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A Realizable Receiver for discriminating arbitrary Coherent States near the Quantum Limit

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***Abstract*—Discriminating coherent states of light is an important instance of quantum state discrimination that is central to all applications of laser light. We obtain the ultimate quantum limit on the error probability exponent for discriminating among any M multimode coherent-state signals via the recently developed theory of the quantum Chernoff exponent in M-ary multi-copy state discrimination. A receiver called the Sequential Waveform Nulling (SWN) receiver is proposed for discriminating an arbitrary coherent-state ensemble using only auxiliary coherent-state fields, beam splitters, and non-number-resolving single photon detectors. An explicit error probability analysis of the SWN receiver is used to show that it achieves the quantum limit on the error probability exponent, which is shown to be a factor of four greater than the error probability exponent of an ideal heterodyne-detection receiver on the same ensemble. Apart from being of fundamental interest, these results are relevant to communication, sensing, imaging, and quantum information processing systems that use laser light.**

The task of optimally discriminating between multiple nonorthogonal quantum states by making appropriate quantum measurements [1]–[4] is a fundamental primitive underlying many quantum information processing tasks, including communication, sensing, and metrology. The paradigmatic problem of the Bayesian formulation of quantum detection theory [1] is to determine the quantum measurement, specified by a positive operator-valued measure (POVM) [2], that minimizes the average error probability in discriminating a given ensemble of states. The mathematical solution to the problem is known in terms of necessary and sufficient conditions that the optimal POVM must satisfy [5], [6], although for discriminating between more than two states, the explicit solution of these conditions has been obtained only in very specific cases – see, e.g., refs. [4], [7] and references therein.

In view of the difficulty of obtaining the optimal quantum measurement in M-ary quantum detection, it is of great interest to obtain bounds on the optimal error probability. Such bounds are of basic importance in quantum information theory [2]. From a practical perspective, it is also important to bound the performance of concrete measurements whose physical realization is known. In this work, we develop both these approaches for the discrimination of arbitrary ensembles of *coherent states* of light [8]. Coherent states of light and their random mixtures are the most ubiquitous quantum states of light and their discrimination is central to optical communication [9], [10] and sensing with laser light, which is in a coherent state to an excellent approximation. For optical systems, the natural resource measure is the average number of photons in a given ensemble, i.e., the average energy. In the particular cases of coherent-state ensembles that have been studied, the optimal error probability of discriminating a given ensemble of coherent states decreases exponentially with the average energy in the high-photon-number limit [1], [11]–[15]. This is also true for receivers that perform the standard direct, homodyne, and heterodyne detections [9], [10] that correspond to particular POVMs that are realizable in the laboratory. However, the exponent of the optimal receiver allowed by quantum mechanics is in general greater than that of the conventional measurements, leaving a gap between the optimal error probability (popularly called the *Helstrom limit*) and the minimum achievable by conventional measurements, viz., homodyne, heterodyne, and direct detection (loosely called *standard quantum limits*).

The subject of coherent-state receiver design has a long history and remains an active area of research – see refs. [11]–[14], [16], [17] and ref. [18] for a somewhat broader overview. Here, we mention some developments of particular relevance to this paper. For discriminating any two coherent states, Kennedy proposed a receiver design [11] that is *exponentially optimal*, i.e., it achieves the maximum error probability exponent in the high-photon-number regime. Bondurant proposed a receiver for the 4-ary Phase Shift Keying (QPSK) constellation that is exponentially optimal [14]. Recently, Becerra and collaborators proposed a feedforward receiver structure for M-ary PSK with arbitrary M and demonstrated that it closely approximates the Helstrom limit for 4-PSK [16], [17].

This paper is organized as follows. After fixing notation and definitions in Section I, in Section II, we use the quantum Chernoff exponent in the recently developed theory of M-ary multi-copy state discrimination [19], [20] to obtain the maximum error probability exponent allowed of *any* coherent-state receiver by quantum mechanics. We also show that the error probability exponent of a multimode heterodyne receiver is smaller in general than that of the SWN receiver by a factor of four. The Kennedy [11], Bondurant [14], and Becerra [16], [17] receivers rely on a strategy of attempting to null, i.e., displace to the vacuum state, the input state by successively subtracting the fields corresponding to the possible hypotheses. In Section III, we adapt this strategy to any multimode M-ary coherent-state set and propose a receiver for discriminating them which we call the *Sequential Waveform Nulling* (SWN) receiver. Like its precursors, its operation requires only auxiliary coherent-state generation, beam splitters and single photon detection. We then compute the error probability of the SWN receiver applied to any state set, and using an upper bound on it, show that the error probability exponent approaches the maximum allowable value, establishing the exponential optimality of the SWN receiver. In Section IV, the performance of the SWN receiver is compared to other known receivers for QPSK and PPM signals. We conclude in Section V with a summary and suggestions for future work.

I. Coherent-state discrimination – Notation and Definitions

We set up the hypothesis testing problem that is the subject of this paper staring from a semiclassical description of the optical fields to be discriminated – see, e.g., ref. [10] for a review of the formalism used here. We are given M co-polarized quasi-monochromatic complex-valued spatiotemporal field waveforms $\{\mathcal{E}_m(\boldsymbol{\rho},t)\}_{m=1}^{M}$, where $\boldsymbol{\rho} \in \mathcal{A}$ is the transverse spatial coordinate in the receiver aperture plane $\mathcal{A}$ and $t \in \mathcal{T} = [0,T]$ denotes time within the signaling interval $\mathcal{T}$. For the quasi-monochromatic case of interest, the waveforms may be specified in units of $\sqrt{\mathrm{photons/m^2/s}}$ and can be readily translated to electric field or intensity units. In the quantum description, the M waveforms correspond to coherent states $\left\{|\boldsymbol{\alpha}_m\rangle = \left|\alpha_m^{(1)}\right\rangle \otimes \cdots \left|\alpha_m^{(S)}\right\rangle\right\}_{m=1}^{M}$ supported on $S \leq M$ orthonormal spatiotemporal modes $\{\phi_s(\boldsymbol{\rho},t)\}_{s=1}^{S}$ that span the waveform space. The m-th waveform can then be represented as the point $\boldsymbol{\alpha}_m \in \mathbb{C}^S$ in an S-mode phase space. We define

$$E_m := \int_{\mathcal{A}}\int_{\mathcal{T}} |\mathcal{E}_m(\boldsymbol{\rho},t)|^2 \, d\boldsymbol{\rho}\, dt = \| \boldsymbol{\alpha}_m \|^2 \tag{1}$$

to be the average energy of the m-th waveform in photons, and

$$\begin{aligned}\Delta_{m,m'} &:= \int_{\mathcal{A}}\int_{\mathcal{T}} |\mathcal{E}_m(\boldsymbol{\rho},t) - \mathcal{E}_{m'}(\boldsymbol{\rho},t)|^2 \, d\boldsymbol{\rho}\, dt \\ &= \| \boldsymbol{\alpha}_m - \boldsymbol{\alpha}_{m'} \|^2 \end{aligned}\tag{2}$$

to be the energy in the difference of the m-th and m'-th waveforms. Also define

$$\underline{\Delta} := \min_{m,m':m\neq m'} \Delta_{m,m'}. \tag{3}$$

If the M hypotheses are distributed according to the probability distribution $\{\pi_m\}_{m=1}^{M}$, the average energy in the waveform ensemble, denoted N, is given by

$$N := \sum_{m=1}^{M} \pi_m E_m. \tag{4}$$

Analgous to the situation in classical digital communication, the *error probability exponent* (EPE) $\xi^{\#}$ of a coherent-state receiver $\#$, where $\#$ may denote, e.g., the optimal Helstrom (Hel) receiver, the heterodyne (Het) receiver, or the SWN receiver, is defined as

$$\xi^{\#}\left[\{\boldsymbol{\alpha}_m\}\right] := -\lim_{N\to\infty} \frac{1}{N} \ln P_E^{\#}\left[\{\boldsymbol{\alpha}_m\}^{(N)}\right], \tag{5}$$

where $P_E^{\#}\left[\{\boldsymbol{\alpha}_m\}^{(N)}\right]$ is the average error probability of the receiver $\#$ used to discriminate the coherent-state ensemble $\{|\boldsymbol{\alpha}_m\rangle\}^{(N)}$ consisting of waveforms proportional to the given set of waveforms $\{\boldsymbol{\alpha}_m\}_{m=1}^{M}$ but scaled so as to have average energy N.

We now recall the definition of the M-ary quantum Chernoff exponent (QCE) for multi-copy state discrimination from ref. [19]. For any ensemble $\mathcal{F} = \{\rho_m\}_{m=1}^{M}$ of states from an arbitrary Hilbert space $\mathcal{H}$, consider the n-copy ensemble $\mathcal{F}^{\otimes n} = \{\rho_m^{\otimes n}\}_{m=1}^{M}$ with the same prior probabilites $\{\pi_m\}_{m=1}^{M}$ as $\mathcal{F}$. The *quantum Chernoff exponent* (QCE) $\xi_{\mathrm{QC}}[\mathcal{F}]$ of $\mathcal{F}$ is defined as

$$\xi_{\mathrm{QC}}[\mathcal{F}] := -\lim_{n\to\infty} \frac{1}{n} \ln P_E^{\mathrm{Hel}}\left[\mathcal{F}^{\otimes n}\right], \tag{6}$$

where $P_E^{\mathrm{Hel}}\left[\mathcal{F}^{\otimes n}\right]$ is the average error probability of the *Helstrom* receiver for discriminating the ensemble $\mathcal{F}^{\otimes n}$. For pure-state ensembles $\mathcal{F} = \{|\psi_m\rangle\}_{m=1}^{M}$, it was shown in [19] that

$$\xi_{\mathrm{QC}}[\mathcal{F}] = \min_{m,m':m\neq m'} -\ln|\langle\psi_m|\psi_{m'}\rangle|^2. \tag{7}$$

II. Optimal and Heterodyne-Detection Exponent in Coherent-state discrimination

For coherent-state ensembles and the Helstrom measurement, the above two notions of error probability exponents can be related as follows. The EPE of the Helstrom measurement on a coherent-state ensemble $\{\boldsymbol{\alpha}_m\}$ is, by definition,

$$\xi^{\mathrm{Hel}}[\{\boldsymbol{\alpha}_m\}] : = -\lim_{N\to\infty} \frac{1}{N} \ln P_E^{\mathrm{Hel}}\left[\{\boldsymbol{\alpha}_m\}^{(N)}\right] \tag{8}$$

$$= -\lim_{n\to\infty} \frac{1}{n} \ln P_E^{\mathrm{Hel}}\left[\{\boldsymbol{\alpha}_m\}^{(n)}\right] \tag{9}$$

$$= -\lim_{n\to\infty} \frac{1}{n} \ln P_E^{\mathrm{Hel}}\left[\otimes^n\{\boldsymbol{\alpha}_m\}^{(1)}\right] \tag{10}$$

$$= \xi_{\mathrm{QC}}\left[\{\boldsymbol{\alpha}_m\}^{(1)}\right] \tag{11}$$

$$= \min_{m,m':m\neq m'} -\ln\left|\langle\boldsymbol{\alpha}_m^{(1)}|\boldsymbol{\alpha}_{m'}^{(1)}\rangle\right|^2 = \underline{\Delta}/N \equiv \kappa. \tag{12}$$

Here, in eq. (9), n is restricted to integer values and the equality of (8) and (9) follows from the existence of the limit of eq. (8). Eq. (10) follows because a coherent-state ensemble can be split into n identical and independent copies using a unitary beam-splitter transformation and because this action cannot change the error probability of the Helstrom receiver. We are now in the multi-copy discrimination framework and the left-most term in eq. (12) follows from eq. (7), and we have used the coherent-state overlap $|\langle\boldsymbol{\alpha}_m|\boldsymbol{\alpha}_{m'}\rangle|^2 = e^{-\|\boldsymbol{\alpha}_m-\boldsymbol{\alpha}_{m'}\|^2}$ and introduced the N-independent constant κ that is a function of the prior

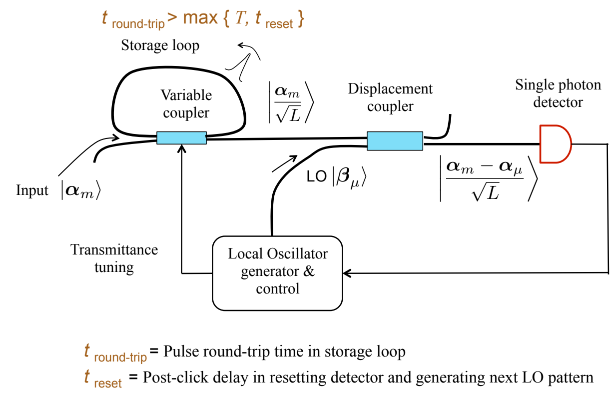

Fig. 1. Schematic of a possible implementation of the SWN receiver. The input field is split into L equal-amplitude "slices". Each slice is displaced by the negative of one of the scaled hypotheses at the displacement coupler and the local oscillator (LO) field pattern is switched to match the next hypothesis for the next slice whenever the single photon detector registers a click. Additional elements necessary to keep the LO amplitude and phase coherent with those of the input are not shown.

probability distribution and the geometry of the coherent-state constellation. It is unaffected by scaling all the waveforms of the ensemble by a common factor.

For a coherent-state ensemble $\{|\boldsymbol{\alpha}_m\rangle\}$ supported on S modes, we may in principle heterodyne each of the S modes to get, conditional on the state $|\boldsymbol{\alpha}_m\rangle$, an observation in $\mathbb{C}^S$ which equals $\boldsymbol{\alpha}_m$ with added white Gaussian noise of variance $1/2$ in each of $2S$ orthogonal quadratures of the S modes [9], [10]. For this essentially classical situation involving a fixed measurement, it is known that the M-ary Chernoff exponent equals the worst-case binary Chernoff exponent among all pairs m and m' of the hypotheses [20], [21]. Since the latter quantity equals $d^2/8\sigma^2$ for two one-dimensional Gaussian distributions with the same variance σ^2 and mean values separated by distance d [22], we get in the heterodyne case

$$\xi^{\text{Het}}\left[\{\boldsymbol{\alpha}_m\}\right] = \min_{m,m':m\neq m'} \frac{\parallel \boldsymbol{\alpha}_m - \boldsymbol{\alpha}_{m'} \parallel^2}{4N} = \frac{\kappa}{4}, \tag{13}$$

so that the heterodyne EPE is a factor of 4 worse than the Helstrom EPE regardless of the ensemble $\{|\boldsymbol{\alpha}_m\rangle\}$.

III. Sequential Waveform Nulling receiver - Operation and Performance

We now describe the operation of the Sequential Waveform Nulling (SWN) receiver (refer Fig. 1).

First, the signal field over $\mathcal{A} \times \mathcal{T}$ is split into L equal-amplitude portions or *slices* (where L should be as large as possible and at least equal to $(M-2)$ – see below) that are placed in storage of some kind, e.g., the fiber loop in Fig. 1, with a view to access the slices sequentially. The receiver operates as follows.

1) Initialize the *slice number* l to $l = 1$.
2) Initialize the *nulled hypothesis* μ to $\mu = 1$.
3) While $l \leq L$
   a) Displace the l-th slice of the input field by the field $-\frac{\mathcal{E}_\mu(\boldsymbol{\rho},t)}{\sqrt{L}}$ and direct-detect the output field in $\mathcal{A} \times \mathcal{T}$ on a single photon detector.
   b) If the detector clicks, set $\mu := \mu + 1$.
   c) $l := l + 1$.
4) Set the receiver's decision $\hat{m} := \mu$.

The L-fold slicing strategy of the SWN receiver, apart from simplifying the theoretical analysis, also serves a practical purpose. The limitations on the speed of electro-optic switching of the LO waveform, as well as the finite dead time of single photon detectors such as avalanche photodiodes (APDs) following the detection of a photon mean that detection cannot continue immediately after a detector click. If the next slice is held in storage until the LO waveform and detector are reset, we need not lose the portion of the input state in this reset period. In effect, such a splitting strategy was already employed, albeit with a different architecture than Fig. 1, in the Becerra *et al.* experiments of ref. [16], [17].

The error probability analysis of the SWN receiver requires only the semiclassical theory of photodetection [9], [10] and proceeds as follows. From the way the receiver operates, it is apparent that when the m-th hypothesis is true, we cannot get more than $m-1$ total clicks over the L slices. Further, if $m-1$ clicks are observed, we declare correctly that hypothesis m is true. We thus have for the conditional probability of error given that hypothesis m is true –

$$\begin{aligned} P^{\text{SWN}}[E|m] &= \Pr\left[\text{Fewer than } m-1 \text{ clicks are observed} \,|\, m\right] \\ &= \sum_{K=0}^{m-2} \Pr\left[K \text{ clicks are observed} \,|\, m\right], \end{aligned} \tag{14}$$

where the $m = 1$ case may be included by agreeing that sums in which the starting value of the summation index exceeds the ending value are zero. For $K > 0$, the summand may be written as follows – the $K = 0$ case is dealt with later. Define a length-K vector $\mathbf{l} = (l_1, \ldots, l_K)$ whose k-th component l_k is the slice number in the detection of which the k-th click occured. The possible instances of $\mathbf{l}$ are the increasing sequences of K integers chosen from $\{1, \ldots, L\}$, and are thus $\binom{L}{K}$ in number. When the nulled hypothesis is $\mu < m$, the average number of photons incident on the detector in one slice is $\Delta_{\mu,m}/L$. We may then write, using the Poisson nature of the count in each slice together with the conditional statistical independence of photodetection in successive slices:

$$\begin{aligned} &\Pr\left[K \text{ clicks are observed} \,|\, m\right] \\ &= \sum_{\text{allowed } \mathbf{l}} \exp\left\{-\Delta_{1,m}\frac{(l_1-1)}{L}\right\}\left(1-\exp\left\{-\frac{\Delta_{1,m}}{L}\right\}\right) \times \\ &\exp\left\{-\Delta_{2,m}\frac{(l_2-l_1-1)}{L}\right\}\left(1-\exp\left\{-\frac{\Delta_{2,m}}{L}\right\}\right)\cdots \\ &\cdots\exp\left\{-\Delta_{K,m}\frac{(l_K-l_{K-1}-1)}{L}\right\}\left(1-\exp\left\{-\frac{\Delta_{K,m}}{L}\right\}\right) \\ &\times\exp\left\{-\Delta_{K+1,m}\frac{(L-l_K)}{L}\right\}, \end{aligned} \tag{15}$$

where factors of the form $\exp\{\cdot\}$ are probabilities that no clicks are obtained in the intervals between the click locations indicated by $\mathbf{l}$ while factors of the form $(1-\exp\{\cdot\})$

are probabilites of obtaining a click in the click locations. We may bound the above expression as

$$\begin{aligned}
&\Pr[K \text{ clicks are observed} \,|\, m\,] \\
&\leq \sum_{\text{allowed } \mathbf{l}} \exp\left\{-\Delta_{1,m}\frac{(l_1-1)}{L}\right\}\exp\left\{-\Delta_{2,m}\frac{(l_2-l_1-1)}{L}\right\} \\
&\cdots \exp\left\{-\Delta_{K+1,m}\frac{(L-l_K)}{L}\right\} \qquad (16) \\
&\leq \sum_{\text{allowed } \mathbf{l}} \exp\left\{-\underline{\Delta}\frac{(l_1-1)}{L}\right\}\exp\left\{-\underline{\Delta}\frac{(l_2-l_1-1)}{L}\right\} \\
&\cdots \exp\left\{-\underline{\Delta}\frac{(L-l_K)}{L}\right\} \qquad (17) \\
&= \binom{L}{K}\exp\left\{-\underline{\Delta}\frac{(L-K)}{L}\right\}. \qquad (18)
\end{aligned}$$

For $K = 0$ and for $m > 1$, we have $\Pr[K \text{ clicks are observed} \,|\, m\,] = \exp(-\Delta_{1,m}) \leq \exp(-\underline{\Delta})$. The upper bound of eq. (18) is therefore also valid in this case. For $m = 1$, we have $P^{\text{SWN}}[E|m = 1] = 0$. Therefore,with the summation convention adopted above, we may write, for all values of m,

$$P^{\text{SWN}}[E|m] \leq \sum_{K=0}^{m-2}\binom{L}{K}e^{-\underline{\Delta}\,\frac{(L-K)}{L}}, \qquad (19)$$

so that the total error probability of the SWN receiver is bounded by

$$P_E^{\text{SWN}}\left[\{\boldsymbol{\alpha}_m\}\right] \leq \sum_{m=1}^{M}\pi_m\sum_{K=0}^{m-2}\binom{L}{K}e^{-\underline{\Delta}\,\frac{(L-K)}{L}}. \qquad (20)$$

A lower bound on the EPE $\xi^{\text{SWN}}\left[\{\boldsymbol{\alpha}_m\}\right]$ of the SWN receiver can be obtained by inserting the right-hand side of (20) into (5), resulting in

$$\xi^{\text{SWN}}\left[\{\boldsymbol{\alpha}_m\}\right] \geq \frac{\underline{\Delta}}{N}\left(1-\frac{M-2}{L}\right) = \kappa\left(1-\frac{M-2}{L}\right), \qquad (21)$$

Because we must have $\xi^{\text{SWN}}\left[\{\boldsymbol{\alpha}_m\}\right] \leq \xi^{\text{Hel}}\left[\{\boldsymbol{\alpha}_m\}\right]$ by definition of the Helstrom receiver, we conclude that the EPE of the SWN receiver is at most a factor of $(1-(M-2)/L)$ away from the Helstrom receiver. If $L < (M-2)$, we lack slices to see enough clicks to ever declare the M-th hypothesis (and perhaps other hypotheses as well), and therefore, $\xi^{\text{SWN}}[\{\boldsymbol{\alpha}_m\}] = 0$. On the other hand, in the limit of $L \to \infty$, the two exponents must be identical, establishing the optimality of the SWN receiver exponent in this limit. For a given M, L need not be very large for the EPE to be close to optimal, as seen in the examples of Section IV.

IV. Examples

The performance of various receivers on the single-mode quadrature phase-shift keyed signal set is compared in Fig. 3, with the hypotheses assumed *a priori* equally likely. The exponential scaling of all the receivers is apparent in the straight-line dropoff of the error probability already evident at rather low photon numbers of $N \sim 5$. The Helstrom measurement (which in this case implements the square-root measurement [15]) and the Bondurant receiver [14], which is equivalent to the $L = \infty$ version of the SWN receiver, have the same EPE. The exact error probability of the SWN receiver and the upper bound of eq. (20) are shown in Fig. 3 for $L = 4, 8$ and 12. Note that the difference in slopes of the $L = 12$ error probability and the Helstrom error probability is slight. Finally, the heterodyne receiver error probability is seen to have a slower dropoff with N than the other receivers, consistent with eq. (13).

The performance of various receivers for pulse position modulation with coherent states [13], [23], [24] with $M = 6$ hypotheses assumed *a priori* equally likely is shown in Fig. 4. The optimal error probability, the error probability

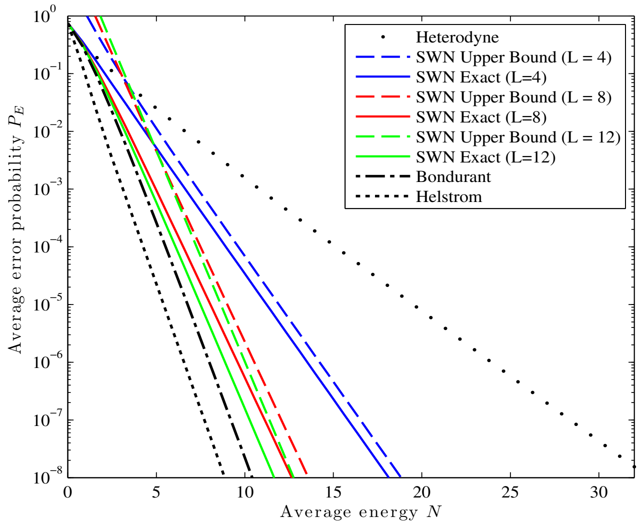

Fig. 2. Error probability behavior of various receivers for an ensemble of equally likely quadrature phase-shift-keyed coherent states as a function of the average ensemble energy. The dashed lines are, for $L = 4, 8$ and 12, the error probability bound eq. (20), with the corresponding solid lines being the exact error probability (see appendix) of the SWN receiver. See Section IV for discussion.

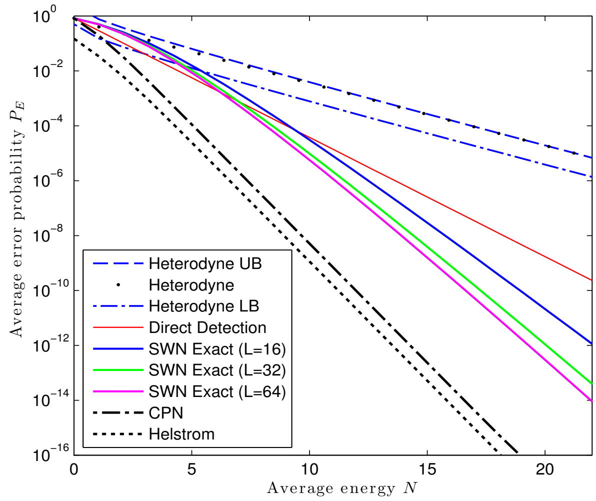

Fig. 3. Error probability behavior of various receivers for an ensemble of equally likely PPM states with $M = 6$ pulses as a function of the average ensemble energy. See Section IV for discussion.

under direct detection, and that of the conditional pulse nulling (CPN) receiver of ref. [24] are known exactly, with the CPN receiver having the same EPE as the Helstrom measurement. We note that the SWN receiver outperforms standard direct detection at photon numbers greater than $N \sim 10$ for all the values of L shown. The CPN outperforms the SWN at all N and L, which may be expected as the former receiver is specifically tailored to the PPM configuration. The exact error probability for heterodyne detection, the union upper bound, and the lower bound of eq. (3.7.3) of ref. [25] are also shown in Fig. 4.

V. Conclusion

We have obtained the ultimate limit allowed by quantum mechanics on the error probability exponent for general M-ary coherent-state discrimination. This exponent was previously known for the binary case and for some M-ary cases (e.g., PPM) with sufficient symmetry, but not in full generality as established here. We showed that this exponent is four times that of a multimode heterodyne-detection receiver. We proposed a sequential nulling receiver for discriminating arbitrary coherent-state ensembles, derived an upper bound on its error probability, and used it to show that, with ideal devices, its error probability exponent approaches the quantum-optimal one. The SWN receiver uses only beam splitters, the ability to engineer arbitrary coherent-state waveforms and single photon detection. We mention that the philosophy of the SWN receiver can be generalized to discriminating multiple copies of pure states in any Hilbert space, i.e., to the scenario of refs. [19], [20]. It has been shown that the resulting receiver attains the M-ary quantum Chernoff exponent It does so using only LOCC operations, in particular, adaptive copy-by-copy binary projective measurements [18], thereby improving on the receiver of [19] that uses joint measurement. In further research, apart from exploring various potential applications of the SWN receiver, it would be useful to study the degrading effects of system non-idealities such as errors in the nulling process arising from imperfect control over the phase and amplitude of the LO waveforms and of excess losses in the storage loop.

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