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Question 3
a.
Find the limits of

i. $\lim_{n \to \infty} \frac{\ln(n)+4}{5n^4+7n^2+6}$

differentiate the function & use L'hôpitals rule

$= \lim_{n \to \infty} \frac{\frac{1}{n}+0}{20n^3+14n+0}$ # divide through by the highest degree

$= \lim_{n \to \infty} \frac{\frac{\frac{1}{n}}{n^3}}{\frac{20n^3}{n^3}+\frac{14n}{n^3}}$

$= \lim_{n \to \infty} \frac{\frac{1}{n^4}}{20+\frac{14}{n^2}}$

$= \frac{\frac{1}{\infty^4}}{20+\frac{14}{\infty^2}}$

$= \frac{0}{20+0}$

$= \underline{\underline{0}}$

$\therefore \lim_{n \to \infty} \frac{\ln(n)+4}{5n^4+7n^2+6} = 0$

ii. $\lim_{n \to \infty} \frac{2^n}{\log_2(n)}$

differentiate the function & use L'hôpitals rule

$\frac{d(2^n)}{dn} = 2^n \ln 2$

$$\frac{d(\log_2 n)}{dn} = \frac{1}{n.\ln(2)}$$

$$= \lim_{n \to \infty} \frac{2^n \ln 2}{n \ln(2)}$$

$$= \lim_{n \to \infty} 2^n . \ln^2(2) . n$$

$$= \lim_{n \to \infty} \ln^2(2) . 2^n . n$$

ii $\cancel{\lim_{n \to \infty}} = \ln^2(2) \lim_{n \to \infty} 2^n . \lim_{n \to \infty} n$

$$= \lim_{n \to \infty} 2^n = \infty$$

$$= \lim_{n \to \infty} n = \infty$$

$$\therefore \lim_{n \to \infty} \frac{2^n}{\log_2(n)} = \infty$$

iii b. i $\sum_{k=0}^{30} k^2 \simeq \int_0^{30} x^2 dx$

$$= \frac{x^{2+1}}{2+1} \Bigg|_{x=0}^{30}$$

$$= \frac{x^3}{3} \Bigg|_0^{30}$$

$$= \frac{(30)^3}{3}$$

$$= \underline{\underline{9000}}$$

ii. $\sum_{k=0}^{100} k^3 \simeq \int_0^{100} x^3 \, dx$

$$= \left. \frac{x^{3+1}}{3+1} \right|_{x=0}^{100}$$

$$= \left. \frac{x^4}{4} \right|_0^{100}$$

$$= \frac{(100)^4}{4}$$

$$= 25000000$$

c. i. $\sum_{k=0}^{n} k \binom{n}{k} = n 2^{n-1}$

i. $2 \sum_{n=0}^{n} 3^k = 3^{n+1} - 1$

$\Rightarrow 2 \left(3^0 + 3^1 + 3^2 + 3^k + \ldots + 3^n\right) = 3^{n+1} - 1$

when $n = 1$

$\Rightarrow 2(3^0 + 3^1 + 3^n) = 3^{n+1} - 1$

$\Rightarrow 2 \left(3^0 + \ldots + 3^1\right) = 3^{1+1} - 1$

$\Rightarrow 2(1+3) = 3^2 - 1$

$\Rightarrow 2(4) = 8$

$\Rightarrow 8 = 8$

when $n = k$

$2 \left(3^0 + \ldots + 3^n\right) = 3^{k+1} - 1$

when $n = k+1$

$2 \left(3^0 + \ldots + 3^k + 3^{k+1}\right) = 3^{(k+1)+1} - 1$

$2(3^0 + \ldots + 3^n) + 2(3^{k+1}) = 3^{k+2} - 1$

$3^{k+1} - 1 + 2(3^{k+1}) = 3^{k+2} - 1$

$3^{k+1}(1+2) - 1 = 3^{k+2} - 1$

$3^{k+1} \cdot 3^1 - 1 = 3^{k+2} - 1$

$3^{k+2} - 1 = 3^{k+2} - 1$

Question:

i. $T(n) = 7T(\frac{n}{2}) + n^2$; $a = 7, b = 2, f(n) = n^2$

$= n^{\log_b a}$

$= \cancel{n \log} \, n^{\log_2 7}$

$= n^{2.81}$

$= f(n) < n^{\log_b a}$

$= O(n^{\log_b a})$

$= O(n^{2.81})$

ii. $T(n) = 5T(n/3) + O(n)$

$a = 5, b = 3, f(n) = O(n)$

$= n^{\log_b a}$

$= \cancel{n \log} \, n^{\log_3 5}$

$= n^{1.465}$

$= f(n) < n^{\log_b a}$

$= O(n^{\log_b a})$

$= O(n^{1.465})$

iii. $T(n) = 3T(n/2) + \frac{3}{4}n + 1$

$a = 3, b = 2$

$f(n) = \frac{3}{4}n + 1$

$= n^{\log_b a}$

$= n^{\log_2 3}$

$\cancel{= n^{\log 1.58}}$

$= n^{1.585}$

$= f(n) < n^{\log_b a}$

$= O(n^{\log_b a})$

$= O(n^{1.585})$