Descartes' Rule of Signs

- **Descartes's rule of signs**, in algebra, rule for determining the maximum number of positive real number solutions (roots) of a polynomial equation in one variable based on the number of times that the signs of its real number coefficients change when the terms are arranged in the canonical order (from highest power to lowest power).

- The rule of signs was given, without proof, by the French philosopher and mathematician René Descartes in *La Géométrie* (1637).

- Descartes' rule of sign is **used to determine the number of real zeros of a polynomial function**.
  - It tells us that the number of positive real zeros in a polynomial function f(x) is the same or less than by an even numbers as the number of changes in the sign of the coefficients.

Key Concept — Descartes' Rule of Signs

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial function with real coefficients, then

- the number of *positive* real zeros of f is equal to the number of variations in sign of $f(x)$ or less than that number by some even number and
- the number of *negative* real zeros of f is the same as the number of variations in sign of $f(-x)$ or less than that number by some even number.

Example

Describe the possible real zeros of

$$1.\ x^3 - 6x^2 + 11x - 6 = 0$$

Ans.

3 or 1 positive real zeros; 0 negative real zero

Example

Describe the possible real zeros of

$$2.\ x^3 - x^2 - 10x - 8 = 0$$

Ans.

1 positive real zero; 2 or 0 negative real zero

Example

Describe the possible real zeros of

$$3.\ x^3 + 2x^2 - 23x - 60 = 0$$

Ans.

1 positive real zero; 2 or 0 negative real zero

Example

Describe the possible real zeros of

$$4.\ 2x^4 - 3x^3 - 4x^2 + 3x + 2 = 0$$

Ans.

2 or 0 positive real zeros; 2 or 0 negative real zeroes

Example

Describe the possible real zeros of

$$5.\ 3x^4 - 16x^3 + 21x^2 + 4x - 12 = 0$$

Ans.

3 or 1 positive real zeros; 1 negative real zero

Example

Describe the possible real zeros of

$$6. -2x^4 + 13x^3 - 21x^2 + 2x + 8 = 0$$

Ans.

3 or 1 positive real zeros; 1 negative real zero