# High-SNR Asymptotics of Mutual Information for Discrete Constellations

Alex Alvarado, Fredrik Brännström[‡], Erik Agrell[‡], and Tobias Koch[§]
Department of Engineering, University of Cambridge, UK
[‡]Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden
[§]Signal Theory and Communications Department, Universidad Carlos III de Madrid, 28911 Leganés, Spain
*alex.alvarado@ieee.org, {fredrik.brannstrom, agrell}@chalmers.se, koch@tsc.uc3m.es*

***Abstract*—The asymptotic behavior of the mutual information (MI) at high signal-to-noise ratio (SNR) for discrete constellations over the scalar additive white Gaussian noise channel is studied. Exact asymptotic expressions for the MI for arbitrary one-dimensional constellations and input distributions are presented in the limit as the SNR tends to infinity. Asymptotics of the minimum mean-square error (MMSE) are also developed. It is shown that for any input distribution, the MI and the MMSE have an asymptotic behavior proportional to a Gaussian Q-function, whose argument depends on the minimum Euclidean distance of the constellation and the SNR. Closed-form expressions for the coefficients of these Q-functions are calculated.**

## I. Introduction

In this paper we consider the real additive white Gaussian noise (AWGN) channel

$$Y = \sqrt{\rho}X + Z, \tag{1}$$

where $X$ is the transmitted symbol and $Z$ is a Gaussian random variable, independent of $X$, with zero mean and unit variance. The capacity of the channel in (1) is given by [1]

$$\mathsf{C}(\rho) = \frac{1}{2}\log(1+\gamma), \tag{2}$$

where $\gamma = \rho\mathbb{E}[X^2]$ is the signal-to-noise ratio (SNR) and $\rho > 0$ is an arbitrary scale factor. Although inputs distributed according to the Gaussian distribution attain the capacity, they suffer from several drawbacks which prevent them from being used in practical systems. Among them, especially relevant are the unbounded support and the infinite number of bits needed to represent signal points. In practical systems, discrete distributions with a bounded support are typically preferred.

The mutual information (MI) between the channel input $X$ and the channel output $Y$ of (1), where the input distribution is constrained to be a probability mass function (PMF) over a discrete constellation, represents the maximum rate at which information can be reliably transmitted over (1) using that particular distribution. While the low-SNR asymptotics of the MI for discrete constellations are well understood (see [1]–[4] and references therein), to the best of our knowledge, only upper and lower bounds are known for the high-SNR behavior. For example, upper and lower bounds on the MI and/or the minimum mean-square error (MMSE) in the high-SNR regime were derived in [5]–[7]. In [8, Appendix E] two constellations with different minimum Euclidean distances (MEDs) are compared, and it is shown that, for sufficiently high SNR, the constellation with larger MED gives a higher MI. Upper and lower bounds on the MI and MMSE for multiple-antenna systems over fading channels can be found in [9]–[11]. Using the Mellin transform method, asymptotic expansions for the MMSE and MI for scalar and vectorial coherent fading channels were recently derived in [12].

In this paper, we study the high-SNR asymptotic behavior of the MI for discrete constellations. In particular, we consider arbitrary constellations and input distributions (independent of $\rho$) and find exact asymptotic expressions for the MI in the limit as the SNR tends to infinity. Exact asymptotic expressions for the MMSE are also developed. Our contribution is two-fold. First, we show that, at high SNR, both the MI and the MMSE have an asymptotic behavior proportional to $\mathsf{Q}\left(\sqrt{\rho}d/2\right)$, where $\mathsf{Q}(\cdot)$ is the Gaussian Q-function and $d$ is the MED of the constellation. While this asymptotic behavior has been demonstrated for uniform input distributions (e.g., [6, eqs. (36)–(37)], [6, Sec. II-C], [9, Sec. III], [11, Sec. III]), we show that it holds for any discrete input distribution that does not depend on the SNR. Second, in contrast to previous works, we provide closed-form expressions for the coefficients before these Q-functions, thereby characterizing the asymptotic behavior of the MI and MMSE more accurately.

## II. Preliminaries

### A. Notation Convention

Row vectors are denoted by boldface letters $\boldsymbol{x} = [x_1, x_2, \ldots, x_M]$ and sets are denoted by calligraphic letters $\mathcal{C}$. All the logarithms are natural logarithms and all the MIs are therefore given in nats. Probability density functions (PDFs) and conditional PDFs are denoted by $f_Y(y)$ and $f_{Y|X}(y|x)$, respectively. Analogously, PMFs are denoted by $P_X(x)$. Expectations are denoted by $\mathbb{E}[\cdot]$. Finally, the Gaussian

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Q-function is defined as

$$\mathsf{Q}(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \mathrm{e}^{-\frac{1}{2}\xi^2}\,\mathrm{d}\xi. \tag{3}$$

### B. Model

We consider the discrete-time, real-valued AWGN channel in (1), where the transmitted symbols $X$ are constrained to the set $\mathcal{X} \triangleq \{x_1, x_2, \ldots, x_M\}$ with cardinality $|\mathcal{X}| = M = 2^m$. The set of indices that enumerates all the constellation symbols in $\mathcal{X}$ is defined as $\mathcal{I}_\mathcal{X} \triangleq \{1, \ldots, M\}$.

We focus on one-dimensional constellations and assume, without loss of generality, that the symbols are different and ordered, i.e., $x_1 < x_2 < \cdots < x_M$. Each of the symbols is transmitted with probability $p_i \triangleq P_X(x_i)$, $0 < p_i < 1$. While the transmitted symbols are fully determined by the PMF $P_X$, we shall use *constellation* to denote the support $\mathcal{X}$ of the PMF and *input distribution* to denote the probabilities $\boldsymbol{p} = [p_1, \ldots, p_M]$ associated with the symbols. We assume that neither the constellation nor the input distribution depends on $\rho$.

The transmitted average symbol energy is finite and given by

$$E_\mathrm{s} \triangleq \mathbb{E}[X^2] = \sum_{i\in\mathcal{I}_\mathcal{X}} p_i x_i^2. \tag{4}$$

It follows that the SNR $\gamma$ in (2) is $\gamma = \rho E_\mathrm{s}$.

An $M$-ary pulse-amplitude modulation ($M$PAM) constellation having $M$ equally spaced symbols (separated by $2\Delta$) is denoted by $\mathcal{E} \triangleq \{x_i = -(M-2i+1)\Delta : i = 1, \ldots, M\}$. A uniform distribution of $X$ (i.e., $p_i = 1/M$, $\forall i \in \mathcal{I}_\mathcal{X}$) is denoted by $P_X^\mathrm{u}$. A uniform input distribution over $\mathcal{X} = \mathcal{E}$ is denoted by $P_X^\mathrm{eu}$, where in this case

$$\Delta^2 = \frac{3E_\mathrm{s}}{(M^2-1)}. \tag{5}$$

We define the entropy of the random variable $X$ as

$$H_{P_X} \triangleq -\mathbb{E}\left[\log\left(P_X(X)\right)\right], \tag{6}$$

the MI between $X$ and $Y$ as

$$\mathsf{I}_{P_X}(\rho) \triangleq \mathbb{E}\left[\log\left(f_{Y|X}(Y|X)/f_Y(Y)\right)\right], \tag{7}$$

and the MMSE as

$$\mathsf{M}_{P_X}(\rho) \triangleq \mathbb{E}[(X - \hat{X}^\mathrm{ME}(Y))^2], \tag{8}$$

where $\hat{X}^\mathrm{ME}(y) \triangleq \mathbb{E}[X|Y=y]$ is the conditional (posterior) mean estimator.

### C. Discrete Constellations

The MED of the constellation is defined as

$$d \triangleq \min_{x_i, x_j \in \mathcal{X} : i \neq j} |x_i - x_j|. \tag{9}$$

We define the counting function

$$A_\mathcal{X}^{(i)}(\delta) \triangleq \begin{cases} 1, & \text{if } \exists x \in \mathcal{X} : x_i - x = \delta, \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where $\delta \in \mathbb{R}$. Since $x_i \in \mathcal{X}$, we have $A_\mathcal{X}^{(i)}(0) = 1$ $\forall i \in \mathcal{I}_\mathcal{X}$. We further define $A_\mathcal{X}$ as twice the number of pairs of constellation points at MED, i.e.,

$$A_\mathcal{X} \triangleq \sum_{i\in\mathcal{I}_\mathcal{X}} \sum_{w\in\{\pm1\}} A_\mathcal{X}^{(i)}(wd). \tag{11}$$

By using the fact that for any real-valued constellation, there are at least one and at most $M-1$ pairs of constellation points at MED, we obtain the bound

$$2 \leq A_\mathcal{X} \leq 2(M-1). \tag{12}$$

The upper bound is achieved by an $M$PAM constellation, for which

$$A_\mathcal{E} = 2(M-1). \tag{13}$$

Analogous to $A_\mathcal{X}^{(i)}(\delta)$, we define $B_{P_X}^{(i)}(\delta)$ as

$$B_{P_X}^{(i)}(\delta) \triangleq \begin{cases} \sqrt{p_j p_i}, & \text{if } \exists x_j \in \mathcal{X} : x_i - x_j = \delta, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Clearly $B_{P_X}^{(i)}(0) = p_i, \forall i \in \mathcal{I}_\mathcal{X}$. For a given $P_X$, we define the constant

$$B_{P_X} \triangleq \sum_{i\in\mathcal{I}_\mathcal{X}} \sum_{w\in\{\pm1\}} B_{P_X}^{(i)}(wd). \tag{15}$$

For a uniform input distribution, $P_X = P_X^\mathrm{u}$ and $MB_{P_X}^{(i)}(\delta) = A_\mathcal{X}^{(i)}(\delta)$, so that

$$B_{P_X^\mathrm{u}} = \frac{A_\mathcal{X}}{M}. \tag{16}$$

*Example 1:* Consider an unequally spaced 4-ary constellation with $x_1 = -4$, $x_2 = -2$, $x_3 = 2$, and $x_4 = 4$, and the input distribution $p_i = i/10$ with $i = 1, 2, 3, 4$. The MED in (9) is $d = 2$, $E_\mathrm{s}$ in (4) is $E_\mathrm{s} = 10$, $A_\mathcal{X}$ in (11) is $A_\mathcal{X} = 4$ (two pairs of constellation points at MED), and $B_{P_X}$ in (15) is $B_{P_X} = 2\sqrt{p_1 p_2} + 2\sqrt{p_3 p_4} \approx 0.98$.

## III. High-SNR Asymptotics

Upper and lower bounds on the MI and MMSE for discrete constellations at high SNR can be found in, e.g., [5]–[7], [9]–[12]. While these bounds describe the correct asymptotic behavior, they are, in general, not tight in the sense that the ratio between them does not tend to one as $\rho \to \infty$. In what follows, we present exact asymptotic expressions for the MI and MMSE for an arbitrary $P_X$.

### A. Asymptotics of the MI and MMSE

For any given input distribution $P_X$, the MI $\mathsf{I}_{P_X}(\rho)$ tends to $H_{P_X}$ as $\rho$ tends to infinity. In the following we study how fast it converges towards its maximum $H_{P_X}$ by analyzing the difference $H_{P_X} - \mathsf{I}_{P_X}(\rho)$.[1] Theorems 1 and 2 are the main results of this paper and characterize the high-SNR behavior of $H_{P_X} - \mathsf{I}_{P_X}(\rho)$ and $\mathsf{M}_{P_X}(\rho)$. The proofs, which are omitted due to space limitations, can be found in [13] (Appendices A and B).

[1] The quantity $H_{P_X} - \mathsf{I}_{P_X}(\rho)$ is the conditional entropy of $X$ given $Y$.

*Theorem 1:* For any $P_X$

$$\lim_{\rho\to\infty} \frac{H_{P_X} - \mathsf{I}_{P_X}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \pi B_{P_X} \tag{17}$$

where $B_{P_X}$ is given by (15).

*Theorem 2:* For any $P_X$

$$\lim_{\rho\to\infty} \frac{\mathsf{M}_{P_X}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \frac{\pi d^2}{4} B_{P_X} \tag{18}$$

where $B_{P_X}$ is given by (15).

Theorems 1–2 reveal that, at high SNR, the MI and the MMSE behave as

$$\mathsf{I}_{P_X}(\rho) \approx H_{P_X} - \pi B_{P_X} \mathsf{Q}\left(\sqrt{\rho}d/2\right), \tag{19}$$

$$\mathsf{M}_{P_X}(\rho) \approx \frac{\pi d^2}{4} B_{P_X} \mathsf{Q}\left(\sqrt{\rho}d/2\right). \tag{20}$$

The results in (19)–(20) show that for any input distribution, the MI and the MMSE have the same high-SNR behavior. Specifically, they are both proportional to a Gaussian Q-function,[2] where the proportionality constants depend on the input distribution (and also on the MED of the constellation in the case of the MMSE).

*Remark 1:* While the results presented in this section hold for any one-dimensional constellation, they directly generalize to multidimensional constellations that are constructed as *ordered direct products* [14, eq. (1)] of one-dimensional constellations. For example, the results directly generalize to rectangular quadrature amplitude modulation constellations.

## B. Discussion and Examples

We start by noting that Theorems 1 and 2 combined give

$$\lim_{\rho\to\infty} \frac{\mathsf{M}_{P_X}(\rho)}{H_{P_X} - \mathsf{I}_{P_X}(\rho)} = \frac{d^2}{4}. \tag{21}$$

Thus, for any $P_X$, the limiting ratio between the MMSE and the conditional entropy does not depend on the input distribution.

For a uniform input distribution ($P_X = P_X^{\mathrm{u}}$), Theorems 1–2 particularize to the following result.

*Corollary 1:* For any $\mathcal{X}$ with a uniform input distribution

$$\lim_{\rho\to\infty} \frac{\log M - \mathsf{I}_{P_X^{\mathrm{u}}}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \pi \frac{A_{\mathcal{X}}}{M}, \tag{22}$$

$$\lim_{\rho\to\infty} \frac{\mathsf{M}_{P_X^{\mathrm{u}}}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \frac{\pi d^2}{4} \frac{A_{\mathcal{X}}}{M}, \tag{23}$$

where $A_{\mathcal{X}}$ is given in (11).

*Proof:* From Theorems 1–2 and (16). ■

Corollary 1 shows that for a uniform input distribution, the MI and the MMSE for discrete constellations in the high-SNR regime are determined by the MED of the constellation and by the number of pairs of constellation points at MED.

*Remark 2:* It follows from Corollary 1 that the constellation that maximizes the MI (or equivalently, the constellation that minimizes the MMSE) at high SNR is the constellation that first maximizes the MED and then minimizes $A_{\mathcal{X}}$. For one-dimensional constellations, the unique constellation that maximizes the MED is an $M$PAM constellation ($\mathcal{X} = \mathcal{E}$), in which case there is no need for optimizing $A_{\mathcal{X}}$.

TABLE I

SUMMARY OF ASYMPTOTICS OF MI AND MMSE.

| Input Distribution | $P_X$ | $P_X^{\mathrm{u}}$ | $P_X^{\mathrm{eu}}$ |
|---|---|---|---|
| $\lim_{\rho\to\infty} \frac{H_{P_X} - \mathsf{I}_{P_X}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)}$ | $\pi B_{P_X}$ | $\pi \frac{A_{\mathcal{X}}}{M}$ | $\frac{2\pi(M-1)}{M}$ |
| $\lim_{\rho\to\infty} \frac{\mathsf{M}_{P_X}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)}$ | $\frac{\pi d^2}{4} B_{P_X}$ | $\frac{\pi d^2}{4} \frac{A_{\mathcal{X}}}{M}$ | $\frac{6\pi E_{\mathrm{s}}}{M(M+1)}$ |

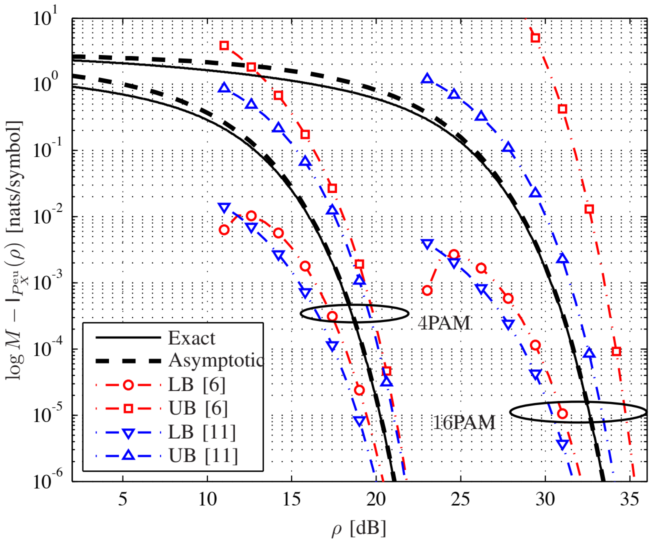


Fig. 1. $\log M - \mathsf{I}_{P_X^{\mathrm{eu}}}(\rho)$ for 4PAM and 16PAM (solid lines) constellations (normalized to $E_{\mathrm{s}} = 1$) and the asymptotic expression in (26) (thick dashed lines). The lower and upper bounds [6, eq. (34)–(35)] and [11, eq. (17)–(19)] are also shown.

*Corollary 2:* For $M$PAM and a uniform input distribution

$$\lim_{\rho\to\infty} \frac{\log M - \mathsf{I}_{P_X^{\mathrm{eu}}}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \frac{2\pi(M-1)}{M}, \tag{24}$$

$$\lim_{\rho\to\infty} \frac{\mathsf{M}_{P_X^{\mathrm{eu}}}(\rho)}{\mathsf{Q}\left(\sqrt{\rho}d/2\right)} = \frac{6\pi E_{\mathrm{s}}}{M(M+1)}. \tag{25}$$

*Proof:* From Corollary 1 combined with (13) and (5), where $d = 2\Delta$. ■

The results obtained in this section are summarized in Table I.

*Example 2:* In Fig. 1, we plot the difference $\log M - \mathsf{I}_{P_X^{\mathrm{eu}}}(\rho)$ (i.e., the conditional entropy) for 4PAM and 16PAM with uniform input distributions[3] together with the approximation

$$\log M - \mathsf{I}_{P_X^{\mathrm{eu}}}(\rho) \approx \frac{2\pi(M-1)}{M} \mathsf{Q}\left(\sqrt{\rho}d/2\right), \tag{26}$$

[2]Disregarding the "offset" $H_{P_X}$ in (19).

[3]Calculated numerically using Gauss–Hermite quadratures with 300 quadrature points [15, Sec. III].

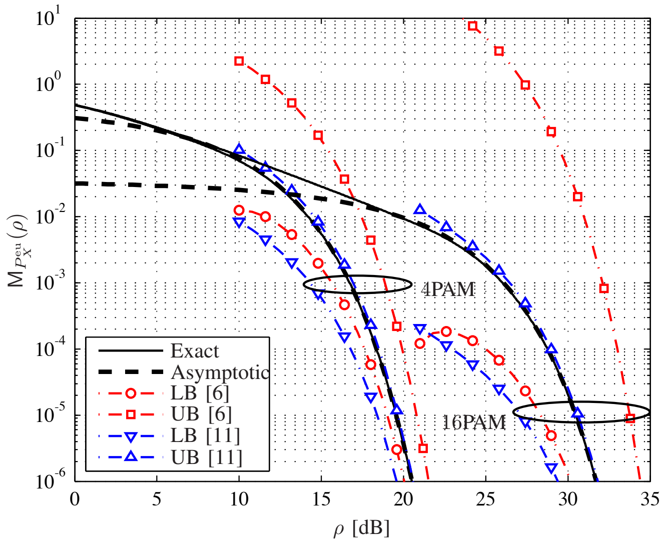


Fig. 2. $\mathsf{M}_{P_X^{\text{eu}}}(\rho)$ for 4PAM and 16PAM (solid lines) constellations (normalized to $E_{\text{s}} = 1$) and the asymptotic expression in (27) (thick dashed lines). The lower and upper bounds [6, eq. (30)–(31)] and [11, eq. (13)–(15)] are also shown.

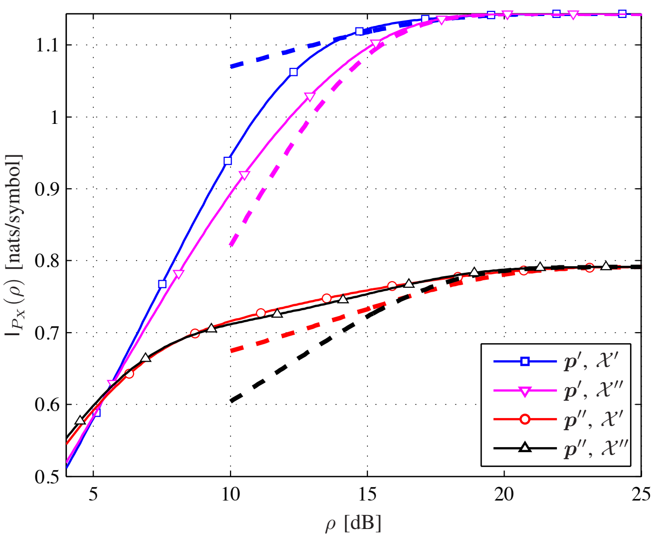


Fig. 3. $\mathsf{I}_{P_X}(\rho)$ for the four combinations of constellations (normalized to $E_{\text{s}} = 1$) and distributions shown in Table II (solid lines with markers) and the asymptotic expression in (19) (thick dashed lines).

which is asymptotically exact according to (24). We also show the lower and upper bounds derived in [6, eq. (34)–(35)] and [11, eq. (17)–(19)]. Observe that (26) approximates $\log M - \mathsf{I}_{P_X^{\text{eu}}}(\rho)$ accurately for a large range of SNR.

In Fig. 2, analogous results for the MMSE are presented, where the bounds derived in [6, eq. (30)–(31)] and [11, eq. (13)–(15)] are also included. Again our asymptotic expression

$$\mathsf{M}_{P_X^{\text{eu}}}(\rho) \approx \frac{6\pi E_{\text{s}}}{M(M+1)} \mathsf{Q}\left(\sqrt{\rho}d/2\right), \tag{27}$$

which corresponds to the limit in (25), approximates the MMSE accurately for a large range of SNRs.

*Example 3:* Consider the following two constellations and input distributions

$$\begin{aligned}
\mathcal{X}' &= \{-2, -1, 1, 3\},\\
\mathcal{X}'' &= \{-2, -1, 1, 2\},\\
\boldsymbol{p}' &= [0.01, 0.33, 0.33, 0.33],\\
\boldsymbol{p}'' &= [0.49, 0.01, 0.01, 0.49].
\end{aligned}$$

Table II shows $H_{P_X}$ in (6), $A_{\mathcal{X}}$ in (11), and $B_{P_X}$ in (15) for all four combinations. The last column in Table II shows the MED when the constellations are normalized to unit energy, i.e., $E_{\text{s}} = 1$ in (4).

Fig. 3 shows the MI for the four cases in Table II together with the asymptotic expression in (19). This figure demonstrates how the developed expressions are not upper or lower bounds to the true MI. Fig. 3 also illustrates how the MI for high SNR converges towards its maximum $H_{P_X}$, listed in Table II. To visualize which constellation (for the same input distribution) converges faster to $H_{P_X}$, we plot in Fig. 4 the corresponding conditional entropy $H_{P_X} - \mathsf{I}_{P_X}(\rho)$. Observe that for the same $H_{P_X}$ (or input distribution) the constellation with

TABLE II
DIFFERENT PARAMETERS FOR THE CONSTELLATIONS AND INPUT DISTRIBUTIONS IN EXAMPLE 3.

| $\boldsymbol{p}$, $\mathcal{X}$ | $H_{P_X}$ | $A_{\mathcal{X}}$ | $B_{P_X}$ | $d$ |
|---|---|---|---|---|
| $\boldsymbol{p}'$, $\mathcal{X}'$ | 1.1436 | 2 | 0.1149 | 0.5220 |
| $\boldsymbol{p}'$, $\mathcal{X}''$ | 1.1436 | 4 | 0.7749 | 0.7036 |
| $\boldsymbol{p}''$, $\mathcal{X}'$ | 0.7912 | 2 | 0.1400 | 0.3956 |
| $\boldsymbol{p}''$, $\mathcal{X}''$ | 0.7912 | 4 | 0.2800 | 0.5038 |

larger MED results in a MI that converges faster to $H_{P_X}$. Once more, we see that the asymptotic expression (17) accurately predicts the high-SNR behavior for a large range of SNR.

## IV. CONCLUSIONS

In this paper, we studied discrete constellations with arbitrary input distributions over the scalar AWGN channel in the high-SNR regime and derived exact asymptotic expressions for key quantities in information theory (MI) and estimation theory (MMSE). Our results show that, as the SNR tends to infinity, and disregarding offsets, both quantities are proportional to $\mathsf{Q}\left(\sqrt{\rho}d/2\right)$, where $d$ is the MED of the constellation. The results further show that the proportionality constant for the MI depends on the number of constellation points at MED and the input distribution, while the proportionality constant for the MMSE depends also on the MED of the constellation.

The results presented in this paper can be used to prove, at high SNR, the long standing conjecture that Gray codes are the binary labelings that maximize the generalized mutual information for bit-interleaved coded modulation [16, Sec. III-C]; see [13, Theorem 8].

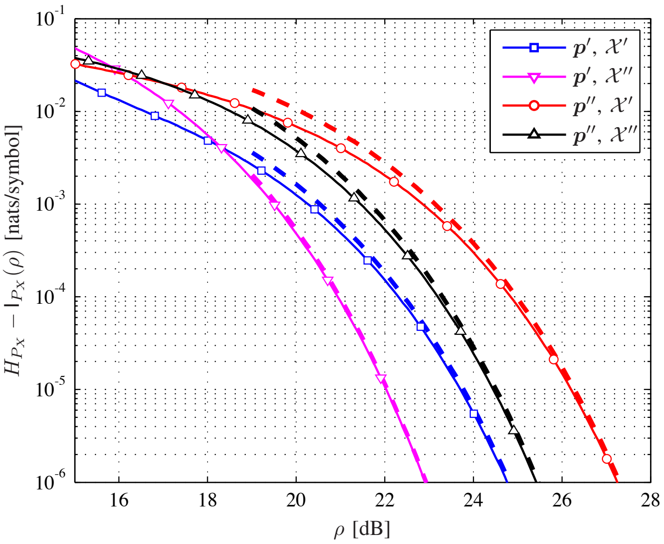


Fig. 4. $H_{P_X} - \mathsf{I}_{P_X}(\rho)$ for the four combinations of constellations (normalized to $E_\mathrm{s} = 1$) and distributions shown in Table II (solid lines with markers) and the asymptotic expression in (19) (thick dashed lines), i.e., $H_{P_X} - \mathsf{I}_{P_X}(\rho) \approx \pi B_{P_X} \mathsf{Q}\left(\sqrt{\rho} d/2\right)$.

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