On the Adaptive Sum-Capacity of Fading MACs with Distributed CSI and Non-identical Links

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***Abstract*—We consider a two-user block-fading MAC with distributed channel state information (CSI), where each user has access to only its own fading coefficients. The average rate-pairs of communication while employing within-block coding is known as the adaptive capacity region, where each user adapts the rate based on its perceived link gain. We evaluate the adaptive sum-capacity of MAC channels with general fading distributions, for discrete as well as continuous valued ones.**

I. Introduction

The multiple access channel (MAC) is a fundamental model for several multiple-transmitter-single-receiver systems, such as the uplink of a cellular network. Fading due to multipath and shadowing, and the inherent time variability of the channel create interesting challenges. With bandwidth becoming costly, schemes to improve the rates of transmission in multiuser networks are of particular interest. It is well known that the effective data-rates over a MAC system depends on the availability of channel state information (CSI). We consider the distributed CSI model described in [1] (page 590–593), see also [2], [3]. We refer to it as the individual CSI model, i.e. each transmitter knows only its own fading coefficients. This should be contradicted with the full CSI case in [4]. In both cases, the receiver has perfect access to the CSI. For more details of the individual CSI model and its practical importance, see [1].

When coding over multiple fading blocks are allowed, the ergodic capacity-region under individual CSI is analyzed in [5]. But the capacity region there is expressed in terms of power-allocation functions, which are not easy to optimize. In contrast, our concern here is within-block coding, which is suitable for slow-fading models [1]. Thus coding and decoding have to be completed in every block. The utility in this setting is the average sum-rate across fading blocks, which is termed as **adaptive sum-capacity**. The name stems from the fact that users can adapt their rates according to the individual channel coefficients. There also is another degree of freedom available in adapting the transmit power, the resulting utility is known as power-controlled adaptive sum-capacity [1], [6]. The power controlled adaptive sum-capacity was explicitly evaluated for identical channel statistics in [6], while the case of asymmetric CSI is analyzed in [7].

The current paper primarily addresses the adaptive sum-capacity in the presence of individual CSI for arbitrary channel distributions, and does not consider power-control. Note that the techniques and solutions for identical channel statistics in [3], [6] are not as such applicable in the presence of non-identical links. In particular, distributed rate-adaptation with respect to different probability laws needs novel access schemes. In [1] (page 593), [2], a single-letter characterization to the adaptive sum-capacity is described, and numerical solutions to a small number of identical fading states are also performed. Extending this computation to channels with several possible fading states was considered a complex problem. We present an explicit solution to the adaptive sum-capacity for general channels, including discrete fading states as well as continuous valued ones, which is the highlight of the current paper (see Theorems 5 and 7).

In addition to its practical relevance in many MAC systems, another motivation to study distributed CSI is its connection to random access, see also [2]. Consider a two-user MAC with fixed fading links used to communicate data. The communication takes place in a slot by slot basis. In addition to two regular streams of independent information at the two users, there also are fixed-size packet arrivals at the two transmitters. The packet arrival law at user i in each slot is modeled by a Bernoulli random variable of parameter p_i. Each packet needs to be communicated to the receiver within a slot. One can think of user i employing a transmit power P_i to send the regular stream, while $P_i + P_i'$ can be used in the slots where there is additional random data to be sent. If no or very little transmitted information is to be lost, and the data packets needs to be decoded at the end of each slot, what will be the maximum throughput (total rate) that the system can deliver? This problem is similar to a two user fading MAC, with each link taking two fading states. We have mentioned this only as a motivating example, and do not elaborate further on the details, primarily due to space-constraints.

A. Paper Organization

The organization of the paper is as follows. Section II will give the detailed system model, definitions and some notations. We derive the adaptive sum-capacity of a two-user MAC with discrete fading states and individual CSI in Section III. Extensions to continuous valued distributions on the fading coefficients is given in Section IV. Section V concludes the paper with some suggestions for future work.

II. System Model

Our system model and objectives are similar to that in [1] (see Section 23.5.2). In here, two transmitters have independent data to be sent to a common receiver. We use the subscript $i \in \{1, 2\}$ to represent variables associated with user i. The communication model is that of a block-fading MAC, in which the instantaneous real-valued received samples can be represented as,

$$y = H_1X_1 + H_2X_2 + Z. \tag{1}$$

User i transmits real-valued signals $X_i \in \mathbb{R}$ and encounters real-valued multiplicative fades $H_i \in \mathcal{H}_i$ (clearly $\mathcal{H}_i$ is a sub-set of $\mathbb{R}$). H_1 and H_2 are assumed to be independent of each other. The additive noise process Z is iid zero mean Gaussian, independent of X_1, X_2 and H_1, H_2 . Without loss of generality (w.l.o.g) we assume that the additive noise has unit variance. The slow fading nature of the medium is captured by the block-fading assumption, i.e. the fading vector (H_1, H_2) remains constant within a block and varies independently across blocks. The full CSI information is available at the receiver.

We assume individual CSI at the transmitters[6], i.e. each transmitter knows its own fading coefficient, but has no idea of the other link, except for the fading statistics. The transmitters have the freedom to adapt their rate according to the available knowledge of the channel state, however the choice of rates should ensure no outage in every transmission block. Here, *outage* is used in the sense of having a small error probability in decoding messages from both the users on a block by block basis[1]. Let the i^{th} user transmit at a fixed power P_i. The next few definitions specialize the ones in [6].

Definition 1. *A rate strategy is a collection of mappings* $R_i : \mathcal{H}_i \to \mathbb{R}^+; i = 1, 2.$

Thus, in the global fading-state (H_1, H_2), the i^{th} user employs a codebook of rate $R_i(H_i)$.

Let $C_{MAC}(h_1, h_2)$ denote the capacity region of a two user Gaussian multiple-access channel with fixed fading gains of h_1 and h_2 respectively at the users. It is well known that $C_{MAC}(h_1, h_2)$ is the collection of all rate-pairs of the form (R_1, R_2) such that

$$\begin{aligned} R_i &\le \frac{1}{2}\log(1 + h_i^2 P_i),\ i \in \{1, 2\}, \\ R_1 + R_2 &\le \frac{1}{2}\log(1 + h_1^2 P_1 + h_2^2 P_2). \end{aligned} \tag{2}$$

Definition 2. *A rate-strategy* $(R_1(\cdot), R_2(\cdot))$ *is termed as outage free if*
$\forall (h_1, h_2) \in \mathcal{H}_1 \times \mathcal{H}_2, (R_1(h_1), R_2(h_2)) \in C_{MAC}(h_1, h_2).$

Thus each user, without knowing the link gain of the other, will pick its own data-rate. The distributed strategy that we devise should ensure that the chosen rate-pair be inside the pentagon formed by decodable (R_1, R_2) pairs given in (2). The challenge is to maximize the total average sum-rate at which data can be transported through this MAC system.

[1]this is different from having an arbitrarily small error probability in the Shannon sense, which may need infinite block-lengths, see [1](page 587)

We will denote the collection of all rate strategies which are outage free by Θ_{MAC}. Let $\psi_i(h)$ denote the cumulative distribution function (cdf) of the fading gains of the i^{th} user.

Definition 3. *The adaptive sum-capacity* $C_{sum}(\psi_1, \psi_2)$ *is the maximum(average) throughput achievable,.i.e.*

$$C_{sum}(\psi_1, \psi_2) = \max_{\theta \in \Theta_{MAC}} \mathbb{E}_{H_1,H_2}(\sum_{i=1}^{2} R_i(H_i)).$$

In order to view the expectations over fading statistics in a unified way, we define generalized cdf inverse functions.

Definition 4. *For any* $x \in [0, 1]$, *the inverse cdf function for user* i *is*

$$h_i(x) = \sup\{h|\psi_i(h) \le x\}. \tag{3}$$

By this definition, we can write (even when H is discrete)

$$E\left[R_i(H_i)\right] = \int_0^1 R_i(h_i(x))dx,\ i = 1, 2. \tag{4}$$

We will now present the adaptive sum-capacity of MACs with discrete fading states and then extend this to continuous-valued fading distributions.

III. Discrete Fading States

In this section, we consider a two-user fading MAC with fading cdfs $\psi_1(h)$ and $\psi_2(h)$. Transmitter i uses a fixed power P_i for all fading states. In presence of individual transmitter CSI, we can assume the fading values to be non-negative. We first consider a MAC with two states for each link to illustrate how the optimal rate allocation is done. The weaker of the states is referred as the bad (B) state and the stronger state is referred as the good (G) state. For link i, these are denoted by respectively B_i and G_i. Fig. 1 shows the MAC capacity regions for each pair of states of the links. For example, the inner pentagon is the capacity region for the state-pair (B_1, B_2), and outer pentagon is the capacity region for (G_1, G_2). Our rate-allocation (in Theorem 5) first chooses any point on the dominant face of the pentagon for the (B_1, B_2) state-pair and assigns the respective co-ordinate values to the rates for the Bad states. This point is marked as ①. Suppose that G_1 has a higher probability than the G_2. Suppose the horizontal line through the point ① intersects the pentagon for the (G_1, B_2) state on the dominant face at point ②. The horizontal coordinate of this point is assigned as the rate for the G_1 state of user 1. Now suppose the vertical line through ② intersects the pentagon for the (G_1, G_2) state at point ③ on its dominant face. The vertical coordinate of this point determines the rate of user 2 for the state G_2. The allocation ensures (as will be shown in Lemma 6) that the operating rate-pair ④ for the state-pair (B_1, G_2) is also inside the corresponding capacity region, as shown in Fig. 1.

Now we discuss the rate-allocation for arbitrary discrete states. Let user 1 have k_1 channel states with probabilities $p_i; 1 \le i \le k_1$ and let user 2 have k_2 channel states with

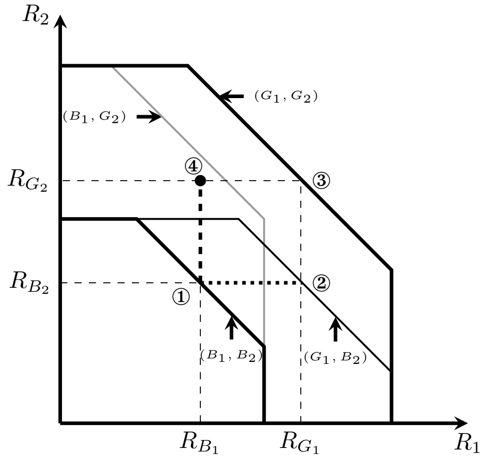

Fig. 1. Illustrating the rate-choice for a 2 state MAC

probabilities $q_i; 1 \leq i \leq k_2$. Let us denote the cdf values of the channels as

$$\alpha_i = \sum_{j=1}^{i} p_j \ , \ 0 \leq i \leq k_1 - 1,$$

$$\beta_i = \sum_{j=1}^{i} q_j \ , \ 0 \leq i \leq k_2 - 1,$$

and let $\{\gamma_i | 0 \leq i < k_1 + k_2 - 1\} = \{\alpha_i | 0 \leq i < k_1\} \cup \{\beta_i | 0 \leq i < k_2\}$ be these values indexed in the increasing order. Here the empty sum is defined to be 0.

Now for $j = 1, 2$, let us define the inverse cdf of user j as

$$h_{ji} = \sup\{h | \psi_j(h) \leq \gamma_i\}. \tag{5}$$

We now state our main result for discrete fading states.

Theorem 5. *For any ρ in the positive interval $\left[\frac{1}{2}\log(1 + \frac{h_{10}^2 P_1}{1 + h_{20}^2 P_2}), \frac{1}{2}\log(1 + h_{10}^2 P_1)\right]$, the rate-strategy given by*

$$R_1(h_{10}) = \rho \tag{6}$$

$$R_2(h_{2i}) = \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2i}^2 P_2) - R_1(h_{1i}) \tag{7}$$

$$R_1(h_{1j}) = \frac{1}{2}\log(1 + h_{1j}^2 P_1 + h_{2(j-1)}^2 P_2) - R_2(h_{2(j-1)}), \tag{8}$$

where $0 \leq i < k$, $1 \leq j < k$ and $k = k_1 + k_2 - 1$, is an outage-free strategy achieving the adaptive sum-capacity.

In the above theorem, the rates for the users are assigned iteratively, alternating between the users. More precisely, they are assigned to h_{ji} in the lexicographic order of the pair (i, j). At a given state of a user, the rate assigned guarantees the maximum sum-rate with a state of the other user where it was last assigned a rate. This sequence of assignment is illustrated in Figure 2, where two cdfs $\psi_1(h)$ and $\psi_2(h)$ are shown. The fading state H_1 takes values in $\{h_{10}, h_{11}, h_{12}\}$ and $H_2 \in \{h_{20}, h_{21}, h_{22}\}$. The iterative rate-assignment is shown at the right, where the rate-choice at the base of each arrow determines the rate for the state at the head/front of the arrow. For example, the rate-choice $R_2(h_{21})$ as well as $R_2(h_{22})$ are determined the choice of $R_1(h_{11})$, and that of $R_1(h_{12})$ is determined by the choice made for $R_2(h_{22})$.

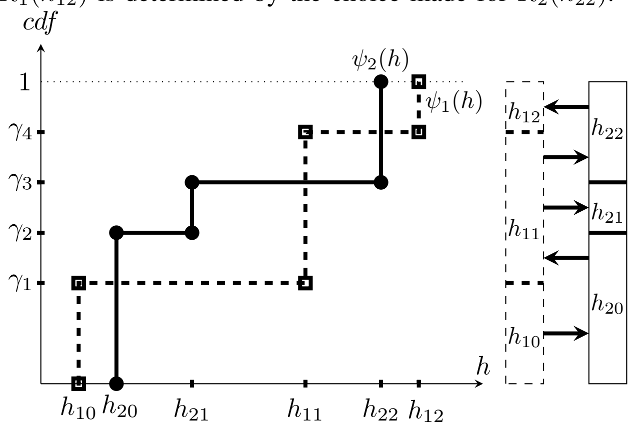

Fig. 2. Illustrating the rate-assignment

The theorem claims that the above assignment of rates is outage-free, see Definition 2. The following lemma will provide a building-block for the proof.

Lemma 6. *Let h_1 and $h_1' \geq h_1$ be two channel states of user 1, and let h_2 and $h_2' \geq h_2$ be two channel states of user 2. If*

$$R_1(h_1) + R_2(h_2) \leq \frac{1}{2}\log(1 + h_1^2 P_1 + h_2^2 P_2),$$

$$R_1(h_1') + R_2(h_2') \leq \frac{1}{2}\log(1 + h_1'^2 P_1 + h_2'^2 P_2),$$

$$\textit{and } R_1(h_1) + R_2(h_2') = \frac{1}{2}\log(1 + h_1^2 P_1 + h_2'^2 P_2),$$

then

$$R_1(h_1') + R_2(h_2) \leq \frac{1}{2}\log(1 + h_1'^2 P_1 + h_2^2 P_2).$$

Proof: For the fading states given in the theorem,

$$\begin{aligned} R_1(h_1') &+ R_2(h_2) \\ &= (R_1(h_1) + R_2(h_2)) + (R_1(h_1') + R_2(h_2')) \\ &\qquad - (R_1(h_1) + R_2(h_2')) \\ &\leq \frac{1}{2}\log(1 + h_1^2 P_1 + h_2^2 P_2) + \frac{1}{2}\log(1 + h_1'^2 P_1 + h_2'^2 P_2) \\ &\qquad - \frac{1}{2}\log(1 + h_1^2 P_1 + h_2'^2 P_2) \end{aligned} \tag{9}$$

Now, let us denote $\tilde{H}_1 = h_1^2 P_1 + h_2^2 P_2$, $\tilde{H}_2 = h_2^2 P_1 + h_2'^2 P_2$, $\tilde{H}_3 = h_1^2 P_1 + h_2'^2 P_2$, and $\tilde{H}_4 = h_2^2 P_1 + h_2^2 P_2$. By the hypothesis, $\tilde{H}_1 \leq \tilde{H}_3, \tilde{H}_4 \leq \tilde{H}_2$, that is, $\tilde{H}_3, \tilde{H}_4$ both lie between $\tilde{H}_1$ and $\tilde{H}_2$. Then, by the concavity of logarithm, we have

$$\begin{aligned} \frac{1}{2}\log(1 + \tilde{H}_1) &+ \frac{1}{2}\log(1 + \tilde{H}_2) \\ &\leq \frac{1}{2}\log(1 + \tilde{H}_3) + \frac{1}{2}\log(1 + \tilde{H}_4). \end{aligned} \tag{10}$$

Lemma 6 is proved by applying (10) to (9). ■

Proof of Theorem 5: First, let us prove that the given rate-strategy is outage-free. Let h and $\bar{h}$ be arbitrary states of user 1 and user 2 respectively. We will show that the chosen rate-pair is inside the corresponding MAC pentagon, and thus outage-free. By the definition in (5), for some i and j, $h = h_{1i}$ and $\bar{h} = h_{2j}$. To check the sum-rate constraint of (2), let us assume w.l.o.g that $i \leq j$. If $i = j$, then by (7), $R_1(h) + R_2(\bar{h}) \leq \frac{1}{2}\log(1 + h^2 P_1 + \bar{h}^2 P_2)$. Now, suppose for some $j \geq i$, it holds that $R_{2j} + R_{1i} \leq \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2j}^2 P_2)$. Using this along with (7) and (8) appropriately in Lemma 6, it follows that $R_{2(j+1)} + R_{1i} \leq \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2(j+1)}^2 P_2)$. Thus, by induction on j, the statement holds true for any $j > i$. An example allocation is presented at the end of this section.

In order to show that there is no outage, we should also prove $R_j(h_{ji}) \leq \frac{1}{2}\log(1+h_{ji}^2 P_j)$, for $j = 1, 2$. We do this by induction on i. The proof is given for the case where $j = 1$. The case of $j = 2$ is similar. Eq. (6) gives the initial step ($i = 0$) for $j = 1$. Now let us consider $i > 0$. By (7) and (8),

$$
\begin{aligned}
&R_1(h_{1i}) \\
&= \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2(i-1)}^2 P_2) \\
&\quad - \frac{1}{2}\log(1 + h_{1(i-1)}^2 P_1 + h_{2(i-1)}^2 P_2) + R_1(h_{1(i-1)}) \\
&\leq \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2(i-1)}^2 P_2) + \frac{1}{2}\log(1 + h_{1(i-1)}^2 P_1) \\
&\quad - \frac{1}{2}\log(1 + h_{1(i-1)}^2 P_1 + h_{2(i-1)}^2 P_2) \qquad (11) \\
&= \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2(i-1)}^2 P_2) \\
&\quad - \frac{1}{2}\log\left(1 + \frac{h_{2(i-1)}^2 P_2}{1 + h_{1(i-1)}^2 P_1}\right) \\
&\leq \frac{1}{2}\log(1 + h_{1i}^2 P_1 + h_{2(i-1)}^2 P_2) - \frac{1}{2}\log\left(1 + \frac{h_{2(i-1)}^2 P_2}{1 + h_{1i}^2 P_1}\right) \qquad (12) \\
&= \frac{1}{2}\log(1 + h_{1i}^2 P_1).
\end{aligned}
$$

The inequality (11) follows from the induction hypothesis, whereas (12) uses the fact that $h_{1i} \geq h_{1(i-1)}$.

Let us now prove that our rate-strategy maximizes the expected sum-rate. The key is to notice that, using the inverse cdf definitions of (3), our rate-allocation ensures that for any $x \in [0, 1)$, $R_1(h_1(x)) + R_2(h_2(x)) = \frac{1}{2}\log(1 + h_1^2(x)P_1 + h_2^2(x)P_2)$. But any outage-free rate-allocation $(R_1(\cdot), R_2(\cdot))$ satisfies

$$
\begin{aligned}
&E(R_1(H_1)) + E(R_2(H_2)) \\
&\qquad = \int_0^1 (R_1(h_1(x)) + R_2(h_2(x)))dx \\
&\qquad \leq \frac{1}{2}\int_0^1 \log(1 + h_1^2(x)P_1 + h_2^2(x)P_2)dx. \qquad (13)
\end{aligned}
$$

The first expression uses (4), and the inequality above follows from (2). Clearly, the proposed scheme achieves this upper bound and this completes the proof of the theorem. ■

IV. Continuous fading state channels

When the fading coefficients take continuous values, it is interesting to see whether our results from the discrete case will hold in a limiting sense. Apart from its technical merit, a result stating the explicit rate allocations will find wide applicability, since continuous-valued distributions like Rayleigh are popularly used to model wireless links. Consider two continuous valued fading distributions $\psi_1(h)$ and $\psi_2(h)$. We will show an elegant solution for the optimal rate allocations. Furthermore, our results can be adapted to combinations of continuous-valued and discrete states. For simplicity, we assume positive valued fading cdfs which admit respective densities[2]. Let $\psi_j^{-1}(\cdot)$ be the inverse cdf of user j, as in (3).

Theorem 7. *For a two user Gaussian MAC with $\psi_1(\cdot)$ and $\psi_2(\cdot)$ as the fading distributions, and with respective transmit powers P_1 and P_2, the adaptive sum-capacity is achieved by the rate-allocation,*

$$
R_i(h) = \int_0^h \frac{yP_i}{1 + y^2 P_i + (\psi_j{}^{-1}(\psi_i(y)))^2 P_j}\,\mathrm{d}y, j \neq i \qquad (14)
$$

Proof: Let us first find an upper bound for the expected sum-rate of any achievable scheme.

$$
\sum_{i=1}^{2} E[R_i(H_i)] = \int_0^\infty R_1(h)d\psi_1(h) + \int_0^\infty R_2(h)d\psi_2(h). \qquad (15)
$$

By the same steps as the discrete-state derivation in (13),

$$
\sum_{i=1}^{2} E[R_i(H_i)] \leq \int_0^1 \frac{1}{2}\log(1 + h_1(x)^2 P_1 + h_2(x)^2 P_2)\,dx,
$$

thus obtaining an upperbound to the achievable sum-rate.

We will also show that this upperbound is in fact achieved by the rate-allocations prescribed in Theorem 7. The remaining part of the proof is contained in Lemmas 8 and 9 below. ■

Lemma 8. *The rate allocation given in* (14) *is outage-free.*

Proof: We will show that $\forall(h_1, h_2)$

$$
R_1(h_1) + R_2(h_2) \leq \frac{1}{2}\log(1 + h_1^2 P_1 + h_2^2 P_2).
$$

The proof is relegated to Appendix A. ■

Lemma 9. *For $x \in [0, 1]$ and the rate allocation in* (14)*,*

$$
R_1(h_1(x)) + R_2(h_2(x)) = \frac{1}{2}\log(1 + h_1^2(x)P_1 + h_2^2(x)P_2).
$$

Proof: See Appendix B. ■

A. Simulation Study

We demonstrate the advantage of the solution that we proposed by an example. We have taken $\psi_1(h)$ as Rayleigh distributed, and $\psi_2(h)$ as a uniform distribution in $[0, a]$. The parameters were chosen such that the second moment remains the same. Figure 3 shows the adaptive sum-capacity when the transmit power is varied while maintaining $P_1 = P_2$, with the fading gains having unit power. For comparison, we also show

[2]with individual CSIT, only fading magnitudes are important

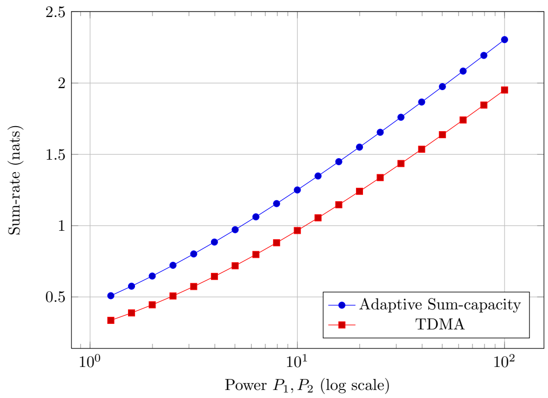

Fig. 3. Adaptive Sum-capacity with $P_1 = P_2$

the sum-rate achieved by the conventional strategy of time division multiplexing (TDMA), where the time is divided into equal-sized slots. Clearly, the proposed solution outperforms the conventional strategy.

V. Conclusion

We have presented the adaptive sum-capacity of fading MACs with non-identical channel statistics. Our solution has an elegant form for continuous valued, as well as discrete fading states. The strategies in fact extend to multiple users, but we have only described the two-user version. Finding the full adaptive capacity region is an interesting direction which we will pursue, along with the power-controlled capacity counterparts.

Appendix A
Proof of Lemma 8

Let R_{12} denote the sum-rate $R_1(h_1) + R_2(h_2)$, where the rate allocations are chosen as in (14). Under the transformation $\psi_2^{-1}(\psi_1(y)) = z$,

$$R_{12} = \int_0^{\psi_2^{-1}(\psi_1(h_1))} \frac{P_1\psi_1^{-1}(\psi_2(z))(\psi_1^{-1}(\psi_2(z))'}{1+z^2P_2+(\psi_1^{-1}(\psi_2(z)))^2P_1}\,dz + \int_0^{h_2} \frac{yP_2}{1+y^2P_2+(\psi_1^{-1}(\psi_2(y)))^2P_1}\,dy.$$

Consider the case when $\psi_2^{-1}(\psi_1(h_1)) < h_2$. Combining terms of the two integrals above,

$$R_{12} = \int_0^{\psi_2^{-1}(\psi_1(h_1))} \frac{P_1\psi_1^{-1}(\psi_2(z))(\psi_1^{-1}(\psi_2(z)))' + zP_2}{1+z^2P_2+(\psi_1^{-1}(\psi_2(z)))^2P_1}\,dz + \int_{\psi_2^{-1}(\psi_1(h_1))}^{h_2} \frac{yP_2}{1+y^2P_2+(P_1\psi_1^{-1}(\psi_2(y)))^2}\,dy$$

By substituting the lower limit of integration in the denominator of the second integral, and denoting $h^* = (\psi_2^{-1}(\psi_1(h_1)))^2P_2 + (h_1)^2P_1 + 1$,

$$R_{12} \le \int_1^{h^*} \frac{1}{2p}\,dp + \int_{\psi_2^{-1}(\psi_1(h_1))}^{h_2} \frac{yP_2}{1+y^2P_2+h_1^2P_1}\,dy$$

Evaluating the integral will give the desired result. The case when $\psi_2^{-1}(\psi_1(h_1)) \ge h_2$ can be handled in a similar fashion.

Appendix B
Proof of Lemma 9

For the given rate-allocation

$$\sum_{i=1}^{2} R_i(h_i(x)) = \int_0^{h_1(x)} \frac{yP_1}{1+y^2P_1+(\psi_2^{-1}(\psi_1(y)))^2P_2}\,dy + \int_0^{h_2(x)} \frac{yP_2}{1+y^2P_2+(\psi_1^{-1}(\psi_2(y)))^2P_1}\,dy \quad (16)$$

Denoting, $\psi_1^{-1}(\psi_2(y)) = z$,

$$\begin{aligned}\sum_{i=1}^{2} R_i(h_i(x)) &= \int_0^{h_1(x)} \frac{yP_1}{1+y^2P_1+(\psi_2^{-1}(\psi_1(y)))^2P_2}\,dy + \int_0^{h_1(x)} \frac{P_2\psi_2^{-1}(\psi_1(z))(\psi_2^{-1}(\psi_1(z))'}{1+z^2P_1+(\psi_2^{-1}(\psi_1(z)))^2P_2}\,dz \\ &= \int_0^{h_1(x)} \frac{P_2\psi_2^{-1}(\psi_1(z))(\psi_2^{-1}(\psi_1(z))' + zP_1}{1+z^2P_1+(\psi_2^{-1}(\psi_1(z)))^2P_2}\,dz \\ &= \int_1^{1+h_1(x)^2P_1+h_2(x)^2P_2} \frac{1}{2p}\,dp \\ &= \frac{1}{2}\log(1+h_1(x)^2P_1+h_2(x)^2P_2).\end{aligned}$$

Since this is true $\forall x \in [0,1]$, we have the desired result.

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