Rate Maximization of Multilayer Transmission over Rayleigh Fading Channels

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***Abstract*—In this paper we consider joint optimization of rate and power for communication systems that use multilayer superposition source coding with successive refinement of information. We assume a Rayleigh fading channel, where rates and power are jointly and optimally allocated between the source layers based on channel statistics information, with the objective of maximizing the expected total received rate at the end user. We show that the optimization problem possesses a strong duality, and hence we use the dual form to show that the optimal solution can be obtained using a two-dimensional bisection search for any number of layers. The outer bisection search is over the Lagrangian dual variable and the inner bisection search is over the decoding SNR threshold of the last layer. Moreover, we show that with a small number of layers, we can approach the performance upper bound, that is achieved by transmitting an infinite number of layers.**

***Index Terms*—Broadcast approach, multilayer transmission, joint optimization.**

I. Introduction

Wireless communication systems are characterized by varying instantaneous channel capacity due to the fading effects caused by the multi-path propagation over the wireless medium. In many practical scenarios, where instantaneous channel-state information (CSI) are not available at the transmitter, a fundamental trade-off exits between the source transmission rate and the outage probability at the receiver. In these situations, it was shown that conventional single-layer transmission schemes are suboptimal while multilayer transmission schemes have the ability to maximize the long-term expected capacity of the channel (see e.g. [1] and references therein).

In Multilayer transmission, the multimedia source is encoded into two or more layers with each layer successively refining the description of the previous layers [2]–[4]. The layers would be simultaneously transmitted but with more protection to the higher priority layers. One common and optimal scheme for multilayer transmission is the broadcast approach [5] in which each source layer is protected using an independent capacity-achieving single-layer code, and all codewords are added together (i.e. superposition coding) with proper power allocation. The receiver applies successive decoding to retrieve the source layers starting with the highest priority layer up to the number of layers that can be decoded reliably based on the instantaneous channel condition. The expected total received rate at the end user depends on the statistics of channel. Therefore, the knowledge of the channel statistics should be utilized in the allocation of the intrinsic optimization parameters of these transmission schemes; namely, the information rate (bits/sec/Hz) and the transmission power that are allocated to each source layer. These two parameters affect the signal-to-noise ratio (SNR) thresholds above which the source layers can be decoded reliably at the destined receiver. Reducing the SNR threshold of a layer makes it more likely to be decoded reliably and vice versa. We can reduce the SNR threshold of a layer by either reducing its rate, and this affects the prospected service quality, or by allocating more of the total power budget to this layer, and this has a negative impact on the SNR thresholds of the remaining layers, which can consequently reduce the probability of reliably decoding the remaining layers at the receiver. These trade-offs have been the motivation behind the work presented in this paper.

The joint power and rate optimization problem was considered in [6, Section IV] for the case of infinite number of layers, which provides an upper bound for the system performance. The more practical case, which is of interest in this paper, is the finite number of layers case. Under the assumption of finite number of layers, most contributions, e.g., [1], [6]–[8], involved fixing some of the degrees of freedom available in the joint optimization problem, which resulted in suboptimal solutions. In [6] and independently in [7], the authors solved the problem of minimizing the expected distortion of a Gaussian source with predetermined and fixed SNR thresholds of the source layers. Fixing the SNR thresholds renders the rate and power variables dependent and hence finding one of them yields the other one. In [7] the power was obtained first followed by the rate, while the opposite way was used in [6]. Alternatively, in [1] the rates of the layers were predetermined, with no restrictions on the SNR thresholds.

Other contributions include [9] in which the joint optimization problem was considered and the numerical results for maximizing the expected rate with two layers were shown.

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However, there was no discussion regarding the general solution of the problem with a finite number of layers, and whether convergence to the optimal solution is guaranteed or not. Furthermore, in [10] it was suggested to alternately optimize the powers (for fixed rates) and the rates (for fixed power), and repeat this process until convergence. However, convergence to the globally optimal solution was not guaranteed. Moreover, the rates were selected from a discrete set of possible rate allocations in order to limit the computational complexity.

In this paper, we solve the joint optimization problem for finite number of layers with no restrictions on the source rates or SNR decoding thresholds. The objective is to maximize the expected total received rate at the end user. In order to achieve this objective, we slightly alter the change of variables step adopted in [1] by using the reciprocal of the SNR thresholds in order to produce a symmetry in the problem between the two optimization variables. We apply the Lagrangian dual problem and we obtain the solution by using a two-dimensional bisection searches. The outer bisection search is over the Lagrangian dual variable and the inner bisection search is over the threshold of the last layer to be decoded. Our results show that with a small number of layers, we can approach the performance upper bound, that is achieved by transmitting an infinite number of layers.

II. System Model and Problem Formulation

We consider a Gaussian source that is encoded into independent M layers, $\mathbf{x} = [x_1, x_2, \cdots, x_M]$, with rates $\mathbf{R} = [R_1, R_2, \cdots, R_M]$, power ratios $\alpha = [\alpha_1, \alpha_2, \dots \alpha_M]$ of the total power, respectively, and with each layer successively refining the description of the previous ones. Therefore, layer x_i has a rate R_i and a power $P_i = \alpha_i \bar{P}$, where $\bar{P}$ is the total source power. The layers are transmitted simultaneously using superposition coding at the transmitter and successive decoding is applied at the receiver. We use the notation $\gamma = \frac{|h|^2 \bar{P}}{\sigma^2}$ to represent the SNR of the channel, where $|h|$ is the magnitude of the channel gain and σ^2 is the received noise variance. We assume that the magnitude of the channel is distributed according to a Rayleigh distribution, and hence the probability density function (PDF) of the received SNR follows an exponential distribution $f_\gamma(\gamma) = \frac{1}{G} e^{-\gamma/G}$, where G is the mean. We also assume that the parameter G is known at the transmitter side.

The problem that we consider in this paper is how to optimally and jointly allocate the rates (R_i's) and power ratio (α_i's) to each layer in order to maximize the expected total rate of information decoded by the end user; $E(\bar{R})$, which depends on how many layers are decoded successfully at the receiver. Therefore, $\bar{R} \in \{\bar{R}_1, \bar{R}_2, \dots, \bar{R}_M\}$, where $\bar{R}_i = \sum_{j=1}^{i} R_j$. This problem can be formulated as:

$$\max_{\{R_i, \alpha_i\}} \int_0^\infty f_\gamma(\gamma) \;\; \bar{R}\left(\gamma, \alpha, \mathbf{R}\right) \; d\gamma, \tag{1a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \tag{1b}$$

where the notation $\bar{R}\left(\gamma, \alpha, \mathbf{R}\right)$ signifies that the total rate that can be decoded reliably at a given SNR value γ depends on the rates and power ratios of the layers. Since we have M layers, we can write (1) similar to [1]:

$$\max_{\{R_i, \alpha_i, \bar{\gamma}_i\}} \sum_{i=1}^{M} \bar{R}_i \Big(F_\gamma(\bar{\gamma}_{i+1}(\alpha, \mathbf{R})) - F_\gamma(\bar{\gamma}_i(\alpha, \mathbf{R})) \Big), \tag{2a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \; \alpha_i \geq 0 \qquad \forall i, \tag{2b}$$

$$\bar{\gamma}_i = \max \left\{ \bar{\gamma}_j, \frac{1}{\frac{\alpha_i}{2^{R_i} - 1} - \sum_{m>i}^{M} \alpha_m} \right\} \quad \forall\, j < i, \tag{2c}$$

where $\bar{\gamma}_i$ is the SNR threshold required to decode all layers up to Layer x_i, F_γ is the cumulative distribution function (CDF) of the SNR distribution, where for a maximum number of layers M, we define $F_\gamma(\bar{\gamma}_{M+1}) = 1$.

The power constraints (2b) and (2c) can be equivalently replaced by the following three[1] constraints [1, Section II-B]:

$$\sum_{i=1}^{M} \frac{b_i - b_{i-1}}{\bar{\gamma}_i} \leq 1, \tag{3a}$$

$$\bar{\gamma}_M \geqslant \bar{\gamma}_{M-1} \geqslant \dots \geqslant \bar{\gamma}_1 > 0, \tag{3b}$$

$$b_M \geqslant b_{M-1} \geqslant \dots \geqslant b_1 \geqslant 0, \tag{3c}$$

where $b_i = 2^{\bar{R}_i} - 1$ and $b_0 = 0$.

With the aid of this alternative way to write the power constraint, the problem in (2) can be reformulated as:

$$\max_{\{b_i, \bar{\gamma}_i\}} \quad \sum_{i=1}^{M} \bar{R}_i\left(b_i\right) \; \Big(F_\gamma(\bar{\gamma}_{i+1}) - F_\gamma(\bar{\gamma}_i) \Big) \tag{4}$$

$$\text{subject to} \qquad \text{(3a), (3b) and (3c)}$$

where $\bar{R}_i(b_i) = \log_2(1 + b_i)$ which is a function of only b_i, and we replace the optimization variable α_i by functions of b_i, and the SNR thresholds $\bar{\gamma}_i$. This reduces the number of variables and renders the problem easier to solve.

By solving (4) for the optimal b_i's and $\bar{\gamma}_i$'s, we can then obtain the optimal rates of each layer using $\bar{R}_i = \log_2(1 + b_i)$, and hence $R_i = \bar{R}_i - \bar{R}_{i-1}$. Similarly, we can obtain the optimal power ratio α_i of each layer using the following relation [1], which can be evaluated successively starting with α_1:

$$\alpha_i = \frac{b_i - b_{i-1}}{1 + b_i} \left(1 - \sum_{j=1}^{i-1} \alpha_j + \frac{1}{\bar{\gamma}_i} \right). \tag{5}$$

For further simplicity of the analysis, we propose an additional modification to the optimization problem, by replacing the SNR decoding thresholds $\bar{\gamma}_i$ by their reciprocals denoted as $\bar{n}_i = \frac{1}{\bar{\gamma}_i}$. Therefore, we can write the optimization problem

[1] In [1], the constraint (3c) was not used since the rates were predetermined and not subject to optimization.

in the following form:

$$\max_{\{b_i,\bar{n}_i\}} \quad \sum_{i=1}^{M} \bar{R}_i(b_i)\ \left[F_n(\bar{n}_i) - F_n(\bar{n}_{i+1})\right] \tag{6a}$$

subject to

$$\sum_{i=1}^{M} b_i \left[\bar{n}_i - \bar{n}_{i+1}\right] \leqslant 1, \tag{6b}$$

$$b_M \geqslant b_{M-1} \geqslant \ldots \geqslant b_1 \geqslant 0, \tag{6c}$$

$$0 \leqslant \bar{n}_M \leqslant \bar{n}_{M-1} \leqslant \ldots \leqslant \bar{n}_1, \tag{6d}$$

where F_n is the CDF of $\bar{n}$, and $\bar{n}_{M+1} = 0$.

It is clear that $\bar{R}_i$ is a concave increasing function in b_i. However, $F_n(\bar{n})$ is not concave in general. For many fading channel models, F_n is convex increasing over $[0, \bar{n}^*]$ and turns into concave increasing for $\bar{n} \geq \bar{n}^*$, where $\bar{n}^*$ is the inflection point. For the special case of Rayleigh Fading with average SNR equals G, which is our interest in this work:

$$F_n(\bar{n}) = \exp\left(\frac{-1}{G\bar{n}}\right), \qquad f_n(\bar{n}) = \frac{1}{G\bar{n}^2}\exp\left(\frac{-1}{G\bar{n}}\right), \tag{7}$$

and $\bar{n}^* = \frac{1}{2G}$.

In the next section, we propose a solution to this problem that is based on applying the Karush-Kuhn-Tucker (KKT) conditions to the dual problem, e.g. [11].

III. Solution Structure

We start by removing the constraints (6c) and (6d) from the problem in (6), and will implicitly consider them when searching for the optimal values of b_i and $\bar{n}_i$. Therefore, the problem in (6) can now be written as

$$\max_{\{b_i,\bar{n}_i\}} \quad \sum_{i=1}^{M} \bar{R}_i(b_i)\ \left[F_n(\bar{n}_i) - F_n(\bar{n}_{i+1})\right] \tag{8a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} b_i \left[\bar{n}_i - \bar{n}_{i+1}\right] - 1 \leqslant 0. \tag{8b}$$

We note that at optimality, the constraint in (8b) must be satisfied with equality, otherwise the values of b_i's can be increased to have (8b) satisfied with equality while obtaining a higher value for the objective. The Lagrangian dual problem of (8) can be written as

$$\min_{\lambda} \quad g(\lambda), \tag{9}$$

subject to $\lambda \geqslant 0$, where

$$g(\lambda) = \max_{\{b_i,\bar{n}_i\}} \mathcal{L}(\mathbf{b}, \bar{\mathbf{n}}, \lambda), \tag{10}$$

and where $\mathcal{L}$ is the Lagrangian which can be written in two equivalent forms $\mathcal{L}(\mathbf{b}, \bar{\mathbf{n}}, \lambda) = \lambda + \sum_{i=1}^{M} L_i$, and $\mathcal{L}(\mathbf{b}, \bar{\mathbf{n}}, \lambda) = \lambda + \sum_{i=1}^{M} \tilde{L}_i$, where L_i and $\tilde{L}_i$ are expressed as

$$L_i = \bar{R}_i(b_i)\ \Delta F_i - \lambda b_i \Delta\bar{n}_i \tag{11a}$$

$$\tilde{L}_i = F_n(\bar{n}_i)\ \Delta\bar{R}_i - \lambda\bar{n}_i\Delta b_i, \tag{11b}$$

and where b_i's and $\bar{n}_i$'s are constrained to the region in (6c) and (6d) respectively, and λ is the Lagrangian dual variable. The following notations were applied $\Delta F_i = [F_n(\bar{n}_i) - F_n(\bar{n}_{i+1})]$, $\Delta\bar{n}_i = \bar{n}_i - \bar{n}_{i+1}$, $\Delta\bar{R}_i = \log_2\left(\frac{1+b_i}{1+b_{i-1}}\right)$, and $\Delta b_i = b_i - b_{i-1}$.

In the dual optimization problem (9), we have $2M$ optimization variables $(b_1, \cdots, b_M, \bar{n}_1, \cdots, \bar{n}_M)$ in addition to the dual variable λ. Therefore, the solution of the dual problem, which is obtained by equating the gradient of the Lagrangian to zero, consists of the following $2M + 1$ equations

$$\frac{\Delta\bar{R}_i}{\Delta b_i}\ f_n(\bar{n}_i) = \lambda \qquad \forall\ i, \tag{12a}$$

$$\frac{\Delta F_i}{\Delta\bar{n}_i}\ \bar{R}_i'(b_i) = \lambda \qquad \forall\ i, \tag{12b}$$

$$\sum_{i=1}^{M} \bar{n}_i \left[b_i - b_{i-1}\right] = 1, \tag{12c}$$

where $\bar{R}_i' = \frac{d\ \bar{R}_i}{d\ b_i}$.

The following theorem represents the major contribution of this paper because it can be used to develop an efficient algorithm to obtain the optimal solution of the joint rate and power allocation problem as explained in Section IV.

Theorem 1 (Existence and Uniqueness of the Solution): For Rayleigh fading channels, a strong duality exists between the primary and the dual problems, and hence the solution of (12) is always unique.

The proof of this theorem is based on the results of [1], where it was shown that for the case of fixed rates of the layers (fixed b_i's) under Rayleigh fading, (12a) can be written as:

$$\frac{\Delta\bar{R}_i}{\Delta b_i}\exp(-\frac{1}{G\bar{n}_i}) = G\bar{n}_i^2\lambda. \tag{13}$$

It was also shown that for a given λ, the solutions of (13) are shown in Fig. 1, where we plot the function $\frac{\Delta\bar{R}_i}{\Delta b_i}\frac{1}{G\bar{n}_i^2}\exp(-\frac{1}{G\bar{n}_i})$ for the different layers against n_i. Note that different layers have different $\frac{\Delta\bar{R}_i}{\Delta b_i}$.

In order to solve the Lagrangian dual problem in (9), we need to find λ that minimizes $g(\lambda)$. It was also shown in [1] that weak duality only occurs when the optimal value of λ results in two optimal sets of power allocations, one set is allocating power to a number of layers N such that $\sum_{i=1}^{N}\bar{n}_i\Delta b_i \leqslant 1$ and the other set is allocating zero power for Layer N and allocating the power between the remaining $N-1$ layers, such that $\sum_{i=1}^{N-1}\bar{n}_i\Delta b_i \geqslant 1$. The following lemma characterized the threshold values of $\bar{n}$ and λ at which this transition from any number of layers N to a number of layers less by one, $N-1$, occurs.

Lemma 1 (A threshold to decide number of layers):

For a given value of λ, if $\bar{n}_i \leq 1/G$, then it is optimal to set $\bar{n}_i = 0$. Therefore, there exists a unique threshold $\bar{n}^{(th)} = 1/G$, which indicates the optimal number of transmitted layers for any given value of λ.

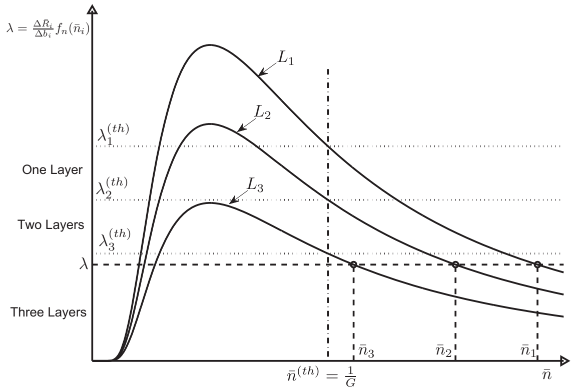

Fig. 1. The solutions $\bar{n}_i$'s for a given λ and the threshold value $\bar{n}^{(th)}$ used to determine the optimal number of layers.

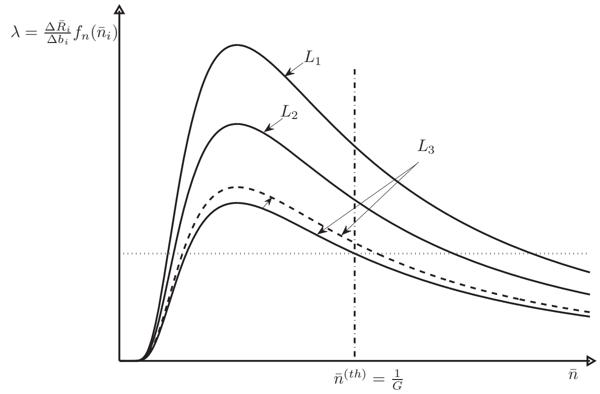

Fig. 2. Increasing $\frac{\Delta \bar{R}_3}{\Delta b_3}$ in Case 1 with $N = 3$.

Also if we define $\lambda_i^{(th)} = \frac{\Delta \bar{R}_i}{\Delta b_i} f(\bar{n}^{(th)})$ as the value of λ that corresponds to $\bar{n}^{(th)}$ for layer i, then for a given value of λ, if $\lambda > \lambda_i^{(th)}$, it is optimal to set $\bar{n}_i = 0$, and if $\lambda < \lambda_i^{(th)}$, it is optimal to assign a finite value to $\bar{\gamma}_i$ according to (12a). These threshold values of λ are shown as the dotted lines in Fig. 1. Next we show that this case is impossible since for each of such cases of multiple solutions, there will be another set of layers/rates that gives a higher value for the Lagrangian than the multiple solutions.

Case 1: We will start with the case of assuming that for a given λ the Lagrangian can be maximized by $N-1$ layers or N layers (both give the same value of the Lagrangian) with the same set of rates for the first $N-1$ layers. As indicated in Lemma 1, this case may happen when $\bar{n}_N = 1/G$, as shown in Fig. 2. Also, in this case the Lagrangian $L_N = 0$. For this case, we can show that this assumption is incorrect. This can be shown by increasing $\frac{\Delta \bar{R}_N}{\Delta b_N}$, and this can be done by decreasing the rate allocated to Layer N. This will lead to increasing the value of $\bar{n}_N$. Noting that L_N can be written as

$$L_N = \Delta \bar{R}_N \left(F_n(\bar{n}_N) - \bar{n}_N f_n(\bar{n}_N) \right), \tag{14}$$

and it can be shown that for a Rayleigh fading,

$$\frac{\partial}{\partial \bar{n}_N} \left(F_n(\bar{n}_N) - \bar{n}_N f_n(\bar{n}_N) \right) = \left(\frac{2G\bar{n}_N - 1}{G^2 \bar{n}_N^3} \right) \exp\left(\frac{-1}{G\bar{n}_N}\right). \tag{15}$$

The differentiation in (15) is positive as long as $\bar{n}_N \geqslant \frac{1}{2G}$. This means that increasing $\bar{n}_N$ above $1/G$ will result in increasing $F_n(\bar{n}_N) - \bar{n}_N f_n(\bar{n}_N)$ from zero to a positive value. Noting that $\Delta \bar{R}_N$ is always positive, this will in turn increase L_N from zero to a positive value, which means that the maximum value of the Lagrangian for the given value of λ can be achieved using N layers and a different set of rates, which contradicts the original assumption.

Case 2: In this case we assume that for a given λ the Lagrangian can be maximized by N layers or K layers (both give the same value of the Lagrangian) with completely different set of rates. Without loss if generality, let's assume that $K < N$. It can be shown that if we choose to transmit with $K+1$ layers, where the rate of Layer $K+1$ is chosen such that $\frac{\Delta \bar{R}_{K+1}}{\Delta b_{K+1}}$ satisfies that $\lambda = \frac{\Delta \bar{R}_{K+1}}{\Delta b_{K+1}} f(\bar{n}^{(th)})$, it can be shown that in this case $L_{K+1} = 0$ and the value of the Lagrangian of the $K+1$ layers is equal to the value of the Lagrangian of the K layers and the N layers. Now, if we increase $\frac{\Delta \bar{R}_{K+1}}{\Delta b_{K+1}}$ of Layer $K+1$, this will increase $\bar{n}_{K+1}$, which will in turn increase L_{K+1} from zero to a positive value similar to Case 1. This means that for this value of λ a higher value can be achieved with $K+1$ layers, which contradicts the original assumption.

Case 3: In case 2, we assumed that we can always increase the smaller number of layers by one. But what about the assumption that for a given λ the Lagrangian can be maximized by the maximum number of layers; i.e., M layers, with two different sets of rates, and that we can not increase the number of layers more than M? In this case we can show that for a given λ, it is impossible to have more than one solution to the equations in (12a) and (12b) with the same number of layers. The proof is omitted due to space limitation.

IV. Search Algorithm and Numerical Results

We propose the following procedure to solve the dual problem. We use a bisection search over λ to find the value of λ that achieves the power constraint (12c) at equality. To execute this one-dimensional bisection search, we need to solve the $2M$ equations in (12a) and (12b) at each step for the given value of λ. Therefore, we use an inner one-dimensional bisection search over $\bar{n}_M$. By setting a value of $\bar{n}_M$ we can obtain b_M from (12b) with $i = M$. Then, using $\bar{n}_M$ and b_M, we can obtain b_{M-1} from (12a) with $i = M$. Repeating the same procedure we can then obtain $\bar{n}_{M-1}$ and b_{M-2} and so on until we obtain $\bar{n}_1$. There will be one equation left which is (12a) with $i = 1$. Therefore, the inner bisection search is done in order to find the solution that satisfies this last equation.

The described algorithm involves a two-level of one-dimensional searches. In the outer bisection search, the computation of the power constraint involves M additions, M

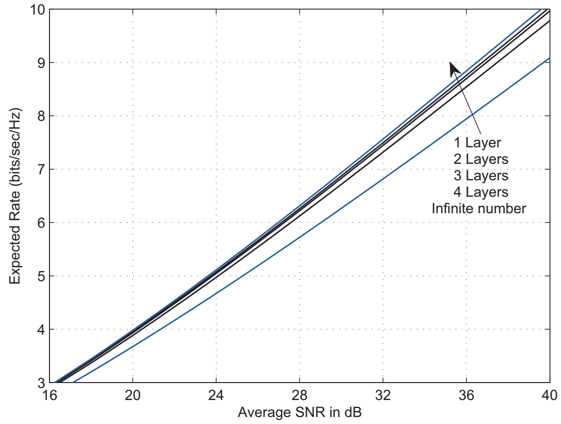

Fig. 3. The maximum expected sum rate versus the average SNR value of a Rayleigh fading channel for one, two, three, four or infinite number of layers.

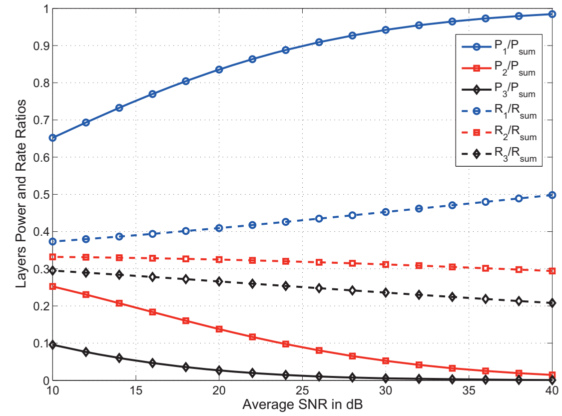

Fig. 4. The optimal relative power and rate ratios of the layers to maximize the expected sum rate versus the average SNR value of a Rayleigh fading channel in a three-layer system.

subtractions and M multiplications yielding a linear order $O(M)$ for the computation load. In the inner bisection search, we apply $2M$ equations yielding also to a linear order $O(M)$. Therefore, while executing the bisection search over λ, we need for any given value of λ a linear order of computation to obtain the $2M$ parameters and then we need another linear order of computation to check the power constraint. The sum of the two linear order operations will result in a linear order of complexity overall. The number of λ values needed while executing the outer bisection search is independent of the number of layers. Thus, the overall complexity of the proposed solution is $O(M)$ with respect to the number of layers M.

Fig. 3 shows a comparison between the maximum sum rate for multi-layer transmission with different number of layers, where the rate and power are optimized jointly to maximize the sum rate. We observe that even with a low number of layers, like two or three layers, we can achieve an expected rate that is close to the maximum possible rate achieved by an infinite number of layers.

Fig. 4 shows the optimal power and rate distribution over the layers in order to maximize the expected rate in a three-layer scenario. The layer that is decoded first is allocated most of the power and the layer that is decoded last is allocated the smallest share of the total power. Similarly, the layer that is decoded first has the highest rate, and the layer that is decoded last has the smallest rate among the three layers. However, the rates of the layers are relatively close to each other, unlike the power ratios of the layers.

V. Conclusion

In this paper we considered a multilayer transmission system that applied superposition coding at the transmitter and successive interference cancelation at the receiver over Rayleigh fading channel. We solved the problem of joint optimal allocation of the source power and the rates of the layers in order to maximize the expected total received rate, with the assumption of only channel statistics being available at the source node. We showed that strong duality exist, and hence, we proposed a two-dimensional bisection search algorithm to solve the problem for any number of layers. Our numerical results demonstrated that with a small number of layers, we can approach the upper bound of the expected rate, which is achieved by transmitting an infinite number of layers.

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