

# Formulas – Lines in the Cartesian Plane

## 1. Types of Lines

Lines parallel to the  $y$ -axis

$$x = k \quad k \in \mathbb{R}$$

Lines parallel to the  $x$ -axis

$$y = k \quad k \in \mathbb{R}$$

## 2. Line Equations

Implicit form

$$ax + by + c = 0 \quad a, b, c \in \mathbb{R}$$

Explicit form

$$y = mx + q$$

Where:  $m$  = slope  $q$  =  $y$ -intercept

## 3. Distances and Notable Points

Distance between two points

$$d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

Midpoint

$$\begin{cases} x_M = \frac{x_a + x_b}{2} \\ y_M = \frac{y_a + y_b}{2} \end{cases}$$

Centroid of a triangle

$$\begin{cases} x_G = \frac{x_a + x_b + x_c}{3} \\ y_G = \frac{y_a + y_b + y_c}{3} \end{cases}$$

## 4. Line through Points

Line through two points

$$\frac{y - y_a}{y_b - y_a} = \frac{x - x_a}{x_b - x_a}$$

Line through a point and slope  $m$

$$y - y_a = m(x - x_a)$$

## 5. Intersections and Distances

Intersection of two lines

$$\begin{cases} r_1 : a_1x + b_1y + c_1 = 0 \\ r_2 : a_2x + b_2y + c_2 = 0 \end{cases}$$

Point-line distance

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

## 6. Relationships Between Lines

Parallel lines

$$m_1 = m_2$$

Perpendicular lines

$$m_1 = -\frac{1}{m_2}$$

Otherwise: intersecting lines

## 7. Slope ( $m$ )

From implicit form

$$m = -\frac{a}{b}$$

From two points

$$m = \frac{y_b - y_a}{x_b - x_a}$$