Throughput Maximization for Energy Harvesting Two-Hop Networks

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***Abstract*—In wireless networks, management of harvested energy is important due to limited and stochastic energy sources. In this paper, throughput maximization for energy harvesting two-hop communication with half-duplex relays is considered. Optimal transmission policies are found for one relay and two parallel relays under the assumption of known energy arrivals at the source and the relays. For each case, a convex optimization problem is formulated to efficiently solve and identify properties of the optimal transmission policies. Performance comparisons are provided to investigate the impact of multiple relays and energy harvesting.**

I. Introduction

One of the important applications of sensor networks is providing long-term monitoring of environment and wildlife. In such applications, sensor nodes are typically small and lightweight, with limited battery capacity. Energy harvesting (EH) enables continuous operation of such networks by ensuring that the energy supply is adequately replenished from the environment. Since most sensor networks rely on multiple hops for range extension, it is important to study how the harvested energies on a route can be efficiently used and the transmissions can be scheduled when nodes are powered by EH.

In recent years, EH communication systems have been studied under various assumptions. Within the offline optimization framework, which assumes perfect non-causal knowledge of all energy arrivals, optimal transmission policies are investigated for single and multi-user networks [1]-[4]. Note that the offline optimization is applicable to predictable energy sources and provides performance upper bound for stochastic energy sources. Gunduz and Devillers consider offline throughput maximization for two-hop communication with a full-duplex relay and with a half-duplex relay for single energy arrival at the source and multiple energy arrivals at the relay [5]. Similarly, multiple energy arrivals at the source and single energy arrival at the half-duplex relay is studied in [6]. Our previous works [7]-[8] also focus on a half-duplex relay, and for two energy arrivals at the source and multiple energy arrivals at the relay, identify necessary properties of an optimal transmission policy using heuristic arguments. Gurakan et. al [9] consider energy harvesting multi-hop communication with energy cooperation, where the source can transfer some of its harvested energy to the relay.

In this paper, we consider offline throughput maximization for a general two-hop EH communication system with half-duplex relay nodes. We first formulate a convex optimization problem for the single relay case in Fig. 1(a) for arbitrary number of energy arrivals at the nodes. Along with providing a solution for the most general case, this framework allows an efficient computation of the optimal transmission policy, consisting of power allocation and scheduling, and results in a number of useful properties that can be used to devise online policies.

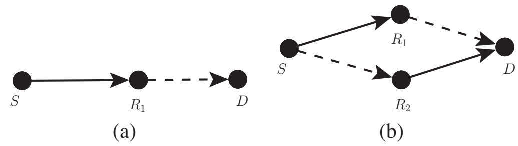

Figure 1. Two-hop communication with (a) one relay, (b) two parallel relays (diamond relay channel).

We then extend the convex formulation to the two parallel relay case in Fig. 1(b), also known as the diamond relay channel [12]. For the diamond relay channel four transmission modes have been studied [13], [14]: i) broadcast mode, source (S) transmits and relays (R_1 and R_2) listen; ii) multi-access mode, R_1 and R_2 transmit and destination (D) listens; iii) successive relaying Phase I, S and R_2 transmit, and R_1 and D listen; iv) successive relaying Phase II, S and R_1 transmit, and R_2 and D listen. Through various combinations of these four modes, information can be delivered from S to D. In this paper, we only consider successive decode-and-forward relaying with Phase I and II, also known as *multihop with spatial reuse*, which is known to perform well for a wide range of channel conditions [13], and is capacity achieving in some special cases [14]. Investigation of other schemes such as broadcast mode followed by the multi-access mode is left for future work. Using the convex optimization framework, we show that the optimal transmission policy for the parallel relay case exhibits some characteristics that are different than its single relay counterpart. Finally, solving the optimization problems, we illustrate the effect of multiple relays and energy harvesting on the throughput.

II. System Model

We consider two-hop communication with one and two parallel relays as in Fig. 1. The relays are half-duplex, and use multihop with spatial reuse in the two relay case [13]. We assume that S, R_1, and R_2 (R for one relay) harvest energy with arbitrary and finite amounts at arbitrary times until a given deadline T sec. We also assume that the nodes have infinite size batteries and there are no losses associated with storing and retrieving energy to and from the battery. We assume that all energy amounts and arrival times are known at the nodes before transmission starts. For ease of exposition, we merge energy arrivals at the nodes in a single time series $(t_0, \dots, t_K)$ by allowing zero energy arrivals at some t_i's at which only one of the nodes harvests energy. We set $t_0 = 0$ and $t_K = T$. We denote harvested energy amounts of S, R_1, and R_2 at $t = t_i$ as $E_{s,i}$, $E_{r_1,i}$, and $E_{r_2,i}$, respectively, ($E_{r,i}$ for one relay), $i = 0, ..., K-1$. The inter-arrival times of energy packets are denoted by $\tau_i \triangleq t_i - t_{i-1}$. We assume that the channels are constant throughout transmission. The channel gain between node k and l is h_{kl} where $k = s, r_1, r_2, r$, and $l = r_1, r_2, r, d$. Note that there are no direct links between S and D and between R_1 and R_2. We also assume independent additive white Gaussian noise of variance one at each receiver.

We consider the throughput maximization problem, that is, we maximize the total data delivered to the destination by a given deadline $t = T$. Assuming that all the energy arrivals are known at all nodes before the transmission starts, we identify optimal power allocation for each node and the transmission schedule which are referred to as *offline optimal transmission policy*. The transmission schedule indicates which node transmits when, and is necessary to coordinate the operation of the half-duplex relays. We assume that energy consumption of the nodes includes only the transmission power[1], and we consider Shannon capacity as the rate-power function of a given link, i.e., $r(p(t)) \triangleq \frac{1}{2}\log(1 + hp(t))$ where h is the real valued channel gain of the link and $p(t)$ is transmission power at time t. As a result of energy arrivals over time, any feasible transmission policy must satisfy *energy causality*, i.e., total consumed energy of a node at time t cannot exceed the total harvested energy at that node by that time. In addition, data transmitted by any of the relays up to time t should not exceed total data received by that relay up to that time, which is referred to as *data causality* constraint.

III. Properties of Optimal Transmission Policy

In this section, we identify some common properties of the optimal transmission policy which will be used to formulate convex optimization problems in Section IV and V for the one and two relay case, respectively.

Lemma 1: The transmission rate of a node is constant between two consecutive energy arrivals when the node is transmitting.

Proof: This is proved for the point to point case in [1, Lemma 2]. The proof can be extended for two hops. ■

Lemma 2: Given a feasible transmission policy for which a relay is not *on* i.e., not transmitting or receiving data, all the time, we can find another feasible transmission policy that ensures the relays are always on without decreasing the throughput.

Proof: The proof is given for the one relay case in [7, Lemma 3]; however this proof cannot be directly extended to the two relay case. In the case of two relays consider a feasible transmission policy for which one of the relays is not always on. We can remove the idle times by increasing transmission duration of one of the nodes (source or the relays) while keeping total transmitted data the same. Due to concavity of the rate-power function, the new policy delivers the same amount of data to the destination and consumes less energy; hence, it is feasible. ■

Remark 3.1: From Lemma 2 we can argue that in the two relay case there exists an optimal strategy for which both S and D are always *on*. Note that because of the half-duplex constraint, this is not the case for a single relay or for two parallel relays using another strategy such as the broadcast mode followed by the multi-access mode.

IV. Two-hop Communication with One Relay

In this section, we consider the one relay case as shown in Fig. 1(a). As argued in Lemma 1, the transmission rate between a pair of nodes remains constant between the energy arrivals (or is zero when the nodes are not communicating); therefore, for each node we consider a single transmission power between energy arrivals to be used when that node is scheduled to transmit. Accordingly, we denote the non-negative transmission powers of S and R as $p_{s,i}$ and $p_{r,i}$ with transmission durations $l_{s,i}$ and $l_{r,i}$, $i = 1, ..., K$, respectively. In addition, due to the half-duplex constraint, $l_{s,i}$ and $l_{r,i}$ must satisfy the condition $l_{r,i} + l_{s,i} \leq \tau_i$. Note that since postponing relay transmission allows the relay to store more energy and data [7, Remark 3.2], we can schedule transmissions such that between two energy arrivals, first the source then the relay transmit. Hence the data causality constraint is active only at energy arrival times.

The throughput optimization problem can be formulated as follows, where the maximization is over $p_{r,i}, p_{s,i}, l_{r,i}, l_{s,i}$:

$$\max \quad \sum_{i=1}^{K} l_{r,i} \log(1 + h_{rd} p_{r,i}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{j=1}^{i} l_{r,j} p_{r,j} \leq \sum_{j=1}^{i} E_{r,j-1}, \ \forall i, \tag{1b}$$

$$\sum_{j=1}^{i} l_{s,j} p_{s,j} \leq \sum_{j=1}^{i} E_{s,j-1}, \ \forall i, \tag{1c}$$

$$\sum_{j=1}^{i} l_{r,j} \log(1 + h_{rd} p_{r,j}) \leq \sum_{j=1}^{i} l_{s,j} \log(1 + h_{sr} p_{s,j}), \ \forall i, \tag{1d}$$

$$l_{r,i} + l_{s,i} \leq \tau_i, \ \forall i, \tag{1e}$$

$$0 \leq p_{r,i}, \ 0 \leq p_{s,i}, \ 0 \leq l_{r,i}, \ 0 \leq l_{s,i}, \ \forall i. \tag{1f}$$

Here the constraints in (1b), (1c) are due to energy causality at R and S, respectively, and the constraint in (1d) is due to data causality at R. Note that by (1d) evaluated at $i = K$, the total amount of data delivered to D is equal to

[1] Extensions to include other types of processing energies can be done as in [10].

the amount of data transmitted by R, hence the throughput maximization problem corresponds to maximization of the total data transmitted by R as in (1a). The above optimization problem is not convex because of the constraints in (1b)-(1d). We define new variables $\alpha_{r,i} = l_{r,i}\log(1+h_{rd}p_{r,i})$ and $\alpha_{s,i} = l_{s,i}\log(1+h_{sr}p_{s,i})$, and rewrite the optimization problem in (1) in terms of $\alpha_{r,i}, \alpha_{s,i}, l_{r,i}, l_{s,i}$ as follows:

$$\max \quad \sum_{i=1}^{K} \alpha_{r,i} \tag{2a}$$

$$\text{s.t.} \quad \sum_{j=1}^{i} \frac{l_{r,j}}{h_{rd}}\left(e^{\frac{\alpha_{r,j}}{l_{r,j}}} - 1\right) \le \sum_{j=1}^{i} E_{r,j-1}, \ \forall i, \tag{2b}$$

$$\sum_{j=1}^{i} \frac{l_{s,j}}{h_{sr}}\left(e^{\frac{\alpha_{s,j}}{l_{s,j}}} - 1\right) \le \sum_{j=1}^{i} E_{s,j-1}, \ \forall i, \tag{2c}$$

$$\sum_{j=1}^{i} \alpha_{r,j} \le \sum_{j=1}^{i} \alpha_{s,j}, \ \forall i, \tag{2d}$$

$$l_{r,i} + l_{s,i} \le \tau_i, \ \forall i, \tag{2e}$$

$$0 \le \alpha_{r,i}, \ 0 \le \alpha_{s,i}, \ 0 \le l_{r,i}, \ 0 \le l_{s,i}, \ \forall i. \tag{2f}$$

Note that $l_{r,j}e^{\frac{\alpha_{r,j}}{l_{r,j}}}$ is perspective of the convex function $e^{\alpha_{r,j}}$, hence it is a convex function of $l_{r,j}$ and $\alpha_{r,j}$ [15]. Similarly, $l_{s,j}e^{\frac{\alpha_{s,j}}{l_{s,j}}}$ in (2c) is a convex function of $l_{s,j}$ and $\alpha_{s,j}$. Therefore, the optimization problem in (2) is convex.

Next, we identify properties of the optimal transmission policy using KKT conditions which are both necessary and sufficient due to convexity of the optimization problem in (2). These properties provide the optimal structure of the transmission policy and are useful in designing online algorithms; for example see [2]. Numerical illustration of the optimal policy and throughput will be carried out in Section VI.

The Lagrangian of (2) with $\lambda_{j,i} \ge 0$, $j = 1, \ldots, 8$ as Lagrange multipliers is the following:

$$\begin{aligned}
\mathcal{L} = &\sum_{i=1}^{K} \alpha_{r,i} - \sum_{i=1}^{K} \lambda_{1,i}\left(\sum_{j=1}^{i} \frac{l_{r,j}}{h_{rd}}\left(e^{\frac{\alpha_{r,j}}{l_{r,j}}} - 1\right) - \sum_{j=1}^{i} E_{r,j-1}\right) \\
&- \sum_{i=1}^{K} \lambda_{2,i}\left(\sum_{j=1}^{i} \frac{l_{s,j}}{h_{sr}}\left(e^{\frac{\alpha_{s,j}}{l_{s,j}}} - 1\right) - \sum_{j=1}^{i} E_{s,j-1}\right) \\
&- \sum_{i=1}^{K} \lambda_{3,i}\left(\sum_{j=1}^{i} \alpha_{r,j} - \sum_{j=1}^{i} \alpha_{s,j}\right) \\
&- \sum_{i=1}^{K} \lambda_{4,i}\left(l_{r,i} + l_{s,i} - \tau_i\right) + \sum_{i=1}^{K} \lambda_{5,i} l_{r,i} \\
&+ \sum_{i=1}^{K} \lambda_{6,i} l_{s,i} + \sum_{i=1}^{K} \lambda_{7,i} \alpha_{r,i} + \sum_{i=1}^{K} \lambda_{8,i} \alpha_{s,i}.
\end{aligned} \tag{3}$$

Differentiating the Lagrangian with respect to $\alpha_{r,i}$ and $\alpha_{s,i}$, we obtain the following:

$$\frac{\partial \mathcal{L}}{\partial \alpha_{r,i}} = 1 - \frac{e^{\frac{\alpha_{r,i}}{l_{r,i}}}}{h_{rd}} \sum_{j=i}^{K} \lambda_{1,j} - \sum_{j=i}^{K} \lambda_{3,j} + \lambda_{7,i} = 0, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{s,i}} = -\frac{e^{\frac{\alpha_{s,j}}{l_{s,j}}}}{h_{sr}} \sum_{j=i}^{K} \lambda_{2,j} + \sum_{j=i}^{K} \lambda_{3,j} + \lambda_{8,i} = 0. \tag{5}$$

Using (4) and replacing $\alpha_{r,i}$ with $l_{r,i}\log(1+h_{rd}p_{r,i})$, we can obtain the optimal relay transmission power $p^*_{r,i}$ as:

$$p^*_{r,i} = \left[\frac{1-\sum_{j=i}^{K}\lambda_{3,j}}{\sum_{j=i}^{K}\lambda_{1,j}} - \frac{1}{h_{rd}}\right]^+. \tag{6}$$

Similarly using (5) and replacing $\alpha_{s,i}$ with $l_{s,i}\log(1+h_{sr}p_{s,i})$, the optimal source transmission power $p^*_{s,i}$ becomes:

$$p^*_{s,i} = \left[\frac{\sum_{j=i}^{K}\lambda_{3,j}}{\sum_{j=i}^{K}\lambda_{2,j}} - \frac{1}{h_{sr}}\right]^+. \tag{7}$$

The optimal transmission durations $l_{s,i}$ and $l_{r,i}$ can be similarly found.

Lemma 3: The optimal transmission power of R is non-decreasing, i.e., $p^*_{r,i} \le p^*_{r,i+1}$, and whenever $p^*_{r,i}$ strictly increases, either the battery or the data buffer of R must be empty at $t = t_i$.

Proof: In [7, Lemmas 4, 5, 7], the proof was given by contradiction. Here, we provide an alternative proof using the KKT conditions. From the complementary slackness conditions, we can argue that whenever $\lambda_{1,i} > 0$, the battery of R must be empty at time t_i, and whenever $\lambda_{3,i} > 0$, the data buffer of R must be empty at time t_i. From (6), we observe that whenever $p^*_{r,i} < p^*_{r,i+1}$, either $\lambda_{1,i} > 0$ or $\lambda_{3,i} > 0$ or both, hence proving the lemma. In addition, since $\lambda_{1,i}$ and $\lambda_{3,i}$ are non-negative, $p^*_{r,i}$ is non-decreasing. ■

Lemma 4: The optimal transmission power of S is non-decreasing, i.e., $p^*_{s,i} \le p^*_{s,i+1}$, and whenever $p^*_{s,i}$ strictly increases, either the battery of S must be empty or both the battery of S and the data buffer of R must be empty and $t = t_i$.

Proof: The proof is based on the Lagrange multipliers $\lambda_{2,i}$ and $\lambda_{3,i}$. From the complementary slackness conditions, we can argue that whenever $\lambda_{2,i} > 0$, the battery of S must be empty at time t_i, and whenever $\lambda_{3,i} > 0$, the data buffer of R must be empty at time t_i. Below we investigate different cases for $\lambda_{2,i}$ and $\lambda_{3,i}$. Note that $(\lambda_{2,i} = 0, \lambda_{3,i} = 0)$ and $(\lambda_{2,i} > 0, \lambda_{3,i} = 0)$ were studied by contradiction in [7, Lemma 5]; here we provide a simpler proof using (7).

1) If $\lambda_{2,i} = 0$ and $\lambda_{3,i} = 0$, $p^*_{s,i} = p^*_{s,i+1}$.
2) If $\lambda_{2,i} > 0$ and $\lambda_{3,i} = 0$, $p^*_{s,i} < p^*_{s,i+1}$.
3) For the remaining cases ($\lambda_{2,i} > 0$ and $\lambda_{3,i} > 0$), and ($\lambda_{2,i} = 0$ and $\lambda_{3,i} > 0$), we argue by contradiction that $p^*_{s,i} \le p^*_{s,i+1}$. Note that $\lambda_{2,i} = 0$ and $\lambda_{3,i} > 0$ implies $p^*_{s,i} > p^*_{s,i+1}$ by (7), hence the argument below also suggests that this case never happens.
   Suppose $p^*_{s,i} > p^*_{s,i+1}$. We can then equalize the power levels $p^*_{s,i}$ and $p^*_{s,i+1}$ such that the new transmission durations and power levels are $l'_{s,i} = (l_{s,i} + l_{s,i+1})\frac{l_{s,i}p^*_{s,i}}{l_{s,i}p^*_{s,i}+l_{s,i+1}p^*_{s,i+1}}$, $l'_{s,i+1} = l_{s,i} + l_{s,i+1} - l'_{s,i}$,

and $p'_{s,i} = p'_{s,i+1} = \frac{p^*_{s,i}+p^*_{s,i+1}}{2}$. The new policy has the same total consumed energy but S transmits more data due to the concavity of the rate-power function. Since we assume that $p^*_{s,i} > p^*_{s,i+1}$, the new transmission duration of $p'_{s,i}$ must increase, i.e., $l'_{s,i} > l_{s,i}$. Note that the above operation is always feasible since we can delay the relay transmission, [7, Remark 3.2]. For the equalized powers, we can obtain another feasible transmission policy by increasing total transmission duration of R and decreasing transmission duration of S and equalizing the transmitted data. As a result, this leads to a policy with higher throughput than the original one, which is a contradiction. Hence, $p^*_{s,i} \le p^*_{s,i+1}$. ∎

V. Two-hop Communication with Two Parallel Relays

In this section, we consider the two parallel relay case shown in Fig. 1(b). As argued in Section I, we only consider multihop with spatial reuse. Using Lemma 2, without loss of optimality, we can restrict our attention to transmission policies for which the relays R_1 and R_2 are always on. Therefore, we consider the case that total transmission time between S and R_1, and between R_2 and D are the same, and denote the duration of Phase I between t_{i-1} and t_i by $l_{I,i}$. Similarly, we consider the same transmission time between S and R_2, and between R_1 and D in Phase II, and denote this by $l_{II,i}$, $i = 1, \dots, K$. Since constant transmission rate for a pair of nodes (when they are communicating) is optimal between energy arrivals from Lemma 1, the transmission power of S for Phases I and II are constant and given by $p_{sI,i}$ and $p_{sII,i}$, respectively. Similarly, $p_{r_1,i}$ and $p_{r_2,i}$ refer to the transmission power of R_1 and R_2, respectively.

For the single relay case, it is shown in [7, Lemma 2] that both S and R deplete their batteries by the deadline. This is accomplished by adjusting transmission durations of S and R to equalize the two hop rates until both batteries are empty. However, in the two relay case, since S is on all the time, the maximum total rate transmitted from S to R_1 and R_2 can sometimes exceed the total rate R_1 and R_2 can deliver to D, resulting in excess energy at S at $t = T$. Similar arguments can be made for the remaining energy at R_1 and/or at R_2 at $t = T$. The lemma below discusses this excess energy case.

Lemma 5: In the optimal transmission policy, if S has positive energy in its battery at $t = T$, then the batteries of both R_1 and R_2 must be empty.

Proof: The proof is by contradiction. Without loss of generality, assume that in an optimal policy both S and R_1 have positive energy in their batteries at $t = T$. Then, we can increase the total data delivered to from S to R_1 and from R_1 to D by increasing the last transmission powers $p_{sI,K}$ and $p_{r_1,K}$, such that all the energies depleted. This results in a contradiction, hence proving the lemma. ∎

We next provide a convex optimization formulation for the throughput maximization problem, and identify additional properties of the optimal transmission policy using KKT optimality conditions. Since R_2 initially has no data to transmit in phase I, without loss of generality, we assume it starts transmission by delivering $\epsilon > 0$ amount of dummy information. By keeping ϵ small and scheduling phases I and II in succession, we can ensure that there is no further loss in the throughput. Then, omitting ϵ for convenience, the throughput optimization problem can be formulated as follows,where the maximization is over $l_{I,i}, l_{II,i}, \alpha_{sI,i}, \alpha_{sII,i}, \alpha_{r_2,i}, \alpha_{r_1,i}$:

$$\max \sum_{i=1}^{K} \alpha_{r_1,i} + \alpha_{r_2,i} \tag{8a}$$

$$\text{s.t.} \sum_{j=1}^{i} \frac{l_{I,j}}{h_{sr_1}} \left(e^{\frac{\alpha_{sI,j}}{l_{I,j}}} - 1 \right) + \frac{l_{II,j}}{h_{sr_2}} \left(e^{\frac{\alpha_{sII,j}}{l_{II,j}}} - 1 \right) \le \sum_{j=1}^{i} E_{s,j-1}, \ \forall i, \tag{8b}$$

$$\sum_{j=1}^{i} \frac{l_{I,j}}{h_{r_2d}} \left(e^{\frac{\alpha_{r_2,j}}{l_{I,j}}} - 1 \right) \le \sum_{j=1}^{i} E_{r_2,j-1}, \ \forall i, \tag{8c}$$

$$\sum_{j=1}^{i} \frac{l_{II,j}}{h_{r_1d}} \left(e^{\frac{\alpha_{r_1,j}}{l_{II,j}}} - 1 \right) \le \sum_{j=1}^{i} E_{r_1,j-1}, \ \forall i, \tag{8d}$$

$$\sum_{j=1}^{i} \alpha_{r_1,j} \le \sum_{j=1}^{i} \alpha_{sI,j}, \ \forall i, \tag{8e}$$

$$\sum_{j=1}^{i} \alpha_{r_2,j} \le \sum_{j=1}^{i} \alpha_{sII,j}, \ \forall i, \tag{8f}$$

$$l_{I,i} + l_{II,i} \le \tau_i, \quad \forall i, \tag{8g}$$

$$0 \le \alpha_{sI,i}, \ 0 \le \alpha_{sII,i}, \ 0 \le \alpha_{r_1,i}, \ \forall i, \tag{8h}$$

$$0 \le \alpha_{r_2,i}, \ 0 \le l_{I,i}, \ 0 \le l_{II,i}, \ \forall i, \tag{8i}$$

where $\alpha_{sI,i} = l_{I,i} \log(1 + h_{sr_1} p_{sI,i})$, $\alpha_{sII,i} = l_{II,i} \log(1 + h_{sr_2} p_{sII,i})$, $\alpha_{r_1,i} = l_{II,i} \log(1 + h_{r_1d} p_{r_1,i})$ and $\alpha_{r_2,i} = l_{I,i} \log(1 + h_{r_2d} p_{r_2,i})$. In the above optimization problem, the constraints in (8b)-(8d) are due to energy causality. The constraints in (8e) and (8f) are due to data causality at R_1 and R_2, respectively. In addition, the constraint in (8g) indicates the half-duplex nature of the relays.

Then, formulating the Lagrangian as in the single relay case we obtain:

$$p^*_{r_1,i} = \left[\frac{1 - \sum_{j=i}^{K} \lambda_{4,j}}{\sum_{j=i}^{K} \lambda_{3,j}} - \frac{1}{h_{r_1d}} \right]^+, \tag{9}$$

$$p^*_{r_2,i} = \left[\frac{1 - \sum_{j=i}^{K} \lambda_{5,j}}{\sum_{j=i}^{K} \lambda_{2,j}} - \frac{1}{h_{r_2d}} \right]^+, \tag{10}$$

$$p^*_{sI,i} = \left[\frac{\sum_{j=i}^{K} \lambda_{4,j}}{\sum_{j=i}^{K} \lambda_{1,j}} - \frac{1}{h_{sr_1}} \right]^+, \tag{11}$$

$$p^*_{sII,i} = \left[\frac{\sum_{j=i}^{K} \lambda_{5,j}}{\sum_{j=i}^{K} \lambda_{1,j}} - \frac{1}{h_{sr_2}} \right]^+, \tag{12}$$

where $\lambda_{1,i}$, $\lambda_{2,i}$, $\lambda_{3,i}$, $\lambda_{4,i}$, and $\lambda_{5,i}$, $i = 1, ..., K$ are the Lagrange multipliers for the constraints in (8b)-(8f), respectively.

Lemma 6: The optimal transmission power of each relay $p^*_{r_j,i}$, $j = 1, 2$ is non-decreasing in i and whenever the power strictly increases, either the battery or the data buffer of that relay must be empty.

Proof: The proof is similar to that of Lemma 3 and is omitted. ■

Lemma 7: Whenever the optimal transmission power of S in Phase I (Phase II) strictly increases, i.e. $p^*_{sI,i} < p^*_{sI,i+1}$ ($p^*_{sII,i} < p^*_{sII,i+1}$), the battery of S must be empty at $t = t_i$, and whenever it decreases, i.e. $p^*_{sI,i} > p^*_{sI,i+1}$ ($p^*_{sII,i} > p^*_{sII,i+1}$), the data buffer of R_1 (R_2) must be empty at $t = t_i$.

Proof: From the complementary slackness conditions, we can argue that whenever $\lambda_{1,i} > 0$, the battery of S is empty at $t = t_i$. In addition, whenever $\lambda_{4,i} > 0$, the data buffer of R_1 is empty at $t = t_i$. From (11), we see that $p^*_{sI,i} < p^*_{sI,i+1}$ implies $\lambda_{1,i} > 0$ and hence the battery of S is empty. Similarly, $p^*_{sI,i} > p^*_{sI,i+1}$ implies $\lambda_{4,i} > 0$ and hence the data buffer of R_1 is empty. The same argument can be made for R_2 as well. ■

Remark 5.1: Lemma 6 suggests that the structure of the optimal relay transmission power for the two relay case is similar to that of a single relay established in Lemma 3. However, comparing Lemma 7 with Lemma 4, we observe that unlike the single relay case where the source power is non-decreasing, in the two relay scenario the transmission power of the source may decrease when the data buffer of the respective relay is empty.

VI. Illustration of Results

In this section, we provide numerical results to show the effect of multiple relays and energy harvesting on the optimal throughput. We assume that the total harvested energy of S is E, and the total harvested energies of R_1 and R_2 are $(1-\lambda)E$ and λE, respectively, with $0 \le \lambda \le 1$. Note that $\lambda = 0$ corresponds to single relay with R_1 only, and $\lambda = 1$ with R_2 only. The channel gains are $h_{sr_1} = 1$, $h_{sr_2} = 4$, $h_{r_1d} = 4$, $h_{r_2d} = 1$ and the deadline is $T = 5$ sec. For the chosen channel gains, multihop with spatial reuse is capacity achieving when all nodes have the same power [14]. We compare the average throughput of two scenarios with $E = 5$: (i) there is a single energy arrival at $t = 0$ (battery-run nodes), (ii) there are five energy arrivals at $t = i$ sec, $i = 0, 1, \ldots, 4$ for each node such that source energies are $[1, 1, 1, 1, 1]$, and R_1 and R_2 energies are $[1, 0, 0, 2, 2]$ (EH nodes). The average throughput as a function of λ for both battery-run and EH nodes is shown in Fig. 2. As expected, the battery-run system with the same total energy performs better than the EH system even when the total energies are the same. For the channel gains in this particular example, having two relays is always better than one for the battery-run system even though this is not true for arbitrary channel gains due to the energy sharing variable λ. Again for a battery run system, having only R_1 or R_2 results in the same throughput, as indicated by $\lambda = 0$ or $\lambda = 1$. However, when EH is present, R_1, which has a lower channel gain from the source is preferred, resulting in a higher throughput for $\lambda = 0$. This is because for the particular energy arrival pattern, a lower h_{sr} results in a longer transmission of

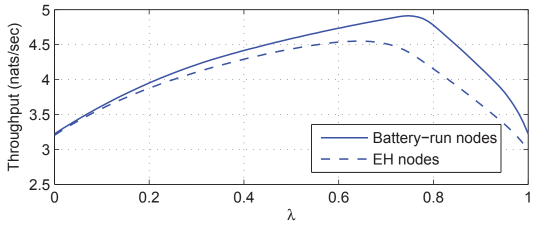

Figure 2. Throughput versus energy allocation among the relays. R_1 has total energy λE, R_2 has $(1-\lambda)E$.

the source, allowing the relay to collect enough energy for the second hop transmission.

VII. Conclusion

In this paper, we have studied EH two hop communication with half-duplex relays for one relay and two parallel relays employing multihop with spatial reuse.. Under the assumption of known energy arrivals, we have formulated two convex optimization problems and identified optimal transmission policies to maximize the total data delivered by a deadline. Using the KKT optimality conditions of the optimization problems, we have identified various properties of the optimal policies. Finally, we have provided numerical results to investigate the effect of multiple relays and energy harvesting on the average throughput. Future work includes finding efficient online policies using the properties of the optimal policies derived in this paper.

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