# Characterization and Optimization of the Constrained Capacity of Coherent Fading Channels Driven by Arbitrary Inputs: A Mellin Transform Based Asymptotic Approach

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***Abstract*—We unveil asymptotic characterizations of the average minimum mean-squared error (MMSE) and the average mutual information in scalar fading coherent channels, where the receiver knows the exact fading channel state but the transmitter knows only the fading channel distribution, driven by a range of inputs both in the regimes of low-SNR – and at the heart of the novelty of the contribution – high-SNR. We also unveil connections to and generalizations of the MMSE dimension. By capitalizing on the characterizations, we conclude with applications of the results to the optimization of the constrained capacity of a bank of parallel independent coherent fading channels.**

## I. Introduction

The characterization of the constrained capacity of fading coherent channels, where the receiver knows the exact fading channel realization but the transmitter knows only the fading channel distribution, driven by arbitrary inputs is a problem with significant practical interest. The importance relates to the fact that this quantity defines the highest information transmission rate between the transmitter and the receiver with a message error probability that approaches zero in systems that employ arbitrary and practical signalling schemes, such as $m$-phase shift keying (PSK) or $m$-quadrature amplitude modulation (QAM) constellations. Unfortunately, the characterization of the constrained capacity, which – assuming that the fading channel variation over time is stationary and ergodic – is given by the average over the fading channel gain distribution of the mutual information between the input and the output of the channel conditioned on the channel gain, is complex due to the absence of closed-form expressions for the mutual information as well as the average mutual information.

The recent years however have witnessed the emergence of an innovative approach that leads to the characterization – in asymptotic regimes – of the constrained capacity of key communications channels [1], [2], [3], [4]. The approach, which has been mainly applied to channels that are not subject to fading, capitalizes on connections between mutual information and minimum mean-squared error (MMSE) [5], [6], in order to express the mutual information or bounds to the mutual information in terms of the MMSE or bounds to the MMSE, respectively.

In this paper, instead, we leverage connections between the average value of the mutual information and the average value of the MMSE in a coherent fading channel to obtain characterizations of the average mutual information from characterizations of the average MMSE. We also leverage key Mellin transform based techniques [7] that lead to the asymptotic expansions of the quantities not only in the regime of low signal-to-noise ratio (SNR) but also - at the heart of the novelty of the contribution - in the regime of high SNR. The asymptotic analysis, which bypasses the difficulty associated with the construction of general non-asymptotic results, often applies to a variety of practical scenarios leading to considerable insight.

We then capitalize on the asymptotic characterizations to solve an optimal power allocation problem in a bank of parallel independent coherent fading channels driven by discrete inputs, and subject to Rayleigh or Ricean fading. Via the new asymptotic characterizations it is then possible to generalize the optimal power allocation results in [2].

Due to space limitations, the technical proofs are referred to [8], where we also present various additional results.

## II. Model and Definitions

We consider a standard frequency-flat fading channel, which for a single time instant, can be modeled as follows:

$$y = \sqrt{snr}hx + n \tag{1}$$

where $y \in \mathbb{C}$ represents the channel output, $x \in \mathbb{C}$ represents the channel input, $h$ is a complex scalar random variable such that $E\{|h|^2\} < +\infty$ which represents the random channel fading gain between the input and the output of the channel, and $n \in \mathbb{C}$ is a circularly symmetric complex scalar Gaussian random variable with zero mean and unit variance which represents standard noise. The scaling factor $snr > 0$ relates to the signal-to-noise ratio. We assume that $x$, $h$ and $n$

are independent random variables. We also assume that the receiver knows the exact realization of the channel gain but the transmitter knows only the distribution of the channel gain.

In particular, we consider two conventional fading models: *i*) the Rayleigh and *ii*) the Ricean fading models. In the Rayleigh fading model, the channel gain is $h \sim \mathcal{CN}\left(0, 2\sigma^2\right)$, where $\sigma > 0$, and hence $|h| \sim \text{Rayleigh}\left(\sigma\right)$, i.e.,

$$f_{|h|}\left(r\right) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right). \tag{2}$$

In the Ricean fading model, the channel gain is $h \sim \mathcal{CN}\left(\mu, 2\sigma^2\right)$, where $\mu \in \mathbb{C} \setminus \{0\}$ and $\sigma > 0$, and hence $|h| \sim \text{Rice}\left(|\mu|, \sigma\right)$, i.e.,

$$f_{|h|}\left(r\right) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2 + |\mu|^2}{2\sigma^2}\right) I_0\left(\frac{r|\mu|}{\sigma^2}\right) \tag{3}$$

where $I_0\left(\cdot\right)$ is the modified Bessel function of the first kind with order zero.

We now introduce a series of quantities which will be used throughout the paper. We define the average value of the MMSE and the average value of the mutual information as follows:

$$\overline{mmse}\left(snr\right) :=: \overline{mmse}\left(x;y\right) := E\left\{\left|hx - hE\left\{x|y,h\right\}\right|^2\right\} \tag{4}$$

$$\overline{I}\left(snr\right) :=: \overline{I}\left(x;y\right) := E\left\{\log\frac{f_{x,y|h}\left(x,y|h\right)}{f_{x|h}\left(x|h\right)f_{y|h}\left(y|h\right)}\right\} \tag{5}$$

It will also be relevant to define the MMSE and the mutual information associated with the *canonical* additive white Gaussian noise (AWGN) channel model given by:

$$y = \sqrt{snr}x + n \tag{6}$$

where $y \in \mathbb{C}$ represents the channel output, $x \in \mathbb{C}$ represents the channel input and $n \in \mathbb{C}$ is a circularly symmetric complex scalar Gaussian random variable with zero mean and unit variance which represents standard noise. The scaling factor $snr > 0$ also relates to the signal-to-noise ratio. We assume that $x$ and $n$ are independent random variables.

Now, the MMSE associated with the estimation of the input given the output of the *canonical* AWGN channel model in (6) is defined as:

$$mmse\left(snr\right) :=: mmse\left(x;y\right) := E\left\{\left|x - E\left\{x|y\right\}\right|^2\right\} \tag{7}$$

and the mutual information between the input and the output of the *canonical* AWGN channel model in (6) is defined as:

$$I\left(snr\right) :=: I\left(x;y\right) := E\left\{\log\frac{f_{x,y}\left(x,y\right)}{f_x\left(x\right)f_y\left(y\right)}\right\} \tag{8}$$

Therefore, it is very simple to express the average MMSE in (4) in terms of the *canonical* MMSE in (7) as:

$$\overline{mmse}\left(snr\right) = E\left\{|h|^2 mmse\left(snr|h|^2\right)\right\} \tag{9}$$

and the average mutual information in (5) in terms of the *canonical* mutual information in (8) as:

$$\overline{I}\left(snr\right) = E\left\{I\left(snr|h|^2\right)\right\} \tag{10}$$

The objective is to characterize the asymptotic behavior, as $snr \to \infty$ or as $snr \to 0^+$, of the average MMSE and the average mutual information, which, when the channel variation over time is stationary and ergodic, leads to the constrained capacity of a fading coherent channel driven by a specific input distribution. We adopt a two-step procedure: *i*) We first obtain, via Mellin transform expansion techniques, a characterization of the asymptotic behavior of the average MMSE in (4); *ii*) We then obtain a characterization of the asymptotic behavior of the average mutual information in (5) by capitalizing on the now well-known relation between average MMSE and average mutual information given by [6]:

$$\frac{d\overline{I}\left(snr\right)}{dsnr} = \overline{mmse}\left(snr\right) \tag{11}$$

We note that the Mellin transform of a function $f(\cdot)$ [7]:

$$M\left[f; 1+z\right] := \int_0^{+\infty} t^z f\left(t\right) dt,$$

plays a key role in the definition of the asymptotic expansions.

## III. High-$snr$ Regime

We now consider the construction of high-$snr$ asymptotic expansions of the average MMSE and the average mutual information in a fading coherent channel driven by inputs that conform to arbitrary discrete distributions with finite support.

The construction of the high-$snr$ expansions capitalizes on the fact that the average MMSE, which can be written as

$$\overline{mmse}\left(snr\right) = \int_0^{+\infty} \frac{\sqrt{t} f_{|h|}\left(\sqrt{t}\right)}{2} mmse\left(snr \cdot t\right) dt \tag{12}$$

can be seen as a certain type of transform with a kernel of monotonic argument [7]. This enables the use of a key asymptotic expansion of integrals technique, which exploits Mellin transforms [7], that leads to a dissection of the asymptotic behavior of the quantities in a large variety of scenarios. In fact, the construction of the high-$snr$ expansions of the integral representation in (12) can be effected by exploiting a range of techniques, such as the Mellin transform method [7] or the integration by parts methods [7]. It is important to emphasize though that the Mellin transform technique – when compared to the integration by parts technique – is able to produce expansions for a wider range of fading and input distributions.

The construction of the high-$snr$ expansions of the average MMSE associated with arbitrary discrete inputs with finite support capitalizes on the characterization of the decay of the *canonical* MMSE in the regime of high-$snr$ in [1].

The following Theorem, which capitalizes on the Mellin transform method [7], applies to general fading models.

**Theorem 1.** *Consider the fading coherent channel in* (1) *driven by an arbitrary discrete input with finite support and define the following quantities which relate to the distribution* *of the fading process*

$$\gamma := \inf \left\{ \gamma^* : f_{|h|}(t) = O\left(t^{-2\gamma^*-1}\right), t \to 0^+ \right\},$$
$$\delta := \sup \left\{ \delta^* : f_{|h|}(t) = O\left(t^{-2\delta^*-1}\right), t \to +\infty \right\},$$
$$C := (0,+\infty) \cap (1-\delta, 1-\gamma).$$

*If $\gamma < \delta$, $C \neq \emptyset$, the fading distribution behaves as*

$$f_{|h|}(t) \sim 2e^{-qt^{-2\mu}} \sum_{m=0}^{+\infty} \sum_{n=0}^{\overline{N}(m)} p_{mn} t^{2a_m-1} (2\log t)^n, t \to 0^+$$

*where $q \geq 0$, $\mu > 0$, $\overline{N}(m) < +\infty$, $p_{mn} \in \mathbb{R}$, $a_m \in \mathbb{R}$ and $a_m \uparrow +\infty$, and there exists $c \in C$ such that for every $x \geq c$,*

$$M\left[f_{|h|}; 3 - 2(x+iy)\right] = O\left(|y|^{-2}\right), |y| \to +\infty$$

*then, in the regime of high-$snr$, for $q = 0$ it follows that*

$$\begin{aligned} &\overline{mmse}(snr) \sim \\ &\sim \sum_{m=0}^{+\infty} snr^{-1-a_m} \sum_{n=0}^{\overline{N}(m)} p_{mn} \times \\ &\quad \times \sum_{j=0}^{n} \binom{n}{j} \left(-\log(snr)\right)^j M^{(n-j)}\left[mmse; 1+a_m\right] \end{aligned}$$

*and for $q \neq 0$ it follows that*

$$\overline{mmse}(snr) = o\left(snr^{-R}\right), \forall R > 0$$

Theorem 1 reveals that in the important scenario which accommodates for the most common fading models (i.e., $q = 0$) the asymptotic behavior as $snr \to +\infty$ of the average MMSE depends mainly on the asymptotic behavior as $t \to 0^+$ of $f_{|h|}(t)$. Interestingly, the fact that the behavior of some quantities around zero determine the behavior of other quantities in the infinity has already been pointed out [9], [10], [11]. Theorem 1 also reveals that the asymptotic behavior as $snr \to +\infty$ of the average MMSE depends on the input distribution via the Mellin transform of the *canonical* MMSE.

Theorem 1 can also now be immediately specialized to the most common fading distributions.

**Corollary 2.** *Consider a Rayleigh* (2) *or Ricean* (3) *fading coherent channel* (1) *driven by an arbitrary discrete input with finite support. Then, in the regime of high-$snr$,*

$$\overline{mmse}(snr) \sim \frac{e^{-\frac{|\mu|^2}{2\sigma^2}}}{snr^2} \sum_{m=0}^{+\infty} \frac{\tau_m^{|\mu|,\sigma} M\left[mmse; 2+m\right]}{snr^m}$$

*where*

$$\tau_m^{|\mu|,\sigma} := \sum_{n=0}^{m} \left( \frac{(-1)^{m-n}}{(m-n)!\,(2\sigma^2)^{m+n+1}} \frac{|\mu|^{2n}}{(n!)^2} \right)$$

The high-$snr$ asymptotic expansion of the average MMSE embodied in Corollary 2 now leads immediately to a high-$snr$ asymptotic expansion of the average mutual information, via the integral representation of (11) as follows:

$$\overline{I}(snr) = \log(m) - \int_{snr}^{\infty} \overline{mmse}(\epsilon)\, d\epsilon$$

**Corollary 3.** *Consider a Rayleigh* (2) *or Ricean* (3) *fading coherent channel* (1) *driven by an arbitrary discrete input with finite support. Then, in the regime of high-$snr$,*

$$\overline{I}(snr) \sim \log(m) - \frac{e^{-\frac{|\mu|^2}{2\sigma^2}}}{snr} \sum_{m=0}^{+\infty} \frac{\tau_m^{|\mu|,\sigma} M\left[mmse; 2+m\right]}{(m+1) snr^m}$$

*where $\tau_m^{|\mu|,\sigma}$ is as in Corollary 2.*

It is now relevant to reflect on the nature of the asymptotic expansions embodied in Corollaries 2 and 3. For Rayleigh and Ricean fading the $(m+1)$-th term in the average MMSE expansion is $O\left(snr^{-m-2}\right)$, the $(m+1)$-th term in the average mutual information expansion is $O\left(snr^{-m-1}\right)$, so that a first-order high-$snr$ expansion of the quantities obeys:

$$\overline{mmse}(snr) = \epsilon \cdot \frac{1}{snr^2} + O\Big(\frac{1}{snr^3}\Big)$$
$$\overline{I}(snr) = \log(m) - \epsilon \cdot \frac{1}{snr} + O\Big(\frac{1}{snr^2}\Big)$$

where

$$\epsilon := \lim_{snr \to +\infty} snr^2 \cdot \overline{mmse}(snr) = \frac{e^{-\frac{|\mu|^2}{2\sigma^2}}}{2\sigma^2} M\left[mmse; 2\right]$$

It turns out that the rate (on a $\log(snr)$ scale) at which the average MMSE or the average mutual information tend to the infinite-$snr$ value is given by:

$$-\lim_{snr \to +\infty} \frac{\log\left(\overline{mmse}(snr)\right)}{\log(snr)} = 2$$
$$-\lim_{snr \to +\infty} \frac{\log\left(\log(m) - \overline{I}(snr)\right)}{\log(snr)} = 1$$

respectively, both for Rayleigh and Ricean fading channels. In contrast, the parameter $\epsilon$, which is akin to the MMSE dimension put forth in [12], provides a more refined representation of the high-$snr$ asymptotics.

Figure 1 depicts the average MMSE and the average mutual information in Rayleigh and Ricean fading coherent channels driven by QPSK inputs. We observe that the expansions capture very well the high-$snr$ behavior of the quantities. We also observe that a single term expansion is sufficient to approximate well the high-$snr$ behavior of the quantities in channels subject to Rayleigh fading. However, expansions with a higher number of terms are necessary to approximate the high-$snr$ behavior of the quantities in channels subject to Ricean fading. This phenomenon is specially pronounced in the regime $|\mu| \gg \sigma$.

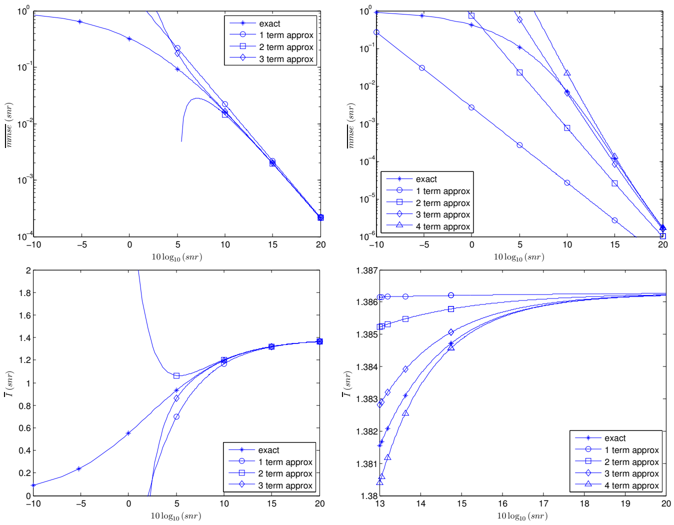


Fig. 1. Average MMSE (above) and average mutual information (below) in a Rayleigh ($\sigma = \frac{1}{\sqrt{2}}$) (left) and Ricean ($|\mu| = \sqrt{\frac{9}{10}}$, $\sigma = \frac{1}{2\sqrt{5}}$) (right) fading coherent channel driven by a QPSK input.

## IV. Low-snr Regime

We now consider the construction of low-$snr$ asymptotic expansions of the average MMSE and the average mutual information in a fading coherent channel driven by arbitrary discrete inputs with finite support. The element of novelty, in view of the fact that examples of low-$snr$ asymptotic expansions of a series of estimation- and information-theoretic quantities are plentiful (see e.g. [13]), relates to the use of Mellin transform expansions techniques to study the behavior of the quantities in such an asymptotic regime. This unconventional approach, which is distinct from the other approaches in the literature, also illustrates that with Mellin transform expansions techniques one can often derive with little effort the asymptotic expansions as $snr \to 0^+$ from asymptotic expansions as $snr \to +\infty$ and vice versa thereby coupling, via the same mathematical formalism, the regimes [7].

The following Theorem applies to relatively general fading models beyond the Rayleigh and Ricean models.

**Theorem 4.** *Consider the fading coherent channel in* (1) *driven by inputs that conform to a discrete distribution with finite support and define the following quantities which relate to the distribution of the fading process*

$$\alpha_1 := \inf\left\{\alpha_1^* : f_{|h|}(t) = O\left(t^{-2\alpha_1^*-1}\right), t \to 0^+\right\}$$
$$\beta_1 := \sup\left\{\beta_1^* : f_{|h|}(t) = O\left(t^{-2\beta_1^*-1}\right), t \to +\infty\right\}$$
$$C_1 := (\alpha_1, \beta_1) \cap (-\infty, 1)$$

*If $\alpha_1 < \beta_1$, $C_1 \neq \emptyset$, the fading distribution behaves as*

$$f_{|h|}(t) = O\left(\exp\left(-k_1 t^{2v_1}\right) t^{-1}\right), t \to +\infty$$

*where $k_1 > 0$ and $v_1 > 0$, and there exists $c_1 \in C_1$ such that for every $x \geq c_1$,*

$$M\left[f_{|h|}; 1 + 2(x + iy)\right] = O\left(|y|^{-2}\right), |y| \to +\infty$$

*then, in the regime of low-$snr$,*

$$\overline{mmse}(snr) \sim \sum_{m=0}^{+\infty} E\left\{|h|^{2(m+1)}\right\} mmse^{(m)}(0) \frac{snr^m}{m!}$$

Theorem 4 can also be immediately specialized to the most common fading distributions.

**Corollary 5.** *Consider a Rayleigh* (2) *or Ricean* (3) *fading coherent channel* (1) *driven by an arbitrary discrete input with finite support. Then, in the regime of low-$snr$,*

$$\overline{mmse}(snr) \sim e^{-\frac{|\mu|^2}{2\sigma^2}} \sum_{m=0}^{+\infty} \tau_m^{|\mu|,\sigma} mmse^{(m)}(0) snr^m$$

*where*

$$\tau_m^{|\mu|,\sigma} := (m+1)\left(2\sigma^2\right)^{m+1} {}_1F_1\left(2+m; 1; \frac{|\mu|^2}{2\sigma^2}\right)$$

*and ${}_1F_1(a; b; c)$ is the confluent hypergeometric series.*

The low-$snr$ asymptotic expansion of the average MMSE embodied in Corollary 5 also lead immediately to a low-$snr$ asymptotic expansion of the average mutual information, via an integral representation of (11) as follows:

$$\overline{I}(snr) = \int_0^{snr} \overline{mmse}(\epsilon)\, d\epsilon$$

**Corollary 6.** *Consider a Rayleigh* (2) *or Ricean* (3) *fading coherent channel* (1) *driven by an arbitrary discrete input with finite support. Then, in the regime of low-$snr$,*

$$\overline{I}(snr) \sim e^{-\frac{|\mu|^2}{2\sigma^2}} \sum_{m=0}^{+\infty} \frac{\tau_m^{|\mu|,\sigma}}{m+1} mmse^{(m)}(0) snr^{m+1}$$

*where $\tau_m^{|\mu|,\sigma}$ is as in Corollary 5.*

It is interesting to stress the role that the *canonical* MMSE plays in the definition of the asymptotic expansions of the average MMSE and the average mutual information as $snr \to 0^+$ and $snr \to +\infty$. The high-$snr$ behavior of the quantities is dictated via the Mellin transform and the higher-order derivatives of the Mellin transform of the *canonical* MMSE. In contrast, the low-$snr$ behavior of the quantities is dictated via the derivatives of the *canonical* MMSE as unveiled in previous contributions [5]. This role follows naturally from our mathematical formalism.

The availability of the derivatives of the canonical MMSE or the Mellin transform and the higher-order derivatives of the Mellin transform of the canonical MMSE for various input classes, which are unveiled in [8] then provides the means to evaluate the asymptotic expansions.

## V. Practical Applications

We conclude by considering a problem of optimal power allocation in a bank of $k$ parallel independent fading coherent channels driven by arbitrary discrete inputs, in order to showcase the application of the asymptotic characterizations. The channel model is:

$$y_i = \sqrt{snr} h_i \sqrt{p_i} x_i + n_i, \quad i = 1, \ldots, k \tag{13}$$

where $y_i \in \mathbb{C}$ represents the output of sub-channel $i$, $x_i \in \mathbb{C}$ represents the input of sub-channel $i$, $h_i \in \mathbb{C}$ represents the gain of sub-channel $i$ and is such that $E\{|h_i|^2\} < +\infty$ and $n_i \in \mathbb{C}$ is a circularly symmetric complex scalar Gaussian random variable with zero mean and unit variance. The scaling factor $p_i \in \mathbb{R}_0^+$ represents the power injected into sub-channel $i$. Again, $snr > 0$ relates to the signal-to-noise ratio. We assume that $x_i, i = 1, \ldots, k$, $h_i, i = 1, \ldots, k$ and $n_i, i = 1, \ldots, k$ are independent random variables and that the receiver knows the exact realization of the sub-channel gains but the transmitter knows only the distribution of the sub-channel gains. This channel model is applicable to a OFDM and multi-user OFDM communications system [1], [2].

We denote the average MMSE and the *canonical* MMSE of sub-channel $i$ in (13) as $\overline{mmse}_i(\cdot)$ and $mmse_i(\cdot)$, respectively. We also denote the average mutual information and the *canonical* mutual information of sub-channel $i$ in (13) as $\overline{I}_i(\cdot)$ and $I_i(\cdot)$, respectively.

The objective is to determine the power allocation policy that maximizes the constrained capacity given by:

$$\overline{I}(snr; p_1, \ldots, p_k) := \sum_{i=1}^{k} \overline{I}_i(snr \cdot p_i)$$

subject to $\sum_{i=1}^{k} p_i \leq P$ and $p_i \geq 0$.

The following Theorem which is based on the previous asymptotic expansions, offers a characterization of the optimal power allocation procedure in the regime of high-SNR.

**Theorem 7.** *Consider a bank of $k$ parallel independent fading coherent channels as in* (13) *driven by arbitrary discrete inputs with finite support subject to Rayleigh or Ricean fading so that $h_i \sim \mathcal{CN}(\mu_i, 2\sigma_i^2)$ with $\mu_i = 0$ or $\mu_i \neq 0$, and $\sigma_i > 0$, $i = 1, \ldots, k$. Then, in the regime of high-$snr$, the optimal power allocation policy obeys:*

$$p_i^* = \sqrt{\exp\left(-\frac{|\mu_i|^2}{2\sigma_i^2}\right) \cdot \frac{M[mmse_i; 2]}{2\sigma_i^2} \cdot \frac{1}{\lambda snr} + O\left(\frac{1}{snr}\right)}$$

*where $\lambda$ is such that $\sum_{i=1}^{k} p_i^* = P$.*

Theorem 7 reveals the impact of the nature of the fading distribution and the input distribution on the high-$snr$ optimal power allocation policy. In Rayleigh fading channels, given equal sub-channel inputs, it can be seen that the higher the average sub-channel strength (i.e., the higher $2\sigma_i^2$) then the lower the allocated power. In Ricean fading channels, it can also be seen that the presence of line-of-sight components affects dramatically the power allocation policy. It is interesting to note that, as expected, the nature of the inputs affects the optimal power allocation policy via the Mellin transform of the *canonical* MMSE. It is also interesting to note that the power allocation policies embodied in Theorem 7 in fact represent a generalization of the power allocation policy put forth in [2], in the sense that – in the single-user setting – it applies to Ricean fading in addition to Rayleigh fading and to scenarios where the different input signals conform to different discrete constellations. Figure 2 confirms that the optimal power allocation converges to the high-$snr$ power allocation uncovered by Theorem 7 for a bank of two parallel independent fading coherent channels.

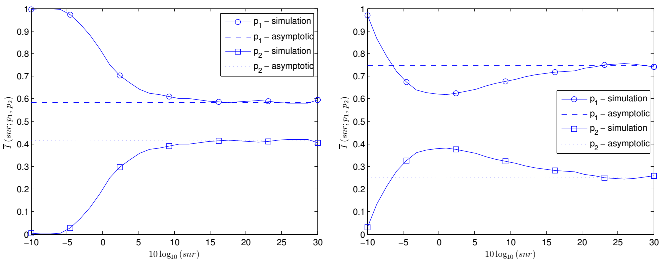


Fig. 2. Optimal and asymptotic power allocation in a bank of two parallel independent Rayleigh fading ($2\sigma_1^2 = 4$ and $2\sigma_2^2 = 1$) (left) and Ricean fading ($\mu_1 = 1 + i$, $2\sigma_1^2 = 4$, $\mu_2 = 1 + i$ and $2\sigma_2^2 = 1$) (right) channels, with maximum allowed power constraint $P = 1$. First sub-channel driven by equiprobable unit-power 16-QAM and second sub-channel driven by equiprobable unit-power QPSK.

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