EMO Algorithms on Correlated Many-Objective Problems with Different Correlation Strength

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***Abstract*—It has been pointed out in many studies that standard Pareto dominance-based EMO (evolutionary multi-objective optimization) algorithms do not work well on many-objective problems with four or more objectives. However, it has also been demonstrated in some studies that many-objective problems are not always difficult for such an EMO algorithm when many objectives are highly correlated or dependent. In this paper, we examine the performance of well-known EMO algorithms on many-objective problems with weakly correlated objectives as well as those with highly correlated objectives. As test problems, we generate many-objective problems with 4-10 objectives from a two-objective knapsack problem with randomly generated two objectives. Each of the other objectives is correlated to one of the two objectives. The strength of the correlation can be arbitrarily specified by a parameter value used for generating correlated objectives. Performance of well-known EMO algorithms such as NSGA-II and MOEA/D is examined by applying them to our test problems with different correlation strength.**

Keywords - Evolutionary computation, evolutionary algorithms, evolutionary multi-objective optimization, many-objective problems.

I. INTRODUCTION

Since the suggestion by Goldberg [1], Pareto dominance relation has been widely used for fitness evaluation in EMO (evolutionary multi-objective optimization) algorithms [2]-[4]. Whereas Pareto dominance-based EMO algorithms such as NSGA-II [5] and SPEA2 [6] usually work very well on multi-objective problems with two or three objectives, their search ability is often severely degraded by the increase in the number of objectives [7]-[9]. This is because almost all individuals become non-dominated with each other in early generations when EMO algorithms are applied to many-objective problems. In such a situation, Pareto dominance relation cannot generate strong selection pressure toward the Pareto front, which leads to severe deterioration of the convergence property of Pareto dominance-based EMO algorithms. Various approaches have been proposed for improving the convergence property of Pareto dominance-based EMO algorithms on many-objective problems (e.g., [10]-[14]). However, the improvement in the convergence property often causes the decrease in the diversity of obtained non-dominated solutions [15], [16].

Recently it has also been demonstrated that many-objective problems are not always difficult for Pareto dominance-based EMO algorithms when the objectives are highly correlated or dependent [13], [14], [17]-[19]. If most objectives of a many-objective problem are highly correlated, it is not likely that its Pareto front spreads over a wide range of the high-dimensional objective space. If almost all objectives are dependent on a few objectives, the dimensionality of the Pareto front may be much smaller than the number of objectives. As a result, the search for Pareto optimal solutions is not so difficult even for many-objective problems if the objectives are correlated or dependent.

In this paper, we examine the performance of four well-known EMO algorithms (NSGA-II [5], SPEA2 [6], MOEA/D [20], SMS-EMOA [21]) on many-objective test problems with correlated objectives. In our former study [18], we used many-objective knapsack problems with highly correlated objectives. As an extension to [18], we examine correlated many-objective knapsack problems with different correlation strength among objectives. That is, we examine the relation between the performance of each EMO algorithm on many-objective test problems and the correlation strength among the objectives.

This paper is organized as follows. In Section II, we explain our test problems with different correlation strength. In Section III, we visually examine the behavior of each EMO algorithm by monitoring the population at each generation in a two-dimensional subspace of the high-dimensional objective space. In Section IV, we examine the search ability of each EMO algorithm using performance measures such as hypervolume. Finally we conclude this paper in Section V.

II. TEST PROBLEMS

As in our former study [18], we generate many-objective test problems from the two-objective 500-item knapsack problem of Zitzler & Thiele [22], which is written as

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))\,, \tag{1}$$

$$\text{subject to } \sum_{j=1}^{500} w_{ij} x_j \le c_i\,,\ \ i = 1, 2, \tag{2}$$

$$x_j = 0 \text{ or } 1,\ \ j = 1, 2, ..., 500, \tag{3}$$

$$\text{where} \quad f_i(\mathbf{x}) = \sum_{j=1}^{500} p_{ij} x_j\,, \quad i = 1, 2. \tag{4}$$

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In this formulation, $\mathbf{x}$ is a 500-dimensional binary vector, p_{ij} is the profit of item j according to knapsack i, w_{ij} is the weight of item j according to knapsack i, and c_i is the capacity of knapsack i (i = 1, 2 and j = 1, 2, ..., 500). This two-objective 500-item knapsack problem is referred to as the 2-500 problem in this paper. All of our test problems are 500-item knapsack problems with the two constraint conditions in (2). Thus all of our test problems have exactly the same feasible solution set.

By randomly specifying the profit p_{ij} of each item j for each objective i as an integer in the interval [10, 100], we generate other eight objectives as follows:

$$f_i(\mathbf{x}) = \sum_{j=1}^{500} p_{ij} x_j \,, \quad i = 3, 4, ..., 10. \tag{5}$$

Using the two objectives in (1) and the randomly generated eight objectives in (5), we define correlated ten objectives as

$$g_i(\mathbf{x}) = f_i(\mathbf{x}) \,, \quad i = 1, 2 \,, \tag{6}$$

$$g_i(\mathbf{x}) = \alpha \cdot f_1(\mathbf{x}) + (1-\alpha) \cdot f_i(\mathbf{x}) \,, \quad i = 3, 5, 7, 9, \tag{7}$$

$$g_i(\mathbf{x}) = \alpha \cdot f_2(\mathbf{x}) + (1-\alpha) \cdot f_i(\mathbf{x}) \,, \quad i = 4, 6, 8, 10, \tag{8}$$

where α is a real number in the unit interval [0, 1]. The value of α can be viewed as the correlation strength. Let us consider two extreme cases: 0 and 1. On the one hand, when $\alpha = 0$, $g_i(\mathbf{x})$ is the same as the randomly generated objective $f_i(\mathbf{x})$. Thus the correlation among $\{g_1(\mathbf{x}), ..., g_{10}(\mathbf{x})\}$ is minimum. On the other hand, when $\alpha = 1$, $g_i(\mathbf{x})$ is the same as $f_1(\mathbf{x})$ or $f_2(\mathbf{x})$. Thus the correlation among $\{g_1(\mathbf{x}), ..., g_{10}(\mathbf{x})\}$ is maximum. In our former study [18], we examined many-objective problems with highly correlated objectives corresponding to $\alpha = 0.9$. In this paper, we examine various specifications of α in [0, 1].

In this paper, we use the following test problems:

$$\begin{aligned} &\text{4-500 problem: } (g_1(\mathbf{x}), g_2(\mathbf{x}), ..., g_4(\mathbf{x}))\,, \\ &\text{6-500 problem: } (g_1(\mathbf{x}), g_2(\mathbf{x}), ..., g_6(\mathbf{x}))\,, \\ &\text{8-500 problem: } (g_1(\mathbf{x}), g_2(\mathbf{x}), ..., g_8(\mathbf{x}))\,, \\ &\text{10-500 problem: } (g_1(\mathbf{x}), g_2(\mathbf{x}), ..., g_{10}(\mathbf{x}))\,. \end{aligned}$$

Each problem is denoted as the k-500-(α) problem where k is the number of objectives. When the value of α is clear or irrelevant, each problem is also denoted as the k-500 problem.

All test problems have the same constraint conditions in (2) of the 2-500 problem. Thus, in our test problems, we always use the greedy repair method in [22] for the 2-500 problem independent of the number of objectives and the value of α.

III. Visual Examination of Algorithm Behavior

We apply NSGA-II, SPEA2 and MOEA/D to our k-500-(α) test problems where k = 4, 6, 8, 10 and α = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0. However, SMS-EMOA is applied only to the 4-500 and 6-500 problems due to its heavy computation load.

A. *Parameter Specifications*

We use the following parameter specifications:

Coding: Binary string of length 500,
Termination condition: 400,000 solution evaluations,
Crossover probability: 0.8 (Uniform crossover),
Mutation probability: 1/500 (Bit-flip mutation),
Population size in NSGA-II, SPEA2 and SMS-EMOA: 100,
Population size in MOEA/D: 100 (2-500), 120 (4-500),
126 (6-500), 120 (8-500), 220 (10-500),
Scalarizing function in MOEA/D:
Weighted Tchebycheff (2-500, 4-500),
Weighted Sum (6-500, 8-500, 10-500),
Neighborhood size in MOEA/D: 10 solutions.

The above choice of a scalarizing function in MOEA/D is based on our former study [23]. We also examined the penalty-based boundary intersection approach, which did not work well on our knapsack problems. In MOEA/D, the reference point is specified for each objective by 1.1 × (Maximum objective value in each population). In SMS-EMOA, the origin of the objective space is used as the reference point. MOEA/D uses a different population size for each test problem due to the combinatorial nature of its weight vector specifications (for details, see [20]). For fair comparison, we always choose 100 solutions randomly from each population when MOEA/D is compared with the other EMO algorithms. This sampling of 100 solutions is iterated 10 times to calculate the average value.

B. *Visual Examination of Each EMO Algorithm*

Since it is not easy to visually examine the behavior of each EMO algorithm in a high-dimensional objective space, we use its two-dimensional subspace with $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$ (i.e., with $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$). That is, we examine the behavior of each EMO algorithm by projecting individuals in each generation onto the two-dimensional subspace. In Fig. 1, we show experimental results of a single run of NSGA-II on the 2-500 problem. We also show its experimental results on the 4-500-(0.8) and 10-500-(0.8) problems in Fig. 2 and Fig. 3, respectively. The value 0.8 of α means strong correlation among the objectives.

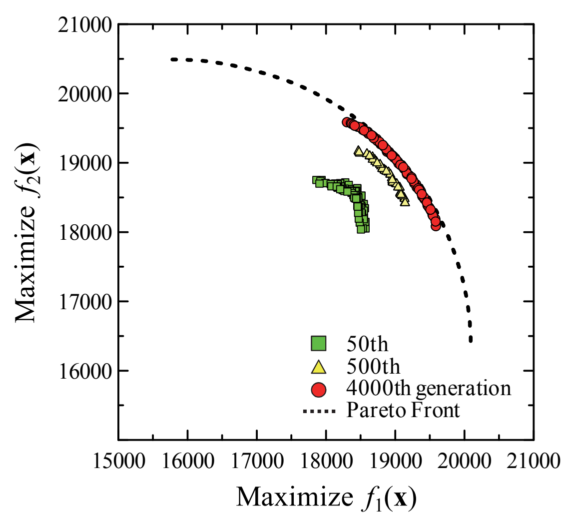

Figure 1. Single run of NSGA-II on the 2-500 problem.

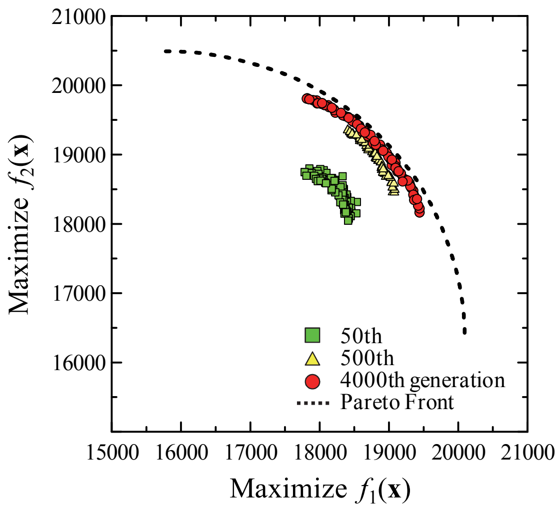

Figure 2. Single run of NSGA-II on the 4-500-(0.8) problem.

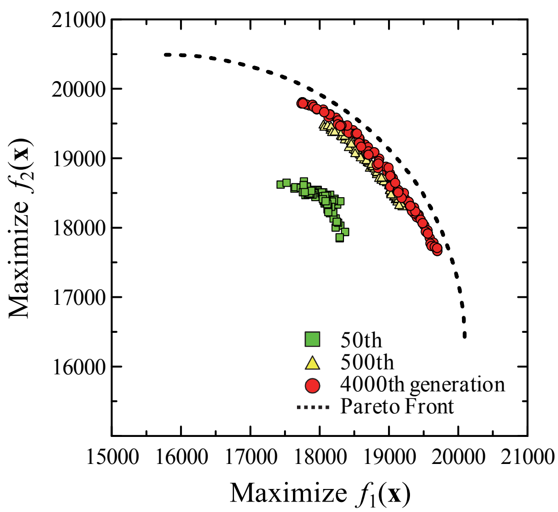

Figure 3. Single run of NSGA-II on the 10-500-(0.8) problem.

We can see from Figs. 1-3 that the behavior of NSGA-II is not so different among its applications to the three test problems with two, four and ten objectives. This is because the added objectives (i.e., $g_3(\mathbf{x})$, ..., $g_{10}(\mathbf{x})$) are strongly correlated to $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ (since $\alpha = 0.8$ in (7) and (8)). We can also see from Figs. 1-3 that the addition of the strongly correlated objectives to the original 2-500 problem increases the diversity of individuals while slightly degrading their convergence to the Pareto front of the original 2-500 problem.

In order to demonstrate the effect of correlation among the objectives on the behavior of NSGA-II, we show experimental results of its single run on the 10-500-(0.4) and 10-500-(0.0) problems in Fig. 4 and Fig. 5, respectively. From Figs. 3-5 with different values of α, we can see that the decrease in the correlation strength increases the diversity of individuals while degrading their convergence toward the Pareto front of the original 2-500 problem. Figs. 1-5 also show the difficulty to search for a solution set that approximates the entire Pareto front with a wide range of objective values. We obtain similar observations from SPEA2 while its results are not shown.

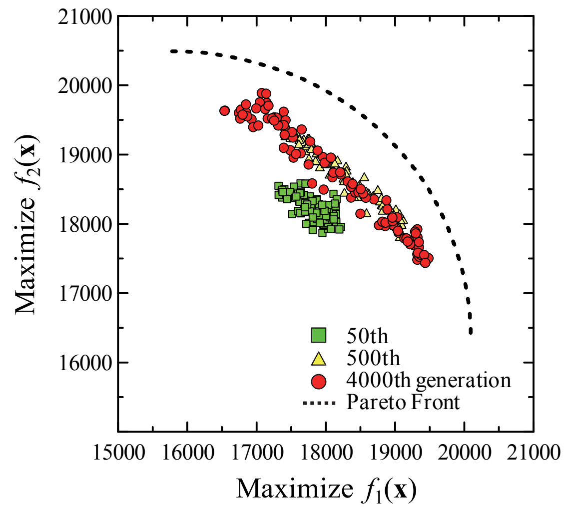

Figure 4. Single run of NSGA-II on the 10-500-(0.4) problem.

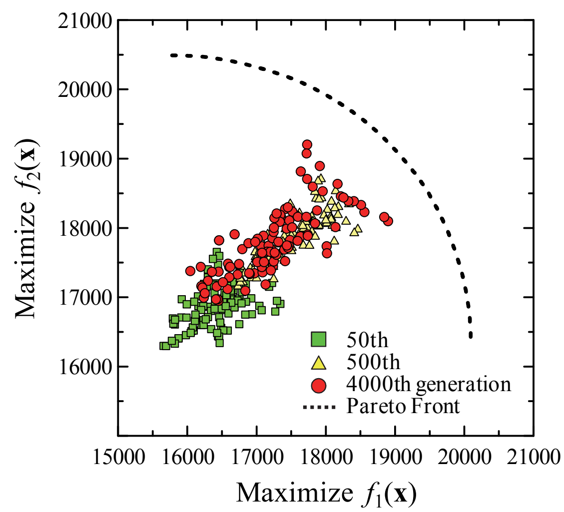

Figure 5. Single run of NSGA-II on the 10-500-(0.0) problem.

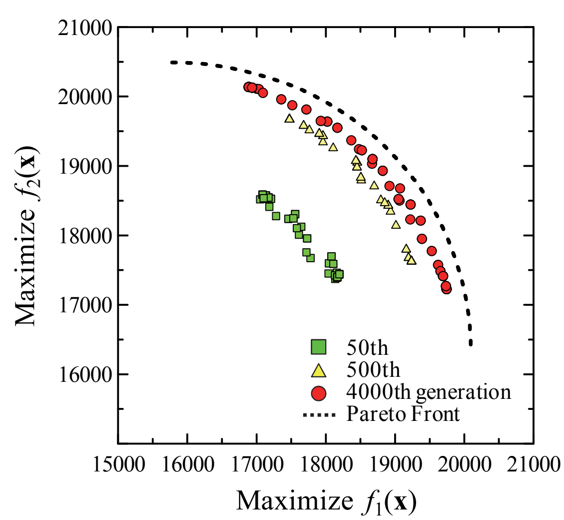

Figure 6. Single run of MOEA/D on the 4-500-(0.8) problem.

Behavior of MOEA/D and SMS-EMOA is different from NSGA-II and SPEA2. For example, we show experimental results of their single run on the 4-500-(0.8) problem in Fig. 6 and Fig. 7, respectively. In Fig. 6, we can see that the population of MOEA/D has large diversity along the Pareto front while its convergence toward the Pareto front is not so good. Experimental results in Fig. 7 by SMS-EMOA show very good converge while the diversity is small.

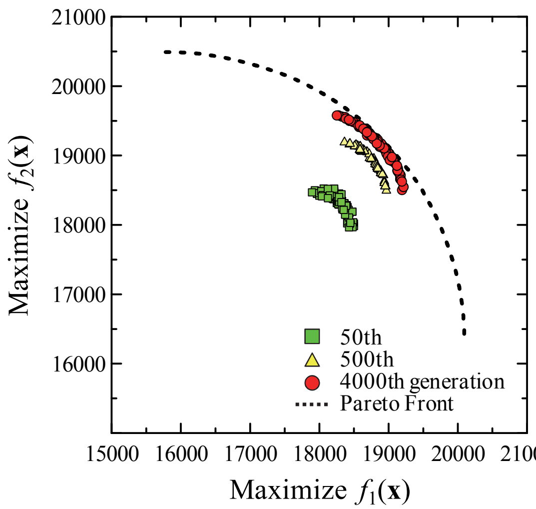

Figure 7. Single run of SMS-EMOA on the 4-500-(0.8) problem.

IV. Examination Using Performance Measures

In this section, we examine the behavior of each EMO algorithm on our test problems with different correlation strength using some performance measures.

A. Convergence Property around the Center Region

First we examine the behavior of each EMO algorithm with respect to its convergence property toward the center region of the Pareto front of each test problem. We evaluate this property by the following very simple measure:

$$\mathrm{MaxSum}(\Psi) = \max_{\mathbf{x}\in\Psi} \sum_{i=1}^{k} f_i(\mathbf{x}), \tag{9}$$

where Ψ denotes the current population, and k is the number of objectives in each test problem (k = 2, 4, 6, 8, 10).

For comparison, we also apply a single-objective genetic algorithm (SOGA) to the following single-objective problem:

$$\text{Maximize } f(\mathbf{x}) = \sum_{i=1}^{k} f_i(\mathbf{x}). \tag{10}$$

Our SOGA is basically the same as NSGA-II except for its fitness evaluation mechanism. The weighted sum $f(\mathbf{x})$ in (10) is used as a fitness function in SOGA while the Pareto ranking and the crowding distance are used in NSGA-II. All the other parts are the same between NSGA-II and SOGA including their parameter specifications.

Average experimental results over 100 runs on the 4-500-(0.0), 4-500-(0.4) and 4-500-(0.8) problems are shown in Figs. 8-10, respectively. In each figure, MaxSum in (9) is normalized so that its average value is 100 at the initial population with randomly generated 100 individuals over 100 runs. In each figure, the number of generations of MOEA/D with a different population size is converted into the equivalent one with the population size 100. From Figs. 8-10, we can see that the increase in the correlation strength among the objectives improves the convergence property of each EMO algorithm.

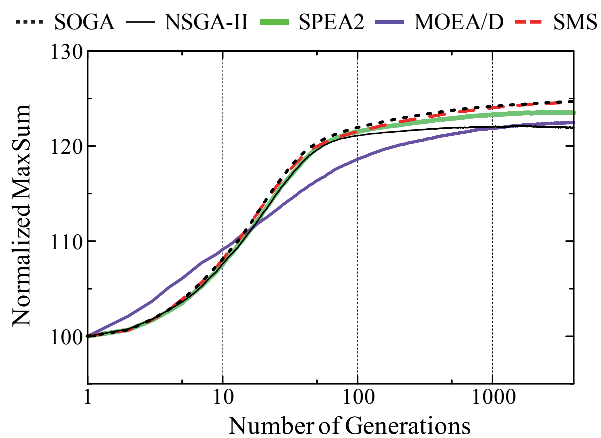

Figure 8. MaxSum on the 4-500-(0.0) problem.

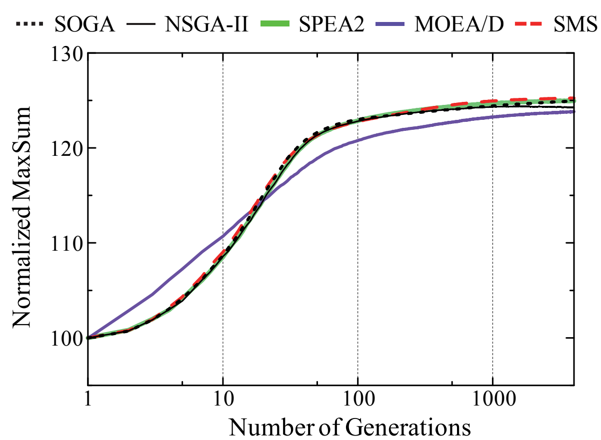

Figure 9. MaxSum on the 4-500-(0.4) problem.

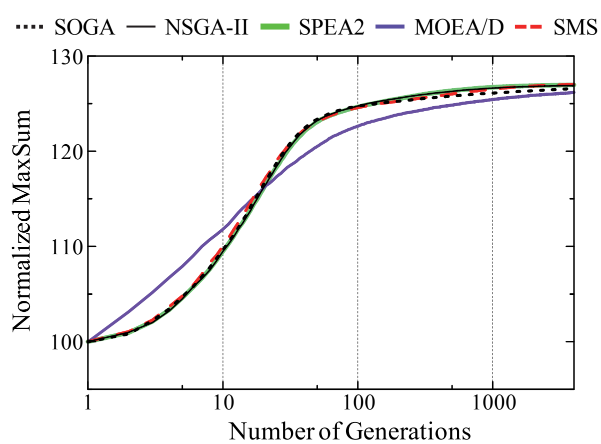

Figure 10. MaxSum on the 4-500-(0.8) problem.

B. Search Ability around the Edges of the Pareto Front

Next we examine the behavior of each EMO algorithm with respect to its search ability around the edges of the Pareto front of each test problem. We evaluate this property using the following very simple measure:

$$\mathrm{SumMax}(\Psi) = \sum_{i=1}^{k} \max_{\mathbf{x} \in \Psi} f_i(\mathbf{x}) \,. \tag{11}$$

This measure can be viewed as evaluating the search ability of each EMO algorithm to find a near optimal solution for each objective. For comparison, we also use SOGA to optimize each objective. Our SOGA with the population size $100/k$ is applied to the maximization problem of $f_i(\mathbf{x})$ for i = 1, 2, ..., k. That is, SOGA is independently used k times in order to maximize the k objectives. The population size in each run is specified as $100/k$ so that SOGA is compared with the EMO algorithms under the same computation load. The SumMax measure in (11) is calculated from those k runs at each generation by combining the k populations into a single population Ψ.

Average experimental results over 100 runs of each EMO algorithm are shown in Figs. 11-13. SumMax in each figure is normalized as in the previous subsection.

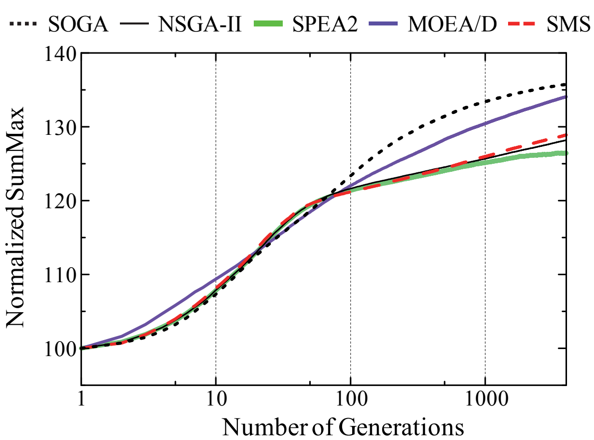

Figure 11. SumMax on the 4-500-(0.0) problem.

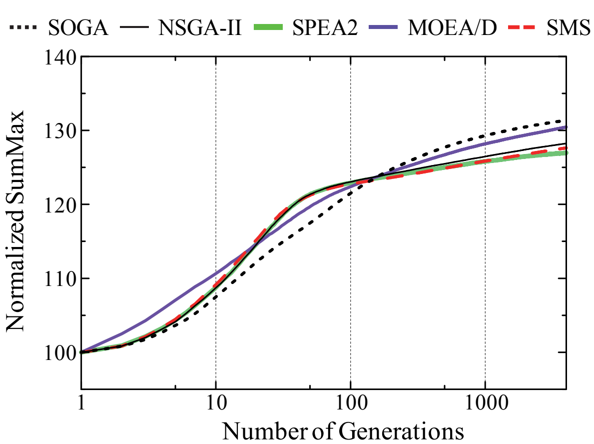

Figure 12. SumMax on the 4-500-(0.4) problem.

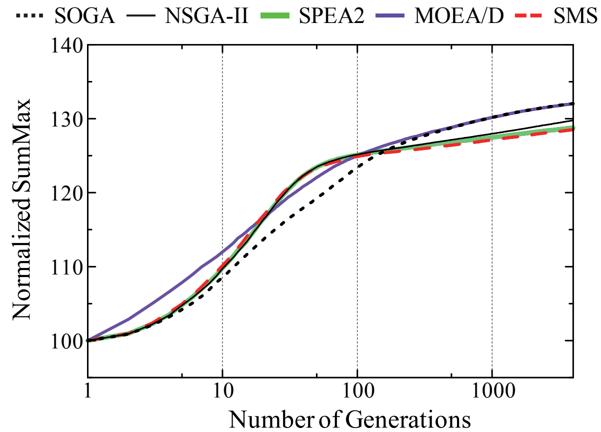

Figure 13. SumMax on the 4-500-(0.8) problem.

From Figs. 11-13, we can see that it is not easy for the EMO algorithms (except for MOEA/D) to search around the edges of the Pareto front even when the objectives are strongly correlated in Fig. 13. This difficulty seems to be more serious in the case of randomly generated objectives in Fig. 11. Among the examined four EMO algorithms, MOEA/D seems to outperform the other EMO algorithms in Figs. 11-13 with respect to the convergence property around the edges of the Pareto front. It should be noted that MOEA/D seems to be outperformed by the other EMO algorithms in Figs. 8-10 with respect to the convergence toward the center region of the Pareto front.

C. Performance Evaluation using Hypervolume Measure

The hypervolume of the final population is calculated in the k-dimensional objective space using its origin (0, 0, ..., 0) as the reference point. The average value of the hypervolume measure is calculated over 100 runs of each EMO algorithm on each test problem. Only in the case of SMS-EMOA on the 6-500-(α) problems, the hypervolume measure is calculated from its single run on each test problem. SMS-EMOA is not applied to the 8-500-(α) and 10-500-(α) problems due to its heavy computation load. Experimental results are summarized in Table I where the average hypervolume is normalized so that the average result of NSGA-II is 100 for each test problem. In Table I, SOGA(A) means SOGA in Subsection IV.A with the weighted sum fitness function while SOGA(B) means SOGA in Subsection IV.B (i.e., k runs of SOGA with the population size $100/k$ for the maximization of each objective).

From Table I, we can see that MOEA/D works well on almost all test problems (except for the 10-objective problems with strong correlation: 10-500-(0.8) and 10-500-(1.0)). When all objectives are randomly generated (i.e., k-500-(0.0) for k=4, 6, 8, 10), much better results are obtained from MOEA/D than NSGA-II and SPEA2. In this case, SMS-EMOA and SOGA(B) also outperform NSGA-II and SPEA2. However, the difference in the performance between NSGA-II and MOEA/D becomes smaller by increasing the correlation strength among the objectives. Actually the best results were obtained from NSGA-II for the 10-objective test problems with strong correlation (i.e., 10-500-(0.8) and 10-500-(1.0)).

TABLE I. NORMALIZED AVERAGE HYPERVOLUME

Test Problem	NSGA-II	SPEA2	MOEA/D	SMS-EMOA	SOGA (A)	SOGA (B)
2-500	100.0	99.2	105.3	99.2	94.8	102.6
4-500-(0.0)	100.0	96.8	116.3	104.3	86.6	104.6
4-500-(0.2)	100.0	97.2	110.6	101.9	87.9	103.8
4-500-(0.4)	100.0	97.1	105.7	99.2	88.4	101.1
4-500-(0.6)	100.0	96.9	103.4	96.9	88.5	99.2
4-500-(0.8)	100.0	97.4	105.2	96.8	88.8	98.6
4-500-(1.0)	100.0	97.7	106.4	96.7	89.3	99.4
6-500-(0.0)	100.0	91.2	128.4	107.9	83.7	106.1
6-500-(0.2)	100.0	92.6	117.3	103.5	84.6	104.0
6-500-(0.4)	100.0	94.2	108.1	100.5	84.7	99.7
6-500-(0.6)	100.0	94.8	103.3	95.6	83.8	95.4
6-500-(0.8)	100.0	95.9	102.9	95.8	83.9	93.4
6-500-(1.0)	100.0	96.7	104.8	96.6	84.8	94.4
8-500-(0.0)	100.0	89.6	137.4	-	85.5	105.6
8-500-(0.2)	100.0	91.3	122.3	-	84.2	102.5
8-500-(0.4)	100.0	93.0	110.0	-	82.9	97.1
8-500-(0.6)	100.0	93.5	102.7	-	80.4	90.6
8-500-(0.8)	100.0	94.5	100.1	-	79.2	87.3
8-500-(1.0)	100.0	96.0	100.5	-	80.8	88.5
10-500-(0.0)	100.0	87.0	149.6	-	85.6	103.2
10-500-(0.2)	100.0	88.9	129.8	-	83.7	99.9
10-500-(0.4)	100.0	91.7	114.8	-	81.9	95.0
10-500-(0.6)	100.0	92.8	103.9	-	78.4	87.2
10-500-(0.8)	100.0	93.8	98.9	-	75.4	81.4
10-500-(1.0)	100.0	95.4	98.4	-	77.3	82.1

V. CONCLUSIONS

In this paper, we examined the performance of well-known EMO algorithms (i.e., NSGA-II, SPEA2, MOEA/D and SMS-EMOA) on many-objective knapsack problems with different correlation strength among their objectives. Such a many-objective test problem was generated by adding new objectives to a two-objective knapsack problem. When the newly added objectives were strongly correlated to the two objectives of the original two-objective problem, the increase in the number of objectives did not severely degrade the search ability of NSGA-II and SPEA2. However, it degraded the search ability of MOEA/D and increased the computation time of SMS-EMOA. The decrease in the correlation strength among the objectives degraded the search ability of all the examined EMO algorithms especially with respect to the convergence property toward the center region of the Pareto front. With respect to the hypervolume measure, the decrease in the correlation strength degraded more severely the search ability of NSGA-II and SPEA2 than that of MOEA/D and SMS-EMOA.

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