Chapter 1

# The propagation of light

## 1.1 INTRODUCTORY

The branch of Science known as **optics** is concerned with the study of light and vision; it may also embrace the study of other radiations closely allied to light.

Light is a form of radiant energy, being similar in its essential nature to other forms, such as heat and radio waves. It is radiated out through space at an enormous speed by **luminous bodies**. Most of these are capable of emitting light by virtue of their exceedingly high temperature, as a result of which their constituent atoms are in a condition of considerable agitation, the effects of which are transmitted outwards from the body in all directions.

A poker, when cold, emits no radiation that we can detect. When gradually heated, it sets up a disturbance in the form of vibrations or waves in the surrounding medium, which radiate outwards at a speed of 186 000 miles per second; at some distance away we can 'feel' the effect as heat, i.e., we detect this form of radiation by our sense of touch. As the temperature rises and the vibrations become more rapid, the poker is seen to become red; the radiation is such that we can 'see' its effects; it is in the form of light. We detect this form of radiation by our sense of sight, the eye acting as detector. With further rise in temperature the poker passes to a 'yellow' and then to a 'white' heat.

The exact nature of light is not completely known; but a clear working idea of these 'wave motions' can be derived from the ripples set up on the surface of water into which a stone is dropped. The important characteristics of the disturbance are: the **velocity** with which it travels outwards, the **frequency** of the rise and fall of the undulations, and the distance between successive wave crests, called the **wave-length**. Clearly the velocity is equal to the product of the wave-length and the frequency or number of vibrations arriving at a given point in unit time.

In the case of light, heat and electrical radiations, the velocity has been found by experiment (Chapter 12) to be 186 000 miles or 300 million metres per second. As we should expect from the poker experiment, the frequency of the vibrations is greater for light than for heat, and greater for some colours of light than others. The wave-length is consequently longer for heat than light and longer for red light (which appeared first) than for other colours. The figures for a few selected radiations are given in the table.

In the table, only one typical value is given for each group of radiations. Actually, each group covers a wide range, and a more complete statement will be found in *Figure 12.8*.

| Radiation | | Velocity (metres per sec) | Frequency (per sec)† | Wave-length (nanometres*) |
|---|---|---|---|---|
| Radio | | $3 \times 10^8$ | $1 \times 10^6$ | $3 \times 10^{11}$ |
| Heat | Thermal Infra-red | $3 \times 10^8$ | $30 \times 10^{12}$ | 10000 |
| | Near Infra-red | $3 \times 10^8$ | $300 \times 10^{12}$ | 1000 |
| Light | Red | $3 \times 10^8$ | $395 \times 10^{12}$ | 759 |
| | Yellow | $3 \times 10^8$ | $509 \times 10^{12}$ | 589 |
| | Violet | $3 \times 10^8$ | $764 \times 10^{12}$ | 393 |
| Ultra-violet | | $3 \times 10^8$ | $1000 \times 10^{12}$ | 300 |
| X-rays | | $3 \times 10^8$ | $3 \times 10^{18}$ | 0.1 |

*1 nanometre equals $1 \times 10^{-9}$ metre
†Frequency per sec is Hertz

## 1.2 RECTILINEAR PROPAGATION OF LIGHT

Any space through which light travels is an **optical medium**. Most optical media have the same properties in all directions and are said to be **isotropic**—although there are a few optical substances, notably certain crystals, where this is not the case and in which particular phenomena arise (Chapter 16). Most media, moreover, possess these same properties throughout their mass, so that they are **homogeneous**.

When light starts from a point source B (*Figure 1.1*) in an isotropic medium, it spreads out uniformly at the same speed in all directions; the position it occupies at a given moment will be a sphere having the source at its centre. Such imaginary spherical surfaces will be called **light fronts** or **wavefronts**. In the case of the water ripples, the disturbance is propagated in one plane only and the wavefronts are *circular*.

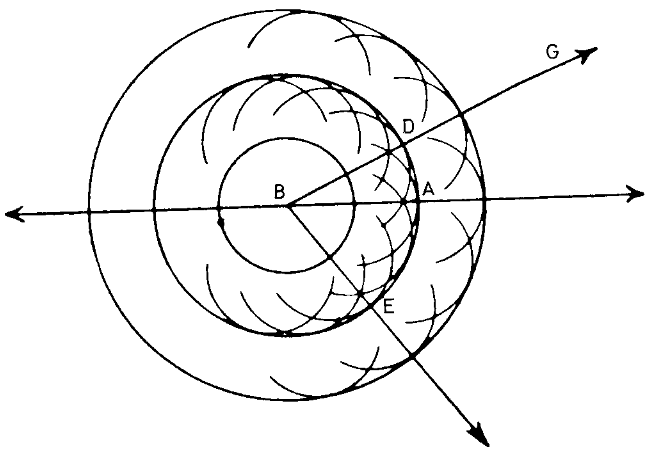


*Figure 1.1 Wavefront. Huygens' Principle*

Huygens (1629-1695), who is usually considered as the founder of the wave theory of light, assumed that any point on a wave surface behaved as a new source from which spherical 'secondary waves' or 'wavelets' spread out in a forward direction. The surface DAE, which at any instant touches or envelops all these wavelets, forms a new wavefront, and it is evident that while the light remains in the same isotropic medium, each new wavefront will be a sphere with its centre at B.

**Huygens' Principle** assumes that the wavelets travel out in a forward direction only, and that the effect of each wavelet is limited to that part which touches the enveloping new wave front. The first of these assumptions may be explained by the fact that the points from which the wavelets arise are not independent sources, but are set in motion as the result of a wavefront coming from the original source; while Fresnel (1788-1827) has shown (Chapter 15) by considering the 'interference' that takes place between the wavelets, that the second assumption is approximately correct.

From *Figure 1.1* it is evident that the path, such as BDG, travelled by any part of the disturbance, is a straight line perpendicular to the wavefront. We speak of such a line along which light travels as a **ray**. Huygens' construction for the new wavefront enables us to find the change in the form or direction of the wavefront on passing into a new medium, or on reflection at a surface, and will be used in this way in subsequent chapters.

A luminous source, such as B (*Figure 1.2*), will be seen only if light from it enters the eye. Thus an eye placed in the region between C and K, or between D and L, of the two screens CD and KL will see B, because the light reaches these regions without interruption. An eye placed between *c* and *d* will not see the object B, because the opaque obstacle *a b* shields the region *c a b d*, which is said to be in **shadow**. B will be visible, however, to an eye at *e*, just beyond the limits of the shadow region.

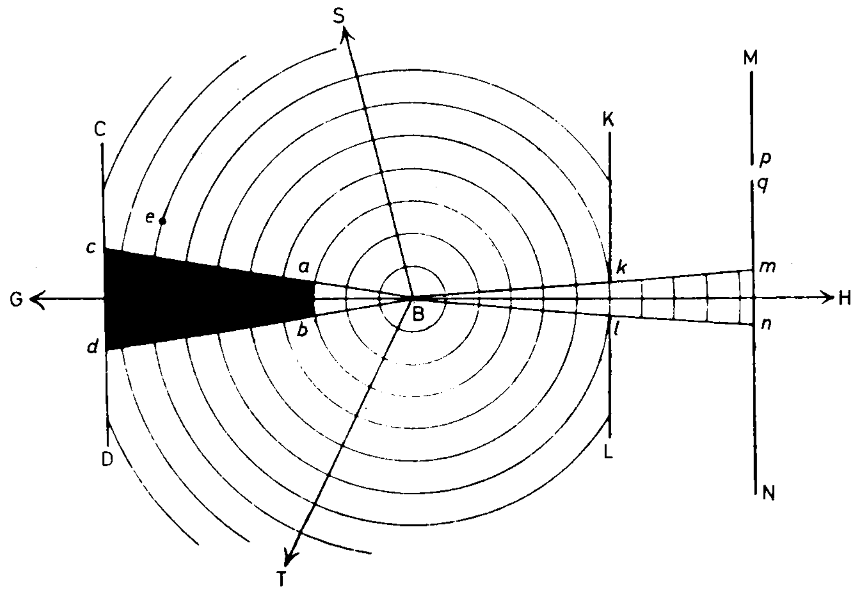


*Figure 1.2 Rectilinear propagation of light. Aperture at kl; obstacle at ab*

If we have two opaque screens at KL and MN, with circular apertures at *k l* and *p q* respectively, the screen MN will be illuminated over a circular patch *m n*, but will be dark elsewhere, and an eye placed at the aperture *p q* will not see the source B.

The fact that B is invisible to an eye placed between *c d* and visible to one placed within the region *k m n l* shows, as was also seen from Huygens' principle, that the light effect at a given point is due sensibly to that portion of the light that has travelled along the straight line joining the point to the source.

If the conditions within the regions *c d* and *m n* were to be studied more closely, it would be found that the patch *m n* is not uniformly illuminated, nor is the patch *c d* completely dark and separated from the surrounding brightness on the screen CD by a sharp line of demarcation, especially if *a b* and *k l* are small. The light waves, in passing the edges of these apertures, bend round into the space behind them in the same way as water waves may be seen curving round the end of a breakwater. Light waves are so very small however, that this bending or **diffraction**, as it is called, takes place to an extent so minute, that special precautions have to be taken to observe it. (See Chapter 15).

## 1.3 PENCILS AND BEAMS

The light from a luminous point, or from any one point on a large source or illuminated object, after passing through a limiting aperture such as *k l* (*Figure 1.2*) constitutes a **pencil** of light. The word **bundle** of rays is often used to mean the same thing. Ray bundles are more commonly associated with computer ray tracing and the calculation of geometrical aberrations (Chapter 18). We will use the word pencil for most of this book. The aperture (*Figure 1.2*) may be simply an opening in an opaque screen or the edges of a lens, mirror, etc. The pencil increases in width as the distance from the source increases, and the light is said to be **divergent**. Under certain circumstances, however, as for example, after the light has passed through a convex lens, the pencil may be modified in such a way that its width is gradually decreasing; the light is then said to be **convergent**, and it converges to a point or **focus**. This focus will be the **image** of the object point from which the light started. Beyond the focus the pencil again diverges, its width still being limited by the original aperture. When the object point or the focus is at a great distance as compared with the width of the aperture the edges of the pencil will be practically parallel. Divergent, convergent and parallel pencils, with the corresponding wavefronts, are illustrated in *Figure 1.3*. The ray passing through the centre of the aperture, will be the **principal** or **chief ray** of the pencil.

The collection of pencils arising from an extended source or object (*Figure 1.4a*) constitutes a **beam**. The edges of the beam may be divergent or convergent, irrespective of the divergence or convergence of the pencils constituting the

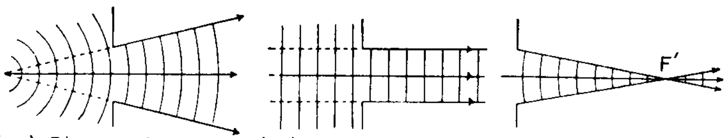


*Figure 1.3 Pencils showing light fronts*

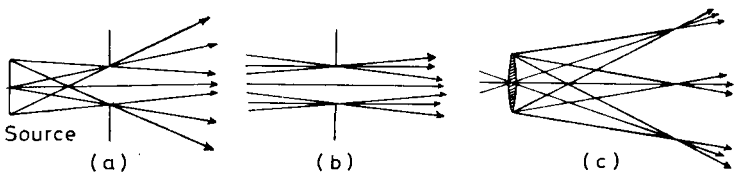


*Figure 1.4 Pencils and beams*

beam. Thus, if light from the sun passes through an aperture (*Figure 1.4b*) the individual pencils from each point on the sun will be parallel, but the pencils are not parallel to each other, and the edges of the beam are divergent. Similarly, the beam from a lens (*Figure 1.4c*) may be divergent, while its constituent pencils are convergent. *The terms divergent, convergent, or parallel light refer to the form of the pencils and not to that of the beam.*

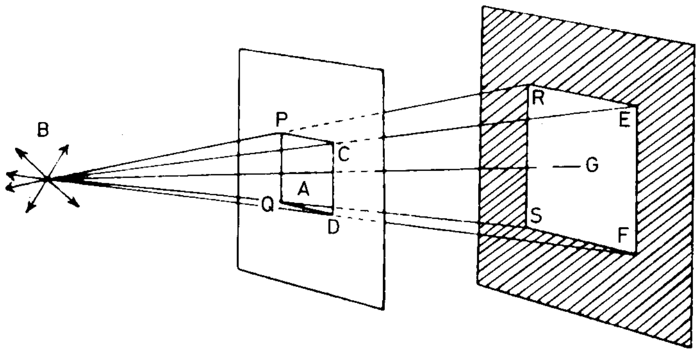


*Figure 1.5 Width of a bundle or pencil*

In *Figure 1.5* BEF is the section of a pencil diverging from a luminous point B, and limited by an aperture CD, EF being the width of the cross section of the pencil in a plane parallel to the plane of the aperture. In the similar triangles CDB and EFB

$$\frac{\mathrm{EF}}{\mathrm{CD}} = \frac{\mathrm{BG}}{\mathrm{BA}} \tag{1.01}$$

As the light spreads out uniformly in all directions,

$$\frac{\text{area EFSR}}{\text{area CDQP}} = \left(\frac{\mathrm{BG}}{\mathrm{BA}}\right)^2 \tag{1.02}$$

that is, *the area of the cross section of a pencil varies as the square of the distance from the source.*

Since the amount of light in a pencil depends only upon the amount given out by the source, the amount falling upon a unit area of an illuminated surface—the **illumination**—varies inversely as the area of the cross section of the pencil; hence, *the illumination of a surface placed perpendicularly to the direction in which the light is travelling varies inversely as the square of the distance of the surface from the source.* This is the well-known **Law of Inverse Squares**, which is of great importance in questions of light measurement, illumination, etc.

## 1.4 VERGENCE

For a given aperture, the divergence or convergence of a pencil and the curvature of the wavefront at the aperture decrease as the distance of the luminous point or of the focus from the aperture is increased. As the effect of a lens or curved mirror is to change the divergence or convergence of light, it is necessary to have some means of expressing the amount of divergence or convergence of a pencil in any given position. This is done by defining the divergence or convergence of a pencil at any particular position as the reciprocal of the distance *from* that position *to* the luminous point or the focus. It will be seen in Chapter 5 that this quantity also represents the curvature of the wavefront. The general term **vergence** has been suggested to denote either divergence or convergence, the difference being shown by a difference of sign. The unit of vergence is the **dioptre**, the vergence of a pencil one metre from the luminous point or focus (see section 5.1).

## 1.5 THE PINHOLE CAMERA

We have an interesting verification of the law of rectilinear propagation in the pinhole camera. The light from each point on an illuminated object, on passing through a small aperture in an opaque screen, gives rise to a narrow pencil, and if the light is received on a second screen at some distance from and parallel to the plane of the aperture, each pencil produces a patch of light of the same shape as the aperture. As the light travels in straight lines, the patches of light on the screen occupy similar relative positions to those of the corresponding points on the object, and the illuminated area of the screen is similar in shape to the original object, but inverted (*Figure 1.6a*). If the aperture is made quite

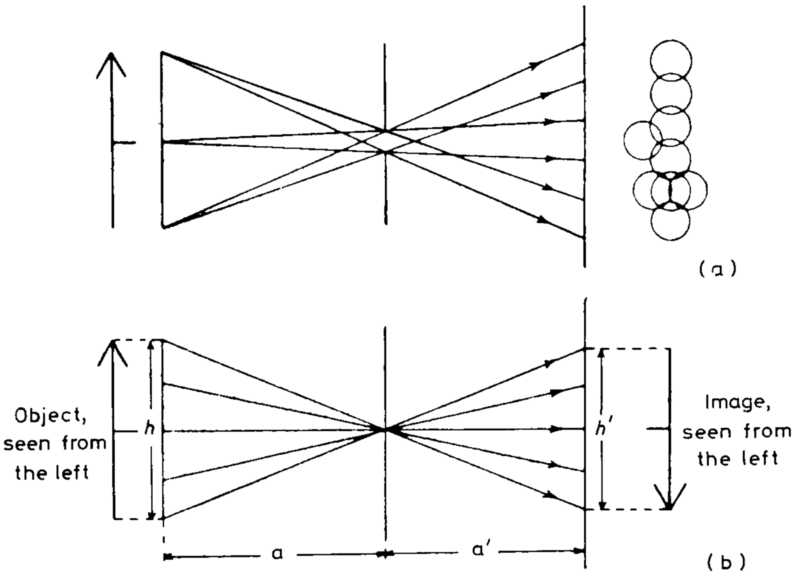


*Figure 1.6 Pinhole camera*

small, the individual patches of light will overlap to only a small extent, and a fairly well defined picture or image of the object is formed (*Figure 1.6b*). As will be seen from *Figure 1.6b*, the size of the inverted image of any object will depend on the distances of object and image from the aperture, thus

$$\frac{h'}{h} = \frac{a'}{a}$$

The degree of sharpness of the pinhole image can never be of a very high order, because if the diameter of the aperture is reduced beyond a certain amount, additional blurring becomes evident, due to diffraction effects. Also, the illumination of the image is very low as compared with that formed by a lens, owing to the very small aperture used. The image is free from distortion and the depth of focus is very great, i.e., images of objects at greatly varying distances are reasonably sharp in one plane.

## 1.6 SHADOWS AND ECLIPSES

The properties of shadows may easily be deduced from the law of rectilinear propagation. If the source is a point, the boundary of the shadow is sharply defined (neglecting diffraction) and its section, as received on a screen perpendicular to the typical direction of the light, will be the same shape as the opaque obstacle would appear when viewed from the position of the source. Thus the shadow of a sphere will have a circular section, while that of a flat circular body will be circular or elliptical according to how it is tilted. The size of the shadow in any position may be determined in the same way as the cross section of a pencil (section 1.3).

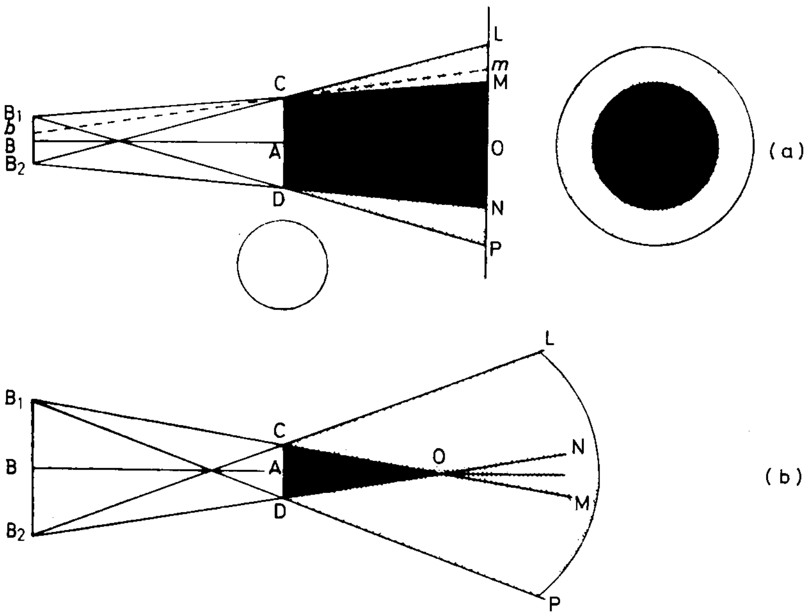


*Figure 1.7 Shadows. Umbra and penumbra*

Usually, the light will be coming from an extended source and we must consider the shadow as formed by light from different points on the source. In *Figure 1.7a* $B_1B_2$ is a circular source, CD a circular opaque object and LP a screen. No light from the source can enter the space CDNM, and on the screen there is a circular area of diameter MN in total shadow, known as the **umbra.** Surrounding the umbra is a space, CLM, DNP, in partial shadow, known as the **penumbra.** In the penumbra the illumination gradually increases from total darkness at the edges of the umbra to the full illumination at the outer edges of the shadow; for example, the point $m$ is receiving light from the portion $B_1b$ of the source. The shadow received on the screen consists, then, of a central dark portion surrounded by an ill-defined shaded edge. The diameters of the umbral and penumbral portions of the shadow received on a screen, may be found from similar triangles.

When the source is larger than the opaque object (*Figure 1.7b*), the umbra is in the form of a cone having the object as its base and at a certain distance from the object the umbra disappears, and the shadow is wholly penumbral. This may readily be seen in the case of the shadows of objects in sunlight. As the sun subtends an angle at the earth of 32 minutes, the length of the umbra in sunlight is 105 times the diameter of the opaque object.

In *Figure 1.7b*,

$$\frac{AO}{BO} = \frac{CD}{B_1B_2} \tag{1.03}$$

$$AO = \frac{CD \times BA}{B_1B_2 - CD} \tag{1.04}$$

## Eclipse of the sun

At certain times the moon comes between the sun and the earth and as the distance between the moon and the earth is less than the length of the umbra of the moon's shadow, the shadow on the earth's surface will consist of an umbral portion surrounded by a penumbra. An observer in the area covered by the umbra will see the moon completely covering the sun and the eclipse will be total; in the penumbra the eclipse will be partial, only a portion of the sun being covered.

## Eclipse of the moon

When the earth is directly between the sun and the moon, the shadow of the earth is seen projected upon the moon's surface, and the eclipse will be total when the moon lies wholly within the umbra. When the eclipse is partial the penumbra of the shadow may be seen as a partial darkening of the moon's surface outside the complete shadow.

Shadows play an important part in our visual interpretation of the forms of solid objects; for example, a distant white sphere, evenly illuminated, is indistinguishable from a circular flat surface, while an illumination giving a shadow on some part of it at once gives it its solid appearance. Much can be done in disguising the form of objects by darkening various parts to create a false impression of shadows.

## 1.7 THE NATURE OF WHITE LIGHT

In the poker experiment, it was seen that the light emitted changed from red, which appeared first, to yellow and finally to white when the temperature was sufficiently high. The nature of white light was investigated by Sir Isaac Newton (1642-1727) in his classical experiments in 1666. On passing a narrow beam of sunlight through a glass prism, Newton found that the patch of light received on a screen was broadened out into a band of colours, in which he recognised seven distinct colours in the following order: red, orange, yellow, green, blue, indigo and violet, each colour gradually shading into the next. This coloured band is called a **spectrum** and the white light is said to be **dispersed**. Recombining the various coloured beams by passing them through a similar prism with its base in the opposite direction to the first, or through a convex lens, Newton found that the light was again white: he therefore concluded that white light is composed of a mixture of light of the seven spectrum colours. There is nothing magical about the number seven. In fact, Newton was colour blind! His assistant named the colours. Recently the spectrum has been divided into 15 colours. These are listed in Chapter 13.

It is now known that each of these colours consists of vibrations of a given range of frequency, red having the lowest frequency and therefore the longest wave-length, and violet the highest frequency and the shortest wave-length, the other colours occupying intermediate positions in the scale of frequencies according to their position in the spectrum. Any solid body, when raised to a 'white' heat, is emitting vibrations of all these frequencies and, in addition, a certain amount which lie outside the range of frequencies to which the eye is sensitive. Of these, those of shorter wave-length than the violet are known as the **ultra-violet** radiations and those of longer wave-length than the red, as the **infra-red** (see Chapter 13).

*CHAPTER 1* *EXERCISES*

1. The velocity of light *in vacuo* is $3 \times 10^{10}$ cm per second. What will be the wave-lengths corresponding to the following frequencies?

| | |
|---|---|
| Orange | $4.5 \times 10^{14}$ Hertz |
| Green | $5.69 \times 10^{14}$ Hertz |
| Blue | $6.96 \times 10^{14}$ Hertz |

2. Describe an experiment to show that light travels in approximately straight lines.

3. A person holding a tube 6 inches long and 1 inch in diameter in front of his eye just sees the whole of a tree through the tube. What is the apparent (angular) height of the tree, and what is its distance if its actual height is 40 feet?

4. Explain the terms, convergent, divergent and parallel light. Illustrate your answer with diagrams showing the form of the wavefronts in each case.

5. A pencil of light diverges from a point source through a rectangular aperture 3 cm $\times$ 4 cm at 30 cm from the source. Find the area of the patch of light on a screen parallel to, and 120 cm from the aperture.

6. A pencil of light on passing through a lens of 40 mm diameter converges to a point 60 cm from the lens. Find the areas of the cross sections of the pencil at 20, 50 and 75 cm from the lens.

7. What are meant by pencils and beams of light? Illustrate your answer with diagrams. A spherical source of 5 cm diameter is placed 30 cm from a circular aperture of 8 cm diameter in an opaque screen; find the size and nature of the patch of light on a white screen 50 cm from and parallel to the plane of the aperture.

8. A small source of light is 30 cm from a rectangular aperture 18 cm × 8 cm. A screen is placed at 105 cm from and parallel to the plane of the aperture; what will be the area of the illuminated patch on the screen?

9. What is the height of a tower that casts a shadow 65 feet 3 inches in length on the ground, the shadow of the observer who is 6 feet high being at the same time 7 feet 6 inches in length?

10. A man 5 feet 11 inches in height is standing at a distance of 4 feet 6 inches from a street lamp. What will be the length of the man's shadow, if the lamp is 9 feet above the roadway?

11. A circular opaque disc of 2 feet diameter is 6 feet from an arc lamp; find the size of the shadow on a screen 20 feet from and parallel to the disc. What will be the form and size of the shadow if the arc is surrounded by a diffusing globe of 1 foot 6 inches diameter?

12. Explain carefully the formation of the image in a pinhole camera. How does the character and size of the image depend on:

(*a*) the size of the aperture.
(*b*) the shape of the aperture,
(*c*) the distance of the aperture from the screen,
(*d*) the distance from the aperture to the object.

13. If the distance between an object and its image formed by a pinhole is 5 feet, what will be the position of the pinhole for the image to be one-tenth the size of the object?

14. A pinhole camera produces an image 2.25 inches diameter of a circular object, and when the screen is withdrawn 3 inches further from the pinhole, the diameter of the image increases to 2.75 inches. What was the original distance between the pinhole and the screen?

15. If the sun subtends an angle of 32 minutes at the earth, what must be the distance between the aperture and the screen of a pinhole camera in order that the image of the sun shall have a diameter of $^1/_2$ inch?

16. Explain the formation of an image in a pinhole camera. Sunlight is reflected from a small plane mirror about 2 mm square and the reflected light falls normally on a white screen. Describe and explain the patch of light formed on the screen when this is (*a*) a few inches from the mirror, (*b*) about 10 feet from the mirror.

17. A circular opaque object 3 inches diameter is placed 12 inches from and parallel to a circular source of 5 inches diameter. Find the nature and size of the shadow on a screen perpendicular to the line passing through the centres of the source and the object and 3 feet from the object.

18. Explain with diagrams the way in which total and partial eclipses of (*a*) the sun and (*b*) the moon are produced.

19. Find the diameter of the umbra and penumbra of the earth at the distance of the moon (240 000 miles) taking the earth's diameter as 8000 miles, and the visual angle subtended by the sun from the earth as 32 minutes. (Assume the light from the sun to be parallel light).

20. Why are shadows much sharper in the case of an arc lamp without a surrounding diffusing globe than with one? Explain with a diagram.

21. A dark room 10 feet square, with white walls, has a small hole in one wall. The image of a man 6 feet high outside the room is formed on the wall and is 5 inches high. How far away is the man? What will be the size of the image of a tree 50 feet high and 300 feet away?

22. An opaque circular disc is interposed between a luminous disc of larger diameter and a screen so that the shadow consists of a central umbra and surrounding penumbra. The discs and screen are parallel and the line joining the centres of the discs is perpendicular to them. Show that the width of the penumbra ring is independent of the size of the opaque disc.