1. Battle Of Bismark Sea (1943)

Here we are going to look at a real life example of a two person total conflict game that happened back in 1943. As some back story, this is set in the South Pacific, and there are two generals, Imamura who needs to transport Japanese troops, and Kenney who needs to bomb Japanese transports. There are two different paths or routes, one North and one South. The North path is much shorter, while the South path is much longer. Imamura needs to figure out which path to take and Kenny needs to figure out which path to send the bombers. If Kenny picks the wrong path, he can recall the bombers later but the number of bombing days would be reduced. Here is table 10.3 from the book:

Table 10.3 The Battle of the Bismarck Sea with payoffs to (Kenney, Imamura)

		Imamura	
		North	South
Kenney	North	$(2, -2)$	$(2, -2)$
	South	$(1, -1)$	$(3, -3)$

As you can see the total payout for each cell adds up to 0, which confirms that this game is a total conflict game. Therefore we can simply drop all of the second outcomes and only list Kenny's outcomes.

Table 10.4 The Battle of the Bismarck Sea as a zero-sum game

		Imamura	
		North	South
Kenney	North	2	2
	South	1	3

Recall that Imamura is trying to make these values as small as possible, therefore Imamura has a dominate strategy of always picking North. Therefore because of this Kenny will always also choose North. Therefore we have a Nash Equilibrium at North North, and the point (2,2). Indeed what happened back in 1943 was both generals picked North. However lets now look what happens if communication is allowed. We have two cases on who goes first:

- If Imamura goes first, we already know that he will pick the North strategy since that is the dominate strategy for Imamura, therefore we end up at the Nash equilibrium 2.

- If Kenny goes first, we already have from minimax that Kenny will pick North. Now we actually have two outcomes, North North for a payout of 2, or North South for a payout of 2. In this case both outcomes have the same value so we could end up at either of them.

2. Penalty as a game

Here we will look at penalty kicks in soccer as a game from both the view of the penalty kicker, and the view of the goalie. We are going to simplify this game as look at this as the kicker can either kick to the left or kick to the right. Similarly the goalie can either attempt to dive to the left or dive to the right. We are simplifying this game by ignoring any kicks in the middle or choice for the goalie to not move at all. From the perspective if the kicker kicks the ball to where the goalie is not, then the kicker gets a payout of 1 and the goalie gets a payout of -1. On the other hand if the kicker kicks the ball to where the goalie is, then the kicker gets a payout of -1, and the goalie gets a payout a 1.

		Goalie	
		Dive left	Dive right
Kicker	Kick left	$(-1, 1)$	$(1, -1)$
	Kick right	$(1, -1)$	$(-1, 1)$

Just like before this is a total conflict game since all of the cells add up to 0. Therefore replace the table with only the kicker's values.

		Goalie	
		Dive left	Dive right
Kicker	Kick left	-1	1
	Kick right	1	-1

Minimax and Maximin both end up with the conclusion that neither choice matters. Now the question is what happens with this game?

We can use the Method of Oddments. Let x be the probability of the kicker kicking to the left.
$\Delta everything$=1 in this case, and the total is therefore 2. Therefore the probability of kicks that should go left = $\frac{1}{2}$ = the probability of kicks

that should go right.

```
A=-1;
B=1;
C=1;
D=-1;
Delta1=abs(A-B);
Delta2=abs(C-D);
Total=Delta1+Delta2;
x=Delta2/Total
1-x
```

The code for method of augments is very simple only needs a few calculations and finds x and 1-x, and similarly could be used from the goalie's perspective. What would have to change for the goalie's perspective? This game isn't that interesting has there is no benefit from either side. In the real world we don't get this exact game. Ignacio Palacios-Huerta did a study about the Italian Football League in 2002. Based on the analysis of 1400 penalty kicks, he came up with the following table:

		Goalie	
		Dive left	Dive right
Kicker	Kick left	0.58, –0.58	0.95, –0.95
	Kick right	0.93, –0.93	0.70, –0.70

A few things to note at the start:
All of the cells still add up to the same number (specifically 0 in this case) so it is still a total conflicts game.
If we look at the sum of the entries on a row we can immediately tell which choice or more worthwhile, however this doesn't give us the odds for playing each position.

So again we go back to the methods of oddments.
We can use the Method of Oddments. Let x be the probability of the kicker kicking to the left, then look at the expected values for the goalie's pure strategies
Using the Method of oddments we get that $x = 0.38\bar{3}$, $1 - x = 0.61\bar{6}$, $y = 0.41\bar{6}$, $1 - y = .58\bar{3}$.