Power Optimization on a Random Wireless Network

Aris L. Moustakas and Nicholas Bambos

***Abstract*—Consider a wireless network of transmitter-receiver pairs. The transmitters adjust their powers to maintain a particular SINR target at the corresponding receiver in the presence of interference from neighboring transmitters. In this paper we analyze the power vector that achieves this target (and hence is optimal) in the presence of randomness in the network. The randomness is realized by randomly turning off a fraction of transmitter-receiver pairs in a regular lattice. We show that the problem is identical to the so-called Anderson model, which describes the motion of electrons in a dirty metal. We show that traditional random matrix theory is only an approximation that, while accurate in some cases, fails to fully describe the system. We apply the coherent potential approximation (CPA), which is equivalent to random matrix theory, to evaluate the average power vector. We also find that although beyond a certain point the infinite system is infeasible with probability one, any arbitrarily large, but finite system has a typically small probability of becoming infeasible. The CPA framework allows us to calculate this outage probability with exponential accuracy by showing that it is proportional to the tails of the eigenvalue distribution of the system.**

I. Introduction

Transmitted power is an important resource in wireless networks and therefore power control has been crucial since the development of legacy networks. Several algorithms been developed that provably allow receivers to obtain e.g. a minimum SINR requirement $\mathrm{SINR}_k \geq \gamma_k$ for link k while minimizing the total power or the power per user, subject to the feasibility of this solution [1]. Importantly, power optimization remains an relevant problem in emerging and future networks. Ad hoc networks are one such class of networks, where substantial effort has been made to analyze their behavior, through metrics such as connectivity and transport capacity [2]–[4]. A number of works have discussed the feasibility conditions of the optimal power vector [3], [5], [7] under general assumptions, without however addressing specific gains from power control. When the feedback between the actions of each transmitter to all neighboring ones is neglected, the Laplace transform method has been shown to be an accurate method to calculate the effects of fading, pathloss and random erasures on the interference to a random receiver in both regular and Poisson random networks [4], [6]. The effect of interference is already a problem in dense WiFi networks and is expected to become an issue in the context of femto-cells. Hence, when each transmitter increases its power in order to compensate for this interference, it may create domino effect of power increases, which need to be addressed.

A. Moustakas is with the Physics Dept., Univ. of Athens, 157 84 Athens, Greece; arislm@phys.uoa.gr. This work was funded by the research program 'CROWN', through the Operational Program 'Education and Lifelong Learning 2007-2013' of NSRF, which has been co-financed by EU and Greek national funds. N. Bambos is with Stanford University, Palo Alto, CA 94305; bambos@stanford.edu

In this paper we introduce an analytical framework to analyze the optimal power performance of a large random network in the presence of interference. Starting from an ordered network structure, we introduce randomness in the network by removing each transmitter-receiver pair with probability e. The resulting network has transceiver pairs randomly located in the original lattice. This thinned network is a model for a realistic cellular network, in which the transmitters are randomly located. It is also a good model for a wireless network with intermittent activity, such that at any time a fraction e of the total transmitters are inactive. We show that although the problem can be reduced to the analysis of a large random matrix, traditional random matrix theory can only be applied as an approximation, which while accurate in some regimes, cannot adequately answer all important questions. Instead we show that the problem is equivalent to the Anderson model, proposed originally by P. W. Anderson to describe the motion of an electron in a random lattice. To obtain the dependence of the average transmitted power, we employ the so-called CPA (coherent phase approximation), which was devised as a self-consistent method to take into account the randomness of the network. The equations obtained through this approximation turn out to be equivalent to those obtained assuming free probability was applicable. This assumption was conjectured to be true by [8] for one-dimensional systems (and by us in a recent paper [11]). However, this is only an approximation and we will show a specific case, in which the analysis fails.

Although the model we use is specific in nature, as we shall see both analytically and numerically, this paradigm is generic for power controlled networks when both interference and randomness are important. In fact, one of the main contributions of the paper is the introduction of tools and methodologies from the physics of disordered metals to analyze such networks.

We now outline the contributions of this paper. In the next section, we describe the network model and show the connection between the Anderson model and the erasure model introduced in [8]. In Section III we introduce the so-called coherent phase approximation (CPA) and show that it is equivalent to the free probability approach. In Section IV we focus on a specific exactly solvable one-dimensional model. In Section V we show that in the infinite system size limit power control in the network is infeasible, while in Section VI we use the framework introduced here to calculate the outage probability for finite networks using the so-called Lifshitz tails.

II. Model Description

A. Network without disorder

We start by defining the *pure* network in our model, i.e. without randomness. We begin with a network of N transmitters on a d-dimensional lattice. The lattice points are equally spaced with distance ℓ between nearest neighbors. Without loss of generality we focus on one-dimensional and two-dimensional square grids (see Fig. 1). Each transmitter is connected via a link to a single receiver located at a distance δ. The channel coefficient between transmitter i and receiver j and averaged over the fading is given by

$$f_{ij} = \frac{\delta^\alpha}{(|\mathbf{m}_i - \mathbf{m}_j|^2\ell^2 + \delta^2)^{\alpha/2}} \tag{1}$$

where $\mathbf{m}_i = [m_{i1} \ldots m_{id}]$ is the lattice vector of integers defining the location of transmitter i. Since the integers m_{ia} fully specify the position $\mathbf{m}_i$, we will drop the index i when not necessary. For simplicity, we have normalized the channel gains to unity for $i = j$. For simplicity we assume periodic boundary conditions on the lattice, hence $m_{ia} \equiv m_{ia}+L$, where $N = L^d$. This means that the distance between any two points is taken as their minimum distance on a toroidal geometry (or a circular geometry for one dimension). It should be emphasized that the effects of this circulant property for the matrix with elements f_{ij} will become negligible for large N. Also, α is the pathloss exponent, which signifies how fast the channel strength decays as a function of distance.

The pathloss function above is somewhat artificial in its dependence on the distances between the receiver and transmitter. Technically, it is strictly correct only when each receiver is located vertically to the line connecting all transmitters (as in any horizontal line of Fig. 1), which is one possible geometry for one dimensional systems [5]. Nevertheless, it has the right behavior for $\mathbf{m}_i = \mathbf{m}_j$ as well as for $|\mathbf{m}_i - \mathbf{m}_j|\ell \gg \delta$. The reason we chose to use this model will become apparent later.

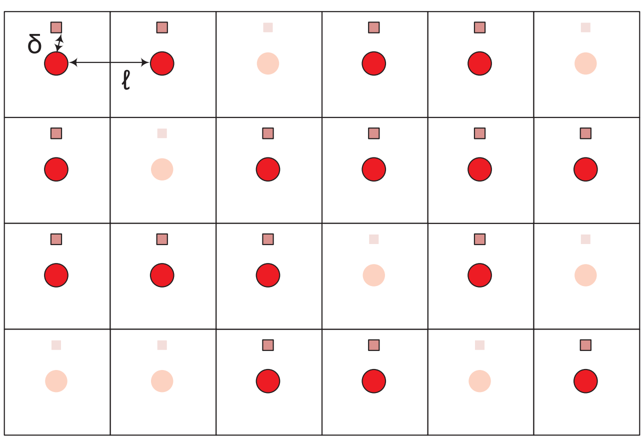

Fig. 1. Schematic figure of wireless network. The circles correspond to transmitters, while the squares receiver users. The opaque squares represent transmitter-receiver pairs that have been "erased" and are thus considered inactive.

The SINR for each connection is given by

$$\mathrm{SINR}_k = \frac{p_k f_{kk}}{n + \sum_{j\neq k} f_{jk} p_j} \tag{2}$$

where p_k is the transmitted power from the k transmitter to the receiver k and n is the thermal noise, assumed equal for all for simplicity. For the connection to be possible, a minimum value of the SINR has to be attainable i.e. $\mathrm{SINR}_k \geq \gamma$. Therefore, each transmitter should adjust its own power to meet this criterion. As a result, the following set of equations should be simultaneously met

$$n^{-1}\gamma^{-1} f_{kk} p_k - \sum_{j\neq k} n^{-1} f_{jk} p_j \geq 1 \tag{3}$$

The above equations constitute a set of linear (planar) constraints on the powers. Equality in the above equation results to the optimal power vector $\mathbf{P}$, which can be written in matrix form as

$$\mathbf{P} = [\mathbf{H}_0 + z_\gamma \mathbf{I}]^{-1}\,\mathbf{J} \tag{4}$$

where $\mathbf{J} = [1, 1, \ldots, 1]^T$, $z_\gamma = n^{-1}\left(\gamma^{-1} - \gamma_c^{-1}\right)$ with

$$\gamma_c^{-1} = \sum_{\mathbf{m}\neq\mathbf{0}} \frac{\delta^\alpha}{(|\mathbf{m}|^2\ell^2 + \delta^2)^{\alpha/2}} \tag{5}$$

and the elements of $\mathbf{H}_0$ given by

$$H_{0,ij} = \begin{cases} (n\gamma_c)^{-1} & i = j \\ -n^{-1} f_{ji} & i \neq j \end{cases} \tag{6}$$

The eigenvalues of circulant matrix $\mathbf{H}_0$ are then given by

$$\epsilon(\mathbf{q}) \;=\; n^{-1} \sum_{\mathbf{m}} \frac{\delta^\alpha \left(1 - e^{i\mathbf{q}^T\mathbf{m}}\right)}{(|\mathbf{m}|^2\ell^2 + \delta^2)^{\alpha/2}} \tag{7}$$

where $\mathbf{q} = \frac{2\pi}{L}[k_1, \ldots, k_d]$ for integer k_a with $0 \leq k_a < L$. The minimum average power is given by

$$p_{ave} = \frac{1}{N}\mathbf{J}^T [\mathbf{H}_0 + z_\gamma \mathbf{I}]^{-1}\,\mathbf{J} = \frac{1}{z_\gamma} \tag{8}$$

where the second equation results from the fact that $\mathbf{J}$ is the $\mathbf{q} = 0$ eigenvector.

B. Randomly Thinned Network

We now introduce disorder by randomly turning off a fraction of transmitters. This can be done in two equivalent ways. In the first, in order to turn off the power p_k of transmitter k we may set f_{kk}, the channel strength between the k-th transmitter and receiver to $f_{kk} = V \to \infty$. Indeed as it can be seen in (3), when f_{kk} becomes arbitrarily large the SINR target constraint for transmitter k may be met with arbitrarily small power p_k. Since the site to be turned off is chosen at random with probability e, we define a random diagonal matrix $\mathbf{V}$ with independent diagonal entries v_i on the ith entry, with $P[v_i = 0] = 1 - P[v_i = 1] = \mathrm{e}$. Thus, in the presence of erasures of transmitters, the matrix $\mathbf{H}_0$ above needs to be replaced by $\mathbf{H} = \mathbf{H}_0 + \mathbf{V}$. Since we are interested only on finite positive power solutions, the limit of $V \to +\infty$ can be taken in the end of the calculation of the inverse matrix. The above matrix $\mathbf{H}$ has deterministic off-diagonal elements and diagonal disorder. In the physics literature it is called the Anderson Hamiltonian (sometimes only when the off-diagonal nearest neighbors are non-zero). It was introduced by P. W. Anderson [10] as a model to explain localization of particles (and waves) in random media. The optimal power vector is then given by

$$\mathbf{P} = [\mathbf{H} + z_\gamma \mathbf{I}]^{-1} \mathbf{J} \tag{9}$$

and its average can be obtained by multiplying from the right by $\mathbf{J}$:

$$p_{ave} = \frac{1}{N(1-\mathrm{e})} \lim_{V\to\infty} \mathbf{J}^T [\mathbf{H} + z_\gamma \mathbf{I}]^{-1} \mathbf{J} \tag{10}$$

It will become useful later on to define the Green's function operator (related with Stieljes transform) as follows

$$\mathbf{G}(E) = [E\mathbf{I} - \mathbf{H}]^{-1} \tag{11}$$

Hence the mean optimal power is simply $p_{ave} = -\mathbf{J}^\dagger \mathbf{G}(-z_\gamma)\mathbf{J}/(N(1-\mathrm{e})$. Another useful expression for p_{ave} as a function of the eigenvalues and eigenfunctions of the random matrix $\mathbf{H}$ is given below.

$$p_{ave} = \frac{1}{N(1-\mathrm{e})} \sum_n \frac{\left|\sum_k u_n(k)\right|^2}{\epsilon_n + z_\gamma} \tag{12}$$

where ϵ_n is the nth eigenvalue of the matrix, while $u_n(k)$ is the value of the nth eigenfunction at location k. We immediately see that the optimal average power is finite and positive as long the eigenvalues of the matrix are large enough, i.e. $\epsilon_n + z_\gamma > 0$.

To make connection with previous work [8], [11] it worth to introduce randomness with another equivalent method. In particular, for every transmitter-receiver pair that is "off" we need to set both its column and row elements to zero. This can be done by modifying the matrix to $\mathbf{H}' = \mathbf{E}\mathbf{H}_0\mathbf{E}$

III. The Coherent Phase Approximation

The diagonal form of the disorder makes the problem easier to handle. In this section we will analyze the system in an approximate way, using a methodology, which was applied extensively in the physics literature to study the movement of electrons in disordered alloys [12], namely the Coherent Potential Approximation (CPA). In particular we will evaluate the average optimal power p_{ave} by calculating an expression for the average Green's function $\overline{\mathbf{G}}(E) = \mathbb{E}[\mathbf{G}(E)]$. The basic idea behind the CPA is to replace each diagonal term Ve_i in $\mathbf{H}$, which can be seen as a local "potential", with a function $\Sigma(E)$, which represents an "mean field" potential, which on average has the same effect on the moving particle as the original potential term. In the physics literature, this function $\Sigma(E)$ is called self-energy. Despite the fact CPA is not an exact result, it has had considerable success in calculating the spectrum of systems with diagonal disorder. It should be mentioned that the CPA result turns out to be identical with the result proposed in [8] using tools from random matrix theory and free probability theory. Essentially, the self-energy $\Sigma(E)$ corresponds to the R-transform in random matrix theory (RMT).

We will now state the CPA equations. For more details the reader may look into a substantial body of references (see [12] and references therein). We start with average CPA Green's function

$$\overline{\mathbf{G}}(E) = \mathbf{G}_0(E) \left[\mathbf{I} - \Sigma(E)\mathbf{G}_0(E)\right]^{-1} \tag{13}$$

where $\mathbf{G}_0$ is the Green's function in the absence of randomness (i.e.$\mathbf{V} = 0$), $\Sigma(E)$ is the self energy, which in the CPA is evaluated from the following self-consistency condition

$$\Sigma(E) = \mathbb{E}_{V_i}\left[\frac{V_i}{1-(V_i - \Sigma(E))g(E)}\right] \tag{14}$$

In the above equation, the expectation is over the values of the diagonal random variables V_i in $\mathbf{V}$, while $g(E)$ is given by

$$g(E) = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{1}{E - \Sigma(E) - \epsilon(\mathbf{q})} \tag{15}$$

The average over V_i in (14) can be readily performed to give $\Sigma(E) = -\frac{\mathrm{e}}{g(E)}$, which gives the following equation for $\Sigma(-z_\gamma)$

$$\mathrm{e} = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\Sigma}{\Sigma + \epsilon(\mathbf{q}) + z_\gamma} \tag{16}$$

This equation is identical to the one derived in [11] under the assumption of free probability. To evaluate the optimal average power we use (10) and the fact that $\mathbf{J}$ is proportional to the eigenstate for $\mathbf{k} = 0$, hence

$$p_{ave} = \frac{1}{1-\mathrm{e}} \frac{1}{\Sigma + z_\gamma} \tag{17}$$

We refer the reader to numerical validation of these formulae in a previous publication [11]. Suffice it to say that our observations there were that while the agreement of the above equations is very good with numerics, the behavior beyond γ_c becomes sample dependent with some samples becoming infeasible at some γ with a certain probability. We will address this later on.

IV. The Wyner model: Exact results

We will now analyze a simple case of the above model, in which we are able to obtain exact results. These results will allow us to later generalize to more complicated situations. The so-called Wyner model consists of a linear array of transmitters located at a fixed distance apart, in such a way that only neighboring transmitters may cause interference to each others' transmissions. Thus $\mathbf{H}_0$ is a tridiagonal matrix with elements $\mathbf{H}_{0,ij} = 2t\delta_{ij} - t\delta_{i,j+1} - t\delta_{i,j-1} - t\delta_{i,1}\delta_{j,N} - t\delta_{i,N}\delta_{j,1}$, with the two terms necessary to keep the matrix circulant. The introduction of the diagonal term $V\mathbf{E}$ to account for the erasures simply sets those sites' power to zero. Hence, in this case the presence of the erasures results to separating the system into independent segments of various lengths.

We start by calculating the eigenvalues of $\mathbf{H}$ for this problem. Clearly, for each segment with length L there are L eigenvalues given by $\epsilon_n = 2t(1-\cos(\frac{\pi n}{L+1}))$ for $n = 1, \ldots, L$. It is easy to realize that in the large N limit the distribution of segment sizes is $P_L = \mathrm{e}(1-\mathrm{e})^L$. Hence the cumulative distribution function can be written in the following form

$$\begin{aligned} Prob(E < x) &= \sum_{q\in\mathbb{Q}, k_0 = q\ell_0} \pi_{\ell_0} \Theta(E - 2t(1-\cos(\pi q))) \\ \pi_\ell &= \frac{\mathrm{e}^2(1-\mathrm{e})^{\ell-2}}{1-(1-\mathrm{e})^\ell} \end{aligned} \tag{18}$$

where k_0 and ℓ_0 are relatively prime. The above CDF of the eigenvalues has two remarkable properties. First, it is nonzero for all energies between, $|E| < 2t$. We shall see below that this behavior is intimately connected with the instability of the system. Second, it forms a "devil's staircase", being discontinuous in all prime values of q and constant inbetween. In contrast the CPA solution is continuous and has support that doesn't extend to $E = 0$.

Indeed, due to the breakdown of the system in segments with no erasures, we may also evaluate the optimal power for each segment. For a segment of length L the power control equations result to

$$\gamma^{-1}p_k - tp_{k-1} - tp_{k+1} = n \tag{19}$$

for $k = 1 \ldots L$, with boundary conditions $p_0 = p_{L+1} = 0$ due to the presence of an erasure at each end of the segment. Interestingly, we find two different types of solutions, depending on the target SINR value γ. For $2t\gamma < 1$, the solution is

$$p_k = \frac{1}{\gamma^{-1} - 2t}\left(1 - \frac{\cosh\left[\left(k - \frac{L+1}{2}\right)z\right]}{\cosh\left(\frac{L+1}{2}z\right)}\right) \tag{20}$$

$$z = \ln\left(\frac{1 - \sqrt{1 - 4t^2\gamma^2}}{2t\gamma}\right) \tag{21}$$

We see that for segment sizes L large compared to the characteristic length z^{-1} the power is roughly constant for most sites, except near the edges of the segment. This parameter region ($2t\gamma < 1$) corresponds to the case when the $\mathrm{e} = 0$ system has finite power. In this case the optimal power per transmitter is always finite and (20) can be summed over k and L to provide an expression for the average power per transmitter.

For larger γ, i.e. when $2t\gamma > 1$ the power at location $k = 1, \ldots, L$ of a segment of size L becomes

$$p_k = \frac{2}{2t - \gamma^{-1}}\frac{\sin\left[(L + 1 - k)\,\phi\right]\sin\left[k\phi\right]}{\cos\left[(L + 1)\,\phi\right]} \tag{22}$$

$$\phi = \arctan\sqrt{4t^2\gamma^2 - 1} \tag{23}$$

In this case we see that p_k can be positive and finite as long as the denominator $\cos(L + 1)\phi > 0$ is positive. This criterion may be equivalently expressed as

$$\cos\phi = \frac{1}{2t\gamma} < \cos\frac{\pi}{L + 1} \tag{24}$$

This inequality holds when the minimum eigenvalue of the segment of length L is less than $-z_\gamma$, hence making the denominator of the corresponding term in (12) negative.

If the above condition is not satisfied power control on this segment and hence the whole system will clearly become infeasible. Hence for $2t\gamma > 1$ power control in a system will be feasible only if all its segments are feasible, which means that they all have to be short enough. This immediately provides an outage criterion for a finite system of size N and target SINR $\gamma > \frac{1}{2t}$, namely $P_{out}(\gamma, N) = Prob(L_{max} > L_c(\gamma))$.

$$L_c(\gamma) = floor\left[\pi\left(\arccos[(2t\gamma)^{-1}]\right)^{-1}\right] \tag{25}$$

where L_{max} is the maximum segment length in a given realization of the system.

Intuitively, in the infinite system size limit $N \to \infty$ the outage probability becomes unity. However, we will also be interested in large but finite systems, for γ close enough to the instability, i.e. $2t\gamma - 1 \ll 1$, the low outage criterion $N(1 - \mathrm{e})^{L_c(\gamma)} \ll 1$ will certainly apply. In this case, we may express the outage probability as

$$P_{out}(\gamma, N) \approx N\mathrm{e}\,(1 - \mathrm{e})^{L_c(\gamma)} \tag{26}$$

Concluding this section, we would like to stress that this very simple but not simplistic one-dimensional Wyner model carries all the qualitative properties of other more general models we will treat below. In particular, we find a discontinuous density of eigenvalues, which is non-zero all the way to the edges of the spectrum without erasures. The system becomes unstable for all $\gamma > \gamma_c$ where γ_c is the maximum γ for the pure system. However, the feasibility of an unstable system is tied to the occurrence of large regions of space without erasures. Also, the density of eigenvalues close to the edge of the spectrum corresponds to the occurrence of such large segments without any erasures.

V. Feasibility Analysis – Infinite Networks

We start by noting that in the infinite size limit, power control in the network is *feasible* only when $\gamma < \gamma_c$, for all erasure probabilities $\mathrm{e} \leq 1$. Indeed, in this case $z_\gamma > 0$ and thus the denominator in (10) is strictly positive definite. Hence the corresponding expression is finite and thus $p_{ave} < \infty$. In fact, generalizing the behavior in the Wyner model discussed above, it is not hard to see that in all cases $\mathrm{e} \geq 0$ the infinite size network is infeasible with probability one. Indeed, for any $\gamma > \gamma_c$ one can construct a erasure-free line or square region of a large enough size which will become infeasible because its lowest eigenvalue, which for large N can be described approximately by (7) will become less than $-z_\gamma$. The presence of interference around it now, will only make things worse. Since the instantiation of such a large but finite erasure free region is guaranteed in an infinite network, the network will always "blow" up in this case.

VI. Outage Probability in Finite Networks

The instability for $\gamma > \gamma_c$ that was shown in the previous section concerns only infinite networks. For finite networks numerical simulations in [11] showed that the instability is probabilistic in nature. It is not hard to see from (10) that the system becomes infeasible when the minimum eigenvalue of $\mathbf{H}$ becomes less that $-z_\gamma > 0$. Therefore, for finite networks the distribution of their minimum eigenvalue is linked with the outage (infeasibility) when $z_\gamma < 0$.

$$P_{out}(\gamma) = \mathbb{P}(E_{min} < -z_\gamma) \tag{27}$$

In this section we will address this quantity drawing from ideas statistical physics of random semiconductors. We will first analyze the behavior of the cumulative density $\mathcal{N}(E)$ in the limit of small eigenvalues of the infinite system and then argue their relation with $\mathbb{P}(E_{min} < -z_\gamma)$. Lifshitz [13], in his study of electronic properties of dirty semiconductors conjectured the correct form of the density of eigenvalues close to the edge of the spectrum. He argued that the very low eigenvalues close to the minimum of the spectrum become exceedingly rare because they need large regions without any impurities (erasures). This is so because erasures create kinks in the profile of the eigenfunction, which tend to increase the eigenvalue of $\mathbf{H}$. The key insight of Lifshitz was that the density of eigenvalues at a given low eigenvalue E is dominated by the probability of having an erasure-free volume $R^d(E)$ in the system, such that E is the minimum eigenvalue in this volume, i.e.

$$\mathcal{N}(E) \sim (1-\mathrm{e})^{R^d(E)} \tag{28}$$

At the boundary of this volume the corresponding eigenfunction has to vanish eventually, due to the appearance of erasures, although the exact details of the boundary conditions are not important. As seen in (7), at small eigenvalues the eigenvalues behave as $\epsilon(\mathbf{q}) \propto |\mathbf{q}|^2$ and hence the minimum eigenvalue E for the region of volume $R^d(E)$ will scale as $E \propto R(E)^{-2}$. Hence $\log \mathcal{N}(E) \propto -E^{-d/2}$ in the low energy regime. This intuitive behavior was proved to be correct in a large volume of works [14]. In particular, in [15] it shown that for the case when f_{ij} in (1) has a finite number of non-zero elements then

$$\lim_{E\to 0^+} \frac{\log(-\log \mathcal{N}(E))}{\log E} = -\frac{d}{2} \tag{29}$$

An important byproduct of this analysis is that the eigenstates of these eigenvalues are *localized* in regions of size $R^d(E)$ and hence the eigenvalues from different regions should be independent. This independence has been shown for one and two-dimensional problems (see discussion in Section 4.6 in [14]) (in contrast with the case of traditional random matrix theory where there is level repulsion. As a result, for large but finite N and small $|z_\gamma|$ we have

$$P_{out}(\gamma) = \mathbb{P}(E_{min} < -z_\gamma) \approx N\mathcal{N}(-z_\gamma) \tag{30}$$

In Fig. 2 we observe that the outage probability predicted above is numerically reproduced with surprising accuracy. This behavior explains the instabilities observed in the average power plots obtaining by solving (17) in [11].

VII. Conclusion

In this paper we have studied the optimal power vector that achieves an SINR target criterion in a wireless network setting where both randomness and interference are relevant. We have indicated that random matrix theory is only an approximation for the behavior, by analyzing a particular exact model. We have mapped the problem to the Anderson model in statistical physics and have introduced a different approach borrowed from statistical physics to analyze the behavior of the system, particularly to describe its outage behavior. In future work we will expand this analysis to the tails of the empirical power distribution in optimal power vector.

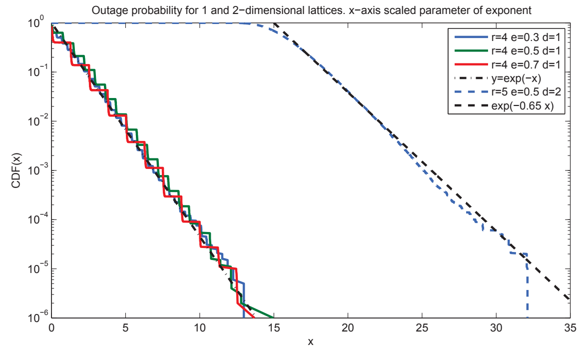

Fig. 2. Plot of probability of outage for a one dimensional chain versus the parameter $x = -\ln(1-\mathrm{e})\sqrt{(t_2\pi^2)/(2|z_\gamma|)}$ for various values of parameters. Nevertheless, the curves fall on top of each other by merely shifting them to the same starting point, indicating that the above is the correct scaling parameter. We also include a 2-dimensional plot of the outage versus $x = -\ln(1-\mathrm{e})(\pi k_0^2)/(2|z_\gamma|)$, where $k_0 \approx 2.4$ is the first zero of $J_0(x)$

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