Technical Report
Completeness-aware Rule Learning from Knowledge Graphs

Thomas Pellissier Tanon[a], Daria Stepanova[a], Simon Razniewski[b], Paramita Mirza[a], and Gerhard Weikum[a]

[a]Max Planck Institute of Informatics
[b]Free University of Bozen-Bolzano
{tpelliss,dstepano,paramita,weikum}@mpi-inf.mpg.de,
razniewski@inf.unibz.it

Abstract. Knowledge graphs (KGs) are huge collections of primarily encyclopedic facts. They are widely used in entity recognition, structured search, question answering, and other important tasks. Rule mining is commonly applied to discover patterns in KGs. However, unlike in traditional association rule mining, KGs provide a setting with a high degree of *incompleteness*, which may result in the wrong estimation of the quality of mined rules, leading to erroneous beliefs such as all artists have won an award, or Hockey players do not have children.
In this paper we propose to use (in-)completeness meta information to better assess the quality of rules learned from incomplete KGs. We introduce completeness-aware scoring functions for relational association rules. Moreover, we show how one can obtain (in-)completeness meta data by learning rules about numerical patterns of KG edge counts. Experimental evaluation both on real and synthetic datasets shows that the proposed rule ranking approaches have remarkably higher accuracy than the state-of-the-art methods in uncovering missing facts.

1 Introduction

Motivation. Advances in information extraction have led to general-purpose knowledge graphs (KGs) containing billions of positive facts about the world (e.g., [3,2,1,21]). KGs are widely applied in semantic web search, question answering, web extraction and many other tasks. Unfortunately, due to their wide scope, KGs are generally incomplete. To account for the incompleteness, KGs typically adopt the Open World Assumption (OWA) under which missing facts are treated as unknown rather than false.

An important task over KGs is rule learning, which is relevant for a variety of applications ranging from knowledge graph curation (completion, error detection) [24,12,10] to data mining and semantic culturonomics. However, since such rules are learned from incomplete data, they might be erroneous and might make incorrect predictions on missing facts. E.g., $r_1 : hasChild(X, Y) \leftarrow worksAt(X, Z), educatedAt(Y, Z)$ could be mined from the KG in Fig. 1, stating that people have children that graduated from the same institution where they work at, as this is frequently the case for popular scientists. While r_1 is clearly not universal and should be ranked lower than the rule $r_2 : hasSibling(X, Z) \leftarrow hasFather(X, Y), hasChild(Y, Z)$, standard rule

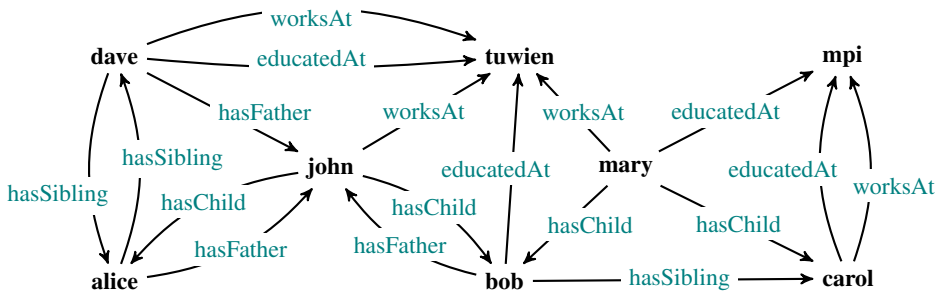

Fig. 1: Example KG

measures like confidence (i.e., conditional probability of the rule's head given its body) incorrectly favor r_1 over r_2 for the given KG.

Recently, efforts have been put into detecting the concrete numbers of facts of certain types that hold in the real world (e.g., Einstein has 3 children) by exploiting Web extraction and crowd-sourcing methods [23,26]. Such meta-data provides a lot of hints about the topology of KGs, and reveals parts that should be especially targeted by rule learning methods. However, surprisingly, despite its obvious importance, to date, no systematic way of making use of such information in rule learning exists.

In this work we propose to exploit meta-data about the number of edge counts in KGs to better assess the quality of learned rules. To further facilitate this approach, we discuss a method for learning edge count information by extracting rules like *"If a person has more than 2 siblings, then his parents are likely to have more than 3 children."*

State of the art and its limitations. In [12] a completeness-aware rule scoring based on the partial completeness assumption (PCA) was introduced. The idea of PCA is that whenever at least one object for a given subject and a predicate is in a KG (e.g., Einstein has a child Eduard), then all objects for that subject-predicate pair (e.g., Einstein's children) are assumed to be known. This assumption was taken into account in rule scoring, and empirically it turned out to be indeed valid in real-world KGs for some topics. However, obviously it does not universally hold, and treats inappropriately cases when edges in a graph are randomly missing. Similarly, whether to count absence of contradiction as confirmation for default rules was discussed in [8]. In [11] new completeness data was learned from a KG by taking as ground truth completeness data obtained via crowd-sourcing. The acquired statements were then used in post-processing step of rule learning to filter out predictions that violate these statements. However, this kind of filtering does not have any impact on the quality of the mined rules and the incorrect predictions for instances about which no completeness information exists.

Contributions. This work presents the first proper investigation of how meta-information about (in-)completeness, more specifically, about the number of edges that should exist for a given subject-predicate pair in a KG, can be used to improve rule learning. The salient contributions of our work are as follows:

1. We present an approach that accounts for meta-data about the number of edges that should exist for given subject-predicate pairs in the ranking stage of rule learning.
2. We discuss a method for the automated acquisition of approximate upper and lower bounds on the number of edges that should exist in KGs.
3. We implement the proposed rule ranking measures and evaluate them both on real-world and synthetic dataset, showing that they outperform previous ones both with respect to the quality of the mined rules and the predictions they produce.[1]

2 Related Work

Rule Learning. The problem of automatically learning patterns from KGs has gained a lot of attention in the recent years. Some relevant works are [12,29], which focus on learning Horn rules and either ignore completeness information, or make use of completeness by filtering out predicted facts violating completeness in a postprocessing step. On the contrary we aim at injecting the statements into the learning process.

In the context of inductive and abductive logic programming, learning rules from incomplete interpretations given as a set of positive facts along with a possibly incomplete set of negative ones was studied in [18]. In contrast to our approach, this work does not exploit the knowledge about the number of missing facts. Learning nonmonotonic rules in the presence of incompleteness was studied in hybrid settings [16,20]. There a background theory or a hypothesis can be represented as a combination of an ontology and Horn or nonmonotonic rules. The main point in these works is the assumption that there might be potentially missing facts in a given dataset. However, it is not explicitly mentioned, which parts of the data are (in)complete like in our setting. Moreover, the emphasis of these works is on the complex reasoning interaction between the components, while we are more concerned with techniques for deriving rules with high predictive quality from large KGs. Recent work by d'Amato *et al*. shows how in the presence of ontologies that allow to determine incorrect facts, rules can be ranked by the ratio of correct versus incorrect predictions [4]; yet, in contrast to our scenario of interest, in this work the knowledge about exact numbers of missing KG facts has not been exploited.

Completeness Information. The idea of bridging the open and closed world assumption by using completeness information was first introduced in the database world in [19,9], and later adapted to the Semantic Web in [5]. For describing such settings, the common approach is to fix the complete parts (and assume that the rest is potentially incomplete), though the converse setting has also been investigated [27].

Recent work on completeness rule mining has extended the AMIE system to mine rules about completeness, i.e., predict in which parts a knowledge graph may be complete or incomplete [11]. The focus of the work is on the learning of association rules like *"If someone has a place of death but no place of death, then a place of death is missing."* In contrast, we reason about the missing edges trying to estimate the exact number (bounds on) the number of edges that should be present in a KG. In [11] it has also been shown that the completeness information can be used to improve the accuracy of fact prediction,

[1] The extended version of this paper is available as a technical report at `https://raw.githubusercontent.com/Tpt/CARL/master/technical_report.pdf`

by pruning out in a post-processing step those facts that are predicted in parts expected to be complete. In the present paper, we aim to inject completeness information into the rule acquisition phase directly, in order to also prune away problematic rules but not only individual incorrect facts.

3 Preliminaries

Knowledge Graphs. Knowledge graphs (KG) represent interlinked collections of factual information, and they are often encoded using the RDF data model [17]. The content of KGs is a set of $\langle subject\ predicate\ object \rangle$ triples, e.g., $\langle john\ hasChild\ alice \rangle$. For encyclopedic knowledge graphs on the semantic web, usually the open world assumption (OWA) is employed, i.e., these graphs contain only a subset of the true information.

For simplicity, we write triples using binary predicates, like $hasChild(john, alice)$. A signature of a KG $\mathcal{G}$ is $\Sigma_{\mathcal{G}} = \langle \mathbf{R}, \mathcal{C} \rangle$, where $\mathbf{R}$ is the set of binary predicates and $\mathcal{C}$ is the set of constants appearing in $\mathcal{G}$. Following [5], we define the gap between the available graph $\mathcal{G}^a$ and the ideal graph $\mathcal{G}^i$, which contains all correct facts over $\mathbf{R}$ and $\mathcal{C}$ that hold in the real world.

Definition 1 (Incomplete data source). *An incomplete data source is a pair $G = (\mathcal{G}^a, \mathcal{G}^i)$ of two KGs, where $\mathcal{G}^a \subseteq \mathcal{G}^i$ and $\Sigma_{\mathcal{G}^a} = \Sigma_{\mathcal{G}^i}$.*

Note that the ideal graph $\mathcal{G}^i$ is a virtual construct that is normally not accessible. Instead, statements about $\mathcal{G}^i \backslash \mathcal{G}^a$, (e.g., *Einstein is missing 2 children and Feynman none*), which we formalize in Sec. 4, describe constraints wrt. the shape of $\mathcal{G}^i$.

Rule Learning. Association rule learning concerns the discovery of frequent patterns in a data set and the subsequent transformation of these patterns into rules. Association rules in the relational format have been subject of intensive research in ILP (see, e.g., [7] as the seminal work in this direction) and more recently in the KG community (see [12] as the most prominent work). In the following we adapt basic notions in relational association rule mining to our case of interest.

A *conjunctive query* Q over $\mathcal{G}$ is of the form $Q(\boldsymbol{X}) \text{:-} \ p_1(\boldsymbol{X_1}), \ldots, p_m(\boldsymbol{X_m})$. Its right-hand side (i.e., body) is a finite set of atomic formulas over $\mathcal{G}$, while the left-hand side (i.e., head) is a tuple of variables occurring in the body. The *answer* of Q on $\mathcal{G}$ is the set $Q(\mathcal{G}) = \{\nu(X) \mid \nu \text{ is a function from variables to } \mathcal{C} \text{ and } \forall i : p_i(\nu(X_i)) \in G\}$. As in [7], the *support* of Q in $\mathcal{G}$ is the number of distinct tuples in the answer of Q on $\mathcal{G}$.

An *association rule* is of the form $Q_1 \Rightarrow Q_2$, such that Q_1 and Q_2 are both conjunctive queries and $Q_1 \subseteq Q_2$, i.e., $Q_1(\mathcal{G}') \subseteq Q_2(\mathcal{G}')$ for any possible KG $\mathcal{G}'$. In this work we exploit association rules for reasoning purposes, and thus (with some abuse of notation) treat them as logical rules, i.e., for $Q_1 \Rightarrow Q_2$ we write $Q_2 \backslash Q_1 \leftarrow Q_1$, where $Q_2 \backslash Q_1$ refers to the set difference between Q_2 and Q_1 seen as sets of atoms.

Classical scoring of association rules is based on *rule support*, *body support* and *confidence*, which in [12] for a rule $r : \boldsymbol{H} \leftarrow \boldsymbol{B}$ with $\boldsymbol{H} = h(X, Y)$ are defined as:

$$supp(r) := \#(x, y) : \exists \boldsymbol{Z} : \boldsymbol{B} \wedge h(x, y) \tag{1}$$

$$supp(\boldsymbol{B}) := \#(x, y) : \exists \boldsymbol{Z} : \boldsymbol{B} \tag{2}$$

$$conf(r) := \frac{supp(r)}{supp(\boldsymbol{B})} \tag{3}$$

where $\#\alpha : \mathcal{A}$ denotes the number of α that fulfill the condition $\mathcal{A}$.

Example 1. Consider the KG in Fig. 1 and the rules r_1 and r_2 mined from it:
- $r_1 : hasChild(X, Y) \leftarrow worksAt(X, Z), educatedAt(Y, Z)$
- $r_2 : hasSibling(X, Z) \leftarrow hasFather(X, Y), hasChild(Y, Z)$

The body and rule supports of r_1 over the KG are $supp(\boldsymbol{B}) = 8$ and $supp(r_1) = 2$ respectively. Hence, we have $conf(r_1) = \frac{2}{8}$. Analogously, $conf(r_2) = \frac{1}{6}$. □

Support and confidence were originally developed for scoring rules over complete data. If data is missing, their interpretation is not straightforward and they can be misleading. In [12], *confidence under the Partial Closed World Assumption* (PCA) has been proposed as measure, which guesses negative facts by assuming that data is usually added to KGs in batches, i.e., if at least one child of John is known then most probably all John's children are present in the KG. Formally, the *PCA confidence* is defined as

$$conf_{pca}(r) := \frac{supp(r)}{\#(x, y) : \exists \boldsymbol{Z} : \boldsymbol{B} \wedge \exists y' : h(x, y') \in \mathcal{G}^a} \tag{4}$$

Example 2. We obtain $conf_{pca}(r_1) = \frac{2}{4}$ because as $carol$ and $dave$ are not known to have any children in the KG, the first four body substitutions are not counted in the denominator. Meanwhile, we have $conf_{pca}(r_2) = \frac{1}{6}$, since all people that are predicted to have siblings by r_2 already have siblings in the available graph. □

Given a rule r and a KG $\mathcal{G}$ the application of r on $\mathcal{G}$ results in a rule-based graph completion defined relying on the Answer Set semantics (see [13] for details).

Definition 2 (Rule-based KG completion). *Let $\mathcal{G}$ be a KG over the signature $\Sigma_{\mathcal{G}} = \langle \mathbf{R}, \mathcal{C} \rangle$ and let r be a rule mined from $\mathcal{G}$, i.e. a rule over $\Sigma_{\mathcal{G}}$. Then the* completion of $\mathcal{G}$ *is a graph $\mathcal{G}_r$ constructed from the answer set of $r \cup \mathcal{G}$.*

Example 3. We have $\mathcal{G}^a_{r_1} = \{\mathcal{G} \cup\ hasChild(john, dave), hasChild(carol, mary), hasChild(dave, dave), hasChild(carol, carol), hasChild(dave, bob), hasChild(mary, dave)$. □

Note that $\mathcal{G}^i$ is the perfect completion of $\mathcal{G}^a$, which is supposed to contain all correct facts with entities and relations from $\Sigma_{\mathcal{G}^a}$ that hold in the current state of the world. The goal of every rule-based KG completion system, is to extract from $\mathcal{G}^a$ a set of rules $\mathcal{R}$, such that $\cup_{r \in \mathcal{R}} \mathcal{G}^a_r$ is as close to $\mathcal{G}^i$ as possible.

4 Completeness-aware Rule Scoring

Scoring and ranking rules is a major step in association rule learning. A variety of measures for ranking rules have been proposed, with some of the most prominent ones being confidence, conviction and lift. The existing (in-)completeness-aware rule measure in the KG context (the PCA confidence (4) [12]) has two apparent shortcomings: First,

as it only counts as counterexamples those x, y pairs for which at least one $h(x, y')$ is in $\mathcal{G}^a$ for some y' and a rule's head predicate h, it may incorrectly give high scores to rules predicting facts for very incomplete relations, e.g., predicting that children work at the same place as their fathers. Second, it is not suited for the data that is not added in batches, such as awards, among which the important ones are added instantly, while others much slowly or even possibly never.

Thus, in this work we focus on the improvements of rule scoring functions by making use of the extra (in-)completeness meta-data. Before dwelling into the details of our approach we discuss the formal representation of such meta-data.

Cardinality Statements. Overall, one can think of 6 different cardinality templates obtained by fixing subject, predicate or object in a triple and report the number of respective facts that hold in $\mathcal{G}^i$. E.g., for $\langle john\ hasChild\ mary \rangle$ we can count (1) children of $john$ (2) edges from $john$ to $mary$; (3) incoming edges to $mary$; (4) facts with $john$ as a subject; (5) facts over $hasChild$ relation; (6) facts with $mary$ as an object.

In practice, numerical statements for templates (1) and (3) can be obtained using web extraction techniques [23], from functional properties of relations or from crowdsourcing. For other templates things get trickier; one might be able to learn them from the data or they could be defined by domain experts in topic-specific KGs. We leave this issue for future work, and focus here only on templates (1) and (3), which could be rewritten as the instances of the template (1) provided that inverse relations can be expressed in a KG. For instance, $\#s : hasChild(s, john) = \#o : hasParent(john, o)$ for the predicates $hasChild$ and $hasParent$, which are inverses of one another.

We represent the (in)completeness meta-data using cardinality statements by reporting (the numerical restriction on) the absolute number of facts over a certain relation in the ideal graph $\mathcal{G}^i$. More specifically, we define the function num that takes as input a predicate p and a constant s and outputs a natural number corresponding to the number of facts in $\mathcal{G}^i$ over p with s as the first argument:

$$num(p, s) := \#o : p(s, o) \in \mathcal{G}^i \tag{5}$$

Naturally, the number of missing facts for a given p and s can be obtained as

$$miss(p, s) := num(p, s) - \#o : p(s, o) \in \mathcal{G}^a \tag{6}$$

Example 4. Consider the KG in Fig. 1. and the following cardinality statements for it:
- $num(hasChild, john) = num(hasChild, mary) = 3$; $num(hasChild, alice) = 1$; $num(hasChild, carol) = num(hasChild, dave) = 0$;
- $num(hasSibling, bob) = 3$; $num(hasSibling, alice) = num(hasSibling, carol) = num(hasSibling, dave) = 2$.

We then have:
- $miss(hasChild, mary) = miss(hasChild, john) = miss(hasChild, alice) = 1$; $miss(hasChild, carol) = miss(hasChild, dave) = 0$;
- $miss(hasSibling, bob) = miss(hasSibling, carol) = 2$; $miss(hasSibling, alice) = miss(hasSibling, dave) = 1$. □

We are now ready to define the *completeness-aware rule scoring problem*.

Problem 1 (Completeness-aware rule scoring). Given a KG and a set of cardinality statements, *completeness-aware rule scoring* aims to score rules not only by their predictive power on the known KG, but also wrt. the number of wrongly predicted facts in complete areas and the number of newly predicted facts in known incomplete areas.

In the following we discuss and compare three novel approaches for completeness-aware rule scoring. These are (i) the *completeness confidence*, (ii) *completeness precision* and *recall*, and (iii) *directional metric*. Henceforth, all examples consider the KG in Fig. 1, rules from Ex. 1, and cardinality statements described in Ex. 4.

4.1 Completeness Confidence

In this work we propose to explicitly rely on incompleteness information in determining whether to consider an instance as a counterexample for a rule at hand or not.

To do that, we first define two indicators for a given rule $r : h(X, Y) \leftarrow \boldsymbol{B}$, reflecting the number of new predictions made by r in incomplete ($npi(r)$) and, respectively, complete ($npc(r)$) KG parts:

$$npi(r) := \sum_x min(\#y : h(x, y) \in \mathcal{G}_r^a \backslash \mathcal{G}^a, miss(h, x)) \tag{7}$$

$$npc(r) := \sum_x max(\#y : h(x, y) \in \mathcal{G}_r^a \backslash \mathcal{G}^a - miss(h, x), 0) \tag{8}$$

Exploiting these additional indicators for $r : h(X, Y) \leftarrow \boldsymbol{B}$ we obtain the following *completeness-aware confidence*:

$$conf_{comp}(r) := \frac{supp(r)}{supp(\boldsymbol{B}) - npi(r)} \tag{9}$$

Example 5. Obviously, the rule r_2 should be preferred over r_1. For our novel completeness confidence, we get $conf_{comp}(r_1) = \frac{2}{6}$ and $conf_{comp}(r_2) = \frac{1}{2}$, resulting in the desired rule ordering, which is not achieved by existing measures (see Ex. 1 and 2). □

Our completeness confidence generalizes both the standard and the PCA confidence:

Proposition 1. *For every KG $\mathcal{G}$ and rule r it holds that*
(i) under the Closed World Assumption (CWA) $conf_{comp}(r) = conf(r)$;
(ii) under the Partial Closed World Assumption (PCA) $conf_{comp}(r) = conf_{pca}(r)$.

Proof. **(i)** Under the CWA, it holds that for all $p \in \mathbf{R}$, $s \in \mathcal{C} : miss(p, s) = 0$. Thus, for all rules r, we have that $npi(r) = 0$, and hence, $conf_{comp}(r) = conf(r)$.

(ii) Under the PCA, it is assumed that for all $p \in \mathbf{R}$ and $s \in \mathcal{C}$ it holds that $miss(p, s) = 0$ if $\exists o : p(s, o) \in \mathcal{G}^a$ and otherwise, $miss(p, s) = +\infty$, since the number of missing facts is unrestricted. Hence, for all r we have $npi(r) = \sum_x predict(r, x)$, where

$$predict(r, x) = \begin{cases} 0, \text{ if } \exists y : h(x, y) \in \mathcal{G}^a \\ \#y : h(x, y) \in \mathcal{G}_r^a \backslash \mathcal{G}^a, \text{ if } \forall y' : h(x, y') \notin \mathcal{G}^a \end{cases}$$

From this we get

$$conf_{comp}(r) = \frac{supp(r)}{supp(\boldsymbol{B}) - \sum_{x:\forall y': h(x,y') \notin \mathcal{G}^a} \#y : h(x,y) \in \mathcal{G}_r^a \backslash \mathcal{G}^a}$$

The denominator of the latter formula counts all rule body and subtracts from them those, for which the head $h(x, y)$ is predicted and $\forall y' : h(x, y') \notin \mathcal{G}^a$. Hence, we end up counting only body substitutions with $h(x, y') \in \mathcal{G}^a$ for at least one y', i.e., $\#(x, y) : \exists \boldsymbol{Z} : \boldsymbol{B} \wedge \exists y' : h(x, y') \in \mathcal{G}^a$, from which the result follows. □

In other words, if the graph is known to be fully complete, i.e., for all $p \in \mathcal{R}, s \in \mathcal{C}$ we have $miss(p, s) = 0$, then $conf_{comp}$ is the same as the standard confidence. Similarly, if $miss(p, s) = 0$ for such p, s pairs that at least one fact $p(s, \_) \in \mathcal{G}^a$ exists and $miss(p, s) = +\infty$ for the rest, then $conf_{comp}$ is the same as the PCA confidence.

4.2 Completeness Precision and Recall

Further developing the idea of scoring rules based on their predictions in complete and incomplete KG parts, we propose to consider the notions of *completeness precision* and *recall*[2] for rules defined in the spirit of information retrieval. Intuitively, rules having high precision are rules that predict few facts in complete parts, while rules having high recall are rules that predict many facts in incomplete ones. Rule scoring could then be based on any weighted combination of these two metrics.

Formally, we define the precision and recall of a rule $r : h(X, Y) \leftarrow \boldsymbol{B}$ as follows:

$$precision_{comp}(r) = 1 - \frac{npc(r)}{supp(\boldsymbol{B})} \tag{10}$$

$$recall_{comp}(r) = \frac{npi(r)}{\sum_s miss(h, s)} \tag{11}$$

The *recall measure* is similar to classical support measures, but now expresses how many facts on KG parts known to be incomplete, are generated by the rule (the more the better). The *precision measure*, in turn, assesses how many of the generated facts are definitely wrong, namely those in complete parts (the more of these, the worse the rule). In fact, this is an upper bound on the precision, as the other facts cannot be evaluated.

Example 6. It holds that $npi(r_1) = 2$, $npc(r_1) = 4$, while $npi(r_2) = 4$, $npc(r_2) = 1$, resulting in $precision_{comp}(r_1) = 0.5$, $recall_{comp}(r_1) \approx 0.67$, and $precision_{comp}(r_2) \approx 0.83$, $recall_{comp}(r_2) \approx 0.67$, which lead to the expected relative rule ordering. □

Limitations. While precision and recall are insightful when there are sufficiently many predictions made in (in-)complete parts, they fail when the number of (in-)completeness statements in comparison with the KG size is small. Consider, for instance, a rule that predicts 1000 new facts over $hasChild$ relation, out of which 2 are in complete, and 2 are in incomplete parts, and overall 1 million children are missing. This would imply a precision of 99.8%, and a recall of 0.0002%, both of which are not very informative.

Therefore, next we propose to look at the difference between expected numbers of predictions in complete and incomplete parts, or simply at their ratio.

[2] For brevity we skip the word "completeness" if clear from the context.

4.3 Directional Bias

If rule mining does make use of completeness information, and both do not exhibit any statistical bias, then intuitively the rule predictions and the (in)complete areas should be statistically independent. On the other hand, correlation between the two indicates that the rule-mining is *(in)completeness-aware*.

Example 7. Suppose in total a given KG stores 1 million humans, and we know that 10,000 (1%) of these are missing some children (incompleteness information), while we also know that 1000 of the persons are definitely complete for children (0.1%). Let the set of rules mined from a KG predict 50,000 new facts for the $hasChild$ relation. Assuming independence between predictions and (in)completeness statements, we would expect 1% out of 50,000, i.e., 500 facts to be predicted in the incomplete areas and 0.1%, i.e., 50 in the complete KG parts. If instead we find 1000 children predicted for people that are missing correspondingly many children, and 10 for people that are not missing these, the former deviates from the expected value by a factor of 2, and the latter by a factor of 5.

Following the intuition from the above example, we propose to look at the extent of the non-independence to quantify the (in)completeness-awareness of rule mining. Let us consider predictions made by rules in a given KG, where *E(#facts)* is the expected number of predictions and $\alpha = 0..1$ is the weight given to completeness versus incompleteness. Then the directional coefficient of a rule r is defined as follows:

$$direct\_coef(r) := \alpha \cdot \frac{E(npc(r))}{npc(r)} + (1-\alpha) \cdot \frac{npi(r)}{E(npi(r))} \tag{12}$$

Intuitively, if the ratio between the KG size and the size of the (in)complete parts is the same as the ratio between the predictions in the (in)complete parts and their total number, i.e., if the directional coefficient is 1, then the rules ignore the statements. The higher is the *directional coefficient*, the more "*completeness-aware*" the rules are.

In practice, expected values might be difficult to compute, and statistical independence is a strong assumption. An alternative that does not require knowledge about expected values is to directly measure the proportion between predictions in complete and incomplete parts. We call this the *directional metric*, which is computed as

$$direct\_metric(r) := \frac{npi(r) - npc(r)}{2 \cdot (npi(r) + npc(r))} + 0.5 \tag{13}$$

The metric is based on the same ideas as the directional coefficient, but does not require knowledge about the expected number of predictions in complete/incomplete KG parts. It returns a value between 0 (predicting only facts in complete areas) and 1 (predicting only facts in incomplete areas). This range allows for convenient combinations with other 0-1 measures. The directional metric of a rule that predicts the same number of facts in incomplete as in complete parts is 0.5, a rule that predicts twice as many facts in incomplete parts has a value of 0.66, and so on.

Since the real-world KGs are often highly incomplete, it might be reasonable to put more weight on predictions in complete parts. This can be done by multiplying predictions made in complete parts by a certain factor. We propose to consider the combination

of a weighted existing association rule measure, e.g., confidence or conviction and the directional metric, with the weighting factor $\beta = 0..1$. Using confidence, we obtain

$$weighted\_dm(r) = \beta \cdot conf(r) + (1 - \beta) \cdot direct\_metric(r) \tag{14}$$

Example 8. We get $direct\_metric(r_1) \approx 0.33$ and $direct\_metric(r_2) = 0.8$. For $\beta = 0.5$ and confidence from Ex. 1, $weighted\_dm(r_1) \approx 0.29$ and $weighted\_dm(r_2) \approx 0.48$. □

5 Acquisition of Numerical Statements

As we have shown, exploitation of numerical (in-)completeness statements is very beneficial for rule quality assessment. A natural question is where to acquire such statements from in real-world settings. Various works have shown that numerical assertions can be frequently spotted on the Web [5], obtained via crowd-sourcing [6], text mining [22] or completeness rule mining [11]. We believe that above all, mining numerical correlations about KG edges, and assembling them into rules, as described next is a valuable and a modular approach to obtain further completeness information.

We start with an available KG $\mathcal{G}^a$ and some statements of the form (5).

Step 1. For every cardinality $num(p, s) = k$, we create the facts $p_{\leq k}(s)$ and $p_{\geq k}(s)$. For the pairs $p \in \mathbf{R}, s \in \mathcal{C}$ with no available cardinality statements we construct the fact $p_{\geq \#o:p(s,o)\in\mathcal{G}^a}(s)$, encoding that outgoing p-edges from s might be missing in $\mathcal{G}^a$, as that latter is believed to be incomplete by default. Here, p_{card} with $card \in \{\leq \_, \geq \_\}$ are fresh unary predicates not present in $\Sigma_{\mathcal{G}^a}$, which describe (bounds on) the number of outgoing p-edges for a given constant. We store all constructed facts over p_{card} in $\mathcal{S}$.

We then complete the domain of each p_{card} predicate as follows. For every $p_{\leq k}(s) \in \mathcal{S}$, if $p_{\leq k'}(s') \in \mathcal{S}$ for some $s' \in \mathcal{C}$ and $k' > k$, we construct the rule $p_{\leq k'}(X) \leftarrow p_{\leq k}(X)$. Similarly, for every $p_{\geq k}(s) \in \mathcal{S}$, if $p_{\geq k'}(s') \in \mathcal{S}$ where $k' < k$, we create $p_{\geq k'}(X) \leftarrow p_{\geq k}(X)$. The constructed rules are then applied to the facts in $\mathcal{S}$ to obtain an extended set $\mathcal{G}^{card}$ of facts over p_{card}.

Step 2. We then apply standard rule learning methods, e.g., [12] on the graph $\mathcal{G}^a \cup \mathcal{G}^{card}$ to mine a set of rules of the forms:

(1) $p_{card}(X) \leftarrow p'_{card}(X)$
(2) $p_{card}(X) \leftarrow p'_{card}(X), p''_{card}(X)$
(3) $p_{card}(X) \leftarrow p'_{card}(X), r(X, Y)$
(4) $p_{card}(X) \leftarrow p'_{card}(X), r(X, Y), p''_{card}(Y)$
(5) $p_{card}(X) \leftarrow r(X, Y), p''_{card}(Y)$

We rank these rules and select the top ones into the set $\mathcal{R}$.

Step 3. Finally, in the last step we use the obtained ruleset $\mathcal{R}$ to derive further numerical statements together with weights assigned to them. For that we compute $\mathcal{G}' = \bigcup_{r\in\mathcal{R}} \{\mathcal{G}^{card} \cup \mathcal{G}^a\}_r$. The weights of the statements are inherited from the rules that deduced them. Given multiple rules predicting the same fact, the highest weight for it is kept. We then post-process predictions made by different rules for the same subject-predicate pair as follows. If $p_{\leq k}(s), p_{\geq k'}(s) \in \mathcal{G}'$ for $k' > k$, we remove from $\mathcal{G}'$ predictions with the lowest weight thus resolving the conflict on the numerical bounds.

Finally, from the obtained graph we reconstruct cardinality statements as follows.

- For $p_{\leq k}(s), p_{\geq k}(s) \in \mathcal{G}'$ with weights w and w' we create a cardinality statement $num(p, s) = k$ with the weight $min(w, w')$ and add it to the set $\mathcal{S}$.
- If $p_{\leq k}(s), p_{\geq k'}(s) \in \mathcal{G}'$ for $k' < k$, then we set $k' \leq num(p, s) \leq k$.
- Among two facts $p_{\leq k}(s), p_{\leq k'}(s)$ (resp. $p_{\geq k}(s), p_{\geq k'}(s)$) with $k < k'$ (resp. $k > k'$) the first kept.

The rest of the facts in $\mathcal{G}'$ are similarly translated into their numerical representations.

Example 9. Consider the KG in Fig. 1 and the following cardinality statements for it: $num(hasChild, john) = num(hasSibling, bob) = 3$. We get the following graph with numerical facts $\mathcal{G}^{card} = \{hasChild_{\geq 3}(john), hasSibling_{\geq 3}(bob), hasChild_{\geq 2}(mary), hasChild_{\geq 2}(john), hasChild_{\geq 2}(bob), hasSibling_{\geq 1}(dave), hasSibling_{\geq 1}(alice), \cdots\}$. On the graph $\mathcal{G}^a \cup \mathcal{G}^{card}$, the rule $hasSibling_{\geq 2}(X) \leftarrow hasFather(X, Y), hasChild_{\geq 3}(X)$ has a confidence of $1/3$ and $hasSibling_{\geq 1}(X) \leftarrow hasFather(X, Y), hasChild_{\geq 3}(Y)$ a confidence of 1. □

Ideally, provided that sufficiently many similar numerical correlations about edge numbers are extracted, one can induce more general hypothesis involving arithmetic functions like the number of person's siblings is bounded by the number of his parents' children plus 1 or the sum of person's brothers and sisters equals the number of his siblings. We leave these more complex generalizations for future work.

6 Evaluation

6.1 Completeness-aware Rule Learning

We have implemented our **c**ompleteness-**a**ware **r**ule **l**earning approach into a C++ system prototype CARL, following a standard relational learning algorithm implementation such as [14].[3] To restrict the rule search space we focus on rules of the form

$$r(X, Z) \leftarrow p(X, Y), q(Y, Z) \tag{15}$$

We aim to compare the predictive quality of the top k rules mined by our completeness-aware approach with the ones learned by standard rule learning methods.

Dataset. We used two datasets for the evaluation: (i) *WikidataPeople*, which is a dataset we have created from the Wikidata knowledge graph, containing 2.4M facts over 9 predicates[4] about biographical information and family relationships of people; and (ii) *LUBM*, which is a synthetic dataset describing the structure of a university [15].

For the WikidataPeople dataset, the approximation of the ideal KG ($\mathcal{G}^i$) is obtained by exploiting available information about inverse relations (e.g., $hasParent$ is the inverse of $hasChild$), functional relations (e.g., $hasFather, hasMother$) as well as manually hand-crafted solid rules from the family domain like

$$hasSibling(X, Y) \leftarrow hasParent(X, Z), hasParent(Y, Z), X \neq Y.$$[5]

[3] The source code and all the data are available at `https://github.com/Tpt/CARL`.

[4] $hasFather$, $hasMother$, $hasStepParent$, $hasSibling$, $hasSpouse$, $hasChild$, $hasBirthPlace$, $hasDeathPlace$, and $hasNationality$

[5] see `https://github.com/Tpt/CARL/tree/master/eval/wikidata` for details

From WikidataPeople $\mathcal{G}^i$ containing 5M facts, we acquired cardinality statements by exploiting properties of functional relations e.g., $hasBirthPlace$, $hasFather$, $hasMother$ and must be uniquely defined, and everybody with a $hasDeathDate$ has a $hasDeathPlace$. The PCA [12] is used for other relations. This resulted in 10M cardinality statements.

LUBM $\mathcal{G}^i$, with 1.2M facts, was constructed by using the LUBM data generator requesting 10 universities, removing all `rdf:type` triples and introducing inverse predicates. 464K cardinality statements were obtained by counting the number of existing objects for each subject-predicate pair, i.e., PCA was assumed on the whole dataset.

Experimental Setup. To assess the effect of our newly introduced measures, we first construct versions of the available KG ($\mathcal{G}^a$) by removing various parts of the data from $\mathcal{G}^i$ and artificially introducing the data bias (i.e., leaving many facts in $\mathcal{G}^a$ over some relations and very few over others). The artificial bias is needed to simulate our scenario of interest, where some parts in $\mathcal{G}^a$ are largely missing while others are fairly complete, which is indeed the case in real world KGs. In Wikidata, for instance, only for 3% of non-living people sibling information is reported, while children data is known for 4%.

We proceed in two steps: First, we define a *global ratio*, which determines a uniform percentage of data retained in the available graph. To further refine this, we then factor in a *predicate ratio* individual to each predicate. For the WikidataPeople KG, this ratio is chosen as *(i)* 0.8 for $hasFather$ and $hasMother$; *(ii)* 0.5 for $hasSpouse$, $hasStepParent$, $hasBirthPlace$, $hasDeathPlace$ and $hasNationality$; *(iii)* 0.2 for $hasChild$; and *(iv)* 0.1 for $hasSibling$. For the LUBM dataset, the predicate ratio is uniformly defined as 1 for regular predicates and 0.5 for inverse predicates.

For a given predicate, the final ratio of facts in $\mathcal{G}^a$ retained from those in $\mathcal{G}^i$ is then computed as $min(1, 2kn)$, where k is the predicate ratio and n is the global ratio.

The assessment of the rules learned from different versions of the available KG is performed by comparing rule predictions with the ideal KG. More specifically, every learned rule is assigned a *quality score*, defined as the ratio of the number of predictions made by the rule in $\mathcal{G}^i \setminus \mathcal{G}^a$ over the number of all predictions outside $\mathcal{G}^a$.

$$quality\_score(r) = \frac{|\mathcal{G}^a_r \cap \mathcal{G}^i \setminus \mathcal{G}^a|}{|\mathcal{G}^a_r \setminus \mathcal{G}^a|} \tag{16}$$

This scoring naturally allows us to control the percentage of rule predictions that hit our approximation of $\mathcal{G}^i$, similar to standard recall estimation in machine learning.

Results. From every version of the available KG we have mined rules of the form (15) and kept only rules with $conf(r) \geq 0.001$ and $supp(r) \geq 10$, whose *head coverage*[6] is greater than 0.001. Figure 2 shows the number of kept rules and their average support (1) for each global ratio used for generating $\mathcal{G}^a$.

We present our evaluation results for WikidataPeople and LUBM datasets in Figure 3. The horizontal axis displays the global ratio used for generating $\mathcal{G}^a$. We compared different rule ranking methods as previously discussed, including standard confidence (3), PCA confidence (4), completeness confidence (9), completeness precision (10), completeness recall (11), directional metric (13) and weighted directional metric ($\beta =$

[6] *Head coverage* is the ratio of the number of predicted facts that are in $\mathcal{G}^a$ over the number of facts matching the rule head.

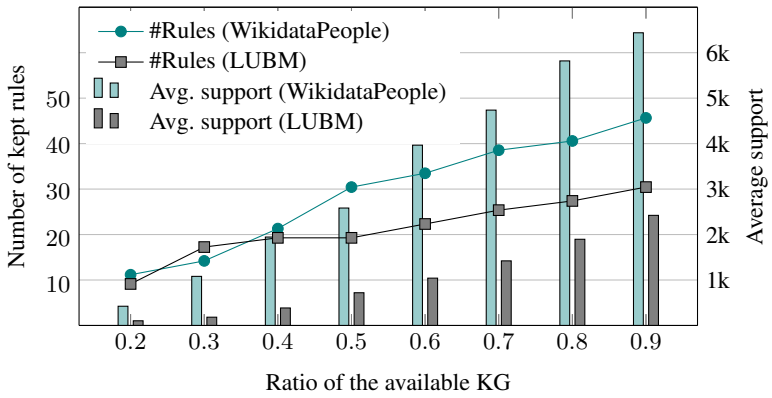

Fig. 2: Number of kept rules (#Rules) and average support for WikidataPeople and LUBM datasets

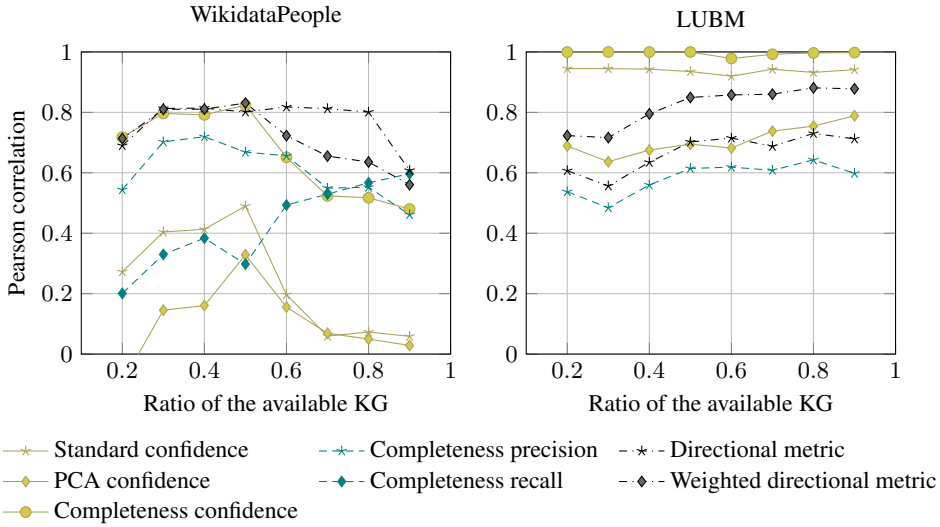

Fig. 3: Evaluation results for WikidataPeople and LUBM datasets

0.5) (14). The Pearson correlation factor[7] (vertical axis) between the ranking measures and the rules quality score (16) is used to evaluate the different measures.

Since facts are missing at random in the considered versions of $\mathcal{G}^a$, the PCA confidence performs worse than the standard for both datasets, while our completeness confidence significantly outperforms the standard confidence (see Tab. 1 for examples).

[7] The Pearson correlation factor between two variables X and Y is defined by $\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$ with cov being the covariance and σ the standard deviation.

Rule r	$conf(r)$	$conf_{pca}(r)$	$conf_{comp}(r)$	$dir\_metric(r)$
$hasSibling(X,Z) \leftarrow hasSibling(X,Y),$ $hasSibling(Y,Z)$	0.10	0.10	0.89	0.98
$hasStepParent(X,Z) \leftarrow hasMother(X,Y),$ $hasSpouse(Y,Z)$	0.0015	0.48	0.0015	0.38

Table 1: Example of rules mined from WikidataPeople with global ratio of 0.5

For the WikidataPeople KG, directional metric, directional metric score, and completeness confidence show the best results followed by completeness precision. For the LUBM KG, the completeness confidence outperforms the rest of the measures, followed by the standard confidence and the directional metric score. Correlation for completeness recall in the LUBM dataset behaved erratic and was slightly negative, thus is not displayed at all. We conjecture that completeness recall is an unsuited measure because it generally rewards rules that predict many facts, irrespective of whether these facts are true or false. Observe that the standard confidence measure is dramatically better for the LUBM KG with correlation factor higher than 0.9 than for the WikidataPeople KG. It is only refined by the completeness confidence which reaches a nearly perfect output with a correlation factor of 0.99. This is maybe because the bias between the different predicates of the LUBM KG is less strong than the one in the WikidataPeople KG, where some predicates are missing a lot of facts, while others just few, possibly leading to the unreliable standard confidence value. Completeness precision, directional metric and directional metric score outperform completeness precision for most settings on the WikidataPeople KG, while they perform slightly worse on the LUBM KG compared to the WikidataPeople KG.

In summary, we see that completeness precision significantly outperforms standard precision and PCA precision, and that the directional metric score can in some settings achieve even better results.

6.2 Automated Acquisition of Cardinality Statements

To evaluate our method for the automated acquisition of cardinality statements from the data we reused the WikidataPeople dataset–without completing the data.

Dataset. We have collected around 282K cardinality statements from various sources:

- Wikidata schema, i.e., $hasFather$, $hasMother$, $hasBirthPlace$, and $hasDeathPlace$ are functional properties and, thus, should have at most one value.
- The 7.5K values of the Wikidata predicate $numberOfChildren$;
- 663 $novalue$ statements from Wikidata;
- 86K cardinality statements from [23] for the *hasChild* predicate of Wikidata;
- 182K cardinality statements are extracted from human-curated and complete Freebase facts (1.6M). The mapping to Wikidata has been done using tools from [25].

Experimental Setup. We randomly set aside 20% of the cardinality statements as the validation set, while the rest are incorporated into the WikidataPeople KG, as explained in Sec. 5. A rule learning algorithm is then run on the KG to mine cardinality rules. The rules r with $supp(r) \leq 200$ or $conf(r) \leq 0.01$, are pruned out.

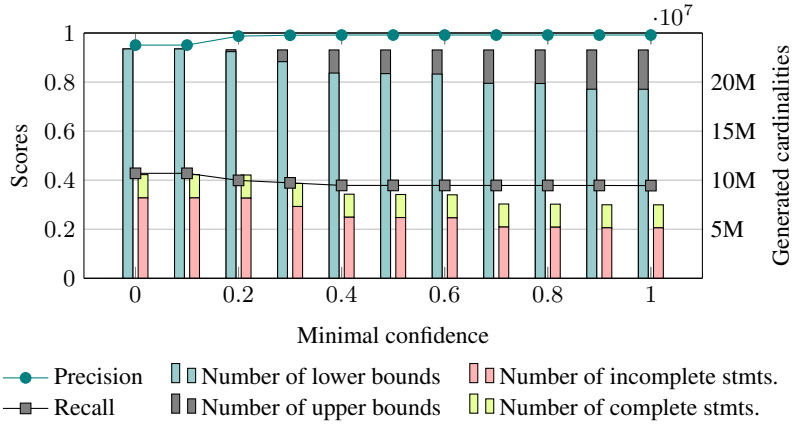

Fig. 4: Evaluation results of predicted cardinalities, number of generated lower and upper bounds, and number of (in)complete statements (when the lower bound matches the upper bound).

Examples of mined rules along with their standard confidences are:

- $hasSibling_{\geq 3}(x) \leftarrow hasSibling(x, y), hasSibling_{\geq 4}(y)$: 0.97
- $hasChild_{\geq 3}(x) \leftarrow hasFather(y, x), hasSibling(y)_{\geq 4}(y)$: 0.90

The learned rules are then applied to the enriched WikidataPeople KG to retrieve new exact cardinalities $num(p, s)$ by only keeping (p, s) pairs where the higher and lower bounds match. The minimum of the standard confidence of the best rules used to get the upper and lower bounds is assigned as the final confidence of each $num(p, s)$.

Results. We aim to evaluate whether we can accurately recover the cardinality statements in the validation set–as the gold standard–by utilizing the learned cardinality rules. For different minimal confidence thresholds, the quality of the predicted cardinalities is measured with standard precision and recall,which is presented in Figure 4. We get a very good precision and a fair recall (around 0.4) on this dataset. If we remove the schema information from the KG, we get lower precision (around 0.7) and recall (around 0.0.1) before a minimal confidence of 0.6 and similar values after.

7 Conclusion and Future Work

We have defined the problem of learning rules from incomplete KGs enriched with the exact numbers of missing edges of certain types, and proposed three novel rule ranking measures that effectively make use of the meta-knowledge about complete and incomplete KG parts: *completeness confidence, precision/recall* and the (weighted) *directional metric*. Our measures have been injected in the rule learning prototype CARL and evaluated on real-world and synthetic KGs, demonstrating significant improvements both with respect to the quality of mined rules and predictions they produce. Moreover, we have proposed a method for acquiring cardinality meta data about edge counts directly from the given KG.

For future work, first, we plan to encode the cardinality information into background knowledge, e.g., using qualified cardinality restrictions in OWL ontologies. Second, an interesting further direction is to learn general correlations about edge counts that include complex mathematical functions, like the number of siblings equals the sum of the number of sisters and brothers.

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