The Capacity Region of the Wireless Ergodic Fading Interference Channel with Partial CSIT to Within One Bit

Reza K. Farsani
School of Cognitive Sciences, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
Email: reza_khosravi@alum.sharif.ir

***Abstract*—Capacity limits are studied for the two-user wireless ergodic fading Interference Channel (IC) with partial Channel State Information at the Transmitters (CSIT) where each transmitter is equipped with an arbitrary deterministic function of the channel state (this model yields a full control over how much state information is available). One of the main challenges in the analysis of fading networks, specifically multi-receiver networks including fading ICs, is to obtain efficient capacity outer bounds. In this paper, a novel capacity outer bound is established for the two-user ergodic fading IC. Besides being well-described, our outer bound is efficient from several aspects. Specifically, it is optimal for the fading IC with uniformly strong interference. Also, it is sum-rate optimal for the channel with uniformly mixed interference. More importantly, it is proved that when each transmitter has access to any amount of CSIT that includes the interference to noise ratio at its non-corresponding receiver, the outer bound differs by no more than one bit from the achievable rate region given by Han-Kobayashi scheme. This result is viewed as a generalization of the Etkin-Tse-Wang "to within one bit" capacity result for the static channel to the time-varying wireless fading case.**

I. INTRODUCTION

Two fundamental characteristics of any multi-user wireless communication network are the interference effect due to concurrent communications from distributed users and the time-varying nature due to the mobility of users. The Gaussian fading Interference Channel (IC) is a basic information theoretic model that simultaneously includes both these characteristics. It is indeed a useful model for many practical applications including ad-hoc wireless networks [1]. In recent years, many papers have been devoted to study capacity bounds for interference channels (see [2] and literature therein for previous and recent results). Nonetheless, even for the simple two-user IC the capacity region is still unknown in general. In 2008, a significant progress was made by Etkin, Tse and Wang (ETW) [3] that characterize the capacity region of the two-user Gaussian static (time-invariant) IC to within one bit. Following up this paper, approximate capacity results were also derived for the Gaussian IC with various types of user cooperation [4-6]. However, despite its great importance from practical viewpoints, there exist very few results for wireless fading ICs. The two-user ergodic fading IC with perfect state information at all terminals is considered in [1] and [7, 8]. In [1], simple capacity inner and outer bounds are analyzed and the corresponding power allocation polices are discussed. In [7], the capacity region for the channel with uniformly strong interference and also the sum-rate capacity for the channel with uniformly mixed interference are established. The ergodic Z-interference channel with no CSIT is studied in [9]. Outage analysis of the two-user Gaussian fading IC is given in [10]. Fading ICs with more than two users have been also studied in literature, however, mostly relying on interference alignment techniques [11-12] and with the purpose of understanding the behavior of capacity in high signal to noise regime.

In this paper, we study capacity limits for the two-user wireless ergodic fading IC with partial Channel State Information at the Transmitters (CSIT). Here, the partial side information at each transmitter is given by a deterministic function (potentially discrete-valued) of channel state. This model for CSIT yields a full control over how much state information is available at a given transmitter. Under such a general model for CSIT, we develop capacity bounds for the channel. A capacity inner bound is directly given based on the Han-Kobayashi (HK) message splitting scheme [13]. One of the main challenges in the analysis of fading networks, specifically multi-receiver networks including fading ICs, is to establish efficient capacity outer bounds. For the Gaussian static (non-fading) IC, several outer bounds have been developed in recent years [3, 14-19]. However, the fact is that the approach of the latter papers is not useful for the Gaussian time-varying fading IC for two reasons. Firstly, each of the outer bounds of [3, 14-19] is established by making the assumption that the static channel is in the weak or the mixed interference regime (for the strong interference regime the capacity region is known) while for the fading case, channel gains vary by time and therefore during the time the channel may alter among weak, mixed, or strong interference regimes. Secondly, in the derivation of the outer bounds of [3, 14-19], mutual information functions including vector random variables are directly evaluated using tools such as worst case additive noise lemma [20] or an extremal inequality given in [21]. A direct application of such techniques for the fading channel, in which the channel coefficients vary by time and the inputs depend on CSITs, is either impossible or involved in too complex computations. In fact, even for the Gaussian static channel with fixed gains, the outer bounds derived in [14-19] have very complex characterizations.

In this paper, we present a novel approach to establish an efficient outer bound for the Gaussian fading IC. In our approach, neither the worst case additive noise lemma [20] nor the extremal inequality [21] is used. Instead, by a subtle combination of broadcast channel techniques (i.e., manipulating mutual information functions composed of vector random variables by Csiszar-Korner identity) and genie-aided techniques, first we derive a single-letter outer bound characterized by mutual information functions including some auxiliary random variables. Then, by novel arguments the derived bound is optimized over its auxiliaries only using the entropy power inequality. In fact, for this latter step, we follow the approach developed in our concurrent paper [22] to evaluate the so-called UV-outer bound for the two-user fading broadcast channel. Besides being well-described, we demonstrate that our outer bound (which holds for the fading IC with any arbitrary amount of CSIT) is efficient from several aspects. Specifically, it is optimal for the fading channel with uniformly strong interference. Also, it is sum-rate optimal for the channel with uniformly mixed interference. More importantly, it is proved that when each transmitter has access to any amount of CSIT that includes the interference to noise ratio of its non-corresponding receiver, the outer bound differs by no more than one bit from the achievable rate region given by Han-Kobayashi (HK) scheme. This result is viewed as a generalization of the ETW "to within one bit" capacity result for the static channel to the wireless ergodic fading case.

In the following section, we present preliminaries and channel model definitions. Our main results are given in Section III.

II. PRELIMINARIES AND DEFINITIONS

In this paper, we use the following notations: $\mathbb{E}[.]$ indicates the expectation operator. The set of real numbers, nonnegative real numbers, and complex numbers are denoted by $\mathbb{R}, \mathbb{R}_+$, and $\mathbb{C}$, respectively. Given a statement F, the indicator function $\mathbb{1}(F)$ is equal to one if F is true and zero otherwise. Finally, the function $\psi(x)$ is defined as: $\psi(x) \equiv \log(1+x)$, for $x \in \mathbb{R}_+$.

The two-user Gaussian fading IC is described by the following:

$$\begin{cases} Y_{1,t} = S_{11,t}X_{1,t} + S_{12,t}X_{2,t} + Z_{1,t} \\ Y_{2,t} = S_{21,t}X_{1,t} + S_{22,t}X_{2,t} + Z_{2,t} \end{cases}, \quad t \geq 1 \tag{1}$$

The sequences $\{X_{1,t}\}_{t\geq 1}$ and $\{X_{2,t}\}_{t\geq 1}$ represent complex-valued transmitted signals by the transmitters, and $\{Y_{1,t}\}_{t\geq 1}$ and $\{Y_{2,t}\}_{t\geq 1}$ represent the received signals at the receivers. The sequences $\{Z_{1,t}\}_{t\geq 1}$ and $\{Z_{2,t}\}_{t\geq 1}$ denote additive noises each of which is an i.i.d. (complex) Gaussian random process with zero mean and unit variance. The state process of the channel is denoted by $\{\boldsymbol{S}_t = (S_{11,t}, S_{12,t}, S_{21,t}, S_{22,t})\}_{t\geq 1}$ where the components $S_{11,t}, S_{12,t}, S_{21,t}$, and $S_{22,t}$ are complex-valued fading coefficients at the time instant t. We assume that the state process of the channel is a stationary and ergodic random which varies in time according to any arbitrary (known) probability distribution. The components of the state $\boldsymbol{S}_t = (S_{11,t}, S_{12,t}, S_{21,t}, S_{22,t}), t \geq 1$ are potentially correlated. We consider a scenario wherein both receivers perfectly know the state information while the transmitters have access to it partially. The partial side information at each transmitter is given by a deterministic function (potentially discrete-valued) of the channel state. Clearly, consider the following two deterministic functions:

$$\xi_i(.): \mathbb{C}^4 \to \mathcal{E}_i, \qquad i = 1,2$$

where $\mathcal{E}_i$ is an arbitrary (potentially finite) set. At each time instant $t, t \geq 1$, the transmitter $X_i, i = 1,2$ is informed of $E_{i,t} = \xi_i(\boldsymbol{S}_t)$ where $\boldsymbol{S}_t$ is the current state of the channel. The transmitters are subject to power constraints: $\mathbb{E}[|X_i|^2] \leq P_i, i = 1,2$. The encoding and decoding schemes for the channel are given in details in [24]. By these preliminaries, we are ready to state our main results.

III. Main Results

We begin by presenting a capacity inner bound for the channel. Let remark that in the following analysis, the random variable $\boldsymbol{S} \triangleq (S_{11}, S_{12}, S_{21}, S_{22}) \in \mathbb{C}^4$ with a given known distribution $P_{\boldsymbol{S}}(\boldsymbol{s})$ represents the channel state. Also, $E_1 = \xi_1(\boldsymbol{S})$ and $E_2 = \xi_2(\boldsymbol{S})$ represent the partial side information at the first and the second transmitters, respectively.

Proposition 1) *Define the rate region $\mathfrak{R}_i^{GFIC}$ as follows:*

$$\mathfrak{R}_i^{GFIC} \triangleq \bigcup_{\substack{\alpha(.):\mathcal{E}_1\to[0,1] \\ \beta(.):\mathcal{E}_2\to[0,1] \\ \varphi_i(.):\mathcal{E}_i\to\mathbb{R}_+, i=1,2 \\ \mathbb{E}[\varphi_i(E_i)]\leq P_i, i=1,2}} \left\{\begin{array}{c} (R_1,R_2)\in\mathbb{R}_+^2 : \\ R_1 \leq \Lambda_1 \\ R_2 \leq \Lambda_2 \\ R_1+R_2 \leq \Lambda_3+\Lambda_4 \\ R_1+R_2 \leq \Lambda_5+\Lambda_6 \\ R_1+R_2 \leq \Lambda_7+\Lambda_8 \\ 2R_1+R_2 \leq \Lambda_3+\Lambda_6+\Lambda_8 \\ R_1+2R_2 \leq \Lambda_4+\Lambda_5+\Lambda_7 \end{array}\right\} \tag{2}$$

where

$$\begin{aligned}
\Lambda_1 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{11}|^2\varphi_1(E_1)}{|S_{12}|^2\varphi_2(E_2)\beta(E_2)+1}\right)\right] \\
\Lambda_2 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{22}|^2\varphi_2(E_2)}{|S_{21}|^2\varphi_1(E_1)\alpha(E_1)+1}\right)\right] \\
\Lambda_3 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{11}|^2\varphi_1(E_1)\alpha(E_1)}{|S_{12}|^2\varphi_2(E_2)\beta(E_2)+1}\right)\right] \\
\Lambda_4 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{21}|^2\varphi_1(E_1)\big(1-\alpha(E_1)\big)+|S_{22}|^2\varphi_2(E_2)}{|S_{21}|^2\varphi_1(E_1)\alpha(E_1)+1}\right)\right] \\
\Lambda_5 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{22}|^2\varphi_2(E_2)\beta(E_2)}{|S_{21}|^2\varphi_1(E_1)\alpha(E_1)+1}\right)\right] \\
\Lambda_6 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{11}|^2\varphi_1(E_1)+|S_{12}|^2\varphi_2(E_2)\big(1-\beta(E_2)\big)}{|S_{12}|^2\varphi_2(E_2)\beta(E_2)+1}\right)\right] \\
\Lambda_7 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{11}|^2\varphi_1(E_1)\alpha(E_1)+|S_{12}|^2\varphi_2(E_2)\big(1-\beta(E_2)\big)}{|S_{12}|^2\varphi_2(E_2)\beta(E_2)+1}\right)\right] \\
\Lambda_8 &\triangleq \mathbb{E}\left[\psi\left(\frac{|S_{21}|^2\varphi_1(E_1)\big(1-\alpha(E_1)\big)+|S_{22}|^2\varphi_2(E_2)\beta(E_2)}{|S_{21}|^2\varphi_1(E_1)\alpha(E_1)+1}\right)\right]
\end{aligned} \tag{3}$$

The set $\mathfrak{R}_i^{GFIC}$ constitutes an inner bound on the capacity region of the two-user Gaussian fading IC in (1).

This achievable rate region is directly derived based on the HK message splitting scheme where each transmitter splits its message into two parts: a common part and a private part. The common parts of the messages are decoded at both receivers while the private parts are decoded only at their respective receivers (the receivers apply joint decoding technique). In the signaling scheme corresponding to the rate region (2), the first transmitter allocates the power $\varphi_1(E_1)\big(1-\alpha(E_1)\big)$ for transmission of the common message and the power $\varphi_1(E_1)\alpha(E_1)$ for transmission of the private message. Similarly, the second transmitter allocates the power $\varphi_2(E_2)\big(1-\beta(E_2)\big)$ for transmission of the common message and the power $\varphi_2(E_2)\beta(E_2)$ for transmission of the private message. A detailed discussion can be found in [24, Prop. 1]. It should be mentioned that in the special case of $E_1 \equiv E_2 \equiv \boldsymbol{S}$, i.e., perfect CSIT, the achievable rate region (2) is also given in [8].

As mentioned in introduction, one of the main contributions of this paper is to establish a novel outer bound on the capacity region of the channel. We derive our result in two steps. In the first step, we establish a single letter outer bound with constraints given by mutual information functions including some auxiliaries. A key feature in the derivation of this bound is novel applications of the Csiszar-Korner identity. This result is presented in the following Lemma .

Lemma) *Define the rate region $\mathfrak{R}_o^{UV\to GFIC}$ as given in* (4)*. The set $\mathfrak{R}_o^{UV\to GFIC}$ constitutes an outer bound on the capacity region of the two-user Gaussian fading IC in* (1).

$$\mathfrak{R}_o^{UV\to GFIC} \triangleq \bigcup_{\substack{P_Q P_{X_1|E_1Q} P_{X_2|E_2Q} \\ \times P_{UV|X_1X_2SQ}, \\ \mathbb{E}[|X_i|^2]\leq P_i, i=1,2}} \left\{\begin{array}{l} (R_1,R_2)\in\mathbb{R}_+^2 : \\ G_1 \triangleq S_{21}X_1+Z_2, \quad G_2 \triangleq S_{12}X_2+Z_1 \\ R_1 \leq \min\left\{\begin{array}{l} I(X_1;Y_1|X_2,\boldsymbol{S},Q), \\ I(U,X_1;Y_1|\boldsymbol{S},Q) \end{array}\right\}, \\ R_2 \leq \min\left\{\begin{array}{l} I(X_2;Y_2|X_1,\boldsymbol{S},Q), \\ I(V,X_2;Y_2|\boldsymbol{S},Q) \end{array}\right\}, \\ R_1+R_2 \leq I(X_1;Y_1|V,X_2,\boldsymbol{S},Q)+I(V,X_2;Y_2|\boldsymbol{S},Q) \\ R_1+R_2 \leq I(X_2;Y_2|U,X_1,\boldsymbol{S},Q)+I(U,X_1;Y_1|\boldsymbol{S},Q) \\ R_1+R_2 \leq I(X_1,X_2;Y_1|G_1,\boldsymbol{S},Q)+I(X_1,X_2;Y_2|G_2,\boldsymbol{S},Q), \\ 2R_1+R_2 \leq I(X_1;Y_1|V,X_2,\boldsymbol{S},Q)+I(V,X_2;Y_2|G_2,\boldsymbol{S},Q) \\ \qquad\qquad +I(X_1,X_2;Y_1|\boldsymbol{S},Q) \\ R_1+2R_2 \leq I(X_2;Y_2|U,X_1,\boldsymbol{S},Q)+I(U,X_1;Y_1|G_1,\boldsymbol{S},Q) \\ \qquad\qquad +I(X_1,X_2;Y_2|\boldsymbol{S},Q) \end{array}\right\} \tag{4}$$

$$
\begin{aligned}
n(R_1+2R_2) &\le I(M_2;Y_2^n,\boldsymbol{S}^n)+I(M_2;Y_2^n,M_1,\boldsymbol{S}^n)+I(M_1;Y_1^n,G_1^n,\boldsymbol{S}^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&= I(X_2^n;Y_2^n|\boldsymbol{S}^n)+I(M_2;Y_2^n|M_1,\boldsymbol{S}^n)+I(M_1;Y_1^n|G_1^n,\boldsymbol{S}^n)+I(X_1^n;G_1^n|\boldsymbol{S}^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&= H(Y_2^n|\boldsymbol{S}^n)-H(Y_2^n|X_2^n,\boldsymbol{S}^n)+I(M_2;Y_2^n|M_1,\boldsymbol{S}^n)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Y_1^n|M_1,\boldsymbol{S}^n)+H(G_1^n|\boldsymbol{S}^n)-H(Z_2^n)\\
&\qquad +n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&\overset{(a)}{=} H(Y_2^n|\boldsymbol{S}^n)+I(M_2;Y_2^n|M_1,\boldsymbol{S}^n)+I(M_1;Y_1^n|\boldsymbol{S}^n)-H(Y_1^n|\boldsymbol{S}^n)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Z_2^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&\overset{(b)}{\le} H(Y_2^n|\boldsymbol{S}^n)+\textstyle\sum_{t=1}^n I\big(M_2;Y_{2,t}\big|Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^n\big)+\sum_{t=1}^n I\big(Y_2^{t-1},M_1;Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)\\
&\qquad -H(Y_1^n|\boldsymbol{S}^n)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Z_2^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&= H(Y_2^n|\boldsymbol{S}^n)+\textstyle\sum_{t=1}^n I\big(M_2;Y_{2,t}\big|Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^{t-1},\boldsymbol{S}_{t+1}^n,\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^{t-1},\boldsymbol{S}_{t+1}^n;Y_{1,t}\big|\boldsymbol{S}_t\big)\\
&\qquad -\textstyle\sum_{t=1}^n I\big(Y_{1,t+1}^n,\boldsymbol{S}^{t-1},\boldsymbol{S}_{t+1}^n;Y_{1,t}\big|\boldsymbol{S}_t\big)-H(Y_1^n|\boldsymbol{S}^n)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Z_2^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&= H(Y_2^n|\boldsymbol{S}^n)+\textstyle\sum_{t=1}^n I\big(X_{2,t};Y_{2,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(U_t,X_{1,t};Y_{1,t}\big|\boldsymbol{S}_t\big)\\
&\qquad -\textstyle\sum_{t=1}^n H\big(Y_{1,t}\big|\boldsymbol{S}_t\big)+\sum_{t=1}^n H\big(Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)-H(Y_1^n|\boldsymbol{S}^n)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Z_2^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&= H(Y_2^n|\boldsymbol{S}^n)+\textstyle\sum_{t=1}^n I\big(X_{2,t};Y_{2,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(U_t,X_{1,t};Y_{1,t}\big|\boldsymbol{S}_t\big)-\sum_{t=1}^n H\big(Y_{1,t}\big|\boldsymbol{S}_t\big)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)-H(Z_2^n)\\
&\qquad +n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&\overset{(c)}{=} \textstyle\sum_{t=1}^n I\big(X_{2,t};Y_{2,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)-\sum_{t=1}^n H\big(Y_{1,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)+H(Y_1^n|G_1^n,\boldsymbol{S}^n)+I(X_1^n,X_2^n;Y_2^n|\boldsymbol{S}^n)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&\le \textstyle\sum_{t=1}^n I\big(X_{2,t};Y_{2,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)-\sum_{t=1}^n H\big(Y_{1,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n H\big(Y_{1,t}\big|G_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(X_{1,t},X_{2,t};Y_{2,t}\big|\boldsymbol{S}_t\big)\\
&\qquad +n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)\\
&\overset{(d)}{=} \textstyle\sum_{t=1}^n I\big(X_{2,t};Y_{2,t}\big|U_t,X_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(U_t,X_{1,t};Y_{1,t}\big|G_{1,t},\boldsymbol{S}_t\big)+\sum_{t=1}^n I\big(X_{1,t},X_{2,t};Y_{2,t}\big|\boldsymbol{S}_t\big)+n\big(\epsilon_{1,n}+2\epsilon_{2,n}\big)
\end{aligned}
\tag{5}
$$

Proof of Lemma) Consider a length-n code with the rate (R_1,R_2) and vanishing average error probability for the channel. Based on the Fano's inequality we have:

$$
H(M_1|Y_1^n,\boldsymbol{S}^n)\le n\epsilon_{1,n}
$$
$$
H(M_1|Y_2^n,\boldsymbol{S}^n)\le n\epsilon_{2,n}
$$

where $\epsilon_{1,n},\epsilon_{2,n}\to 0$ as $n\to\infty$. Define new auxiliary random variables U_t and V_t as follows:

$$
\begin{cases} U_t \triangleq \big(Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^{t-1},\boldsymbol{S}_{t+1}^n\big)\\ V_t \triangleq \big(Y_{1,t+1}^n,Y_2^{t-1},M_2,\boldsymbol{S}^{t-1},\boldsymbol{S}_{t+1}^n\big)\end{cases},\qquad t=1,\dots,n \tag{6}
$$

Note that $X_{1,t}$ and $X_{2,t}$ are given by deterministic functions of $\big(U_t,E_{1,t}=\xi_1(\boldsymbol{S}_t)\big)$ and $\big(V_t,E_{2,t}=\xi_2(\boldsymbol{S}_t)\big)$, respectively. Moreover, define the genie signals $G_{1,t}$ and $G_{2,t}$ as:

$$
\begin{cases} G_{1,t}\triangleq S_{21,t}X_{1,t}+Z_{2,t}\\ G_{2,t}\triangleq S_{12,t}X_{2,t}+Z_{1,t}\end{cases},\qquad t=1,\dots,n \tag{7}
$$

Let remark that the first four constraints of (4) are in fact adapted from the UV-outer bound for the discrete IC that is established in our recent paper [2, Part I, Sec. III. B]. Also, the fifth constraint of (4) is actually the sum-rate capacity of a genie-aided channel with G_1^n and G_2^n given to the first and the second receivers, respectively. This constraint can be represented in the following form, as well:

$$
R_1+R_2\le I(X_1;Y_1,G_1|\boldsymbol{S},Q)+I(X_2;Y_2,G_2|\boldsymbol{S},Q)
$$

In this conference version of the paper, due to the limited space, we only present the derivation of the novel constraint on the linear combination R_1+2R_2, as given in (5). The constraint on $2R_1+R_2$ can also be derived similarly. In equations (5), the equality (a) holds because $H(Y_2^n|X_2^n,\boldsymbol{S}^n)=H(G_1^n|\boldsymbol{S}^n)$; inequality (b) is due to the following:

$$
\begin{aligned}
&I(M_2;Y_2^n|M_1,\boldsymbol{S}^n)+I(M_1;Y_1^n|\boldsymbol{S}^n)\\
&\quad= \textstyle\sum_{t=1}^n I\big(M_2;Y_{2,t}\big|Y_2^{t-1},M_1,\boldsymbol{S}^n\big)+\sum_{t=1}^n I\big(M_1;Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)\\
&\quad\le \textstyle\sum_{t=1}^n I\big(M_2,Y_{1,t+1}^n;Y_{2,t}\big|Y_2^{t-1},M_1,\boldsymbol{S}^n\big)\\
&\qquad +\textstyle\sum_{t=1}^n I\big(Y_2^{t-1},M_1;Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)\\
&\qquad -\textstyle\sum_{t=1}^n I\big(Y_2^{t-1};Y_{1,t}\big|Y_{1,t+1}^n,M_1,\boldsymbol{S}^n\big)\\
&\quad= \textstyle\sum_{t=1}^n I\big(M_2;Y_{2,t}\big|Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^n\big)\\
&\qquad +\textstyle\sum_{t=1}^n I\big(Y_2^{t-1},M_1;Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)\\
&\qquad +\textstyle\sum_{t=1}^n I\big(Y_{1,t+1}^n;Y_{2,t}\big|Y_2^{t-1},M_1,\boldsymbol{S}^n\big)\\
&\qquad -\textstyle\sum_{t=1}^n I\big(Y_2^{t-1};Y_{1,t}\big|Y_{1,t+1}^n,M_1,\boldsymbol{S}^n\big)\\
&\quad= \textstyle\sum_{t=1}^n I\big(M_2;Y_{2,t}\big|Y_{1,t+1}^n,Y_2^{t-1},M_1,\boldsymbol{S}^n\big)\\
&\qquad +\textstyle\sum_{t=1}^n I\big(Y_2^{t-1},M_1;Y_{1,t}\big|Y_{1,t+1}^n,\boldsymbol{S}^n\big)
\end{aligned}
$$

where the last step is derived by Csiszar-Korner identity; equality (c) holds because $H(Y_2^n|X_1^n,X_2^n,\boldsymbol{S}^n)=H(Z_2^n)$, and equability (d) holds because given $\big(X_{1,t},\boldsymbol{S}_t\big)$, the genie signal $G_{1,t}$ is reduced to $Z_{2,t}$ which is independent of $U_t,X_{1,t},Y_{1,t}$, and $\boldsymbol{S}_t$. By applying a standard time-sharing argument to the last expression in (5), we derive the desired bound given in (4). Please refer to [24, Lemma 1] for a detailed proof. ■

Remarks:

1. Consider the novel constraints given in (4) on the linear combinations $2R_1+R_2$ and R_1+2R_2. These constraints are characterized based on both the genie signals and the auxiliaries U and V. The inclusion of the auxiliaries U and V into these constraints (that is derived by a subtle application of the Csiszar-Korner identity) puts them in correlation with the constraints on the sum-rate and the individual rates. As shown in [24], this new technique yields tighter outer bounds than the arguments based on the worst case noise lemma [20].
2. As we see later, the outer bound (4) simultaneously includes several benefits: 1) It is optimal for the channel with strong interference, 2) It is sum-rate optimal for the channel with mixed interference, 3) It is to within one bit of the capacity region when each transmitter has access to any amount of CSIT that includes the interference to noise ratio of its non-

corresponding receiver, 4) It has a single-letter characterization which is not as complex as the ones derived in [14-19]. Moreover, in [24] we analytically show that for the Gaussian IC with fixed gains (nonfading case), it is strictly tighter than both Kramer's outer bounds [23] and ETW outer bounds [3] for all range of channel parameters where the capacity is unknown. In general, the characterization (4) exhibits an efficient combination of the broadcast channel techniques (i.e., manipulating mutual information functions composed of vector RVs by Csiszar-Korner identity) and the genie-aided techniques for establishing capacity outer bounds.

In the second step, the constraints of the region (4) are evaluated to derive a bound with an explicit characterization. A remarkable point is that we only make use of the EPI for this purpose. In fact, we essentially follow a similar approach to the one developed in our concurrent paper [22] for evaluating the UV-outer bound for the two-user fading broadcast channel. Unfortunately, due to the lack of space, here we are not able to present our derivations for this step. We encourage the reader to see [24, Th. 1] where the explicit characterization of the outer bound corresponding to the region (4) is also given.

Let reconsider the outer bound given in (4). In [24], we show that if one relax the following constraints from (4):

$$\begin{cases} R_1 \leq I(U, X_1; Y_1|\boldsymbol{S}, Q) \\ R_2 \leq I(V, X_2; Y_2|\boldsymbol{S}, Q) \end{cases} \tag{8}$$

then, the resultant region can be evaluated with a more simple characterization. This result is formulated in the following Theorem.

Theorem 1) *Define the rate region* $\mathfrak{R}_o^{GFIC}$ *as given in* (10). *The set* $\mathfrak{R}_o^{GFIC}$ *constitutes an outer bound on the capacity region of the two-user Gaussian fading IC in* (1).

The fact is that the outer bound derived by evaluating the region (4) is strictly tighter than $\mathfrak{R}_o^{GFIC}$ in (10); however, it is characterized in terms of some additional ingredients (see [24, Th. 1]). If the constraints (8) are relaxed, these additional ingredients are removed and the corresponding bound is given by (10) which is sufficient for the purposes of this paper.

The outer bound $\mathfrak{R}_o^{GFIC}$ in (10) is uniformly applicable for all channels with arbitrary fading statistics and arbitrary amount of state information at the transmitters, specifically, for the channel with no CSIT. Here, we remark that an outer bound was also reported in [1] for the Gaussian fading IC with perfect CSIT. Note that the outer bound of [1] does not include any constraint on $2R_1 + R_2$ and $R_1 + 2R_2$. Even without considering these constraints, by a simple comparison (equation by equation) one can analytically show that our outer bound in (10) is strictly tighter than that of [1].

We next prove some important results for the channel using the outer bound $\mathfrak{R}_o^{GFIC}$ in (10). First we show that for the case where each transmitter has access to the interference to noise ratio perceived at its non-corresponding receiver, this outer bound differs by no more than one bit from the capacity region. This result is given in the following theorem.

Theorem 2) *Consider the two-user Gaussian fading IC* (1) *with partial side information* E_1 *and* E_2 *at the transmitters* X_1 *and* X_2, *respectively. Assume that* $E_1 \equiv (|S_{21}|, E_1^*)$ *and* $E_2 \equiv (|S_{12}|, E_2^*)$ *where* E_1^* *and* E_2^* *are given by arbitrary deterministic functions of the channel state* $\boldsymbol{S}$. *If* (R_1, R_2) *belongs to the outer bound* $\mathfrak{R}_o^{GFIC}$ *in* (10), *then* $(R_1 - 1, R_2 - 1)$ *is achievable.*

Proof of Theorem 2) Fix two deterministic functions $\varphi_1(.)$ and $\varphi_2(.)$ with $\mathbb{E}[\varphi_i(E_i)] \leq P_i, i = 1,2$. Let $\mathfrak{R}_o^{GFIC}(\varphi_1(E_1), \varphi_2(E_2))$ denotes a subset of $\mathfrak{R}_o^{GFIC}$ that is given by equation (10) for the fixed functions $\varphi_1(.)$ and $\varphi_2(.)$. Also, let $\mathfrak{R}_i^{GFIC}(\varphi_1(E_1), \varphi_2(E_2))$ be a subset of $\mathfrak{R}_i^{GFIC}$ in (2) that is determined by the power allocation policies $\varphi_1(.)$ and $\varphi_2(.)$ and the following assignments:

$$\begin{aligned} \alpha(E_1) = \alpha(|S_{21}|, E_1^*) &\triangleq \min\left\{1, \frac{1}{|S_{21}|^2\varphi_1(E_1)}\right\} \\ \beta(E_2) = \beta(|S_{12}|, E_2^*) &\triangleq \min\left\{1, \frac{1}{|S_{12}|^2\varphi_2(E_2)}\right\} \end{aligned} \tag{9}$$

We show that if (R_1, R_2) belongs to $\mathfrak{R}_o^{GFIC}(\varphi_1(E_1), \varphi_2(E_2))$, then $(R_1 - 1, R_2 - 1)$ belongs to $\mathfrak{R}_i^{GFIC}(\varphi_1(E_1), \varphi_2(E_2))$. This can be proved by an equation-by-equation comparison of the latter regions. To see the derivations, please refer to [24, Th. 2]. ■

$$\mathfrak{R}_o^{GFIC} \triangleq \bigcup_{\substack{\varphi_1(.), \varphi_2(.) \\ \mathbb{E}[\varphi_i(E_i)] \leq P_i, i=1,2}} \left\{ \begin{array}{l} (R_1, R_2) \in \mathbb{R}_+^2 : \\ R_1 \leq \mathbb{E}[\psi(|S_{11}|^2\varphi_1(E_1))] \\ R_2 \leq \mathbb{E}[\psi(|S_{22}|^2\varphi_2(E_2))] \\ R_1 + R_2 \leq \mathbb{E}\left[\left(\psi(|S_{11}|^2\varphi_1(E_1)) + \psi\left(\frac{|S_{22}|^2\varphi_2(E_2)}{|S_{21}|^2\varphi_1(E_1) + 1}\right)\right)\mathbb{1}(|S_{21}| < |S_{11}|)\right] \\ \qquad\qquad + \mathbb{E}[\psi(|S_{21}|^2\varphi_1(E_1) + |S_{22}|^2\varphi_2(E_2))\mathbb{1}(|S_{21}| \geq |S_{11}|)] \\ \qquad\qquad , \\ R_1 + R_2 \leq \mathbb{E}\left[\left(\psi(|S_{22}|^2\varphi_2(E_2)) + \psi\left(\frac{|S_{11}|^2\varphi_1(E_1)}{|S_{12}|^2\varphi_2(E_2) + 1}\right)\right)\mathbb{1}(|S_{12}| < |S_{22}|)\right] \\ \qquad\qquad + \mathbb{E}[\psi(|S_{11}|^2\varphi_1(E_1) + |S_{12}|^2\varphi_2(E_2))\mathbb{1}(|S_{12}| \geq |S_{22}|)] \\ \qquad\qquad , \\ R_1 + R_2 \leq \mathbb{E}\left[\psi\left(|S_{12}|^2\varphi_2(E_2) + \frac{|S_{11}|^2\varphi_1(E_1)}{|S_{21}|^2\varphi_1(E_1) + 1}\right)\right] + \mathbb{E}\left[\psi\left(|S_{21}|^2\varphi_1(E_1) + \frac{|S_{22}|^2\varphi_2(E_2)}{|S_{12}|^2\varphi_2(E_2) + 1}\right)\right] \\ \qquad\qquad , \\ 2R_1 + R_2 \leq \mathbb{E}\left[\left(\psi(|S_{11}|^2\varphi_1(E_1)) - \psi(|S_{21}|^2\varphi_1(E_1))\right)\mathbb{1}(|S_{21}| < |S_{11}|)\right] \\ \qquad\qquad + \mathbb{E}\left[\psi\left(|S_{21}|^2\varphi_1(E_1) + \frac{|S_{22}|^2\varphi_2(E_2)}{|S_{12}|^2\varphi_2(E_2) + 1}\right)\right] + \mathbb{E}[\psi(|S_{11}|^2\varphi_1(E_1) + |S_{12}|^2\varphi_2(E_2))] \\ R_1 + 2R_2 \leq \mathbb{E}\left[\left(\psi(|S_{22}|^2\varphi_2(E_2)) - \psi(|S_{12}|^2\varphi_2(E_2))\right)\mathbb{1}(|S_{12}| < |S_{22}|)\right] \\ \qquad\qquad + \mathbb{E}\left[\psi\left(|S_{12}|^2\varphi_2(E_2) + \frac{|S_{11}|^2\varphi_1(E_1)}{|S_{21}|^2\varphi_1(E_1) + 1}\right)\right] + \mathbb{E}[\psi(|S_{21}|^2\varphi_1(E_1) + |S_{22}|^2\varphi_2(E_2))] \end{array} \right\} \tag{10}$$

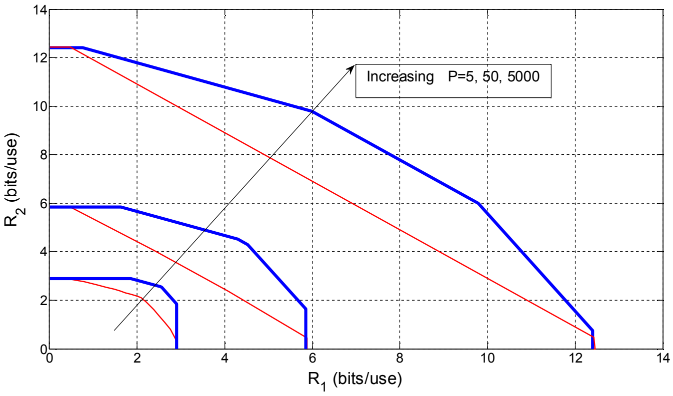

Figure 1. Comparison between the inner bound $\mathfrak{R}_i^{GFIC}$ in (2) and the outer bound $\mathfrak{R}_o^{GFIC}$ in (10) for a Rayleigh fading IC with no CSIT.

Remark: The result of Theorem 2 holds for arbitrary stationary and ergodic fading statistics. Also, from the viewpoint of CSIT quality, this theorem holds for a broad range of possible scenarios because the only requirement is that the first and the second transmitters have access to $|S_{21}|$ and $|S_{12}|$, respectively. It is indeed interesting that to achieve this "to within one bit" capacity result, we need not to impose the restrictive assumption of prefect CSIT (although the result clearly holds for this case, as well).

Then, we present some special cases for which the outer bound $\mathfrak{R}_o^{GFIC}$ in (10) is optimal. Specifically, we consider the class of *uniformly strong* as well as *uniformly mixed* Gaussian fading ICs studied in the recent work [7].

Definition: *The two-user Gaussian fading IC* (1) *is said to have uniformly strong interference provided that:*

$$Pr\{|S_{11}| > |S_{21}|\} = Pr\{|S_{22}| > |S_{12}|\} = 0 \tag{11}$$

Definition: *The two-user Gaussian IC* (1) *is said to have uniformly mixed interference provided that:*

$$Pr\{|S_{11}| < |S_{21}|\} = Pr\{|S_{22}| > |S_{12}|\} = 0 \tag{12}$$

The authors in [7] established the capacity region of the fading IC with uniformly strong interference (11) and also the sum-rate capacity of the channel with uniformly mixed interference (12) for the case where both transmitters have access to perfect state information. In the following, using the outer bound $\mathfrak{R}_o^{GFIC}$ in (10), we extend the capacity results of [7] to the case with any arbitrary amount of state information at the transmitters.

Proposition 2) *Consider the two-user Gaussian fading IC* (1) *with partial CSIT.*

I. The outer bound $\mathfrak{R}_o^{GFIC}$ *in* (10) *is optimal for the channel with uniformly strong interference* (11).

II. The outer bound $\mathfrak{R}_o^{GFIC}$ *in* (10) *is sum-rate optimal for the channel with uniformly mixed interference* (12).

Proof of Proposition 2) For the channel with uniformly strong interference (11), the first four constraints of the outer bound $\mathfrak{R}_o^{GFIC}$ in (10) yield the capacity region. Also, for the channel with uniformly mixed interference (12), the third and the fourth constraints of this outer bound yield the sum-rate capacity. The detailed proof can be found in [24, Prop. 2]. ■

Finally, we provide some numerical examples to compare the derived bounds. Consider the channel with no CSIT, i.e., $E_1 \equiv E_2 \equiv \emptyset$. Assume that $|S_{11}|, |S_{12}|, |S_{21}|$, and $|S_{22}|$ are independent *Rayleigh-distributed* random variables with the parameters $1, .15, .15,$ and 1, respectively. Also, assume that the transmitters are subject to equal power constraints: $P_1 = P_2 = P$. Figure 1 compares the inner bound $\mathfrak{R}_i^{GFIC}$ in (2) and the outer bound $\mathfrak{R}_o^{GFIC}$ in (10) for four values of P. For each case, the red curve depicts the inner bound and the blue curve depicts the outer bound. As we see, for both small and large values of P, the gap between the two curves remains bounded. In fact, we conjecture that our outer bound (10) is within to a constant gap of the capacity region for the general case with any arbitrary amount of CSIT (specifically for the case with no CSIT). This conjecture is under investigation.

Conclusion

We established a novel capacity outer bound for the two-user fading IC with partial CSIT. For the case where each transmitter has access to any amount of CSIT that includes the interference to noise ratio at its non-corresponding receiver, our outer bound differs by no more than one bit from the capacity region.

References

[1] D. Tuninetti, "Gaussian fading interference channels: Power control," in *Proc. 42nd Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Oct. 2008, pp. 701–706.

[2] R. K. Farsani, "Fundamental limits of communications in interference networks-Parts I-V", *2012, available on ArXiv*.

[3] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, pp. 5534–5562, Dec. 2008.

[4] C. Suh and D. Tse, "Feedback capacity of the Gaussian interference channel to within two bits," *IEEE Trans. on Inf. Theory*, 57(5):2667–2685, May 2011.

[5] I-H. Wang and D. N. C. Tse, "Interference mitigation through limited transmitter cooperation," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2941–2965, May 2011.

[6] I-Hsiang Wang and D. Tse, "Interference mitigation through limited receiver cooperation," *In IEEE Trans. on Information Theory*, vol. 57(5), 2011, pp. 2913-2940.

[7] L. Sankar, X. Shang, E. Erkip, and H. V. Poor, "Ergodic fading interference channels: Sum-capacity and separability," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2605–2626, May 2011.

[8] L. Sankar, E. Erkip, and H. V. Poor, "A general coding scheme for two-user interference channels," arXiv:1001.2786, 2010.

[9] Y. Zhu and D. Guo, "Ergodic fading Z-interference channels without state information at transmitters," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2627–2647, May 2011.

[10] Y. Weng and D. Tuninetti, "Outage analysis of block-fading Gaussian interference channels," *IEEE Trans. on Inf. Theory*, vol. 57, no. 10, pp. 6487–6501, Oct. 2011.

[11] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the k-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[12] B. Nazer, M. Gastpar, S. A. Jafar, and S. Vishwanath, "Ergodic interference alignment," *IEEE Trans. Inf. Theory*, vol. 58, pp. 6355–6371, Oct. 2012.

[13] T. S. Han, K. Kobayashi, "A new achievable region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[14] A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel, *IEEE Trans. Inform. Theory*, vol. 55, no. 2, pp. 620-643, Feb. 2009.

[15] X. Shang, G. Kramer, and B. Chen, "A new outer bound and the noisy interference sum-rate capacity for the Gaussian interference channels, *IEEE Trans. Inform. Theory*, vol. 55, no. 2, pp. 689-699, Feb. 2009.

[16] V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. Inform. Theory*, vol. 55, no. 7, pp. 3032-3050, Jul. 2009.

[17] R. H. Etkin, "New sum-rate upper bound for the two-user Gaussian interference channel," in *Proc. IEEE ISIT*, Seoul, Korea, Jun. 2009.

[18] A. Chaaban and A. Sezgin, "An extended etkin-type outer bound on the capacity of the Gaussian interference channel," in *Proc. 45nd Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Nov. 2011.

[19] J. Nam and G. Caire, "A new outer bound on the capacity region of Gaussian interference channels," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, USA. 2012, pp. 2806–2810.

[20] S. Diggavi and T. M. Cover, "Worst additive noise under covariance constraints," *IEEE Trans. Inf. Theory*, vol. 47, no. 7, pp. 3072-3081, Nov. 2001.

[21] T. Liu and P. Viswanath, "An extremal inequality motivated by multiterminal information theoretic problems," *IEEE Trans. Inform. Theory*, vol. 53, no. 5, pp. 1839-1851, May 2007.

[22] R. K. Farsani, "Capacity bounds for wireless ergodic fading broadcast channels with partial CSIT," *IEEE ISIT 2013*.

[23] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 3, pp. 581–586, Mar. 2004.

[24] R. K. Farsani, "The capacity region of the wireless ergodic fading interference channel with partial CSIT to within one bit," 2013, *arXiv:*1302.1836.