Efficient Incremental Relaying

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Abstract—We propose a novel relaying scheme which improves the spectral efficiency of cooperative diversity systems by utilizing limited feedback from destination. Our scheme capitalizes on the fact that relaying is only required when direct transmission suffers deep fading. We calculate the packet error rate for the proposed efficient incremental relaying scheme with both amplify and forward and decode and forward relaying. Numerical results are also presented to verify their analytical counterparts.

I. Introduction

Cooperative diversity [1] is an effective way to take advantage of other user's antenna resources to improve error rate performance. User nodes help a source node in communication by forwarding the received information from the source node to the destination node. In this way, it provides an independent replica of the directly received signal. Both direct and relayed signal are combined at destination. Relaying can be done in different ways. The relaying node can for instance completely decode and re-encode information before relaying which is referred as decode-and-forward (DF) relaying. Alternatively the relaying node can just amplify the received signal before re-transmitting it towards destination which is termed as amplify-and-forward (AF) relaying. What follows, we provide a brief review of the different relaying strategies and we outline the proposed scheme.

In fixed relaying, the relays always forward the received signal after either amplifying or after decoding and re-encoding [1]. On the other hand, selective relaying [1] utilizes the instantaneous channel information to decide between relay forwarding and source re-transmission in the second phase. Whenever the source-to-relay signal-to-noise (SNR) ratio is above a certain threshold, the relay forwards its received signal towards the destination to achieve the benefits of cooperative diversity, otherwise the source is instructed to re-transmit.

In incremental relaying [1]–[3], feedback from the destination about the success or failure of direct transmission is used. Relay is allowed to forward signal only when direct transmission fails otherwise source continues with the next message reducing total time for transmission from two to one time slot. Incremental relaying results in better spectral efficiency. Incremental relaying protocols are extensions of incremental redundancy protocols, or hybrid automatic-repeat-request (ARQ). *Fractional incremental relaying* (FIR) protocol [4] further improves spectral efficiency by re-transmitting partial information from relays. In FIR, the relay divides the received packet into fractions and sends a fraction if destination is not able to decode the packet correctly indicated by a negative acknowledgment (NACK) from the destination. The relay keeps on sending next fraction until it receives an acknowledgment (ACK) from the destination or a maximum number of the relay transmission is reached.

Patil investigates in [5] the use of packet combining in cooperative diversity scenario. A comprehensive study of bit error rate (BER) and throughput performance of the scheme in additive white Gaussian noise (AWGN) and fading channel with and without maximum ratio combining (MRC) is provided. A cross layer approach is used to exploit the inherent time diversity in ARQ retransmissions by MRC of the packet and the retransmission.

In our proposed scheme, instead of relaying all the packets, the relay is engaged only for forwarding the packets for which the SNR of the direct link is very low. A group of packets is sent from the source in the broadcast phase which is received by both the destination and the relay. At the end of each broadcast phase, the destination sends feedback to the relay to indicate the indices of the M packets with lowest SNR in the group. The relay then forwards only a subset of packets as instructed and these re-transmitted packets are then combined with the directly received signal and decoded at the destination using MRC. Therefor in contrast to classical incremental relaying, the proposed scheme avoids complete decoding at destination before feedback is sent.

In this paper, we also present a performance analysis and derive the packet error rate (PER) of the proposed scheme for both AF and DF relaying. The remainder of the paper is organized as follows. Section II presents the model of the system under consideration and preliminary results are provided section III. While section IV gives derivation of the PER, the achievable diversity order and the efficiency of the proposed scheme are discussed in section V. Numerical results are given in section VI and finally, section VII concludes the paper.

II. System Model

We consider a three node system which consists of a source node **S**, a destination node **D**, and a relay node **R**, as shown in Fig. 1. We assume Rayleigh fading channel between transmit and receive nodes with average SNR for **S**→**R**, **R**→**D**, and **S**→**D** links as Γ_{SR}, Γ_{RD}, and Γ, respectively. A frame
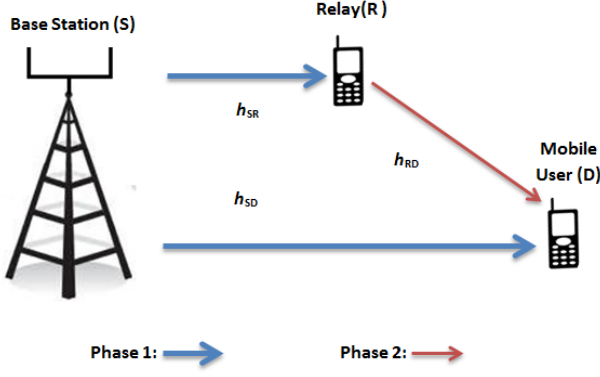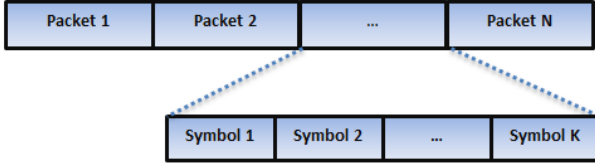
Fig. 1. System model.

Fig. 2. Frame structure.

containing L symbols is transmitted by **S** towards **D** which is over heard by **R**. Each frame contains N packets each of length $K = L/N$ symbols as shown in Fig. 2.

The channel fading remains constant for the duration of each packet and may vary from one packet to another. Let x_{nk} be the kth symbol transmitted by **S** in the nth packet. The signal received by **D** is

$$y_{nk} = \sqrt{E_s} h_{SD_n} x_{nk} + u_{nk}, \qquad (1)$$

where $h_n, n = 1, 2, \cdots, N$ is the channel coefficient during the nth packet and u_{nk} is the AWGN at the receiver. At the end of transmission of a frame, the destination sends back the indices of the M packets for which channel attenuation $|h_{SD_n}|$ is worse. Upon reception of index i, the relay gets involved in communication and forwards the ith packet.

We first consider the AF relaying to re-transmit the signal. In this case, the relay amplifies the signal

$$r_{ik} = \sqrt{E_s} h_{SR_n} x_{ik} + v_{nk} \qquad (2)$$

received from **S** with a fixed gain of

$$G = \sqrt{\frac{E_s}{E_s \Gamma_{SR} + N_0}} \qquad (3)$$

and forwards it to **D**. The resulting signal received in the kth time slot of the mth relayed packet at **D** is then given by:

$$z_{mk}^{AF} = h_{RD} G r_{ik} + w_{mk}, \qquad (4)$$

where h_{RD}, r_{mk}, and w_{mk} are the channel coefficient from **R** to **D**, the signals received at the relay during the mth packet, and the AWGN affecting the destination.

We also consider DF relaying, the relays working in DF mode first decode the received symbols and then re-encode information before forwarding them to the destination. In order to avoid error propagation the relay is allowed to participate in the relaying phase only if the relay is able to correctly decodes the symbol. The signal received at the destination **D** in the kth time slot of the mth relayed packet in DF relaying is

$$z_{mk}^{DF} = h_{RD} x_{mk} + u_{mk}, \qquad (5)$$

where u_{mk} the AWGN affecting the destination.

III. PRELIMINARIES

In this section, we first provide some results from order statistics. These results will be used in next section to derive the PER of the proposed scheme.

Let $\gamma_i = |h_i|^2 \frac{E_s}{N_0}$ be the SNR for the ith packet, where h_i is the channel coefficient, E_s is average symbol power, and N_0 is the noise power. Since Rayleigh fading channel is assumed, the probability density function (PDF) of γ_i is given by:

$$f_{\gamma_i}(\gamma) = \frac{1}{\Gamma_i} e^{-\frac{\gamma}{\Gamma_i}}, \quad 0 < \gamma < \infty, \qquad (6)$$

where $\Gamma_i = E\left[|h_i|^2\right] \frac{E_s}{N_0} = \frac{E_s}{N_0} = \Gamma$ with average fading power $E\left[|h_i|^2\right] = 1$. We denote $\gamma_{(i)}, i = 1, 2, \cdots, N$ as the sorted γ_i such that $\gamma_{(1)} < \gamma_{(2)} < \cdots < \gamma_{(N)}$. The joint PDF of $\gamma_{(i)}$'s is known to be given by [6]:

$$f_\gamma(\gamma_{(1)}, \gamma_{(2)}, \cdots, \gamma_{(N)}) = \frac{N!}{\Gamma^N} e^{-\frac{1}{\Gamma} \sum_{i=1}^N \gamma_{(i)}}. \qquad (7)$$

The ordered SNRs i.e., $\gamma_{(i)}$'s can be represented by the sum of independent exponential random variables as [6], [7]:

$$\gamma_{(i)} = \sum_{m=1}^{i} \frac{\Gamma}{N - m + 1} V_m, \qquad (8)$$

where V_m are independent identically distributed normalized exponential random variables with PDF:

$$f_{V_m}(v) = e^{-v}, \quad 0 < v < \infty. \qquad (9)$$

By using the value of $\gamma_{(i)}$ from (8) and by the taking expectation over V_m's we get:

$$\begin{aligned} \Phi_i(s) &= E_{V_m}\left[e^{-s \sum_{m=1}^{i} \frac{\Gamma}{N-m+1} V_m} \right] \\ &= \prod_{m=1}^{i} \int_0^\infty e^{-\frac{s\Gamma}{(N-m+1)} V_m} e^{-V_m} dV_m \\ &= \prod_{m=1}^{i} \frac{(N - m + 1)}{s\Gamma + (N - m + 1)}. \end{aligned} \qquad (10)$$

IV. PERFORMANCE ANALYSIS

Under the assumption, that the fading remains constant over one packet, the PER for a Binary Phase Shift Keying (BPSK) system with block length of K symbols on a point-to-point communication link can be calculated as:

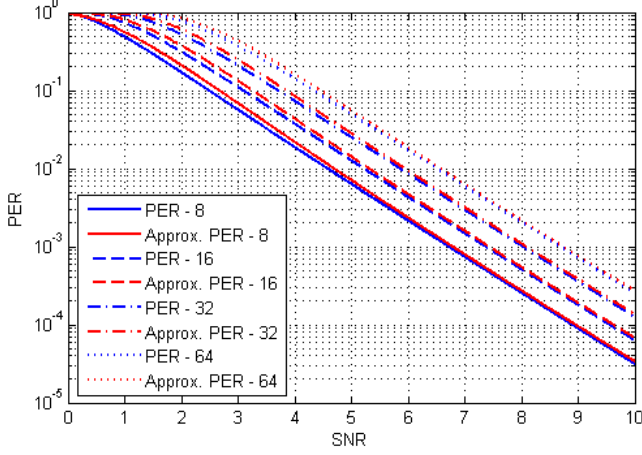$$PER(\gamma) = 1 - \left[1 - Q\left(\sqrt{2\gamma}\right) \right]^K. \qquad (11)$$

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Fig. 3. PER approximation for different values of packet length (K).

where $Q(.)$ is the Gaussian Q-function [8]. In order to simplify the above equation we use the approximation of erfc$(.)$ given in [9] as:

$$\text{erfc}(x) \approx \frac{1}{2}\left(\frac{1}{3}e^{-x^2} + e^{-\frac{4}{3}x^2}\right). \qquad (12)$$

Consequently we get

$$Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right) \approx \frac{1}{4}\left(\frac{1}{3}e^{-\frac{x^2}{2}} + e^{-\frac{4}{3}\frac{x^2}{2}}\right). \qquad (13)$$

We now can calculate the nth power of $Q(x)$ as

$$\begin{aligned} Q^n(x) &\approx \frac{1}{4^n}\left[\frac{1}{3}e^{-\frac{x^2}{2}} + e^{-\frac{4}{3}\frac{x^2}{2}}\right]^n \\ &= \sum_{m=0}^{n} C_{n,m}e^{-\frac{A_{n,m}x^2}{2}}, \end{aligned} \qquad (14)$$

where we define $C_{n,m} = \binom{n}{m}\frac{1}{4^n 3^{n-m}}$ and $A_{n,m} = n + \frac{m}{3}$. Expanding Eq. (11) and using the powers of $Q(x)$, we get the following representation of PER in terms of sum of exponentials:

$$PER(\gamma) \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m}e^{-A_{n,m}\gamma}, \qquad (15)$$

where $D_{K,n} = \binom{K}{n}(-1)^{n+1}$.

Fig. 3 presents a comparison of the original PER expression given by Eq. (11) as well as the PER approximation given by Eq. (15). It is obvious that Eq. (15) provides better approximation for high SNR and large values of K.

The unconditional PER can be calculated by taking expectation over γ yielding

$$PER \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m}\Phi(A_{n,m}), \qquad (16)$$

where $\Phi(s) = E_\gamma\left[e^{-s\gamma}\right]$ is the Moment Generation Function (MGF) of the SNR γ.

A. Packet Error Rate for the Proposed Scheme

The packet error rate PER_T when the relay forwards the M-weakest packets is calculated as

$$PER_T = \frac{1}{N}\sum_{i=1}^{M} PER_{C_i} + \frac{1}{N}\sum_{j=M+1}^{N} PER_j, \qquad (17)$$

where PER_{C_i} is the PER of the ith weakest packet which is decoded at destination after receiving and combining the direct and the forwarded signals using MRC and PER_i is the packet error rate for the ith packet. The following two subsections present now derivations of the PER for AF and DF relaying.

B. Amplify and Forward

We can calculate the values for the PER_{C_i} and PER_i for AF relaying using the approximation given by Eq. (16). If γ_R and $\gamma_{(i)}$ are the SNR of the relayed packet and the direct packet with the lowest SNR respectively, PER_{C_i} for BPSK can be calculated as:

$$PER_{C_i} = E\left[PER(\gamma_R + \gamma_{(i)})\right],$$

where $E_{X,Y}(.)$ denotes expectation with respect to random variables X and Y. Using Eq. (16), we get:

$$PER_{C_i}^{AF} \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m}\Phi_R(A_{n,m})\Phi_i(A_{n,m}), \qquad (18)$$

where $\Phi_R(s)$ and $\Phi_i(s)$ can be calculated as follows.

By defining $\gamma_{SR} = |h_{SR}|^2 \frac{E_s}{N_0}$, $\gamma_{RD} = |h_{RD}|^2 \frac{E_s}{N_0}$, and $c_1 = 1 + \frac{E_s}{N_0}$, for AF relaying with fixed relay gain, we have [10]:

$$\gamma_R = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{RD} + c_1}. \qquad (19)$$

Consequently, we get:

$$\Phi_R(s) = c_2(s)E_{\gamma_{RD}}\left[\frac{\gamma_{RD} + c_1}{\gamma_{RD} + c_1 c_2(s)}\right], \qquad (20)$$

with $c_2(s) = \frac{1}{1 + s\Gamma_{SR}}$. Using [11, p.366, Eq. (3.384)], we can simplify the above equation similar to [12, Eq. (35)] as:

$$\Phi_R(s) = c_2\left\{1 + \frac{c_1 - c_1 c_2}{\Gamma_{RD}}\exp\left(\frac{c_1 c_2}{\Gamma_{RD}}\right)\Gamma\left(0, \frac{c_1 c_2}{\Gamma_{RD}}\right)\right\}, \qquad (21)$$

where $\Gamma(.,.)$ is the incomplete Gamma function [11]. Similarly for PER_i, we can write:

$$PER_i \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m}\Phi_i(A_{n,m}), \qquad (22)$$

where $\Phi_i(s)$ is given by Eq. (10).

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C. Decode and Forward

For a relay system with a decode and forward strategy, we can calculate PER_{C_i} as

$$PER_{C_i} = PER_{SR}PER_i + (1 - PER_{SR})PER_{CO_i}, \quad (23)$$

where PER_{SR}, PER_i, and PER_{CO_i} are the PER at the relay, the PER at destination when relay is unsuccessful in decoding, and the PER at destination when relay is successful in decoding, respectively. The PER for SR link is given by Eq. (16):

$$PER_{SR} \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m} \Phi_{SR}(A_{n,m}), \quad (24)$$

where

$$\Phi_{SR}(s) = \frac{1}{1 + s\Gamma_{SR}}. \quad (25)$$

Similarly, we can calculate PER_{CO_i} using Eq. (16)

$$PER_{CO_i} \approx \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m} \Phi_i(A_{n,m}) \Phi_{RD}(A_{n,m}), \quad (26)$$

with $\Phi_{RD}(s) = \frac{1}{1+s\Gamma_{RD}}$.

V. DIVERSITY ORDER AND EFFICIENCY

From Eq. (10), we observe that for higher values of SNR, we get:

$$\begin{aligned} \Phi_i(s) &= \prod_{p=1}^{i} \frac{(N-p+1)}{s\Gamma + (N-p+1)} \\ &\propto \Gamma^{-i} K_i, \end{aligned} \quad (27)$$

where K_i is a constant of proportionality. Eq. (27) eventually leads to

$$PER_i \propto \Gamma^{-i} \sum_{n=1}^{K} D_{K,n} \sum_{m=0}^{n} C_{n,m} K_i. \quad (28)$$

Now by putting values of c_1 and c_2 in Eq. (21) and using the series representation [11] of the incomplete Gamma function similar to [13, Eq. 35], we can show that $\Phi_C(s)$ decays proportional to Γ. Hence we can easily conclude that for AF in the high SNR range:

$$PER_{C_i}^{AF} \propto \Gamma^{-(i+1)} \qquad i = 1, 2, \dots, M \quad (29)$$

$$PER_i \propto \Gamma^{-i}. \qquad i = M+1, \dots, N. \quad (30)$$

Since the least exponent of SNR which corresponds to $i = 1$, among all the terms of PER_T, is 2 we can deduce that the diversity order achieved is 2 which is also verified in the numerical results presented in the next section. Similarly we can show that the diversity order achieved for DF is also 2.

For efficiency, let us define

$$FR = \frac{M}{N} = \frac{\text{Number of packets forwarded by the relay}}{\text{Total packets}} \quad (31)$$
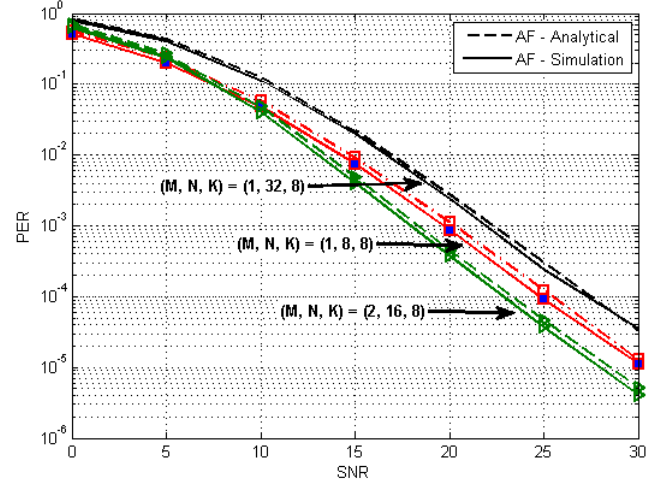

Fig. 4. Comparison of analytical and simulation results for AF with different values of the (M, N, K).

as the forwarding rate (FR) of the proposed scheme, then we can calculate the efficiency of the proposed scheme as

$$\eta = \frac{N}{N + M} = \frac{1}{1 + FR}. \quad (32)$$

VI. NUMERICAL RESULTS

In this section we present some selected numerical results which were validated through Monte Carlo simulations of the proposed scheme. In our simulations, we assume BPSK modulation and a relay which is placed at an equal distance from both the source and the destination. Figure 4 shows a comparison of analytical PER results obtained from (17) and simulated PER for AF. From this figure, we can observe that the analytical results provide a good match with those obtained through simulation for different values of (M, K, N). We observe that Eq. (12) results in very good approximation of the PER for different values of frame, packet and forwarding rates. It can be further noted that the proposed scheme performs better with lower value of packet length K and higher values of M (i.e., number of packets forwarded by the relay).

Figure 5 shows similar comparison of analytical PER and simulated PER for DF. Again note that the analytical results match with those obtained through simulation for DF. In Fig. 6 we present a comparison of AF and DF relaying, we can observe that for $M = 1$, DF performance is slightly better than AF, however DF outperforms AF for $M > 1$.

Fig. 7 presents some numerical results of the proposed scheme with $K = 16$ and different forwarding rates FR to demonstrate the ability of the proposed scheme to achieve better performance with limited involvement of the relay. It can be observed that with a forwarding rate of FR $= 1/8$, we can get more than 6 dB improvement at a target PER of 10^{-2} in comparison to no relaying (i.e., $M = 0$). Note also that the PER performance at FR $= 4/8$ forwarding rate approaches that of conventional schemes in which packets are forwarded
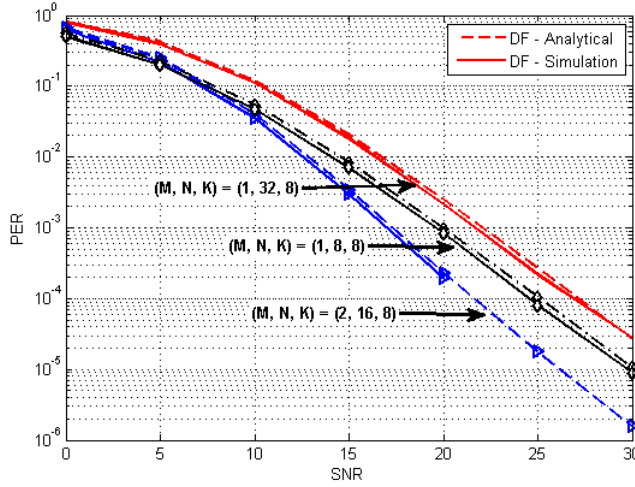
Fig. 5. Comparison of analytical and simulation results for DF with different values of the (M, N, K).
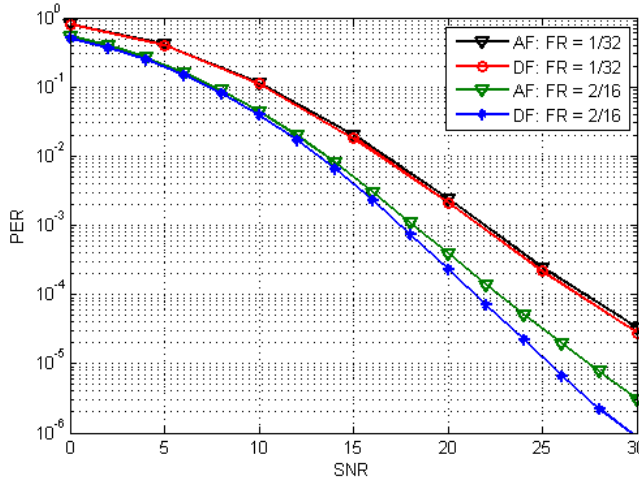

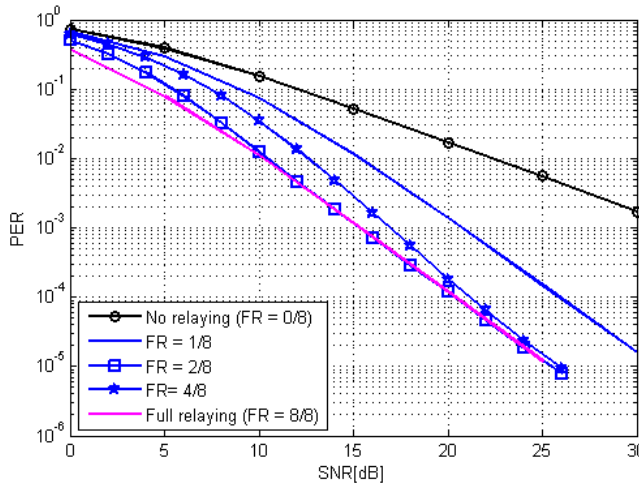Fig. 6. Comparison of AF and DF with different values of the M.

Fig. 7. PER for different forwarding rates FR with DF relaying.

unconditionally (i.e., $M = N$) in the medium and high SNR range.

VII. CONCLUSION

An efficient relaying scheme was proposed for incremental relaying which achieves better throughput with the help of minimum feedback from the destination. Performance analysis based on the packet error rate is presented for the proposed scheme. The results have shown that performance of the proposed scheme with partial relay involvement can approach that of the classical scheme which always uses relay for forwarding all the information.

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REFERENCES

[1] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[2] A. Ibrahim, A. Sadek, W. Su, and K. Liu, "Cooperative communications with partial channel state information: When to cooperate?" in *Proc. IEEE Global Telecommunications Conference (GLOBECOM '05)*, Saint Louis, Missouri, USA, Dec. 2005, pp. 3068 –3072.

[3] S. Ikki and M. Ahmed, "Performance analysis of incremental relaying cooperative diversity networks over Rayleigh fading channels," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC 2008)*, Las Vegas, Nevada, USA, Apr. 2008, pp. 1311 –1315.

[4] H. Long, K. Zheng, W. Wang, and F. Wang, "Approximate performance analysis of the incremental relaying protocol and modification," in *Proc. IEEE 70th Vehicular Technology Conference Fall (VTC 2009-Fall)*, Anchorage, Alaska, USA, Sept. 2009, pp. 1 –5.

[5] R. Patil, "On throughput performance of decode and forward cooperative relaying with packet combining and ARQ," *International Journal of Computer Networks and Communications (IJCNC)*, vol. 4, no. 3, pp. 57–67, May 2012.

[6] M. Z. Win and J. H. Winters, "Virtual branch analysis of symbol error probability for hybrid selection/maximal-ratio combining in Rayleigh fading," *IEEE Trans. Commun.*, vol. 49, pp. 1926–1934, Nov. 2001.

[7] M.-S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalized selection combining over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 401 –415, Mar. 2000.

[8] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels (Wiley Series in Telecommunications and Signal Processing)*. Wiley-IEEE Press, 2005.

[9] M. Chiani and D. Dardari, "Improved exponential bounds and approximation for the Q-function with application to average error probability computation," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM '02)*, Taipei, Taiwan, Nov. 2002, pp. 1399 – 1402.

[10] M. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963 – 1968, Nov. 2004.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. Academic Press, Jan. 1994.

[12] M. M. Fareed and M. Uysal, "BER-optimized power allocation for fading relay channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2350 –2359, June 2008.

[13] R. Nabar, H. Bolcskei, and F. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1099 – 1109, August 2004.