Homework 5: DFA Regular Expressions and Minimization

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Exercise 3.2.1: DFA Regular Expressions and State Elimination

Given Transition Table

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

a) Initial Regular Expressions $R_{ij}^{(0)}$

- $R_{11}^{(0)} = \varepsilon$, $R_{12}^{(0)} = 0$, $R_{13}^{(0)} = \emptyset$
- $R_{21}^{(0)} = 1$, $R_{22}^{(0)} = \varepsilon$, $R_{23}^{(0)} = 0$
- $R_{31}^{(0)} = \emptyset$, $R_{32}^{(0)} = 1$, $R_{33}^{(0)} = 0 \mid \varepsilon$

b) Regular Expressions $R_{ij}^{(1)}$ (using q_1 as intermediate)

Apply: $R_{ij}^{(1)} = R_{ij}^{(0)} \mid R_{i1}^{(0)}(R_{11}^{(0)})^* R_{1j}^{(0)}$

- $R_{11}^{(1)} = \varepsilon$, $R_{12}^{(1)} = 0$, $R_{13}^{(1)} = \emptyset$
- $R_{21}^{(1)} = 1$, $R_{22}^{(1)} = \varepsilon \mid 10$, $R_{23}^{(1)} = 0$
- $R_{31}^{(1)} = \emptyset$, $R_{32}^{(1)} = 1$, $R_{33}^{(1)} = 0$

c) Regular Expressions $R_{ij}^{(2)}$ (using q_1 and q_2 as intermediate)

- $R_{13}^{(2)} = 0(10)^*0$
- $R_{11}^{(2)} = \varepsilon \mid 0(10)^*1$

- $R_{12}^{(2)} = 0 \mid 0(10)^*10$
- $R_{23}^{(2)} = 0 \mid 10(10)^*0$
- $R_{33}^{(2)} = 0 \mid 1(10)^*0$

d) Final Regular Expression

From q_1 (start) to q_3 (final):

$$\boxed{R = 0(10)^*0}$$

e) State Elimination Method

DFA diagram: Eliminate q_2.

Final expression:

$$\boxed{R = (1 \mid 01)00(0)}$$

Exercise 3.2.2

Given Transition Table

	0	1
$\rightarrow q_1$	q_2	q_3
q_2	q_1	q_3
$*q_3$	q_2	q_1

a) Initial Regular Expressions $R_{ij}^{(0)}$

- $R_{11}^{(0)} = \varepsilon$, $R_{12}^{(0)} = 0$, $R_{13}^{(0)} = 1$
- $R_{21}^{(0)} = 0$, $R_{22}^{(0)} = \varepsilon$, $R_{23}^{(0)} = 1$
- $R_{31}^{(0)} = 1$, $R_{32}^{(0)} = 0$, $R_{33}^{(0)} = \varepsilon$

b) $R_{ij}^{(1)}$ (using q_1 as intermediate)

Apply formula: $R_{ij}^{(1)} = R_{ij}^{(0)} \mid R_{i1}^{(0)}(R_{11}^{(0)})^* R_{1j}^{(0)}$

- $R_{12}^{(1)} = 0$, $R_{13}^{(1)} = 1$
- $R_{22}^{(1)} = \varepsilon \mid 00$, $R_{23}^{(1)} = 1 \mid 01$
- $R_{33}^{(1)} = \varepsilon \mid 11$

c) $R_{ij}^{(2)}$ (using q_1 and q_2 as intermediate)

$$R = R_{13}^{(2)} = 1 \mid 0(\varepsilon \mid 00)^*(1 \mid 01)$$

d) Final Regular Expression

$$\boxed{R = 1 \mid 0(00)^*(1 \mid 01)}$$

e) State Elimination

After eliminating q_2, from q_1 to q_3:

$$\boxed{R = (1 \mid 0(00)^*1)}$$

Exercise 4.4.1: DFA Minimization

Transition Table

	0	1
$\rightarrow A$	B	A
B	A	C
C	D	B
$*D$	D	A
E	D	F
F	G	E
G	F	G
H	G	D

a) Distinguishability Table

Final state: D Mark all pairs with one final and one non-final state.

Equivalent states: **E, F, G** are not marked $\rightarrow$ indistinguishable.

b) Minimized DFA

Merge: $E, F, G \rightarrow$ EFG

State	0	1	Accept?
$\rightarrow A$	B	A	No
B	A	C	No
C	D	B	No
D	D	A	Yes
EFG	EFG	EFG	No
H	EFG	D	No

Exercise 4.4.2: DFA Minimization

Transition Table

	0	1
$\to A$	B	E
B	C	F
$*C$	D	H
D	E	I
E	F	H
$*F$	G	B
G	H	B
H	I	C
$*I$	A	E

a) Distinguishability Table

Final states: C, F, I Unmarked (equivalent): **C, F, I**

b) Minimized DFA

Merge: $C, F, I \to$ CFI

State	0	1	Accept?
$\to A$	B	E	No
B	CFI	CFI	No
CFI	D	H	Yes
D	E	CFI	No
E	CFI	H	No
G	H	B	No
H	CFI	CFI	No

DFA minimized from 9 states to 7.