Does correlation help in MOEAs

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ABSTRACT

This research investigates the impact of objective correlations on Multi-Objective Evolutionary Algorithm (MOEA) performance, aiming to guide MOEA selection for problems with varying correlations. Testing different MOEAs on diverse objective correlations, the study finds that decomposition-based algorithms excel in anti-correlated spaces but struggle with complex Pareto fronts. Domination-based algorithms demonstrate consistent performance, making them a reliable choice for unknown Pareto fronts. Indicator-based algorithms, however, show limited effectiveness. The study highlights the need for further research in high-dimensional objective spaces and suggests exploring the generation of custom problems for more controlled testing.

1 INTRODUCTION

A Multi-Objective Evolutionary Algorithm (MOEA) [12] [10] is a computational method used for solving optimisation problems that involve multiple, often conflicting objectives. MOEAs leverage mechanisms inspired by natural evolution, such as selection, crossover, and mutation, to evolve a population of solutions towards an optimal set, typically focusing on achieving a balance among the different objectives.

This study addresses the uncertainty in selecting the most effective MOEA for problems with varying correlations among objectives, aiming to establish guidelines for MOEA selection. In multi-objective optimisation, the aim is to collect a solution or set thereof which provide some trade-off between objectives. We are exploring if different types of correlations - from conflicting (anti-correlated) to harmonious (fully correlated) - significantly influence the performance of MOEAs. There's a lack of clear guidelines on which type of MOEA (decomposition-based [24] [5], domination-based [10] [6], or indicator-based [8]) performs best under specific correlation scenarios. This project aims to fill this gap by systematically testing different MOEAs on problems with diverse objective correlations, thereby providing empirical insights into the optimal MOEA selection based on the nature of objective correlations.

Investigating how objective function correlations affect MOEA performance is vital for efficient, effective multi-objective optimisation. Diverse correlations pose distinct challenges; conflicting objectives demand trade-off balance, while harmonious ones need simultaneous optimisation. We can derive the correlation of objectives given an objective space by using sampling techniques to generate data-points.

Recognising the correlation among objectives in multi-objective optimisation can expedite the problem-solving process. By identifying the most suitable MOEA for a given correlation scenario, this research can streamline optimisation tasks, leading to faster, more efficient solutions. This has implications across diverse sectors, from environmental management to product design, where timely and optimal decisions are crucial. Knowing the correlations beforehand allows for a targeted algorithmic approach, reducing computational time and resource usage. Such advancements can lead to quicker innovation cycles, better resource management, and overall, a more efficient approach to tackling complex problems with multiple objectives.

2 BACKGROUND

Extensive research has been conducted by Ishibuchi, Hisao *et al* on the correlation in MOP objective spaces, primarily focusing on problems featuring correlation rather than anti-correlation. Their findings serve as important benchmarks for the studies presented in this paper, especially regarding the correlation dynamics in MOPs. The following works are discussed:

Effects of the existence of highly correlated objectives on the behavior of MOEA/D [17]: A critical examination was undertaken of MOEAs: NSGA-II [6], SPEA2 [26], and MOEA/D [24]. The study was centred on a six-objective knapsack problem, featuring two trios of highly correlated objectives. Through an analysis based on the average hypervolume [27] measure over 100 runs, it was observed that MOEA/D excelled in general performance. However, its efficacy diminished notably with the introduction of correlated objectives, a trend not observed in NSGA-II and SPEA2. The study also delved into the dynamics of two-dimensional objective space projections and the influence of correlated objectives on algorithm performance. It underscored a marked decline in MOEA/D's efficiency in the presence of high objective correlation, underscoring the potential need for tailored adjustments in weight vectors relative to objective correlation. These findings illuminate varying algorithm behaviours in the realm of multi-objective optimisation, particularly highlighting the distinct challenges faced by MOEA/D.

Behaviour of EMO algorithms on many-objective optimisation problems with correlated objectives [16]: Further investigation into the performance of NSGA-II, SPEA2, and MOEA/D was conducted in the context of many-objective optimisation problems. Unlike MOEA/D, NSGA-II and SPEA2, grounded in Pareto dominance, demonstrated stable efficiency even with an increased count of objectives, more so in cases of high objective correlation. This reinforces the previous findings from the previously discussed paper [17]. The study revealed that integrating highly correlated objectives occasionally bolsters performance in bi-objective problems for Pareto dominance-based algorithms. This observation is pivotal in understanding the impact of objective correlation on the effectiveness of MOEAs, highlighting the essentiality of incorporating objective interrelations in the design and evaluation of these algorithms.

Behaviour of Multi-objective Evolutionary Algorithms on Many-Objective Knapsack Problems [15]: This work provides a comprehensive analysis of three categories of MOEAs algorithms: Pareto dominance-based, scalarising function-based, and hypervolume-based. Utilising NSGA-II, MOEA/D, SMS-EMOA [9], and HypE [1] algorithms, the study navigates through knapsack problems with 2 to 10 objectives, scrutinising algorithm performance across a spectrum of objectives, inclusive of randomly generated, correlated, and dependent types. Key findings indicate that

NSGA-II typically falls behind in performance except in scenarios with highly correlated objectives. Conversely, MOEA/D's performance exhibits significant fluctuation contingent on the employed scalarising functions. This reveals how objective correlation impacts MOEAs' effectiveness, highlighting the necessity of considering objective relationships in algorithmic design and evaluation, thus fortifying the findings of the aforementioned papers.

EMO algorithms on correlated many-objective problems with different correlation strength [14]: This report extends the analysis of the impact of objective correlation on the performance of MOEAs in many-objective contexts. It underscores the challenge Pareto dominance-based MOEAs face when dealing with many-objective problems, and how their performance can be enhanced when the objectives exhibit high correlation. The methodology involved transforming a two-objective knapsack problem into many-objective variants by modulating the correlation strength among objectives. MOEAs, such as NSGA-II and MOEA/D, were then assessed on these varied problems. The investigation focused on elucidating the interplay between the strength of objective correlation and algorithmic performance. The findings indicate that the degree of correlation profoundly influences the search efficiency and convergence behaviour of MOEAs, hinting at reduced performance under negative correlation scenarios, something that will be explored in greater depth during this study. This insight is useful for understanding and optimising MOEA performance in relation to objective correlation dynamics.

Further Research: Although non-specific to algorithm selection for correlation in objective spaces, Verel, Sébastien *et al* [23] have conducted research into correlation between objectives to determine hyperparameters for input variables. They explore efficient set properties (high quality input variables) in ρMNK landscapes by highlighting the impact of objective correlation. It shows that varying correlations can significantly change the efficient set's size and structure, affecting the MOEA's performance. By understanding these correlations, MOEA designers can better tailor their algorithms for specific problem characteristics, potentially improving solution quality and efficiency, especially in high-dimensional or complex objective spaces. This knowledge is crucial for MOEAs to navigate and optimise in multi-objective environments effectively. This information can be used in conjunction with Ishibuchi, Hisao *et al* work, and the research undertaken in this report for highly comprehensive algorithm choice and design. Additionally, the paper *Solution analysis in multi-objective optimisation* [4], provides a method for analysing trade-offs by using simple rank-ordering of objectives and examining their correlation with problem variables. This approach helps to understand how different objectives interact and influence each other in the objective space. By identifying these correlations, you can predict how changes in one objective might affect others. This insight is crucial for selecting optimisation algorithms that are best suited for specific areas of the objective space, directly aiding in your project's goal of algorithm selection based on objective space analysis. This can be useful for adjusting results if correlation changes for a problem during planning, preventing complete re-evaluation of problem.

3 EXPERIMENTAL DESIGN

The objective of this research is to determine if correlation helps in MOEAs, specifically to provide some guidelines on algorithm selection for a given correlation. The research conducted by Ishibuchi, Hisao *et al* provide comprehensive guidelines for positive correlation, but do not distinctly selection for the type of MOEA, nor cover anti-correlation. These areas will be focused upon. To provide a brief summary of the three categories of MOEA, and the algorithms under consideration:

Domination-based: Prioritise solutions that dominate others in multi-objective optimisation, focusing on non-dominated solutions to find an optimal trade-off between objectives. NSGA-II and SPEA2 will be used. **Decomposition-based:** Splits multi-objective optimisation tasks into simpler single-objective problems, solving them collectively to balance competing goals (spread and convergence) efficiently. MOEA/D and NSGA-III [5] will be used. **Indicator-based:** Use quantitative metrics to guide the search towards optimal trade-offs in MOPs. SMS-EMOA will be used.
All MOEAs and problem suites will be implemented using Pym∞ [3]. We will use the ZDT [25] and DTLZ [7] problem suites, and extract problems from these sets. All ZDT problem objectives are two dimensional, whereas the DTLZ suite allow the user to specify the number of objectives. We explore 2D and 3D instances of the DTLZ suite, due to the limited scope of the project. The correlation for each problem will be determined for each problem, allowing us to select a set of problems that possess a range of correlations *a posteriori*.

Determining problem correlation: The first decision to address is the measure of correlation used determine the correlation of a given problem. To decide, we consult Fritsche, Gian *et al* paper on "Capturing relationships in multi-objective optimisation" [11]. The paper finds that Kendall's [18], Pearson's [21], and Spearman's [22] correlations differ in effectiveness for multi-objective optimisation: Kendall's is specific (minimising false positives), Pearson's is sensitive (identifying true interactions), and Spearman's balances both. It concludes that no metric is universally superior. Pearson's strength lies in its sensitivity to actual correlations, making it a valuable tool for accurately understanding and characterising the relationships in an objective space, particularly when accurate detection of these relationships is crucial. This is ideal for our implementation, therefore will be the measure employed in our research. The next decision we must address is our methodology to represent the objective space. We must covert the objective space into some point-based representation, using a sampling technique. Two methods present themselves:

Grid search [20]: This method involves systematically sampling the solution space, incrementing each input variable by small steps. It should provide complete coverage of the objective space at a high computational cost.

Random search [2]: Rather than systematically searching, this method elects to generate random solutions a large number of times, to produce an approximation of the objective space.

Bergstra, James *et al* [2] found that random trials can outperform grid search. Their research showed that random search often yields comparable or superior results to grid search and manual search, but with less computational effort. Due to this, the random search method was chosen.

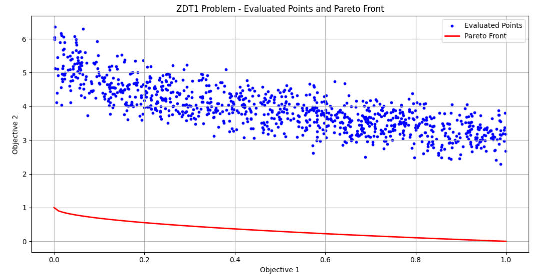


Figure 1: Example of random search point generation

For each problem, the correlation will be deduced by sampling the objective space 1000 times, as seen in 1. Since many of our problems under consideration allow for variation in the number of input variables, we will test a range of input values, to ensure consistency amongst results, before using the default input values (typically 30 input variables, with some problem-specific variation). This allows to to understand the correlation of any given problem from the test suites. Our studies will focus on problems with 2-3 objectives, due to the limited scope of our research, and lack of computational capacity.

Execution: The flow of our program is as follows:

Algorithm 1 Your Algorithm Name

1: **procedure** CORRELATION IN MOEAS(*parameters*)
2: Acquire problems, algorithms
3: Calculate problem correlations
4: **for** *problem* in *problems* **do**
5: **for** *algorithm* in *algorithms* **do**
6: **for** *iteration* = 1 to 10 **do**
7: Run algorithm on problem
8: Measure quality and store instance
9: **end for**
10: Record average quality and spread of runs
11: **end for**
12: Visually represent algorithms given problem
13: **end for**
14: **end procedure**

Each run of the algorithms will be seeded to ensure fairness across runs, and to ensure that the results of the program are replicable. This includes problems being given a static initial set of points, to ensure that the change in hypervolume are equivalent. In line 6, it is shown that we will not evaluate the success of our program based off of one run, rather, we will complete a number of runs and use statistical measures, such as average and range to give a better demonstration of algorithmic performance. This leads to the final decision in our experimental design.

Measures of quality Quality measures in multi-objective optimisation include convergence and spread. Convergence, typically gauged by the generational distance, is represented by a single numerical value denoting the average distance of solutions from the optimal Pareto front. A lower value signifies solutions are closer to the optimal front, reflecting better performance in achieving convergence. Spread is measured by a diversity metric. A higher value typically indicates a wider spread, implying greater diversity in the solution set, while a lower value points to solutions being more closely clustered, denoting less diversity. The hypervolume indicator provides a value that encompasses both convergence and spread, and is the best candidate for a measure of quality. It would be pertinent to consider average execution time of the algorithms under consideration, due to the impact this can have on real world problems.

4 RESULTS AND ANALYSIS

Firstly, we should cover the problems that we consider in our research. The following were selected due to their diversity in correlation, with the final problem being chosen to begin exploration of three dimensional objective spaces.

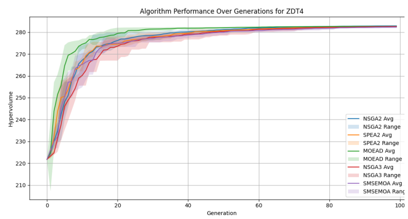
Problem	Correlation	Description
ZDT4	-0.04	Large number of local optima, challenging for algorithms to find the global Pareto-optimal front.
ZDT2	-0.24	Convex Pareto-optimal front, tests an algorithm's ability to maintain diversity.
DTLZ7	-0.52	Disconnected Pareto-optimal front, evaluates an algorithm's ability to converge and maintain diversity.
DTLZ3	-0.79	Large number of local optima, challenging for algorithms to find the global Pareto-optimal front.
DTLZ1	-0.95	Linear Pareto-optimal front, challenging an algorithm's ability to handle flat regions and non-uniformity.
DTLZ5 3D	obj1-2 0.1, obj2-3 -0.5, obj1-1 -0.5	Reduced-dimensionality Pareto-optimal front in a three-dimensional objective space.

Immediately noticeable, none of the problems we explored exhibit positive correlation values, with the exception of DTLZ5 3D, which is slightly correlated between objectives one and two, hence its selection. Although detrimental, since it becomes more difficult to corroborate these results with previously existing research [17] [16] [15], these anti (negative) correlations allow us to fill the gap left by Ishibuchi, Hisao *et al*, whom only studied positive correlations. These problems likely exhibit anti-correlation due to the inherent nature of MOPs, which often provide a trade-off between objectives. Since the majority of problems we examine in this research are two dimensional, this trade-off has to be enforced, otherwise there would be a single optimal solution, rather than a pareto front. We can see that as objective dimensionality increases, so does the rate of occurrence of positive correlation, suggesting that in higher dimensions, problems are more likely to exhibit a range of correlations. Despite this setback, we can proceed to examine algorithmic performance over the selected problems:

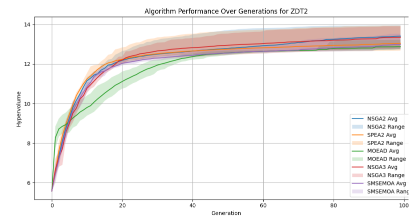
IMGS

Algorithm performance:

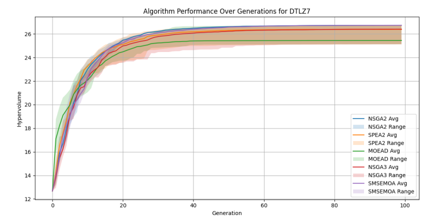
MOEA/D stands out for its rapid convergence across various problems, particularly excelling in small objective spaces like DTLZ1 and DTLZ3. This algorithm is notably effective in situations with low positive correlation, as seen in DTLZ5, but faces challenges in larger objective spaces with high anti-correlation, such as DTZ2 and DTLZ7. However, both of these problems exhibit unusual Pareto fronts, being convex and disconnected respectively, perhaps acting as the reason for this poorer performance. A key consideration is that, despite generally better performance, MOEA/D usually evaluated $\tilde{5}$ times slower than all other algorithms. NSGA3, on the other hand, consistently delivers good performance. It is not always the top performer in negatively correlated contexts and tends to be slightly less effective on disconnected Pareto fronts. This is likely due to the under-utilisation of the reference vectors used to ensure diversity, as many will not hit the Pareto front, resulting in invalid or distant solutions [19]. SMS-EMOA generally underperforms across the board, with a notable exception in DTLZ7,



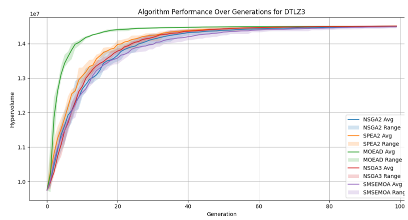
(a) ZDT4



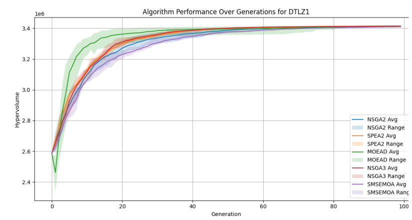
(b) ZDT2



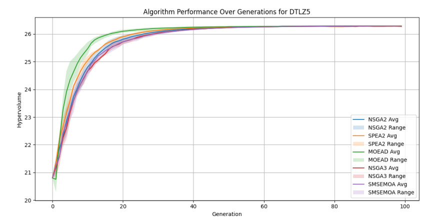
(c) DTLZ7



(d) DTLZ3



(e) DTLZ1



(f) Caption 6

which may be attributed to the disconnected nature of the Pareto front. This suggests indicator-based approaches are not suitable for anti-correlated objective, although we could not back up this with another algorithms performance due to the limitations of pymoo's algorithm selection. NSGA2's performance is mediocre, deteriorating as anti-correlation increases. However, in DTLZ7, it stands out, demonstrating less spread and more consistency. This is likely due to the unusual Pareto front, and is more a testament to the difficulty other approaches faced when solving such a problem. SPEA2 is a model of consistency, regularly ranking in the top three but never first. It is often significantly outperformed by MOEA/D, however, in cases when MOEA/D fails, its spread will often show the fastest potential convergence. However, we cannot attribute the success of an algorithm to correlation if it is proven to have better average performance than another algorithm, since SPEA2 is proven better than NSGA-II [13]. Overall, NSGA3 and SPEA2 are relatively strong competitors, except in scenarios like ZDT4, which involve a very large objective space with almost no correlation.

In the specific case of ZDT5, the 3D objective space, MOEA/D markedly outperforms the others, which exhibit similar levels of performance.

Ishibuchi, Hisao *et al* comparisons: In the analysis, MOEA/D faced challenges with higher correlations, particularly negative correlations, similar to the findings reported in their research [17]. NSGA3 and SPEA demonstrated similar behaviours in this context, performing consistently well but not exhibiting massive variation due to correlation.

Regarding the knapsack problem, NSGA2 showed ineffectiveness in highly anti-correlated scenarios. Similarly, MOEA/D also experienced significant variations, which could be attributed to the scalarising functions it employs, since it becomes harder to accurate scalarise a complex front, as suggested by Ishibuchi, Hisao *et al.*

5 DISCUSSION AND FUTURE WORK

The quality of decomposition-based algorithms was overall successful on anti-correlated objectives, however they became less successful when approached with a more complex Pareto fronts. Domination-based approach performed consistently, and our research aligned with Ishibuchi, Hisao *et al.* Indicator-based algorithms should have been explored in greater depth, but were limited by scope. Despite this, SMS-EMOA performed consistently badly. To categorise into guidelines for algorithmic selection: **Decomposition-based:** Best for anti-correlated objective spaces, provided the Pareto front doesn't exhibit any unusual behaviour. **Domination-based:** Performed consistently well. If the Pareto front form is unknown, this is likely the best approach due to its consistency and slightly lower convergence. **Indicator-based:** Unlikely to perform well, unless specifically tailored. It would be valuable to visualise spread and convergence as separate values, since this could refine the selection process.

Future works: This project opens numerous avenues for future research. A key limitation was the absence of problems with high objective dimensionality. Future studies could delve into this area, using parallelisation to manage higher dimensions more efficiently. Creating custom problems, as done by Ishibuchi, Hisao *et al.* [16], would grant more control over testing and enable continuous assessment of both positive and negative correlations. Another interesting aspect is comparing the correlation of points generated by an algorithm to the problem's correlation. This approach may be less effective for decomposition-based algorithms, but examining trends could be insightful for other algorithms, particularly in selecting the most suitable one. Investigating the correlation within a specific generation and its impact on the algorithm's progress is another potential research direction. Additionally, transforming a multi-objective problem (MOP) into a many-objective problem by dividing each original objective into two – exploitation and exploration – could reveal the compatibility or conflict between different elements of the objectives. Lastly, expanding this research to include Bayesian optimisation or Gaussian processes is a viable path, although assessing correlation in Gaussian processes presents its own challenges.

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6 APPENDICES

Larger versions of previous figures (UNMARKED)

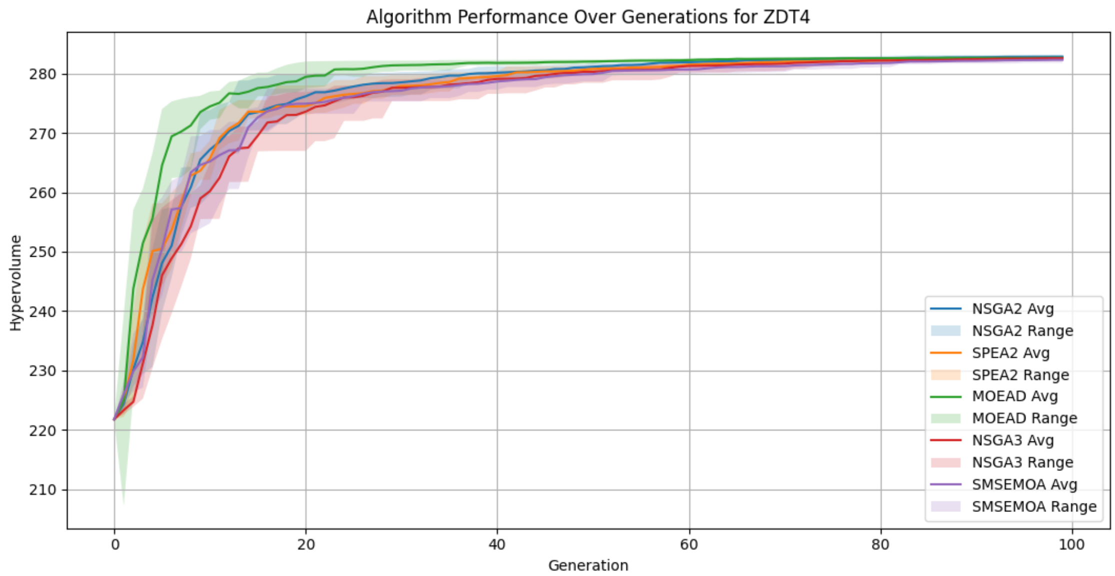

Figure 3: ZDT4

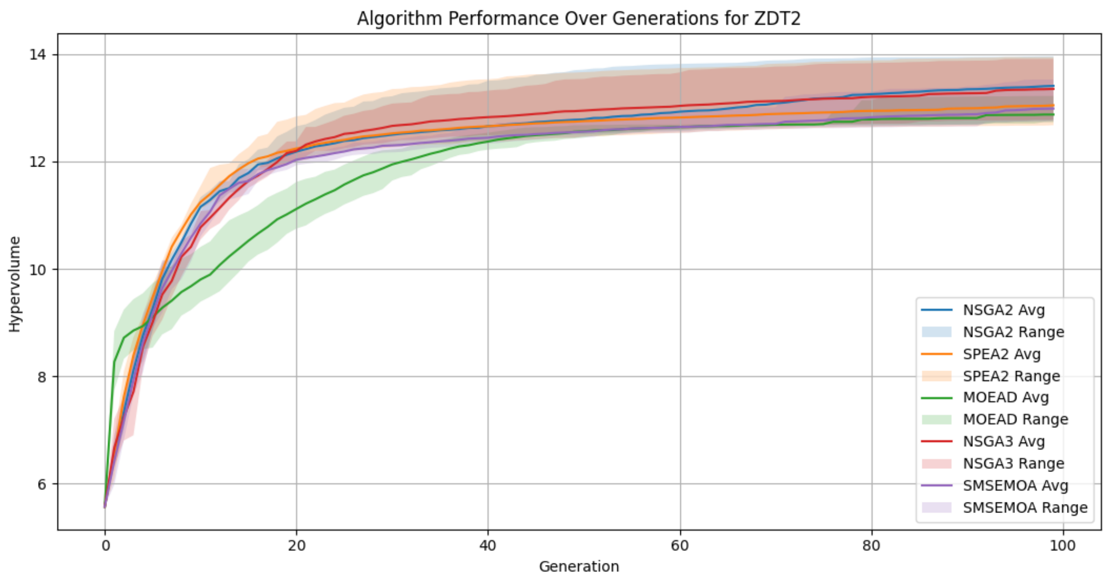

Figure 4: ZDT2

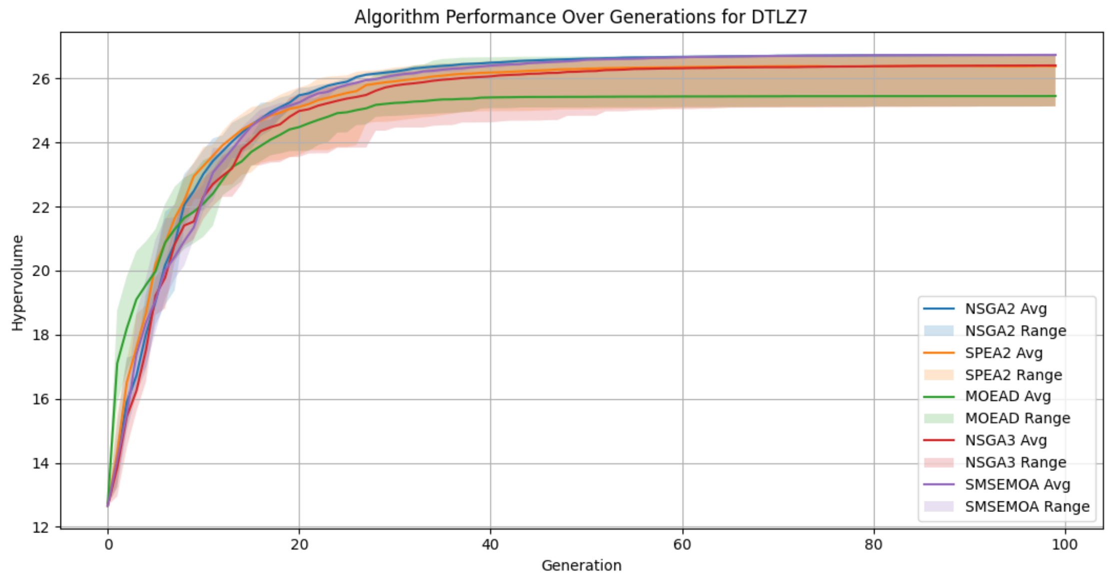

Figure 5: DTLZ7

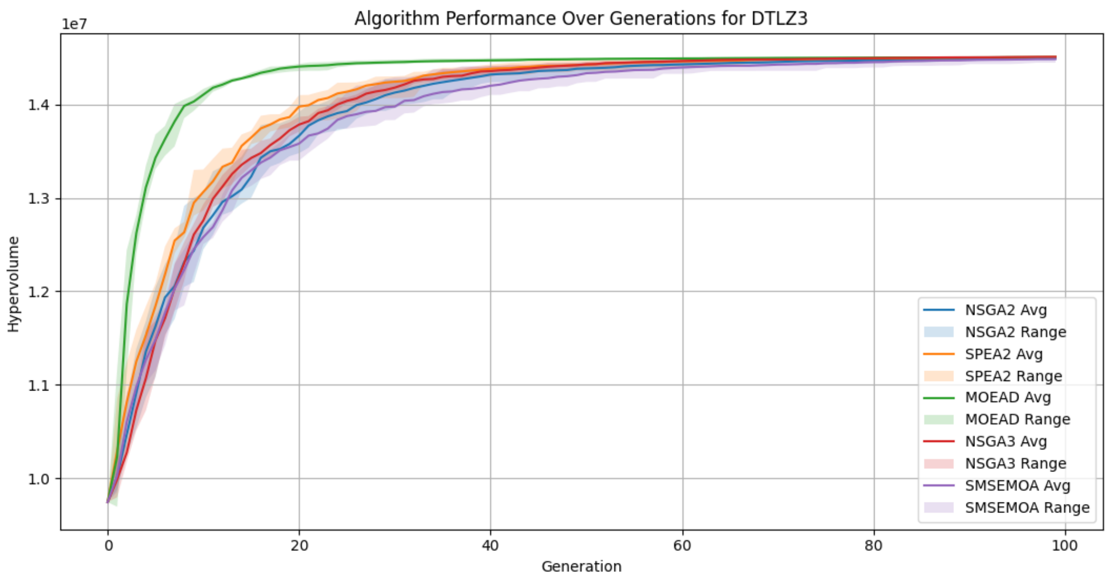

Figure 6: DTLZ3

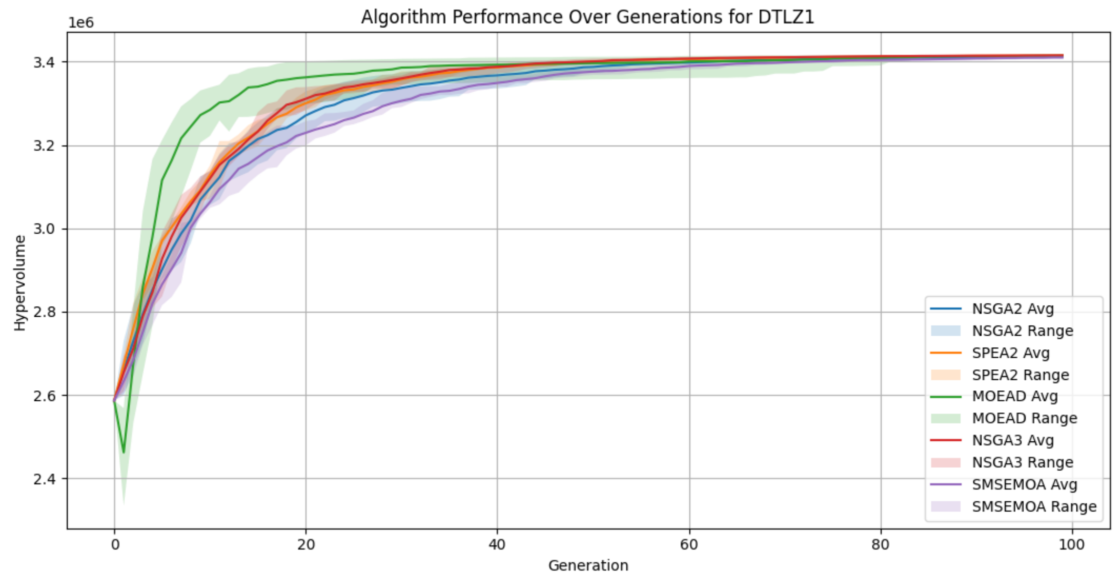

Figure 7: DTLZ1

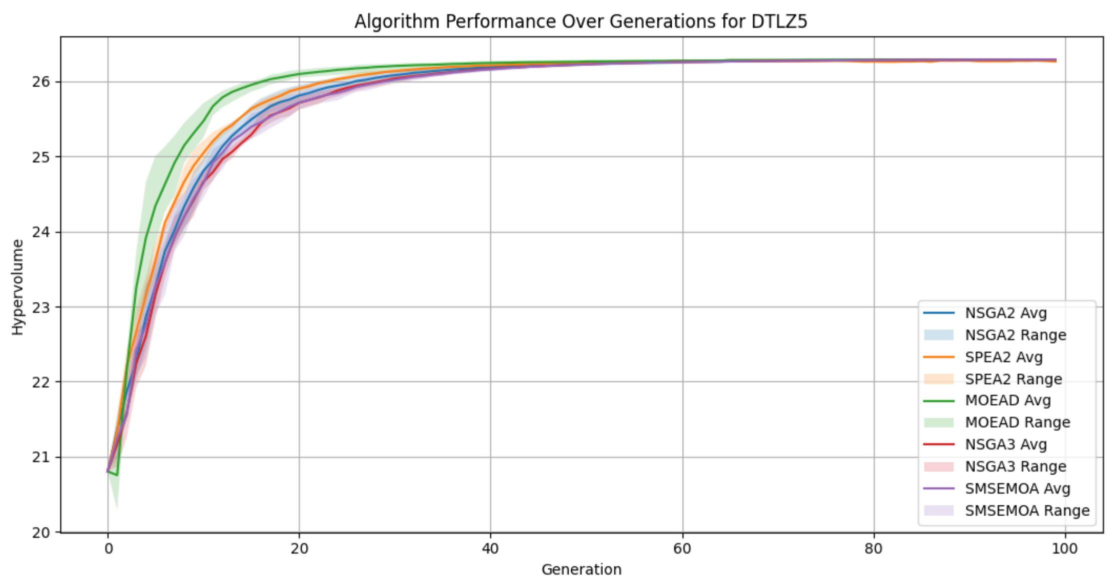

Figure 8: Another DTLZ5 3D, focus on positive objective