Quarantine Effects on Ebola

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MAT3395 Introduction to Mathematical Model
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Introduction to Ebola

Ebola is a serious spreading disease that is usually fatal. The virus was first detected for the first time in 1976, in central Africa, and has repeatedly disappeared and reemerged in outbreaks over the period of time that followed. [1]

In past Ebola outbreaks, the death rate has been between 25 and 90 percent. People are usually infected through contact with infected animals or body fluids from infected humans such as blood and saliva. Thus isolation of the source of infection and associated adjuvant treatment can minimize mortality. There is no accurate account of the origin of the virus, but evidence now suggests that fruit bats may be the host for Ebola. [2]

Research Objective

As previously stated, Ebola can be transmitted through contact with the bodily fluids of an infected person. Therefore, the objective of our study is modelling the development of Ebola virus in Bangladesh (with and without isolation), based on 2020 data.

Naive Assumptions

- The isolation measures are perfect, i.e., that the isolated individuals do not come into contact with the outside world.
- Limited hospital capacity, maximum 133920 hospital beds available (2016). [3]
- People gain antibodies after they recover ($\geq$ 10 years). [4]
- Quarantined population gets additional treatment.

Quarantine Model

$$\frac{dS}{dt} = \pi - \beta SI - \mu S$$

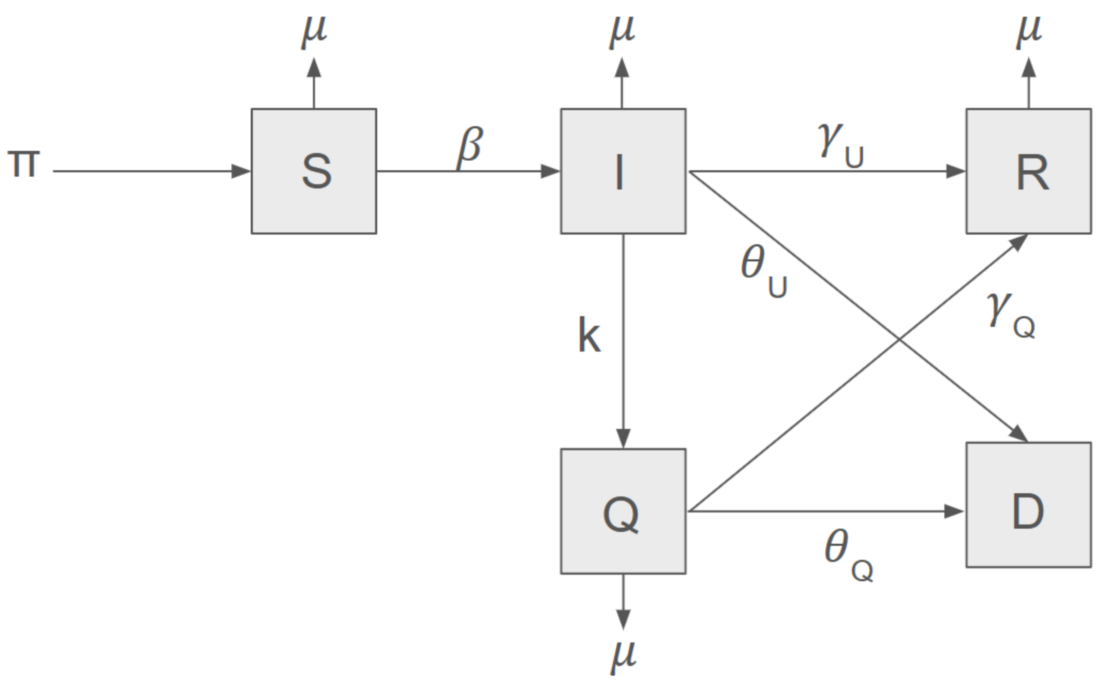
$$\frac{dI}{dt} = \beta SI - kI - \theta_U I - \gamma_U I - \mu I$$

$$\frac{dQ}{dt} = kI - \theta_Q Q - \gamma_Q Q - \mu Q$$

$$\frac{dR}{dt} = \gamma_U I + \gamma_Q Q - \mu R$$

$$\frac{dD}{dt} = \theta_U I + \theta_Q Q$$

Flow Diagram of Quarantine Model



S: Susceptible population

I: Infected population

Q: Quarantine population

R: Recovered population

D: Dead population

Comparison Model

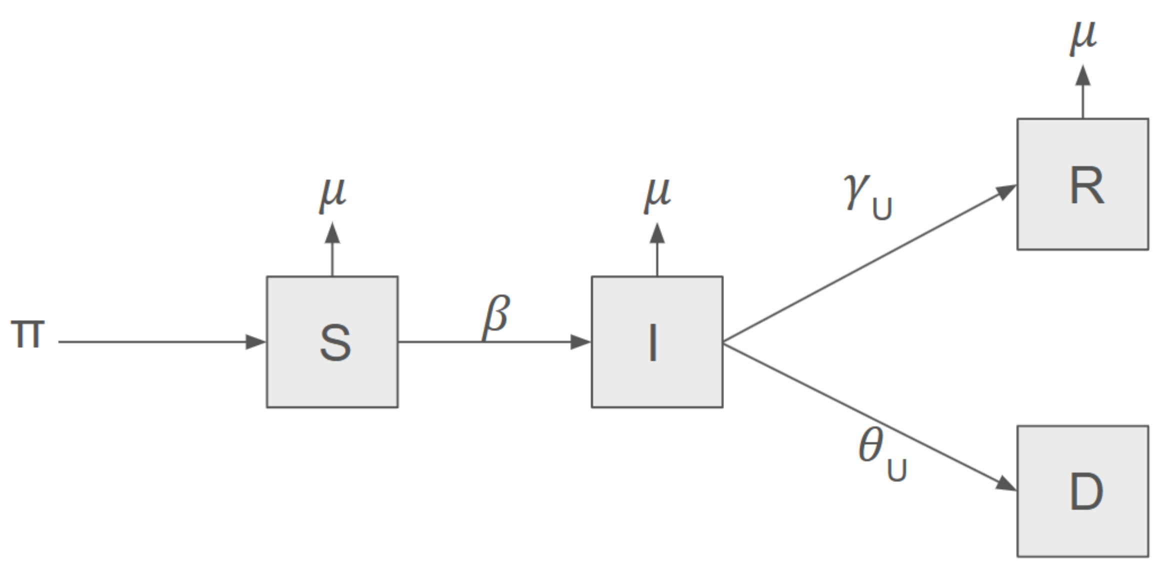
$$\frac{dS}{dt} = \pi - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - (\gamma_U + \theta_U + \mu)I$$

$$\frac{dR}{dt} = \gamma_U I - \mu R$$

$$\frac{dD}{dt} = \theta_U I$$

Flow Diagram of Comparison Model



S: Susceptible population

I: Infected population

R: Recovered population

D: Dead population

Data and Estimated Parameters

Parameter	Description	Value	Source
π	Birth rate	8255.342 humans/day	Birth rate, crude (per 1,000 people) - Bangladesh | Data (worldbank.org)
μ	Death rate	1/26280 day^{-1}	Death rate, crude (per 1,000 people) - Bangladesh | Data (worldbank.org)
k	Quarantine rate of infected population	1/6 day^{-1}	
β	Transmission rate between S and I	1.4e-9 $humans^{-1} \cdot day^{-1}$	
γ_Q	Quarantined recovery rate	1/12 day^{-1}	High Survival Rates and Associated Factors Among Ebola Virus Disease Patients Hospitalized at Donka National Hospital, Conakry, Guinea - PMC (nih.gov)
γ_U	Untreated recovery rate	1/24 day^{-1}	

θ_Q	Quarantine death rate	1/16 day^{-1}	Ebola - Wikipedia
θ_U	Untreated death rate	1/8 day^{-1}	

Parameters explanation

The birth rate(π) and the natural death rate(μ) of Bangladesh in 2020 are given by the World Bank Organization.

The quarantine rate of infected population(k) is based on the method of testing and the time required for testing. We know from CDC that Polymerase chain reaction (PCR) is a very common method for detecting Ebola[5]. And we know from past covid-19 pandemics that PCR usually takes 1 to 3 days to produce results[6], and the average time to locate and isolate an infected person is 3 days. Therefore, we believe that an average of 6 days to isolate an infected person is a reasonable value.

For the untreated infected population, we simply halved and doubled the recovery and death rates based on quarantine parameters.

The transmission rates were taken based on our no-isolation model and an article[7] about the basic reproductive ratio of Ebola. This article, based on data from past Ebola outbreaks, gives a basic reproductive ratio of Ebola of 1.8, we use it as a reference to derive the transmission rate.

Let's find the disease-free equilibrium of our comparison model, that is the model with no interventions. With I = 0, we have the disease -free equilibrium $(\frac{\pi}{\mu}, 0, 0, 0)$. With this, we try to get our R_0 through the Jacobian method.

$$J = \begin{bmatrix} -\beta I - \mu & -\beta S \\ \beta I & \beta S - (\gamma_U + \theta_U + \mu) \end{bmatrix}$$

$$J_{(\frac{\pi}{\mu},0)} = \begin{bmatrix} -\mu & -\beta\frac{\pi}{\mu} \\ 0 & \beta\frac{\pi}{\mu} - (\gamma_U + \theta_U + \mu) \end{bmatrix}$$

$$J_{(\frac{\pi}{\mu},0)} - I\lambda = \begin{bmatrix} -\mu - \lambda & -\beta\frac{\pi}{\mu} \\ 0 & \beta\frac{\pi}{\mu} - (\gamma_U + \theta_U + \mu) - \lambda \end{bmatrix}$$

The third and fourth equations are decoupled from the model, so we only focus on S' and I'. And the two eigenvalues we get are $\lambda = -\mu,\ \beta\frac{\pi}{\mu} - (\gamma_U + \theta_U + \mu)$. The first one is always negative so the second eigenvalue is the one that determines the stability here. Hence $R_0 = \frac{\beta\pi}{\mu(\mu+\theta_U+\gamma_U)}$. In order to find a transmission rate that works under our parameter setting, we substitute the $\mu,\ \theta_U,\ \gamma_U,\ \pi$ in it. With this, we found that when $\beta = 1.4 \times 10^{-9}$, we have a $R_0 \approx 1.8220$ which is pretty close to the number we found in the article. So, we use $\beta = 1.4 \times 10^{-9}$ as our transmission rate for both quarantine and non-quarantine models.

Models Simulations & Analysis

Let's first look at the long-term behaviour of the population if the government and the masses of Bangladesh don't do anything about the spread of Ebola. Here, we assume that the initial number of infections is 100 (I_0).

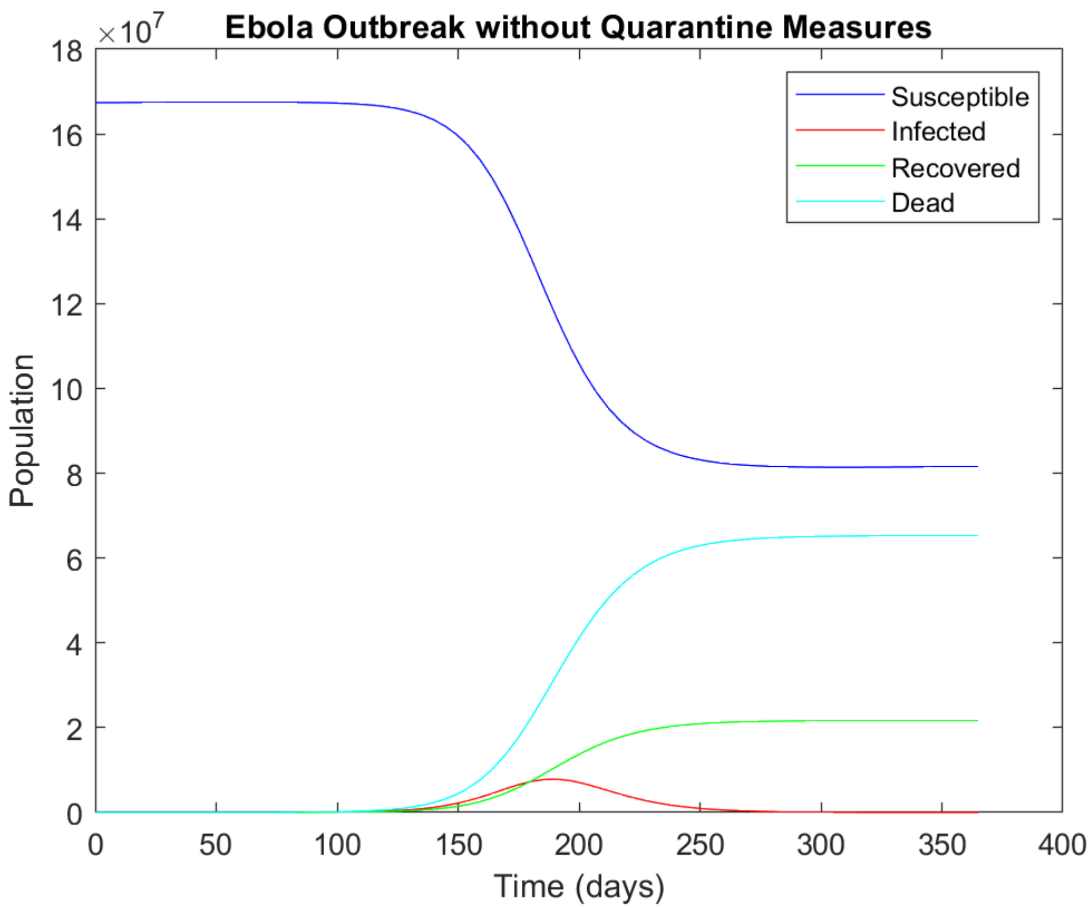


Figure 1. Ebola Outbreak without Quarantine Measures (original setting)

Above figure is the comparison model that does not have any interventions. Unsurprisingly, over the course of a year, almost half of the susceptible population is infected by the Ebola virus, and because of its high mortality rate, the number of deaths from the Ebola virus is close to 70 million, which is a huge catastrophe for a country with a population of 167 million people. Under this model, the number of infections peaks at around 180 days; only 20 million infected people have recovered from Ebola.

We have learned about the horrors of Ebola without any human intervention, so can the spread of the virus be effectively controlled through isolation methods?

Let's see how the population changes in the quarantine model.

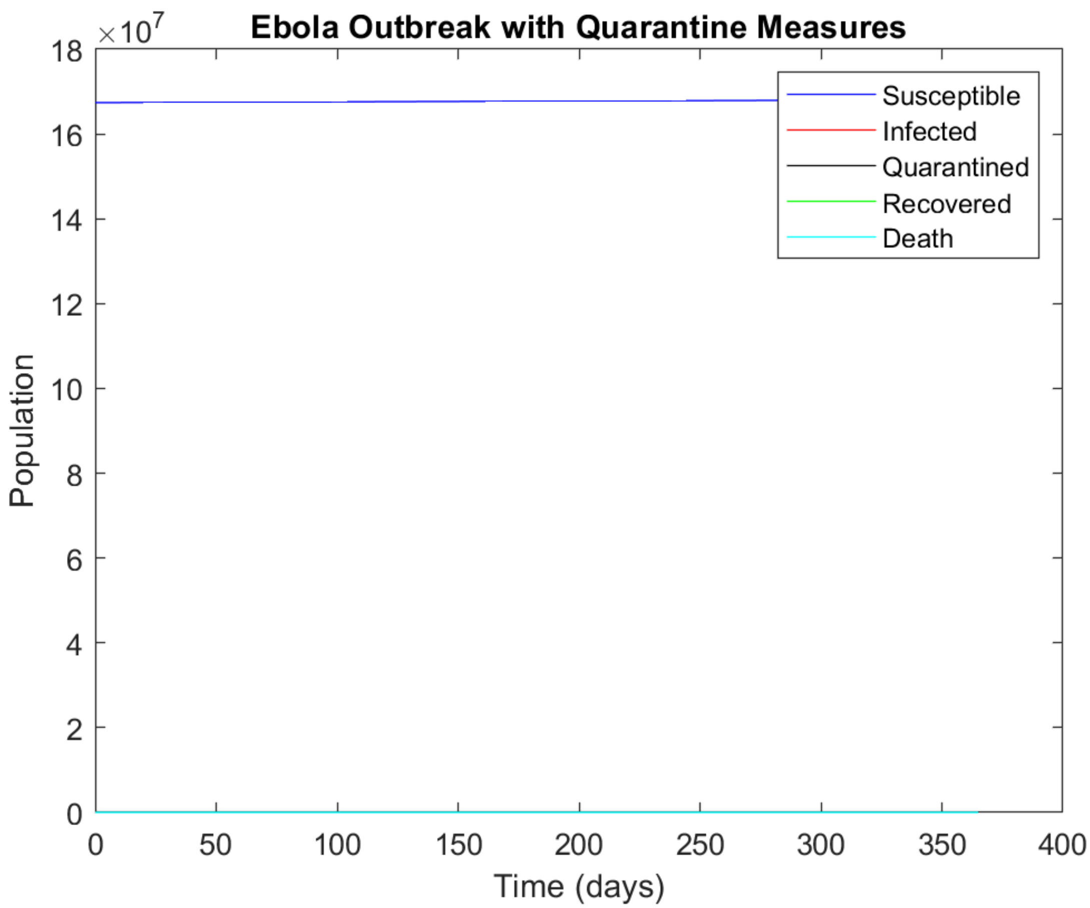


Figure 2. Ebola Outbreak with Quarantine Measures ($k = \frac{1}{6}$)

As can be seen in Figure 2, the susceptible population has not only not declined but has also risen over time, due to the birth rate. While the rest of the population is almost equal to 0. Let's remove the curve for the susceptible population and focus on the changes in the rest of the population.

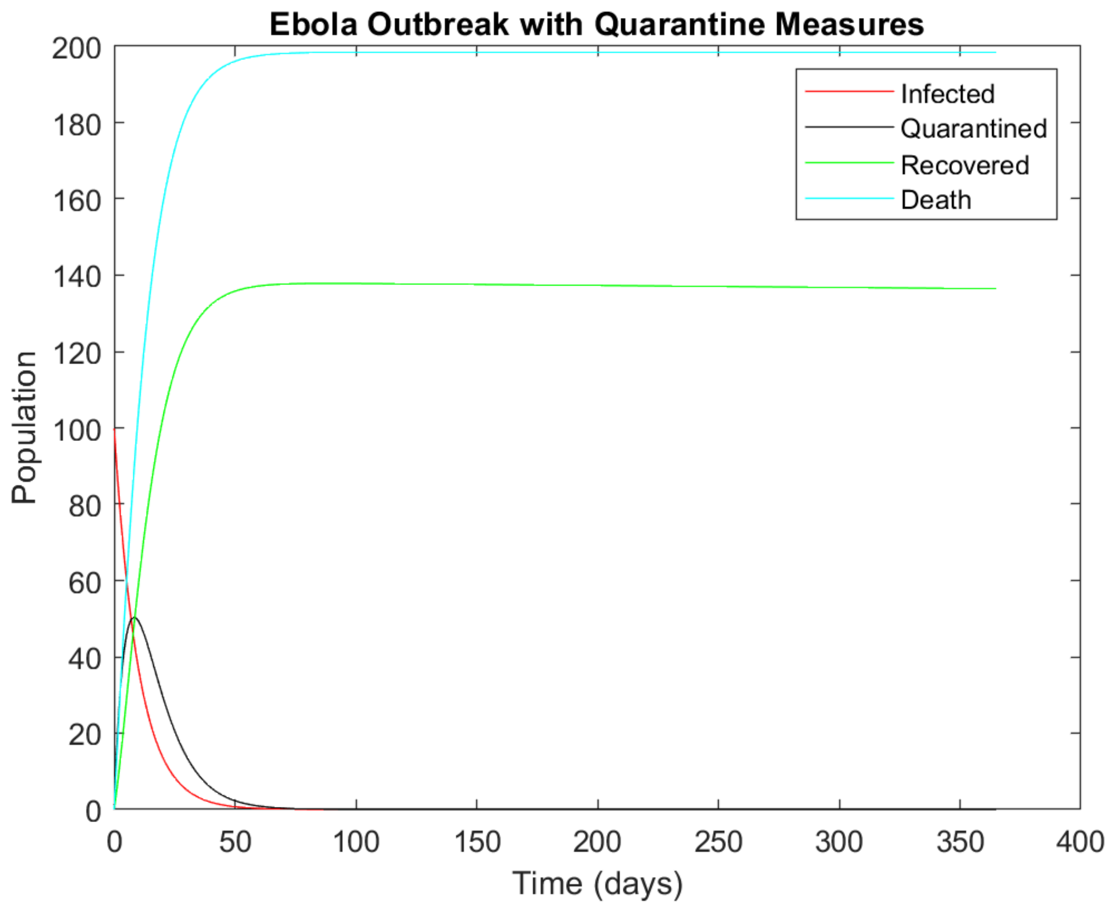


Figure 3. Figure 2 without susceptible population

As can be seen in Figure 3, there are fewer than 200 deaths, otherwise it is clear that the quarantine measures have a felt control over the spread of the Ebola virus. The death toll has plummeted from nearly 65 million to less than 200, but does that make sense in reality?

In reality, hospitals, as well as the government, are unable to detect the first signs of Ebola contagion and are therefore unable to take immediate quarantine measures.

In order to bring our study closer to reality, we imposed additional conditions, that is when the number of infections is greater than 1% of the total population, society becomes concerned about the disease and hospitals begin to accept and isolate Ebola patients.

First, let's look at the impact of isolation on the spread of disease when the number of infections reaches one percent of the total population.

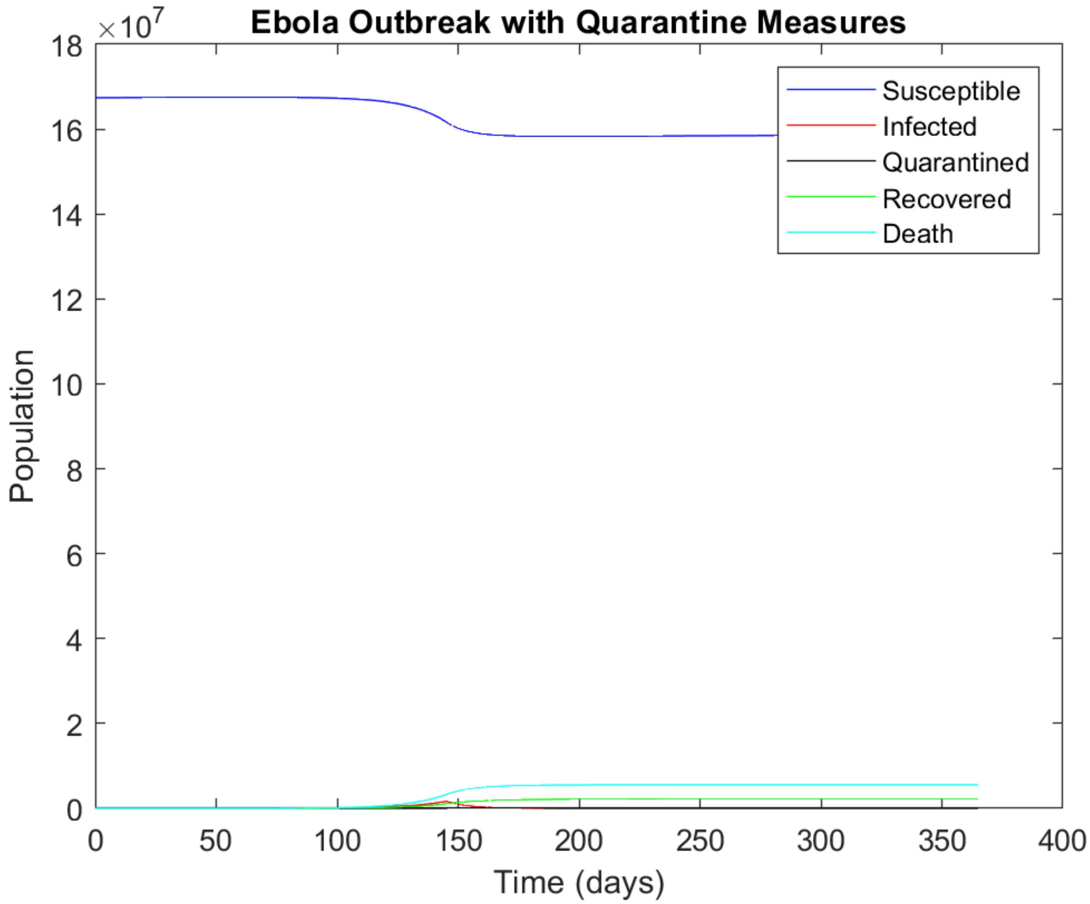


Figure 4. Ebola Outbreak with Quarantine Measures ($k = \frac{1}{6}$)

It is not hard to see from the figure above that the number of deaths has dropped significantly from around 60 million to around 5 million compared to the no-intervention scenario. In order to show the details better, we removed the line of susceptible population and the updated figure is shown below.

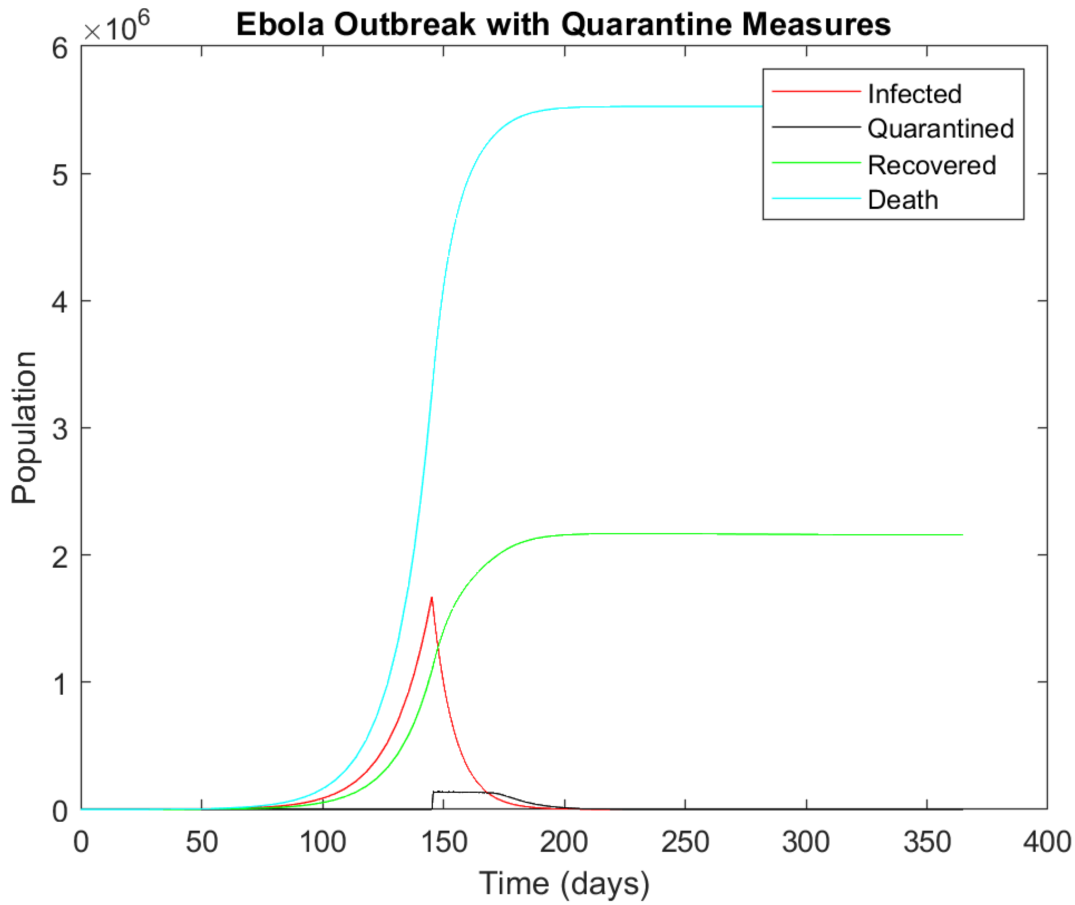


Figure 5. Figure 4 without susceptible population

In figure 5, we still have nearly 5.5 million people who have died from Ebola. When the number of infections reaches 1% of the total population, hospital quarantines intervene and the number of infections drops dramatically in less than 50 days. The situation can get better, because in reality there is great concern in society before the virus infects one percent of the total population. What about the quarantine measure kicks in when there are 0.05% of the total population infected? Let's take a look.

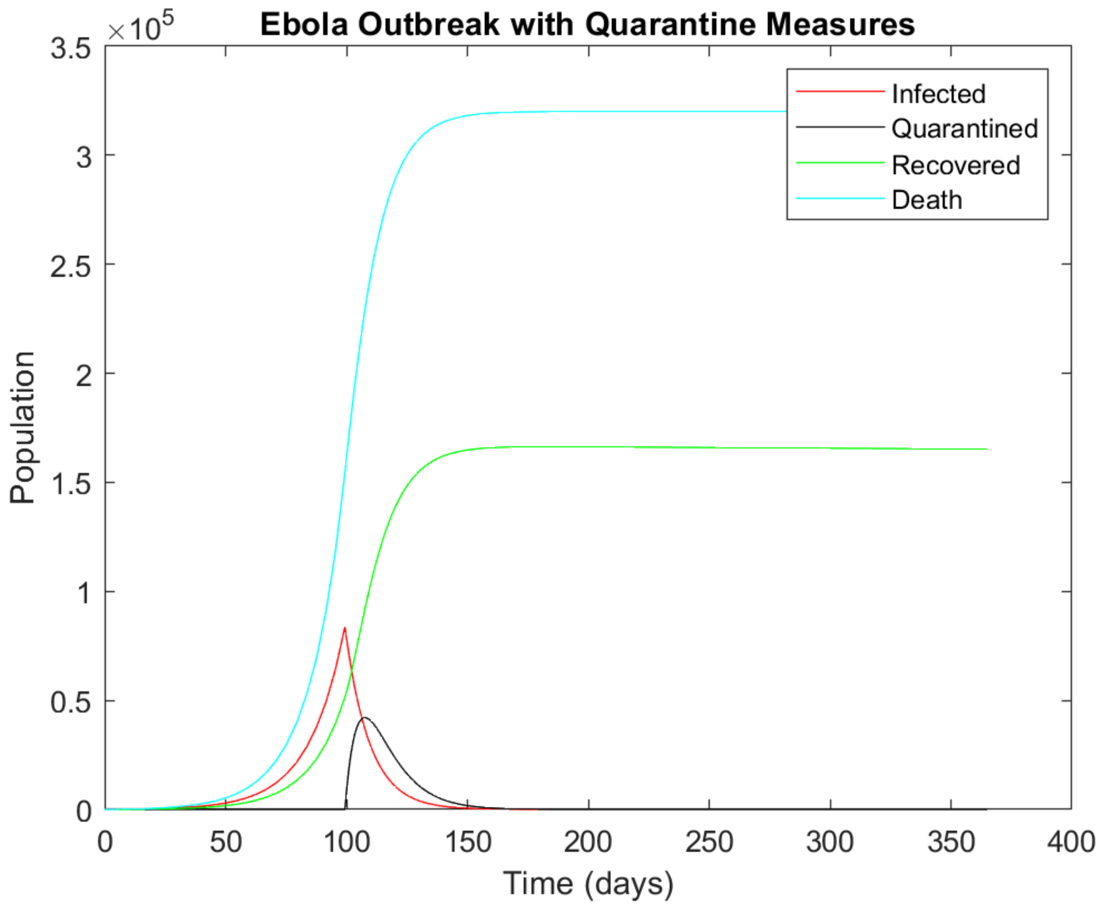


Figure 6. Quarantine after 0.5% total population infected (without susceptible population)

This time, the number of deaths is an order of magnitude less, from 5.5 million to about 0.325 million, and the number of infections is reduced to fewer than 100,000 people. We therefore believe that the earlier human intervention in the disease can maximize the protection of the population from Ebola.

Are there other ways to further mitigate the negative effects of Ebola transmission? We know that the magnitude of the basic reproduction ratio determines whether a virus is able to spread widely in a population. Therefore, we decided to derive the basic reproduction ratio of the quarantine model.

Before deriving the basic reproduction ratio, let's find the disease-free equilibrium first. Again, the fourth and fifth equations are decoupled since S', I' and Q' do not depend on them. The system has no infected population when both I and Q are 0. Thus the disease-free equilibrium is $(\frac{\pi}{\mu}, 0, 0)$. We use the Jacobian method to derive the basic reproductive ratio here.

$$J = \begin{bmatrix} -\beta I - \mu & -\beta S & 0 \\ \beta I & \beta S - (k - \gamma_U + \theta_U + \mu) & 0 \\ 0 & k & -\theta_Q - \gamma_Q - \mu \end{bmatrix}$$

$$J_{(\frac{\pi}{\mu},0,0)} = \begin{bmatrix} -\mu & -\beta\frac{\pi}{\mu} & 0 \\ 0 & \beta\frac{\pi}{\mu} - (k + \gamma_U + \theta_U + \mu) & 0 \\ 0 & k & -\theta_Q - \gamma_Q - \mu \end{bmatrix}$$

$$J_{(\frac{\pi}{\mu},0,0)} - \lambda I = \begin{bmatrix} -\mu - \lambda & -\beta\frac{\pi}{\mu} & 0 \\ 0 & \beta\frac{\pi}{\mu} - (k + \gamma_U + \theta_U + \mu) - \lambda & 0 \\ 0 & k & -\theta_Q - \gamma_Q - \mu - \lambda \end{bmatrix}$$

The three eigenvalues are $\lambda = -\mu,\ \beta\frac{\pi}{\mu} - (k + \gamma_U + \theta_U + \mu),\ -(\theta_Q + \gamma_Q + \mu)$. The first and third eigenvalues are always negative, So the second eigenvalue determines the stability of this system. Thus the basic reproductive ratio is

$$R_o = \frac{\beta\pi}{\mu(k+\gamma_U+\theta_U+\mu)}$$

With our original setting, $R_0 \approx 0.911$. That means that each infected person can only infect 0.911 people, which is less than 1. The coefficient associated with isolation in this ratio is k, the quarantine rate for the infected population. If we want to further control the spread of the virus, we need to make the basic reproductive ratio smaller, we can increase the coefficient k to make the denominator larger and thus the ratio smaller. The original $k = 1/6$ day^{-1}, that is averagely it takes 6 days to quarantine an infected person. If the government and hospitals can respond quickly to Ebola, perhaps they can reduce that time to 3 days? This gives us a $R_0 \approx 0.6074$. Let's see what effect changing the parameter k will have.

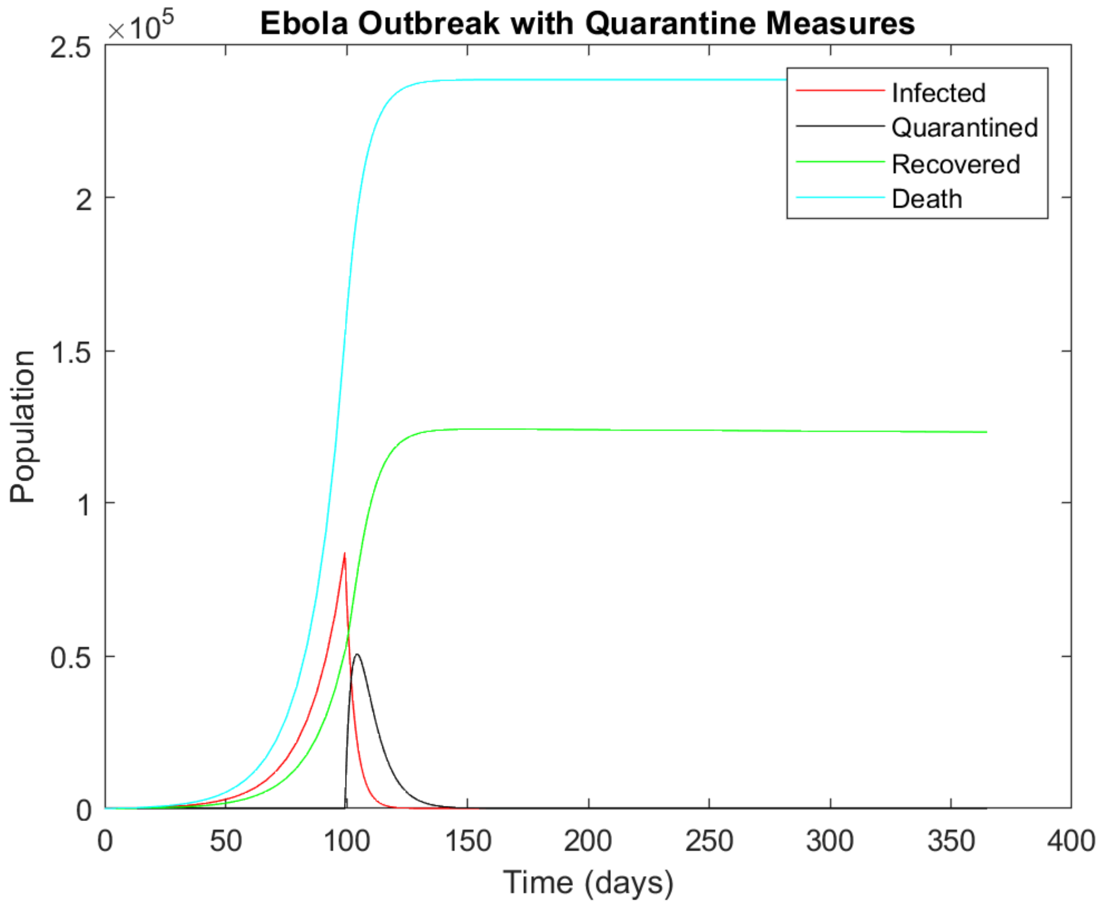


Figure 7. Ebola Outbreak with Quarantine Measures ($k = \frac{1}{3}$, without susceptible population)

Again, society becomes concerned about the Ebola virus when the number of infected persons reaches 0.05% of the total population. Figure 7 looks very similar to Figure 6, but a closer look reveals that the number of deaths has dropped from the previous 325,000 to less than 250,000, and the number of infections has also fallen more rapidly since the quarantine system intervened.

Conclusion

By comparing two different models with no human intervention and with quarantine, we find that when quarantine intervenes, it does ameliorate the negative impact of Ebola on the population, effectively reducing the number of people infected as well as the number of people who die from Ebola, and likewise shrinking the length of the outbreak.

By adjusting the conditions of the quarantine intervention, we have found that the earlier Ebola is detected, the better the spread of the virus can be controlled, which is very conducive to reducing the number of deaths. This means that in reality, Bangladesh government units such as centers for disease control department need to keep a close eye on the spread of Ebola in the country, and it is important to take control measures as early as possible.

Also, we find from the basic reproductive ratio in the quarantine model that adjustments to the quarantine rate can also help us control the spread of Ebola. By decreasing the time required to isolate an infected person, it can effectively reduce the duration of an Ebola outbreak. This suggests that the Bangladesh government and hospitals need to be more efficient in detection as well as isolation in the face of an Ebola outbreak, and that a rapid response to the virus is important.

Reference

1. What is Ebola Disease? | Ebola (Ebola Virus Disease) | CDC
2. Ebola virus disease (who.int)
3. Hospital beds (per 1,000 people) - Bangladesh | Data (worldbank.org)
4. Survivors | Treatment | Ebola (Ebola Virus Disease) | CDC
5. Diagnosis | Ebola (Ebola Virus Disease) | CDC
6. Testing for COVID-19: When to get tested and testing results - Canada.ca
7. A systematic review of early modelling studies of Ebola virus disease in West Africa - PMC (nih.gov)