MAT 2030 W24 Final Exam. Instructor: Frank Okoh. Email: ac2235@wayne.edu.

You must show work for credit. No credit for the whole problem if the requested graph is not presented. Solutions using notation different from the book earn no credit. If you use a source, change the notation to ours.

Follow the guideline for pdf submission in the syllabus. If I cannot read your exam or summary, you score is 0 on the exam or summary or both. We have no time for re-submission.

Give no help and receive no help from other persons.

Failure to comply means Failure (F) in the course.

1. This problem is about Change Of Variables.

(a) State the Lecture Number where Change of Variable for Double Integrals was stated and the page number in the book where it is also stated.

Let T be a transformation from the uv-plane to the xy-plane given by $T(u,v) = (x,y)$ where $x = 2u + v,\ \ y = u + 2v$.

(b) Compute the Jacobian of T.

(c) Verify that $u = \frac{2x-y}{3}$ and $v = \frac{2y-x}{3}$ satisfy the above equations for x and y. You must show work for credit.

This gives the transformation T^{-1} from the xy-plane to the uv-plane.

(d) Draw the triangle $R = ABC$ with vertices $A(0,0)$, $B(1,2)$, and $C(2,1)$ on a scaled xy-plane.

(e) Let $T^{-1}(A) = A_1$, $T^{-1}(B) = B_1$, $T^{-1}(C) = C_1$. Verify that the triangle $S = A_1B_1C_1$ has vertices $(0,0)$, $(0,1)$ and $(1,0)$. Show work. Draw this triangle in the uv-plane and label the vertices.

(f) The triangle S in (e) has the description $0 \leq u \leq 1 - v,\ \ 0 \leq v \leq 1$. Draw this region on the uv-plane. Indicate the range of u as a horizontal strip, type II region (Similar to Figure 16 p.1057). label boundaries $u = 0$ and $u = 1 - v$.

(g) Use (f) and Theorem 9, p. 1112 to write $\int\int_R (x - 3y)dA$ as an iterated integral in the uv-plane. Imitate the last part of Example 3, p. 1114.

(h) **Do not evaluate this iterated integral.**

2. This question is on material from 12.4 and 12.5. We begin with the cross product.

(i) Give the Lecture number in your Course Summary where the cross product of two vectors was defined and its properties stated. Give the page number in the book where the properties are stated.

Let $\mathbf{a} = \langle a_1, a_2, a_3\rangle$, $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, $\mathbf{c} = \langle c_1, c_2, c_3\rangle$ be vectors in $\mathbf{R}^3$.

(ii) Use this notation for the vectors **a**, **b**, and **c** , the definition of $+$, and the definition of $\times$ to write out $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$.

(iii) Let $\mathbf{a} = \langle 1, 0, 6 \rangle$ $\mathbf{b} = \langle 2, -4, 3 \rangle$ and $\mathbf{c} = \langle -7, 5, -1 \rangle$. Compute $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$ and $\mathbf{a} \times \mathbf{b}$. Then use Property 3 p. 859 to compute $\mathbf{a} \times \mathbf{c}$. Show your work.

(iv) Give the Lecture number in your Course Summary where the dot product of two vectors was defined and its properties stated.

(v) What does $\mathbf{v} \cdot \mathbf{w} = 1$ say about the angle between **v** and **w** if **v** and **w** are unit vectors? Illustrate your answer with a diagram regarding **v** and **w** as vectors in $\mathbf{R}^2$.

(vi) (Triple Products) Find the volume of the box spanned by $\mathbf{a}, \mathbf{b}$, and **c** from part (iii).

(vii) Give the height of a cube with the same volume as the box above.

3. This problem is mainly about Section 14.7.

(i) State the Lecture Number where the Second Derivative Test was stated and the page number in the book where it is also stated.

(ii) Let $\mathcal{P}$ be the plane with equation $x + y - z = 2$. Show that the point $A(-1, 3, 5)$ is not on $\mathcal{P}$ and the point $B(1, 1, 0)$ is on $\mathcal{P}$.

(iii) let d be the distance from $A(-1, 3, 5)$ to a point (x, y, z) on $\mathcal{P}$. Show that there is no maximum value of d by drawing a diagram.

(iv) For d in (iii), d^2 is given by the formula

$$d^2 := f(x, y) = (x + 1)^2 + (y - 3)^2 + (x + y - 7)^2.$$

Find the only critical point (a, b) of $f(x, y)$.

(v) Compute f_{xx} and f_{xy}.

(vi) Which theorem gives you f_{yx} from your answer in (v)?

(vii) Use the Second Derivative Test to determine whether $f(x, y)$ has a local maximum, local minimum or a saddle point at (a, b).

(viii) Use Theorem 9, p.1015 and your answers in (iii), (iv), and (v) to show that $f(x, y)$ has an absolute minimum at (a, b). **No other method earns credit.**

(ix) Now, use (viii) only to get the (shortest) distance from $A(-1, 3, 5)$ to the plane $\mathcal{P}$?