Barnes-Wall lattices for the Symmetric Interference Channel

María Constanza Estela and Cong Ling
Department of Electrical and Electronic Engineering
Imperial College London, United Kingdom
E-mail: m.estela09@imperial.ac.uk, cling@ieee.org

Jean-Claude Belfiore
Département Communications et Electronique
Télécom-ParisTech, Paris, France
E-mail: belfiore@telecom-paristech.fr

***Abstract*—In this paper we study the performance of Barnes-Wall lattices in a symmetric interference channel, under different types of interference. We are inspired by the work of Jafar [1], in which a scheme is proposed for each type of interference, using a base Q expression for the transmitted signals. This is similar to the multilevel structure of Barnes-Wall lattices. With the advantage of their good performance and the extension in bigger dimensions that using lattices implies, we propose to use Barnes-Wall lattices to improve the performance of each user, under lattice alignment in a symmetric interference channel.**

I. Introduction

Interference alignment [2] is a new technique that prepares the signals at the transmitters such that at the undesired receivers it aligns, leaving half of the available space for the desired signal. It can be considered from the signal space or from the signal scale perspective. In the later case, structured codes such as lattice codes have been used to align interference. It was shown in [3] that using lattice codes, the interference produced by one interferer is the same as the one produced by many interferers. The sum of all the interference can be made to be in a lattice different from the desired signal lattice. Therefore, the sum of the interference is decoded, and removed to decode the desired signal. This is referred to as *lattice alignment* [3]. In [4] and [5] a symmetric Gaussian K-user interference channel is considered and lattice codes are used. In the first case the authors consider the very strong interference scenario, where a particular condition of the channel coefficient is given to attain the achievable rate. Lattice codes and successive interference decoding are used to achieve the optimal rate. In [5], a layered lattice coding scheme is presented with which the degrees of freedom (DoF) is found for a wider range of indirect channel coefficients. In [6], a deterministic channel approach is applied to an interference channel, where signals are represented in base Q, to construct an interference alignment scheme. With a symmetric interference channel where the indirect channel coefficients are given by Q^{-1} and zero paddings in the signal construction, interference is perfectly aligned, leaving $K/2$ degrees of freedom for the desired signal. In [7] the generalized DoF are found for different types of interference according to the signal-to-noise ratio (SNR) and interference-to-noise ratio (INR). Following with the ideas of [6] and [7], in [1] achievable schemes for the generalized DoF are found for different types of interference, for K users. The signals are represented in base Q, and a detailed scheme is given for the different types of interference. The construction of the schemes can serve as an inspiration to use lattice codes.

All of these works deal with the DoF or achievable rate of the interference channel but we are interested the performance of the interference channel with alignment. In [8], Choi studies a MIMO interference channel where he is concerned with a precoder design with joint detectors at the receiver, where lattices are used to be able to decode. The focus of his paper is from a practical point of view, as to find the performance of the technique. In [9] the practical performance of the interference channel is also of interest. A many-to-one interference channel is considered, defined in [3], where only one user is affected by interference. A lattice code is designed for all users, to obtain a small probability of error at receiver 1. For some conditions a relation between the union bound of the error probability of user 1, and the theta series of the interference and joint lattices is found. In this paper we are concerned about the performance of a symmetric interference channel. We will consider the base Q representation of [1] using higher dimensional lattices. In particular we will use Barnes-Wall lattices, due to their very similar level structure as the one dimensional lattice scheme used in [7] and [1], and their good performance. We will show that it is possible to build similar schemes for different types of interference using Barnes-Wall lattices, with the benefit of improving the performance.

In the next section we will explain the symmetric interference channel and we will summarize the schemes presented by Jafar [1] for some types of interference. Section III will introduce the Barnes-Wall lattices and we will explain the similitudes with the scheme proposed by Jafar. In section IV we will discuss the decoding used for both methods. Section V will show some simulations results on the performance of the Jafar scheme and Barnes-Wall lattices. Finally we will draw some conclusions in section VI.

II. Symmetric interference channel

Consider the K-user symmetric interference channel (IC) model given by [1]:

$$y_k = x_k + g \sum_{j=1, j\neq k}^{K} x_j + z_k, \tag{1}$$

where y_k is the received signal at receiver k, g is the integer and real indirect channel coefficient, x_k is the signal transmitted by transmitter k, x_j is the interference signal from transmitter j, z_k is the additive white Gaussian noise with variance σ^2 and zero mean, and $k = 1, \cdots, K$. In [1] the channel is defined as $g = \sqrt{\frac{\text{INR}}{\text{SNR}}}$. In [7] the interference is classified in types defined by means of a parameter $\alpha = \frac{\log \text{INR}}{\log \text{SNR}}$. Each type of interference is given by [7]:

- Noisy: $0 \leq \alpha \leq 1/2$
- Weak: $1/2 \leq \alpha \leq 2/3$
- Moderately weak: $2/3 \leq \alpha < 1$
- Strong: $1 < \alpha \leq 2$
- Very strong: $\alpha \geq 2$

where there is a discontinuity at 1.

In [6] and [1] the idea of quantizing the signal in *levels* is used. In the paper by Jafar [1] the problem has been addressed using a 1 dimensional lattice and they are interested in a scheme to achieve the DoF found in [7], for each type of interference. The transmitted signal is constructed as $x_k = Q^{N-1}C_{N-1} + \cdots + QC_1 + C_0$, where we will define each level to be represented by Q^i. Here C_i are codewords that have some properties related to Q and K, where K, Q, M and N are positive integers. The parameter Q is also related to the SNR since $\text{SNR} = Q^{\frac{2M}{|\alpha-1|}}$, where $Q \gg K$ and M grows to infinity. The channel is given by $g = Q^{\text{sgn}(\alpha-1)M}$, where $\text{sgn}(x) = 1$ if $x > 0$ and -1 if $x < 0$. The channel acts as a shifter, which may produce that some levels match at the receivers. With some careful properties of the codewords C_i, each level can be decoded sequentially and independently, and interference can be eliminated.

The schemes presented in this paper for each type of interference manage to completely remove the interference using alignment and some interesting codeword constructions for one dimensional symbols. Interference is then cancelled, being able to decode the desired message. Consider for example the very strong interference scheme. Here, the channel is given by $g = Q^M$, and the transmitted signals are given by $x_k = Q^{N-1}C_{N-1} + \cdots + QC_1 + C_0$, where C_i are the codewords. In order to avoid carry-overs, Jafar poses another condition which for very strong interference is given by $C_i \in \{1, \cdots, Q-2\}$. Note that in this case, it means that $Q \geq 4$ for the code to exist. Thus the channel coefficient can not be smaller than 4. Finally the restriction that $N = \lfloor \frac{M}{|\alpha-1|} \rfloor$ is given to satisfy the power constraint. Then $M \geq N$. Consider the following example in moderately weak interference, where $N = M = 1$ and $Q \geq 7$:

$$\begin{aligned} x_1 &= Q^4C_4 + Q^3C_3 + Q^2C_4 + C_0 \\ x_2 &= Q^4D_4 + Q^3D_3 + Q^2D_4 + D_0, \end{aligned}$$

where $g = Q^{-1}$. Thus, at receiver 1 we have:

$$\begin{aligned} y_1 = Q^4C_4 &+ Q^3(C_3 + D_4) + Q^2(C_4 + D_3) \\ &+ QD_4 + C_0 + Q^{-1}D_0 + z_1 \end{aligned}$$

This scheme has some nice properties, each C_i and D_i is built in a codebook where there are no carry-overs, $C_1 = D_1 = 0$, and $C_4 = C_2$, $D_4 = D_2$. All of this makes it easy to decode and eliminate the interference at each receiver. The levels are also related to the SNR. This means that in order to find the DoF the values of Q and N are very big.

The schemes proposed by Jafar are very interesting. However, they have some limitations. The values of Q are very limited in order to be able to construct a codebook, then the values that the channel can take are also very limited. Also, the system is built (for simplicity) in one dimension. We can see an extension in higher dimensions if we apply lattices, in particular Barnes-Wall lattices which seem very useful since their construction is very similar. Finally, we do not know how is the performance of these schemes, which is actually hard to analyse as in some cases the SNR must be very big and it is related to the value of α by $g = \text{SNR}^{\frac{\alpha-1}{2}}$. We can conjecture that using higher dimensional lattices we can find a better performance. To simplify the problem we will assume that the interference is very strong and strong if the channel $g > 1$, while the interference is noisy, weak or moderately weak if $g < 1$.

III. Proposed scheme based on Barnes-Wall lattices

Barnes-Wall (BW) lattices are a family of full rank lattices whose dimension is a power of 2, which correspond to the densest lattices in dimensions 2, 4, 8 and 16. These binary decomposable lattices can be built using Reed-Muller (RM) codes, which are a class of binary error correcting codes, where $\text{RM}(r', m+1)$ corresponds to the RM code of order r', vector length 2^{m+1} and minimum distance $2^{m+1-r'}$. The lattice $\Lambda(0, m)$ is the *m*th member of the BW family of lattices, and can be expressed as a real or a complex lattice of dimension 2^{m+1} or 2^m respectively. For convenience in this paper we will work will real lattices. The family of lattices given by $\Lambda(r, m)$ where $m \geq 0$ and $0 \leq r \leq m$ can be obtained by [10]:

- $m - r$ even:

$$\begin{aligned} \Lambda(r,m) = &\, 2^{\frac{m-r}{2}}\mathbb{Z}^{2^{m+1}} \\ &+ \sum_{\substack{r+1 \leq r' \leq m \\ m-r' \text{ odd}}} 2^{(r'-r-1)/2}\text{RM}\left(r', m+1\right) \end{aligned} \tag{2}$$

- $m - r$ odd:

$$\begin{aligned} \Lambda(r,m) = &\, 2^{\frac{m-r+1}{2}}\mathbb{Z}^{2^{m+1}} \\ &+ \sum_{\substack{r+1 \leq r' \leq m \\ m-r' \text{ even}}} 2^{(r'-r-1)/2}\text{RM}\left(r', m+1\right). \end{aligned} \tag{3}$$

Note that $\Lambda(r, m)$ has minimum distance 2^{m-r}.

As shown in the previous equations, the construction of these lattices is the same as construction D, where lower order codes are nested into bigger ones. This property is useful in the decoding process which will be explained later.

For the rest of the paper, whenever we work with BW lattices, the codewords will be expressed with C_i for user 1 and D_i for user 2, with $i = 0$ for the lowest RM code and increasing up to the highest level represented by $\mathbb{Z}^{2^{m+1}}$.

We can think of an interference channel problem as in (1) where each user transmits a point in the same BW lattice. Let us consider an example using $\Lambda_{16} = \Lambda(0,3) = 4\mathbb{Z}^{16} + 2\text{RM}(3,4) + \text{RM}(1,4)$. Considering only 2 users, each transmitted signal is given by:

$$\begin{aligned}\mathbf{x}_1 &= 4\mathbf{C}_2 + 2\mathbf{C}_1 + \mathbf{C}_0\\ \mathbf{x}_2 &= 4\mathbf{D}_2 + 2\mathbf{D}_1 + \mathbf{D}_0,\end{aligned}$$

where $\mathbf{C}_0, \mathbf{D}_0 \in \text{RM}(1,4)$, $\mathbf{C}_1, \mathbf{D}_1 \in \text{RM}(3,4)$ and $\mathbf{C}_2, \mathbf{D}_2 \in \mathbb{Z}^{16}$. Let us mimic the idea by Jafar for a case where the channel is given by $g = 2^{-1}$. Note that in order to simplify the problem we will assume that the channel is a power of 2. Then:

$$\mathbf{y}_1 = 4\mathbf{C}_2 + 2(\mathbf{C}_1 + \mathbf{D}_2) + (\mathbf{C}_0 + \mathbf{D}_1) + 2^{-1}\mathbf{D}_0 + \mathbf{z}_1.$$

In order to decode, we need some properties like before, otherwise we have some problems with interference. If we say $\mathbf{C}_1 = \mathbf{D}_1 = 0_{16}$ we have:

$$\mathbf{y}_1 = 4\mathbf{C}_2 + 2\mathbf{D}_2 + \mathbf{C}_0 + 2^{-1}\mathbf{D}_0 + \mathbf{z}_1.$$

To keep the problem simple we are using powers of 2. Therefore, we also have some limitations on the channels. Also, for our analysis we will consider only 2 users. The reason is that, even if the schemes can work with K users, the number of users the system can accept depends on Q. This comes from [1] since to define a codebook C_i, to avoid carryovers from one level to the other one, there is a relation with Q. In that case, to prove the DoF for each type of interference Q must be very big, and in particular $Q \gg K$. Then, for a given set of variables, including a given channel gain, there is a given amount of users the system can allocate. For the clarity and the purposes of this paper, we will consider only a 2 user interference channel, to prove that we can mimic the Jafar schemes in higher dimensions, and obtain better performance.

We will try to mimic as close as possible the construction of Jafar, by finding a lattice in a bigger dimension with the same or similar structure. For example, for the very strong interference case the highest level of the Jafar symbol is given by Q^{N-1}. In order to make this easier we will force $Q^{N-1} = 2^{\gamma}$, where from (2) $\gamma = m/2$ if m is even or from (3) $\gamma = (m+1)/2$ if m is odd (note that $r = 0$). For the weak interference case we will force $Q^{2M+N-1} = 2^{\gamma}$. Let us see some examples:

- Very strong interference: Consider $Q = 4$, $N = 2$, $M = 2$. For Jafar we have $x_1 = 4C_1 + C_0$. Then we select a lattice with a similar structure but in higher dimensions. In that case we can see that the lattice $\Lambda(0,3)$ is a lattice we can use. Since $\Lambda_{16} = \Lambda(0,3) = 4\mathbb{Z}^{16} + 2\text{RM}(3,4) + \text{RM}(1,4)$ we can say $\mathbf{x}_1 = 4\mathbf{C}_2 + 2\mathbf{C}_1 + \mathbf{C}_0$, where $\mathbf{C}_1 = 0_{16}$ and $\mathbf{C}_2 \in \mathbb{Z}^{16}/2\mathbb{Z}^{16}$. The same is applied at transmitter 2: $\mathbf{x}_2 = 4\mathbf{D}_2 + 2\mathbf{D}_1 + \mathbf{D}_0$, where $\mathbf{D}_1 = 0_{16}$ and $\mathbf{D}_2 \in \mathbb{Z}^{16}/2\mathbb{Z}^{16}$. Then with the channel given by $g = 16$ at receiver 1 we have:
$$\mathbf{y}_1 = 64\mathbf{D}_2 + 16\mathbf{D}_0 + 4\mathbf{C}_2 + \mathbf{C}_0 + \mathbf{z}_1. \tag{4}$$
- Weak interference: Consider $Q = 4$, $N = 1$, $M = 1$. Then for Jafar we have: $x_1 = 16C_2 + C_0$. A lattice with a similar structure is found by saying $\gamma = \log_2(16)$. Then $\Lambda(0,8)$ satisfies what we need since $\Lambda(0,8) = 16\mathbb{Z}^{512} + 8\text{RM}(7,9) + 4\text{RM}(5,9) + 2\text{RM}(3,9) + \text{RM}(1,9)$. Then $\mathbf{x}_1 = 16\mathbf{C}_4 + \mathbf{C}_0$, where $\mathbf{C}_3 = \mathbf{C}_2 = \mathbf{C}_1 = 0_{512}$ and $\mathbf{C}_4 \in \mathbb{Z}^{512}/2\mathbb{Z}^{512}$. Using the same idea for user 2, $\mathbf{x}_2 = 16\mathbf{D}_4 + \mathbf{D}_0$. With the channel given by $g = 4^{-1}$, at receiver 1 we have:
$$\mathbf{y}_1 = 16\mathbf{C}_4 + 4\mathbf{D}_4 + \mathbf{C}_0 + 2^{-2}\mathbf{D}_0 + \mathbf{z}_1. \tag{5}$$

IV. Decoding and Error Bound

A. Decoding

The decoding of the Jafar schemes take place depending on the type of interference. For most types of interference, the decoding used is a multilevel decoder, starting from the lowest level. Once a level is decoded it is subtracted from the received signal and divided by Q, and the process is repeated. The decoding of each level will be shown next, for the case of weak interference, and it is similar for strong and moderately weak. We will start with the very strong interference case which is different than the other 3 cases. For the very strong interference, Jafar's model is given by:

$$\begin{aligned}x_1 &= Q^{N-1}C_{N-1} + \cdots + QC_1 + C_0\\ x_2 &= Q^{N-1}D_{N-1} + \cdots + QD_1 + D_0,\end{aligned}$$

with the channel given by $g = Q^M$. At receiver 1 we have:

$$\begin{aligned}y_1 = Q^{N+M-1}D_{N-1} + \cdots + Q^M D_0 + Q^{N-1}C_{N-1} +&\\ \cdots + QC_1 + C_0 + z_1.&\end{aligned}$$

Since $M \geq N$ then the interference has been shifted to *higher levels*, allowing the desired signal to be in *lower levels*, with no overlap. To decode we just need:

$$\hat{x}_1 = \lfloor y_1 \rceil \bmod Q^N, \tag{6}$$

where $\lfloor \rceil$ refers to the rounding operation.

Now let us consider the case for weak interference:

$$\begin{aligned}x_1 = Q^{2M+N-1}C_{2M+N-1} + \cdots + Q^{2M}C_{2M}&\\ + Q^{M-1}C_{M-1} + \cdots + C_0&\end{aligned}$$

$$\begin{aligned}x_2 = Q^{2M+N-1}D_{2M+N-1} + \cdots + Q^{2M}D_{2M}&\\ + Q^{M-1}D_{M-1} + \cdots + D_0&\end{aligned}$$

In this case, the channel is given by Q^{-M}. At receiver 1:

$$\begin{aligned}y_1 = Q^{2M+N-1}C_{2M+N-1} + \cdots + Q^{2M}C_{2M} + \cdots&\\ +Q^{M+N-1}D_{2M+N-1} + \cdots + Q^M D_{2M}&\\ +Q^{M-1}C_{M-1} + \cdots + C_0&\\ +Q^{-1}D_{M-1} + \cdots + Q^{-M}D_0 + z_1.&\end{aligned}$$

Algorithm 1

$s = y_1$ ▷ for strong interference
$s = Q^M y_1$ ▷ for moderately weak and weak interference
for all received levels **do** ▷ Starting from the lowest level
  $c = \lfloor s \rceil \bmod Q$
  $\hat{C}_i = c$ (or $\hat{D}_i = c$) ▷ Estimated value for level Q^i
  $s = (s - c)/Q$
end for

Algorithm 2 Sequential BDD for BW Lattices (modified from [12])

$\mathbf{s} = \mathbf{y}_1$ ▷ for very strong and strong interference
$\mathbf{s} = Q^M \mathbf{y}_1$ ▷ for moderately weak and weak interference
for all received levels **do** ▷ Starting from the lowest level
  if $\mathbf{C}_i$ (or $\mathbf{D}_i$) to decode corresponds to a RM code **then**
    $\mathbf{b} = \lfloor \mathbf{s} \rceil \bmod 2$
    $\rho = 1 - 2|\mathbf{s} - \lfloor \mathbf{s} \rceil|$
    $\mathbf{c} = \text{RMdec}(j, \mathbf{b}, \rho)$ ▷ from [12], Algorithm 3; where j corresponds to $\text{RM}(j, m+1)$ to decode
  else If it corresponds to $\mathbb{Z}^{2^{m+1}}/2\mathbb{Z}^{2^{m+1}}$
    $\mathbf{c} = \lfloor \mathbf{s} \rceil \bmod 2$
  end if
  $\hat{\mathbf{C}}_i = \mathbf{c}$ (or $\hat{\mathbf{D}}_i = \mathbf{c}$) ▷ Estimated value for level Q^i
  $\mathbf{s} = (\mathbf{s} - \mathbf{c})/Q$
end for

The desired signal and the interference do not overlap, so decoding can take place by levels. In fact, we decode from the lowest level upwards by applying Algorithm 1.

To compare with the above we will use BW lattices. A low-complexity bounded distance decoding algorithm for Leech lattices was presented in [11], which can also be applied to decode Construction D lattices. Since BW lattices can also be constructed by construction D, this is a useful decoder for this kind of lattices. In particular, we will focus on the low-complexity bounded distance decoder (BDD) algorithm for BW lattices proposed by Micciancio in [12], Algorithm 2. We will modify this algorithm to work with real lattices. The decoding procedure is shown in Algorithm 2.

With this, all levels have been estimated and we can obtain an estimated transmitted version of $\mathbf{x}_1$.

From the examples shown in section III we have:

- Very strong interference: From (4):

$$\mathbf{y}_1 = 64\mathbf{D}_2 + 16\mathbf{D}_0 + 4\mathbf{C}_2 + \mathbf{C}_0 + \mathbf{z}_1.$$

  From $\mathbf{s} = \mathbf{y}_1$ the BDD computes the estimated transmitted RM code from $\text{RM}(1,4)$, to obtain $\hat{\mathbf{C}}_0$. Then, $\hat{\mathbf{C}}_2 = ((\mathbf{s} - \hat{\mathbf{C}}_0)/4) \bmod 2$. The estimated transmitted symbol from user 1 is $\hat{\mathbf{x}}_1 = 4\hat{\mathbf{C}}_2 + \hat{\mathbf{C}}_0$.
- Weak interference: From (5):

$$\mathbf{y}_1 = 16\mathbf{C}_4 + 4\mathbf{D}_4 + \mathbf{C}_0 + 2^{-2}\mathbf{D}_0 + \mathbf{z}_1.$$

  From $\mathbf{s} = 4\mathbf{y}_1$ the BDD computes the estimated transmitted RM code from $\text{RM}(1,9)$, to obtain $\hat{\mathbf{D}}_0$. Then $\mathbf{s} = (\mathbf{s} - \hat{\mathbf{D}}_0)/4$ and proceeds to estimate the transmitted RM code from $\text{RM}(1,9)$ to obtain $\hat{\mathbf{C}}_0$. Then to obtain $\hat{\mathbf{D}}_4$ we say $\mathbf{s} = (\mathbf{s} - \hat{\mathbf{C}}_0)/4$ and $\hat{\mathbf{D}}_4 = \lfloor \mathbf{s} \rceil \bmod 2$. Finally we say $\mathbf{s} = (\mathbf{s} - \hat{\mathbf{D}}_4)/4$ and $\hat{\mathbf{C}}_4 = \lfloor \mathbf{s} \rceil \bmod 2$. The estimated transmitted symbol from user 1 is $\hat{\mathbf{x}}_1 = 16\hat{\mathbf{C}}_4 + \hat{\mathbf{C}}_0$.

B. Bound

In order to decode we have used the sequential BDD for BW lattices, as the one proposed by Micciano [12], which uses a modified RM decoder from Schnabl [13]. The sequential BDD, decodes the lowest level first and then proceeds to the upper ones. Then, the lower bound is given by the RM decoder. According to Schnabl [13] if the system is given by: $\mathbf{y} = \mathbf{c} + \mathbf{e}$, where $\mathbf{e}$ is an error and $\mathbf{c} \in \text{RM}(r', m+1)$, in order to correctly decode we need $\|\mathbf{e}\| < \sqrt{2^{m+1-r'}}$. Given the construction of the BW lattices as previously defined the euclidean code $2^{(i-1)/2}\text{RM}(i, m+1)$ has minimum squared distance equal to 2^m [14]. Then for any $\Lambda(0, m)$ in order to correctly decode we need:

$$\|\mathbf{e}\|^2 < \frac{2^m}{4}. \tag{7}$$

This however, may not be a tight bound.

V. Simulations

In this section we will show some simulations for the proposed idea. Since we are working with lattices, we will work with the term volume-to-noise ratio, which is defined as $\text{VNR} = \frac{V(\Lambda)^{\frac{2}{d}}}{2\pi\sigma^2 e}$, where $V(\Lambda)^{\frac{2}{d}}$ is the normalized fundamental volume of the lattice Λ, which for BW lattices is given by $2^{\frac{m}{2}}$, and $d = 2^{m+1}$. Jafar's scheme symbols are considered as 1 dimensional lattice points. Figs. 1 and 2 show the results for the very strong and weak interference examples given in the previous section. Fig. 3 shows another example for strong interference where $Q = 8$, $M = 1$ and $N = 2$. In that case, the symbol for the Jafar scheme is built as: $x_1 = 64C_2 + 8C_1 + C_0$, where $C_2 = C_0$. We can construct a symbol from the lattice $\Lambda(0,8)$ as $\mathbf{x}_1 = 16\mathbf{C}_4 + 8\mathbf{C}_3 + 4\mathbf{C}_2 + 2\mathbf{C}_1 + \mathbf{C}_0$. First, to mimic the Jafar symbols we need to make $\mathbf{C}_4 = \mathbf{C}_2 = \mathbf{C}_1 = 0$. We have chosen this restriction since at the receiver we want the levels of Jafar and the BW lattice to *coincide*. Since Jafar's symbol is using levels 64, 8 and 1, from $\Lambda(0,8)$ we can only choose levels 1 and 8 to avoid errors in other levels. Then at receiver 1, and considering 2 users: $\mathbf{y}_1 = 64\mathbf{D}_3 + 8(\mathbf{C}_3 + \mathbf{D}_0) + \mathbf{C}_0 + \mathbf{z}_1$. To avoid interference with other users, we have to choose $\mathbf{C}_3 = \mathbf{D}_3 = 0$, leaving us only with level 1. Also for the strong interference case, we can construct the transmitted symbol from $\Lambda(0,11)$ as: $\mathbf{x}_1 = 64\mathbf{C}_6 + 32\mathbf{C}_5 + 16\mathbf{C}_4 + 8\mathbf{C}_3 + 4\mathbf{C}_2 + 2\mathbf{C}_1 + \mathbf{C}_0$. First to mimic the Jafar scheme we say $\mathbf{C}_5 = \mathbf{C}_4 = \mathbf{C}_2 = \mathbf{C}_1 = 0$, and second to avoid interference we say $\mathbf{C}_3 = 0$. The construction of the transmitted signal of user 2 is similar as the one for user one, therefore will be omitted. The error probabilities have been normalized per 2 dimensions as suggested by Forney in [15]. We can observe as expected that using higher dimensional lattices the asymptotic performance is improved.

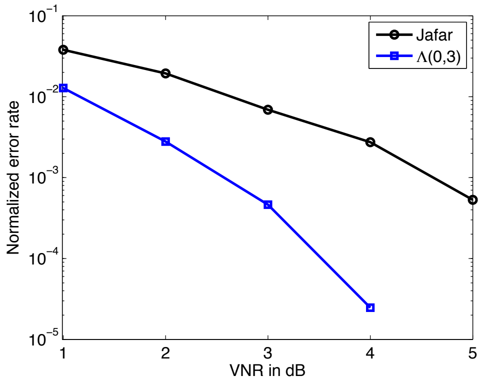

Fig. 1. Error rate for very strong interference, when $Q = 4$, $M = 2$, $N = 2$

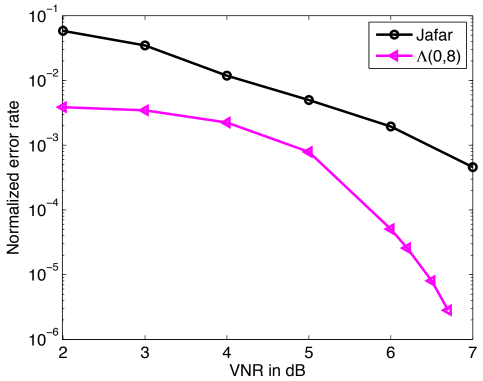

Fig. 2. Error rate for weak interference, when $Q = 4$, $M = 1$, $N = 1$

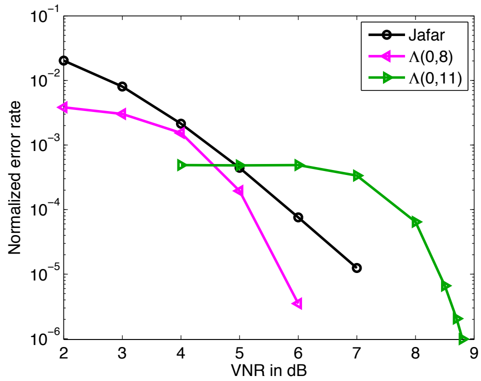

Fig. 3. Error rate for strong interference, when $Q = 8$, $M = 1$, $N = 2$

This should not be a surprise since we are working with BW lattices which have a very good performance. However, the trick is to find the correct lattice to mimic the code construction given by Jafar for any channel and any type of interference. We can conjecture that the improvement on the performance will also be the case for any type of interference.

VI. Conclusion

In this paper, we have shown the work of Jafar and how it is possible to eliminate the interference by building a clever scheme for each type of interference. Inspired by this, we used BW lattices due of its resemblance as multilevel codes, and the extension that lattices imply in higher dimensions. We show that with a proper selection of the lattices, we can improve the performance of the schemes proposed by Jafar.

Acknowledgment

The authors would like to thank Yanfei Yan for providing the sequential BDD code and for fruitful discussions.

References

[1] S. A. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric Gaussian K user interference channel," *IEEE Transactions on Information Theory*, vol. 56, pp. 3297–3303, Jul. 2010.

[2] V. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Transactions on Information Theory*, vol. 54, pp. 3425–3441, Aug. 2008.

[3] G. Bresler, A. Parekh, and D. Tse, "The approximate capacity of the many-to-one and one-to-many Gaussian interference channels," *IEEE Transactions on Information Theory*, vol. 56, pp. 4566–4592, Sep. 2010.

[4] S. Sridharan, S. Vishwanath, and S. A. Jafar, "Capacity of the symmetric K-user Gaussian very strong interference channels," *Proc. IEEE Global Telecommun. Conf.*, vol. 56, Dec. 2008.

[5] S. Sridharan, A. Jafarian, S. Vishwanath, S. Jafar, and S. Shamai, "A layered lattice coding scheme for a class of three user Gaussian interference channel," *46th Annual Allerton Conference on Communication, Control, and Computing, 2008*, pp. 531–538, Sep. 2008.

[6] V. Cadambe, S. Jafar, and S. Shamai, "Interference alignment on the deterministic channel and application to fully connected Gaussian interference networks," *IEEE Transactions on Information Theory*, vol. 55, pp. 269–274, Jan. 2009.

[7] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Transactions on Information Theory*, vol. 54, pp. 5534–5562, Dec. 2008.

[8] J. Choi, "Interference alignment over lattices for MIMO interference channels," *IEEE Communications Letters*, vol. 15, pp. 374–376, Apr. 2011.

[9] M. C. Estela, L. Luzzi, C. Ling, and J.-C. Belfiore, "Analysis of lattice codes for the many-to-one interference channel," *IEEE Information Theory Workshop*, pp. 417–421, Sep. 2012.

[10] A. J. Salomon and O. Amrani, "Augmented product codes and lattices: Reed-Muller codes and Barnes-Wall lattices," *IEEE Transactions on Information Theory*, vol. 51, pp. 3918–3930, Nov. 2005.

[11] G. D. Forney, "A bounded-distance decoding algorithm for the Leech lattice, with generalizations," *IEEE Transactions on Information Theory*, vol. 35, pp. 906–909, Jul. 1989.

[12] D. Micciancio and A. Nicolosi, "Efficient bounded distance decoders for Barnes-Wall lattices," *IEEE International Symposium on Information Theory, 2008*, pp. 2484–2488, Jul. 2008.

[13] G. Schnabl and M. Bossert, "Soft-decision decoding of Reed-Muller codes as generalized multiple concatenated codes," *IEEE Transactions on Information Theory*, vol. 41, pp. 304–308, Jan. 1995.

[14] J. Harshan, E. Viterbo, and J.-C. Belfiore, "Practical encoders and decoders for Euclidean codes from Barnes-Wall lattices," *submitted to IEEE Transactions on Information Theory*.

[15] G. D. Forney, M. D. Trott, and S.-Y. Chung, "Sphere-bound-achieving coset codes and multilevel coset codes," *IEEE Transactions on Information Theory*, vol. 46, pp. 820–850, May 2000.