

# Three-dimensional cutting-tool-path restriction. Application to ruled surfaces approximated by plane bifacets

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## Abstract

The restriction of cutting-tool paths is essential in the generation of the NC programs required to machine parts. In this paper, we propose a method to solve the problem of tool-path restriction during the machining of ruled surfaces. This method is based on the approximation of the surfaces by plane bifacets and the search of machining limit points including the determination of interference zones between cutting tool and bifacets. The idea of approximating surfaces by plane bifacets is not new. The originality of our method lies in the progressive decomposition of the limit surface which reduces computation time especially when several tool paths need not be restricted. The progressive decomposition of the limit surface concerns its subdivision in portions and subportions of surface as well as the accuracy of the approximation. The method proposed can be extended to collision control when machining other types of surfaces. © 1998 Elsevier Science B.V. All rights reserved.

**Keywords:** CAD/CAM; Ruled surface; Tool path; Interference control; Surface decomposition

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## 1. Introduction

The use of Computer-Aided Design (CAD) and Computer-Aided Manufacturing (CAM) systems in the development of part production functions has steadily increased over the past two decades. The recent research works carried out in this field are related to:

- the definition of feature based models to integrate frame-based knowledge in the design process;
- the development of new technical databases to

ensure good processing and consistency of information defined in complex projects;

- the development of techniques for tool-path computation and the generation of Numerical Control (NC) machining programs;
- the development of several Computer-Aided Process Planning (CAPP) approaches based on expert systems and Artificial Intelligence (AI) concepts.

In CAM, a NC machining program defines a set of cutting-tool movements from which parts, molds or dies are machined. The generation of NC machining programs requires the selection of cutting conditions

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(feedrates, thickness, etc.) and the computation of tool paths.

Tool paths can be determined by copying a prototype form [1] or using geometrical methods (for simple shapes) [2] or numerical computation methods [3]. According to the existing surface models, these tool paths can easily be computed for sculptured and free-form surfaces. However, they must be restricted when Cutter Location (CL) points are not gouge-free [4] or in the case of the machining of a surface limited by another. In the example shown in Fig. 1, the generation of tool paths includes the determination of limit points. To avoid interferences with the limit surface, the cutting tool must stop at each limit point.

Several approaches presented in the Section 2 have been developed to compute limitation points. Most of them use geometrical or numerical computation techniques applied directly to surface or solid models and are very expensive in computation time. In this paper we propose another method to determine limit points and then restrict the corresponding tool paths. This method, which has been implemented and tested in research and industrial works [5], is based on the approximation of a ruled surface by plane bifacets. Compared to the existing approaches, the method we propose is designed to give approximate results but with a substantial decrease in computation time. Its originality lies in the progressive decomposition of the portions of the limit surface concerned by the interferences with the cutting tool. The progressive decomposition is aimed at quickly reaching a high accuracy of the approximation of the limit surface when the tool path must be restricted.

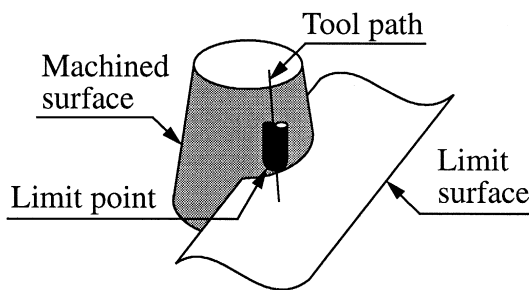


Fig. 1. Interference problem.

## 2. Related work

As depicted in Section 1, a number of approaches have been developed to solve the problem of tool-path limitation. In this section we analyze some of these approaches and show their contribution to our method. According to the main concepts developed in several works, we can sort these approaches into three categories:

- iterative approaches;
- approaches by surface polygonization;
- approaches by offset surface.

Iterative approaches are generally used to determine an exact solution which corresponds to the tangent point between the tool and the limit surface. The 'part-drive and check surface' method [6] developed with the APT (Automatically Programmed Tools) language is one of the first examples of the determination of exact solutions. Due to their iterative aspect, these approaches can be quite time-consuming [7] and lead to convergence problems [8]. In fact, since the algorithm used in iterative approaches relies on Newton's method, it only converges to the correct solution if it has a starting position which is close to the correct one. The choice of the starting point is the major problem for the iterative approaches. For this purpose, Bobrow [9] points out that it is difficult to prove the convergence of the algorithm developed for the most general combination of Cutter Contact (CC) curve and check surface.

However, the basic idea of the algorithm proposed by Bobrow has been considered in our work. In this algorithm, the tool moves at each iteration according to a fixed step to reach the CC point  $t$  (Fig. 2). Then the minimal distance between the tool and the limit surface is computed. In the example shown in Fig. 2, the minimal distance is the distance  $\|P_t - P_s\|$  between the point  $P_t$  on the tool and the point  $P_s$  on the limit surface. After some iterations, if the point  $P_t$  is found and  $\|P_t - P_s\| < \delta$  (for some small  $\delta < 0$ ), the corresponding CL point is the limit point from which the tool path must be restricted. In the method we propose, the limit surface is approximated by a set of bifacets such as the representation used in approaches by surface polygonization. In this case, both the point  $P_s$  and the CC point  $t$  are determined directly without iterations for  $\delta = 0$ .

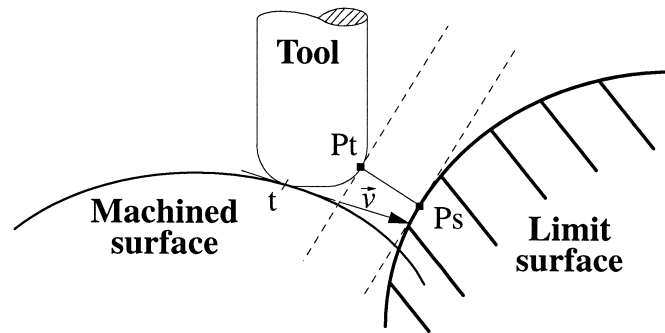


Fig. 2. Curve trim algorithm [9].

In approaches by surface polygonization, surfaces are approximated by a set of polygons to reduce computation time. This representation technique was first suggested by Duncan and Mair [10] who developed a method called polyhedral machining. When a high accuracy is required, the approximation of the limit surface leads to a great number of polygons and an important computation time for restricting the tool paths. Since in the method we propose the approximation by bifacets is gradually limited to a portion of the limit surface, computation time is substantially reduced.

An alternative approach is to compute offset surfaces [11,12] and the Intersection Curve of the offset surfaces (Offset–IC) from which tool paths must be generated. For a given tool path, limit points are determined by searching for their intersection with the Offset–IC. In some derivated approaches, limit points are obtained from the intersection between the tool path and the offset surface of the limit surface [13]. Even though the approach by offset surface is an easily conceivable manner of generating interference-free tool paths [14], it is not always easy to manipulate the mathematical expression defined, especially for ruled surfaces which are a special kind of free-form surfaces [15]. The difficulty in manipulating the mathematical expression of offset surfaces is reinforced by the computation problem of their intersection curve. In fact, tracing the intersection curve is generally a difficult problem [16] as it is ultimately determined by the solution of nonlinear equations [17]. To overcome these problems, Lai and Wang [18] have defined offset triangular meshes for parametric sculptured surfaces. In our work, we have

approximated only the limit surface by triangular facets. This approximation has been adapted for the ruled surfaces used in the manufacturing of dies.

### 3. Surface definition

Forging dies are generally defined by compound surfaces composed of simple surface primitives (cylinders, torus, spheres, etc.), ruled surfaces and other free-form surfaces. In the specific representation of these surfaces adopted by forging mouldmakers [19], a ruled surface is composed of two contours and a set of ruling lines. It may be completely specified by means of a parametric definition of two variables  $u$  and  $v$  (Fig. 3):

$$\vec{OM} = (1 - w) \times \vec{f}(u) + w \times \vec{g}(v) \quad (1)$$

where:

$$\vec{f}(u) = \vec{OA} \quad \vec{f}(0) = \vec{OA}_0 \quad \vec{f}(1) = \vec{OA}_1 \quad (2)$$

$$\vec{g}(v) = \vec{OB} \quad \vec{g}(0) = \vec{OB}_0 \quad \vec{g}(1) = \vec{OB}_1 \quad (3)$$

$$\vec{AM} = w \times \vec{AB} = w \times (\vec{OB} - \vec{OA}) \quad (4)$$

The control law  $u = h(v)$  which links the two contours is computed from the ruling lines. For example, if any ruling line is fixed, we have  $u = v$ . Each contour can be represented by a set of:

- straight lines defined by their starting and finishing points.

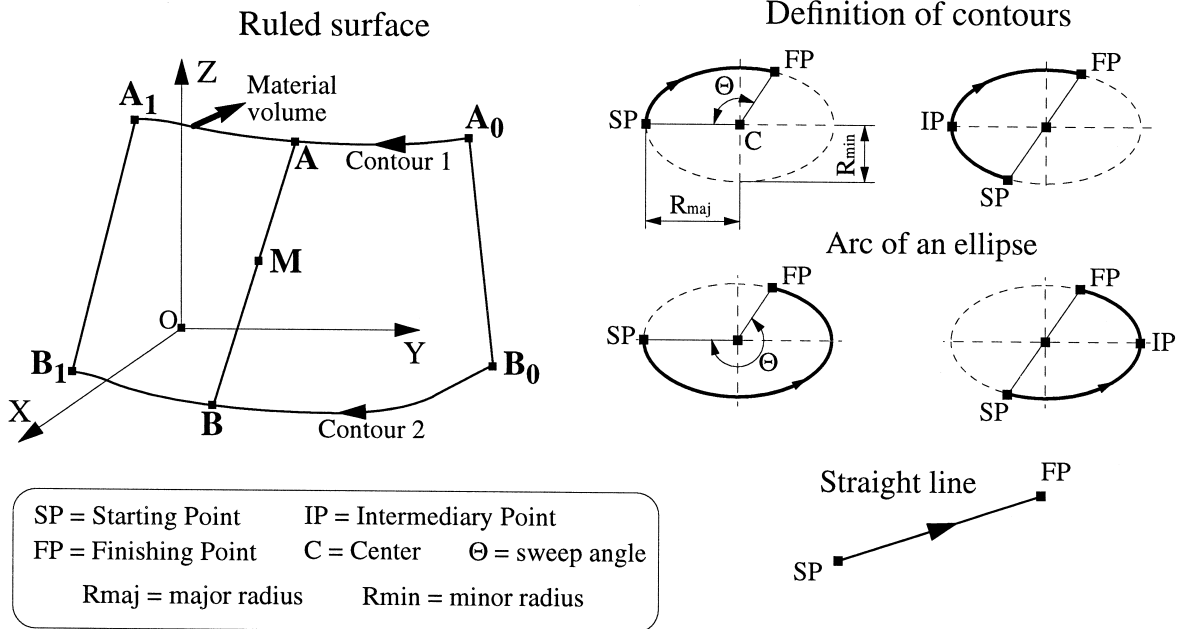


Fig. 3. Ruled surface model.

- elliptical arcs defined by their starting and finishing points, center, minor and major radii, sweep angle and intermediary point (in the case of a semi-ellipse).

As the triangular facets are created out of the material volume of the limit ruled surface, this volume has to be specified especially when the two contours are not closed. By convention, the material volume is confined to the right of someone posted on the contour 1, looking at the contour 2 and fixing the definition direction of the contour 1 from his/her foot to his/her head. When defining a contour, a convex arc of ellipse can be distinguished from a concave one thanks to the value of the sweep angle  $\Theta$  and the position of the intermediary point IP if it exists.

#### 4. Surface decomposition

During the decomposition of a limit ruled surface, bifacets are created from an even number of points and then basic portions and subportions of the surface are identified. Two decomposition levels can be distinguished. The first level corresponds to the rough

decomposition of the limit surface into basic portions. In the second level, the basic portions of surface concerned by the tool-path restriction are iteratively decomposed into subportions. The technique used to create the bifacets is slightly different from the first level (initial facetization) to the second one (fine facetization). With regard to the identification of portions or subportions of surface, the same technique is used in these two levels.

##### 4.1. Facetization

For the rough decomposition of a limit surface, a set of pairs of facets (bifacets) is created from its initial representation. A bifacet is created from two pairs of points, each belonging to one contour of the ruled surface. The determination of these points is carried out in two steps. In the first step named 'subdivision', characteristic points are determined on each contour. In the second step, additional points are created if necessary, on one of the two contours so that both have the same number of points. The creation of additional points prevents bifacet-overlapping and facilitates the identification of portions of the limit surface.

#### 4.1.1. Subdivision of contours

Generally, subdivision techniques are used to determine the intersection lines between parametric surfaces [20]. Before an intersection line is determined, the surfaces are decomposed into portions by inserting new knots into the set of knots. Several methods have been developed from subdivision techniques to approximate curves by polygons or surfaces by a set of plane polygons [21]. The method we propose is similar to the existing methods and based on the characteristic points defined on the contours of a limit ruled surface. Three types of characteristic points are considered (see Fig. 4a):

1. Limit points of the primitives (straight lines and arcs of ellipse) defined on the contours (Ex:  $P_{1,1}$ ,  $P_{1,3}$ ,  $P_{1,5}$ ,  $P_{1,6}$ ,  $P_{2,1}$ ,  $P_{2,2}$ ,  $P_{2,4}$ ) and limit points of rule lines.
2. Upturning points of the arcs of ellipse defined on the contours (Ex:  $P_{1,2}$ ).
3. Intersection points of the tangent lines defined from the limit points and the upturning points which belong to the convex arcs of ellipse (Ex:  $P_{1,4}$ ,  $P_{2,3}$ ).

The polygon created from all the points identified for a contour represents the subdivision line of this contour. Bifacets are created from the two subdivision lines of a limit surface.

In the example shown in Fig. 4a, the two subdivision lines do not have the same number of characteristic points. As we cannot create bifacets without overlapping according to the method described be-

low, we choose to insert additional points on the contour which has the least points. The additional point is created in three steps by:

- searching for the longest line of the approximation line;
- inserting the additional point in the middle of this longest line;
- updating the approximation line.

The procedure is repeated for all the additional points required. In the previous example, two additional points  $P_{2,4}$  and  $P_{2,5}$  are inserted on the contour 2 (See Fig. 4b).

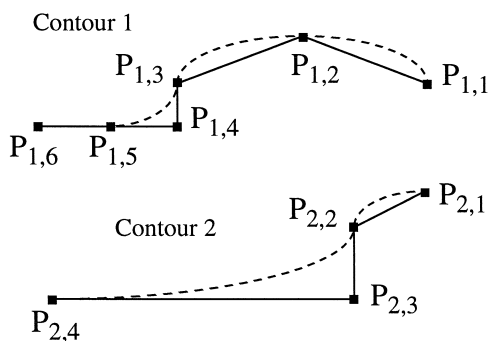
The limit points and the upturning points belong to both a contour and its subdivision line. They can be used for the identification of portions of the limit surface. To determine the composition of a subdivision line  $i$ , each point  $P_{i,j}$  belonging to it is characterised by the value of the function  $\text{Init}(P_{i,j})$  which defines the origin of the point. This function can take the two following values:

$\text{Init}(P_{i,j}) = 1$  for a limit or upturning point

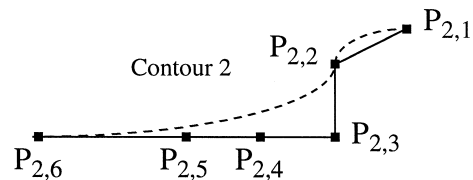
$\text{Init}(P_{i,j}) = 0$  for a point of any other type

The iterative decomposition of portions of surface also comprises the subdivision of the portions of contours, the insertion of additional points and the creation of the bifacets. Before inserting additional points, each subdivision line is made of:

- the two limit points of the portion of contours;



(a) Determination of characteristic points



(b) Insertion of additional points

Fig. 4. Creation of subdivision lines.

- an intermediary point created in between the two limit points;
- the intersection points of the tangent lines defined from the limit points and the intermediary point created on a convex arc of ellipse.

According to Eq. (5),  $\text{Init}(P_{i,j}) = 1$  for the limit points and the intermediary point created on the contour  $i$ .  $\text{Init}(P_{i,j}) = 0$  for the additional points and the intersection points of the tangent lines defined from the limit points and the intermediary point. The two intermediary points created on the contours of a portion of surface divide this latter into two subportions of surface.

#### 4.1.2. Creation of bifacets

When the two subdivision lines have the same number  $n$  of points, bifacets are created. The bifacet  $k$  ( $k \in [1, n-1]$ ) is composed of the two triangular facets whose vertices are the points  $P_{1,k}$ ,  $P_{1,k+1}$ ,  $P_{2,k}$  and  $P_{2,k+1}$  (Fig. 5). The vertices of the first facet of the bifacet  $k$  are  $P_{1,k}$ ,  $P_{1,k+1}$  and  $P_{2,k}$  and those of the second facet are  $P_{2,k}$ ,  $P_{2,k+1}$  and  $P_{1,k+1}$ . Note that all the points of each subdivision line are either on the associated contour or outside of the material volume of the limit surface. This arrangement marks the approximation by excess (of material) of the limit surface. The excess of material will be machined in the polishing stage of dies.

#### 4.2. Basic portions and subportions of surface

##### 4.2.1. Identification

The basic portions of the limit surface are determined after having searched for the interference between the facets created during the rough decomposition and the cutting tool. Consider the interference concerned by one facet of the bifacet  $k$ . The basic portion associated to this bifacet is limited by the four points  $P_{1,a}$ ,  $P_{1,b}$ ,  $P_{2,c}$  and  $P_{2,d}$  such that:

$$\begin{aligned} a \leq k, b \geq k+1, \quad c \leq k, d \geq k+1 \\ \text{Init}(P_{1,a}) = 1, \quad \text{Init}(P_{1,b}) = 1 \\ \text{Init}(P_{2,c}) = 1, \quad \text{Init}(P_{2,d}) = 1 \end{aligned} \quad (6)$$

These four points have to satisfy the following conditions:

$$\text{if } a < k \text{ then } \forall i \in ]a, k], \text{Init}(P_{1,i}) = 0 \quad (7)$$

$$\text{if } b > k+1 \text{ then } \forall j \in [k+1, b[, \text{Init}(P_{1,j}) = 0 \quad (8)$$

$$\text{if } c < k \text{ then } \forall l \in ]c, k], \text{Init}(P_{2,l}) = 0 \quad (9)$$

$$\text{if } d > k+1 \text{ then } \forall m \in [k+1, d[, \text{Init}(P_{2,m}) = 0 \quad (10)$$

The conditions given in Eq. (6) express the fact that

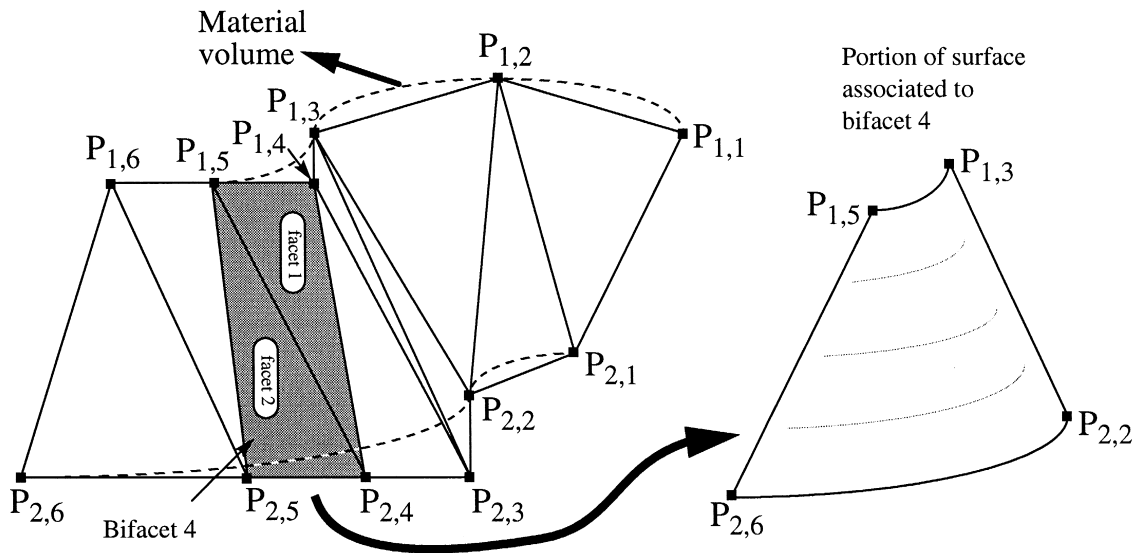


Fig. 5. Principle of facetization.

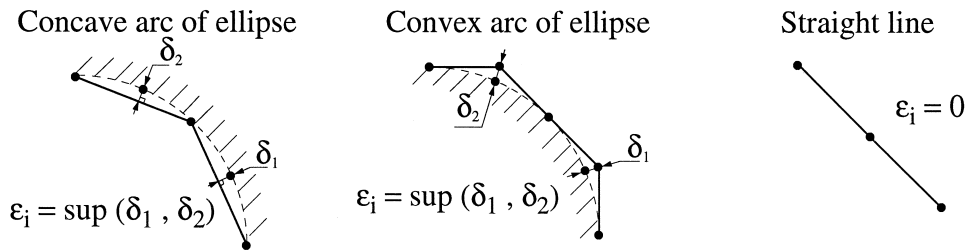


Fig. 6. Approximation error.

the portion of surface includes the bifacet  $k$  and is limited by limit or upturning points. Eqs. (7)–(10) show that the limit points of the portion of surface are the nearest points from the limit points of the bifacet  $k$  which belong to both the contours and the subdivision lines. In the example shown in Fig. 5, the basic portion associated to the bifacet 4 is defined by the points:  $P_{1,3}$ ,  $P_{1,5}$ ,  $P_{2,2}$  and  $P_{2,6}$ .

#### 4.2.2. Decomposition

The iterative decomposition of a basic portion of the limit surface is made by first dividing it into two subportions and then, if necessary, progressively dividing each subportion into two other subportions. The decomposition stops when the approximation error  $\varepsilon_r$  is less than the approximation accuracy wanted. The approximation error is defined by:

$$\varepsilon = \sup(\varepsilon_i) \quad (11)$$

where  $\varepsilon_i$  is the subdivision error of the portion or subportion of the contour  $i$ . This subdivision error is

specific to each primitive used to represent the contours (See Fig. 6).

## 5. Interference avoidance

To avoid interference between the cutting tool and the limit surface, we search for the general interference zone ( $G_1G_2$ ) along the line  $\Delta$  which contains the tool path (Fig. 7) and then restrict the tool path according to this zone. The restricted tool path must be defined so that the cutting tool moves out of the general interference zone and the material volume which is limited rather by a set of bifacets than by the limit-ruled surface. The tool path is represented by a straight line limited by the points  $T_1$  and  $T_2$ . The cutting tool used in our examples is a ball-end mill represented by a sphere.

### 5.1. Evaluation of interference zones

The general interference zone is determined by associating several elementary interference zones,

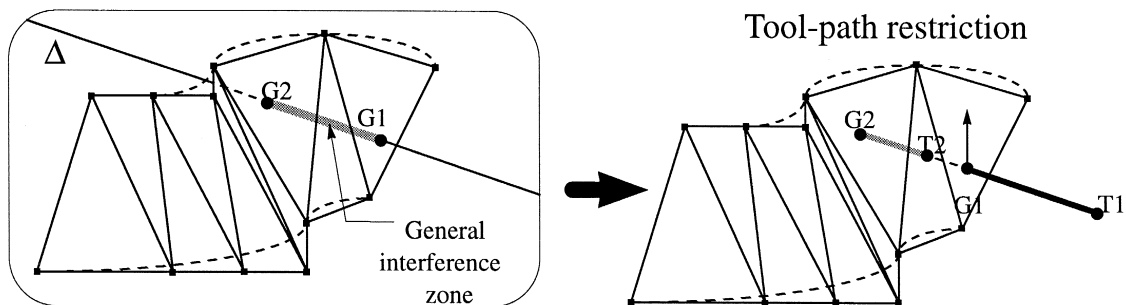


Fig. 7. Interference control procedure.

each corresponding to the interference between the cutting tool and one facet of the limit surface. The elementary interference zones have been pointed out when the approximation accuracy wanted is reached for any portion or subportion of the limit surface.

The elementary interference zone of a facet is the set of CL points which define the positions of the cutting tool when it has an intersection with the facet. This zone is limited by two points  $Q_1$  and  $Q_2$  which can be of two types (See Fig. 8):

Type I: when the cutting tool is tangent to the facet.

Type II: when the cutting tool is tangent to one of the three edges of the facet.

When determining elementary interference zones, the type I points called ‘minimal distance points’ are first searched for. If two type I points have been found, the cutting tool cannot interfere with the edges of the facet and it is therefore unnecessary to search for type II points.

To define an elementary interference zone, consider a CL point  $Q$  which belongs to this zone. If the position of  $Q$  is given by

$$\vec{T_1 Q} = \lambda \times \vec{T_1 T_2} \quad (12)$$

then the elementary interference zone  $\diamond$  can be expressed as:

$$\mathfrak{Z} = [\lambda_1, \lambda_2], (\lambda \in \mathfrak{Z}) \quad (13)$$

where

$$\vec{T_1 Q_1} = \lambda_1 \times \vec{T_1 T_2}; \quad \vec{T_1 Q_2} = \lambda_2 \times \vec{T_1 T_2}$$

Using Eq. (13), the general interference zone can be

defined as the union of all the elementary interference zones.

To determine the points  $Q_1$  and  $Q_2$ , consider a facet limited by the three points  $A(x_a, y_a, z_a)$ ,  $B(x_b, y_b, z_b)$  and  $C(x_c, y_c, z_c)$ , a tool path limited by  $T_1(x_1, y_1, z_1)$  and  $T_2(x_2, y_2, z_2)$  and the cutting tool the radius of which is  $r$ . If the elementary interference zone of the facet is limited by the points of type I, their positions defined by  $\lambda_i$  and the corresponding CC points  $S_j(x_s, y_s, z_s)$  are the solutions of the following equations:

$$x_s = x_1 + \lambda \times (x_2 - x_1) \pm r \times x_n \quad (14)$$

$$y_s = y_1 + \lambda \times (y_2 - y_1) \pm r \times y_n \quad (15)$$

$$z_s = z_1 + \lambda \times (z_2 - z_1) \pm r \times z_n \quad (16)$$

$$x_n \times (x_s - x_a) + y_n \times (y_s - y_a) + z_n \times (z_s - z_a) = 0 \quad (17)$$

where  $\vec{n}(x_n, y_n, z_n)$  is the unit normal vector of the facet. As the CC points must belong to the facet (portion of plane limited by three edges) the solutions of Eqs. (14)–(17) must also satisfy the following condition:

$$\begin{aligned} \text{Sign} \left[ \left( \vec{AB} \wedge \vec{AS_j} \right) \times \vec{n} \right] \\ = \text{Sign} \left[ \left( \vec{BC} \wedge \vec{BS_j} \right) \times \vec{n} \right] \\ = \text{Sign} \left[ \left( \vec{CA} \wedge \vec{CS_j} \right) \times \vec{n} \right] \end{aligned} \quad (18)$$

where  $\text{Sign}(a)$  is equal to 1 if  $a$  is positive and equal to  $-1$  if  $a$  is negative.

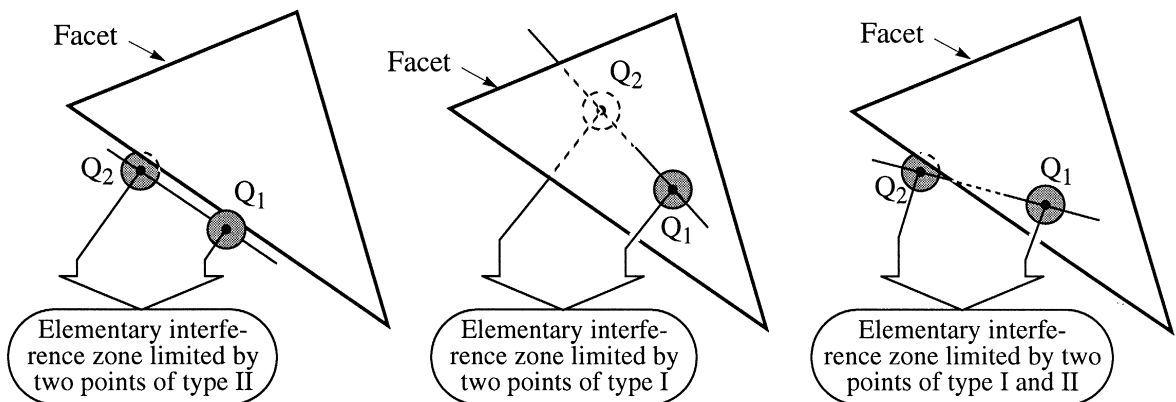


Fig. 8. Elementary interference zone.



If the elementary interference zone is limited by type II points, we must search for the points  $Q_1$  and  $Q_2$  with regard to each edge of the facet. For example if we consider the edge  $(A, C)$ , the positions of the CL points defined by  $\lambda_i$  and the positions of the CC points defined by  $\gamma_j$  are the solutions of the following equations:

$$x_a + \gamma \times (x_c - x_a) = x_1 + \lambda \times (x_2 - x_1) \pm r \times x_u \quad (19)$$

$$y_a + \gamma \times (y_c - y_a) = y_1 + \lambda \times (y_2 - y_1) \pm r \times y_u \quad (20)$$

$$z_a + \gamma \times (z_c - z_a) = z_1 + \lambda \times (z_2 - z_1) \pm r \times z_u \quad (21)$$

$$x_u \times (x_c - x_a) \times y_u \times (y_c - y_a) \times z_u \times (z_c - z_a) = 0 \quad (22)$$

$$x_u^2 + y_u^2 + z_u^2 \quad (23)$$

where  $\vec{U}(x_u, y_u, z_u)$  is a vector which contains both the CC and the CL points. As the CC points must belong to the edge  $(A, C)$ , the solutions of Eqs. (19)–(23) must also satisfy the following condition:

$\gamma \in [0, 1]$ . For each edge, two CL points can be obtained. In this case, the points  $Q_1$  and  $Q_2$  are the CL points which give the longest elementary interference zone.

### 5.2. Restriction examples

The method presented in this paper has been implemented in LURPA-CN3D which is a CAM prototype software developed at LURPA laboratory [19]. Fig. 9 shows an example of its application to the machining of two ruled surfaces which partially represent a die.

To process the tool paths generated for the machining of the surfaces shown in Fig. 9, we distinguish three restriction cases (Fig. 10): (1) No restriction (Ex: tool paths  $TP_1$  and  $TP_2$ ). The tool path is not restricted because it is out of the material volume and the general interference zone does not exist. (2) Partial restriction (Ex: tool paths  $TP_3$  and  $TP_4$ ): the general interference zone ( $G_1, G_2$ ) exists and, for example, the portion ( $G_2, T_2$ ) of the tool path is inside of the material volume. The restricted tool path is  $(T_1, G_1)$ . (3) Total restriction (Ex: tool paths

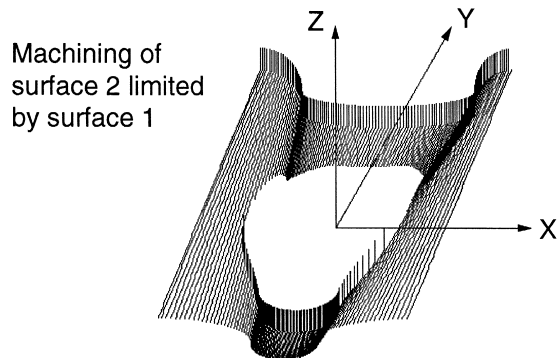
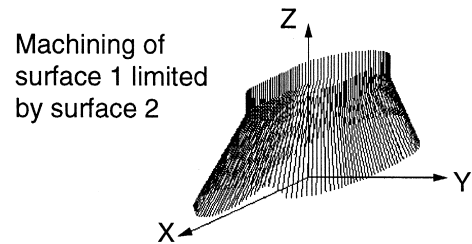
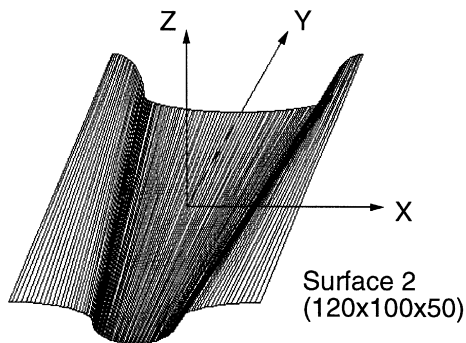
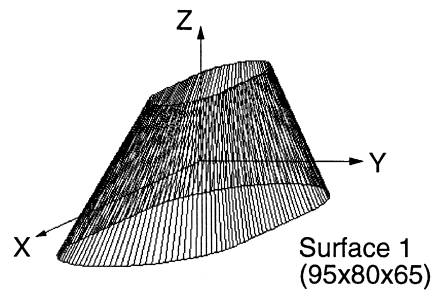


Fig. 9. Machining example.

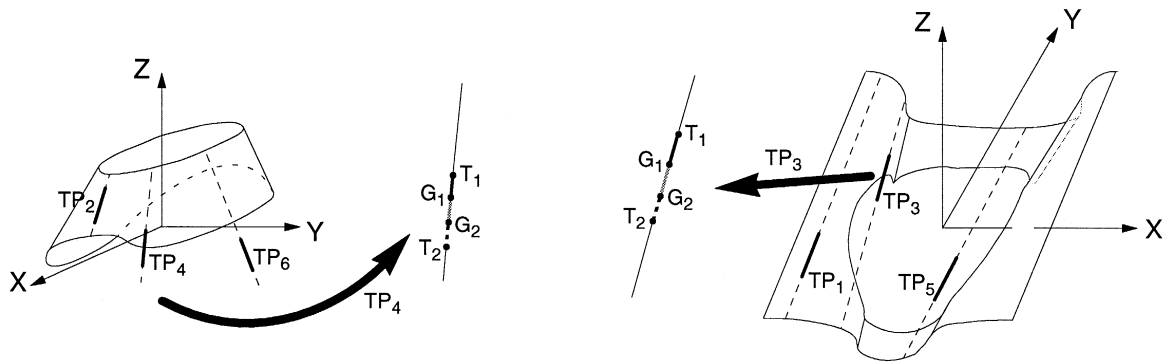


Fig. 10. Example of tool-path restriction.

TP<sub>5</sub> and TP<sub>6</sub>): the tool path is totally inside of the general interference zone or the material volume. So, the cutting tool cannot move along this path. A portion of the tool path which is inside of the material volume is identified by analyzing the position of the points T<sub>1</sub>, T<sub>2</sub>, G<sub>1</sub> and G<sub>2</sub>. This analysis is made with regards to the facets (stemmed from the initial facetization) with which the tool can interfere when it moves along the line  $\Delta$  (See Fig. 7). The position of each point is determined by considering the normal vector  $\vec{n}$  of the facets. Each normal vector is computed according to the position of the material volume defined in Section 3 (see Fig. 3).

### 5.3. Results analysis

To machine the two surfaces shown in Fig. 9, several CL points have been generated along each ruling line using the LURPA-CN3D software. These CL points have been computed according to roughness constraints. In the following, the detailed analysis of the restriction of the tool paths defined from the CL points is presented.

For the example shown in Fig. 9, the machining thickness defined is: 0.5 mm along the ruled lines and 0.3 mm along the contours. The approximation accuracy wanted is 0.05 mm. From these constraints, 40570 tool paths have been defined for the surface 1 (40887 CL points for 317 ruling lines) and 63642 tool paths for the surface 2 (63946 CL points for 304 ruling lines). According to the processing results, the tool paths can be classified into three categories.

(1) Minimal processing: the tool paths of this category are totally inside or outside of the material

volume. The interference control has been suspended after the rough decomposition of the limit surface due to the absence of a portion of surface to process (Ex: tool paths TP<sub>1</sub> and TP<sub>5</sub> shown in Fig. 10).

(2) Average processing: the tool paths of this category are totally outside of the material volume. But the interference control has been suspended during the iterative decomposition of the limit surface before the approximation accuracy wanted is reached. The reason is that after some iterations (1 or 2), there is no more subportion of surface to process. This category corresponds to the tool paths located near the intersection line of the two surfaces.

(3) Total processing: the tool paths of this category are partially or totally restricted. All the iterations necessary to reach the approximation accuracy wanted are made (Ex: tool paths TP<sub>3</sub> and TP<sub>4</sub> shown in Fig. 10).

Table 1 shows how the tool paths have been processed on a SPARC 1 station. The processing time, which is 5 min 49 s for surface 1 and 9 min 6 s

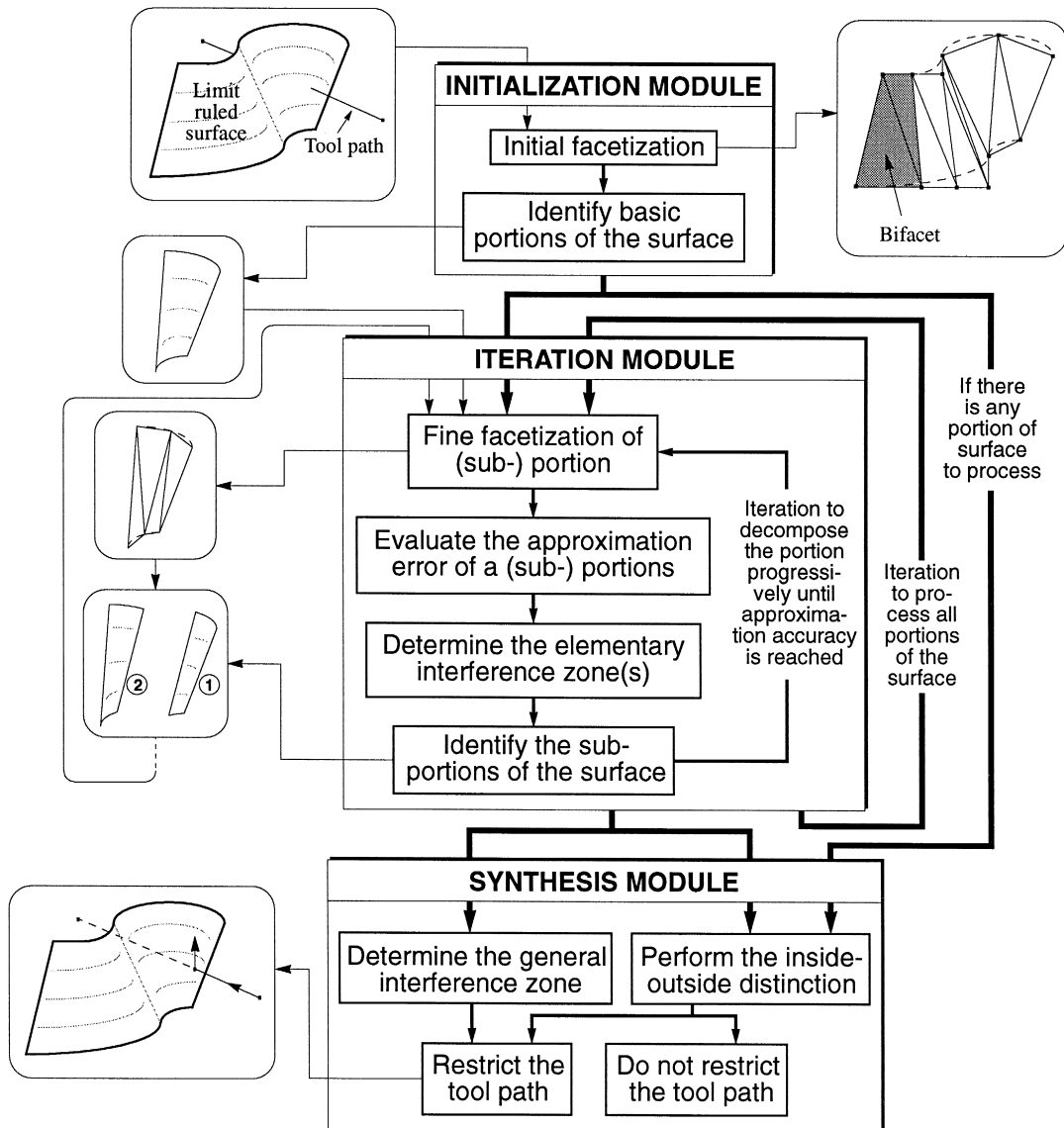
Table 1  
Machining results

|                                  | Machining of<br>surface 1 | Machining of<br>surface 2 |
|----------------------------------|---------------------------|---------------------------|
| Minimal processing<br>tool paths | 33,783                    | 51,571                    |
| Average processing<br>tool paths | 6049                      | 9547                      |
| Total processing<br>tool paths   | 738                       | 2524                      |

for surface 2, depends on the number of tool paths obtained for each category. Suppose that the processing time for one tool path is  $t$ . For  $n$  tool paths, the processing time will be  $n \times t$  with most of the existing methods. Using the method we propose, the processing time will be:

$$(t_1 < t_2 < t_3) (n_1 + n_2 + n_3 = n)$$

where  $t_1$  is the minimal processing time,  $t_2$  is the



## 6. Overview of the tool-path restriction procedure

A ruled surface can be machined either along ruling lines or by using a contour machining approach whereby the tool paths are defined from isoparametric curves. The method we propose to restrict tool paths can be applied to the CL points defined using any one of these two machining approaches. In the following, a tool path is represented by a straight line joining two CL points.

The procedure for restricting tool paths shown in Fig. 11, comprises three modules: the initialization module, the iteration module and the synthesis module. In the initialization module the limit ruled surface is roughly approximated by a set of bifacets (pairs of triangular facets). This decomposition of the surface is the first and corresponds to the initial facetization. After the rough approximation of the limit surface, we look for interferences between the cutting tool and the facets created. For each interference, the portion of the limit surface which corresponds to the facet is identified. Even if no portion of surface is identified, it is necessary to perform the inside–outside distinction i.e., to determine if the entire tool path is inside or outside the material volume limited by the limit surface. The inside–outside distinction is made in the last module.

The portions of the limit surface identified in the initialization module are processed in the iteration module. During this processing each portion of the surface is successively approximated by other bifacets (fine facetization) and decomposed if necessary (when interferences occur) into subportions of the surface. The processing is repeated with the subportions of the surface until the approximation accuracy wanted is reached. In this case, the elementary interference zone determined for each facet is pointed out to the last module. Before the approximation accuracy is reached, the iteration is suspended for a given portion of the surface if there are any subportions to process. This way, the ruled surface is progressively approximated by facets and during this approximation, only the portions and subportions of the surface which may interfere with the cutting tool are processed. This technique permits to reduce processing time as the ruled surface is decomposed into a minimal number of facets according to the accuracy wanted.

At the end of the iteration module, the tool path must be restricted if at least one elementary interference zone has been pointed out. As for the first module, if no elementary interference zone has been pointed out, the inside–outside distinction is performed in the synthesis module. This distinction can lead to total restriction (tool path inside the material volume) or to nonrestriction (tool path outside the material volume). If elementary interference zones have been pointed out, the restriction is made according to their association.

The two main tasks carried out during the tool-path restriction procedure are:

- decomposition of the ruled surface including its facetization and the identification of basic portions and subportions;
- interference control through the determination of the virtual interference zones and the tool-path restriction.

These tasks, which characterize the method we propose in this paper, have been presented in Section 5 and Section 6.

## 7. Conclusion

A method for the restriction of tool paths has been presented. It is based on the progressive decomposition of ruled surfaces and the control of the interference between a cutting tool and limit surfaces. During the decomposition, the limit surfaces are approximated by bifacets according to the approximation accuracy wanted. The surface finish in the intersection area depends on this approximation accuracy. The decomposition process is well adapted to the type of the tool paths so that the processing time can be reduced, for example when the intersection zone is very small compared with the machined surface.

As any surface can be converted into a set of facets [22], we are trying to apply the method presented in this paper to other types of surface. The difficulty in the extension of our method is to have subdivision points which belong to both the subdivision lines and the characteristic curves of the surfaces. These points are required for the progressive decomposition of the surfaces. Another problem (not so difficult) is to define the subdivision error which must specify the stop criteria in the iteration module.

The extension of our method will be presented further.

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