ISIT'2013 1569719837

Pilot-based Product Superposition for Downlink Multiuser MIMO

Yang Li and Aria Nosratinia
The University of Texas at Dallas, Richardson, TX 75080, USA , Email: {yang, aria}@utdallas.edu

***Abstract*— In the fading MIMO broadcast channel, until recently the results under full CSIR and no CSIR indicated that degrees of freedom (DoF) cannot be improved beyond what is available via TDMA. Recently, however, it was discovered that when one node has full CSIR and the other has none, TDMA is no longer DoF-optimal and a product decomposition with noncoherent (Grassmannian) signaling achieves the optimal DoF under certain receiver antenna configurations. This work extends product superposition to the domain of coherent signaling with pilots, showing that it can also achieve the optimal DoF. In addition, this work demonstrates the advantages of product superposition in low-SNR as well as high-SNR, and extends the set of receiver antenna configurations under which product decomposition achieves optimal DoF.**

I. Introduction

Due to varying mobilities, wireless network nodes often have unequal capability to acquire CSIR (channel state information at receiver). Downlink (broadcast) transmission to nodes with unequal CSIR is therefore a subject of practical interest.

It has been known that if all downlink users have full CSIR or none of them do, then orthogonal transmission (e.g. TDMA) achieves the optimal degrees of freedom (DoF) [1], [2] when no CSIT is available. Recently it was discovered [3] that a very different behavior emerges when one user has perfect CSIR and the other has none: in this case TDMA is highly suboptimal and a product superposition can achieve the optimal degrees of freedom (DoF). However, the analysis of [3] was limited to high-SNR, did not demonstrate optimality in all receiver antenna configurations, and more importantly, it required non-coherent Grassmannian signaling.

Most practical systems use pilots and employ channel estimation and coherent detection. Therefore in this paper we extend the product superposition to coherent signaling with pilots. We show the DoF optimality of product superposition for more antenna configurations, and in addition show that it has excellent performance in low-SNR as well as high-SNR.

A downlink scenario with two users is considered in this paper, where one user has a short coherence interval and is referred to as the *dynamic user*, and the other has a long coherence interval and is referred to as the *static user*. The main results of this paper are as follows.

- We propose a new signaling structure that is a product of two matrices representing the signals of the static and dynamic user, respectively, where the data for both users are transmitted using coherent signaling. We show that under this method, at both high SNR and low SNR, the dynamic user experiences almost no degradation due to the transmission of the static user. Therefore in the sense of the cost to the other user, the static user's rate is added to the system "for free."
- We show that the product decomposition is DoF optimal when the dynamic user has either more, less or equal number of antennas as the static user. Previously [3] the DoF optimality was demonstrated only when the dynamic user had fewer or equal number of antennas compared with the static user.

The following notation is used throughout the paper: for a matrix $\mathbf{A}$, the transpose is denoted with $\mathbf{A}^t$, the conjugate transpose with $\mathbf{A}^H$, the pseudo inverse with $\mathbf{A}^\dagger$ and the element in row i and column j with $[\mathbf{A}]_{ij}$. The $k \times k$ identity matrix is denoted with $\mathbf{I}_k$. The set of $n \times m$ complex matrices is denoted with $\mathcal{C}^{n\times m}$. We denote $\mathcal{CN}(0,1)$ as the circularly symmetric complex Gaussian distribution with zero mean and unit variance. For all variables the subscripts "s" and "d" stand as mnemonics for "static" and "dynamic", respectively, and subscripts "τ" and "δ" stand for "training" and "data."

II. System Model and Preliminaries

We consider an M-antenna base-station transmitting to two users, where the dynamic user has N_d antennas and the static user has N_s antennas. The channel coefficient matrices of the two users are $\mathbf{H}_d \in \mathcal{C}^{N_d\times M}$ and $\mathbf{H}_s \in \mathcal{C}^{N_s\times M}$, respectively. In this paper we restrict our attention to $M = \max\{N_d, N_s\}$. The system operates under block-fading, where $\mathbf{H}_d$ and $\mathbf{H}_s$ remain constant for T_d and T_s symbols, respectively, and change independently across blocks. The coherence time T_d is small but T_s is large ($T_s \gg T_d$) due to different mobilities. The difference in coherence times means that the channel resources required by the static user to estimate its channel are negligible compared to the training requirements of the dynamic user. To reflect this in the model, it is assumed that $\mathbf{H}_s$ is known by the static user (but unknown by the dynamic user, naturally), while $\mathbf{H}_d$ is not known *a priori* by either user.

Over T time-slots (symbols) the base-station sends $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_M]^t$ across M antennas, where $\mathbf{x}_i \in \mathcal{C}^{T\times 1}$ is the signal vector sent by the antenna i. The signal at the dynamic and static users is respectively

$$\mathbf{Y}_d = \mathbf{H}_d\mathbf{X} + \mathbf{W}_d, \quad \mathbf{Y}_s = \mathbf{H}_s\mathbf{X} + \mathbf{W}_s, \tag{1}$$

where $\mathbf{W}_d \in \mathcal{C}^{N_d\times T}$ and $\mathbf{W}_s \in \mathcal{C}^{N_s\times T}$ are additive noise with i.i.d. entries $\mathcal{CN}(0,1)$. Each row of $\mathbf{Y}_d \in \mathcal{C}^{N_d\times T}$ (or $\mathbf{Y}_s \in \mathcal{C}^{N_s\times T}$) corresponds to the received signal at an antenna

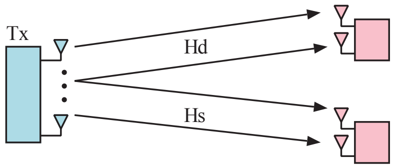

Fig. 1. Channel model.

of the dynamic user (or the static user) over T time-slots. The base-station is assumed to have an average power constraint ρ

$$\mathbb{E}\big[\sum_{i=1}^{M}\text{tr}(\mathbf{x}_i\mathbf{x}_i^H)\big] = \rho\, T_d. \tag{2}$$

In this paper, the base-station does not know $\mathbf{H}_d$ due to its fast variation. The channels $\mathbf{H}_d$ and $\mathbf{H}_s$ have i.i.d. entries with the distribution $\mathcal{CN}(0,1)$. We assume $M = \max(N_d, N_s)$ and $T_d \geq 2N_d$ [4].

A. The Baseline Scheme

For the dynamic user, we consider the following near-optimal baseline method. The base-station activates only N_d out of M antennas [4], sends an orthogonal pilot matrix $\mathbf{S}_\tau \in \mathcal{C}^{N_d \times N_d}$ during the first N_d time-slots, and then sends i.i.d. $\mathcal{CN}(0,1)$ data signal $\mathbf{S}_\delta \in \mathcal{C}^{N_d\times(T_d-N_d)}$ in the following $T_d - N_d$ time-slots [5], that is

$$\mathbf{X} = \left[\sqrt{\frac{\rho_\tau}{N_d}}\,\mathbf{S}_\tau\ \sqrt{\frac{\rho_\delta}{N_d}}\,\mathbf{S}_\delta\right] \tag{3}$$

where $\mathbf{S}_\tau\mathbf{S}_\tau^H = N_d\mathbf{I}$, and ρ_τ and ρ_δ are the average power used for training and data, respectively, and satisfy the power constraint in (2):

$$\rho_\tau N_d + \rho_\delta(T_d - N_d) \leq \rho T_d. \tag{4}$$

The dynamic user employs a linear minimum-mean-square-error (MMSE) estimation on the channel. The normalized channel estimate obtained in this orthogonal scheme is denoted $\overline{\mathbf{H}}_d \in \mathcal{C}^{N_d\times N_d}$. Under this condition, the rate attained by the dynamic user is [5]:

$$R_d \geq (1-\frac{N_d}{T_d})\mathbb{E}\big[\log\det(\mathbf{I}_{N_d} + \frac{\rho_d}{N_d}\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H)\big], \tag{5}$$

where ρ_d is the effective signal-to-noise ratio (SNR)

$$\rho_d = \frac{\rho_\delta\,\rho_\tau}{1+\rho_\delta+\rho_\tau N_d}. \tag{6}$$

For the static user, the channel is known at the receiver, the base-station sends data directly using all M antennas. The rate achieved by the static user is [6]

$$R_s = \mathbb{E}\left[\log\det\left(\mathbf{I}_{N_s} + \frac{\rho}{N_s}\,\mathbf{H}_s\mathbf{H}_s^H\right)\right]. \tag{7}$$

Time-sharing between R_d and R_s yields the rate region

$$\mathcal{R}_{OT} = \big(tR_d,\,(1-t)R_s\big). \tag{8}$$

III. Pilot-Based Product Superposition

A. Signaling Structure

Over T_d symbols (the coherence interval of the dynamic user) the base-station sends $\mathbf{X} \in \mathcal{C}^{N_s\times T_d}$ across N_s antennas:

$$\mathbf{X} = \mathbf{X}_s\mathbf{X}_d, \tag{9}$$

where $\mathbf{X}_s \in \mathcal{C}^{N_s\times N_d}$ is the data matrix for the static user and has i.i.d. $\mathcal{CN}(0,1)$ entries. The signal matrix $\mathbf{X}_d \in \mathcal{C}^{N_d\times T_d}$ is intended for the dynamic user and consists of the data matrix $\mathbf{X}_\delta \in \mathcal{C}^{N_d\times(T_d-N_s)}$ whose entries are i.i.d. $\mathcal{CN}(0,1)$ and the pilot matrix [1] $\mathbf{X}_\tau \in \mathcal{C}^{N_d\times N_s}$ which is *unitary*, and is known to both static and dynamic users.

$$\mathbf{X}_d = \left[\sqrt{c_\tau}\,\mathbf{X}_\tau\ \sqrt{c_\delta}\,\mathbf{X}_\delta\right], \tag{10}$$

where the constant c_τ and c_δ satisfy the power constraint (2):

$$N_sN_d\big(c_\tau + (T_d - N_d)c_\delta\big) \leq \rho T_d. \tag{11}$$

Please make note of the normalization of pilot and data matrices in the product superposition: The pilot matrix is unitary, i.e., the entire pilot power is normalized, while the data matrix is normalized per time per antenna. This is only for convenience of mathematical expressions in the sequel; full generality is maintained via multiplicative constants c_δ and c_τ.

A sketch of the ideas involved in the decoding at the dynamic and static users is as follows. The signal received at the dynamic user is

$$\mathbf{Y}_d = \mathbf{H}_d\mathbf{X}_s\left[\sqrt{c_\tau}\mathbf{X}_\tau\ \sqrt{c_\delta}\mathbf{X}_\delta\right] + \mathbf{W}_d \tag{12}$$

where $\mathbf{W}_d$ is the additive noise. The dynamic user uses the pilot matrix to estimate the equivalent channel $\mathbf{H}_d\mathbf{X}_s$, and then decodes $\mathbf{X}_\delta$ based on the channel estimate.

For the static user, the signal received during the first N_d time-slots is

$$\mathbf{Y}_{s1} = \sqrt{c_\tau}\,\mathbf{H}_s\mathbf{X}_s\mathbf{X}_\tau + \mathbf{W}_{s1} \tag{13}$$

where $\mathbf{W}_{s1}$ is the additive noise at the static user during the first N_d samples. The static user multiplies its received signal by $\mathbf{X}_\tau^H$ from the right and then recovers the signal $\mathbf{X}_s$.

B. Main Result

Theorem 1: Consider an M-antenna base-station, a dynamic user with N_d-antennas and coherence time T_d, and a static user with N_s-antennas and coherence time $T_s \gg T_d$. Assuming the dynamic user does not know its channel $\mathbf{H}_d$ but the static user knows its channel $\mathbf{H}_s$, the pilot-based product superposition achieves the rates

$$R_d = (1-\frac{N_d}{T_d})\mathbb{E}\left[\log\det\left(\mathbf{I}_{N_d} + \frac{\rho_d}{N_d}\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H\right)\right], \tag{14}$$

[1] Each of the dynamic user's codewords includes pilots because it needs frequent channel estimates. No pilots are included in the individual codewords of the static user because it only needs infrequent channel estimate updates. In practice static user's channel training occurs at much longer intervals outside the proposed signaling structure.

$$R_s = \frac{N_d}{T_d}\,\mathbb{E}\bigg[\log\det\bigg(\mathbf{I}_{N_s} + \frac{\rho_s}{N_s}\,\mathbf{H}_s\mathbf{H}_s^H\bigg)\bigg], \tag{15}$$

where $\overline{\mathbf{H}}_d$ is the *normalized* MMSE channel estimate of the equivalent dynamic channel $\mathbf{H}_d\mathbf{X}_s$, and ρ_d and ρ_s are the effective SNRs:

$$\rho_d = \frac{c_\tau c_\delta N_d N_s^2}{1 + c_\tau N_s + c_\delta N_d N_s}, \tag{16}$$

$$\rho_s = c_\tau N_s. \tag{17}$$

Proof: See Appendix I. ∎

For the static user, the effective SNR $\rho_s = c_\tau$ increases linearly with the power used in the training of the dynamic user. This is because the static user decodes based on the signal received during the training phase of the dynamic user. The pre-log factor is $\frac{N_d}{T_d}$ because the static user's signal only occupies N_d out of T_d time-slots.

For the dynamic user, the effective SNR ρ_d is unaffected by superimposing $\mathbf{X}_s$ on $\mathbf{X}_d$. To see this, compare (4) with (11) to arrive at $\rho_\tau = c_\tau N_s$ and $\rho_\delta = c_\delta N_d N_s$, therefore the two SNRs are equal to

$$\rho_d = \frac{c_\tau c_\delta N_d N_s^2}{1 + c_\tau N_s + c_\delta N_d N_s}. \tag{18}$$

Intuitively, the rate available to the dynamic user via orthogonal transmission (Eq. (5)) and via superposition (Eq. (14)) will be very similar: the normalized channel estimate $\overline{\mathbf{H}}_d$ in both cases has uncorrelated entries with zero mean and unit variance.[2] Thus the product superposition achieves the static user's rate "for free" in the sense that the rate for the dynamic user is approximately the same as in the single-user scenario. In the following, we discuss this phenomenon at low and high SNR.

1) Low-SNR Regime: We have $\rho_d, \rho_s \ll 1$. Let the eigenvalues of $\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H$ be denoted $\bar{\lambda}_{1i}^2$, $i = 1, \ldots, N_d$. Using (14) and a Taylor expansion of the log function at low SNR, the achievable rate for the dynamic user is approximately:

$$R_d \approx (1 - \frac{N_d}{T_d})\frac{\rho_d}{N_d}\,\mathbb{E}\big[\sum_{i=1}^{N_d}\bar{\lambda}_{1i}^2\big] \tag{19}$$

$$= (1 - \frac{N_d}{T_d})\frac{\rho_d}{N_d}\,\mathrm{tr}\big(\mathbb{E}[\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H]\big) \tag{20}$$

$$= (1 - \frac{N_d}{T_d})N_d\,\rho_d. \tag{21}$$

Similarly, from (5), the baseline method achieves the rate

$$(1 - \frac{N_d}{T_d})N_d\,\rho_d. \tag{22}$$

Thus, the dynamic user attains the same rate as it would in the absence of the other user and its interference, i.e., a single-user rate. At low SNR, one cannot exceed this performance.

[2] The dynamic channel estimates in the orthogonal and superposition transmissions have the same mean and variance but are not identically distributed, because in the orthogonal case, $\overline{\mathbf{H}}_d$ is an estimate of $\mathbf{H}_d$, a Gaussian matrix, while in the superposition case it is an estimate of $\mathbf{H}_d\mathbf{X}_s$, the product of two Gaussian matrices. Therefore the expectations in Eq. (5) and (14) may produce slightly different results.

The rate available to the static user at low-SNR is obtained via (15), as follows:

$$R_s \approx \frac{\rho_s}{T_d}\,\mathrm{tr}\big(\mathbb{E}[\mathbf{H}_s\mathbf{H}_s^H]\big) \tag{23}$$

$$= \frac{N_s^2\,\rho_s}{T_d}. \tag{24}$$

2) High-SNR Regime: We have $\rho_d, \rho_s \gg 1$, therefore from (14) the achievable rate for the dynamic user is

$$R_d \approx (1 - \frac{N_d}{T_d})\bigg(N_d\log\frac{\rho_d}{N_d} + \mathbb{E}\big[\sum_{i=1}^{N_d}\log\bar{\lambda}_{1i}^2\big]\bigg). \tag{25}$$

The dynamic user attains $N_d(1 - N_d/T_d)$ degrees of freedom, which is the maximum DoF even in the absence of the static user [4]. Superimposing $\mathbf{X}_s$ only affects the distribution of eigenvalues $\bar{\lambda}_{1i}^2$, whose impact is negligible at high-SNR.

For the static user, let the eigenvalues of $\mathbf{H}_s\mathbf{H}_s^H$ be denoted λ_{2i}^2, $i = 1, \ldots, N_s$. From (15), we have

$$R_s \approx \frac{N_d}{T_d}\bigg(N_s\log\frac{\rho_s}{N_s} + \mathbb{E}\big[\sum_{i=1}^{N_s}\log\lambda_{2i}^2\big]\bigg), \tag{26}$$

which implies that the static user achieves $N_d N_s/T_d$ degrees of freedom. Thus, the pilot-based product superposition achieves the optimal DoF obtained in [3] for $N_d \leq N_s$, and for $N_d > N_s$ meets the coherent upper bound.

C. Power Allocation

The effective SNRs of the dynamic and static users depend on c_τ and c_δ. We focus on c_τ and c_δ that maximize R_d (equivalently ρ_d) in a manner similar to [5]. From (55) and (48), the effective SNR is identical to (18). From (11), we have $c_\tau = \rho T_d/(N_d N_s) - c_\delta(T_d - N_d)$. Substitue c_τ into (**??**):

$$\rho_d = \frac{N_d N_s(T_d - N_d)}{T_d - 2N_d}\cdot\frac{c_\delta(a - c_\delta)}{-c_\delta + b}, \tag{27}$$

where

$$a = \frac{\rho T_d}{N_d N_s(T_d - N_d)}, \tag{28}$$

$$b = \frac{N_d + \rho T_d}{N_d N_s(T_d - 2N_d)}. \tag{29}$$

Noting that $0 \leq c_\delta \leq a$, we obtain the value of c_δ that maximizes R_d:

$$c_\delta^* = b - \sqrt{b^2 - ab}, \tag{30}$$

which corresponds to

$$\rho_d^* = \frac{N_d N_s(T_d - N_d)}{T_d - 2N_d}\big(2b - a - 2\sqrt{b^2 - ab}\big), \tag{31}$$

$$\rho_s^* = \frac{\rho T_d}{N_d} - N_s(T_d - N_d)(b - \sqrt{b^2 - ab}). \tag{32}$$

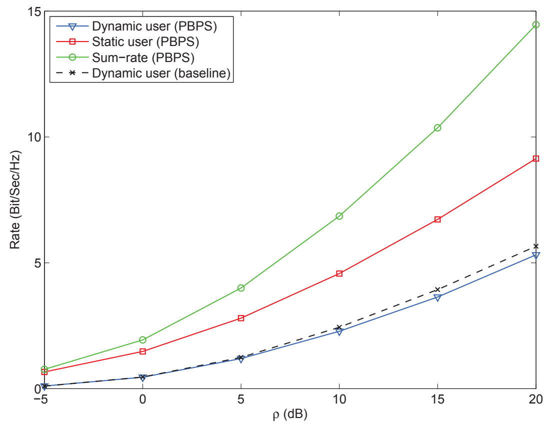

Fig. 2. Rate achieved by the pilot-based product superposition (PBPS): $N_d = 2$, $N_s = M = 4$ and $T_d = 5$.

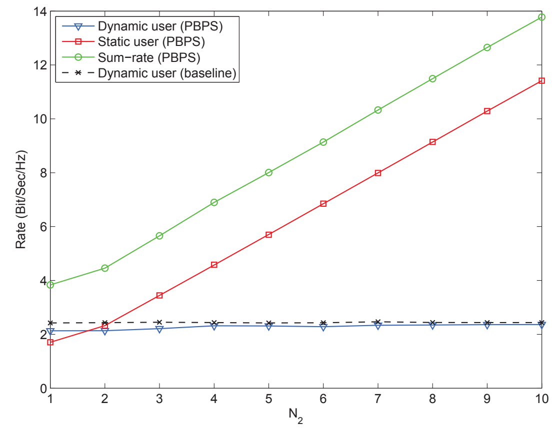

Fig. 3. Impact of the static user's antenna number : $\rho = 10$ dB, $N_d = 2$, $M = N_s$ and $T_d = 5$.

D. Partial CSIT

In the proposed method, the static user operates under an equivalent single-user channel, by inverting either the pilot component or all components of the dynamic user's signal. Thus, any benefits that can be realized in the single-user MIMO can also be available to the static user, including the benefits arising from CSIT. For example, water-filling can be applied to allocate power across multiple eigen-modes of the static user. CSI can also simplify decoding at the static user. To see this, using singular value decomposition (SVD), $\mathbf{H}_s = \mathbf{U}_s \mathbf{\Sigma}_s \mathbf{V}^H$, where $\mathbf{\Sigma}_s = \mathrm{diag}(\lambda_1, \cdots, \lambda_{N_s})$. Then, the base-station sends

$$\mathbf{X} = \mathbf{V}\mathbf{X}_s \, [\sqrt{c_\tau}\mathbf{X}_\tau \;\; \sqrt{c_\delta}\mathbf{X}_\delta]. \tag{33}$$

Since $\mathbf{V}$ is unitary, the entries of $\mathbf{V}_s\mathbf{X}_s$ remain i.i.d. $\mathcal{CN}(0,1)$, and therefore, the performance of the dynamic user is unaffected by precoding with $\mathbf{V}$. Without interference decoding, the static user forms the equivalent diagonal channel

$$\mathbf{U}_s^H \mathbf{Y}_\tau \mathbf{X}_\tau = \sqrt{c_\tau}\,\mathbf{X}_s + \mathbf{W}'_\tau, \tag{34}$$

where $\mathbf{W}'_\tau$ is the noise with i.i.d. $\mathcal{CN}(0,1)$ distribution.

IV. Numerical Results

Unless specified otherwise, a power allocation is assumed (c_τ and c_δ) that maximizes the rate for the dynamic user.

Figure 2 illustrates the rate for dynamic and static users in the pilot-based product superposition, as shown in Theorem 1. We consider $N_d = 2$, $N_s = M = 4$ and $T_d = 5$. Both the baseline method and proposed methods optimize the rate for the dynamic user. In this case, the baseline method cannot provide any rate for the static user. In addition to near-optimal rate for the dynamic user, the proposed method significant rate for the static user. The separation from optimality is negligible in the low-SNR regime, and in the high-SNR regime the rate of the dynamic user has the optimal degrees of freedom (SNR slope).

Figure 3 shows the impact of the available antenna of the static user. Here, $\rho = 10$ dB, $N_d = 2$, $M = N_s$ and $T_d = 5$. The static user's rate (thus the sum-rate) increases linearly with N_s, because the degrees of freedom is $N_d N_s / T_d$, as indicated by Theorem 1. The gap of the dynamic user's rate under the proposed method and the baseline method vanishes as N_s increases. Intuitively, the rate difference is because of the Jensen's loss: in the proposed method the equivalent channel is the product matrix $\mathbf{H}_d\mathbf{X}_s$ and is "more spread" than the channel in the baseline method. As N_s increases, $\mathbf{X}_s$ becomes "more unitary" ($\mathbf{X}_s\mathbf{X}_s^H / N_s \to \mathbf{I}_{N_d}$) and thus less impact on the distribution of $\mathbf{H}_d$.

V. Conclusion

In this paper, we propose and analyze a pilot-based signaling that significantly improves the rate performance of the MIMO broadcast channel with varying CSIR. The proposed method sends a product of two signal matrices for the static and dynamic user, respectively, and each user decodes its own signal in a conventional manner. The method can work without interference cancellation, therefore has low complexity. For the entire SNR range, the static user attains considerable rate almost without degrading the rate for the dynamic user.

Appendix I
Proof Of Theorem 1

A. Rate of the Static User

During the first N_d time-slots, the static user receives

$$\mathbf{Y}_{s1} = \sqrt{c_\tau}\,\mathbf{H}_s\mathbf{X}_s\mathbf{X}_\tau + \mathbf{W}_{s1}. \tag{35}$$

Since $\mathbf{X}_\tau$ is known by the static user, it can be inverted:

$$\mathbf{Y}'_{s1} = \mathbf{Y}_{s1}\mathbf{X}_\tau^H = \sqrt{c_\tau}\,\mathbf{H}_s\mathbf{X}_s + \mathbf{W}'_{s1} \tag{36}$$

where $\mathbf{Y}_{s1} \in \mathcal{C}^{N_s \times N_d}$ and $\mathbf{W}'_{s1}$ is the equivalent noise whose entries remain i.i.d. $\mathcal{CN}(0,1)$. Therefore, the channel seen by the static user becomes a point-to-point MIMO channel. Let

$\mathbf{y}'_{si}$ and $\mathbf{x}_{si}$ be the column i of $\mathbf{Y}'_{s1}$ and $\mathbf{X}_s$, respectively. The mutual information

$$I(\mathbf{Y}_{s1};\mathbf{X}_s) = \sum_{i=1}^{N_d} I(\mathbf{y}'_{si};\mathbf{x}_{si}) \tag{37}$$

$$= N_d \log\det\left(\mathbf{I}_{N_s} + c_\tau\,\mathbf{H}_s\mathbf{H}_s^H\right), \tag{38}$$

so the effective SNR for the static user is $\rho_s = c_\tau$

In the following $T_d - N_d$ time-slots, the static user disregards the received signal and achieves the average rate

$$R_s = \frac{N_d}{T_d}\,\mathbb{E}\left[\log\det\left(\mathbf{I}_{N_s} + \rho_s\,\mathbf{H}_s\mathbf{H}_s^H\right)\right], \tag{39}$$

where the expectation is over the channel realizations of $\mathbf{H}_s$.

B. Rate of the Dynamic User

The dynamic user first estimates the equivalent channel and then decodes its data. During the first N_d time-slots, the dynamic user receives the pilot signal

$$\mathbf{Y}_\tau = \sqrt{c_\tau}\,\mathbf{H}_d\mathbf{X}_s\mathbf{X}_\tau + \mathbf{W}_\tau \tag{40}$$

$$= \sqrt{c_\tau N_s}\,\widetilde{\mathbf{H}}_d\mathbf{X}_\tau + \mathbf{W}_\tau, \tag{41}$$

where $\widetilde{\mathbf{H}}_d \in \mathcal{C}^{N_d\times N_d}$ is the equivalent channel

$$\widetilde{\mathbf{H}}_d \triangleq \frac{1}{\sqrt{N_s}}\,\mathbf{H}_d\mathbf{X}_s \tag{42}$$

Let $\tilde{h}_{ij} = [\widetilde{\mathbf{H}}_d]_{ij}$, then we have $\mathbb{E}[\tilde{h}_{ij}] = 0$ and

$$\mathbb{E}[\tilde{h}_{ij}\,\tilde{h}_{pq}^H] = \begin{cases} 1, & \text{if } (i,j) = (p,q) \\ 0, & \text{else} \end{cases}, \tag{43}$$

i.e., $\tilde{h}_{ij}$ is uncorrelated with zero-mean and unit variance. The dynamic user estimates $\widetilde{\mathbf{H}}_d$ by the MMSE. Let

$$C_{YY} = (1 + c_\tau N_s)\mathbf{I}_{N_d}, \quad C_{YH} = \sqrt{c_\tau N_s}\,\mathbf{X}_\tau^H, \tag{44}$$

we have

$$\widehat{\mathbf{H}}_d = \mathbf{Y}_\tau C_{YY}^{-1} C_{YH} \tag{45}$$

$$= \frac{\sqrt{c_\tau N_s}}{1 + c_\tau N_s}\left(\sqrt{c_\tau N_s}\,\widetilde{\mathbf{H}}_d + \mathbf{W}_\tau\mathbf{X}_\tau^H\right) \tag{46}$$

Because $\mathbf{W}_\tau$ has i.i.d. $\mathcal{CN}(0,1)$ entries, the noise matrix $\mathbf{W}_\tau\mathbf{X}_\tau^H$ also has i.i.d. $\mathcal{CN}(0,1)$ entries. Define $\hat{h}_{1ij} = [\widehat{\mathbf{H}}_d]_{ij}$. Then, we have $\mathbb{E}[\hat{h}_{1ij}] = 0$ and

$$\mathbb{E}[\hat{h}_{ij}\hat{h}_{pq}^H] = \begin{cases} \delta_1^2, & \text{if } (i,j) = (p,q) \\ 0, & \text{else} \end{cases}, \tag{47}$$

where

$$\delta_1^2 = \frac{c_\tau N_s}{1 + c_\tau N_s}. \tag{48}$$

In other words, the estimate of the equivalent channel has uncorrelated elements with zero-mean and variance δ_1^2.

During the remaining $T_d - N_d$ time-slots, the dynamic user regards the channel estimate $\widehat{\mathbf{H}}_d$ as the true channel and decodes the data signal. At the time-slot i, $N_d < i \le T_d$, the dynamic user receives

$$\mathbf{y}_{di} = \sqrt{c_\delta N_s}\,\widehat{\mathbf{H}}_d\mathbf{x}_{di} + \underbrace{\sqrt{c_\delta N_s}\,\widetilde{\mathbf{H}}_e\mathbf{x}_{1di} + \mathbf{w}_{di}}_{\mathbf{w}'_{di}}, \tag{49}$$

where $\widetilde{\mathbf{H}}_e = \widetilde{\mathbf{H}}_d - \widehat{\mathbf{H}}_d$ is the estimation error for $\widetilde{\mathbf{H}}_d$, and $\mathbf{w}'_{di}$ is the equivalent noise that has zero mean and autocorrelation

$$\mathbf{R}_{w_d} = c_\delta N_s\,\mathbb{E}\big[\widetilde{\mathbf{H}}_e\widetilde{\mathbf{H}}_e^H\big] + \mathbf{I}_{N_d} \tag{50}$$

$$= \big(1 + \frac{c_\delta N_d N_s}{1 + c_\tau N_s}\big)\mathbf{I}_{N_d}. \tag{51}$$

Using the argument that Gaussian distribution maximizes the differential entropy with given second moments [7], the mutual information is lower-bounded as

$$I(\mathbf{y}_{di};\mathbf{x}_{di}|\widehat{\mathbf{H}}_d) \ge \log\det\left(\mathbf{I}_{N_d} + \frac{c_\delta N_s\,\widehat{\mathbf{H}}_d\widehat{\mathbf{H}}_d^H}{1 + c_\delta N_d N_s/(1 + c_\tau N_s)}\right) \tag{52}$$

$$= \log\det\left(\mathbf{I}_{N_d} + \frac{c_\delta\delta_1^2 N_s\,\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H}{1 + c_\delta N_d N_s/(1 + c_\tau N_s)}\right), \tag{53}$$

where $\overline{\mathbf{H}}_d$ is the normalized channel whose elements have unit variance

$$\overline{\mathbf{H}}_d = \frac{1}{\delta_1}\widehat{\mathbf{H}}_d. \tag{54}$$

From (53), the effective SNR for the dynamic user is

$$\rho_d = \frac{c_\delta\delta_1^2 N_s}{1 + c_\delta N_d N_s/(1 + c_\tau N_s)}. \tag{55}$$

The average rate that the dynamic user achieves is

$$R_d \ge (1 - \frac{N_d}{T_d})\mathbb{E}\big[\log\det(\mathbf{I}_{N_d} + \rho_d\overline{\mathbf{H}}_d\overline{\mathbf{H}}_d^H)\big], \tag{56}$$

where the expectation is over the dynamic user's channel realizations.

References

[1] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1691 – 1706, July 2003.

[2] C. Huang, S. Jafar, S. Shamai, and S. Vishwanath, "On degrees of freedom region of MIMO networks without channel state information at transmitters," pp. 849 –857, Feb. 2012.

[3] Y. Li and A. Nosratinia, "Product superposition for MIMO broadcast channel," *IEEE Trans. Inform. Theory*, to appear.

[4] L. Zheng and D. N. C. Tse, "Communication on the Grassmann manifold: a geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. Inform. Theory*, vol. 48, no. 2, pp. 359–383, 2002.

[5] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans. Inform. Theory*, vol. 49, no. 4, pp. 951–963, 2003.

[6] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Euro. Trans. on Telecomm.*, vol. 10, no. 6, pp. 585–595, 1999.

[7] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley and Sons, 1991.