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Optimal Path Planning for Mobile Robot Navigation Using FA-TPM in Cluttered Dynamic Environments

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Abstract

Mobile Robot Navigation(MRN) in moving obstacles environment is a very difficult research area in field of robotics. Nowadays, mostly Nature-inspired algorithms are used methods used for autonomous MRN. Many algorithms in MRN are developed by simulating the swarm behavior of various creatures like ants, fireflies, honey bees, cuckoos, spider monkeys, whales and the outcomes are very motivating. The paper presents an hybrid of Firefly algorithm (FA)along with Three Path Method (TPM) in [35] for MRN in cluttered dynamic environment. TPM check the free path in the environment and whenever all the paths are blocked by obstacles, FA is used for avoiding obstacles. The key concept of the proposed method is the searching of brighter firefly from the groups. Several experiments are performed by considering two environments to validate the performance of the proposed hybrid method and the results are compared with existing navigation methods. It is observed that the proposed hybrid method shows superior performance in terms of reduced path length and lower computational cost.

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Keywords: Firefly algorithm; Mobile Robot; Navigation; Obstacle avoidance; Dynamic environment; Three Path Method

1. Introduction

Over the last decades, MRN is the main topics in robotics research. Researchers have developed different navigation algorithms like fuzzy logic, genetic algorithm, neural networks, potential field method, Particle Swarm Optimization, others optimization and hybrids methods. For efficient path planning, it is very necessary to use efficient navigation algorithm.

A mobile robot motion planning and collision avoidance method[1] based on simultaneous ultrasonic sensor and camera range measurements is developed in an indoors environment. A visual navigation method based on a sketched

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semantic map is presented in [2] by an imitating human inquiry navigation method in a unknown and dynamic environment. In [3], InsertBug algorithm is introduced to describe all the planned paths and stored by vectors.

A neural network and reinforcement learning based techniques is addressed [4-5], to enable a mobile robot to learn constructed environments on its own and automatically navigate. Odometry method is introduced in [6] to carry the mobile robot to reach its goal. But it has drifting problems.

A novel control strategy is introduced[7], to allow a class of mobile robot to navigate in a dynamic and potentially cluttered environments. In [8], an effective human comfortable safety framework is presented for mobile robot navigation systems in social environments. An evolutionary based navigation method is developed in [9], to trace the collision free near optimal path for underwater robot in a three-dimensional scenario.

In [10], a hybrid motion planning method is developed based on the invasive weed optimization and neuro-fuzzy controller. The invasive weed optimization is used to train the premise parameters. A new way to combine metric and topological information in a common map for large environment is performed [11] and a new methodology is introduced to solve mobile robot path planning by determining the points of the obstacles and path improvement algorithm.

A singleton type-1 fuzzy logic system(T1-SFLS) controller and Fuzzy-Wind Driven Optimiztion algorithm is introduced [13] for autonomous mobile robot path planning in an unknown static and dynamic environments. In [14], neural network based control system is developed for control of unmanned agricultural vehicles. A biological inspired episodic cognitive map building framework and odometry along with electronic compass are presented in [15] and [16] respectively.

A stable switched system [17] and sampling based algorithm [18] are addressed for navigation of wheeled mobile robot in an unknown environments. A survey is presented on car like mobile robot navigation [19]. A fuzzy logic controller [20] and free segments with turning points algorithm [21] are developed for mobile robot navigation in static environments. Matrix-Binary Codes based genetic algorithms [22] and Artificial Neural Networks along with genetic algorithm [23] are used for path planning of mobile robot in an unknown environments.

In [24], a control scheme for navigation and obstacle avoidance is introduced for autonomous flying agent. It utilizes a PID controller which regulates the error dynamics to stabilize the navigation. A navigational controller has been developed in [25] for a humanoid robot by using fuzzy logic for avoiding the obstacles present and reaching the target safely in environments. Firefly algorithm [26], Particle Filter and FAST-SLAM algorithm [27] and Topical local-metric framework [28] are presented for path planning of mobile robot in dynamic environments.

A mid vehicle collision detection and avoidance system with a constraint free condition is introduced [29]. It produce mid vehicle maneuvers particularly when jammed between the front and rear vehicles. Three path estimation models based on crisp, fuzzy and fuzzy regression logic are blended to formulate a mid vehicle collision detection and avoidance system. Hybrid Cuckoo Search-Bat algorithm [30] and Genetic algorithm [31] are presented for optimal path planning of autonomous mobile robot in an unknown and known environments respectively.

As per from the literature surveys, it is found that most of the existing navigation methods are implemented in static environment and has more path length and more computational cost. And mostly the method used are of single optimization method. In this paper, we addresses a novel hybrid method of FA-TPM for solving the navigational problem of mobile robot in dynamic environments. FA is an meta-heuristic search algorithm developed by Xin-She Yang in [33], which is inspired by the flashing behavior of fireflies. The number of parameters to be tuned in FA is less than that of CS(Cuckoo search), GA(Genetic Algorithm) and PSO (Particle Swarm optimization). An advantage of the FA algorithm is that it is more generic to implement to a wider class of optimization problems. TPM algorithm which is presented in [35] is used. TPM will checked the free path and FA will be implemented when all the three paths are blocked by obstacles. Finally, some simulation and experiments are carried out to prove the effectiveness of the proposed hybrid algorithm.

Section 2 describes the detail discussion about the problem formulation of mobile robot navigation with FA and TPM. Simulation results are presented in section 3. Evaluation and Discussion is presented in Section 4. In Section 5, Experimental results are presented. Finally, conclusion is given out in Section 6.

2. Proposed Method

Our objectives is to find the optimal path from origin to target position by avoiding the different types of obstacles present in the environment. TPM in [35] will check the free paths in the environments. FA will be used to avoid the obstacles when all the three paths are blocked as shown in Fig.1. The parameters that we have used in FA algorithm are shown in Table 1. The information gathered by the sensors from the environment about the goal and obstacles enable to solve the problem by FA algorithm. An objective function equation is developed based on the position of the goal and the obstacles present both in the static and dynamic environments. This minimized the navigation problem. After that, we have implemented FA algorithm and in each iterations locations of globally brighter firefly is chosen and during execution of the process the robot moves to these locations in series.

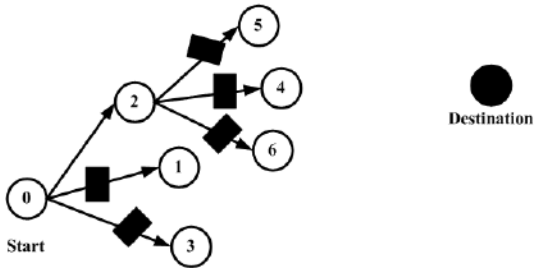


Fig. 1: Situation that used FA algorithm.

2.1. Formulation of the objective function using the FA algorithm

The robot has to reach the goal position with any obstacle avoidance in different environments. The robot workspace is divided into three paths. Whenever there is a free path it will choose the path to move and if all the paths are blocked, then the FA algorithm will be activated to avoid the obstacles.

Some of the key features of the FA algorithm are:

- It can handle both the linear and non-linear problems.
- It does not required any good initial solution to start the iteration process.
- The computational convergence is very high.
- It is easy to adopt hybrid method with other techniques.

And the objectives of our proposed controller are:

- To develop and design the efficient algorithm for optimal path planning in static and dynamic environments.
- To reach the target position as soon as possible by introducing Three Path Method.
- To give the uniqueness between the simulation and experimental results.
- To produce better performance from other existing path planning algorithm in terms of computational path and time in both static and dynamic environments.

2.1.1. Obstacle avoidance behavior

It is used to avoid the robot from hitting against obstacles (stationary or moving obstacles) present in different environments. The FA produces number of random fireflies near the obstacle and selected the brighter one among the group, it has maximum distance from the nearest obstacle. The robot occupies the newly selected firefly and started searching for next brighter fireflies till the optimal path is generated. The Euclidean distance between the best firefly and the nearest obstacle is shown in equation (1) in terms of objective function.

$$(Distance)_{F-OB} = \sqrt{(X_{OB} - X_{F_i})^2 + (Y_{OB} - Y_{F_i})^2} \tag{1}$$

where X_{F_i} and Y_{F_i} are the x and y coordinates of the firefly position respectively, X_{OB} and Y_{OB} are the x and y coordinates of the obstacle position respectively.

The Euclidean distance between the robot and the nearest obstacle is shown in equation (2).

$$(Distance)_{R-OB} = \sqrt{(X_{OB_n} - X_R)^2 + (Y_{OB_n} - Y_R)^2} \tag{2}$$

where X_{OB_n} and Y_{OB_n} are the x and y coordinates of the nearest obstacle position respectively, X_R and Y_R are the x and y coordinates of the robot position respectively.

2.1.2. Goal seeking behavior

As mentioned in the obstacle avoidance behavior, the brighter firefly is selected from the group in such away that it has maximum distance from the nearest obstacle. The position of the brighter firefly must be always at the minimum distance from the goal position in order to reach the goal position with minimum path length and time. The minimum Euclidean distance between the target and firefly is shown in equation (3) in terms of objective function.

$$(Distance)_{F-G} = \sqrt{(X_G - X_{F_i})^2 + (Y_G - Y_{F_i})^2} \tag{3}$$

where X_G and Y_G are the x and y coordinates of the goal position respectively.

Based on the study of above two behaviors, the objective function of fireflies for path planning optimization problem can be expressed as given below:

$$f_i = H_1.\frac{1}{MIN_{OB_n \in OB_d} \parallel (Distance)_{F-OB} \parallel} + H_2. \parallel (Distance)_{F-G} \parallel \tag{4}$$

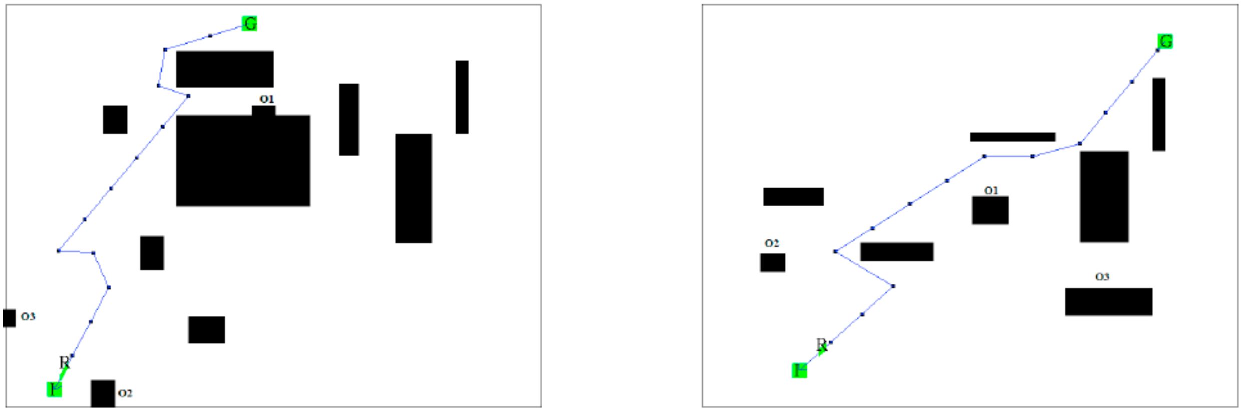
From the objective function, we assumed that the environment is of n numbers of obstacles and represented as $OB_1, OB_2, OB_3, OB_4,...,OB_n$ and their center coordinates are (X_{OB_1}, Y_{OB_1}), (X_{OB_2}, Y_{OB_2}), (X_{OB_3}, Y_{OB_3}), (X_{OB_4}, Y_{OB_4}),...,(X_{OB_n}, Y_{OB_n}).

Due to the threshold sensor value, number of obstacles it can detect in environment during navigation is $OB_d \in (OB_1, OB_2, OB_3, OB_4,...,OB_n)$. It can be seen that, when the number of fireflies(f_i) is away from the obstacle, then the value of $MIN_{OB_n \in OB_d} \parallel (Distance)_{F-OB} \parallel$ will be increased and when the number of fireflies(f_i) is nearer to the goal position, then the value of $\parallel (Distance)_{F-G} \parallel$ will be decreased. So, from the above discussions, we have concluded that the path planning problem for a mobile robot in static and dynamic environments by FA is a minimization problem.

Based on the objective function equation (4), it can be seen that the fitting parameters H_1 and H_2 have influences on robot navigation. When H_1 is maximum then the robot can safely avoid the obstacle whereas when it is minimum, there is chance of hitting the obstacle. Similarly, when H_2 is maximum then the robot path length decreases whereas when it is minimum, path length increases. Therefore, the proper selection of fitting parameters result the success of objective function and elevation of local minima problem. In this paper, we have used the fitting parameters by the trail and error method.

2.2. Steps for FA along with TPM for MRN

- Initialize the start and goal positions of the mobile robot.
- Movement of mobile robot towards the goal with Three Path Concept till all the paths are blocked.
- Activate the FA algorithm when all the paths are blocked.
- Generate the population of fireflies randomly.
- Select the brightest firefly among the population to fit the objective function equation (4).
- Move robot towards the current brighter firefly position.
- Repeat the steps (ii) to (vi) until it reach the goals or avoids obstacles.



(a) Robot path in first environment (b) Robot path in second environment

Fig. 2: Robot reaching the goal position in diffrent environments.

3. Simulation Results

Simulations are presented to demonstrate the effectiveness of hybrid method of FA-TPM for MRN. The simulations results are obtained from workstation running Microsoft Visual C++, 2010 with OpenGL. The simulation workspace which we have considered is (-120, -120) to (120, 120). Proper selection of the parameter is must according to problem domain. The controlling parameters which decide the functioning of FA are shown in Table 1. Robot size is taken as 4×3 square unit (pixel).

Table 1: Parameters for FA algorithm.

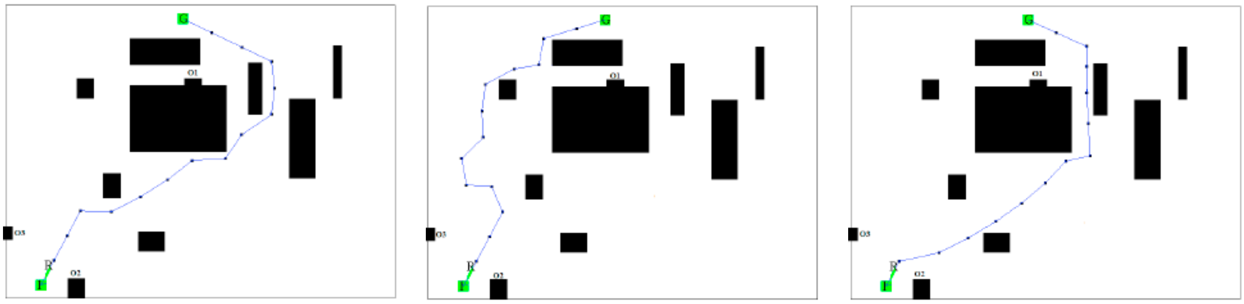
SL.No	Parameters	Values
1	Number of fireflies(N)	5-100
2	Number of generation	50-100
3	Light absorption coefficient(γ)	0.1-1
4	Randomization parameter(α)	0.1-1
5	Attractiveness(β)	0.1-1
6	Fitting parameter(H_1)	0.1-1
7	Fitting parameter(H_2)	0.01-0.0001

The robot can move ahead to distance D = 20 units from current position. At any position, the robot will move straight towards the goal. Other directions are 35 degree above and below the perpendicular direction.

The initial and goal positions are represented by *I* and *G* respectively. Obstacles are represented by black rectangular box. Obstacles labeled with *O1*, *O2* and *O3* are moving obstacles and unlabeled ones are static obstacles.

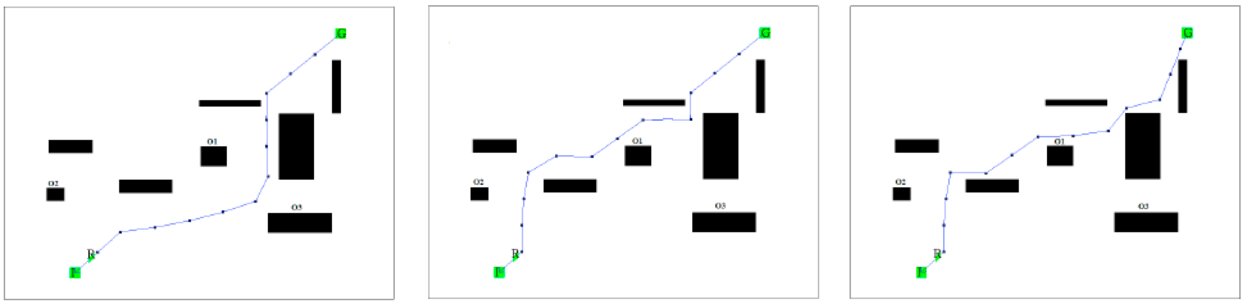
Table 2: Comparison with other methods in first environment.

Methods	Path Length (pixels)	Computational Time (sec)
Firefly algorithm in [26]	179.81	0.0315
Genetic algorithm in [31]	187.25	0.0923
Cuckoo search algorithm in [32]	181.72	0.0563
Proposed Method	**173.23**	**0.0013**



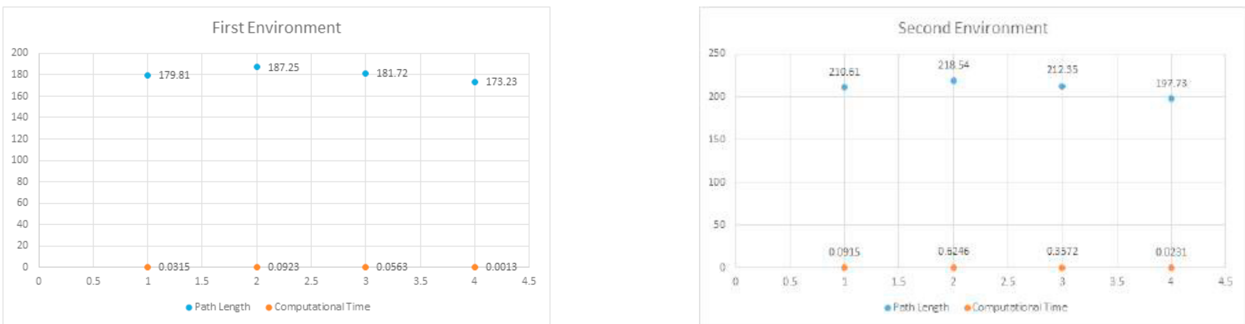
(a) By firefly algorithm in [26] (b) By genetic algorithm in [31] (c) By cuckoo search algorithm in [32]

Fig. 3: Robot reaching the goal position in first environment for other three methods.



(a) By firefly algorithm in [26] (b) By genetic algorithm in [31] (c) By cuckoo search algorithm in [32]

Fig. 4: Robot reaching the goal position in second environment for other three methods.



(a) Result analysis in first environment (b) Result analysis in second environment

Fig. 5: Graphical result anaylsis in different environments.

4. Evaluation and Discussion

The proposed method is evaluated in two different environments. The initial position of the robot in first and second environments are (-90, -100) and (-70, -90) respectively. The goal position of the robot in first and second environments are (-10, 100) and (80, 90) respectively. The path of the robot from initial to goal position in first and second environments are shown in Fig.2.

Comparison of our proposed algorithm with other three standard exiting methods in the first and second dynamic environments are shown in Table 2 and Table 3 respectively. The diagrammatic path length in first environment for other three methods namely firefly algorithm in [26], genetic algorithm in [31] and cuckoo search algorithm in [32]

Table 3: Comparison with other methods in second environment.

Methods	Path Length (pixels)	Computational Time (sec)
Firefly algorithm in [26]	210.61	0.0915
Genetic algorithm in [31]	218.54	0.6246
Cuckoo search algorithm in [32]	212.35	0.3572
Proposed Method	**197.73**	**0.0231**

Table 4: Fire Bird V Robot technical configuration.

SL.No.	Elements	Technical Specifications
1	Sensors	8 analog IR proximity sensors(20 cm), 5 sharp GP2D12 IR range sensor (80 cm)
2	Processor	Atmel ATMEGA8
3	RAM	512 Kbytes Improved
4	Speed (m/s)	0.02-0.6
5	Motion	2 DC brushed servo motors with incremental encoders
6	Size/cm	16(diameter) and 10(Height)
7	Weight/gms	1300
8	Payload/g	Nearly 300

are shown in Fig.3.

Similarly, the diagrammatic path length in second environment for other three methods namely firefly algorithm in [26], genetic algorithm in [31] and cuckoo search algorithm in [32] are shown in Fig.4.

The graphical result analysis of all the methods in different environment are shown in Fig.5.

5. Experimental Results

For testing the proposed algorithm in real world environments we have used an autonomous mobile robot called Fire Bird V which is shown in Fig.5. The said robot was designed by NEX Robotics and Embedded Real-Time Systems Lab, CSE IIT Bombay. Table 4 gives the details technical configuration of the robot.

Fig. 6: Fire Bird V.

(a) Initial position (b) first motion (c) second motion (d) third motion (e) goal position

Fig. 7: Robot motion in real world environment.

Robot motion in real world environment are shown in Fig. 6. The initial position of the robot is (1, 1) and the goal position of the robot is (4, 5). All the units are considered in terms of feet and obstacles in the environment are represented by O1, O2, O3 and O4. During the experiment, we have made four assumptions:

- The workspace is considered as plane ground.
- Inertial effects is not considered.
- Navigation is done without sleeping.
- Goal position is stationary.

6. Conclusion

A novel hybridized FA-TPM method has been developed for the optimal path planning of autonomous mobile robot in a cluttered dynamic environment. Firefly algorithm is used when all the three path are blocked. If anyone of the path is free,then the robot will choose the free path. The proposed hybrid method gives the most effective path for mobile robot without any collision in minimum time. Simulation and Experimental shows that the proposed method has more efficient and novelty than the other existing intelligent navigation approaches in different dynamic environments. In future work, the proposed work can be implemented for the navigation of autonomous vehicles, Drone navigation, public transportation and military warfare in complex dynamic environment.

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