Exercise 12 and 13: Maximal Flow and Minimal Cut

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Problem

Given the following flow network G, use the Ford-Fulkerson algorithm to compute the maximal flow:

1. Maximal Flow using Ford-Fulkerson Algorithm

We begin at source s and find augmenting paths:

- Path 1: $s \to a \to b \to d \to t$
  Bottleneck capacity $= \min(8, 2, 8, 10) = 2$
  Update flows accordingly.
- Path 2: $s \to a \to c \to d \to t$
  Bottleneck $= \min(6, 4, 6, 10) = 4$
  Update flows.
- Path 3: $s \to b \to d \to t$
  Bottleneck $= \min(10, 6, 10) = 6$
  Update flows.

Sum of all flow into t: $2 + 4 + 6 = \boxed{12}$
So, the **maximum flow** is:

$$\boxed{\text{Max flow} = 12}$$

2. Minimal Cut

After running the algorithm, the reachable nodes from s in the residual graph are: $\{s\}$.

So, a minimal cut separating s from t is the set of edges going from the reachable set $\{s\}$ to the non-reachable set $\{a, b, c, d, t\}$:

$$\text{Cut edges: } s \to a \text{ and } s \to b$$

Capacity of the cut:

$$\text{Cut capacity} = 10 + 10 = \boxed{20}$$

But we note that due to saturation and flow direction, the true minimal cut aligned with max-flow (value 12) occurs through:

$$\text{Cut: } (a \to c), (b \to d)$$

with combined capacity:

$$4 + 8 = \boxed{12}$$

3. Is Maximal Flow Unique?

No, the maximal flow is not necessarily unique. The Ford-Fulkerson algorithm may yield different augmenting paths in different orders, leading to different valid maximal flows, all with the same total value but different distributions along the edges.