



## Tool-path generation for machining sculptured surface

F. Li<sup>a</sup>, X. C. Wang<sup>a</sup>, S. K. Ghosh<sup>b</sup>, D. Z. Kong<sup>a</sup>, T. Q. Lai<sup>a</sup> and X. T. Wu<sup>a</sup>

<sup>a</sup>Department of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, China

<sup>b</sup>International College of Engineering, GKN Automotive AG, Postfach 1152, 53784 Lohmar, Germany

In NC machining of sculptured surfaces, the machining efficiency of curvature catering method is much higher than traditional ones using ball-ended or drum-shaped tools. However, in order to get the best result of utilizing curvature catering method, great attentions have to be paid to the arrangement of tool traces, for the sensitivity of machining efficiency with respect to feeding direction is much higher than the traditional ones. A method of generation "optimal" tool-path for 5-axis NC sculptured surface milling with the curvature catering method is presented in the paper. It consists of four sections; 1) an algorithm for determining "optimal" feeding direction; 2) variable cutter step size algorithm; 3) side step determination; 4) the criteria for fractionalizing a surface. A numerical example is also presented to demonstrate the feasibility of the method.

### 1. INTRODUCTION

Sculptured surfaces are widely used in aerospace, marine and mould industry. Traditionally, sculptured surfaces are machined using ball-ended or drum-shaped tools on 3-axis machine tools. During the last two decades, 5-axis NC milling machine tools have been utilized increasingly with considerable success in these fields [1–5]. However, these methods are far from making full use of the potential ability of 5-axis machine tools. Based on these considerations, Prof. X. C. Wang Proposed an entirely new concept of manufacturing sculptured surfaces — Curvature catering method [1–2].

This paper will discuss the arrangement of tool traces on the surface in order that the new method can get the best result. Commonly used tool path planning methods are;

1) to introduce explicitly tool guiding surfaces;

2) to plan tool paths on the  $xy$ -plane of a Cartesian coordinate;

3) to plan tool paths on the parametric space;

The first approach is used in APT III and called the APT-based tool path generation method. The second approach is used for non-parametric surface defined in the form of  $z = f(x, y)$  and is called the Cartesian machining method. The third can be used for parametric surfaces presented in the form  $\bar{r} = \bar{r}(u, w)$ . The third approach is selected to generate tool paths for sculptured surfaces machining with curvature catering method. The following issues should be addressed for planning tool paths for the method;

1) an algorithm for determining "optimal" feeding direction;

2) variable cutter step size algorithm;

3) side step determining method based on

the given tolerance;

4) the criteria for fractionalizing a surface into several areas in order to reduce the machining time;

5) a numerical example is presented at this section to demonstrate the feasibility of the method.

## 2. "OPTIMAL" FEEDING DIRECTION

If a sculptured surface is being machined with traditional method using a ball-ended or drum-shaped tool, the machining time is not changed greatly when different feeding directions are selected for tool path planning, for the maximum width of machined strip on the surface mainly depends on the radius of tool and the scallop height. However, for the curvature catering method, the sensitivity of machining efficiency with respect to the feeding direction is much higher than those methods.

Figs. 1 and 2 are illustrations of machining sculptured surfaces with curvature catering method. The tool-nose-trace circle  $C$  is on the tool end-plane  $S$ . It is supposed that circle  $C$  contacts surface  $\Sigma$  at  $M$ ; In the figures  $T$  denotes the tangent plane of  $\Sigma$  at  $M$ ;  $\vec{l}$  the intersecting line of  $S$  and  $T$ ;  $\vec{e}_1$  and  $\vec{e}_2$  the unit vectors of the principal directions at point  $M$ ; and  $K_1$  and  $K_2$  the corresponding principal curvatures. Let  $\vec{n}$  denote the unit outward normal vector at the same point,  $\vec{e}_1$ ,  $\vec{e}_2$  and  $\vec{n}$  compose a rectangular right-handed system.  $\theta$  is the directed angle for  $\vec{e}_1$  turning to the direction of  $\vec{l}$  in the counter-clockwise direction and  $\eta$  the angle between  $S$  and  $T$ . For the given radius of tool-nose trace  $R$  and a certain point on the surface, the direction of  $\vec{l}$  can be determined using ref. [1-2]. Theoretically, if a tool path is an orthogonal trajectory of  $\vec{l}$  the maximum width of ma-

chined strip on the surface could be obtained, and the best efficiency can be expected in this tool pass. However, a singular tool trace will bring about difficulties not only in

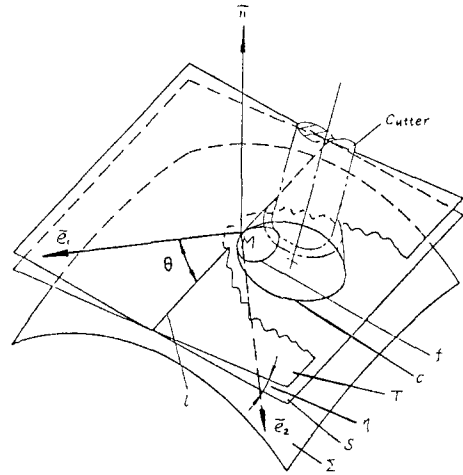


Fig. 1 An illustration of machining a convex surface

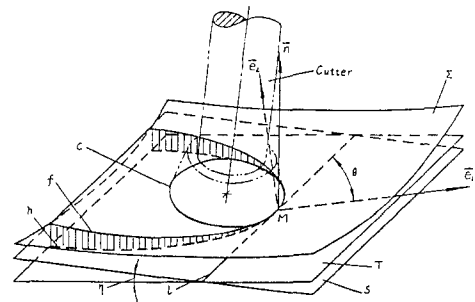


Fig. 2 An illustration of machining a concave surface

the calculation of this tool pass, but also in the arrangement of neighbouring traces.

Thus it is nearly impossible to arrange tool traces for the whole surface according to the ideal condition.

Another issue to be considered in the choice of feeding direction is the interference problem. As shown in Fig. 3, the surface has a shoulder along the feeding direction. If the feeding direction is chosen as shown in Fig. 3 (a), no gouge area exists. On the contrary, the opposite feeding direction (as shown in Fig. 3 (b)) will lead to gouge area.

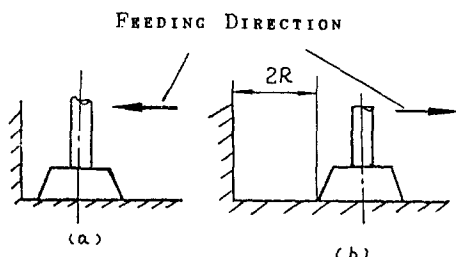


Fig. 3 Interference and feeding direction

The authors suggest the following method can be utilized in practice:

At the selected points, which are evenly and sparsely scattered on the whole surface, calculate the directions of  $\bar{l}$ . The values of  $du, dw$  of their orthogonal directions can be determined from

$$(\bar{\tau}_u du + \bar{\tau}_w dw) \cdot \bar{\tau} = 0$$

Thus

$$\frac{dw}{du} = -\frac{\bar{\tau}_u \cdot \bar{l}}{\bar{\tau}_w \cdot \bar{l}} = C_i$$

If the value does not change too much across the surface, an average value "C" is calculated as follows;

$$C = \frac{\sum_{i=1}^n \frac{C_i}{\sqrt{1+C_i^2}}}{\sum_{i=1}^n \frac{1}{\sqrt{1+C_i^2}}}$$

Where  $n$  express the number of selected points, and this value is used as the feeding direction of the cutter on  $uv$ -plane. Otherwise, the average values are calculated separately in different regions and the tool paths are respectively arranged in these regions. However, the more regions the surface is divided into, the more overlap exists between the regions.

### 3. VARIABLE CUTTER STEP SIZE ALGORITHM

In above section the feeding direction has been determined. The issue of determining step lengths along a cutter pass can now be discussed. For a given straight line tool path on  $uv$ -plane, there exists a space curve on the surface, which is to be approximated by a sequence of arc segments formed by linear interpolation of 5-axis system as shown in Fig. 4. The step length must ensure that the deviation error is within the given tolerance.

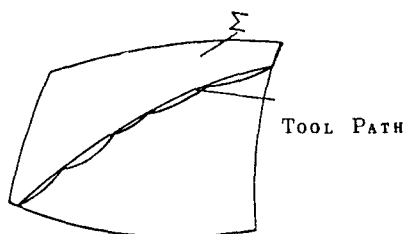


Fig. 4 A curve is approximated by a sequence of arc segments

Depending on the machine type, 5-axis milling machines can be divided into dividing head, circular swing table, and so on. For machining large-scale work pieces, the former is mainly used, and the only type to be discussed here. Let  $\varphi$  denote the angle between  $\vec{l}$  and the feeding direction, as shown in Fig. 5, the trace of  $M$  in one interpolation step can be written in the form.

$$\begin{cases} y = y_0 - R \sin \varphi \sin \lambda - l_t \cos \lambda \\ \varphi = \frac{\varphi_B - \varphi_A}{x_B - x_A} (x - x_A) + \varphi_A \\ \lambda = \frac{\lambda_B - \lambda_A}{x_B - x_A} (x - x_A) + \lambda_A \end{cases}$$

Where

$$y_0 = \left[ \frac{R(\sin \varphi_B \sin \lambda_B - \sin \varphi_A \sin \lambda_A)}{x_B - x_A} + \frac{l_t(\cos \lambda_B - \cos \lambda_A)}{x_B - x_A} \right] (x - x_A) + y_A + R \sin \varphi_A \sin \lambda_A + l_t \cos \lambda_A$$

and  $l_t$  denotes the distance from the center of cutter to the center of swing.

Suppose the maximum deviation is at  $x = (x_A + x_B)/2$ , the deviation caused by tool swing is

$$\begin{aligned} \delta_n &= y|_{x=(x_A+x_B)/2} - y_A \\ &= \frac{R}{2} (\sin \varphi_A \sin \lambda_A + \sin \varphi_B \sin \lambda_B) \\ &\quad + \frac{S_0}{2} (\cos \lambda_A + \cos \lambda_B) \\ &\quad - R \sin \frac{\varphi_A + \varphi_B}{2} \sin \frac{\lambda_A + \lambda_B}{2} \\ &\quad - S_0 \cos \frac{\lambda_A + \lambda_B}{2} \\ &\simeq -\frac{R}{8} [(\varphi_B - \varphi_A)^2 + (\lambda_B - \lambda_A)^2] \\ &\quad \times \sin \frac{\varphi_A + \varphi_B}{2} \sin \frac{\lambda_A + \lambda_B}{2} \end{aligned}$$

$$-\frac{l_t}{8} (\lambda_B - \lambda_A)^2 \cos \frac{\lambda_A + \lambda_B}{2}$$

Let  $\Delta s$  denote the step size, then

$$\begin{aligned} \delta_n &\simeq -\frac{R}{8} \left[ \left( \frac{d\varphi}{ds} \right)^2 + \left( \frac{d\lambda}{ds} \right)^2 \right] (\Delta s)^2 \sin \varphi \sin \lambda \\ &\quad - \frac{l_t}{8} \left( \frac{d\lambda}{ds} \right)^2 (\Delta s)^2 \cos \lambda \end{aligned}$$

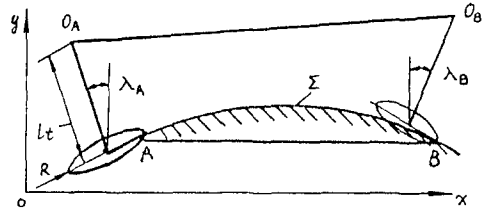
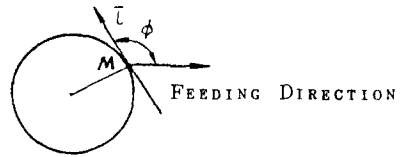


Fig. 5 Error caused by 5-axis interpolation

Let  $k_f$  denotes the normal curvature of the surface along the feeding direction, the deviation caused by straight line approximation will be

$$\delta_t = \frac{k_f}{8} (\Delta s)^2$$

In order that the total deviation is within the given tolerance,

$$|\delta_n + \delta_t| \leq \varepsilon$$

We have

$$\Delta S \leq 2 \sqrt{\frac{2\varepsilon}{|K_f - R[(\frac{dp}{ds})^2 + (\frac{d\lambda}{ds})^2] \sin\eta \sin\lambda - l_c(\frac{d\lambda}{ds}) \cos\lambda|}}$$

#### 4. DETERMINATION OF PATH INTERVALS

The distance between two neighbouring tool traces is called the path interval. In curvature catering method, it is calculated as follows:

1) Starting at one corner of the surface, according to the feeding direction and the initial path interval, the first tool path is initially arranged on the surface.

2) Select some points sparsely scattered on the first tool path and calculate the axis and position vector of the cutter. Let  $\theta$  denote the angle between  $M$  and  $P$  on  $C$ , the corresponding point on the surface can be determined as follows:

$$\Delta \vec{r}_i = \vec{P} - \vec{r}_i$$

$$Edu + Fdw = \Delta \vec{r}_i \cdot \vec{r}_u$$

$$Fdu + Gdw = \Delta \vec{r}_i \cdot \vec{r}_w$$

$$u_{i+1} = u_i + du, w_{i+1} = w_i + dw$$

$$\vec{r}_{i+1} = \vec{r}(u_{i+1}, w_{i+1})$$

$$\text{until } |\Delta \vec{r}_i \cdot \vec{r}_u| < \varepsilon, |\Delta \vec{r}_i \cdot \vec{r}_w| < \varepsilon$$

The distance between  $P$  and the surface is

$$\delta = (\vec{r}(u_i + du, w_i + dw) - \vec{R}_c) \cdot \vec{n}$$

Change  $\theta$ , until  $\delta$  is equal to designated scallop height. The projected length of the two points on the orthogonal trajectory of the feeding direction is the practical width of the machined strip. Because the two points are usually not symmetrical to the contact point, need to be calculated separately. Along the whole pass, the minimum width of machined

strip can be searched on the two sides respectively. The DLmin and DRmin are defined as the two minimum width of them.

3) By changing the initial path interval, the first pass with DRmin being just equal to the initial path interval can be determined.

4) Based on the DLmin of the first path, the second path can be determined.

5) Checking the amount of the overlapping of adjacent strip envelopes if they have a small amount of overlap.

#### 5. FRACTIONALIZING A SURFACE

At Section 3 the feeding direction has been discussed. If the value of  $du : dw$  changes too much across the surface, the surface have to be fractionalized into different regions. This criteria is according to the different feeding directions. Some type of sculptured surface such as turbine blades and rotary wings, lines of curvatures are generally very close to the parametric line, or even coincide with each other. Thus for these surfaces the tool traces can be arranged along a family of parametric lines corresponding to smaller normal curvatures. For most of these surfaces, there exist a demarcation line near to the advancing edge where the order of the values of the normal curvature along both parametric line is reversed, so the tool paths should be separately arranged for both side of the component.

The second criteria is based on the width of machined strip. For some surfaces, there is a considerable change in the width of the valid area formed in a tool pass. In order to reduce the machining time, the tool traces had better be separately arranged in different regions, even though the feeding directions are the same. For instance, when machining the convex side of an air foil, the width of valid area is usually limited by the small cur-

vature radius at the advancing edge. If the tool paths on the whole convex side are arranged in accordance with the width of valid area at the advancing edge, the advantage of the new method cannot be exploited fully. Therefore, the tool paths should be arranged separately in different regions in this case, and a high efficiency can be expected.

## 6. NUMERICAL EXAMPLE

If the surface, as shown in Fig. 6, is machined with the new method. The diameter of the tool is 60mm, the given tolerance is

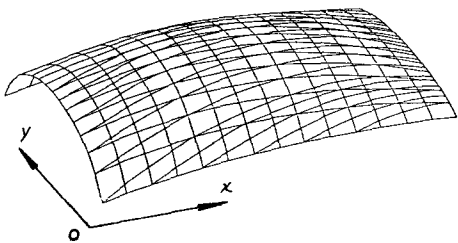


Fig. 6 The tool paths on the surface

0.005mm, The best feeding direction which is calculated by the new method is  $C = du : du = 4$ , the average width is about 0.8 on  $uv$ -plane, the projecting length is about 16mm. The width and the length of workpiece are both 260mm, so the tool path steps needed to machine the whole surface is 23. If the same surface is machined by using ball-ended tool, the maximum diameter of the tool is about 30mm, the average width is 0.7746mm, it needs about 336 steps.

## 7. CONCLUSIONS

The essential algorithms about the tool-path planning of manufacturing sculptured surfaces with curvature catering method on 5-axis milling tool are presented in this paper. They are; 1) algorithm for determining "optimal" feeding direction; 2) variable cutter step size algorithm; 3) side step determining method based on the given tolerance; 4) the criteria for fractionalizing a surface. The validity and the feasibility of these algorithms have been verified by a numerical example. These algorithms can also be used to plan paths for rootics system and manipulator with five freedoms.

## REFERENCES

1. Wang, X. C, Li, Y. B, Ghosh, S. K and Wu, X. T. "Curvature Catering- A New Approach in the Manufacture of Sculptured Surfaces (Part 1. Theorem)", *J. Mats. Proc. Tech.*, 1993, pp. 159-176.
2. Wang, X. C, Li, Y. B, Ghosh, S. K & Wu, X. T. "Curvature Catering- A New Approach in the Manufacture of Sculptured Surfaces (Part 2. Methodology)", *J. Mats. Proc. Tech.*, 1993, pp. 177-194.
3. Marciniak, K. "Influence of Surface Shape on Admissible Tool Positions in 5-axis Face Milling", *Computer Aided Design*, Vol. 19, No. 5, 1987, pp. 223-236.
4. Choi, B. K, Park, J. W & Jun, C. S. "Cutter-Location Data Optimization in 5-axis Surface Machining", *Computer Aided Design*, Vol. 25, No. 6, 1993, pp. 337-386.
5. Yuan, Z. J, Liu, X. W, Liu, H. M. "The Theory Analysis for 5-axis NC End Milling", *Chinese J. Mech. Engineering*. Vol. 29, No. 1, 1993, pp. 31-37, (in chinese).