$ identifier
9. $F \rightarrow (E)$

- Try deriving 1 + 2 * 3

Derivation Order

- Derivation orders refer to the order in which variables are replaced in the current partial derivation.
- The two most common ones are:
  - **Leftmost derivation** replaces the leftmost variable in each step
  - **Rightmost derivation** replaces the rightmost variable in each step

Leftmost Derivation Example 1+2*3

$E \Rightarrow \mathbf{E} + T$

$\Rightarrow \mathbf{T} + T$

$\Rightarrow \mathbf{F} + T$

$\Rightarrow 1 + \mathbf{T}$

$\Rightarrow 1 + \mathbf{T} * F$

$\Rightarrow 1 + \mathbf{F} * F$

$\Rightarrow 1 + 2 * \mathbf{F}$

$\Rightarrow 1 + 2 * 3$

Grammar

1. E → T
2. E → E + T
3. E → E − T
4. T → F
5. T → T * F
6. T → T / F
7. F → number
8. F → identifier
9. F → (E)

Rightmost Derivation Example 1+2*3

$$
\begin{aligned}
E &\Rightarrow E + \mathbf{T} \\
&\Rightarrow E + T * \mathbf{F} \\
&\Rightarrow E + \mathbf{T} * 3 \\
&\Rightarrow E + \mathbf{F} * 3 \\
&\Rightarrow \mathbf{E} + 2 * 3 \\
&\Rightarrow \mathbf{T} + 2 * 3 \\
&\Rightarrow \mathbf{F} + 2 * 3 \\
&\Rightarrow 1 + 2 * 3
\end{aligned}
$$

Grammar

1. E → T
2. E → E + T
3. E → E − T
4. T → F
5. T → T * F
6. T → T / F
7. F → number
8. F → identifier
9. F → (E)

Where Are We?

- CFGs are used to specify programming language syntax
- Parsing finds the parse tree of the program (token stream)
- CFGs for programming languages must unambiguously capture the program structure.
- Parsers must be efficient:
  - A parser can be generated from a CFG that runs in $O(n^3)$ time
  - We prefer (require) linear time.
- How do we get this?

Regular Grammars: A Brief Aside

- A CFG is *right-linear* if all productions are of the form
  - $A \rightarrow \sigma B$, $\sigma \in \Sigma^*$, $B \in V$
  - $A \rightarrow \sigma$, $\sigma \in \Sigma^*$
- A CFG is *left-linear* if all productions are of the form
  - $A \rightarrow B\sigma$, $\sigma \in \Sigma^*$, $B \in V$
  - $A \rightarrow \sigma$, $\sigma \in \Sigma^*$
- A CFG is regular if it is right-linear or left-linear
- Regular grammars specify exactly the set of regular languages
- Regular grammars are too weak to specify most programming languages
- But, parsers generated from them run in linear time!
  - Why?
  - Are there more complex grammars for which linear time parsers exist?

LL and LR Grammars

Two kinds of unambiguous grammars that can be parsed efficiently

- LL(k) grammars
  - Are scanned Left-to-right and generate a Leftmost derivation
  - If the first letter in the current sentential form is a variable, k tokens look-ahead in the input suffice to decide which production to use to expand it.
- LR(k) grammars
  - Are scanned Left-to-right and generate a Rightmost derivation
  - The next k tokens in the input suffice to choose the next step the parser should perform.
- The syntax of almost every programming language can be described by LL(1) or LR(1) grammars!
  - How? Why?

S-Grammars

- First let's consider a very simple grammar
- An *S-grammar* or *simple grammar* is a special case of an LL(1)-grammar
- A CFG is an S-grammar if
  - Every production starts with a terminal
  - Productions for the same LHS start with different terminals

  E.g., If G contains $A \to aA$ and $A \to a$ then G is not simple!
- Idea: When using S-Grammars, selecting which rule to apply is easy.

Example: LL(1) Parsing (top-down) S-Grammar for Polish Notation

1. $S \rightarrow + SS$
2. $S \rightarrow - SS$
3. $S \rightarrow * SS$
4. $S \rightarrow / SS$
5. $S \rightarrow$ neg S
6. $S \rightarrow$ integer

Expression:

`– * + 1 2 3 4`

Is interpreted as:

`(1 + 2) * 3 – 4`

- How do we derive
  $\boxed{-}\boxed{*}\boxed{+}\boxed{1}\boxed{2}\boxed{3}\boxed{4}$

$$
\begin{aligned}
S &\Rightarrow - \mathbf{S}\ S \\
&\Rightarrow - * \mathbf{S}\ S\ S \\
&\Rightarrow - * + \mathbf{S}\ S\ S\ S \\
&\Rightarrow - * + 1\ \mathbf{S}\ S\ S \\
&\Rightarrow - * + 1\ 2\ \mathbf{S}\ S \\
&\Rightarrow - * + 1\ 2\ 3\ \mathbf{S} \\
&\Rightarrow - * + 1\ 2\ 3\ 4
\end{aligned}
$$

- This is an example of LL(1) parsing
- How does a parser do this?

LL(1) Parsing of S-Grammars

```
# Use a stack to store the
# current sentential form
push(S)  # push start variable
t = next_token()
loop until no more tokens:
    x = pop()
    if x == t:
        t = next_token()
        continue
    elseif x ∈ V:
        select production x → t α
        add children t α to node x
        push(tα)
    else:
        error
```

Grammar

1. S → + SS
2. S → − SS
3. S → ∗ SS
4. S → / SS
5. S → neg S
6. S → integer

Parse Expression:

`- * + 1 2 3 4`

This takes linear time!

Example: LR(1) Parsing (bottom-up) S-Grammar for Polish Notation

1. $S \rightarrow + SS$
2. $S \rightarrow - SS$
3. $S \rightarrow * SS$
4. $S \rightarrow / SS$
5. $S \rightarrow \text{neg } S$
6. $S \rightarrow \text{integer}$

Expression:

```
– * + 1 2 3 4
```

Is interpreted as:

```
(1 + 2) * 3 — 4
```

- How do we derive
  $\boxed{-}\boxed{*}\boxed{+}\boxed{1}\boxed{2}\boxed{3}\boxed{4}$

$\Leftarrow - * + \mathbf{S}\ 2\ 3\ 4$ [use rule 6]

$\Leftarrow - * + S\ \mathbf{S}\ 3\ 4$ [use rule 6]

$\Leftarrow - * \mathbf{S}\ 3\ 4$ [use rule 1]

$\Leftarrow - * S\ \mathbf{S}\ 4$ [use rule 6]

$\Leftarrow - \mathbf{S}\ 4$ [use rule 3]

$\Leftarrow - S\ \mathbf{S}$ [use rule 6]

$\Leftarrow \mathbf{S}$ [use rule 2]

- This is an example of LR(1) parsing
- How does a parser do this?

LR(1) Parsing of S-Grammars

\# Use a stack to store the
\# what has been seen so far
push(next_token()) # init stack
loop until no more tokens:
  if ∃(P → α) such that α ∈ Stack
    # reduce operation
    pop(α)
    push(P)
    add children α to node P
  else:
    # shift operation
    push(next_token())

Grammar

1. S → + SS
2. S → − SS
3. S → ∗ SS
4. S → / SS
5. S → neg S
6. S → integer

Parse Expression:

`– * + 1 2 3 4`

This takes linear time!

Building Parsers

- We now have some intuition about parsing algorithms
- But …
  - The above algorithms are for S-Grammars (too simple)
  - Want to generate parser given a grammar
- So ...
  - Assume that we will be using more complex grammars
  - How do we generate the parser?

Building an LL(1) Parser

- Basic Challenge: Given current token, which production does the parser select if next item in sentential form is a nonterminal

  E.g., if S is on the stack and input is +, then parser must select production $S \rightarrow +SS$

- In general: for input **a**, sentential form A . . ., either
  - $A \Rightarrow \alpha \Rightarrow^* \mathbf{a}\beta$
  - $A \Rightarrow \alpha \Rightarrow^* \varepsilon$ and derivation of A is succeeded by **a**.
- Intuitively, **a** is in the *predictor set* of $A \rightarrow \alpha$

  if $A\beta \Rightarrow \alpha\beta \Rightarrow^* a\gamma$, for $\beta,\gamma \in \Sigma^*$

  I.e., the parser selects $A \rightarrow \alpha$ if **a** is the input and in the *predictor set* of $A \rightarrow \alpha$

LL(1) Grammars

- **Definition**: A grammar is LL(1) if the predictor sets of all productions with the same LHS are disjoint.
- E.g. S-Grammars are LL(1)

  Grammar

  1. $S \rightarrow + SS$
  2. $S \rightarrow - SS$
  3. $S \rightarrow * SS$
  4. $S \rightarrow / SS$
  5. $S \rightarrow neg\ S$
  6. $S \rightarrow integer$

Production	Predictor Set
S → + S S	{+}
S → - S S	{-}
S → * S S	{*}
S → / S S	{/}
S → neg S	{neg}
S → integer	{integer}