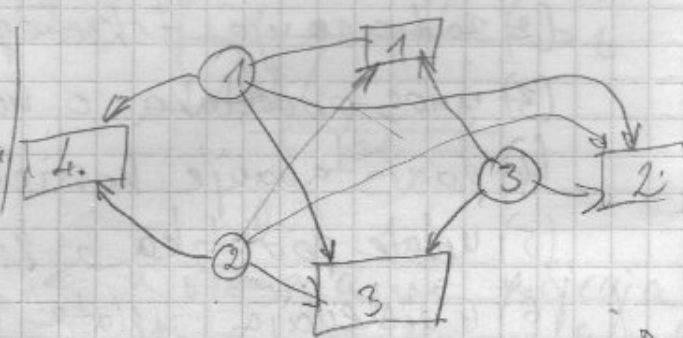


ORERST

30.10.2003

- izvor (skladište) $i = 1, m$
- odredište (prodatnica) $j = 1, n$



$$\min Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

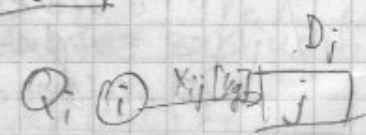
uz

$$\sum_{j=1}^n x_{ij} \leq Q_i \quad i = 1, m$$

$$\sum_{i=1}^m x_{ij} \geq D_j \quad j = 1, n$$

$$\sum_{i=1}^m Q_i \geq \sum_{j=1}^n D_j$$

$$c_{ij} \left[\frac{\text{kn}}{\text{kg}} \right]$$



Transportni problem

- raspoloživih ima malo više od potrebnih ($230 > 220$),

	1	2	3	raspoloživo
1	3	2	4	100
2	4	5	6	70
3	2	1	3	60
potreba	80	50	90	

Zatvoriti transp. problem:

$$\sum_{i=1}^m Q_i = \sum_{j=1}^n D_j$$

C_{ij}	1	2	3	4	fiktivno odredište raspoloživo Q_i
1	3	2	1	0	100
2	4	5	6	0	70
3	2	1	3	0	60

potreba 80 50 90 10 $230 - 220 = 10$
 $\underbrace{j=1..n}_{D_j}$

$$\min Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

$$\left. \begin{aligned} \sum_{j=1}^n X_{ij} &= Q_i \quad i=1..m \\ \sum_{i=1}^m X_{ij} &= D_j \quad j=1..n \end{aligned} \right\} \begin{aligned} &m+n \text{ jednačini} \\ &m+n-1 \text{ linearno nezav. jedu.} \end{aligned}$$

$$\forall X_{ij} \geq 0$$

- toliko ima i različit var.

	1	2	3	4	
1	80	20	+		100
2		30	40		40
3			50	10	60
	80	50	80	10	

$\frac{Dz}{DX_{12}} = f_{12} = 1 - 6 + 5 - 2 = -2$ (23 se smanji za 1 (23 je u origina. tablici = 6))

$DX_{13} = 1$

$f_{32} = 1 - 3 + 6 - 5 = -1$

Sistematski postupak:

	1	2	3	4	u_i
1	3	2	1	0	3
2	4	5	2	0	6
3	2	1	3	0	3

	1	2	3	4	v_j
0	-1	0	3		

za bi li: $x_{ij} : u_i + v_j = c_{ij}$

ta ide 0 jer od baz. var. (zaokr.) najvedi trosak ima (tj. je) 5

za nebazicne varijable: $f_{ij} = c_{ij} - (u_i + v_j)$

$$f_{13} = 1 - (3 + 0) = -2$$

$$f_{14} = 0 - (3 + (-3)) = 0$$

$$f_{21} = 4 - (6 + 0) = -2$$

$$f_{24} = 0 - (6 + (-3)) = 3 \quad \text{---} \text{pozici najveće note du } (-3)$$

$$f_{31} = 2 - (3 + 0) = -1$$

$$f_{32} = 1 - (3 + (-1)) = -1$$

novi baz. rješenje:

80	20			100
	30	30	10	70
		60		60
80	50	90	10	

3	2	1	0	3
4	5	6	0	6
2	1	3	0	3
0	1	0	-6	

kandidati
D=90

$$f_{13} = 1 - 3 = -2$$

$$f_{14} = 0 - (-2) = 2$$

$$f_{21} = 4 - 6 = -2 \quad D=30 \quad \underline{f_{21}}$$

$$f_{31} = 2 - 3 = -1$$

$$f_{32} = 1 - 2 = -1$$

$$f_{34} = 0 - (-3) = 3$$

50 > 20
90 > 10
30 > 10

50	50			100
30		30	10	70
		60		60
80	50	30	10	

	1	2	3	4	
1	3	2	7	6	5
2	4	5	6	0	6
3	2	1	3	0	3
	-2	-3	0	-6	

$$f_{13} = 1 - 5 = -4$$

$$f_{14} = 0 + 1 = 1$$

$$f_{22} = 5 - 3 = 2$$

$$f_{31} = 2 - 1 = 1$$

$$f_{32} = 1 - 0 = 1$$

$$f_{34} = 0 + 3 = 3$$

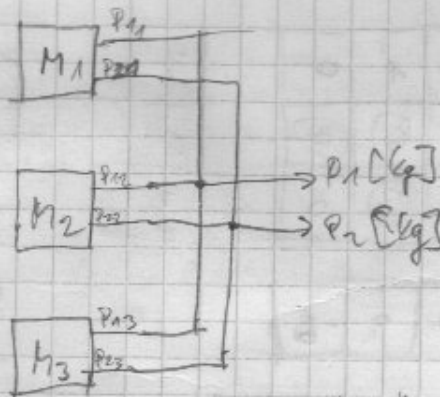
20	50	30	
60			10
	60		

(...)

problem pridruživanja

$$\min z =$$

$$\begin{array}{ll} p_{11} & p_{12} \\ p_{12} & p_{22} \\ p_{13} & p_{23} \end{array}$$



$$\min z = \sum_{i=1}^m \sum_{j=1}^n p_{ij} \cdot x_{ij}$$

$$M_1[u] + p[u] = M_{1uk}[u] \leq 40$$

$$M_2[u] + p[u] = M_{2uk}[u] \leq 40$$

$$M_3[u] + p[u] = M_{3uk}[u] \leq 0$$

$$\max p = p$$

Metoda grananja i ogradjivanja

n poslova

n strojeva

$O(n!)$

jedan na jedan

A1

trošak
7

$$DG = 7 + 40 + 10 + 33 + 14 = 104$$

$$H4 = 7 + 54 + 14 + 61 + 33 =$$

$$7 + 56 + 48 + 29 + 61 = 204$$

13. 11. 2009.

Ovo se može riješiti kao transp. problem:

ovo se riješ. post.

Gran. i ogradj.:

	1	2	3	4	
A	8	2	1	3	1
B	4	5	6	5	1
C	2	1	3	6	1
D	5	3	8	4	1

transf. u transf.
→ probl.

	1	2	3	4	
A	8	2	1	3	1
B	4	5	6	5	1
C	2	1	3	6	1
D	5	3	8	4	1
	1	1	1	1	

	1	2	3	4	
A	$1 + e_1$				
B		$1 + e_2$			
C			$1 + e_3$		
D				1	
	1	1	1	1	
	e_1	e_2	e_3		

$$4 = e_1 + e_2 + e_3 = 4 \checkmark$$

u jednostojno

cjelobrojno programiranje

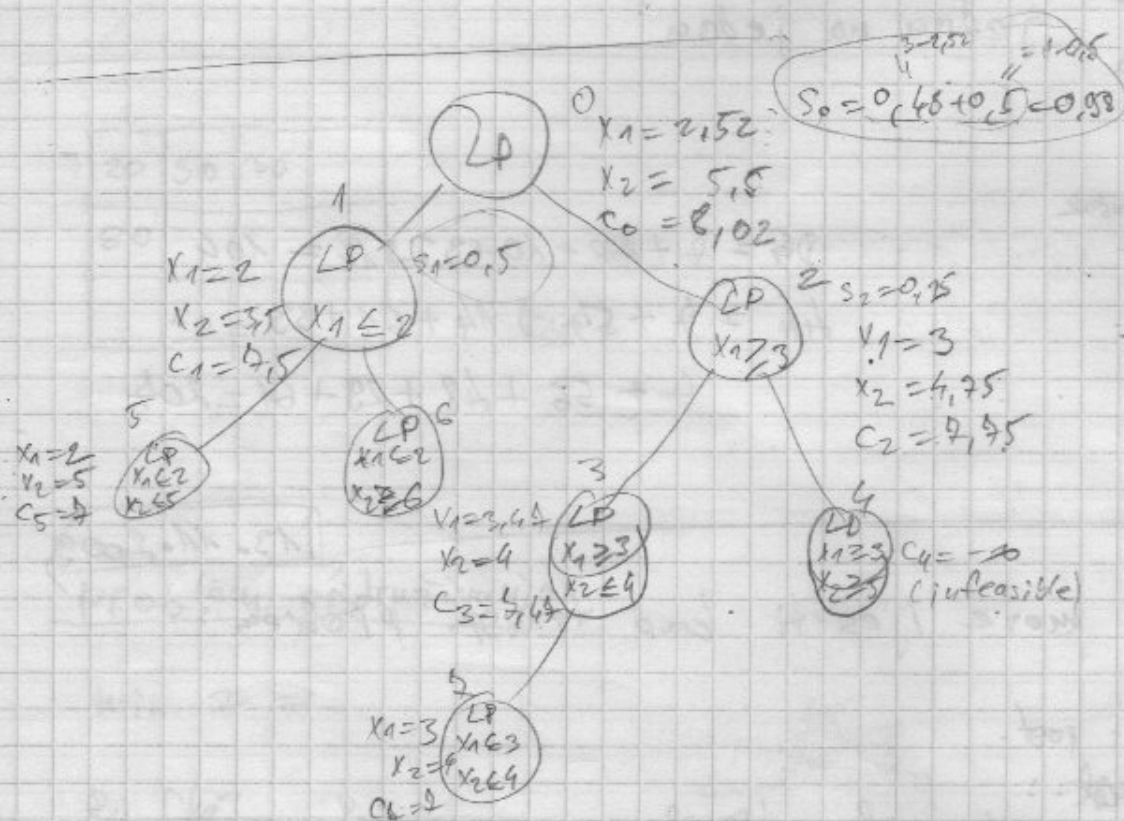
$$\max z = x_1 + x_2$$

← slajd 52

$$x_1 = 2,52$$

$$x_1 \leq 2$$

$$x_1 \geq 3$$



← 2 ima
 najbifin
 cijnu (2,5)
 pa ideemo
 dalje iz 2

- rjesenje (cijelo) je na slajdu 56 - na kraju usimamo
 bilo koje rjesenje (sua su ista) - alternativni
 optimizirani.

$$x_1 = 0, 1, 2, 3, \dots$$

$$x_i = f_0 + 2f_1 + 4f_2$$

$$x = \sum_{i=0}^m f_i \cdot 2^i$$

$j=1, n$ čvorova
 $i=1, m$ grana

[16. 11. 2009.]

$$gubitak = g\% \cdot S_{naga}$$

→ tijekom cijelog životnog vijeka voda

l_i - dužina grane i

$f_i = 0, 1$ - treba li i -ta grana graditi ili ne

$$\min z = \sum_{i=1}^m l_i (f_i + g \cdot f_i + g l_i)$$

$$p_i + l_i \leq C_i \cdot f_i$$

+ idući u jednom
 ili u drugom smjeru

$$k(i, j) = \begin{cases} -1 & i \text{ izlazi iz } j \\ 0 & \text{ne koincidiraju} \\ 1 & i \text{ ulazi u } j \end{cases}$$

← matrica koinkidencije

$$\sum_{i=1}^m k(i, j) \cdot p_i - \sum_{i=1}^m k(i, j) l_i = w_j ; j=1, n$$

$$p_i \geq 0, l_i \geq 0, i=1, m$$

$$f_i = 0, 1, i=1, m$$

Separabilno programiranje:

$$x = \sum_{i=1}^n y_i$$

$$f(x) = y_0 + \sum_{i=1}^n s_i y_i ; y_i \leq f_i ; i=1, n$$

Vještoviće cjelobro. progr. : $q_1 \leq g_1$

$$q_1 \geq g_1 \cdot f_1$$

$$f_1 = 0$$

$$f_1 = 1$$

$$y_1 \leq g_1$$

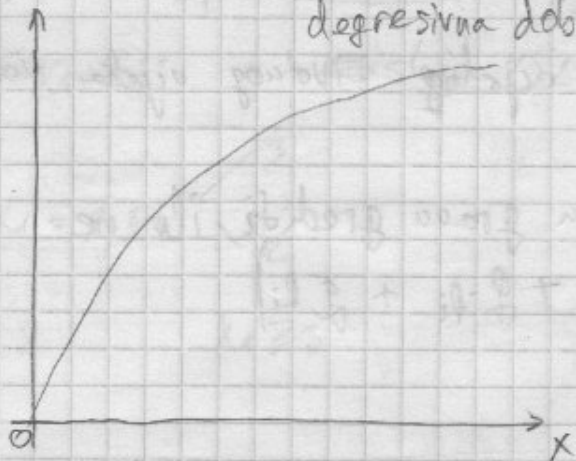
$$y_1 = g_1$$

$$y_1 \geq 0$$

$$q_2 \leq g_2 f_2$$

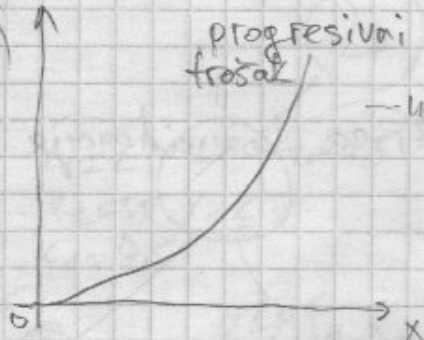
$$y_2 \geq g_2 f_2 \dots$$

max
 $f(x)$



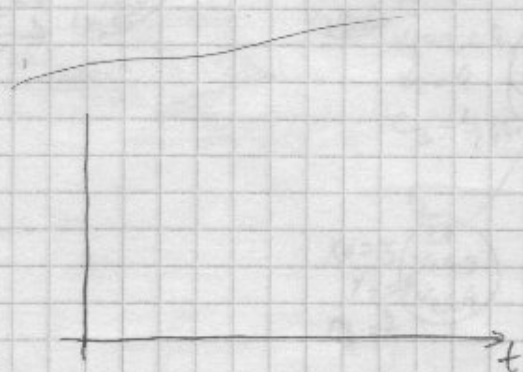
degresivna dobit \rightarrow može LP, ne mora SEP

min
 $f(x)$



progresivni
trošak

\rightarrow može LP, ne mora SEP



Uredništit - do (uključujući) separabilnog program.
(do slojda 62 (bez vjeqa))

LP - odgovor na pitanje: Što roditi, čime, iz čega, koliko
MP (mrežno planiranje) - kada?