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Bounds on the Capacity Region of a Class of Gaussian Broadcast Interference Channels

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***Abstract*—In this paper, a class of Gaussian broadcast interference channels is investigated, where one of the two broadcast users is subject to the interference coming from a point-to-point transmission. Channel parameters are categorized into three regimes. For the first two, where an ordering of the decodability of the broadcast users exists, inner bounds based on superposition and rate splitting are obtained. Entropy-power-inequality-based outer bounds are derived by combining bounding techniques for Gaussian broadcast and interference channels. These inner and outer bounds lead to either exact or approximate characterizations of the capacity region and sum capacity under various conditions. For the remaining complementing regime, inner and outer bounds are also provided.**

I. Introduction

The broadcast channel (BC) and interference channel (IC) are two fundamental building blocks of wireless networks that have drawn considerable research efforts in the past few decades. Traditionally BC and IC are studied separately, mostly due to the technical challenges each channel presents and their relevance to a practical network. However due to recent interest in heterogeneous cellular networks with frequency reuse factor one, both broadcasting and interference phenomena become equivalently prevalent. In this paper, we consider a channel model inherits features from both BC and IC, which is named as broadcast interference channel (BIC).

We focus on a Gaussian broadcast interference channel (GBIC) with a specific interference pattern as shown in Fig. 1. For example, X_1 could represent the signal transmitted by a macro base station that wishes to communicate with two mobiles, one of which is interfered by a nearby femto base station transmitting X_2 and communicating with its own receiver. In our previous work [1], we have studied a class of discrete memoryless broadcast interference channels (DM-BICs), exhibiting the same interference pattern. In this paper, we adapt achievable schemes from [1] to a GBIC. The main contribution of this work is the derivation of the outer bound for the GBIC. To achieve this, we use the standard entropy power inequality (EPI) [2] and Costa's EPI [3] to deal with interference for $b < 1$ and $b \geq 1$ respectively. This technique is an extension of the one originated in [4] (also see [5] for discussion of bounds in [4]). Furthermore, we incorporate the broadcast bounding technique to address the trade-off between the rates of broadcast users. As a result, the derived outer bound is parameterized by a power splitting factor. Note that similar bounding techniques have been previously employed in [6] for a class of GBIC that is more general than the one studied here. Although parts of the results in paper (Theorem 1 and the second part of Theorem 2) can be deduced from [6], the fact that our outer bounds address the broadcasting trade-off in an explicit way by including a power splitting factor, as opposed to having rates appear on both sides of the inequalities in [6], reveals the connection between inner and outer bounds, leading to new capacity results that cannot be readily derived from [6].

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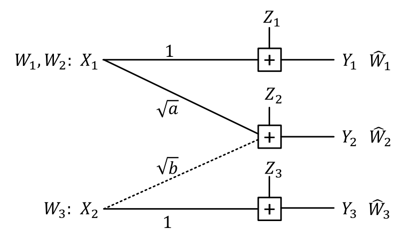

Fig. 1: Gaussian broadcast interference channel

This paper is organized as follows. We describe the system model in Section II. In Section III, channel parameters are divided into three regimes, for each of which inner and outer bounds are derived. In Section IV, numerical results are presented, demonstrating the tightness of the derived inner and outer bounds. This paper is concluded in Section V.

Notation: $\mathcal{C}(x) \triangleq \frac{1}{2}\log_2(1+x)$. $\bar{x} \triangleq 1-x$.

II. System Model

As shown in Fig. 1, transmitter 1 sends X_1 to two receivers, each receiving Y_i, $i \in \{1,2\}$. Similarly, transmitter 2 sends X_2 to a receiver observing output Y_3. However, X_2 also interferes the second receiver of transmitter 1. Mathematically, the channel is given by

$$Y_1 = X_1 + Z_1, \tag{1a}$$

$$Y_2 = \sqrt{a}X_1 + \sqrt{b}X_2 + Z_2, \tag{1b}$$

$$Y_3 = X_2 + Z_3, \tag{1c}$$

where $Z_i \sim \mathcal{N}(0,1)$ for $i \in \{1,2,3\}$ is the i.i.d. Gaussian noise process and X_i, $i \in \{1,2\}$, are subject to power constraints: $E(X_i^2) \leq P_i$. Note that a GBIC with arbitrary channel parameters can always be normalized to the above form. For a given rate triple (R_1, R_2, R_3) and number of channel uses n, the broadcast transmitter encodes messages W_j,

$W_j \in \{1, ..., 2^{nR_j}\}$ for user j, $j = \{1, 2\}$, into a codeword X_1^n while the interference transmitter encodes message W_3 for user 3 into a codeword X_2^n. User j, $j = \{1, 2, 3\}$, produces an estimate $\hat{W}_j$ of message W_j based on the channel output Y_j^n via a decoding function. Rate triple (R_1, R_2, R_3) is said to be achievable if there exits encoding and decoding functions such that the error probability $P_e = \max_{j=\{1,2,3\}} P_{e,j}$, where $P_{e,j} = P_r(\hat{W}_j \neq W_j)$, approaches zero as n approaches infinity. The capacity region is the union of all achievable rate triples.

III. Inner and Outer Bounds

For a regular BC, establishing the capacity region resides on notions of degradedness, being less noisy or more capable [7], which reflect an ordering of the decodability among broadcast users. Under the presence of interference, however, these notions need to be revisited. For a DM-BIC with channel transition probability given by $p(y_1, y_2, y_3|x_1, x_2) = p(y_1|x_1)p(y_2|x_1, x_2)p(y_3|x_2)$, we proposed two related notions in [1]. We say user 1 is *interference-cognizant less noisy* than user 2, denoted by $Y_1 \succ_c Y_2$, if user 1 is less noisy [7] than user 2 even when it knows the exact interfering signal, or mathematically $I(U_1; Y_1) \geq I(U_1; Y_2|X_2)$ for all $p(u_1, x_1)p(x_2)$ such that $U_1 \rightarrow (X_1, X_2) \rightarrow (Y_1, Y_2)$ form a Markov chain. On the other hand, user 2 being *interference-oblivious less noisy* than user 1, denoted by $Y_1 \prec_o Y_2$, presents a reverse scenario where even if interference is unattended, user 2 is still less noisy than user 1. Mathematically, this is represented as $I(U_1; Y_1) \leq I(U_1; Y_2)$ for all $p(u_1, x_1)p(x_2)$ such that $U_1 \rightarrow (X_1, X_2) \rightarrow (Y_1, Y_2)$ form a Markov chain. As we shall see in the remainder of this section, similar notions can be defined for the GBIC as well. To guide the discussion, we divide channel parameters into three regimes according to user 2's broadcast link strength.

A. GBIC with $0 \leq a \leq 1$

Channel condition $0 \leq a \leq 1$ can be viewed as the Gaussian counterpart of interference-cognizant less noisiness. To see this, we note that, conditioned on X_2, Y_2 is stochastically degraded [2] with respect to Y_1, which implies $I(U_1; Y_1) \geq I(U_1; Y_2|X_2)$ for all $p(u_1, x_1)p(x_2)$ such that $U_1 \rightarrow (X_1, X_2) \rightarrow (Y_1, Y_2)$ form a Markov chain. The following inner bound for the GBIC is a direct consequence of this relation.

Corollary 1: For some $\alpha, \gamma \in [0, 1]$, let $\mathcal{S}_1(\alpha, \gamma)$ denote the set of non-negative (R_1, R_2, R_3) satisfying

$$\begin{aligned} R_1 &\leq \mathcal{C}(\bar{\alpha}P_1) \\ R_2 &\leq \mathcal{C}\left(\tfrac{a\alpha P_1}{1+a\bar{\alpha}P_1+b\bar{\gamma}P_2}\right) \\ R_3 &\leq \mathcal{C}(P_2) \\ R_2 + R_3 &\leq \mathcal{C}\left(\tfrac{a\alpha P_1+b\gamma P_2}{1+a\bar{\alpha}P_1+b\bar{\gamma}P_2}\right) + \mathcal{C}(\bar{\gamma}P_2). \end{aligned}$$

Then $\mathcal{S}_1 = \bigcup_{\alpha,\gamma\in[0,1]} \mathcal{S}_1(\alpha, \gamma)$ is an achievable rate region for the GBIC in (1) with $0 \leq a \leq 1$.

Proof: The inner bound is obtained by evaluating $\mathcal{R}_{(2)}$ in [1], an inner bound for a DM-BIC with $Y_1 \succ_c Y_2$. The proof is relegated to [8], the longer version of this paper. ■

For the achievability, the broadcast transmitter employs superposition coding, with α representing the portion of power for user 2. The interference transmitter employs rate splitting, with γ representing the power distribution between the common and private signals. In fact, with the above coding scheme, one can directly obtain an inner bound without resorting to $\mathcal{R}_{(2)}$. However, when deriving $\mathcal{R}_{(2)}$ in [1], we used the condition $Y_1 \succ_c Y_2$ to remove redundant constraints. This leads to a concise representation of the inner bound as shown in Corollary 1. Furthermore, because the inner bound in [1] for a DM-BIC can be reduced to the best known inner bounds for a discrete memoryless BC and IC respectively, $\mathcal{S}_1$ also can be reduced to those for a Gaussian BC (GBC) and a Gaussian IC (GIC).

It was observed in [1] that the usual strong but not very strong interference condition in an IC does not carry over to a DM-BIC with $Y_1 \succ_c Y_2$, similarly to the GBIC with $0 \leq a \leq 1$. This is another example of the entangled interaction between BC and IC components of the channel. In a regular GIC, the strong interference condition is obtained by comparing the interference-to-noise-ratio (INR) at the interfered receiver and the signal-to-noise-ratio at the desired receiver. However in the GBIC, the dynamic of the trade-off between R_1 and R_2 for the broadcast part causes the effective noise level at user 2 to vary, amounting to a varying INR. Therefore the usual notion of strong interference does not apply to the GBIC. However, it is easy to see that the very strong interference condition, i.e. $b \geq 1 + aP_1$, still applies.

Corollary 2: The capacity region of the GBIC in (1) with $0 \leq a \leq 1$ and $b \geq 1 + aP_1$ is given by $\{(R_1, R_2, R_3) : 0 \leq R_1 \leq \mathcal{C}(\bar{\alpha}P_1), 0 \leq R_2 \leq \mathcal{C}(\frac{a\alpha P_1}{1+a\bar{\alpha}P_1}), 0 \leq R_3 \leq \mathcal{C}(P_2)\}$.

For the general case, in particular when $b < 1 + aP_1$, we next present an EPI-based outer bound. In the derivation, we combine bounding techniques for a GBC and a GIC, where EPI is used for both dealing with interference and quantifying the trade-off between the broadcast rates.

Theorem 1: Let $\mathcal{O}_1(\alpha)$ denote the set of non-negative (R_1, R_2, R_3) satisfying

$$R_1 \leq \mathcal{C}(\bar{\alpha}P_1)$$

$$R_2 \leq \mathcal{C}\left(\frac{a\alpha P_1 + bP_2 + (1-a)(1-2^{2\xi(\frac{b}{1-a})})}{a + a\bar{\alpha}P_1 + (1-a)2^{2\xi(\frac{b}{1-a})}}\right) \tag{2}$$

$$R_2 \leq \mathcal{C}\left(\frac{a\alpha P_1}{1+a\bar{\alpha}P_1}\right) \tag{3}$$

$$R_3 \leq \mathcal{C}(P_2),$$

for some $\alpha \in [0, 1]$, where

$$\xi(x) \triangleq \begin{cases} \mathcal{C}(x(2^{2R_3} - 1)), & x < 1 \\ R_3, & x \geq 1 \end{cases}. \tag{4}$$

Then $\mathcal{O}_1 = \bigcup_{\alpha\in[0,1]} \mathcal{O}_1(\alpha)$ is an outer bound on the capacity region of the GBIC in (1) with $0 \leq a \leq 1$.

Proof: See Appendix A. ■

Remark 1: Although it is possible to deduce Theorem 1 from Theorem 4 in [6], here we choose to explicitly show the derivation with an emphasis on including a power splitting factor α. The reason is that similar steps apply to the proofs of other outer bounds presented in Section III.B and Section III.C, which cannot be deduced from [6].

For any achievable rate triple (R_1, R_2, R_3) in Corollary 1, there exists a $\beta \in [0,1]$ such that $R_3 = \mathcal{C}(\beta P_2)$. In the following, we show that when β is sufficiently small: $\beta \leq \frac{\min\{1,b\}}{1+aP_1}$, or $\beta = 1$ (The case of $\beta = 1$ was covered in [6]), the achievable scheme in Corollary 1 is capacity achieving.

Theorem 2: The following rate triples are on the boundary of the capacity region of the GBIC in (1) with $0 \leq a \leq 1$:

$$\left\{ \begin{array}{l} (R_1, R_2, R_3): \ R_1 = \mathcal{C}(\bar{\alpha} P_1), \ R_2 = \mathcal{C}\left(\frac{a\alpha P_1}{1+a\bar{\alpha}P_1}\right), \\ R_3 = \mathcal{C}(\beta P_2), \ \text{for some } \alpha \in [0,1], \ \beta \in [0, \frac{\min\{1,b\}}{1+aP_1}] \end{array} \right\} \tag{5}$$

and if $a + b \leq 1$

$$\left\{ \begin{array}{l} (R_1, R_2, R_3): \ R_1 = \mathcal{C}(\bar{\alpha} P_1), \ R_2 = \mathcal{C}\left(\frac{a\alpha P_1}{1+a\bar{\alpha}P_1+bP_2}\right), \\ R_3 = \mathcal{C}(P_2), \ \text{for some } \alpha \in [0,1] \end{array} \right\} \tag{6}$$

Proof: To prove (5), we evaluate outer bound $\mathcal{O}_1$ in Theorem 1 for $a + b < 1$ and $a + b \geq 1$ respectively. In either case, the resultant outer bound coincides with inner bound $\mathcal{S}_1$ in Corollary 1 with $\gamma = 1$. Similarly (6) can be proved. The details can be found in [8]. ■

The sum capacity for $0 \leq a \leq 1$ is straightforward. Since user 2 is always stochastically degraded with respect to user 1, to maximize the broadcast sum rate, the broadcast transmitter should transmit with full power to user 1, i.e. setting $\alpha = 0$ in Corollary 1, which at the same time renders interference irrelevant.

Corollary 3: The sum capacity of the GBIC in (1) with $0 \leq a \leq 1$ is $\mathcal{C}(P_1) + \mathcal{C}(P_2)$.

B. GBIC with $a \geq 1 + bP_2$

Assuming X_2 is Gaussian distributed, it is easy to see that, when $a \geq 1 + bP_2$, $I(U_1; Y_1) \leq I(U_1; Y_2)$ for all $p(u_1, x_1)$ such that $U_1 \to (X_1, X_2) \to (Y_1, Y_2)$ form a Markov chain. Hence we obtain an inner bound for a GBIC by specializing that, with Gaussian inputs, for a DM-BIC with $Y_1 \prec_o Y_2$.

Corollary 4: For some $\alpha, \gamma \in [0,1]$, let $\mathcal{S}_2(\alpha, \gamma)$ denote the set of non-negative (R_1, R_2, R_3) satisfying

$$R_1 \leq \mathcal{C}\left(\frac{\bar{\alpha}P_1}{1+\alpha P_1}\right)$$
$$R_2 \leq \mathcal{C}\left(\frac{a\alpha P_1}{1+b\bar{\gamma}P_2}\right)$$
$$R_3 \leq \mathcal{C}(P_2)$$
$$R_2 + R_3 \leq \mathcal{C}\left(\frac{a\alpha P_1 + b\gamma P_2}{1+b\bar{\gamma}P_2}\right) + \mathcal{C}(\bar{\gamma}P_2).$$

Then $\mathcal{S}_2 = \bigcup_{\alpha,\gamma \in [0,1]} \mathcal{S}_2(\alpha, \gamma)$ is an achievable rate region of the GBIC in (1) with $a \geq 1 + bP_2$.

Proof: The proof is similar to that for Corollary 1 and is given in [8]. ■

Remark 2: The difference between S_1 and S_2 is that the roles of user i's signal, $i \in \{1,2\}$, in the superposition coding are switched, with user 1's signal as the cloud center for S_2 and user 2's signal for S_1.

Unlike the achievability part, the converse result presented in [1] for a DM-BIC with $Y_1 \prec_o Y_2$ cannot be specialized to the GBIC with $a \geq 1 + bP_2$. This is because the GBIC with $a \geq 1 + bP_2$ does not satisfy the condition of $Y_1 \prec_o Y_2$ for an arbitrary distribution of X_2. Next, we derive an outer bound for the GBIC with $a \geq 1 + bP_2$ using similar arguments as for $\mathcal{O}_1$ except one crucial difference. In order to invoke EPI to relate broadcast rates R_1 and R_2, we need first to quantify the loss of optimality when fixing X_2 to be Gaussian. To achieve this, we use a result from [9], which says that the Gaussian input incurs no more than half-bit loss compared to the optimal distribution for an arbitrary noise channel.

Theorem 3: Let $\mathcal{O}_2(\alpha)$ denote the set of non-negative (R_1, R_2, R_3) satisfying the following

$$R_1 \leq \mathcal{C}\left(\frac{\bar{\alpha}P_1}{1+\alpha P_1}\right) \tag{7}$$
$$R_2 \leq \mathcal{C}(a\alpha P_1 + bP_2) - \xi(b) + 0.5 \tag{8}$$
$$R_2 \leq \mathcal{C}(aP_1 + bP_2) - \xi(b) \tag{9}$$
$$R_2 \leq \mathcal{C}(a\alpha P_1) \tag{10}$$
$$R_3 \leq \mathcal{C}(P_2), \tag{11}$$

where $\alpha \in [0,1]$ and $\xi(\cdot)$ is defined in (4). Then $\mathcal{O}_2 = \bigcup_{\alpha \in [0,1]} \mathcal{O}_2(\alpha)$ is an outer bound on the capacity region of the GBIC in (1) with $a \geq 1 + bP_2$.

Proof: The proof is similar to that for Theorem 1 and is given in [8]. ■

When interference is strong but not very strong, i.e. $1 \leq b < 1 + aP_1$, the outer bound $\mathcal{O}_2(\alpha)$ (without (9)) differs from the inner bound $\mathcal{S}_2(\alpha, \gamma)$ with $\gamma = 1$ only by a constant 0.5 in inequality (8). Hence having receiver Y_2 decoding interference as a whole is approximately capacity achieving.

Corollary 5: The inner bound $\mathcal{S}_2$ given in Corollary 4, is within 0.5 bits to the capacity region of the GBIC in (1) with $a \geq 1 + bP_2$ and $1 \leq b < 1 + aP_1$.

It is also straightforward to obtain the capacity region when interference is very strong, i.e. $b \geq 1 + aP_1$, where interference can be decoded and removed at receiver Y_2 by treating the desired signal as noise.

Corollary 6: The capacity region of the GBIC in (1) with $a \geq 1 + bP_2$ and $b \geq 1 + aP_1$ is given by $\{(R_1, R_2, R_3) : 0 \leq R_1 \leq \mathcal{C}(\frac{\bar{\alpha}P_1}{1+\alpha P_1}), 0 \leq R_2 \leq \mathcal{C}(a\alpha P_1), 0 \leq R_3 \leq \mathcal{C}(P_2)\}$.

In the following, we focus on sum rate/capacity analysis.

Proposition 1: The sum rate of the achievable scheme given in Corollary 4 is

$$R_s = \begin{cases} \mathcal{C}\left(\frac{aP_1}{1+bP_2}\right) + \mathcal{C}(P_2), & \text{if } 0 \leq b < 1 \\ \min\{\mathcal{C}(aP_1 + bP_2), \mathcal{C}(aP_1) + \mathcal{C}(P_2)\}, & \text{if } b \geq 1 \end{cases} \tag{12}$$

Proof: R_s is achieved by setting $\alpha = 1$, $\gamma = 0$ if $b < 1$ and $\alpha = 1$, $\gamma = 1$ if $b \geq 1$ for $\mathcal{S}_2$ in Corollary 4. The sum rate optimality of such choice of parameters is shown in [8]. ■

Theorem 4: The sum rate R_s given in (12) is within 0.5 bits to the sum capacity C_s of the GBIC in (1) with $a \geq 1 + bP_2$, i.e. $C_s \leq R_s + 0.5$.

Proof: The proof is based on the genie bounding technique introduced in [5]. Details can be found in [8]. ■

Remark 3: For the GBIC with $a \geq 1 + bP_2$, to achieve (approximately) the sum capacity, the broadcast transmitter should transmit with full power to user 2. Depending on the strength of interference link, user 2 takes different actions. When interference is very strong, i.e. $b \geq 1 + aP_1$, user 2 decodes interference by treating the desired signal as noise. When interference is strong but not very strong, i.e. $1 \leq b < 1 + aP_1$, user 2 jointly decodes interference and the desired signal. When interference is weak, i.e. $b < 1$, user 2 treats interference as noise.

C. GBIC with $1 < a < 1 + bP_2$

With $1 < a < 1 + bP_2$, there is no immediate ordering of the decodability between the two broadcast users given the interference from user 3. We consider two superposition inner bounds, where the signal for each of the broadcast users serves as the cloud center respectively, and then take convex hull of the two regions.

Corollary 7: For some $\alpha_1, \alpha_2, \gamma_1, \gamma_2 \in [0, 1]$, let $\mathcal{S}_3(\alpha, \gamma)$ denote the set of non-negative (R_1, R_2, R_3) satisfying

$$
\begin{aligned}
R_1 &\leq \mathcal{C}\left(\tfrac{\bar{\alpha}_1 P_1}{1+\alpha_1 P_1}\right) \\
R_3 &\leq \mathcal{C}(P_2) && (13) \\
R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{aP_1}{1+b\bar{\gamma}_1 P_2}\right) && (14) \\
R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{\bar{\alpha}_1 P_1}{1+\alpha_1 P_1}\right) + \mathcal{C}\left(\tfrac{a\alpha_1 P_1}{1+b\bar{\gamma}_1 P_2}\right) && (15) \\
R_1 + R_2 + R_3 &\leq \mathcal{C}\left(\tfrac{aP_1 + b\gamma_1 P_2}{1+b\bar{\gamma}_1 P_2}\right) + \mathcal{C}(\bar{\gamma}_1 P_2) && (16) \\
R_1 + R_2 + R_3 &\leq \mathcal{C}\left(\tfrac{\bar{\alpha}_1 P_1}{1+\alpha_1 P_1}\right) + \mathcal{C}\left(\tfrac{a\alpha_1 P_1 + b\gamma_1 P_2}{1+b\bar{\gamma}_1 P_2}\right) + \mathcal{C}(\bar{\gamma}_1 P_2), && (17)
\end{aligned}
$$

and $\mathcal{S}_4(\alpha, \gamma)$ be the set of non-negative (R_1, R_2, R_3) satisfying

$$
\begin{aligned}
R_2 &\leq \mathcal{C}\left(\tfrac{a\alpha_2 P_1}{1+a\bar{\alpha}_2 P_1 + b\bar{\gamma}_2 P_2}\right) \\
R_3 &\leq \mathcal{C}(P_2) && (18) \\
R_1 + R_2 &\leq \mathcal{C}(\bar{\alpha}_2 P_1) + \mathcal{C}\left(\tfrac{a\alpha_2 P_1}{1+a\bar{\alpha}_2 P_1 + b\bar{\gamma}_2 P_2}\right) && (19) \\
R_1 + R_2 &\leq \mathcal{C}(P_1) && (20) \\
R_2 + R_3 &\leq \mathcal{C}\left(\tfrac{a\alpha_2 P_1 + b\gamma_2 P_2}{1+a\bar{\alpha}_2 P_1 + b\bar{\gamma}_2 P_2}\right) + \mathcal{C}(\bar{\gamma}_2 P_2) \\
R_1 + R_2 + R_3 &\leq \mathcal{C}(\bar{\alpha}_2 P_1) + \mathcal{C}\left(\tfrac{a\alpha_2 P_1 + b\gamma_2 P_2}{1+a\bar{\alpha}_2 P_1 + b\bar{\gamma}_2 P_2}\right) + \mathcal{C}(\bar{\gamma}_2 P_2). && (21)
\end{aligned}
$$

We further let $\mathcal{S}_3 = \bigcup_{\alpha_1, \gamma_1 \in [0,1]} \mathcal{S}_3(\alpha, \gamma)$ and $\mathcal{S}_4 = \bigcup_{\alpha_1, \gamma_1 \in [0,1]} \mathcal{S}_4(\alpha, \gamma)$. Then the convex hull of $\mathcal{S}_3$ and $\mathcal{S}_4$ is an achievable rate region of the GBIC in (1) with $1 < a < 1 + bP_2$.

Proof: Similar to $\mathcal{S}_1$ in Corollary 1 and $\mathcal{S}_2$ in Corollary 4, $\mathcal{S}_3$ and $\mathcal{S}_4$ are obtained by evaluating corresponding inner bounds for a DM-BIC given in [1]. The proof is relegated to [8]. ■

Proposition 2: The sum rate of the inner bound in Corollary 7 is given by $R_s = \max\{R_{s,1}, R_{s,2}\}$, where

$$
\begin{aligned}
R_{s,1} = \max_{\alpha_1, \gamma_1 \in [0,1]} \min\{&\text{rhs}(13) + \text{rhs}(14), \text{rhs}(13) + \text{rhs}(15), \\
&\text{rhs}(16), \text{rhs}(17)\}, \\
R_{s,2} = \mathcal{C}(P_1) &+ \mathcal{C}(P_2),
\end{aligned}
$$

and $\text{rhs}(x)$ denotes the right-hand side of inequality (x).

Proof: It is straightforward to show that the sum rates of $\mathcal{S}_3(\alpha, \gamma)$ and $\mathcal{S}_4(\alpha, \gamma)$ are given by $\min\{\text{rhs}(13) + \text{rhs}(14), \text{rhs}(13) + \text{rhs}(15), \text{rhs}(16), \text{rhs}(17)\}$ and $\min\{\text{rhs}(18) + \text{rhs}(19), \text{rhs}(18) + \text{rhs}(20), \text{rhs}(21)\}$ respectively. Since $\text{rhs}(18) + \text{rhs}(20)$ is indeed achievable for $\mathcal{S}_4(\alpha, \gamma)$ with $\alpha_2 = \gamma_2 = 0$, $R_{s,2} = \mathcal{C}(P_1) + \mathcal{C}(P_2)$. ■

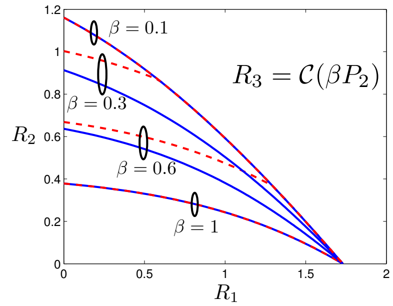

Fig. 2: Inner (solid line) and outer bounds (dashed line) for the GBIC in (1) with $P_1 = 10$, $P_2 = 8$, $a = 0.4$, $b = 0.6$

Theorem 5: Let $\mathcal{O}_3(\alpha)$ denote the set of non-negative (R_1, R_2, R_3) satisfying the following

$$
\begin{aligned}
R_1 &\leq \mathcal{C}\left(\tfrac{\bar{\alpha} P_1}{1+\alpha P_1}\right) \\
R_2 &\leq \mathcal{C}(a\alpha P_1) \\
R_2 &\leq \mathcal{C}(aP_1 + bP_2) - \xi(b) && (22) \\
R_3 &\leq \mathcal{C}(P_2),
\end{aligned}
$$

where $\alpha \in [0, 1]$ and $\xi(\cdot)$ is defined in (4). Then $\mathcal{O}_3 = \bigcup_{\alpha \in [0,1]} \mathcal{O}_3(\alpha)$ is an outer bound on the capacity region of the GBIC in (1) with $1 < a < 1 + bP_2$.

Proof: The proof is similar to those for Theorem 1 and Theorem 3. The details can be found in [8]. ■

IV. Numerical Results

We first compare inner bound $\mathcal{S}_1$ and outer bound $\mathcal{O}_1$ for $0 \leq a \leq 1$ in Fig. 2, where $R_3 = \mathcal{C}(\beta P_2)$ is fixed at different values to investigate the trade-off between R_1 and R_2. The result agrees with Theorem 2. We observe that these bounds are tight when the interference rate is either small or equal to $\mathcal{C}(P_2)$.

For $a \geq 1 + bP_2$, we provide a numerical result in Fig. 3, which compares inner bound $\mathcal{S}_2$ and outer bound $\mathcal{O}_2$ when $b < 1$. We observe that for all values of β, the inner bound is always within 0.5 bits to the outer bound, confirming Corollary 5.

For $1 < a < 1 + bP_2$, the sum rate from Proposition 2 is plotted in Fig. 4, where sum rate upper bound R_o is obtained from $\mathcal{O}_3$ in Theorem 5. We observe that depending on the strength of broadcast link a, one of the two regions, $\mathcal{S}_3$ and $\mathcal{S}_4$, gives rise to R_s, ensuring the necessity of the convex hull operation in the achievable scheme given by Corollary 7.

V. Conclusion

In this paper, we have investigated the GBIC in (1) for three regimes of channel parameters. Inner and outer bounds on the capacity regions have been obtained for each of these regimes. Noticeably, the outer bounds extend previously known bounds in the literature [6] by combing bounding techniques for a

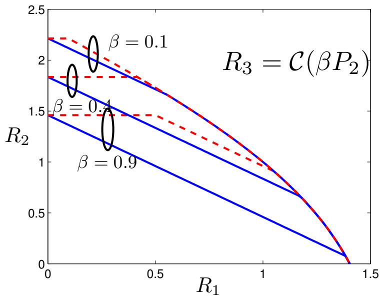

Fig. 3: Inner (solid line) and outer bounds (dashed line) for the GBIC in (1) with $P_1 = 6$, $P_2 = 3$, $a = 4$, $b = 1$

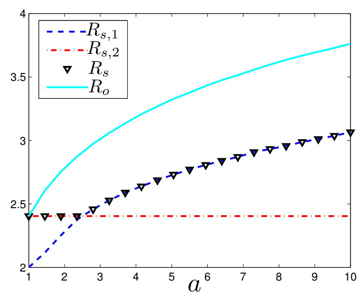

Fig. 4: Sum rate for the GBIC in (1) with $P_1 = 6$, $P_2 = 3$, $b = 3$, $1 < a < 1 + bP_2$

GBC and a GIC. The capacity region and sum capacity have subsequently been determined either exactly or approximately (within a half-bit gap) under various conditions.

Appendix A
Proof of Theorem 1

Proof: The bound $R_3 \leq \mathcal{C}(P_2)$ is straightforward. We next consider bounding R_1. Since $\frac{n}{2}\log 2\pi e = h(Z_1^n) \leq h(Y_1^n|W_2) \leq h(X_1^n + Z_1^n) \leq \frac{n}{2}\log[2\pi e(1+P_1)]$, we must have $h(Y_1^n|W_2) = \frac{n}{2}\log[2\pi e(1+\bar{\alpha}P_1)]$ for some $\alpha \in [0,1]$. Hence we have

$$n(R_1 - \epsilon_n) \leq I(X_1^n; Y_1^n|W_2) = h(Y_1^n|W_2) - h(Z_1^n) = n\mathcal{C}(\bar{\alpha}P_1),$$

where $\lim_{n\to\infty}\epsilon_n = 0$. Before bounding R_2, let us first examine the term $h(\sqrt{b}X_2^n + Z_2^n)$.

Case I: $b < 1$

In this case, we have

$$\begin{aligned} h(\sqrt{b}X_2^n + Z_2^n) &= h(\sqrt{b}(X_2^n + Z_3^n) + \sqrt{1-b}\tilde{Z}^n) \quad (23)\\ &\geq \frac{n}{2}\log\left[2^{\frac{2}{n}h(\sqrt{b}(X_2^n+Z_3^n))} + 2\pi e(1-b)\right] \quad (24)\end{aligned}$$

where in (23), $\tilde{Z} \sim \mathcal{N}(0,1)$ is independent of Z_3, and (24) is due to EPI [2]. Since

$$n(R_3 - \epsilon_n) \leq I(X_2^n; Y_3^n) = h(X_2^n + Z_3^n) - \frac{n}{2}\log(2\pi e),$$

we have $h(\sqrt{b}(X_2^n + Z_3^n)) \geq nR_3 + \frac{n}{2}\log(2\pi eb)$ (ϵ_n is removed for conciseness). Continuing on (24), we have

$$\begin{aligned} h(\sqrt{b}X_2^n + Z_2^n) &\geq \frac{n}{2}\log\left[2\pi eb2^{2R_3} + 2\pi e(1-b)\right]\\ &= n\xi(b) + \frac{n}{2}\log 2\pi e.\end{aligned}$$

Case II: $b \geq 1$

In this case, we have

$$\begin{aligned} &h(\sqrt{b}X_2^n + Z_2^n)\\ &= h(X_2^n + \tfrac{1}{\sqrt{b}}Z_2^n) + \frac{n}{2}\log b\\ &\geq \frac{n}{2}\log\left[(1-\tfrac{1}{b})2^{\frac{2}{n}h(X_2^n)} + \tfrac{1}{b}2^{\frac{2}{n}h(X_2^n+Z_2^n)}\right] + \frac{n}{2}\log b \quad (25)\\ &\geq \frac{n}{2}\log\left[2^{\frac{2}{n}h(X_2^n+Z_2^n)}\right]\\ &\geq nR_3 + \frac{n}{2}\log 2\pi e = n\xi(b) + \frac{n}{2}\log 2\pi e \quad (26)\end{aligned}$$

where (25) is due to Costa's EPI [3] and (26) is due to $h(X_2^n + Z_3^n) \geq nR_3 + \frac{n}{2}\log 2\pi e$.

Now consider

$$\begin{aligned} &h(Y_2^n|W_2)\\ &= h(X_1^n + \tfrac{\sqrt{b}}{\sqrt{a}}X_2^n + \tfrac{1}{\sqrt{a}}Z_2^n)|W_2) + \frac{n}{2}\log a\\ &\geq \frac{n}{2}\log\left[2^{\frac{2}{n}h(Y_1^n|W_2)} + \tfrac{1-a}{a}2^{\frac{2}{n}h(\sqrt{\frac{b}{1-a}}X_2^n+\tilde{Z}_2^n)}\right] + \frac{n}{2}\log a \quad (27)\\ &\geq \frac{n}{2}\log\left[2\pi e(1+\bar{\alpha}P_1) + \tfrac{1-a}{a}2\pi e2^{2\xi(\frac{b}{1-a})}\right] + \frac{n}{2}\log a, \quad (28)\end{aligned}$$

where $\tilde{Z}_1, \tilde{Z}_2 \sim \mathcal{N}(0,1)$ are i.i.d. Gaussian noise processes, (27) is due to EPI and (28) follows from the bounds on $h(\sqrt{b}X_2^n + Z_2^n)$ we just showed above.

We are now in position to bound R_2. To obtain (2), consider

$$\begin{aligned} n(R_2 - \epsilon_n) &\leq h(Y_2^n) - h(Y_2^n|W_2)\\ &\leq \frac{n}{2}\log[2\pi e(1 + aP_1 + bP_2)] - \frac{n}{2}\log\Big[2\pi e(1+\bar{\alpha}P_1)\\ &\quad + \tfrac{1-a}{a}2\pi e2^{2\xi(\frac{b}{1-a})}\Big] - \frac{n}{2}\log a \quad (29)\\ &= n\mathcal{C}\left(\frac{a\alpha P_1 + bP_2 + (1-a)(1-2^{2\xi(\frac{b}{1-a})})}{a + a\bar{\alpha}P_1 + (1-a)2^{2\xi(\frac{b}{1-a})}}\right),\end{aligned}$$

where (29) is due to (28) and the fact that Gaussian distribution maximizes entropy given a covariance constraint. The bound (3) follows standard GBC converse [7], where interference X_2 is known at Y_2. ■

References

[1] Y. Liu and E. Erkip, "On a class of discrete memoryless broadcast interference channels," in *Proc. Int. Symposium Inf. Theory*, August 2012.

[2] T. Cover and J. Thomas, *Elements of Information Theory 2nd Edt*, New York: Wiley, 2006.

[3] M. Costa, "A new entropy power inequality," *IEEE Trans. Inf. Theory*, vol. 31, pp. 751-760, Nov. 1985.

[4] G. Kramer, "Outer bounds on the capacity region of Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 3, pp. 581-586, Mar. 2004.

[5] V. S. Annapureddy and V. Veeravalli, "Gaussian interference networks: sum capacity in the low interference regime and new outer bounds on the capacity region," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032-035, July 2009.

[6] X. Shang and H. V. Poor, "On the capacity of Gaussian broadcast channels that receive interference," in *Proc. 45th Conference on Information Sciences and Systems*, Baltimore, MD, pp. 1-5, March 2011.

[7] A. El Gamal and Y. Kim, *Network Information Theory*, Cambridge University Press, 2011.

[8] Y. Liu and E. Erkip, "Bounds on the capacity region of a class of Gaussian broadcast interference channels," longer version available at http://wireless.poly.edu/wiki/YuanpengWebPage.

[9] R. Zamir and U. Erez, "Gaussian input is not too bad," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1362-1367, June 2004.