Turing Machine Solutions for Exercise A

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4/18/2025

Exercise A Solutions

1. TM for $L = \{10^n : n \in \mathbb{N}\}$ and return 10^{n+1}

Idea: The TM reads the input 10^n, moves to the end of the tape, appends an extra 0.

- States: q_0 (start), q_1 (move right to the end), q_2 (write extra 0), q_{accept}
- Alphabet: $\{0, 1, B\}$
- Transitions:

$$\begin{aligned}\delta(q_0, 1) &= (q_1, 1, R)\\ \delta(q_1, 0) &= (q_1, 0, R)\\ \delta(q_1, B) &= (q_2, 0, S)\\ \delta(q_2, 0) &= (q_{\text{accept}}, 0, S)\end{aligned}$$

2. TM for $L = \{10^n : n \in \mathbb{N}\}$ and return 1

Idea: Erase all 0s, then move to beginning and keep only 1.

- States: q_0 (start), q_1 (erase 0s), q_2 (go back left), q_3 (halt with 1), q_{accept}
- Alphabet: $\{0, 1, B\}$
- Transitions:

$$\begin{aligned}\delta(q_0, 1) &= (q_1, 1, R)\\ \delta(q_1, 0) &= (q_1, B, R)\\ \delta(q_1, B) &= (q_2, B, L)\\ \delta(q_2, 1) &= (q_3, 1, S)\\ \delta(q_3, 1) &= (q_{\text{accept}}, 1, S)\end{aligned}$$

3. TM that swaps 0's and 1's in a binary string

Idea: Read input from left to right, replacing 0 with X (temp), 1 with Y. Then go back and finalize: X $\to$ 1, Y $\to$ 0.

- States: q_0 (marking), q_1 (return), q_2 (finalizing), q_{accept}
- Transitions:

$$\begin{aligned}
\delta(q_0, 0) &= (q_0, X, R) \\
\delta(q_0, 1) &= (q_0, Y, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, X) &= (q_1, X, L) \\
\delta(q_1, Y) &= (q_1, Y, L) \\
\delta(q_1, B) &= (q_2, B, R) \\
\delta(q_2, X) &= (q_2, 1, R) \\
\delta(q_2, Y) &= (q_2, 0, R) \\
\delta(q_2, B) &= (q_{\text{accept}}, B, S)
\end{aligned}$$