Form / Recipe Master Method

(simplified version)

Given recurrence: T(n) =

1 Look and interpret

2 Perform pattern matching: T(n) = a · T(n/b) + f(n)

a = ______, b = ______, f(n) = ________________

Pattern matching successful? ☐ yes ☐ no ⟶ *abort / try different method*

3 Check parameters

Parameter	*Criterion*	*Evaluation*
a	constant, $a \geq 1$	☐ ok ☐ not ok
b	constant, $b > 1$	☐ ok ☐ not ok
f(n)	asymptotically positive	☐ ok ☐ not ok

Master method applicable? ☐ yes ☐ no ⟶ *abort / try different method*

4 Determine case

Calculate $x = \log_b(a)$ = ________________________ (leaves in recursion tree: n^x)

Compare asymptotically f(n) = _______________ with n^x = ______________:

Comparison	*Description*	*Case*	*Result*	*Work*
f(n) grows polynomially slower than n^x.	$f(n)$ "$\preccurlyeq$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in O(n^{x-\varepsilon})$	① ☐	$T(n) \in \Theta(n^x)$	mainly in the leaves
f(n) grows (about) as fast as n^x.	$f(n)$ "$\cong$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Theta(n^{x-\varepsilon} \cdot \log(n))$	② ☐	$T(n) \in \Theta(n^x \cdot \log(n))$	distributed evenly
f(n) grows polynomially faster than n^x.	$f(n)$ "$\succcurlyeq$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Omega(n^{x-\varepsilon})$	③ ☐	$T(n) \in \Theta(f(n))$	mainly in the root node

4a Check regularity for case ③

Find constants $c < 1$ and $n > n_0$ such that it holds for any n big enough that:

$$a \cdot f(n/b) \leq c \cdot f(n)$$

________________________ ≤ ______ · ______________

Regularity check successful? ☐ yes ☐ no ⟶ *abort / try different method*

5 Write down solution

$T(n) \in \Theta($______________$)$

Form / Recipe Master Method

(simplified version)

Given recurrence: $T(n) = 7 \cdot T(n/2) + n^2$

1 Look and interpret

The problem is split up in 7 subproblems each having half the size oft he original problem. The work which needs to be done outside of the recursive call depends quadratically on the size n of the problem.

2 Perform pattern matching: $T(n) = a \cdot T(n/b) + f(n)$

a = **7**, b = **2**, f(n) = $\mathbf{n^2}$

Pattern matching successful? ☒ yes ☐ no → *abort / try different method*

3 Check parameters

Parameter	*Criterion*	*Evaluation*	
a	constant, $a \geq 1$	☒ ok	☐ not ok
b	constant, $b > 1$	☒ ok	☐ not ok
f(n)	asymptotically positive	☒ ok	☐ not ok

Master method applicable? ☒ yes ☐ no → *abort / try different method*

4 Determine case

Calculate $x = \log_b(a) = \mathbf{\log_2(7) \approx 2.807}$ (leaves in recursion tree: n^x)

Compare asymptotically $f(n) = \mathbf{n^2}$ with $n^x = n^{\log_2(7)} \approx n^{2.807}$:

Comparison	*Description*	*Case*	*Result*	*Work*
f(n) grows polynomially slower than n^x.	$f(n)$ "$\lessdot$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in O(n^{x-\varepsilon})$	① ☒	$T(n) \in \Theta(n^x)$	mainly in the leaves
f(n) grows (about) as fast as n^x.	$f(n)$ "$\cong$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Theta(n^{x-\varepsilon} \cdot \log(n))$	② ☐	$T(n) \in \Theta(n^x \cdot \log(n))$	distributed evenly
f(n) grows polynomially faster than n^x.	$f(n)$ "$\gtrdot$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Omega(n^{x-\varepsilon})$	③ ☐	$T(n) \in \Theta(f(n))$	mainly in the root node

4a Check regularity for case ③

Find constants $c < 1$ and $n > n_0$ such that it holds for any n big enough that:

$$a \cdot f(n/b) \leq c \cdot f(n)$$

$$\_\_\_\_\_\_ \leq \_\_\_\_ \cdot \_\_\_\_\_\_$$

Regularity check successful? ☐ yes ☐ no → *abort / try different method*

5 Write down solution

$T(n) \in \Theta(\mathbf{n^{\log_2(7)}})$

Form / Recipe Master Method

(simplified version)

Given recurrence: $T(n) = 32 \cdot T(n/4) + n^3$

1 Look and interpret

The problem is split up in 32 subproblems each having a quarter the size oft he original problem. The work which needs to be done outside of the recursive call depends cubically on the size n of the problem.

2 Perform pattern matching: T(n) = a · T(n/b) + f(n)

a = **32**, b = **4**, f(n) = **n^3**

Pattern matching successful? ☒ yes ☐ no → *abort / try different method*

3 Check parameters

Parameter	*Criterion*	*Evaluation*
a	constant, $a \geq 1$	☒ ok ☐ not ok
b	constant, $b > 1$	☒ ok ☐ not ok
f(n)	asymptotically positive	☒ ok ☐ not ok

Master method applicable? ☒ yes ☐ no → *abort / try different method*

4 Determine case

Calculate $x = \log_b(a) =$ **$\log_4(32) = 2.5$** (leaves in recursion tree: n^x)

Compare asymptotically $f(n) =$ **n^3** with $n^x = n^{\log_4(32)} = n^{2.5}$:

Comparison	*Description*	*Case*	*Result*	*Work*
f(n) grows polynomially slower than n^x.	$f(n)$ "$\precneqq$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in O(n^{x-\varepsilon})$	① ☐	$T(n) \in \Theta(n^x)$	mainly in the leaves
f(n) grows (about) as fast as n^x.	$f(n)$ "$\cong$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Theta(n^{x-\varepsilon} \cdot \log(n))$	② ☐	$T(n) \in \Theta(n^x \cdot \log(n))$	distributed evenly
f(n) grows polynomially faster than n^x.	$f(n)$ "$\succneqq$" $\Theta(n^x)$ $\exists \varepsilon > 0$: $f(n) \in \Omega(n^{x-\varepsilon})$	③ ☒	$T(n) \in \Theta(f(n))$	mainly in the root node

4a Check regularity for case ③

Find constants $c < 1$ and $n > n_0$ such that it holds for any n big enough that:

$$a \cdot f(n/b) \leq c \cdot f(n)$$

$$\mathbf{32 \cdot (n/4)^3 = 0.5 \cdot n^3} \leq \mathbf{0.5} \cdot \mathbf{n^3}$$

→ The equation is fulfilled for c = 0.5 and any arbitrary n.

Regularity check successful? ☒ yes ☐ no → *abort / try different method*

5 Write down solution

$T(n) \in \Theta(\mathbf{n^3})$