Two Way Communication over Exponential Family Type Channels

Lav R. Varshney
IBM Thomas J. Watson Research Center

***Abstract*—The capacity region of the additive exponential noise two-way channel is established. Adaptation is not necessary for optimal communication, and the rate region is simply a function of the one-way capacity. The result is extended to two-way channels of exponential family type, using a saddle point theorem.**

I. Introduction

When initiating network information theory, Shannon introduced the problem of two-way communication and established inner and outer bounds for the capacity region of any discrete memoryless two-way channel [1]. Jelinek proposed canonical decompositions of channels into independent one-way channels and special noiseless channels [2]. Much research effort has focused on finding the capacity region of the binary multiplier channel, see e.g. [3] as well as references therein and thereto.

The reason Shannon's bounds do not coincide in general is that optimal two-way communication requires *adaptation*: coding strategies must depend on previously received symbols transmitted by the other terminal. In certain special cases, however, strategies without adaptation are optimal, making an equivalence to systems with two independent one-way channels. These include the mod-2 adder channel, the class of *symmetric* discrete memoryless two-way channels [1] and the additive white Gaussian noise (AWGN) two-way channel [4]. Optimal strategies for AWGN two-way channels with additional interference and transmitter side information use dirty paper coding, also without adaptation [5].

Since communication strategies without adaptation simplify engineering system design, there is growing interest in establishing other settings where adaptation is useless from a capacity point of view, and self-interference cancellation is optimal [6]. This is especially true since full-duplex wireless communication is gaining popularity [7]–[9].

This paper considers the two-way exponential noise channel and finds the capacity region, showing that adaptation is useless. That is to say, the two-way channel in Fig. 1(a) is essentially equivalent to two parallel one-way channels in Fig. 1(b), when using appropriate coding techniques. This result extends to the class of two-way channels of exponential family type (E-type), including the exponential and AWGN two-way channels. While following Han's approach to proof [4], a new technical tool used is the saddle-point theorem for exponential families [10], [11].

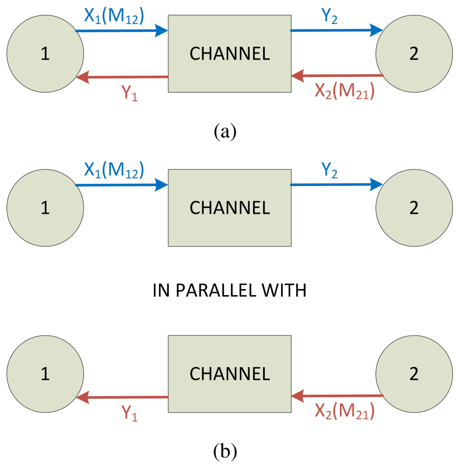

Fig. 1. A two-way channel of E-type is essentially equivalent to two one-way channels of E-type.

II. System Model and Definitions

As depicted in Fig. 1(a), a two-way channel has two terminals, each of which has an input and an output. Each terminal tries to reliably communicate a message to the other terminal. Terminal 1 sends channel input x_1, which is received by terminal 2 as channel output y_2 at terminal 2. Simultaneously, terminal 2 sends channel input x_2 which is received by terminal 1 as channel output y_1.

A memoryless two-way channel is completely specified by transition probability assignment $p(y_1, y_2|x_1, x_2)$ for $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$, $y_1 \in \mathcal{Y}_1$, and $y_2 \in \mathcal{Y}_2$. Herein, the input alphabets $\mathcal{X}_1, \mathcal{X}_2$ and output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$ are the real line $\mathbb{R}$.

Encoding at each terminal depends on both the message to be transmitted and the sequence of symbols previously received at that terminal. Likewise, decoding at each terminal depends on both the message transmitted and the sequence of symbols received at that terminal.

An (n, K_{21}, K_{12}) code for a communication system with a two-way channel is defined as the tuple $(\phi_1, \phi_2, \psi_1, \psi_2, \mathcal{M}_{12}, \mathcal{M}_{21})$, where $\mathcal{M}_{12} = \{1, 2, \ldots, K_{12}\}$ and $\mathcal{M}_{21} = \{1, 2, \ldots, K_{21}\}$ are the message sets available for communication from terminal 1 to 2 and from terminal 2 to 1, respectively.

The encoding functions ϕ_1 and ϕ_2 at terminals 1 and 2 respectively are communication strategies that depend on the message to be transmitted and the previously received channel output sequence. Hence ϕ_1 should be thought of as a sequence

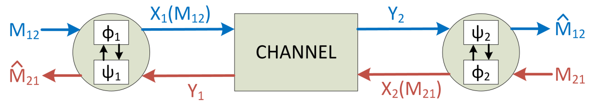

Fig. 2. A communication system for the two-way channel of E-type.

of codes for each possible block length:

$$\phi_1 = (\phi_{11}(\cdot), \phi_{12}(\cdot,\cdot), \cdots, \phi_{1n}(\cdot,\cdot,\cdots,\cdot)), \tag{1}$$

where $\phi_{11} : \mathcal{M}_{12} \mapsto \mathbb{R}$, $\phi_{12} : \mathcal{M}_{12} \times \mathbb{R} \mapsto \mathbb{R}$, and so on until $\phi_{1n} : \mathcal{M}_{12} \times \mathbb{R}^{n-1} \mapsto \mathbb{R}$. Likewise for ϕ_2.

The decoding functions ψ_1 and ψ_2 at terminals 1 and 2 respectively are defined as follows:

$$\psi_1 : \mathcal{M}_{12} \times \mathbb{R}^n \mapsto \mathcal{M}_{21} \tag{2}$$
$$\psi_2 : \mathcal{M}_{21} \times \mathbb{R}^n \mapsto \mathcal{M}_{12}. \tag{3}$$

The operation of ψ_1 is the following. At terminal 1, if message $m_{12} \in \mathcal{M}_{12}$ was sent and sequence $\vec{y}_1 \in \mathbb{R}^n$ was received, the decoder ψ_1 declares that $\hat{m}_{21} = \psi_1(m_{12}, \vec{y}_1) \in \mathcal{M}_{21}$ was received. Likewise for ψ_2.

This is all depicted in Fig. 2.

Probabilities of error P_{e1}, P_{e2} are defined as:

$$P_{e1} = \frac{1}{K_{12}K_{21}} \sum_{m_{21}=1}^{K_{21}} \sum_{m_{12}=1}^{K_{12}} \Pr[\hat{m}_{21} \neq m_{21} | m_{12}, m_{21} \text{ sent}] \tag{4}$$

$$P_{e2} = \frac{1}{K_{12}K_{21}} \sum_{m_{21}=1}^{K_{21}} \sum_{m_{12}=1}^{K_{12}} \Pr[\hat{m}_{12} \neq m_{12} | m_{12}, m_{21} \text{ sent}]. \tag{5}$$

and their maximum is $P_e = \max(P_{e1}, P_{e2})$.

Definition 1: A pair of nonnegative numbers (R_1, R_2) is an *achievable rate* for the two-way channel if for any $\eta > 0$ and any $0 < \lambda < 1$ there exists an (n, K_{21}, K_{12}) code such that

$$\frac{1}{n} \log K_{21} \geq R_1 - \eta, \tag{6}$$
$$\frac{1}{n} \log K_{12} \geq R_2 - \eta, \tag{7}$$

and $P_e \geq \lambda$.

Definition 2: The set of all achievable rates is the *capacity region* $\mathcal{C}$ of the two-way channel.

The goal is, of course, to determine a simple expression for the capacity region of general two-way channels, however this has been elusive, cf. [12]. Here we restrict attention to a particular class of noise models $p(y_1, y_2 | x_1, x_2)$: the two-way channels of E-type. Of particular note in these models is the additive way in which transmission in one direction interferes with transmission in the opposite direction.

Before proceeding, we review Shannon's inner and outer bounds for two-way channels.

III. Shannon's Bounds

Shannon established an inner bound and an outer bound for the capacity region of discrete memoryless two-way channels.

Let X_1 and X_2 be any input random variables (that meet cost constraints) with corresponding output random variables Y_1 and Y_2 induced by the two-way channel transition probability assignment. Then the probability distribution of $X_1X_2Y_1Y_2$ is given by:

$$\begin{aligned} &p_{X_1,X_2,Y_1,Y_2}(x_1, x_2, y_1, y_2) \\ &\quad = p_{X_1,X_2}(x_1, x_2) p_{Y_1,Y_2|X_1,X_2}(y_1, y_2 | x_1, x_2), \end{aligned} \tag{8}$$

where $p_{X_1,X_2}(x_1, x_2)$ is a general input distribution. For such an $X_1X_2Y_1Y_2$, let $G(X_1X_2Y_1Y_2)$ be the set of all rate pairs (R_1, R_2) that satisfy:

$$R_1 \leq I(X_1; Y_2 | X_2) \tag{9}$$
$$R_2 \leq I(X_2; Y_1 | X_1). \tag{10}$$

Separately, denote the set of all $X_1X_2Y_1Y_2$ with probability distribution of the form (8) as $\mathcal{Z}_O$ and denote as $\mathcal{Z}_I$ the subset of $X_1X_2Y_1Y_2$ with an input distribution that takes product form:

$$\begin{aligned} &p_{X_1,X_2,Y_1,Y_2}(x_1, x_2, y_1, y_2) \\ &\quad = p_{X_1}(x_1) p_{X_2}(x_2) p_{Y_1,Y_2|X_1,X_2}(y_1, y_2 | x_1, x_2). \end{aligned} \tag{11}$$

Now bringing things together, let

$$G_I = \text{convex closure of} \bigcup_{X_1X_2Y_1Y_2 \in \mathcal{Z}_I} G(X_1X_2Y_1Y_2) \tag{12}$$

and

$$G_O = \text{closure of} \bigcup_{X_1X_2Y_1Y_2 \in \mathcal{Z}_O} G(X_1X_2Y_1Y_2). \tag{13}$$

Theorem 1 ([1]): The rate regions G_I and G_O are an inner bound and an outer bound to the capacity region $\mathcal{C}$ of a discrete memoryless two-way channel:

$$G_I \subset \mathcal{C} \subset G_0. \tag{14}$$

Note the inner bound, defined with a product input distribution, is proven using an achievability scheme where the two terminals transmit symbols without adaptation to received symbols. Hence, if the inner and outer bounds coincide, the capacity region is achievable without adaptation. Further, the capacity region will simply be determined by the parallel one-way capacities of the channels, as in Fig. 1(b).

With the inner bound proven using Shannon's techniques and the outer bound reproven specifically for the AWGN two-way channel, the bounds do in fact coincide [4, Thm. 5]. We aim to expand this result to the entire class of E-type two-way channels, but first consider the specific example of the additive white exponential noise (AWEN) two-way channel to see the basic idea in a simpler setting and get an explicit rate region.

IV. AWEN Two-Way Channel

A. Definition

An AWEN two-way channel is defined by the channel mapping:

$$Y_1 = X_1 + X_2 + N_1 \tag{15}$$
$$Y_2 = X_1 + X_2 + N_2, \tag{16}$$

where N_1 and N_2 are (possibly dependent) exponential random variables with means m_1 and m_2 respectively.

Further there are expected amplitude constraints A_1 and A_2 on the channel input sequences $\{x_{1t}(m_{12})\}_{t=1}^n$ and $\{x_{2t}(m_{21})\}_{t=1}^n$, respectively:

$$E\left[\sum_{t=1}^{n}\{x_{1t}(m_{12})\}\right] \le nA_1 \tag{17}$$
$$E\left[\sum_{t=1}^{n}\{x_{2t}(m_{21})\}\right] \le nA_2, \tag{18}$$

where $m_{12} \in \mathcal{M}_{12}$ and $m_{21} \in \mathcal{M}_{21}$ are the messages to be conveyed and the expectations are taken to include the statistics of the output feedback.

The goal is to find the capacity region $\mathcal{C}(A_1, A_2)$ of this two-way channel.

B. Coding Theorem

As with the AWGN two-way channel [4], we need to prove the validity of Shannon's outer bound; arguments for the inner bound carry over directly from the discrete memoryless setting.

Let $G_O(A_1, A_2)$ and $G_I(A_1, A_2)$ be rate regions as defined in Sec. III, appropriately restricted to meet the average amplitude constraints.

Theorem 2: The rate region $G_O(A_1, A_2)$ is an outer bound and the rate region $G_I(A_1, A_2)$ is an inner bound on the capacity region $\mathcal{C}(A_1, A_2)$ of the AWEN two-way channel with average amplitude constraints A_1, A_2.

Proof: Directly applying Shannon's argument [1] yields the achievability of any rate $G_I(A_1, A_2)$.

Now we want to show $G_O(A_1, A_2)$ is an outer bound, following Han [4, Lemma 1]. We have message sets $\mathcal{M}_{12}, \mathcal{M}_{21}$ and a coding system satisfying amplitude constraints (17) and (18) with rate:

$$R_a = \tfrac{1}{n}\log|\mathcal{M}_a|,\ a \in \{12, 21\} \tag{19}$$

and attaining arbitrarily small P_e. Assume that messages m_{12}, m_{21} are equiprobably generated from their message sets and denote them by random variables W_1, W_2, respectively. Let the corresponding n-letter input and output variables be $X_1^n, X_2^n, Y_1^n, Y_2^n$.

By Fano's inequality,

$$H(W_1|X_2^n Y_2^n) \le n\epsilon \tag{20}$$
$$H(W_2|X_1^n Y_1^n) \le n\epsilon \tag{21}$$

where ϵ is arbitrarily small.

This leads to the following result.

$$\begin{aligned} nR_1 &= H(W_1) \\ &\le H(W_1|W_2) - H(W_1|X_2^n Y_2^n W_2) + n\epsilon \\ &= H(W_1|W_2) - H(W_1|Y_2^n W_2) + n\epsilon \\ &= I(W_1; Y_2^n|W_2) + n\epsilon \\ &\overset{(a)}{=} \sum_{t=1}^{n} I(W_1; Y_{2t}|W_2 Y_2^{t-1} X_{2t} X_2^{t-1}) + n\epsilon \\ &\le \sum_{t=1}^{n} I(W X_{1t} W_1; Y_{2t}|W_2 Y_2^{t-1} X_{2t} X_2^{t-1}) + n\epsilon \\ &\overset{(b)}{=} \sum_{t=1}^{n} I(X_{1t}; Y_{2t}|W_2 Y_2^{t-1} W_{2t} X_2^{t-1}) + n\epsilon \\ &\le \sum_{t=1}^{n} I(X_{1t} W_2 Y_2^{t-1} X_2^{t-1}; Y_{2t}|X_{2t}) + n\epsilon \\ &\overset{(c)}{=} \sum_{t=1}^{n} I(X_{1t}; Y_{2t}|X_{2t}) + n\epsilon \end{aligned} \tag{22}$$

where (a) follows since $W_2 Y_2^{t-1}$ determines $X_{2t} X_2^{t-1}$; (b) follows since W_1 can affect Y_{2t} only through (X_{1t}, X_{2t}); and (c) follows since $W_2 X_2^{t-1} Y_2^{t-1}$ is independent of Y_{2t} given $X_{1t} X_{2t}$.

Let Q be a random variable taking values $1, \ldots, n$ equiprobably, and define

$$X_1 = X_{1t} \text{ given } Q = t$$
$$X_2 = X_{2t} \text{ given } Q = t$$
$$Y_1 = Y_{1t} \text{ given } Q = t$$
$$Y_2 = Y_{2t} \text{ given } Q = t.$$

Then (22) can be rewritten as follows.

$$\begin{aligned} R_1 &\le I(X_1; Y_2|X_2 Q) + \epsilon \le I(Q X_1; Y_2|X_2) + \epsilon \\ &= I(X_1; Y_2|X_2) + \epsilon \end{aligned} \tag{23}$$

since Q is independent of Y_2 given X_1, X_2. Identically:

$$R_2 \le I(X_2; Y_1|X_1) + \epsilon. \tag{24}$$

Noticing (17) and (18) imply $E(X_1) \le A_1$ and $E(X_2) \le A_2$, (23) and (24) yield the result:

$$(R_1 - \epsilon, R_2 - \epsilon) \in G_O(A_1, A_2). \tag{25}$$

Since ϵ is arbitrary and $G_O(A_1, A_2)$ is closed by definition, $(R_1, R_2) \in G_O(A_1, A_2)$. ■

C. Characterization

Let us now compare the inner and outer bounds for the AWEN two-way channel, to find they coincide and yield a simple expression for the capacity region.

Theorem 3:

$$\mathcal{C}(A_1, A_2) = \begin{cases} R_1 \le \log\left(1 + \frac{A_2}{m_1}\right) \\ R_2 \le \log\left(1 + \frac{A_1}{m_2}\right) \end{cases} \tag{26}$$

is the capacity region for the two-way AWEN channel with amplitude constraints A_1, A_2.

Proof: The basic idea is to show that the inner and outer bounds coincide.

INNER BOUND*:* Use independent one-way capacity-achieving inputs X_1 and X_2 that meet the amplitude constraint $E[X_1] \leq A_1$ and $E[X_2] \leq A_2$. By the saddle-point result of [10], this is as follows:

$$\Pr[X_1 = 0] = \frac{m_1}{A_1 + m_1} \tag{27}$$

$$\Pr[X_1 = x_1 | X_1 > 0] = e^{-\frac{x_1}{m_1 + A_1}} \tag{28}$$

and

$$\Pr[X_2 = 0] = \frac{m_2}{A_2 + m_2} \tag{29}$$

$$\Pr[X_2 = x_2 | X_2 > 0] = e^{-\frac{x_2}{m_2 + A_2}}. \tag{30}$$

Then since the effect of X_2 on Y_2 can be subtracted:

$$I(X_1; Y_2 | X_2) = \log\left(1 + \tfrac{A_1}{m_2}\right) \tag{31}$$

by the one-way saddle-point computation of [10].

Likewise,

$$I(X_2; Y_1 | X_1) = \log\left(1 + \tfrac{A_2}{m_1}\right). \tag{32}$$

OUTER BOUND*:* By definition of the outer bound:

$$R_1 \leq I(X_1; Y_2 | X_2) \tag{33}$$

$$= H(Y_2 | X_2) - H(Y_2 | X_1 X_2) \tag{34}$$

by the definition of conditional mutual information.

Since X_1 and X_2 are independent of the noise N_2, it follows from the channel definition that

$$H(Y_2 | X_1 X_2) = H(N_2) = \log(m_2 e). \tag{35}$$

Now consider the following (since one can subtract away the effect of X_2 with knowledge of it).

$$\begin{aligned} H(Y_2|X_2) &= H(X_1 + N_2 | X_2) \\ &= \int_0^\infty p(x_2) H(X_1 + N_2 | X_2 = x_2) dx_2 \\ &= \int_0^\infty p(x_2) H(S | X_2 = x_2) dx_2, \end{aligned}$$

where S is defined to be $S = X_1 + N_2$.

Note that S is non-negative and governed by an expected amplitude constraint since X_1 has constrained expected amplitude and N_2 has known mean.

So using an "entropy-amplitude" inequality, which arises from the maximum entropy principle for non-negative random variable with expected amplitude constraint, we get

$$H(S | X_2) \leq \log\left(E_X[S] e\right), \tag{36}$$

where $E_X[\cdot]$ is expectation conditioned on $X_2 = x$.

So

$$\begin{aligned} H(Y_2|X_2) &\leq \int_0^\infty p(x_2) \log\left(E_X[S]e\right) dx_2 \\ &\stackrel{(a)}{\leq} \log\left(e \int_0^\infty p(x_2) E_X[S] dx_2\right) \\ &= \log\left(eE[S]\right) \\ &\leq \log\left(e(m_2 + A_1)\right) \end{aligned}$$

where (a) is due to the concavity of the log function.

Thus we end up with the following.

$$\begin{aligned} R_1 &\leq -\log(m_2 e) + \log(m_2 e + A_1 e) \\ &= \log\left(\frac{m_2 e + A_1 e}{m_2 e}\right) \\ &= \log\left(1 + \frac{A_1}{m_2}\right) \end{aligned}$$

By a symmetric argument,

$$R_2 \leq \log\left(1 + \frac{A_2}{m_1}\right). \tag{37}$$

Since the inner and outer bounds coincide, we have the capacity region. ■

Note that adaptation is not required to achieve the capacity region for the AWEN two-way channel.

V. EXPONENTIAL FAMILY TYPE TWO-WAY CHANNELS

A. Definition

Consider cost function $\rho : \mathbb{R} \mapsto \mathbb{R}^+$ and define a corresponding E-type one-way channel as

$$S = X + N, \tag{38}$$

where N is additive noise independent of X and distributed according to

$$p_N(x) = \frac{1}{Z(\beta)} e^{-\beta \rho(x)}, \tag{39}$$

for any $\beta \in \mathbb{R}$, where partition function $Z(\beta) = \int e^{-\beta\rho(x)} dx$.

An E-type two-way channel is defined by:

$$Y_1 = X_1 + X_2 + N_1 \tag{40}$$

$$Y_2 = X_1 + X_2 + N_2, \tag{41}$$

where N_1 and N_2 are (possibly dependent) exponential family random variables as above, with ρ-moments $E[\rho(N_1)] = m_1$ and $E[\rho(N_2)] = m_2$, respectively.

Further there are expected ρ-moment constraints A_1 and A_2 on the channel input sequences $\{x_{1t}(m_{12})\}_{t=1}^n$ and $\{x_{2t}(m_{21})\}_{t=1}^n$, respectively:

$$E\left[\sum_{t=1}^n \rho\left\{x_{1t}(m_{12})\right\}\right] \leq nA_1 \tag{42}$$

$$E\left[\sum_{t=1}^n \rho\left\{x_{2t}(m_{21})\right\}\right] \leq nA_2, \tag{43}$$

where $m_{12} \in \mathcal{M}_{12}$ and $m_{21} \in \mathcal{M}_{21}$ are the messages to be conveyed and expectations include output feedback statistics.

B. Coding Theorem

Let $G_O(A_1, A_2)$ and $G_I(A_1, A_2)$ be rate regions as in Sec. III, appropriately restricted to meet ρ-moment constraints.

Theorem 4: The rate region $G_O(A_1, A_2)$ is an outer bound and the rate region $G_I(A_1, A_2)$ is an inner bound on the capacity region $\mathcal{C}(A_1, A_2)$ of the E-type two-way channel with cost ρ and any β with ρ-moment constraints A_1, A_2.

Proof: Follows proof of Thm. 2, *mutatis mutandis*. ∎

C. Characterization

Now we see that here too, adaptation is not necessary to achieve capacity, as the inner and outer bounds coincide. The result requires use of saddle-point theorems for exponential families.

Definition 3: Consider an input distribution p_X and an independent noise p_N, with corresponding output $S = X + N \sim p_S$. Then p_X and p_N are *ρ-saddle-point admissible* if:

1) p_N is of the form (39) for some β,
2) p_S is of the form (39) for some $\beta = \gamma$, and
3) $I(X; N) = I(X; X + N) < \infty$.

Theorem 5 ([11]): Let an input distribution p_X and an additive noise p_N be ρ-saddle-point admissible. Then for any other input p_{X^o}:

$$I(X^o; N) \leq I(X; N), \tag{44}$$

with equality if and only if $p_{X^o} = p_X$.

It should be noted further that since S is of the exponential family, it is maximum entropy under the ρ-moment constraint.

Theorem 6: The capacity region $\mathcal{C}(A_1, A_2)$ for the E-type two-way channel with cost ρ with ρ-moment constraints A_1, A_2 is given by $G_I(A_1, A_2)$.

Proof: The inner bound $G_I(A_1, A_2)$ will be determined by the one-way capacities $C(A_1), C(A_2)$, and in particular due to Thm. 5 the capacity-achieving input distributions and the noise distribution will be ρ-saddle-point admissible.

We proceed as in the OUTER BOUND part of the proof of Thm. 3. By definition:

$$R_1 \leq I(X_1; Y_2|X_2) = H(Y_2|X_2) - H(Y_2|X_1 X_2). \tag{45}$$

Since X_1 and X_2 are independent of the noise N_2:

$$H(Y_2|X_1 X_2) = H(N_2). \tag{46}$$

Since effects of X_2 can be subtracted, with knowledge of it:

$$\begin{aligned} H(Y_2|X_2) &= H(X_1 + N_2|X_2) \\ &= \int_0^\infty p(x_2) H(S|X_2 = x_2) dx_2, \end{aligned}$$

where $S = X_1 + N_2$.

Note that S is governed by a ρ-moment constraint since X_1 has constrained ρ-moment and N_2 has known ρ-moment. So using the maximum entropy principle, we know that $H(S|X_2)$ is upper-bounded by the differential entropy of the maximum entropy distribution with ρ-moment $E_X[\rho(S)]$, where $E_X[\cdot]$ is expectation conditioned on $X_2 = x$. But we also know the maximum entropy distribution is from the exponential family. So we can then say that $H(Y_2|X_2)$ is upper-bounded by the differential entropy of the exponential family distribution with ρ-moment given by $m_2 + A_1$.

Recalling $R_1 \leq H(Y_2|X_2) - H(N_2)$ and what we have just derived, Thm. 5 implies $R_1 \leq C(A_1)$. By a symmetric argument, $R_2 \leq C(A_2)$.

Since the outer bound coincides with the inner bound, we have the capacity region. ∎

VI. Conclusion

Exponential family distributions have a property called *convolution divisibility* which leads to the saddle point theorem of Coleman and Raginsky [11, Rem. 1]. This paper has applied that technical machinery to prove a structural result for the seminal network information theory problem of two-way communication: adaptation is useless for the class of E-type two-way channels. This extends the results of Han [4] beyond the special case of Gaussian two-way channels.

Going forward it is of interest to see whether this technical machinery sheds light on other network information theory problems, whether closely related ones like two-way communication with interference [5] and two-way relaying, or further afield where results for Gaussian settings are known, but perhaps not for general exponential families [13].

Acknowledgment

Thanks to D. V. Oppenheim for pushing me to think about adaptation and coordination and to the 2012 School of Information Theory organizing committee (P. Cuff, A. B. Wagner, and A. S. Avestimehr) for bringing two-way channels to mind.

References

[1] C. E. Shannon, "Two-way communication channels," in *Proc. 4th Berkeley Symp. Math. Stat. Probab.*, J. Neyman, Ed., vol. 1. Berkeley: University of California Press, 1961, pp. 611–644.
[2] F. Jelinek, "Coding for and decomposition of two-way channels," *IEEE Trans. Inf. Theory*, vol. IT-10, no. 1, pp. 5–17, Jan. 1964.
[3] Z. Zhang, T. Berger, and J. P. M. Schalkwijk, "New outer bounds to capacity regions of two-way channels," *IEEE Trans. Inf. Theory*, vol. IT-32, no. 3, pp. 383–386, May 1986.
[4] T. S. Han, "A general coding scheme for the two-way channel," *IEEE Trans. Inf. Theory*, vol. 30, no. 1, pp. 35–44, Jan. 1984.
[5] R. Khosravi-Farsani and M. Rostami, "Two-way writing on dirty paper," *IEEE Commun. Lett.*, vol. 15, no. 7, pp. 689–691, Jul. 2011.
[6] Z. Cheng and N. Devroye, "Two-way networks: when adaptation is useless," arXiv:1206.6145v1 [cs.IT]., Jun. 2012.
[7] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving single channel, full duplex wireless communication," in *Proc. 16th Annu. Int. Conf. Mobile Comput. Netw. (MobiCom'10)*, Sep. 2010, pp. 1–12.
[8] M. Jain, J. I. Choi, T. M. Kim, D. Bharadia, S. Seth, K. Srinivasan, P. Levis, S. Katti, and P. Sinha, "Practical, real-time, full duplex wireless," in *Proc. 17th Annu. Int. Conf. Mobile Comput. Netw. (MobiCom'11)*, Sep. 2011, pp. 301–312.
[9] M. Duarte, "Full-duplex wireless: Design, implementation and characterization," Ph.D. dissertation, Rice University, Houston, TX, Apr. 2012.
[10] S. Verdú, "The exponential distribution in information theory," *Probl. Inf. Transm.*, vol. 32, no. 1, pp. 100–111, Jan.-Mar. 1996.
[11] T. P. Coleman and M. Raginsky, "Mutual information saddle points in channels of exponential family type," in *Proc. 2010 IEEE Int. Symp. Inf. Theory*, Jun. 2010, pp. 1355–1359.
[12] G. Kramer, "Capacity results for the discrete memoryless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 4–21, Jan. 2003.
[13] A. El Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge: Cambridge University Press, 2011.