On Degrees of Freedom Scaling in Layered Interference Networks with Delayed CSI

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***Abstract*—The multi-hop layered interference network is investigated with delayed knowledge of channel state information (CSI) at all nodes. It is demonstrated how multi-hopping can be utilized to increase the achievable degrees of freedom (DoF). In particular, for the K-user $2K$-hop interference network, a multi-phase transmission scheme is proposed which systematically exploits the layered structure of the network and delayed CSI to achieve DoF values which scale with K. As such, this result provides the first example of a network with distributed transmitters and delayed CSI whose DoF scales with the number of users, although sub-linearly.**

I. Introduction

Interference management is a central challenge in the design and operation of wireless networks. Following the surprising result of [1], which showed even completely expired channel state information at transmitter (CSIT), a.k.a. delayed CSIT, can provide degrees of freedom (DoF) gain in multiple-input single-output (MISO) broadcast channels (BC), there has been a growing interest in investigating the delayed CSIT model in different networks. Specifically, K-user interference channel (IC) and X channel were considered in [2], [3], where it was shown that both channels can achieve more than one DoF with delayed CSIT. Despite the remarkable DoF gains offered by the delayed CSI in BC, its DoF benefits so far reported over no CSI in the networks with distributed transmitters are quite marginal. In particular, the achieved DoFs for both IC and X channel are less than 2 for any number of users.

Motivated by recent results which demonstrate that, with instantaneous and perfect CSI at all nodes, multi-hopping can significantly impact the DoF of interference networks, e.g., [4]–[8], in this paper we investigate the DoF of K-user multi-hop layered interference networks with delayed CSI. For 2-user interference networks with delayed CSI at sources and no CSI at relays, it was previously shown in [9] that one layer of relays can increase the DoF to $4/3$ (as opposed to one). In this paper, we focus on $K \geq 3$ and seek whether it is feasible to utilize multi-hopping to attain DoF scaling, i.e., scaling with the number of users, with delayed CSI at all nodes.

Specifically, we consider the K-user $2K$-hop interference network and propose a K-phase transmission scheme which systematically utilizes multi-hopping and delayed CSI to increase the achievable DoF of the K-user interference network. Among its other ingredients, the proposed scheme possesses two key ingredients, namely, *hop-distributed partial scheduling and interference nulling (PSIN)* and *symbol offloading*, which will be highlighted through a numerical example.

As a surprising result, the achievable DoF of the proposed transmission scheme scales with K as fast as $\frac{1}{2}f^{-1}(K)$, where f^{-1} is the inverse function of $f(x) \triangleq x^x$. Although this achievable DoF scales very slowly with the number of users, the importance of this result is that it is the first example of a single-antenna network with distributed transmitters wherein the delayed CSI yields DoF scaling with number of users.

II. Problem Statement and Main Results

A K-user N-hop layered interference network is a set of K source nodes $\{S_i\}_{i=1}^K$, a set of K destination nodes $\{D_i\}_{i=1}^K$, and $N-1$ sets of intermediate nodes $\{V_i^{(n)}\}_{i=1}^K$, $2 \leq n \leq N$, called *relays*. There is a communication channel between each two consecutive layers of the network, called a hop, as depicted in Fig. 1. Each relay operates in full-duplex mode, i.e., it can transmit and receive simultaneously. For any $1 \leq n \leq N$ and $1 \leq i \leq K$, during time slot t in hop n, $V_j^{(n)}$ transmits $x_j^{(n)}(t) \in \mathbb{C}$ and $V_i^{(n+1)}$ receives $y_i^{(n)}(t) \in \mathbb{C}$, where

$$y_i^{(n)}(t) = \sum_{j=1}^{K} h_{ij}^{(n)}(t) x_j^{(n)}(t) + z_i^{(n)}(t),$$

and $h_{ij}^{(n)}(t) \in \mathbb{C}$ is the channel coefficient between $V_j^{(n)}$ and $V_i^{(n+1)}$, and $z_i^{(n)}(t)$ is zero-mean unit-variance additive complex Gaussian noise at the input of $V_i^{(n+1)}$. It is assumed that the noise and channel coefficients are i.i.d. over time and nodes, and channel coefficients are drawn according to a continuous distribution. All transmitted signals are subject to a power constraint P. We consider delayed CSI model in which the whole network CSI is known at all nodes after a finite delay, which for simplicity is assumed one time slot.

Each source node S_i has a message to communicate with its corresponding destination node D_i. We also consider a more general traffic demand setting, namely, X network, in which each source node S_i has a message to communicate with each destination node D_j. The block code, probability of error, achievable rate, and capacity of the network are defined in a standard way. We consider sum-DoF, or simply DoF, of the network, which is defined as the slope of the network sum-capacity versus $\log_2 P$ as $P \to \infty$.

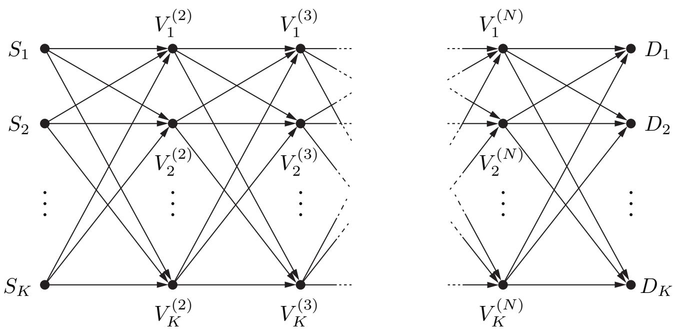

Fig. 1. K-user N-hop interference network.

Our main result provides an achievable DoF for the K-user $2K$-hop interference network with delayed CSI. More formally, we have the following theorem whose proof is outlined in Section III.

Theorem 1. *The DoF of the K-user $2K$-hop interference network with $K \geq 3$ and delayed CSI satisfies*

$$\mathsf{DoF}^{\text{IC}}(K, 2K) \geq \frac{1}{t_1(q, K) + t_2(q, K)}, \tag{1}$$

where

$$t_1(q, K) \triangleq \frac{1}{q-1}\left(\frac{\Gamma(q^{-1})(K-1)!}{\Gamma(K+q^{-1})} - \frac{1}{K}\right), \tag{2}$$

$$t_2(q, K) \triangleq \frac{Kq+1}{q(q+1)K} + \frac{(2q-1)(K-1)}{2K[(K-1)q+1]}, \tag{3}$$

and $2 \leq q \leq K-1$ is an arbitrary integer, and $\Gamma(\cdot)$ is the gamma function.

This result yields the following corollary, whose proof is referred to [10], and demonstrates the achievable scaling of DoF with our proposed transmission strategy.

Corollary 1. *The DoF of the K-user $2K$-hop interference network with delayed CSI scales with K. Specifically, the following inequality provides an asymptotic lower bound to $\mathsf{DoF}^{\text{IC}}(K, 2K)$.*

$$\mathsf{DoF}^{\text{IC}}(K, 2K) \geq \frac{1}{2} f^{-1}(K)(1 - \delta_K), \tag{4}$$

where f^{-1} is the inverse function of $f(x) \triangleq x^x$ and $\delta_K > 0$ goes to zero as $K \to \infty$.

This result demonstrates that in networks with distributed transmitters and delayed CSI, it is plausible to achieve DoF scaling with the number of users. However, since the gap between our achievable DoF and the best known upper bound ($\approx K/\ln K$, i.e., the DoF scaling rate of K-user MISO BC with delayed CSIT [1]) scales with the number of users, characterization of the optimal DoF scaling rate of this network with delayed CSI remains open. Also, the lower bound of (1) is not tight. For example, although we show $15/11 = 1.\overline{36}$ DoF is achievable for the 3-user case in Section III, achievability of $22/15 = 1.4\overline{6}$ DoF is shown in [10].

III. Overview of Transmission Strategy for the K-user $2K$-hop Interference Network

In this section, we provide a proof sketch for Theorem 1. We will explain the main building blocks of the achievability strategy and derive the achievable DoF for the 3-user case. We refer the reader to [10] for details of the scheme for general K.

We consider the K-user $2N$-hop interference network as a cascade of two K-user N-hop X networks. We then have the following lemma, whose proof is referred to [10].

Lemma 1. *Any symmetric achievable DoF in the K-user N-hop X network is also achievable in the K-user $2N$-hop interference network which is formed by cascading two copies of the X network.*

According to the above lemma, it is sufficient to achieve the lower bound of (1) in the K-user K-hop X network with delayed CSI. Hence, in what follows, we describe the main building blocks of our transmission scheme for this network. Also, the analytical details of each building block are provided for the case of $K = 3$, i.e., 3-user 3-hop X network, for which we show achievability of $15/11$ DoF.

As summarized in Table I, our scheme for the K-user K-hop X network operates in K phases which are performed sequentially. Each phase involves a subset of hops, from a specific hop to hop K. We first have the following definition.

Definition 1 (Order-m Symbol)**.** *Order-1 symbols are simply the original information symbols. For any $2 \leq m \leq K$, an order-m symbol is defined as a piece of information which is desired by a subset of cardinality m of destination nodes, and is available at a source or relay node.*

Notation 1. *$u^{[i|j]}$ denotes an information symbol of S_i for D_j. An order-2 symbol which is available at S_i and desired to be decoded by D_j and D_k is denoted by $u^{[i|j,k]}$. Time duration of hop k in phase m of the scheme is denoted by $T_m^{(k)}$.*

A. Phase 1

▷ *Hop 1 (PSIN)*:

The information symbols (order-1 symbols) are fed to the network by the source nodes in hop 1. This is accomplished using a *partial scheduling and interference nulling (PSIN)* operation. The role of PSIN in hop 1 is twofold. First, it controls the number of interferers which contribute to each linear combination obtained by each layer-2 relay (and eventually by each destination). Second, it will enable generation of order-2 symbols at the destination nodes by yielding linear combinations which contain an appropriate mixture of the order-1 symbols. In particular, PSIN has the following ingredients.

- *Partial transmitter/destination scheduling*: By scheduling $L \leq K$ source nodes and one destination per time slot, the number of interferers at the non-scheduled destinations is partially controlled and also the scheduled destination will not eventually receive any interference.
- *Partial interference nulling*: The L scheduled source nodes transmit some redundancy together with order-1

TABLE I
OPERATION OF DIFFERENT HOPS IN THE K-PHASE TRANSMISSION SCHEME FOR THE K-USER K-HOP X NETWORK

Phase	Hop 1	Hop 2	Hop 3	$\cdots$	Hop $K-2$	Hop $K-1$	Hop K
1	PSIN	AF	AF	$\cdots$	AF	AF	order-2 symbol generation
2	symbol offloading	PSIN	AF	$\cdots$	AF	AF	order-3 symbol generation
3	silent	symbol offloading	PSIN	$\cdots$	AF	AF	order-4 symbol generation
$\vdots$	$\vdots$			$\ddots$			$\vdots$
$K-1$	silent	silent	silent	silent	symbol offloading	PSIN	order-K symbol generation
K	silent	silent	silent	silent	silent	symbol offloading	final delivery

symbols so that the layer-2 relays can null out the effect of one of the L interferers from their received signals.

As elaborated on in [10], the number of spent time slots of hop 1 in phase 1 is equal to

$$T_1^{(1)} = N_1 \times \frac{K(L-1)+1}{KL(L-1)}, \tag{5}$$

where N_1 is the number of order-1 symbols transmitted over the network in phase 1.

<u>*3-user case*</u>: Let $L=3$ be fixed. In hop 1, 18 information symbols, all desired by a specific destination, say D_1, are transmitted in 7 time slots as follows. During the first 6 time slots, each source node transmits a fresh information symbol in each time slot, i.e.,

$$x_j^{(1)}(t) = u_t^{[j|1]}, \qquad 1 \le j \le 3, \quad 1 \le t \le 6. \tag{6}$$

In time slot $t=7$, each source node transmits summation of its 6 previously transmitted symbols.

$$x_j^{(1)}(7) = u_1^{[j|1]} + u_2^{[j|1]} + \cdots + u_6^{[j|1]}, \quad 1 \le j \le 3.$$

Hence, ignoring the noise, for any $1 \le i \le 3$ we have

$$y_i^{(1)}(t) = \sum_{j=1}^{3} h_{ij}^{(1)}(t) u_t^{[j|1]}, \qquad 1 \le t \le 6, \tag{7}$$

$$y_i^{(1)}(7) = \sum_{j=1}^{3} h_{ij}^{(1)}(7) \sum_{t=1}^{6} u_t^{[j|1]}. \tag{8}$$

We note that each source node contributes exactly 6 information symbols to the 7 received signals of each relay. Therefore, each relay can apply three different linear transformations on its 7 received signals to obtain three different linear combinations, in each of which the contribution of one source node is nulled out. In particular, relay $V_i^{(2)}$, $1 \le i \le 3$, obtains the following linear combinations from (7) and (8).

$$\frac{y_i^{(1)}(7)}{h_{i1}^{(1)}(7)} - \sum_{t=1}^{6} \frac{y_i^{(1)}(t)}{h_{i1}^{(1)}(t)} = L_{i\backslash 1}(\mathbf{u}^{[2|1]}) + L_{i\backslash 1}(\mathbf{u}^{[3|1]}), \tag{9}$$

$$\frac{y_i^{(1)}(7)}{h_{i2}^{(1)}(7)} - \sum_{t=1}^{6} \frac{y_i^{(1)}(t)}{h_{i2}^{(1)}(t)} = L_{i\backslash 2}(\mathbf{u}^{[3|1]}) + L_{i\backslash 2}(\mathbf{u}^{[1|1]}), \tag{10}$$

$$\frac{y_i^{(1)}(7)}{h_{i3}^{(1)}(7)} - \sum_{t=1}^{6} \frac{y_i^{(1)}(t)}{h_{i3}^{(1)}(t)} = L_{i\backslash 3}(\mathbf{u}^{[1|1]}) + L_{i\backslash 3}(\mathbf{u}^{[2|1]}), \tag{11}$$

where we have denoted by $\mathbf{u}^{[j|1]} \triangleq [u_1^{[j|1]}, \cdots, u_6^{[j|1]}]^T$ the transmitted vector of S_j and by $L_{i\backslash j'}(\mathbf{u}^{[j|1]})$ its contribution in the received signal of $V_i^{(2)}$ after nulling the effect of $S_{j'}$.

Since the channel coefficients have continuous distributions, for any $1 \le j \le 3$, six partial linear combinations $\{L_{i\backslash j'}(\mathbf{u}^{[j|1]})\}_{i=1}^3$, $j' \in \{1,2,3\}\backslash\{j\}$, are linearly independent almost surely. Thus, if all these 6 linear combinations are delivered to D_1, it will be able to solve them for $\mathbf{u}^{[j|1]}$. The rest of transmission scheme is dedicated to this goal. Equivalently, for an arbitrary N_1, N_1 symbols can be transmitted in $T_1^{(1)} = 7 \times \frac{N_1}{18}$ time slots in hop 1 and $9 \times \frac{N_1}{18} = \frac{N_1}{2}$ linear combinations are generated at $\{V_i^{(2)}\}_{i=1}^3$ as in (9) to (11).

$\triangleright$ <u>*Hops* $2, \cdots, K-1$ *(AF)*</u>:

Each of these hops performs an amplify-and-forward (AF) operation to forward appropriate linear combinations of the order-1 symbols to the destination nodes. The number of spent time slots in each of these hops is equal to [10]

$$T_1^{(k)} = \frac{N_1}{K(L-1)}, \quad 2 \le k \le K-1. \tag{12}$$

<u>*3-user case*</u>: AF operation is performed by $\{V_i^{(2)}\}_{i=1}^3$ in hop 2. In particular, $\{V_i^{(2)}\}_{i=1}^3$ amplify-and-forward the linear combinations obtained in hop 1, i.e., (9) to (11), over hop 2 at 3 linear combinations per time slot. For instance, in one time slot, $\{L_{i\backslash 1}(\mathbf{u}^{[2|1]}) + L_{i\backslash 1}(\mathbf{u}^{[3|1]})\}_{i=1}^3$ are transmitted respectively by $\{V_i^{(2)}\}_{i=1}^3$ and $\{L'_{j\backslash 1}(\mathbf{u}^{[2|1]}) + L'_{j\backslash 1}(\mathbf{u}^{[3|1]})\}_{j=1}^3$ are received respectively by $\{V_j^{(3)}\}_{j=1}^3$, where

$$L'_{j\backslash 1}(\mathbf{u}^{[k|1]}) \triangleq \sum_{i=1}^{3} h_{ji}^{(2)}(t) L_{i\backslash 1}(\mathbf{u}^{[k|1]}). \tag{13}$$

Hence, this hop takes $T_1^{(2)} = \frac{N_1}{2} \times \frac{1}{3} = \frac{N_1}{6}$ time slots.

$\triangleright$ <u>*Hop K (Order-2 symbol generation)*</u>:

In this hop, the layer-K relays forward their received linear combinations to the destinations. Then, appropriate parts of the received signals at non-intended destination nodes are considered as order-2 symbols. Each order-2 symbol, if delivered to its respective pair of destinations, will partly align the past interference at one destination node while providing a useful piece of information for the other. As detailed in [10], the number of spent time slots of this hop is equal to

$$T_1^{(K)} = \frac{N_1}{(K-1)(L-1)+1}, \tag{14}$$

and the number of generated order-2 symbols is equal to

$$N_2 = N_1 \times \frac{(K-1)\,(2(L-1)-1)}{2\,((K-1)(L-1)+1)}. \tag{15}$$

<u>*3-user case*</u>: Relays $\{V_j^{(3)}\}_{j=1}^3$ amplify-and-forward the signals received during hop 2, cf. (13). Recall that if all the $L_{i\backslash j'}(\mathbf{u}^{[k|1]})$'s, or equivalently all the $L'_{i\backslash j'}(\mathbf{u}^{[k|1]})$'s, are delivered to D_1, then it will be able to decode all its information symbols. Let $\{V_j^{(3)}\}_{j=1}^3$ spend one time slot to transmit $\{L'_{j\backslash 1}(\mathbf{u}^{[2|1]})+L'_{j\backslash 1}(\mathbf{u}^{[3|1]})\}_{j=1}^3$ respectively (which include 6 quantities of type L'). Then, $\{L''_{j\backslash 1}(\mathbf{u}^{[2|1]})+L''_{j\backslash 1}(\mathbf{u}^{[3|1]})\}_{j=1}^3$ are received respectively by $\{D_j\}_{j=1}^3$, where

$$L''_{j\backslash 1}(\mathbf{u}^{[k|1]}) \triangleq \sum_{i=1}^{3} h_{ji}^{(3)}(t) L'_{i\backslash 1}(\mathbf{u}^{[k|1]}). \tag{16}$$

First, we note that D_1 receives an entirely desired linear combination, i.e., $L''_{1\backslash 1}(\mathbf{u}^{[2|1]}) + L''_{1\backslash 1}(\mathbf{u}^{[3|1]})$. Moreover, if we deliver $L''_{2\backslash 1}(\mathbf{u}^{[2|1]})$ to both D_1 and D_2, then D_1 obtains another desired linear combination whereas D_2 can cancel it out to obtain $L''_{2\backslash 1}(\mathbf{u}^{[3|1]})$ which in turn is desired by D_1. Hence, we consider $L''_{2\backslash 1}(\mathbf{u}^{[2|1]})$ as an order-2 symbol, since it can be reconstructed by S_2 using delayed CSI, and denote it by $u^{[2|1,2]}$. Then, $L''_{2\backslash 1}(\mathbf{u}^{[3|1]})$ will be a side information available at D_2 and desired by D_1, which can be reconstructed by S_3 using delayed CSI. As such, we denote it by $u^{[3|1;2]}$. Similarly, the order-2 symbol $u^{[2|1,3]} \triangleq L''_{3\backslash 1}(\mathbf{u}^{[2|1]})$ and side information $u^{[3|1;3]} \triangleq L''_{3\backslash 1}(\mathbf{u}^{[3|1]})$ are generated at D_3.

After delivering $u^{[2|1,2]}$, $u^{[3|1;2]}$, $u^{[2|1,3]}$, and $u^{[3|1;3]}$ to D_1, it will obtain 5 desired equations (including its own received one) in terms of the 6 transmitted quantities of type L'. Hence, it still needs another equation in terms of the transmitted L''s. To provide it with the desired equation, the above time slot needs to repeated $6/5$ times. Equivalently, 5 time slots are spent similarly by transmitting 5 *distinct* sets of L' quantities. Then, another time slot is spent by transmitting summation of all the 5 previously transmitted signals by each relay node.

In total, $T_1^{(3)} = \frac{N_1}{6} \times \frac{6}{5} = \frac{N_1}{5}$ time slots are spent in hop 3 during phase 1 and $\frac{N_1}{5} \times 2$ order-2 symbols together with $\frac{N_1}{5} \times 2$ side information quantities are generated. Finally, since *each* source node has information symbols for *each* destination, one can easily show that by scheduling all the destination nodes in this phase, an equal number of side information quantities of both types $u^{[i|j;k]}$ and $u^{[i|k;j]}$ are generated for any $1 \le i,j,k \le 3$. Then, new order-2 symbols are defined as follows.

$$u^{[i|j,k]} \triangleq u^{[i|j;k]} + u^{[i|k;j]}. \tag{17}$$

Therefore, the total number of generated order-2 symbols by the end of phase 1 is $N_2 = \frac{2N_1}{5} + \frac{N_1}{5} = \frac{3N_1}{5}$.

B. Phase m, $2 \le m \le K-1$

▷ <u>*Hops $1, \cdots, m-2$ (Silent)*</u>:

These hops are silent during phase m (for $m = 2$, the set of these hops is empty). Therefore,

$$T_m^{(k)} = 0, \quad 3 \le m \le K-1, \quad 1 \le k \le m-2. \tag{18}$$

▷ <u>*Hop $m-1$ (Symbol offloading)*</u>:

The goal of phase m is to transmit the order-m symbols, generated by the end of phase $m-1$. Transmission in this phase starts with hop $m-1$, over which K order-m symbols per time slot are offloaded by $\{V_i^{(m-1)}\}_{i=1}^K$ to $\{V_i^{(m)}\}_{i=1}^K$. The key idea behind the proposed symbol offloading is that, instead of delivering the original order-m symbols, linear functions of them are offloaded as *new* order-m symbols. While the former is equivalent to transmission over a single-hop X channel with delayed CSIT, the proposed offloading is accomplished at the maximum DoF of K symbols per channel use. The spent time slot of this hop is therefore equal to

$$T_m^{(m-1)} = \frac{N_m}{K}, \quad 2 \le m \le K. \tag{19}$$

<u>*3-user case*</u>: The goal of phase 2 is to transmit the order-2 generated by the end of phase 1. To this end, over hop 1, the order-2 symbols are offloaded from the source nodes to $\{V_i^{(2)}\}_{i=1}^3$. Each time slot of this hop is dedicated to a pair of destination nodes. During the time slot dedicated to (D_i, D_j), $u^{[1|i,j]}$, $u^{[2|i,j]}$, and $u^{[3|i,j]}$ are transmitted by S_1, S_2, and S_3 respectively. During this time slot, relay $V_k^{(2)}$, $1 \le k \le 3$, receives a linear combination

$$y_k^{(1)}(t) = h_{k1}^{(1)}(t)u^{[1|i,j]} + h_{k2}^{(1)}(t)u^{[2|i,j]} + h_{k3}^{(1)}(t)u^{[3|i,j]}$$

of the three transmitted order-2 symbols, where t is the corresponding time slot. If all these three linear combinations are eventually delivered to both D_i and D_j, then both nodes will be able to decode $u^{[1|i,j]}$, $u^{[2|i,j]}$, and $u^{[3|i,j]}$. Therefore, $y_1^{(1)}(t)$, $y_2^{(1)}(t)$, and $y_3^{(1)}(t)$ can be considered as three "new" order-2 symbols which are now available at the relay side (not the source side). Hence, N_2 new order-2 symbols are generated at layer-2 relays in this hop during $T_2^{(1)} = \frac{N_2}{3} = \frac{N_1}{5}$ time slots.

▷ <u>*Hop m (PSIN)*</u>:

PSIN over hop m of phase m is accomplished similar to that proposed for hop 1 in phase 1, and the following number of time slots is spent [10].

$$T_m^{(m)} = N_m \times \frac{K(L-1)+1}{KL(L-1)}, \quad 2 \le m \le K-1. \tag{20}$$

<u>*3-user case*</u>: The same PSIN scheme proposed for hop 1 of phase 1 is performed for transmission of (new) order-2 symbols over hop 2 of phase 2, and thus, $T_2^{(2)} = \frac{7N_2}{18} = \frac{7N_1}{30}$.

▷ <u>*Hops $m+1, \cdots, K-1$ (AF)*</u>:

The AF operation in each of these hops is accomplished in the same way as that performed in hops $2, \cdots, K-1$ during phase 1, and thus, we have

$$T_m^{(k)} = \frac{N_m}{K(L-1)}, \quad m+1 \le k \le K-1. \tag{21}$$

$\triangleright$ *Hop K (Order-$(m+1)$ symbol generation)*:

The order-2 symbol generation ideas proposed for hop K in phase 1 are generalized to generate order-$(m+1)$ symbols in hop K of phase m. As shown in [10],

$$T_m^{(K)} = \frac{N_m}{(K-m)(L-1)+1}, \qquad 1 \leq m \leq K-1, \quad (22)$$

and the number of generated order-$(m+1)$ symbols equals

$$N_{m+1} = N_m \times \frac{(K-m)\left((m+1)(L-1)-1\right)}{(m+1)\left((K-m)(L-1)+1\right)}. \quad (23)$$

3-*user case*: The order-3 symbol generation process in hop 3 of phase 2 generalizes the order-2 symbol generation proposed in phase 1. The details are omitted here due to lack of space. The number of spent time slots and generated order-3 symbols will then be $T_2^{(3)} = \frac{N_2}{3} = \frac{N_1}{5}$ and $N_3 = \frac{5N_2}{9} = \frac{N_1}{3}$.

C. Phase K

$\triangleright$ *Hops $1, \cdots, K-2$ (Silent)*:

These hops are silent during phase K, and therefore,

$$T_K^{(k)} = 0, \quad 1 \leq k \leq K-2. \quad (24)$$

$\triangleright$ *Hop $K-1$ (Symbol offloading)*:

This hop performs the same symbol offloading operation proposed in previous hops during previous phases, now on the order-K symbols, and thus, The spent time slot of this hop is therefore equal to

$$T_K^{(K-1)} = \frac{N_K}{K}, \quad (25)$$

which results in $T_3^{(2)} = \frac{N_3}{3} = \frac{N_1}{9}$ for the 3-user case.

$\triangleright$ *Hop K (Final delivery)*:

Hop K in phase K is responsible for delivering order-K symbols to all K destination nodes. Using a time division scheme, one order-K symbol per time slot is delivered to the destination nodes, and thus, one can write

$$T_K^{(K)} = N_K, \quad (26)$$

which results in $T_3^{(3)} = N_3 = \frac{N_1}{3}$ in the 3-user case.

D. DoF Calculation for the K-user K-hop X Network

For the general K, by appropriately interleaving B rounds of the proposed transmission scheme, one can deliver $N_1 B$ information symbols in $(B + \frac{K^2+3K}{2} - 2)\max_{1\leq k\leq K} T^{(k)}$ time slots as shown in [10]. This implies achievability of

$$\frac{N_1}{\max_{1\leq k\leq K} T^{(k)}} \quad (27)$$

DoF as $B \to \infty$, where $T^{(k)} = \sum_{m=1}^{K} T_m^{(k)}$ is the total time duration of hop k. For the 3-user case, N_1 information symbols were delivered to the destinations by spending the following time slots.

$$\text{Hop 1: } T_1^{(1)} + T_2^{(1)} + T_3^{(1)} = \frac{7N_1}{18} + \frac{N_1}{5} + 0 = \frac{53N_1}{90},$$

$$\text{Hop 2: } T_1^{(2)} + T_2^{(2)} + T_3^{(2)} = \frac{N_1}{6} + \frac{7N_1}{30} + \frac{N_1}{9} = \frac{23N_1}{45},$$

$$\text{Hop 3: } T_1^{(3)} + T_2^{(3)} + T_3^{(3)} = \frac{N_1}{5} + \frac{N_1}{5} + \frac{N_1}{3} = \frac{11N_1}{15}.$$

Therefore, in view of (27), the proposed scheme achieves $\frac{15}{11}$ DoF for the 3-user 3-hop X network.

For general K, using (5), (12), (14), (15) and (18) to (26), the lower bound of (1) results from the following lemma, which is proved in Section VII and Appendix B of [10].

Lemma 2. *The achievable DoF of the proposed scheme for the K-user K-hop X network with delayed CSI, given by* (27), *is lower bounded by $\frac{N_1}{T^{(1)}+T^{(K)}}$. Moreover, this is equal to* (1) *by defining $t_1(q,K) \triangleq T^{(K)}/N_1$ and $t_2(q,K) \triangleq T^{(1)}/N_1$ with $q \triangleq L-1$, where $3 \leq L \leq K$ is an arbitrary integer.*

This completes the proof of Theorem 1.

IV. Conclusion

The impact of multi-hopping on the DoF of interference networks with delayed CSI was investigated in this paper. For the K-user $2K$-hop interference network, a multi-phase transmission scheme was proposed whose achievable DoF does not saturate with increasing the number of users. Since the gap between our achievable DoF and the best known upper bound scales with the number of users, the problem of characterizing the DoF scaling rate of this network with delayed CSI remains open. Also, another open problem is to find the minimum number of hops and relays per hop required to achieve DoF scaling.

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