Equivalence of Two Proof Techniques for Non-Shannon-type Inequalities

Tarik Kaced
Institute of Network Coding
The Chinese University of Hong Kong, Shatin, N.T.
email: tarik@inc.cuhk.edu.hk

Abstract—We compare two different techniques for proving non-Shannon-type information inequalities. The first one is the original Zhang-Yeung's method, commonly referred to as the copy/pasting lemma/trick. The copy lemma was used to derive the first conditional and unconditional non-Shannon-type inequalities. The second technique first appeared in Makarychev *et al* paper [6] and is based on a coding lemma from Ahlswede and Körner works. We first emphasize the importance of balanced inequalities and provide a simpler proof of a theorem of Chan's for the case of Shannon-type inequalities. We compare the power of various proof systems based on a single technique.

Index Terms—Information inequalities; non-Shannon-type; Balanced inequalities; proof techniques;

I. Introduction

Information inequalities are the inequalities that are valid for the Shannon entropy of random variables. They play a central role in information theory for they tell us how much can information be compressed, and are useful in many converse coding theorems. Determining all the inequalities satisfied by the joint entropy, and thus describing the so-called space of entropic vectors, has become a major challenge in information theory. Apart from evident applications in all kinds of information-theoretic problems, more fundamental connections are known to exist with matroid theory, Kolmogorov complexity, determinantal inequalities, combinatorics, or group theory.

Shannon's seminal works [10, 11] of the 1940's introduced, amid many other things, the first information inequality commonly called *the basic inequality*:

$$H(AC) + H(BC) \geq H(ABC) + H(C).$$

Which, in the language of information theory, means that the conditional mutual information $I(A{:}B|C)$ is non-negative.

Positive linear combinations of instances of the basic inequalities are called *Shannon-type* inequalities. The question of whether these Shannon-type inequalities are the only valid ones or not was raised by Pippenger [8] in 1986, yet only answered more than 10 years later. The first non-Shannon-type inequality was proven by Z. Zhang and R. W. Yeung in [17] using the copy trick. Their technique has subsequently been used to find infinite families of non-Shannon-type inequalities (see [4, 7, 13]). A few years later, a different technique was discovered by K. Makarychev, Y. Makarychev, A. Romashchenko and N. Vereshchagin (see [6]) based on results on sub-achievable entropy vectors for the entropy characterization problem (see [3,

p. 352]). This new technique proved a 5-variable generalization of the original 4-variable Zhang-Yeung inequality.

To the author's knowledge, these two techniques are the only ones known, to-date, for proving non-Shannon-type inequalities. The aim of this paper is to study and compare the power of these two techniques. The task of proving Shannon-type inequalities is known to be a LP problem and is better left to computer programs, e.g. ITIP [14] or Xitip [9]. Rules could be added to these programs for the derivation of non-Shannon-type inequalities using the two techniques we mentioned.

We show that each technique can prove the same inequalities modulo rewriting inequalities in some equivalent form. Indeed, a result of Chan's work in [1] states that every information inequality can be equivalently put in balanced form. We present an elementary proof of this result for the particular case of Shannon-type inequalities, and argue that balanced inequalities play an important role in the comparison of the two techniques.

After fixing notations, the rest of the paper is organized as follows. Section II explains Chan's balanced inequalities, Zhang-Yeung and Makarychev *et al* respective techniques are presented in Section III. Various proof systems involving the two different techniques are compared in Section IV.

A. Preliminaries

Let $\{X_i\}_{i \in \mathcal{N}}$ be a collection of random variables indexed by a set $\mathcal{N}$ of n elements. For a non-empty subset $J \subseteq \mathcal{N}$, we denote by X_J the set of random variables $\{X_j : j \in J\}$.

An unconditional linear information inequality for a set of n random variables is a linear form with $2^n - 1$ real coefficients $(c_J)_{\varnothing \neq J \subseteq \mathcal{N}}$ such that for all jointly distributed random variables $\{X_i\}_{i \in \mathcal{N}}$,

$$\sum_{\varnothing \neq J \subseteq \mathcal{N}} c_J H(X_J) \geq 0.$$

We call *Shannon-type* the inequalities of the set of all positive linear combinations of instances of the basic inequality. That is, a valid inequality that can be put in the form

$$\sum_{\substack{\varnothing \neq J \subseteq \mathcal{N} \\ \varnothing \neq K \subseteq \mathcal{N} \\ L \subseteq \mathcal{N}}} c_{J,K,L} I(X_J{:}X_K|X_L) \geq 0, \tag{1}$$

where all $c_{J,K,L}$ are non-negative.

II. Balanced Inequalities

Definition 1 (Balanced Inequalities)**.** *An n-variable information inequality is said* balanced *for variable X_i if the sum of the coefficients involving X_i is zero:*

$$\sum_{i \in J \subseteq \mathcal{N}} c_J = 0.$$

An n-variable information is simply called balanced *if it is balanced for all of its n variables.*

Given a valid linear information inequality, can one obtain a balanced counterpart that is also a valid information inequality? This question was answered in a paper of Chan's (see [1]).

Theorem 1 (Balanced Inequalities, Chan [1])**.** *Let $(c_J)_{\varnothing \neq J \subseteq \mathcal{N}}$ be a list of coefficients, the following are equivalent:*
 1) *The inequality*

$$\sum_{\varnothing \neq J \subseteq \mathcal{N}} c_J H(X_J) \geq 0$$

 is a valid information inequality.
 2) *The inequality*

$$\sum_{\varnothing \neq J \subseteq \mathcal{N}} c_J H(X_J) - \sum_{j \in \mathcal{N}} r_j H(X_j | X_{\mathcal{N}-j}) \geq 0,$$

 where r_j is the sum of all c_J involving j, is a valid balanced information inequality.

The previous result states that any information inequality can be *balanced* by subtracting the corresponding terms. Obviously, the coefficients r_j must be non-negative, hence the balanced inequality appears to be stronger.

Example 1. *The 3-variable inequality*

$$H(X_2, X_3) \geq 0$$

balances into the following inequality

$$I(X_1 : X_2 X_3) + I(X_2 : X_3 | X_1) \geq 0.$$

The original proof of Theorem 1 involves a random coding argument and Chan–Yeung's technique of entropic vectors approximation using quasi-uniform distributions (see [2]). While the original proof is quite involved, we present hereafter a simpler proof for the case of Shannon-type inequalities.

A. Balancing the Basic Inequality

For a set of n random variables $X_{\mathcal{N}}$, an instance of the basic inequality has the form:

$$I(X_J : X_K | X_L) \geq 0, \tag{2}$$

for any non-empty subsets J, K and any subset L of $\mathcal{N}$.

Proposition 1. (2) *is balanced $\Leftrightarrow J \cap K \subseteq L$*

Proof. Assume (2) is balanced and let $\alpha \in J \cap K$. Since $r_\alpha = 0$, we must have $\alpha \in L$.

Conversely, assume $J \cap K \subseteq L$. This ensures that for any $\alpha \in J \cap K$ we have $r_\alpha = 0$. Moreover, (2) is already balanced for any single variable in the set X_L, for they appear in each term of the inequality (twice with coefficient 1 and twice with coefficient -1). By a similar argument, for any $\beta \in J \setminus K$, we have $r_\beta = 0$. A symmetric remark holds for any $\beta \in K \setminus J$. Therefore, inequality (2) is balanced. $\square$

Whenever $W = J \cap K \setminus L$ is non-empty, inequality (2) rewrites to:

$$H(X_W | X_L) + I(X_{J-W} : X_{K-W} | X_{W \cup L}) \geq 0,$$

which balances into

$$I(X_W : X_{\mathcal{N}-W} | X_L) + I(X_{J-W} : X_{K-W} | X_{W \cup L}) \geq 0.$$

This inequality is the sum of two (other) instances of the basic inequality, it is thus a valid Shannon-type inequality. Thereforem we have just proven the following proposition:

Proposition 2. *Theorem 1 holds for instances of the basic inequality.*

Example 2. *For 3-variables information inequalities, the only balanced instances of the basic inequality are the following ones:*

$$I(X_1 : X_2 X_3) \geq 0, \ I(X_2 : X_1 X_3) \geq 0, \ I(X_3 : X_1 X_2) \geq 0,$$
$$I(X_1 : X_2 | X_3) \geq 0, I(X_1 : X_3 | X_2) \geq 0, I(X_2 : X_3 | X_1) \geq 0,$$
$$I(X_1 : X_2) \geq 0, \qquad I(X_1 : X_3) \geq 0, \qquad I(X_2 : X_3) \geq 0.$$

(Note that we can recover the first line from the last two.)

B. Balancing Shannon-type Inequalities

By definition, a Shannon-type inequality (of the form (1)) is simply a (weighted) sum of instances of the basic inequality. Since the balanced property is stable by sums, balancing a Shannon-type inequality is the same as balancing each of the instances of the basic inequality in (1). By Proposition 2, the balanced inequality thus obtained is valid:

Corollary 1. *Theorem 1 holds for Shannon-type information inequalities.*

Notice that the argument of Subsection II-A shows that the balanced inequality we obtain is Shannon-type.

C. Balancing General Information Inequalities

For a general (non-Shannon-type) information inequality, we should still rely on the original proof of Theorem 1, though a more direct proof is not excluded. Note, however, that most of, if not all, the known non-Shannon-type inequalities are already balanced.

Remark 1. *Checking if a given inequality is balanced and balancing an inequality have linear complexity in the length of the inputted inequality (as a sum of joint entropies).*

III. Techniques for non-Shannon-type inequalities

We describe the two main techniques for proving non-Shannon-type information inequalities.

A. Zhang-Yeung's Technique

RULE ZY

(A) Let f and g be linear maps on entropies such that

$$f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) + \alpha I(Z{:}X_\mathcal{N}|Y_\mathcal{M}) \geq 0,$$

for some $\alpha \geq 0$;

(B) then the following (stronger) inequality is also valid:

$$f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) \geq 0.$$

The correctness of this rule is based on the following lemma.

Lemma 1 (Copy lemma, [4])**.** *Let A, B, C be three jointly distributed random variables. There exists a fourth random variable A' such that:*

- *(A, B) and (A', B) have the same distribution;*
- *A' is independent of (A, C) given B.*

Such an A' is called a C-copy of A over B.

Proof of Correctness of RULE ZY*:* Take Z' to be a $X_\mathcal{N}$-copy of Z over $Y_\mathcal{M}$, and apply the inequality of step (A) for $Z = Z'$. By Lemma 1, we obtain the inequality of step (B). ∎

This technique has been extensively used to obtain constrained and unconstrained non-Shannon-type inequalities (e.g. [4, 5, 7, 13, 16, 17]). As an example, we show how to obtain the very first non-Shannon-type inequality using this rule.

Theorem 2 (Zhang and Yeung, [17])**.** *The following is a 4-variable information inequality:*

$$I(C{:}D) \leq I(C{:}D|A) + I(C{:}D|B) + I(A{:}B) + \\ + I(C{:}D|A) + I(A{:}C|D) + I(A{:}D|C).$$

Proof: Apply Rule ZY to the following Shannon-type information inequality (which can be verified using a computer program):

$$I(C{:}D|A) + I(C{:}D|B) + I(A{:}B) + \\ + I(C{:}D|Z) + I(Z{:}C|D) + I(Z{:}D|C) - I(C{:}D) + \\ + 3I(Z{:}AB|CD) \geq 0$$

Take $Z = A$ in the inequality we obtain. ∎

B. Makarychev *et al* *Technique*

RULE MMRV

(A) Let f and g be linear maps on entropies such that

$$f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) \geq 0;$$

(B) then the following (stronger) inequality is also valid:

$$f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) - r_Z H(Z|Y_\mathcal{M}) \geq 0,$$

where r_Z is the sum of coefficients of g involving Z.

The correctness of this rule is based on a result from the works of Ahlswede–Gács–Körner. The general result is presented in a book by Csiszár and Körner [3] and rediscovered in [1, 15] . The relevance of this result was also underlined by Wyner in [12]. We state here a special case suited to our needs which is a corollary of [3, p.352] written in the style of [15, Theorem 5] .

Lemma 2 (Ahlswede–Körner Lemma, [3])**.** *Let $y_1, \ldots, y_n, z$ be $n + 1$ jointly distributed random variables. Consider their respective M i.i.d. copies $Y_1, \ldots, Y_n, Z$. Then there exists a random variable Z' such that:*

- *$H(Z'|Y_1, \ldots, Y_n) = 0$,*
- *$H(Y_J|Z') - M \cdot H(y_J|z) = o(M)$, for all $\varnothing \neq J \subseteq \mathcal{N}$.*

Denote this Z' by $AK(Z{:}Y_1, \ldots, Y_n)$.

Proof of Correctness of RULE MMRV*:* Consider the joint M i.i.d copies $X_\mathcal{N}^M, Y_\mathcal{M}^M, Z^M$ of variables $X_\mathcal{N}, Y_\mathcal{M}, Z$. Let $Z' = AK(Z^M{:}Y_\mathcal{M}^M)$ be the variable obtained using Lemma 2. Apply the inequality of step (A) to the corresponding M independent copies except take $Z = Z'$. Entropy terms not involving Z' are thus M times greater. Let us compute the entropy terms involving Z' (from g) using Lemma 2:

$$\begin{aligned}
H(Z') &= I(Z'{:}Y_\mathcal{M}^M) + H(Z'|Y_\mathcal{M}^M) \\
&= H(Y_\mathcal{M}^M) - H(Y_\mathcal{M}^M|Z') \\
&= M \cdot H(Y_\mathcal{M}) - M \cdot H(Y_\mathcal{M}|Z) + o(M) \\
&= M \cdot [H(Z) - H(Z|Y_\mathcal{M})] + o(M).
\end{aligned}$$

Let $J \subseteq \mathcal{M}$,

$$\begin{aligned}
H(Z', Y_J^M) &= H(Z') + H(Y_J^M|Z') \\
&= M \cdot [H(Z) - H(Z|Y_M) + H(Y_J|Z)] + o(M) \\
&= M \cdot [H(Z, Y_J) - H(Z|Y_M)] + o(M).
\end{aligned}$$

Rewriting our instance of inequality (A) thus gives

$$M \cdot [f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) - r_Z H(Z|Y_M)] + o(M) \geq 0,$$

where r_Z is the sum of coefficients of g involving Z. Dividing the last inequality by M and making M tend to infinity gives the inequality of step (B). ∎

As an example, we retrieve Makarychev *et al* proof of the generalization of Zhang and Yeung 4-variable inequality (see Theorem 2).

Theorem 3 (Makarychev *et al*, [6])**.** *The following is a 5-variable information inequality:*

$$I(C{:}D) \leq I(C{:}D|A) + I(C{:}D|B) + I(A{:}B) + \\ + I(C{:}D|E) + I(E{:}C|D) + I(E{:}D|C)$$

Proof: Apply RULE MMRV to the Shannon-type inequality:

$$I(C{:}D|A) + I(C{:}D|B) + I(A{:}B) + \\ + 2H(Z|C) + 2H(Z|D) - H(Z) \geq 0$$

and take $Z = E$ which gives (up to some rewriting) :

$$I(C, D{:}E) \leq I(C{:}D|A) + I(C{:}D|B) + I(A{:}B) + \\ + 2I(D{:}E|C) + 2I(C{:}E|D)$$

Rewriting

$$I(C, D{:}E) = I(C{:}D) + I(D{:}E|C) + I(C{:}E|D) - I(C{:}D|E)$$

finishes the proof. ∎

Note that Theorem 1 can be proven using Lemma 2. Since balancing will appear to be important in the sequel, we state simple properties about the two rules.

Proposition 3.

- *Suppose inequality* (B) *is inferred from* (A) *by* RULE ZY *and* V *is a variable, then*

  (A) *is balanced for* V *iff* (B) *is balanced for* V.

- *Suppose inequality* (B) *is inferred from* (A) *by* RULE MMRV *and* $V \neq Z$ *is a variable, then:*
  - (A) *is balanced for* V *iff* (B) *is balanced for* V.
  - (B) *is balanced for* Z.

The proof follows immediately from the statements of the rules and the definition of balanced inequalities. Notice that RULE MMRV is only useful when applied to inequalities that are not balanced for Z. However, the rule balances for Z afterwards.

IV. COMPARISON OF PROOFS SYSTEMS

In the spirit of information inequality provers, we will consider and compare various proof systems based on the two rules described above.

Definition 2. *A* proof system *(for inequalities) consists of a pool* P *of inequalities and a rule* T. *A (computation)* step *in a proof system is described as follows:*

1) Pick an inequality (A) *from the convex closure of* P;
2) Apply rule T *to* (A) *and infer inequality* (B);
3) Add (B) *to the pool* P.

A derivation *is a sequence of valid steps in a system. An inequality* $(\mathcal{I})$ *is provable in system* S *if it belongs to the convex closure of the pool of* S *after a derivation.*

Note that in the previous rules, the naming of the variables is unimportant. The special variable Z may change for each application of a rule. We want to compare the following systems:

- SYSTEM ZY: the system using RULE ZY.
- SYSTEM ZY+B: the system using RULE ZY and balancing at each step.
- SYSTEM MMRV: the system using RULE MMRV.
- SYSTEM MMRV+B: the system using RULE MMRV and balancing at each step.

Usually, a proof system will be initialized with a starting pool of inequalities: the (elemental) Shannon-type inequalities.

We now show the main result, namely that the two inference rules of Section III are in a sense equivalent if we keep in mind Theorem 1 about balanced inequalities.

Theorem 4 (Equivalence modulo balancing)**.**

Suppose (B_1) *can be inferred from* (A_1) *by* RULE ZY, *where* (A_1) *is balanced for* Z. *Then there is an* (A_2) *such that:*

- (B_1) *can be inferred from* (A_2) *by* RULE MMRV;
- (A_2) *follows from* (A_1);

Suppose (B'_1) *can be inferred from* (A'_1) *by* RULE MMRV. *Then there is an* (A'_2) *such that:*

- (B'_1) *can be inferred from* (A'_2) *by* RULE ZY;
- (A'_2) *balances for* Z *into* (A'_1).

Proof:

RULE ZY $\Rightarrow$ RULE MMRV: Suppose

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g(Y_{\mathcal{M}}, Z) + \alpha I(Z{:}X_{\mathcal{N}}|Y_{\mathcal{M}}) \geq 0, \quad (A_1)$$

for some $\alpha \geq 0$, is a valid information inequality that is balanced for Z. By RULE ZY, the stronger

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g(Y_{\mathcal{M}}, Z) \geq 0 \quad (B_1)$$

is also valid. Let us show that inequality (B_1) can also be obtained using RULE MMRV. Start from the inequality

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g'(Y_{\mathcal{M}}, Z) \geq 0 \quad (A_2)$$

defined using $g' = g + \alpha H(Z|Y_{\mathcal{M}})$. This inequality is valid since α is non-negative, thus (A_2) follows from (A_1) and $H(Z|Y_{\mathcal{M}}) \geq I(Z{:}X_{\mathcal{N}}|Y_{\mathcal{M}})$.

By applying RULE MMRV we get

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g'(Y_{\mathcal{M}}, Z) - r'_Z H(Z|Y_{\mathcal{M}}) \geq 0, \quad (B_2)$$

where r'_Z is the sum of coefficients of g' involving Z. By definition of g' we have $r'_Z = \alpha + r_Z$, where r_Z is the sum of coefficients of g involving Z. Thus inequality (B_2) rewrites to

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g(Y_{\mathcal{M}}, Z) - r_Z H(Z|Y_{\mathcal{M}}) \geq 0,$$

and since (A_1) is balanced for Z, *i.e.*, $r_Z = 0$, the previous inequality is exactly inequality (B_1).

RULE MMRV $\Rightarrow$ RULE ZY: Suppose

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g(Y_{\mathcal{M}}, Z) \geq 0 \quad (A'_2)$$

is a valid information inequality. By RULE MMRV, the stronger

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + g(Y_{\mathcal{M}}, Z) - r_Z H(Z|Y_{\mathcal{M}}) \geq 0 \quad (B'_2)$$

is also valid. Let us show that inequality (B'_2) can also be inferred using RULE ZY and balancing.

Notice first that

$$H(Z|Y_{\mathcal{M}}) = H(Z|X_{\mathcal{N}}Y_{\mathcal{M}}) + I(Z{:}X_{\mathcal{N}}|Y_{\mathcal{M}}),$$

therefore (A'_2) rewrites to

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + [g(Y_{\mathcal{M}}, Z) - r_Z H(Z|Y_{\mathcal{M}})] + \\ + r_Z H(Z|X_{\mathcal{N}}Y_{\mathcal{M}}) + r_Z I(Z{:}X_{\mathcal{N}}|Y_{\mathcal{M}}) \geq 0,$$

where r_Z is the sum of the coefficients of g involving Z. Balancing this inequality for Z gives:

$$f(X_{\mathcal{N}}, Y_{\mathcal{M}}) + [g(Y_{\mathcal{M}}, Z) - r_Z H(Z|Y_{\mathcal{M}})] + \\ + r_Z I(Z{:}X_{\mathcal{N}}|Y_{\mathcal{M}}) \geq 0 \quad (A'_1)$$

Applying the inference rule of RULE ZY to (A_1') gives

$$f(X_\mathcal{N}, Y_\mathcal{M}) + g(Y_\mathcal{M}, Z) - r_Z H(Z|Y_\mathcal{M}) \geq 0, \qquad (B_1')$$

which is exactly inequality (B_2'). ∎

This result shows the importance of balancing non-Shannon-type inequalities for an inequality prover. For a Shannon-type inequality, its balanced counterpart is also Shannon-type and thus already belongs to the pool. However, the balanced counterpart of a non-Shannon-type inequality may not belong to the pool.

Corollary 2. *Let $(\mathcal{I})$ be an information inequality. The following are equivalent:*

- $(\mathcal{I})$ *is provable in* SYSTEM MMRV+B.
- $(\mathcal{I})$ *is provable in* SYSTEM ZY+B.
- $(\mathcal{I})$ *is provable in* SYSTEM MMRV *when using only inequalities balanced for all variables but Z.*
- $(\mathcal{I})$ *is provable in* SYSTEM ZY *when using only balanced inequalities.*

Proof: Follows immediately from Proposition 3 and Theorem 4. ∎

V. CONCLUSION

We have shown that it does not matter which of the two rules to implement in an information inequality prover, as long as it applies them to balanced inequalities. Since the cost of checking and balancing an inequality is minor, and balanced inequalities are stronger than their counterparts, they should be useful for such programs. Moreover, we have seen that balancing is not compulsory at each step because the rules can only improve balancing (Proposition 3). A last argument in favour of balanced inequalities might be the fact that they are the only inequalities valid for continuous entropy (see [1, Theorem 2]).

REFERENCES

[1] T. H. Chan. Balanced information inequalities. *IEEE Trans. Inf. Theor.*, 49(12):3261–3267, December 2003.

[2] Terence H. Chan and Raymond W. Yeung. On a Relation Between Information Inequalities and Group Theory. *IEEE Trans. on Inform. Theory*, 48:1992–1995, 2002.

[3] Imre Csiszar and Janos Korner. *Information theory : coding theorems for discrete memoryless systems.* Academic Press ; Akademiai Kiado, New York : Budapest :, 1981.

[4] Randall Dougherty, Christopher Freiling, and Kenneth Zeger. Six new non-shannon information inequalities. In *in Proc. IEEE Int. Symp. Inf. Theory*, pages 233–236, 2006.

[5] Tarik Kaced and Andrei Romashchenko. On Essentially Conditional Information Inequalities. *Proceedings IEEE ISIT*, pages 1935–1939, 2011.

[6] K. Makarychev, Y. Makarychev, A. Romashchenko, and N. Vereshchagin. A new class of non-shannon-type inequalities for entropies. *Communications in Information and Systems*, 2(2):147–166, 2002.

[7] František Matúš. Infinitely many Information Inequalities. *Proceedings ISIT 2007*, pages 41–44, 2007.

[8] Nicholas Pippenger. What are the laws of information theory. *Special Problems on Communication and Computation Conference, Palo Alto, California*, 1986.

[9] Rethnakaran Pulikkoonattu, Etienne Perron, and Suhas Diggavi. Information theoretic Inequality prover (Xitip).

[10] Claude E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, July 1948.

[11] Claude E. Shannon. A mathematical theory of communication (continued). *Bell System Technical Journal*, 27(4):623–656, October 1948.

[12] A. Wyner. On source coding with side information at the decoder. *IEEE Trans. Inf. Theor.*, 21(3):294–300, September 2006.

[13] Weidong Xu, Jia Wang, and Jun Sun. A projection method for derivation of non-shannon-type information inequalities. In *ISIT'08*, pages 2116–2120, 2008.

[14] R. W. Yeung and Y. O. Yan. Information Theoretic Inequality Prover (ITIP).

[15] Z. Zhang. On a new non-shannon-type information inequality. *Communications in Information and Systems 3*, pages 47–60, 2003.

[16] Zhen Zhang and Raymond W. Yeung. A non-Shannon-type Conditional Information Inequality. *IEEE Trans. on Inform. Theory*, 43:1982–1986, 1997.

[17] Zhen Zhang and Raymond W. Yeung. On characterization of entropy function via information inequalities. *IEEE Trans. Inf. Theor.*, 44(4):1440–1452, July 1998.