Problem Set 4

Applied Stats II

Due: April 12, 2024

Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in R, please include the code you used to get your answers. Please also include the .R file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- Your homework should be submitted electronically on GitHub in .pdf form.
- This problem set is due before 23:59 on Friday April 12, 2024. No late assignments will be accepted.

Question 1

We're interested in modeling the historical causes of child mortality. We have data from 26855 children born in Skellefteå, Sweden from 1850 to 1884. Using the "child" dataset in the eha library, fit a Cox Proportional Hazard model using mother's age and infant's gender as covariates. Present and interpret the output.

```r
# Install required packages
install.packages("survival")
install.packages("survminer")
install.packages("ggplot2")
install.packages("stargazer")
install.packages("eha")

# Load required libraries
library(survival)
library(ggplot2)
library(ggfortify)
library(survminer)
library(stargazer)
library(eha)


```

```
# Load the dataset and examine its structure
data(child)

# Display the first few rows of the dataset
head(child)
```

Table 1: head(child)

id	m.id	sex	socBranch	birthdate	enter	exit	event	illeg	m.age
3	9	male	farming	1853-05-23	0	15.000	0	no	35.009
42	150	male	farming	1853-07-19	0	15.000	0	no	30.609
47	158	male	worker	1861-11-17	0	15.000	0	no	29.320
54	178	male	farming	1872-11-16	0	15.000	0	no	41.183
78	263	female	worker	1855-07-19	0	0.559	1	no	42.138
102	342	male	farming	1855-09-29	0	0.315	1	no	32.931

```
# Examine the structure of the dataset
str(child)
```

Table 2: str(child)

id	m.id	sex	socBranch	birthdate	enter	exit	event	illeg	m.age
9	246606	male	farming	1853-05-23	0	15.000	0	no	35.009
150	377744	male	farming	1853-07-19	0	15.000	0	no	30.609
158	118277	male	worker	1861-11-17	0	15.000	0	no	29.320
178	715337	male	farming	1872-11-16	0	15.000	0	no	41.183
263	978617	female	worker	1855-07-19	0	0.559	1	no	42.138
342	282943	male	farming	1855-09-29	0	0.315	1	no	32.931
363	341341	male	farming	1858-07-25	0	15.000	0	no	38.670
393	840879	male	farming	1871-04-29	0	15.000	0	no	35.059
408	586140	female	farming	1859-07-06	0	15.000	0	no	33.515
486	564736	female	farming	1862-08-16	0	15.000	0	yes	29.525

```
# Display information about the dataset
help("child")
```

A data frame with 26855 children born 1850-1884.

id: An identification number.
m.id: Mother's id.
sex: Sex.
socBranch: Working branch of family (father).
birthdate: Birthdate.

enter: Start age of follow-up, always zero.
exit: Age of departure, either by death or emigration.
event: Type of departure, death = 1, right censoring = 0.
illeg: Born out of marriage ("illegitimate")?
m.age: Mother's age.

```r
# Check the dimensions of the dataset
dim(child)
```

[1] 26574 10
The dataset 'child' has 26574 rows and 10 columns.

```r
# Summary statistics for mother's age
summary(child$m.age)
```

```
Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
15.83   27.18   31.79   32.03   36.74   50.86
```

```r
# Fit Cox Proportional Hazards model
fit <- coxreg(Surv(enter, exit, event) ~ sex + m.age, data = child, coxph =
    TRUE)

# Summarize the results of the Cox Proportional Hazards model
summary(fit)
```

```
Covariate             Mean       Coef     Rel.Risk   S.E.    LR p
sex                                                          0.002
            male      0.510     0         1 (reference)
          female      0.490    -0.082     0.921      0.027
m.age                32.010     0.008     1.008      0.002   0.000

Events                     5616
Total time at risk         325030
Max. log. likelihood       -56503
LR test statistic          22.52
Degrees of freedom         2
Overall p-value            1.28921e-05
```

In this case, we observed the following:

The sex of the infants significantly affects their survival rate. The relative risk (or hazard) ratio for female infants compared to male infants is 0.921, indicating a decrease in relative risk by 8.1%. This suggests that female infants are more likely to survive.

Mother's age also influences the infant's survival rate. For each unit increase in age (e.g., one year), the relative risk increases by 0.8%.

The LR test statistic is 22.52, indicating the degree of improvement in model fit after adding variables. In this case, the large value of the LR test statistic suggests a significant improvement in model fit after adding variables.

The p-value is 1.28921e-05, close to zero, indicating that the overall model fit is significant, meaning that at least one covariate has a significant impact on survival time.

```r
# a) Plotting Kaplan-Meier survival curve
# Creating survival object:
# Using the Surv() function
# From the enter (entry time), exit (exit time), and event (event occurrence) variables
# in the child dataset
# Created a survival object child_surv.
# This function is used to represent the survival time and event status for each observation.
child_surv <- with(child, Surv(enter, exit, event))


# Fitted the Kaplan-Meier survival curve using the survfit() function
# child_surv ~ 1 indicates considering only time without any other explanatory variables,
km <- survfit(child_surv ~ 1, data = child)

# Summary statistics and plotting
# Summary statistics of the Kaplan-Meier survival curve were obtained using the summary() function,
# specifying the times of interest with times = seq(0, 15, 1). Starting from time 0, every year (in units of years) until 15 years.
summary(km, times = seq(0, 15, 1))
```

```
Call: survfit(formula = child_surv ~ 1, data = child)

time n.risk n.event survival std.err lower 95% CI upper 95% CI
    0  26574       0    1.000 0.00000        1.000        1.000
    1  24319    2161    0.919 0.00168        0.915        0.922
    2  23450     778    0.889 0.00193        0.885        0.893
    3  22766     596    0.867 0.00209        0.862        0.871
    4  22269     430    0.850 0.00220        0.846        0.854
    5  21859     365    0.836 0.00228        0.832        0.841
    6  21533     261    0.826 0.00233        0.822        0.831
    7  21266     214    0.818 0.00238        0.813        0.823
```

8	21077	151	0.812	0.00241	0.807	0.817
9	20915	117	0.808	0.00243	0.803	0.812
10	20777	103	0.804	0.00245	0.799	0.808
11	20655	81	0.801	0.00246	0.796	0.805
12	20531	91	0.797	0.00248	0.792	0.802
13	20404	89	0.794	0.00250	0.789	0.798
14	20277	95	0.790	0.00251	0.785	0.795
15	20141	84	0.787	0.00253	0.782	0.792

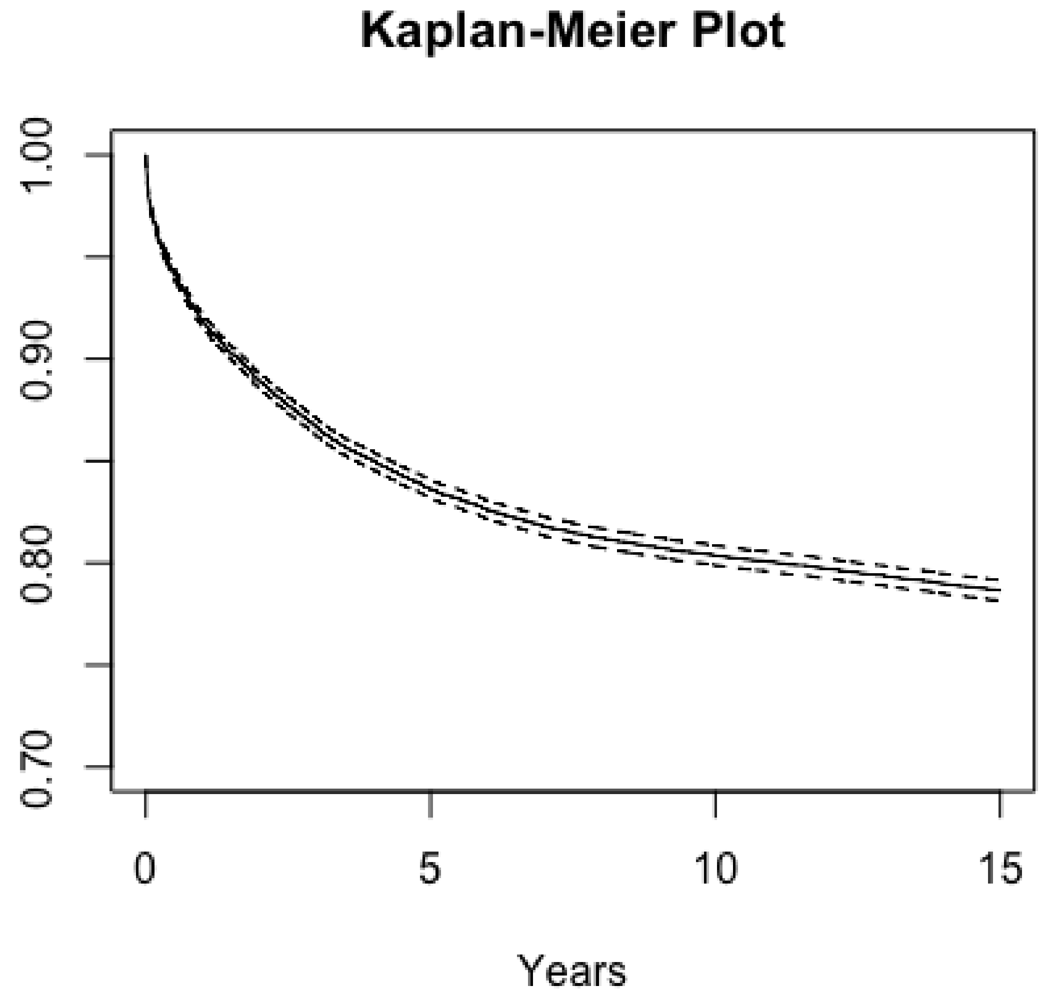
As time progresses, the survival rate gradually decreases.
For example, at the first time point (time = 0), the survival rate is 1, indicating that all individuals are alive at the starting time point;

whereas at the 15th time point, the survival rate decreases to 0.787, indicating that approximately 78.7% of individuals are still alive at that time point.

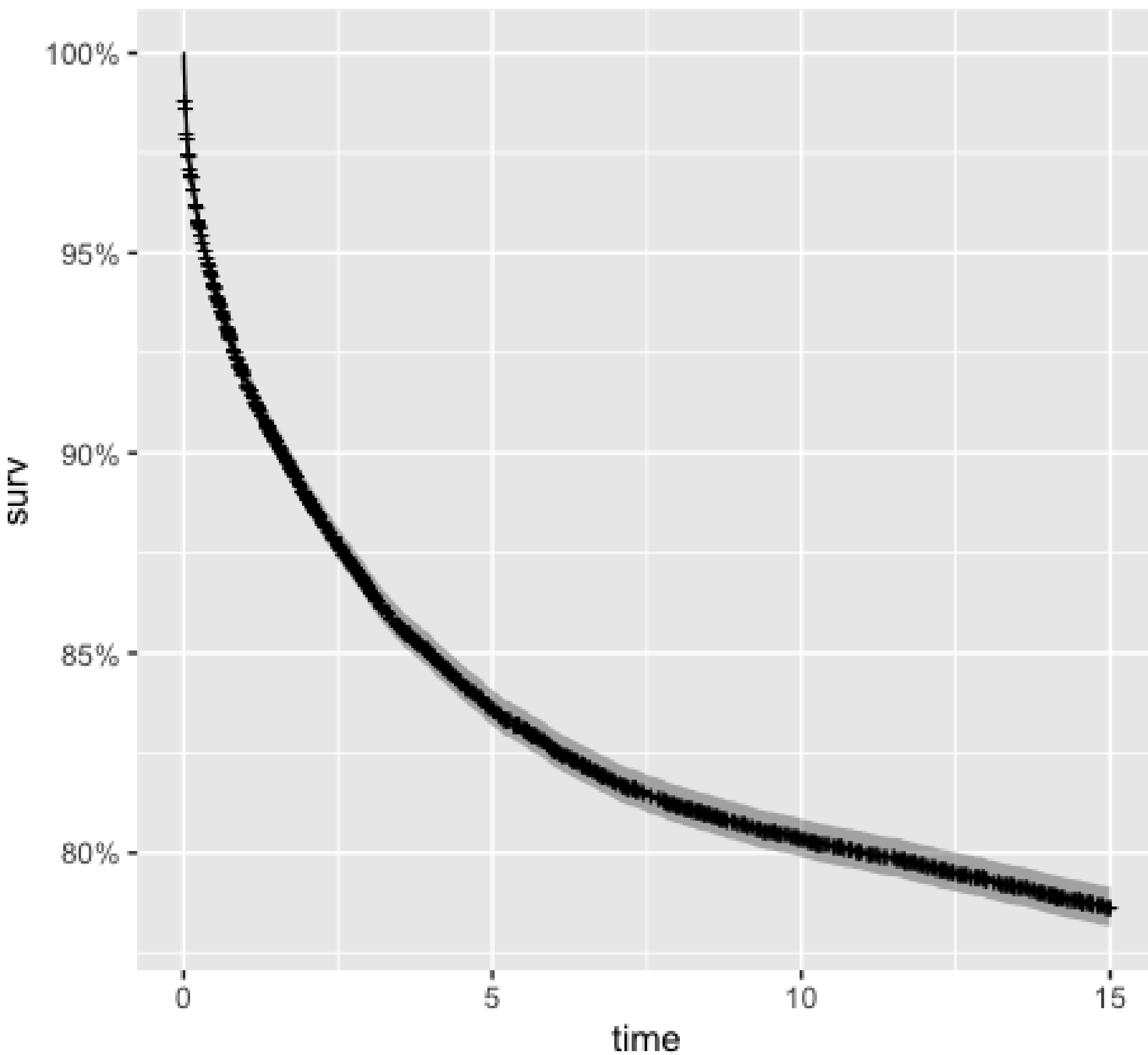
The decline in survival rate implies an increase in mortality rate.

The survival rate gradually decreases, indicating that at each time point, more individuals experience the event (such as death), thus the mortality rate increases gradually.

```r
# Plot the Kaplan-Meier curve with the main title "Kaplan-Meier Plot", x-axis label "Years", and y-axis limited from 0.7 to 1.
plot(km, main = "Kaplan-Meier Plot", xlab = "Years", ylim = c(0.7, 1))
```




```
autoplot(km)
```




```
# Plot the survival curves based on m.age variable
# Fit Kaplan-Meier survival curves for different ages using survfit() function
km_mAge <- survfit(child_surv ~ m.age, data = child)





# b) Fitting and interpretation of the Cox proportional hazards model
cox <- coxph(child_surv ~ sex + m.age, data = child)
# Summary statistics of the Cox model are obtained using the summary()
    function
summary(cox)
```

```
Call:
```

```
coxph(formula = child_surv ~ sex + m.age, data = child)

  n= 26574, number of events= 5616

               coef exp(coef)  se(coef)      z Pr(>|z|)
sexfemale -0.082215  0.921074  0.026743 -3.074 0.002110 **
m.age      0.007617  1.007646  0.002128  3.580 0.000344 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

          exp(coef) exp(-coef) lower .95 upper .95
sexfemale    0.9211     1.0857     0.874    0.9706
m.age        1.0076     0.9924     1.003    1.0119

Concordance= 0.519  (se = 0.004 )
Likelihood ratio test= 22.52  on 2 df,   p=1e-05
Wald test            = 22.52  on 2 df,   p=1e-05
Score (logrank) test = 22.53  on 2 df,   p=1e-05
```

Interpretation of summary statistics output:

coef: In this example, the coef for sexfemale is -0.0822, indicating a decrease in risk by -0.0822 units relative to male infants. The coef for m.age is 0.0076, suggesting that for each additional year of mother's age, the risk of death for her infant increases by 0.0076 units.

exp(coef): The exponentiated coefficients represent the relative hazard ratios for each variable, indicating changes in risk relative to the reference level.

In this example, the exp(coef) for sexfemale is 0.9211, indicating a reduction in hazard ratio by 0.9211 units compared to male infants. For m.age, exp(coef) is 1.0076, implying that relative to the reference level (which is the baseline for m.age, likely the median or mean age), the hazard ratio for infants increases by a factor of 1.0076.

```r
# Variable deletion:
# The drop1() function was used to perform variable deletion testing on the model, assessing the significance of each explanatory variable.
drop1(cox, test = "Chisq")
```

```
Single term deletions

Model:
child_surv ~ sex + m.age
       Df    AIC     LRT  Pr(>Chi)
<none>    113011
```

```
sex      1 113018  9.4646 0.0020947 **
m.age    1 113022 12.7946 0.0003476 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

In this output, we observe the results of the deletion tests for two explanatory variables:

sex and m.age:
The deletion test results for the sex variable indicate that when the sex variable is removed from the model, the model's Log Likelihood Ratio Test (LRT) statistic is 9.4646, with a corresponding p-value of 0.0020947.

This suggests that the sex variable has a significant effect in the model, indicating that gender has a significant impact on infant survival time.

The deletion test results for the m.age variable indicate that when the m.age variable is removed from the model, the model's LRT statistic is 12.7946, with a corresponding p-value of 0.0003476.

Similarly, this indicates that the m.age variable has a significant effect in the model, suggesting that mother's age also has a significant impact on infant survival time.

Therefore, this test result informs us that both gender and mother's age are significant explanatory variables in the model, and they both have important effects on infant survival time.

```r
# Using the `stargazer()` function to output the results of the Cox model in text format for easy viewing and comparison of different models' effects.
stargazer(cox, type = "text")
```

```
================================================
                         Dependent variable:
                     ---------------------------
                             child_surv
------------------------------------------------
sexfemale                     -0.082***
                               (0.027)

m.age                          0.008***
                               (0.002)

------------------------------------------------
Observations                   26,574
R2                              0.001
```

```
Max. Possible R2                 0.986
Log Likelihood                -56,503.480
Wald Test                 22.520*** (df = 2)
LR Test                   22.518*** (df = 2)
Score (Logrank) Test      22.530*** (df = 2)
================================================
Note:                *p<0.1; **p<0.05; ***p<0.01
```

The results of this Cox proportional hazards model are crucial for understanding the impact on infant survival time, particularly concerning gender and maternal age.

1. Effect of Gender (sexfemale) on Infant Survival Time:
- The coefficient (Coef) is -0.082, indicating that the survival time risk for female infants decreases by 0.082 relative to male infants. This suggests that, all other factors being equal, female infants are more likely to survive.
- In practical terms, female infants exhibit higher survival rates compared to male infants.

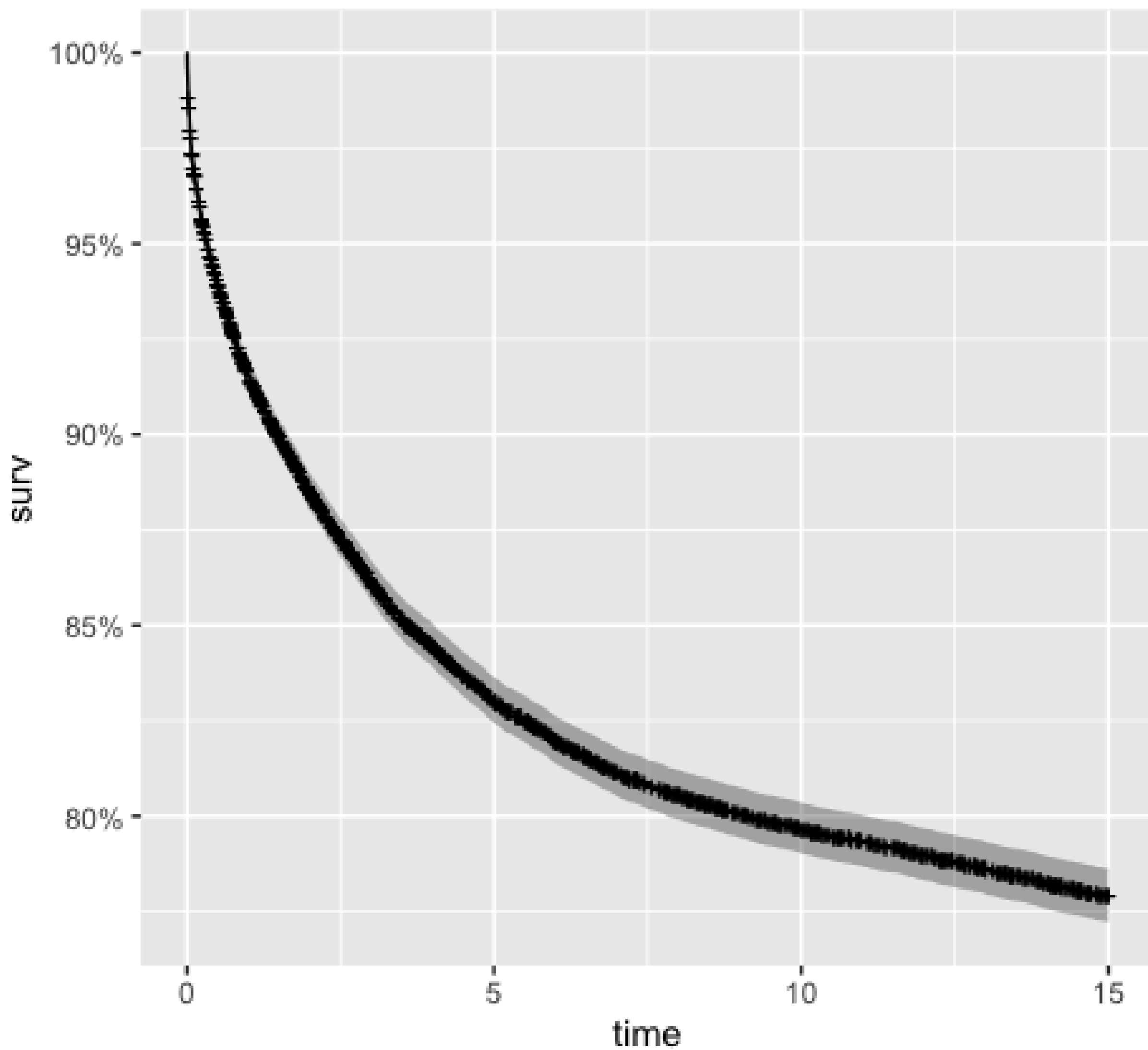
2. Effect of Maternal Age (m.age) on Infant Survival Time:
- The Coef is 0.008, signifying that for each additional year of maternal age, the infant's survival time risk increases by 0.008. This implies that infants born to older mothers are more likely to face mortality risks.

```r
# exponentiate parameter estimates to obtain hazard ratios
exp(-0.082215)
```

[1] 0.9210739

The hazard ratio for female infants is 0.92, indicating a lower risk of mortality compared to male infants.
This can be interpreted as approximately 92 female infants potentially dying for every 100 male infants.
This suggests that the risk for female infants is reduced by approximately 9.21% compared to male infants.

```r
# Fit the Cox model using survfit() and generate an automated survival curve
   plot using autoplot().
cox_fit <- survfit(cox)
autoplot(cox_fit)
```




```r
# Use the median() function to calculate the median age of the mother
median_age <- median(child$m.age, na.rm = TRUE)
print(median_age)
```

[1] 31.789
setting the median age as the reference line

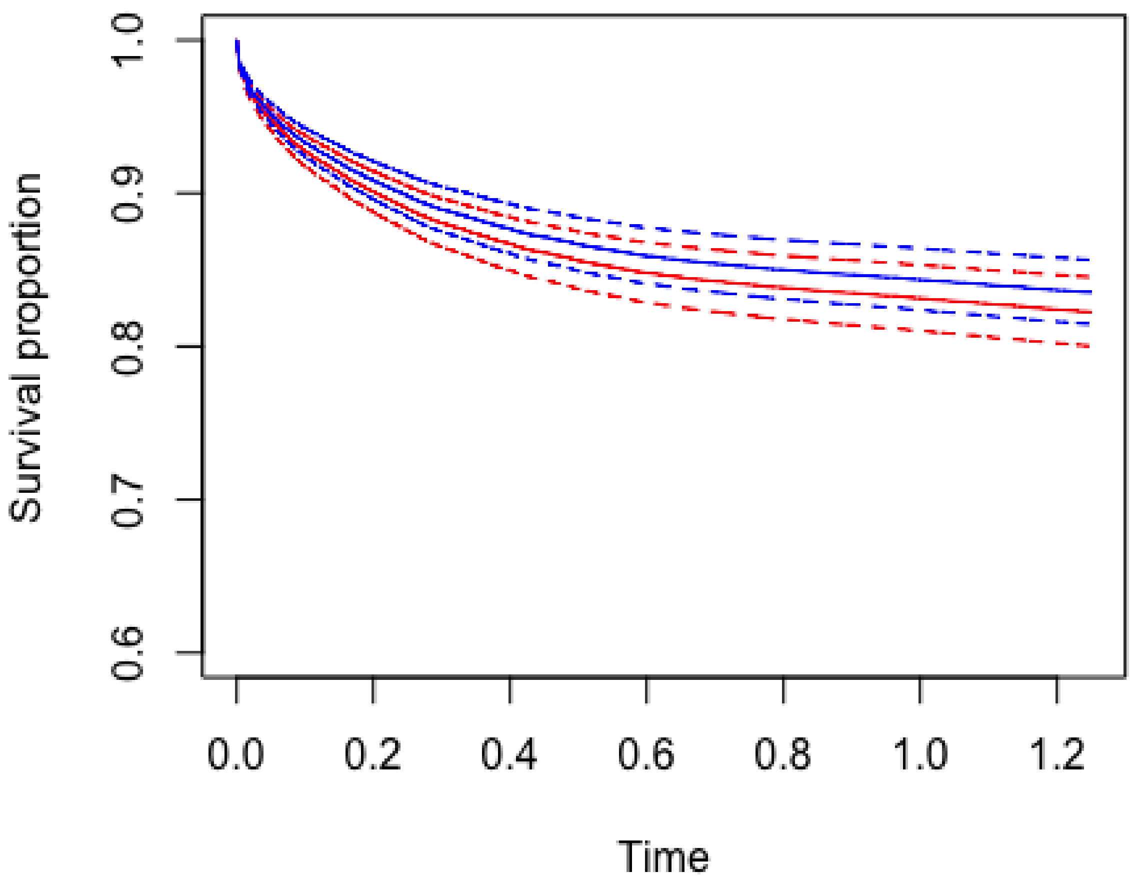
```r
# Create a new data box newdat that contains both genders and median age
newdat <- data.frame(
  sex = c("male", "female"),
  m.age = median_age
)


# Convert m.age to a factor variable and give it two levels
```

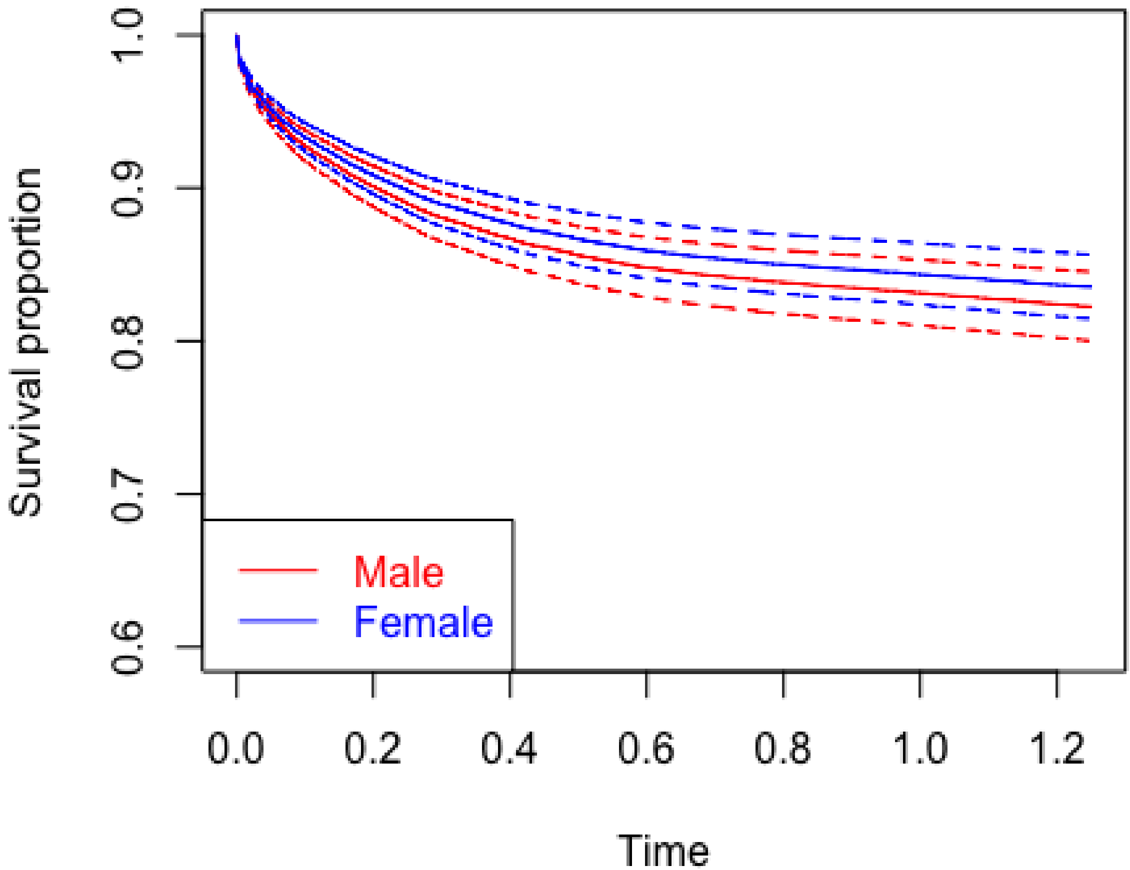
```r
newdat$m.age <- factor(newdat$m.age, levels = c("median_age", "other_level"))

# Generate survival curve
surv_fit <- survfit(cox, newdata = newdat)


# plot the survival curve using the plot function
# This code is an instruction to plot a survival curve using the plot function.
# It plots the survival curve with time (x axis) and survival ratio (y axis). Survival proportion means survival proportion,
# represents the percentage of individuals who survive over a period of time.
plot(surv_fit, xscale = 12, conf.int = TRUE,
     ylim = c(0.6, 1),
     col = c("red", "blue"),
     xlab = "Time",
     ylab = "Survival proportion",
     main = "")
```




```
# Legend is drawn, showing the curve colors of different genders
# The legend is marked with the confidence interval for the predicted value,
    not the standard error for the term in the model
legend("bottomleft",
       legend=c("Male", "Female"),
       lty = 1,
       col = c("red", "blue"),
       text.col = c("red", "blue"))
```




```r
# Adding an interaction
# The coxph() function was used to fit a Cox proportional risk model
    containing the interaction of mother's age and infant sex
cox.int <- coxph(child_surv ~ sex * m.age, data = child)


# summary statistics are performed on the interaction model using the summary
    () function
summary(cox.int)
```

```
Call:
coxph(formula = child_surv ~ sex * m.age, data = child)

  n= 26574, number of events= 5616
```

```
                    coef exp(coef)  se(coef)      z Pr(>|z|)
sexfemale       -0.127105  0.880641  0.140304 -0.906   0.3650
m.age            0.006963  1.006987  0.002925  2.380   0.0173 *
sexfemale:m.age  0.001389  1.001390  0.004263  0.326   0.7445
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

                exp(coef) exp(-coef) lower .95 upper .95
sexfemale          0.8806     1.1355    0.6689     1.159
m.age              1.0070     0.9931    1.0012     1.013
sexfemale:m.age    1.0014     0.9986    0.9931     1.010

Concordance= 0.519  (se = 0.004 )
Likelihood ratio test= 22.62  on 3 df,   p=5e-05
Wald test            = 22.53  on 3 df,   p=5e-05
Score (logrank) test = 22.56  on 3 df,   p=5e-05
```

An interaction term has been added here to examine the joint effect of mother's age and infant gender.

Specifically:
This Cox proportional hazards model considers the interaction between infant gender and mother's age on infant survival time.
'sexfemale' and 'm.age' represent the two main explanatory variables in the model, which are gender and mother's age, respectively.
For 'sexfemale', the coefficient is -0.127105, indicating that female infants have a lower risk compared to male infants, but the coefficient is not significant ($p = 0.365$).
Whereas for 'm.age', the coefficient is 0.006963, indicating that for each additional year of mother's age, the infant's risk increases by 0.006963.
This coefficient is statistically significant ($p = 0.0173$), suggesting that mother's age has a certain impact on infant survival time.
'sexfemale:m.age' is the coefficient for the interaction term between gender and mother's age, which is 0.001389, but this coefficient is not significant ($p = 0.7445$).
This indicates that the interaction between gender and mother's age does not significantly affect infant survival time when considering both factors.

The statistical test results of the model show that the p-values for Likelihood ratio test, Wald test, and Score (logrank) test are all less than 0.05, indicating that the overall model is significant, and at least one variable has a significant effect on infant survival time.

```r
# The Chi-square test of model fitting was performed using drop1() function
drop1(cox.int, test = "Chisq")
```

```
Single term deletions

Model:
child_surv ~ sex * m.age
          Df    AIC     LRT Pr(>Chi)
<none>       113013
sex:m.age  1 113011 0.10623   0.7445
```

The ‘drop1()‘ function is used here to perform a chi-square test of model fitting, which aims to systematically remove each explanatory variable from the model and compare the difference between the reduced model and the full model.

The results here indicate that after removing the interaction term ‘sex:m.age‘, there is no significant change in the model fit (p = 0.7445), meaning that removing this interaction term does not significantly affect the difference between the reduced model and the full model.

In other words, considering the interaction between mother's age and infant gender in the model does not significantly improve the explanatory power for survival time.

```r
# Use the stargazer() function to output the results of the interaction model as text
stargazer(cox.int, type = "text")
```

```
================================================
                         Dependent variable:
                     ---------------------------
                             child_surv
------------------------------------------------
sexfemale                      -0.127
                               (0.140)

m.age                          0.007**
                               (0.003)

sexfemale:m.age                 0.001
                               (0.004)

------------------------------------------------
Observations                   26,574
R2                              0.001
Max. Possible R2                0.986
Log Likelihood               -56,503.430
Wald Test                 22.530*** (df = 3)
```

```
LR Test                    22.624*** (df = 3)
Score (Logrank) Test       22.562*** (df = 3)
===================================================
Note:                  *p<0.1; **p<0.05; ***p<0.01
```

The output presents the results of the Cox proportional hazards model in tabular form.

Explain:

sexfemale: This is the coefficient for gender, -0.127, but not significant with a p-value of 0.365. This indicates that the change in risk for female infants compared to male infants is -0.127, but this change is not significant.

m.age: This is the coefficient for mother's age, 0.007, and significant with a p-value of 0.0173. This means that for every one year increase in mother's age, the change in risk for infants is 0.007.

sexfemale:m.age: This is the coefficient for the interaction between gender and mother's age, 0.001, and not significant with a p-value of 0.7445. This suggests that the interaction between gender and mother's age is not significant in this model.

Overall, the model fit is not ideal, with an R^2 of only 0.001, indicating that the model does not explain much of the variation in the data.

Summary:

In this case, we used the Cox proportional hazards model to study the survival time of infants born in the Skellefte region of Sweden from 1850 to 1884, and investigated the effects of mother's age and infant gender on infant survival time.

Firstly, we found that gender has a significant impact on survival time. Female infants have a survival time risk reduction of approximately 8.21% compared to male infants. This suggests that, all else being equal, female infants are more likely to survive.

Secondly, mother's age also has a significant impact on infant survival time. For every one year increase in mother's age relative to the median age (approximately 31.789 years), the infant's survival time risk increases by approximately 0.76%. This implies that infants born to older mothers are more likely to face a risk of death.

In other words, for every one year decrease in mother's age relative to the median age (approximately 31.789 years), the infant's survival time risk decreases by approximately 0.76%.

This suggests that infants born to younger mothers are more likely to enjoy longer survival times.

Additionally, we also explored the interaction between infant gender and mother's age on infant survival time. The results showed that, after considering gender and mother's age, the interaction between them did not significantly affect infant survival time.