

Uneven offset method of NC tool path generation for free-form pocket machining

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Abstract

This paper presents a new method of generating contour-parallel tool path for free-form pocket machining. With the so-called “uneven offset” technique developed in the paper, NC tool path intervals are evidently increased without leaving an uncut area. As a result, the length of the tool path is reduced and the producing efficiency is effectively improved. © 2000 Published by Elsevier Science B.V.

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1. Introduction

The majority of industrial milling tasks can be performed using 2.5-dimensional (2.5D) milling. This is partially due to the fact that a surprisingly large number of mechanical parts are 2.5D and even the more complicated objects are usually produced from a billet by a 2.5D roughing and 3D–5D finishing. Thus the computation of the tool path for pocket machining is one of the most important issues in CAM.

A popular tool path style in pocket machining is called contour-parallel path, which is generated by a

successive offset of the original boundary. Many procedures have been developed to compute contour-parallel paths [1–7]. However, only a few of them can handle the pockets bounded with free-form curves [1].

Milling efficiency has always been an important criterion for evaluating various milling tool paths, and there should be no exception for a 2.5D milling task. Though the currently used computation methods can deal with arbitrary-shaped pocket machining problems, few works have been done on milling efficiency. As a result, the milling efficiencies of the tool paths generated with these methods are not as high as one might have expected. Fig. 1 shows the case.

As shown in Fig. 1, if we generate a current path with an offset distance of $2R$, there will be an uncut area left. Here R is the tool radius. The currently

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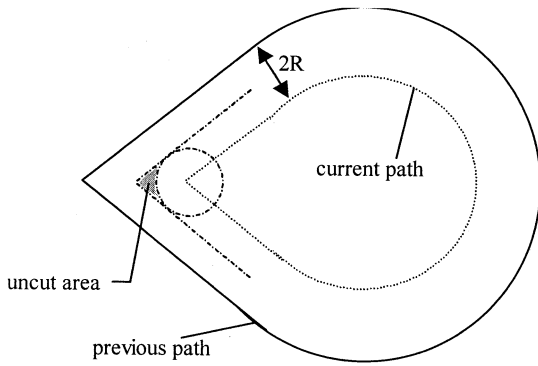


Fig. 1. Generation of current path with offset distance $2R$.

used methods handle the problem by reducing the offset distance. But this will result in an awful payment of producing efficiency, especially in consideration of the arc part of the path where the reduction of offset distance is unnecessary.

Fig. 2 shows the determination of offset distance in computing a line segment $P'_i P'_{i+1}$ of a current tool path from the corresponding segment $P_i P_{i+1}$ of previous path. The offset distance of R is certainly undesired in consideration of the producing efficiency. Czeranowsky [2] has proposed to compute the offset distance as $R + R \sin(\theta/2)$, where θ is an internal angle of a pair of consecutive path segments. However, when computing a segment of the current path, as shown in Fig. 2, the smaller one of the two concerned internal angles (θ_1, θ_2) had to be employed; the cost of efficiency is also considered.

In this paper, we present a method to generate high-efficiency tool paths for free-form pocket ma-

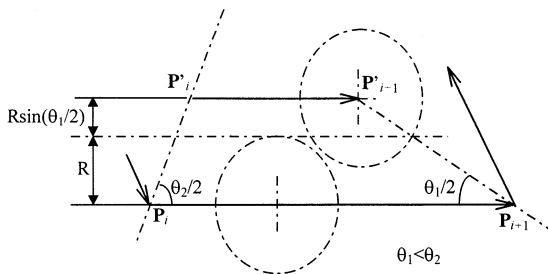


Fig. 2. Determination of offset distance in computation of a path segment.

chining. The approach is composed of two basic steps: (1) compute the initial tool path by offsetting the discrete boundary with distance of tool radius R ; and (2) compute each successive path by offsetting the CL points on a previous path with various distances. The offset distance of each CL point is evaluated with the formula developed in this paper.

2. Generation of initial tool path

There are three steps in generating an initial tool path: (1) discrete the boundary curves into a group of knots under the specially designed criterion; (2) compute the CL points on an initial tool path; and (3) detect and remove the invalid loops of the tool path.

2.1. Subdivision of the boundary curves

As shown in Fig. 3, we note a boundary curve as $P(u)$ and the knots on it as P_i ($i = 0, 1, \dots$). The unit normal vector is noted by n_i . The angle between n_i and the vertical direction of line $P_i P_{i+1}$ is noted by α_i . The angle between n_{i+1} and the vertical direction of line $P_i P_{i+1}$ at P_{i+1} is noted by β_i . The deviation from $P(u)$ to $P_i P_{i+1}$ is noted by δ_i .

The knots P_i ($i = 0, 1, \dots$) are generated through recursive subdivision of $P(u)$. The criterion for subdivision is designed as the following equations show.

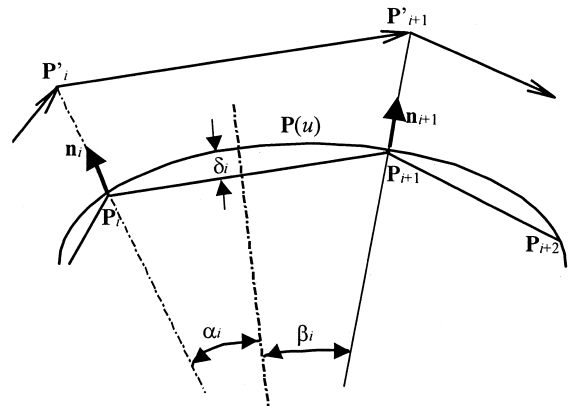


Fig. 3. Computation of knots on initial tool path.

For each line segment $\mathbf{P}_i\mathbf{P}_{i+1}$, there is

$$\delta_i < \varepsilon/2$$

and also there is

$$\min(\cos \alpha_i, \cos \beta_i) > \frac{R}{R + \varepsilon/2}$$

where ε is the permitted error.

The details of subdividing the most commonly used free-form curves such as Bezier curve or B-spline curve can be found in many publications [8,9] and will not be presented here.

2.2. Computation of the CL points on an initial tool path

As shown in Fig. 3, for a single G^1 continuous boundary curve $\mathbf{P}(u)$, the CL points \mathbf{P}'_i ($i = 0, 1, \dots$) on an initial tool path can be computed as:

$$\mathbf{P}'_i = \mathbf{P}_i + R\mathbf{n}_i$$

Supposing ε is the permitted error, it is easy to prove with Fig. 3 that the offset error of polyline $\{\mathbf{P}'_i\}$ from $\mathbf{P}(u)$ is smaller than ε .

However, in practice a boundary of pocket is often composed of a group of curves, and the connection of these curves may not be G^1 continuous. This will lead to the break of an initial tool path. The cases are shown in Fig. 4.

In Fig. 4, two boundary curves $\mathbf{P}(u)$ and $\mathbf{Q}(v)$ connect at \mathbf{O} . Their unit normal vectors at \mathbf{O} are noted as \mathbf{n} and \mathbf{m} . The offsetting of \mathbf{O} in direction \mathbf{n} and \mathbf{m} results in two separate CL points \mathbf{P}_0 and \mathbf{Q}_0 . In the case of Fig. 4(a) where $\mathbf{P}(u)$ and $\mathbf{Q}(v)$ connect in convex relation, additional CL points \mathbf{A}_i ($i = 0, \dots, n$) should be inserted along a transitional arc

to avoid overcut. Here, \mathbf{O} is the centre of the arc, R is its radius, $\mathbf{A}_0 = \mathbf{P}_0$ and $\mathbf{A}_n = \mathbf{Q}_0$. Obviously the deviation of polyline $\{\mathbf{A}_i\}$ from the arc should be smaller than the permitted error ε . Therefore the angle θ_i between vectors \mathbf{OA}_i and \mathbf{OA}_{i+1} should satisfy:

$$\theta_i < 2\arccos\left(\frac{R - \varepsilon}{R}\right) \text{ for } i = 0, \dots, n - 1$$

In Fig. 4(b) where $\mathbf{P}(u)$ and $\mathbf{Q}(v)$ connect in concave relation, \mathbf{P}_0 and \mathbf{Q}_0 are directly connected without inserting any additional CL point.

2.3. Detection and removal of invalid loops

The tool paths generated through the offsetting process may have self-intersections at which the offset profiles can be decomposed into a set of simple loops. Some of the loops may be invalid, such as the loop $\mathbf{S}-\mathbf{P}_0-\mathbf{Q}_0$ in Fig. 4(b). By determining the loop orientation we can detect the invalid loops and remove them. The details can be found in Refs. [1] and [4]. No further improvement on this issue will be presented in this paper.

3. Generation of tool path with uneven offset technique

Once the initial tool path has been generated, we can compute the successive tool paths from it. According to the currently used approaches, CL points are obtained by computing the intersections between offsetting segments of previous path. In this paper, however, CL points on current path are computed in a different way by directly offsetting those CL points on previous path. The technique is named as uneven offset for the fact that the actual offset distances of CL points may be different from each other because they are decided by different local situations and different nominal offset distances. The following are the details.

3.1. Computation of CL points

In Fig. 5, \mathbf{P}_i ($i = 0, 1, \dots$) are CL points on a previous path, and \mathbf{Q}_j ($j = 0, 1, \dots$) are the current CL points to be computed. In the case of Fig. 5(a)

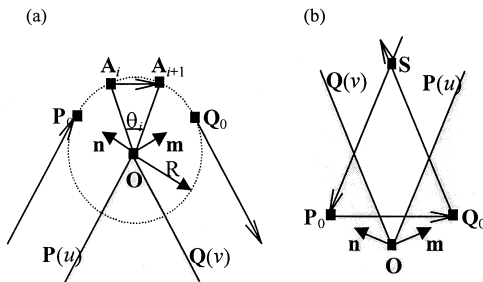


Fig. 4. Break of initial tool path.

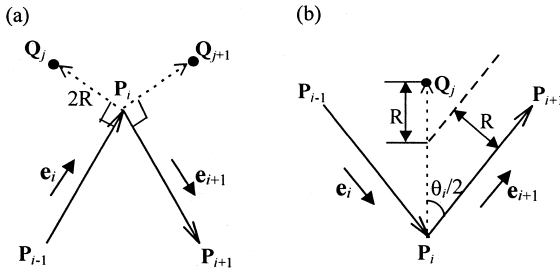


Fig. 5. Offsetting of CL points.

where \mathbf{P}_i is the joint of two convexly connecting path segments $\mathbf{P}_{i-1}\mathbf{P}_i$ and $\mathbf{P}_i\mathbf{P}_{i+1}$, two CL points on current path are generated through offsetting of \mathbf{P}_i in different directions by

$$\begin{aligned} \mathbf{Q}_j &= \mathbf{P}_i + 2R\mathbf{G}\mathbf{e}_i \\ \mathbf{Q}_{j+1} &= \mathbf{P}_i + 2R\mathbf{G}\mathbf{e}_{i+1} \end{aligned} \quad (1)$$

where

$$\mathbf{e}_i = \frac{\mathbf{P}_i - \mathbf{P}_{i-1}}{\|\mathbf{P}_i - \mathbf{P}_{i-1}\|}, \quad \mathbf{G} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

In Fig. 5(b), $\mathbf{P}_{i-1}\mathbf{P}_i$ and $\mathbf{P}_i\mathbf{P}_{i+1}$ connect in a concave relation, and one CL point on the current path is generated as

$$\mathbf{Q}_j = \mathbf{P}_i + \left(R + R/\sin(\theta_i/2) \right) \frac{\mathbf{e}_{i+1} - \mathbf{e}_i}{\|\mathbf{e}_{i+1} - \mathbf{e}_i\|} \quad (2)$$

where θ_i is the angle between $\mathbf{P}_{i-1}\mathbf{P}_i$ and $\mathbf{P}_i\mathbf{P}_{i+1}$.

Fig. 6 shows a current path segment $\mathbf{Q}_j\mathbf{Q}_{j+1}$ computed with Eq. (2) while the previous path is given by Fig. 2. It can be observed that the valid cutting area of $\mathbf{Q}_j\mathbf{Q}_{j+1}$ is larger than that of $\mathbf{P}'_i\mathbf{P}'_{i+1}$ in Fig. 2. This means the improvement of producing efficiency.

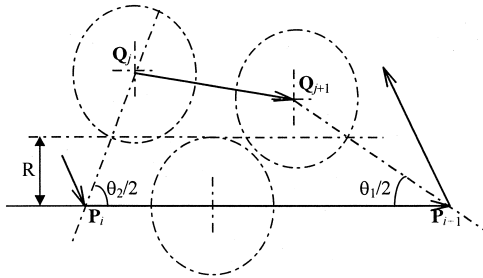
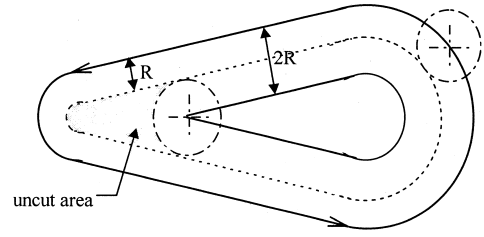


Fig. 6. Generation of a path segment with uneven offset method.

Fig. 7. Uncut area owing to nominal offset distance of $2R$.

3.2. Modification of nominal offset distance

In this paper, the maximum value that the distance between two adjacent tool paths can reach in computation of CL points is defined as the *nominal offset distance*. For example, the distance between two adjacent tool paths computed with formula (1) and (2) can reach $2R$. Thus the nominal offset distance of formula (1) and (2) is $2R$.

In view of producing efficiency, it may be desirable to compute CL points with a nominal offset distance of $2R$. But in practice it may result in an uncut area after the removal of invalid loops of path, as illustrated in Fig. 7.

To deal with this problem, we modify formula (1) and (2) for those CL points on the detected invalid loops as

$$\begin{aligned} \mathbf{Q}_j &= \mathbf{P}_i + R\mathbf{G}\mathbf{e}_i \\ \mathbf{Q}_{j+1} &= \mathbf{P}_i + R\mathbf{G}\mathbf{e}_{i+1} \end{aligned} \quad (3)$$

and

$$\mathbf{Q}_j = \mathbf{P}_i + \left(R/\sin(\theta_i/2) \right) \frac{\mathbf{e}_{i+1} - \mathbf{e}_i}{\|\mathbf{e}_{i+1} - \mathbf{e}_i\|} \quad (4)$$

Here, the nominal offset distance has been reduced to R , as shown in Fig. 8.

Because the offset distances are reduced only for those CL points on invalid loops of path, the producing efficiency of tool path is furthest preserved.

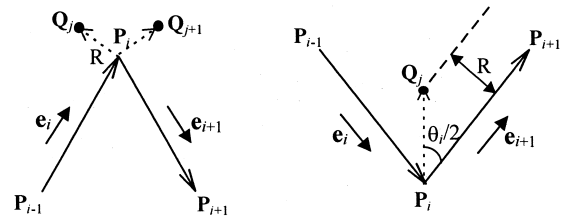


Fig. 8. Modification of CL points.

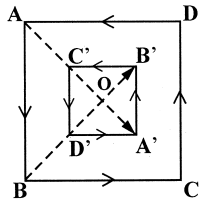


Fig. 9. The second case of terminal condition.

3.3. Termination test

The offsetting of tool path has to stop when one of the following two cases occurs. The first case is when all the obtained path loops are detected to be invalid. The second case is when the direction of every segment of tool path is reversed after an offset, as illustrated in Fig. 9.

4. Process of tool path generation

In this section, we summarize the process of our tool path generation method as follows.

- step 1: Discrete the boundary curves into a group of knots;
- step 2: Compute the CL points on initial tool path;
- step 3: Detect and remove the invalid loops of initial tool path;
- step 4: Do termination test. If the terminal condition is satisfied then terminate the process and output the results;
- step 5: Compute the CL points on current path with formula (1) and (2);
- step 6: Detect the invalid loops of current path. Instead of removing them, modify the CL points on them with formula (3) and (4);
- step 7: Detect and remove the invalid loops of the modified current path;
- step 8: Go back to step 4.

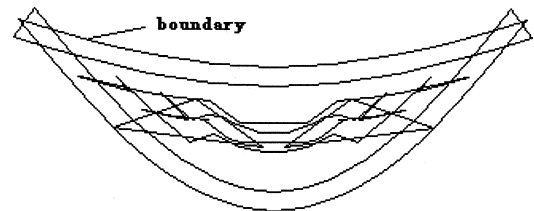
5. Examples

The method described above has been realized for the pockets bounded with Bezier curves. Two examples are presented here to show the difference of

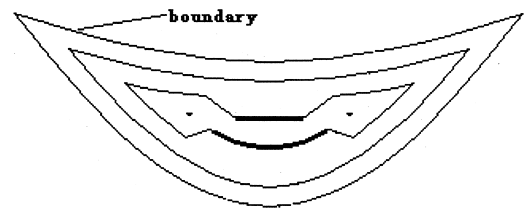
milling efficiency between the new path style and the currently used one. The examples are not designed to be very complex so as to demonstrate the main progress made in this paper more clearly.

Fig. 10 gives an example of tool path generation for machining a luniform pocket. The tool radius $R = 3$ mm and the permitted error is 0.2 mm. Fig. 10(a) shows the tool path of new style generated by uneven offset technique without removing the invalid loops. Fig. 10(b) shows the final result. Fig. 10(c) shows the tool path generated by the commonly used methods.

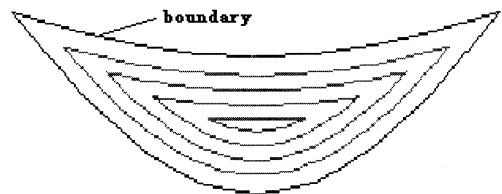
It can be seen that with uneven offset technique, the path intervals are closed to $2R$ at the boldly displayed segments in Fig. 10(b), and then fewer paths are generated.



(a) Tool path of new style with invalid loops



(b) Tool path of new style without invalid loops



(c) Tool path of commonly used style

Fig. 10. Example of tool path generation for a luniform pocket machining.

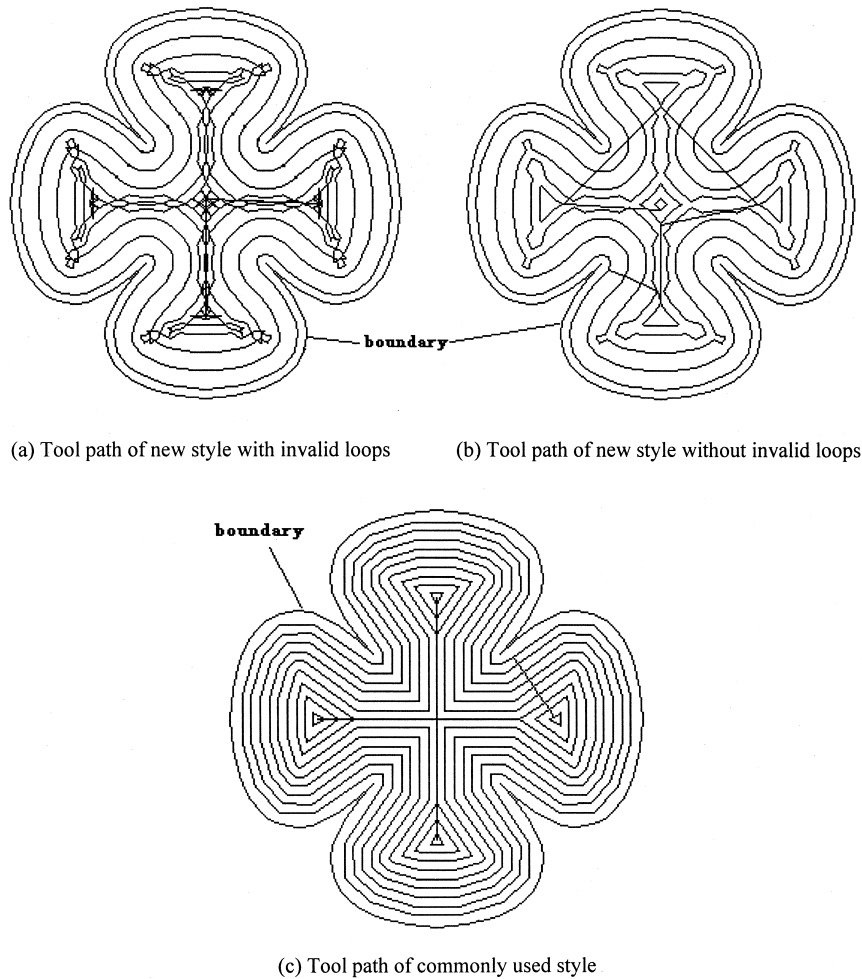


Fig. 11. Example of tool path generation for a badge-shaped pocket machining.

Fig. 11 gives another example of tool path generation for machining a badge-shaped pocket. The tool radius $R = 1.5$ mm and the permitted error is 0.2 mm. Fig. 11(a) shows the new style of tool path without removing the invalid loops. Fig. 11(b) shows the final result. Fig. 11(c) shows the tool path generated by the commonly used methods.

It can be seen that the path intervals are closed to $2R$ for most of the path segments in Fig. 11(b), while the path intervals in Fig. 11(c) are only R . The total lengths of the paths in Fig. 11(b) and (c) are 1.258 m and 1.945 m, respectively. It can be concluded for the example in Fig. 11 that the producing

efficiency of tool path can be improved with 35% by adopting uneven offset technique.

6. Conclusion

This paper presents a new method of generating contour-parallel tool path for free-form pocket machining. With the uneven offset technique developed in the paper, the NC tool path intervals are evidently increased without leaving an uncut area. As a result, the length of the tool path is reduced and the machining efficiency is effectively improved.

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