# On the Degrees of Freedom Region of General MIMO Broadcast Channel with Mixed CSIT

Bo Yang, Desheng Wang, Jun Sun, and Yingzhuang Liu
The Department of Electronics and Information Engineering
Huazhong University of Science and Technology (HUST), Wuhan, China
Email: yangbo@hust.edu.cn, dswang@mail.hust.edu.cn, francissunj@gmail.com, liuyz@hust.edu.cn

***Abstract*—The two-user multiple-input multiple-output (MIMO) Gaussian Broadcast Channel (BC) with mixed CSIT (i.e., delayed CSIT plus estimated current CSIT) under general antenna configuration is considered in this paper. Unlike the MISO scenario, obtaining tight characterization of the Degrees of Freedom (DoF) region seems difficult for the general MIMO BC settings. Despite that, the inner bound we provide in this paper is believed to be the best achievable bound so far. One novel ingredient of our scheme is the rate-splitting of the interference-encoded symbols, which provides the possible benefit of DoF gain by accommodating the transmission to the asymmetric receivers. Moreover, the outer bound of DoF region is also presented, which coincides our proposed inner bound in some cases.**

***Index Terms*—Broadcast Channel, DoF Region, Mixed CSIT, MIMO.**

## I. Introduction

It is well known that channel state information at transmitter(CSIT) plays a crucial role for the multiple-antenna Broadcast Channel (BC). Taking a MISO BC for example, when perfect CSIT is available, the Degrees of Freedom are $\min(M, K)$ [1], where $M$ and $K$ denote the number of antennas at the transmitter and the number of users, respectively. For the cases of no CSIT, however, the DoF will collapse to 1 [2].

To bridge the huge gap between the above two extremes, Maddah Ali and Tse pioneered to study the scenario where perfect delayed CSI is available at the transmitter [3]. Their seminal work shows that even completely outdated CSIT can provide DoF gain, which is in sharp contrast to the conventional wisdom. The main idea of their scheme is to exploit the side information available at the receivers, which is obtained via overhearing. Later on, Vaze and Varanasi generalize Maddah Ali and Tse's work to the MIMO setting from the MISO one for the two-user broadcast channel [4].

Considering the temporal correlation is totally ignored in the Maddah-Ali-Tse (MAT) scheme, Yang *et al.* [5] proposed to exploit this correlation to further enhance the obtained DoF. They and Gou *et al.* [6] both fully characterize the DoF region of the (2,1,1) BC, which is considerably higher than that of MAT scheme, owing to the interference reduction via partial zero-forcing.

In this paper we intend to generalize the above results to the case of arbitrary antenna configurations, i.e., the general two-user MIMO BC with mixed CSIT. For this channel, we present an outer bound and an inner bound of the DoF region. The outer and inner bound have a small gap and in some ranges of the parameter $\alpha$, which will be defined shortly, they actually coincide partially. The main difficulty of tackling the general MIMO BC with mixed CSIT lies in the redundancy and asymmetry of the receiver antennas, which is not encountered in the MISO BC settings [5],[6]. To address this problem, we propose a new achievable scheme whose key ingredient is the rate-splitting of the interference-encoded symbols in the core stage of broadcasting the interference signals, whose main role is to help resolving interference and providing new observations. Our proposed scheme is believed to be the best achievable scheme so far. Due to space limitation, in this paper we focus on a representative case, i.e., the (4,3,2) setting, where the transmitter has 4 antennas while the receivers have 3 and 2 antennas, respectively.

The rest of this paper is organized as follows. In Section II the channel model and some notations used in this paper are provided. The outer bound of DoF region is given in Section III. Section IV is devoted to present the inner bound and explain the achievable scheme. Finally, Section V concludes this paper.

## II. System Model

We consider a MIMO BC, where the transmitter is equipped with $M$ antennas and the two receivers are equipped with $N_1$ and $N_2$ antennas respectively. For notational brevity, we will denote this channel as $(M, N_1, N_2)$ throughout this paper. The received signals are given by

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{n}_1 \tag{1}$$
$$\boldsymbol{z} = \mathbf{G}\boldsymbol{x} + \boldsymbol{n}_2 \tag{2}$$

where $\boldsymbol{x}$ is the transmitted signal, $\mathbf{H}$ and $\mathbf{G}$ are the $N_1 \times M$ and $N_2 \times M$ channel coefficients matrices between the transmitter and the two receivers, $n_1 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_1})$ and $n_2 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_2})$ denote the additive complex Gaussian noise vectors at the two receivers, $\boldsymbol{y}, \boldsymbol{z}$ are the signals at each receiver. The power constraint is given by $\mathbb{E}\|\boldsymbol{x}^2\| \leq P$. We omit all time indexes in (1) and (2) for simplicity.

The channels are assumed to be Rayleigh fading, and all components of channel matrices are independent, each being a complex Gaussian distribution $\mathcal{CN}(0, 1)$. Throughout the paper, we assume that the transmitter has access to perfect CSI of the past time slots and possesses an estimation of the current slot. The channel estimation is modeled as

$$\mathbf{H} = \widehat{\mathbf{H}} + \widetilde{\mathbf{H}} \tag{3}$$

$$\mathbf{G} = \widehat{\mathbf{G}} + \widetilde{\mathbf{G}} \tag{4}$$

where $\widehat{\mathbf{H}}, \widehat{\mathbf{G}}$ are the estimated value and $\widetilde{\mathbf{H}}, \widetilde{\mathbf{G}}$ denote the estimation error of the channel matrices. As in [5], we assume that elements in $\widehat{\mathbf{H}}, \widehat{\mathbf{G}}$ are i.i.d complex Gaussian distributed $\mathcal{CN}(0, 1-\sigma^2)$, and elements in $\widetilde{\mathbf{H}}, \widetilde{\mathbf{G}}$ are i.i.d complex Gaussian distributed $\mathcal{CN}(0, \sigma^2)$, where $0 \le \sigma^2 \le 1$. Furthermore, the channel estimation exponent $\alpha$ is defined as

$$\alpha = -\lim_{P\to\infty} \frac{\log \sigma^2}{\log P} \tag{5}$$

i.e. $\sigma^2 = \Theta(P^{-\alpha})$. Note that for this definiton $\alpha = 0$ and $\alpha = 1$ correspond to the two extreme cases where no current CSIT and perfect current CSIT is available.

## III. The Converse

The DoF region of a two-user MIMO broadcast channel is defined as:

$$\begin{aligned}\mathcal{D} = \Bigg\{ &(d_1, d_2) \in \mathbb{R}_+^2 : \forall (w_1, w_2) \in \mathbb{R}_+^2 \\ &w_1 d_1 + w_2 d_2 \\ &\le \limsup_{P\to\infty} \left[ \sup_{\mathbf{R}(P)\in\mathcal{C}(P)} \frac{w_1 R_1(P) + w_2 R_2(P)}{\log P} \right] \Bigg\}\end{aligned} \tag{6}$$

where $\mathbf{R}(P) = (R_1(P), R_2(P))$ is the achievable rate tuple and $\mathcal{C}(P)$ is the capacity region, $d_1$ and $d_2$ are DoF of Receiver 1(R1) and Receiver 2(R2), respectively.

We present the outer bound for the DoF region of general BC on the top of next page.

This outer bound can be obtained in nearly the same spirit of [5] and [7], hence the detailed proof is omitted here, and we only give an outline of the proof. The proof starts with a genie providing the received signal of one receiver to the other, thus making the channel degraded. Then by using the Fano's inequality and the extremal inequality in [8] to upperbound the difference of two entropy terms, and moreover, by exploiting the isotropic property of the Gaussian vectors, we can reach the weighted sum DoF inequalities at last.

It is worth noting that in the general MIMO BC settings, the sum constraint (6c) might be active for some ranges of $\alpha$. In contrast, in the MISO BC the sum constraint keeps inactive. This can be seen as one manifestation of the additional difficulty for the general MIMO BC.

## IV. The achievable scheme

### A. Inner bound

The following lemma is of much importance to the following work.

**Lemma 1.** *Suppose that $M \ge N_1 \ge N_2$, for a $(M, N_1, N_2)$ broadcast channel with mixed CIST, the DoF pair $((M-N_2)\alpha, N_2)$ can be achieved.*

The scheme to achieve the DoF pair $((M-N_2)\alpha, N_2)$ is motivated by the scheme in [6], in which the transmitter send a superposition of one common symbol and two private symbols for each receiver.

The main result of this work is summarized in the following theorem.

**Theorem 1** (Inner bound)**.** *For a $(4,3,2)$ BC with mixed CSIT, the DoF region characterized by the corner points $(0,2)$,$(2\alpha,2)$,$\left(\frac{10+4\alpha}{5}, \frac{4+2\alpha}{5}\right)$,$\left(\frac{10+5\alpha}{5}, \frac{4+\alpha}{5}\right)$ and $(3,0)$ is achievable for $\alpha > 1/2$. While for $\alpha \le 1/2$, the DoF region characterized by the corner points $(0,2)$,$(2\alpha,2)$,$\left(\frac{12}{5}, \frac{4+2\alpha}{5}\right)$,$\left(\frac{12+3\alpha}{5}, \frac{4-\alpha}{5}\right)$ and $(3,0)$ is achievable .*

An illustration of the achievable DoF region for the above inner bound and its comparison with the other DoF regions, which include: the DoF region with only delayed CSIT, the outer bound of DoF region with mixed CSIT as well as the DoF region with perfect CSIT, is provided in Fig 1, where the point $\left(\frac{10+5\alpha}{5}, \frac{4+\alpha}{5}\right)$ is omitted because it is very close to the point $\left(\frac{10+4\alpha}{5}, \frac{4+2\alpha}{5}\right)$. It can be seen that the inner bound is very close to the outer bound of the DoF region. In fact, when $\alpha = 1$, the two bounds coincide in a part of line-segment which consists of the sum-DoF optimal points. Moreover, the inner bound of DoF region is much better than that with only delayed CSIT, which manifests the benefit of the estimated current CSIT.

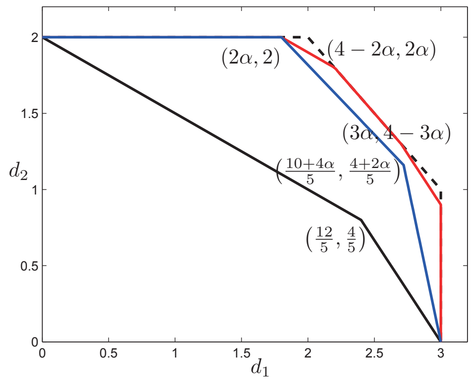



Fig. 1: Comparison of DoF regions of a $(4,3,2)$ with mixed CSIT where $\alpha = 0.9$. The four curves from the left to the right correspond to the DoF region with only delayed CSIT, the inner bound and outer bound of DoF region with mixed CSIT and DoF region with perfect CSIT, respectively.

We now elaborate on the achievable scheme to meet the above inner bound. First some notations are introduced. We let $a_{t,s}, b_{t,s}, c_{t,s}$ denote the symbols intended for R1, R2 and the common message, respectively, with $\boldsymbol{u}_{t,s}, \boldsymbol{v}_{t,s}, \boldsymbol{w}_{t,s}$ being the corresponding precoding vectors. The subscript $t$ indicates a precoder or a symbol belongs to slot $t$, and the subscript $s$ serves to distinguish different precoders or symbols in a same slot. Moreover, $\mathbf{H}_t, \mathbf{G}_t$ are the channel matrices of R1 and R2 in slot $t$, and we let $\mathbf{H}_t^s$ be the $s^{\text{th}}$ row vector of $\mathbf{H}_t$.

Our achievable scheme consists of 3 phases for both $\alpha > 1/2$ and $\alpha \le 1/2$. The number of slots for each phase will be

$$d_1 \le N_1 \tag{6a}$$
$$d_2 \le N_2 \tag{6b}$$
$$d_1 + d_2 \le M \tag{6c}$$
$$\frac{d_1}{\min\{M, N_1\}} + \frac{d_2}{\min\{M, N_1+N_2\}} \le 1 + \frac{\min\{M, N_1+N_2\} - \min\{M, N_1\}}{\min\{M, N_1+N_2\}}\alpha \tag{6d}$$
$$\frac{d_1}{\min\{M, N_1+N_2\}} + \frac{d_2}{\min\{M, N_2\}} \le 1 + \frac{\min\{M, N_1+N_2\} - \min\{M, N_2\}}{\min\{M, N_1+N_2\}}\alpha \tag{6e}$$

specified later. An overview of the scheme is given in Fig. 2.

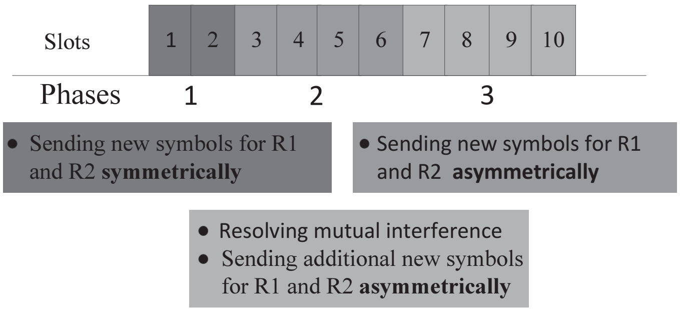


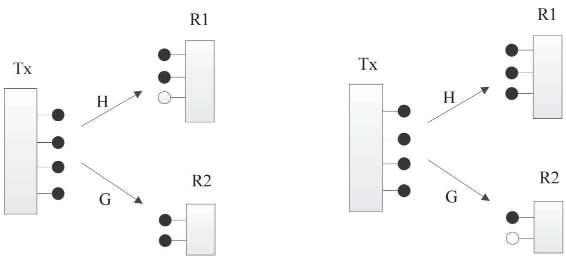

Fig. 2: Overview of the achievable scheme

### B. *Scheme for* $\alpha > 1/2$

By Lemma 1, we know that DoF pair $(2\alpha, 2)$ is always achievable for any $\alpha$. Besides, the DoF pairs $(0, 2)$ and $(3, 0)$ can be trivially achieved. So we only need to show that for $\alpha > 1/2$, DoF pairs $(\frac{10+4\alpha}{5}, \frac{4+2\alpha}{5})$ and $(\frac{10+5\alpha}{5}, \frac{4+\alpha}{5})$ are achievable.

<u>*Phase 1*</u>: This phase consists of $N_2(M - N_1) = 2$ slots, which are devoted to send new symbols to both receivers.

Since there are more receiver antennas than transmitter antennas, some receiving signal is redundant. So we only use the received signal on the first two antennas of R1, hence resulting in an effective $(4, 2, 2)$ BC, as illustrated in Fig 3a.




(a) Treated as $(4, 2, 2)$ (b) Treated as $(4, 3, 1)$

Fig. 3: $(4, 3, 2)$ broadcast channel, where the antenna denoted by white circle is unused.

In Slot 1 the transmitter send $a_{1,1}, \cdots, a_{1,4}$ and $b_{1,1}, \cdots, b_{1,4}$ to R1 and R2, respectively. The power allocation of these symbols is given in Table I.

TABLE I: Power allocation in Slot 1

| Symbols | $a_{1,1}, a_{1,2}$ | $a_{1,3}, a_{1,4}$ | $b_{1,1}, b_{1,2}$ | $b_{1,3}, b_{1,4}$ |
|---|---|---|---|---|
| Power level | $P$ | $P^{1-\alpha}$ | $P$ | $P^{1-\alpha}$ |
| Rates/log $P$ | 1 | $1-\alpha$ | 1 | $1-\alpha$ |

Besides, the normalized precoding vectors $\boldsymbol{u}_{1,1}, \cdots, \boldsymbol{u}_{1,4}$, $\boldsymbol{v}_{1,1}, \cdots, \boldsymbol{v}_{1,4}$ are assigned to symbols $a_{1,1}, \cdots, a_{1,4}$ and $b_{1,1}, \cdots, b_{1,4}$, respectively, where we choose $\boldsymbol{u}_{1,1}, \boldsymbol{u}_{1,2} \in null(\widehat{\mathbf{G}}_1)$ and $\boldsymbol{v}_{1,1}, \boldsymbol{v}_{1,2} \in null(\widehat{\mathbf{H}}_1)$, the other vectors are chosen randomly. Thus, the received signals are given by

$$\begin{aligned}\boldsymbol{y}_1 = {} & \mathbf{H}_1 \sum_{i=1}^{4} \boldsymbol{u}_{1,i} a_{1,i} \\ & + \left( \widetilde{\mathbf{H}}_1 \sum_{i=1,2} \boldsymbol{v}_{1,i} b_{1,i} + \mathbf{H}_1 \sum_{i=3,4} \boldsymbol{v}_{1,i} b_{1,i} \right)\end{aligned} \tag{8}$$

$$\begin{aligned}\boldsymbol{z}_1 = {} & \mathbf{G}_1 \sum_{i=1}^{4} \boldsymbol{v}_{1,i} b_{1,i} \\ & + \left( \widetilde{\mathbf{G}}_1 \sum_{i=1,2} \boldsymbol{u}_{1,i} a_{1,i} + \mathbf{G}_1 \sum_{i=3,4} \boldsymbol{u}_{1,i} a_{1,i} \right)\end{aligned} \tag{9}$$

where $\boldsymbol{y}_1$ is a $2 \times 1$ vector since we discard the received signal on the third antenna of R1. The additive noise is omitted in the above equations for the sake of brevity, since noise is insignificant in high SNR scenarios.

It is worth noting that the interference terms in $\boldsymbol{y}_1$ and $\boldsymbol{z}_1$ might serve two-fold purposes: On one hand, it could be used to cancel the interference at the unintended receiver. On the other hand, it could provide a new observation for the intended receiver. In light of this, decoding is not conducted in this phase. Rather, the receivers will wait until they have obtained the two interference terms and then perform decoding.

Owing to the ZF processing (by $\boldsymbol{u}_1, \boldsymbol{u}_2$), the power level of $\widetilde{\mathbf{G}}_1 \sum_{i=1,2} \boldsymbol{u}_{1,i} a_{1,i}$ is reduced to $P^{1-\alpha}$. On the other hand, since the power allocated to $a_{1,3}, a_{1,4}$ is $P^{1-\alpha}$, the power level of $\mathbf{G}_1 \sum_{i=3,4} \boldsymbol{u}_{1,i} a_{1,i}$ is also $P^{1-\alpha}$. Hence the total interference signal are of power level $P^{1-\alpha}$.

At the end of Slot 1, the transmitter is able to reconstruct all the interference signals at both receivers in Slot 1. The transmitter quantizes and encodes them as done in [5].

The resulting encoded symbols are denoted as $c_{1,1},\cdots,c_{1,4}$[1], which will be transmitted in Phase 3.

The procedures in the second slot of Phase 1 is the same as that of the first slot. Another 4 new symbols for R1 as well as 4 new symbols for R2 are transmitted. And the common symbols generated at the end of this slot are denoted as $c_{2,1}$,$c_{2,2}$,$c_{2,3}$,$c_{2,4}$.

*Phase 2*: This phase consists of $M(N_1-N_2)=4$ slots. It mainly serves to send some new symbols to R1, as well as comparatively fewer symbols to R2.

In this phase, the second antenna of R2 is unused, thus resulting in a $(4,3,1)$ BC as illustrated in Fig 3b.

In Slot 3, 4 symbols for R1 and 1 symbol for R2 are transmitted, which are denoted as $a_{3,1}$,$\cdots$,$a_{3,4}$ and $b_{3,1}$. The corresponding power allocation is given in Table II.

TABLE II: Power allocation in Slot 3

| Symbols | $a_{3,1},a_{3,2},a_{3,3}$ | $a_{3,4}$ | $b_{3,1}$ |
|---|---|---|---|
| Power level | $P$ | $P^{1-\alpha}$ | $P$ |
| Rates/log $P$ | 1 | $1-\alpha$ | $\alpha$ |

The normalized precoding vectors $\boldsymbol{u}_{3,1}$,$\cdots$,$\boldsymbol{u}_{3,4}$, $\boldsymbol{v}_{3,1}$ are assigned to symbols $a_{3,1}$,$\cdots$,$a_{3,4}$ and $b_{3,1}$, respectively, where we choose $\boldsymbol{u}_{3,1},\boldsymbol{u}_{3,2},\boldsymbol{u}_{3,3}\in null(\widehat{\mathbf{G}}_3)$ and $\boldsymbol{v}_{3,1}\in null(\widehat{\mathbf{H}}_3)$. Hence, the received signals are given by

$$\boldsymbol{y}_3=\mathbf{H}_3\sum_{i=1}^{4}\boldsymbol{u}_{3,i}a_{3,i}+\widetilde{\mathbf{H}}_3\boldsymbol{v}_{3,1}b_{3,1} \tag{10}$$

$$\boldsymbol{z}_3=\mathbf{G}_3\boldsymbol{v}_{3,1}b_{3,1}+\left(\widetilde{\mathbf{G}}_3\sum_{i=1}^{3}\boldsymbol{u}_{3,i}a_{3,i}+\mathbf{G}_3\boldsymbol{u}_{3,4}a_{3,4}\right) \tag{11}$$

where $\boldsymbol{z}_3$ is $1\times 1$ vector. As explained in Slot 1, we also discard the noise terms for simplicity.

Notice that interference at R1 is caused by only one symbol, so if R1 obtains one component of the interference vector, say $\widetilde{\mathbf{H}}_3^1\boldsymbol{v}_{3,1}b_{3,1}$, it can cancel all the interference on its 3 antennas. Meanwhile, for the received signal $\boldsymbol{z}_3$, considering that the desired signal $\mathbf{G}_3\boldsymbol{v}_{3,1}b_{3,1}$ is of power level $P$ while the interference signal is of power level $P^{1-\alpha}$, R2 can therefore decode $b_{3,1}$ by treating interference as noise. The decoding success is guaranteed since $R_{b_{3,1}}\approx\log\left(\frac{P}{P^{1-\alpha}}\right)=\alpha\log P$.

At the end of this slot, the transmitter applies quantization and encoding as in the first phase. For the interference in the received signal at R2, the resulting symbol is denoted as $d_{3,1}$ and will only be useful for R1, since R2 has already decoded $b_{3,1}$.

Next, the transmitter quantizes and encodes $\widetilde{\mathbf{H}}_3^1 v_{3,1}b_{3,1}$, which results in approximately $(1-\alpha)\log P$ bits. These bits are then evenly encoded into 3 symbols via a Gaussian codebook. Those resulting symbols are denoted as $d_{3,2}$,$d_{3,3}$,$d_{3,4}$, each containing approximately $\frac{1}{3}(1-\alpha)\log P$ bits. The interference symbol splitting is illustrated in Fig. 4.

[1]See [5] for validation of the above quantization and encoding procedures.

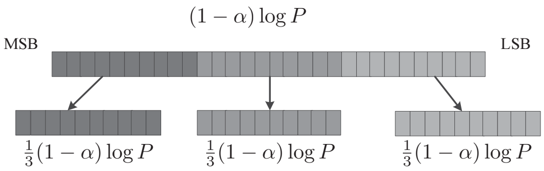



Fig. 4: Interference symbol splitting.

Here we highlight that in the above procedure we deliberately split the interference signal in $\boldsymbol{y}_3^1$ into three parts, for the purpose of fully utilizing the DoF potential (i.e., 3 DoF) of the $4\times 3$ channel of R1.

In the remaining 3 slots (i.e. Slot 4 to 6) we conduct the same procedures as in Slot 3, resulting in symbols $d_{4,1}$,$\cdots$,$d_{4,4}$,$d_{5,1}$,$\cdots$,$d_{5,4}$,$d_{6,1}$,$\cdots$,$d_{6,4}$, respectively.

*Phase 3*: This phase consists of $M(M-N_1)=4$ slots. It aims at transmitting the symbols encoded from the interference signals in previous slots, and in addition, sending new private symbols at the same time.

In specific, we have in total 8 common symbols that are generated in Phase 1 i.e., $c_{1,1}$,$\cdots$,$c_{1,4}$, $c_{2,1}$,$\cdots$,$c_{2,4}$, and 16 interference-encoded symbols $d_{3,1}$,$\cdots$,$d_{3,4}$, $d_{4,1}$,$\cdots$, $d_{4,4}$,$d_{5,1}$,$\cdots$, $d_{5,4}$,$d_{6,1}$,$\cdots$, $d_{6,4}$ produced in Phase 2 which are only useful to R1.

In Slot 7 we transmit $c_{1,1}$,$c_{1,2}$,$d_{3,1}$,$\cdots$,$d_{3,4}$ and 3 new private symbols $a_{7,1}$,$a_{7,2}$,$a_{7,3}$ for R1 and 1 new private symbols $b_{7,1}$ for R2 according to the power allocation given in Table III. Since $\alpha>1/2$, we have $0<\frac{4}{3}\alpha-\frac{1}{3}<\alpha$, which implies the power level of $d_{3,2}$,$d_{3,3}$,$d_{3,4}$ is higher than that of $a_{7,1}$,$a_{7,2}$,$a_{7,3}$.

TABLE III: Power allocation in Slot 7

| Symbols | $c_{1,1},c_{1,2},d_{3,1}$ | $d_{3,2}\cdots d_{3,4}$ | $a_{7,1}\cdots a_{7,3}$ | $b_{7,1}$ |
|---|---|---|---|---|
| Power level | $P$ | $P^{\alpha}$ | $P^{\frac{4}{3}\alpha-\frac{1}{3}}$ | $P^{\alpha}$ |
| Rates/log $P$ | 1 | $\frac{1}{3}(1-\alpha)$ | $\frac{4}{3}\alpha-\frac{1}{3}$ | $\alpha$ |

Next, randomly chosen precoding vectors $\boldsymbol{w}$ and $\boldsymbol{q}$ are assigned to the common symbols and inference-encoded symbols, respectively. As for the precoders for the private symbols, we choose $\boldsymbol{u}_{7,1},\boldsymbol{u}_{7,2},\boldsymbol{u}_{7,3}\in null(\widehat{\mathbf{G}}_7^1)$, and $\boldsymbol{v}_{7,1}\in null(\widehat{\mathbf{H}}_7)$. So the received signals are given by

$$\begin{aligned}\boldsymbol{y}_7=&\mathbf{H}_7\left(\sum_{i=1,2}\boldsymbol{w}_{7,i}c_{1,i}+\boldsymbol{q}_{7,1}d_{3,1}\right)+\mathbf{H}_7\sum_{i=2}^{4}q_{7,i}d_{3,i}\\&+\mathbf{H}_7\sum_{i=1}^{3}\boldsymbol{u}_{7,i}a_{7,i}+\widetilde{\mathbf{H}}_7\boldsymbol{v}_{7,1}b_{7,1}\end{aligned} \tag{12}$$

$$\begin{aligned}\boldsymbol{z}_7=&\mathbf{G}_7\left(\sum_{i=1,2}\boldsymbol{w}_{7,i}c_{1,i}+\boldsymbol{q}_{7,1}d_{3,1}\right)+\mathbf{G}_7\sum_{i=2}^{4}q_{7,i}d_{3,i}\\&+\mathbf{G}_7\boldsymbol{v}_{7,1}b_{7,1}+\begin{bmatrix}\widetilde{\mathbf{G}}_7^1\sum_{i=1}^{3}\boldsymbol{u}_{7,i}a_{7,i}\\ \mathbf{G}_7^2\sum_{i=1}^{3}\boldsymbol{u}_{7,i}a_{7,i}\end{bmatrix}\end{aligned} \tag{13}$$

In the following we describe the decoding procedures for R1, which is mainly based on the technique of successive cancellation, that is, the symbols will be subtracted out from the received signal once they are decoded.

1) Decode $c_{1,1}$,$c_{1,2}$,$d_{3,1}$ simultaneously by treating other symbols as noise. Notice that the highest power level of the remaining symbols is $P^{\alpha}$, therefore, the total achievable rate of $c_{1,1}$,$c_{1,2}$,$d_{3,1}$ is $3\log\left(\frac{P}{P^{\alpha}}\right)=3(1-\alpha)\log P$. which is the same as the sum bits of $c_{1,1}$,$c_{1,2}$,$d_{3,1}$, hence guaranteeing the decoding success.
2) Decode $d_{3,2}$,$d_{3,3}$,$d_{3,4}$ simultaneously by treating other symbols as noise. Now the power level of the effective noise is $P^{\frac{4}{3}\alpha-\frac{1}{3}}$ ,hence, the achievable rate of $d_{3,2}$,$d_{3,3}$,$d_{3,4}$ is $3\log\left(\frac{P^{\alpha}}{P^{\frac{4}{3}\alpha-\frac{1}{3}}}\right)=(1-\alpha)\log P$.
3) Decode $a_{7,1}$,$a_{7,2}$ and $a_{7,3}$.

The decoding procedure of R2 is as follows.

1) Subtract $d_{3,1}$,$\cdots$,$d_{3,4}$ out from the received signal since R2 has full knowledge of $d_{3,1}$,$\cdots$,$d_{3,4}$.
2) Decode $c_{1,1}$,$c_{1,2}$ by treating other symbols as noise. Notice that the power level of the effective noise is $P^{\alpha}$, hence the total achievable rate is $2\times\log\left(\frac{P}{P^{\alpha}}\right)=2\times(1-\alpha)\log P$. Subtract them out after decoding.
3) Decode $b_{7,1}$.

In the remaining 3 slots, the transmitter will send the remaining common symbols and interference-encoded symbols together with new private symbols as in Slot 7. The decoding procedures for the receivers are the same as described above.

Equipped with those decoded common symbols and interference symbols, R1 can now turn to resolve the signals it received in the first two phases. Since $c_{1,1}$,$c_{1,2}$,$c_{1,3}$,$c_{1,4}$ are already known, R1 can decode $a_{1,1}$,$\cdots$,$a_{1,4}$ in the first phase, which results in a total information bits of $2\times\log P+2\times\log P+\alpha\log P^{1-\alpha}=(4-2\alpha)\log P$. Likewise, R2 can decode $b_{1,1}$,$\cdots$,$b_{1,4}$, whose sum rate is $(4-2\alpha)\log P$ bits. Symbols sent in Slot 2 are similarly decoded.

After knowing $d_{3,2}$,$d_{3,3}$,$d_{3,4}$, R1 can use them to cancel the interference in Slot 3. In addition, $d_{3,1}$ serves as a fourth observation for R1, hence R1 can decode $a_{3,1}$,$\cdots$,$a_{3,4}$ correctly. The total information bits are $3\times\log P+\log P^{1-\alpha}=(4-\alpha)\log P$. Similarly, R1 can resolve the symbols in Slot 4, Slot 5, and Slot 6.

Collecting all the symbols in these 10 slots, the total DoF are $d_1=(4-2\alpha)\times2+(4-\alpha)\times4+3\times(\frac{4}{3}\alpha-\frac{1}{3})\times4=20+8\alpha$ and $d_2=(4-2\alpha)\times2+\alpha\times4+\alpha\times4=8+4\alpha$, respectively, implying that the DoF pair $(\frac{10+4\alpha}{5},\frac{4+2\alpha}{5})$ is achievable.

We note that in the scheme described above, the channel is treated as a $(4,2,2)$ BC in the first phase. Alternatively, it could be treated as a $(4,3,1)$ BC, which will result in an achievable DoF pair $\left(\frac{10+5\alpha}{5},\frac{4+\alpha}{5}\right)$ following the same spirit of the above scheme. An illustration of the achievable DoF region is provided in Fig 1.

### C. Scheme for $\alpha\leq 1/2$

For the case $\alpha\leq 1/2$, since the procedures of the achievable scheme are very similar to those of the case $\alpha>1/2$, with a small difference that in the former case: no symbol is sent to R2 in Phase 2, hence the rate-splitting of the interference-encoded symbols is not performed. Due to space limitation, we'll skip the achievable scheme. One thing we'd like to note is as follows.

*Remark 1*: The difference between the schemes for $\alpha>1/2$ and $\alpha\leq 1/2$ is that in Phase 2, for $\alpha>1/2$ the transmitter will send 1 symbol with rate $\alpha\log P$ to R2 per slot, while the transmitter send nothing to R2 for $\alpha\leq 1/2$. However, those additional symbols intending to R2 also incur cost: each symbol causes interference to R1 with power level $P^{1-\alpha}$, which will consume a total of $(1-\alpha)$ DoF of R1 via the rate-splitting technique mentioned before. So overall, sending 1 symbol to R2 can gain $\alpha$ DoF for R2 while losing $(1-\alpha)$ DoF for R1. When $\alpha>1/2$, net DoF gain is available.

## V. Conclusion

We studied the DoF region of general MIMO BC with arbitrary antenna configurations in this paper. The main contribution of this work is that we employ rate-splitting in encoding the quantized interference signals, which enables exploiting the full potential of the MIMO channel, thus leading to better DoF region than a simple generalization of previous schemes. Actually, when the channel estimation quality is near perfect (i.e., $\alpha=1$), our scheme is sum-DoF optimal, which can not be achieved by naive generalization of existing schemes. To reduce or close the gap between the inner bound and outer bound of DoF region remains our future work.

## Acknowledgment

This work was supported by Natural Science Foundation of China under grant 60802009 and 61231007 as well as the National Science and Technology Major Project under grant 2010ZX03003-001-02 and 2013ZX03003002-004. The corresponding author is Desheng Wang.

## References

[1] H. Weingarten, Y. Steinberg, and S. Shamai, "The Capacity Region of the Gaussian Multiple-input Multiple-output Broadcast Channel," *IEEE Trans. Inform. Theory*, vol. 52, pp. 3936–3964, Sept. 2006.
[2] C. Huang, S. Jafar, S. Shamai, and S. Vishwanath, "On Degrees of Freedom Region of MIMO Networks Without Channel State Information at Transmitters," *IEEE Trans. Inform. Theory*, vol. 58, no. 2, pp. 849–857, 2012.
[3] M. A. Maddah-Ali and D. Tse, "Completely Stale Transmitter Channel State Information is Still Very Useful," *IEEE Trans. Inform. Theory*, vol. 58, pp. 4418–4431, July 2012.
[4] C. Vaze and M. Varanasi, "The Degrees of Freedom Region of the Two-user MIMO Broadcast Channel with Delayed CSIT," in *Proc. 2011 IEEE Int. Symp. on Inform. Theory*, July 31-Aug. 5, pp. 199–203.
[5] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, "Degrees of Freedom of Time Correlated MISO Broadcast Channel with Delayed CSIT," *IEEE Trans. Inform. Theory*, vol. 59, pp. 315–328, Jan.2013.
[6] T. Gou and S. Jafar, "Optimal Use of Current and Outdated Channel State Information: Degrees of Freedom of the MISO BC with Mixed CSIT," *Communications Letters, IEEE*, vol. 16, pp. 1084 – 1087, July 2012.
[7] X. Yi, S. Yang, D. Gesbert, and M. Kobayashi, "The Degrees of Freedom Region of Temporally-Correlated MIMO Networks with Delayed CSIT," *arXiv:1211.3322*, 2012.
[8] T. Liu and P. Viswanath, "An Extremal Inequality Motivated by Multi-terminal Information-Theoretic Problems," *IEEE Trans. Inform. Theory*, vol. 53, pp. 1839–1851, May 2007.