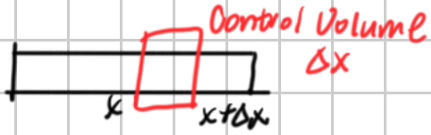
Heat conduction in 1D and IR modelling:

ct708@cam.ac.uk.

1. Background:



① Conservation of Energy:

$$q_x - q_{x+\Delta x} = \rho c \Delta x \frac{\partial T}{\partial t}. \quad (i)$$

② Fourier's Law:

$$q = -k\frac{\partial T}{\partial x} \quad (ii)$$

where k is thermal conductivity

plug (ii) into (i):

$$-k\frac{\partial T}{\partial x}\Big|_x - \left(-k\frac{\partial T}{\partial x}\Big|_{x+\Delta x}\right) = \rho c \Delta x \frac{\partial T}{\partial t}$$

$$\Rightarrow k\left(\frac{\partial T}{\partial x}\Big|_{x+\Delta x} - \frac{\partial T}{\partial x}\Big|_x\right) = \rho c \Delta x \frac{\partial T}{\partial t} \quad (iii).$$

③ Take limit: $\Delta x \to 0$.

(iii) gives:

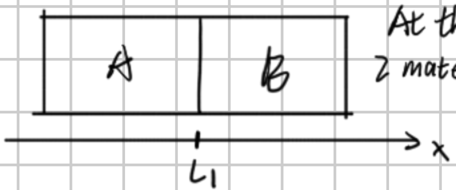
$$k\left(\frac{\partial T}{\partial x}\Big|_{x+\Delta x} - \frac{\partial T}{\partial x}\Big|_x\right) = \rho c \cdot \frac{\partial T}{\partial t}(dx)$$

$$\therefore k\left(\frac{\frac{\partial T}{\partial x}\Big|_{x+\Delta x} - \frac{\partial T}{\partial x}\Big|_x}{\partial x}\right) = \rho c\left(\frac{\partial T}{\partial t}\right)$$

$$\lim_{x\to 0}: k\left(\frac{\partial T^2}{\partial^2 x}\right) = \rho c\left(\frac{\partial T}{\partial t}\right)$$

(1D Transient Heat Conduction Equation).

④ Boundary Conditions:



At the interface of 2 materials A.B:

1a) Temperature is continuous:

$$T_1(L_1, t) = T_2(L_1, t)$$

1b). Heat flux is continuous:

$$-k_1\frac{\partial T_1}{\partial x} = -k_2\frac{\partial T_2}{\partial x}$$

2. Experiment: Implementing IR camera to visualize heat transfer,

* Reference: ① youtube: ICI, "what is Emissivity and why is it Important for Thermal Imaging".

② youtube: TEquipment, "A Complete guide to Emissivity for Thermal Imaging"

(1) How IR cameras work:

The IR camera actually reads the intensity of radiation. And uses radiation to calculate the temperature by:

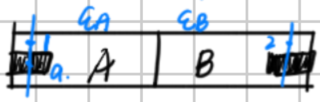
$$R = \varepsilon_{real} \cdot \sigma \cdot T_{real}^4$$

R → radiation intensity. ε_{real} → real emissivity. T_{real} → real temperature.

But different materials have different emissivity of surface. This leads to significant error.

(2) Caliberation:

A. Use a black tape whose ε is known:



B. Set $\varepsilon_{tape} = 0.95$. And measure the temperature of taped region a. Then, measure the temperature at 1 directly above/below a. Adjust ε_A until T_a and T_1 are approximately the same.

C. Use another IR camera. Repeat the process with B. (2 cameras enable simultaneous true reading for comparison) ⟹ $\varepsilon_{real} T_{real}^4 = T_{cam}^4 \varepsilon_{cam}$.

A demo is also built to virtually simulate this interesting phenomenon.
Please see my Github Page attached!