We consider a batter-pitcher duel between Ryan Howard of the Philadelphia Phillies and various pitcher's in the national league. The pitcher will throw a fastball(FB), a splitfingered fastball(SF), a curve ball(CB), and a changeup(CH). In this example, we obtained the data from the Internet (www.STATS.com). We consider both a right-handed pitcher (RHP) and a left-handed pitcher (LHP) separately in this analysis.

| Howard/RHP | FB | CB | CH | SF |
|---|---|---|---|---|
| **FB** | .337 | .246 | .220 | .200 |
| **CB** | .283 | .571 | .339 | .303 |
| **CH** | .188 | .347 | .714 | .227 |
| **SF** | .200 | .227 | .154 | .500 |

```
import nashpy as nash
import numpy as np
from scipy.optimize import linprog

A = np.array([[0.337, 0.283, 0.188, 0.200], [0.246, 0.571, 0.347, 0.227],
 [0.220, 0.339, 0.714, 0.154], [0.200, 0.303, 0.227, 0.500]])
B = 1-A
prisoners_dilemma = nash.Game(B, A)

equilibria1=prisoners_dilemma.support_enumeration()
equilibria2=prisoners_dilemma.vertex_enumeration()
equilibria3=prisoners_dilemma.lemke_howson_enumeration()
for eq1 in equilibria1:
    print(eq1)
    outcome=prisoners_dilemma[eq1]
    print(outcome,'outcome1')
for eq2 in equilibria2:
    print(eq2)
    outcome=prisoners_dilemma[eq2]
    print(outcome,'outcome2')
for eq3 in equilibria3:
    print(eq3)
    outcome=prisoners_dilemma[eq3]
    print(outcome,'outcome3')

G=1-A

(n,m) = np.shape(G)

A_ub = -np.transpose(G)
# we add an artificial variable to maximize, present in all inequalities
A_ub = np.append(A_ub, np.ones((m,1)), axis = 1)
# all inequalities should be inferior to 0
b_ub = np.zeros(m)

# the sum of all variables except the artificial one should be equal to one
A_eq = np.ones((1,n+1))
A_eq[0][n] = 0
b_eq = 1

c = np.zeros(n + 1)
# -1 to maximize the artificial variable we're going to add
```

```
c[n] = -1

res = linprog(c, A_ub=A_ub, b_ub=b_ub, A_eq=A_eq, b_eq=b_eq,
 bounds=(0,None),method='revised simplex')

print((res.x[:-1], res.fun))
```

First off there is a lot of repeated code for solving this question. eq1, eq2, eq3, and the bottom code as well are all solving and getting the exact same solution:
0.65935396, 0. , 0.03241071, 0.30823533
This first part of the output means that the pitcher should throw approximately 66% FB, 0 CB, 3% CH, 31% SF, for an overall batting average of at most about .291, which is found in the second value of the output:
0.70902029 0.29097971
This means the batter can at most hit .291

Now lets look at the same questions form the batter perspective:
Three changes to the code:

```
A=A.transpose()
prisoners_dilemma = nash.Game(A, B)
G=A
```

The top ones is to switch the perspective for the first few solvers, and the second line is to show that we are now looking at the other game for the solution. The G=A shows that we are now looking at the batter instead of G=1-A for the pitcher.
We now get FB 27%, guess CB 64%, and guess SF 8% of the time to obtain a .291 batting average.

## 1. Cuban Missile Crisis

We will highlight the scenario from 1962. The Cuban missile crisis was precipitated by a Soviet attempt in October 1962 to install medium-range and intermediate-range nuclear armed ballistic missiles in Cuba that were capable of hitting a large portion of the United States. The goal of the United States was immediate removal of the Soviet missiles, and U.S. policy makers seriously considered two strategies to achieve this end: naval blockade or air strikes.

In President Kennedy's speech to the nation, he explained the situation as well as the goals for the United States. He set several initial steps. First, to halt the offensive buildup, a strict quarantine on all offensive military equipment under shipment to Cuba was being initiated. He went on to say that any launch of missiles from Cuba at anyone would be considered an act of war by the Soviet Union against the United States, resulting in a full retaliatory nuclear strike against the Soviet Union. He called on Khrushchev to end this threat to the world and restore world peace.

Assumptions: We assume the two opponents are rational players. Model Building: The goal of the United States was the immediate removal of the Soviet missiles, and U.S. policy makers seriously considered two strategies to achieve this end:

- 1. A naval blockade (B), or "quarantine" as it was euphemistically called, to prevent shipment of more missiles, possibly followed by stronger action to induce the Soviet Union to withdraw the missiles already installed.
- 2. A "surgical" air strike (A) to wipe out the missiles already installed, insofar as possible, perhaps followed by an invasion of the island.

The Soviet policy had the following choices:

- Withdrawl the missiles
- Maintenance of the missiles

This leads to the following payout diagram:

| | | Soviet Union | |
|---|---|---|---|
| | | Withdraw missiles (W) | Maintain missiles (M) |
| United States | Blockade (B) | (3, 3) | (2, 4) |
| | Air strike (A) | (4, 2) | (1, 1) |

What does the movement diagram look like, and are there any Nash Equilibrium?

As in Chicken, if both players attempt to get to their equilibrium, the outcome of the games ends up at (1, 1). This is disastrous for both countries and their leaders. The best solution is the (3, 3) compromise position. However, (3, 3) is not stable. This will eventually put us back at (1, 1). In this situation, one way to avoid this Chicken dilemma is to try strategic moves.

Both sides did not choose their strategies simultaneously or independently. Soviets responded to the US blockade after it was imposed. The United States held out the chance of an air strike as a viable choice even after the blockade. If the Soviet Union would agree to remove the weapons from Cuba, the United States would agree (a) to remove the quarantine and (b) not to invade Cuba. If the Soviets maintained their missiles, the United States preferred the air strike to the blockade. Attorney General Robert Kennedy said, "If they do not remove the missiles, then we will". The United States used a combination of promises and threats. The Soviets knew our credibility in both areas were high (strong resolve). Therefore, they withdrew the missiles, and the crisis ended. Khrushchev and Kennedy were wise.

Both sides considered more than the two alternatives listed, as well as several variations on each. The Soviets, for example, demanded withdrawal of American missiles from Turkey as a quid pro quo for withdrawal of their own missiles from Cuba, a demand publicly ignored by the United States.

Although in one sense the United States "won" by getting the Soviets to withdraw its missiles, Premier Nikita Khrushchev of the Soviet Union at the same time extracted from President Kennedy a promise not to invade Cuba, which seems to indicate that the eventual outcome was a compromise of sorts. But, this is not game theory's prediction for Chicken because the strategies associated with compromise do not constitute a Nash equilibrium.

To see this, assume play is at the compromise position (3, 3), that is, the United States blockades Cuba, and the Soviet Union withdraws its missiles. This strategy is not stable because both players would have an incentive to defect to their more belligerent strategy. If the United States were to defect by changing its strategy to air strike, play would move to (4, 2), improving the payoff the United States received; if the Soviet Union were to defect by changing its strategy to maintenance, play would move to (2, 4), giving the Soviet Union a payoff of 4. (This classic game theory setup gives us no information about which outcome would be chosen because the table of payoffs is symmetric for the two players. This is a frequent problem in interpreting the results of a game

theoretic analysis, where more than one equilibrium position can arise.) Finally, should the players be at the mutually worst outcome of (1, 1), that is, nuclear war, both would obviously desire to move away from it, making the strategies associated with it, like those with (3, 3), unstable.

## 2. Writers Guide Strike

The 2007-2008 Writers Guild of America strike was a strike by the Writers Guild of America, East (WGAE) and the Writers Guild of America, West (WGAW) that started on November 5, 2007. The WGAE and WGAW were two labor unions representing film, television, and radio writers working in the United States. Over 12,000 writers joined the strike. These entities will be referred to in the model as the Writers Guild.

The strike was against the Alliance of Motion Picture and Television Producers (AMPTP), a trade organization representing the interests of 397 American film and television producers. The most influential of these are eight corporations: CBS Corporation, Metro-Goldwyn-Mayer, NBC Universal, News Corp/Fox, Paramount Pictures, Sony Pictures Entertainment , the Walt Disney Company, and Warner Brothers. We refer to this group as Management.

The Writers Guild indicated their industrial action would be a "marathon." AMPTP negotiator Nick Counter indicated negotiations would not resume as long as strike action continued, stating, "We're not going to negotiate with a gun to our heads-that's just stupid." The last such strike in 1988 lasted 21 weeks and 6 days, costing the American entertainment industry an estimated $500 million ($870 million in 2007 dollars). According to a report on the January 13, 2008, edition of NBC Nightly News, if one takes into account everyone affected by the current strike, the strike had cost the industry $1 billion so far; this is a combination of lost wages to cast and crew members of television and film productions and payments for services provided by janitorial services, caterers, prop and costume rental companies, and the like. The TV and movie companies stockpiled "output" so that they could possibly outlast the strike rather than work to meet the demands of the writers and avoid the strike.

The strategies for the Writers Guild (Rose):

- Strike (S)
- Not to Strike (NS)

The strategies for the Management (Colin):

- Increase net Salary (IN)
- Status Quo (SQ)

Now for the rankings for both sides. The Writers Rankings from 1 to 4 where 4 is the best:

- S SQ (1)
- NS SQ (2)
- S IN (3)
- NS IN (4)

The Management Rankings from 1 to 4 where 4 is the best:

- S IN (1)
- NS IN (2)
- S SQ (3)
- NS SQ (4)

| | | Management (Colin) | |
|---|---|---|---|
| | | $SQ$ | $IN$ |
| Writers Guild (Rose) | $S$ | (1, 3) | (3, 2) |
| | $NS$ | (2, 4) | (4, 1) |

What does the movement diagram look like, and are there any Nash Equilibrium?

The Writers Guild considers a threat and tells Management that if it chooses SQ that it will strike, putting the players at (1, 3). This result is indeed a threat as it is worse for both the Writers Guild and Management. However, the options for Management under IN are both worse than (1, 3), so Management does not accept the threat. The guild does not have a promise. At this point, we might involve an arbiter.