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Two-user MISO Broadcast Channel: Synergistic Benefits of Alternating CSIT

Ravi Tandon
Department of ECE
Virginia Tech
Blacksburg, VA
tandonr@vt.edu

Syed Ali Jafar
CPCC, Department of EECS
UC Irvine
Irvine, CA
syed@uci.edu

Shlomo Shamai
Department of EE
The Technion
Haifa, Israel
sshlomo@ee.technion.ac.il

H. Vincent Poor
Department of EE
Princeton University
Princeton, NJ
poor@princeton.edu

***Abstract*—The degrees of freedom (DoF) of the two-user multiple-input single-output (MISO) broadcast channel (BC) are studied under the assumption that the form, $I_i, i = 1, 2$, of the channel state information at the transmitter (CSIT) for each user's channel can be either perfect (P), delayed (D) or not available (N), i.e., $I_1, I_2 \in \{P, N, D\}$, and therefore the overall CSIT can alternate between the 9 resulting states I_1I_2. The fraction of time associated with CSIT state I_1I_2 is denoted by the parameter $\lambda_{I_1I_2}$ and it is assumed throughout that $\lambda_{I_1I_2} = \lambda_{I_2I_1}$, i.e., $\lambda_{PN} = \lambda_{NP}, \lambda_{PD} = \lambda_{DP}, \lambda_{DN} = \lambda_{ND}$. Under this assumption of symmetry, the main contribution of this paper is a complete characterization of the DoF region of the two user MISO BC with alternating CSIT. The results highlight the synergistic benefits of alternating CSIT and the tradeoffs between various forms of CSIT for any given DoF value.**

I. Introduction

The availability of channel state information at transmitters (CSIT) is a key ingredient for interference management techniques [1]. It affects not only the capacity but also the degrees of freedom (DoF) of wireless networks. Perhaps the simplest setting that exemplifies the critical role of CSIT is the two-user vector broadcast channel, also known as the multiple input single output broadcast channel (MISO BC), in which a transmitter equipped with two antennas sends independent messages to two receivers, each equipped with a single antenna. Degrees of freedom characterizations for the MISO BC are available under a variety of CSIT models, including full (perfect and instantaneous) CSIT [2], no CSIT [3]–[6], delayed CSIT [7], [8], compound CSIT [9]–[11], quantized CSIT [12]–[14], mixed (perfect delayed and partial instantaneous) CSIT [15]–[17], asymmetric CSIT (perfect CSIT for one user, delayed CSIT for the other) [18], [19] and with knowledge of only the channel coherence patterns available to the transmitter [18], [20]. Yet, the understanding of the role of CSIT for the MISO BC is far from complete, even from a DoF perspective, as exemplified by the Lapidoth-Shamai-Wigger conjecture [21], which is but one of the many open problems along this research avenue.

In this work we focus on an aspect of CSIT that has so far received little direct attention – *that it can vary over time*. Consider the MISO BC for the case in which perfect CSIT is available for one user and no CSIT is available for the other user. Incidentally, the DoF are unknown for this problem. Now, staying within the assumption of full CSIT for one user and none for the other, suppose we allow the CSIT to vary, in the sense that half the time we have full CSIT for user 1 and none for user 2, and for the remaining half of the time we have full CSIT for user 2 and none for user 1. This is one example of what we call the *alternating CSIT* setting. In general terms, the defining feature of the alternating CSIT problem is a joint consideration of multiple CSIT states.

We motivate the alternating CSIT setting by addressing three natural questions — 1) is it practical, 2) is it a trivial extension, and 3) is it desirable/beneficial, relative to the more commonly studied non-alternating/fixed CSIT settings?

To answer the first question, we note that alternating CSIT may be already practically unavoidable due to the time varying nature of wireless networks. However, more interestingly, the form of CSIT may also be *deliberately* varied as a design choice, often with little or no additional overhead. For example, acquiring perfect CSIT for one user and none for the other for half the time and then switching the role of users for the remaining half of the time, carries little or no additional overhead relative to the non-alternating case in which perfect CSIT is acquired for the same user for the entire time while no CSIT is obtained for the other user. Thus, alternating CSIT is as practical as the non-alternating CSIT setting.

The second question relates to the novelty of the alternating CSIT setting with respect to the non-alternating CSIT setting. Is the former just a direct extension of the latter? As we will show in this work, this is not the case. Surprisingly, we find that the lack of a direct relationship between the alternating and non-alternating settings works in our favor. Indeed, we are able to solve the alternating CSIT DoF problem in several cases for which the non-alternating case remains open. In particular, this includes the above mentioned case of full CSIT for one user and none for the other. As mentioned previously, for this problem the DoF remain open in the non-alternating CSIT setting. However, we are able to find the DoF for the same problem under the alternating CSIT assumption.

The third question, whether there is a benefit of alternating CSIT relative to non-alternating CSIT, is perhaps the most interesting question. Here, we will show that the constituent fixed-CSIT settings in the alternating CSIT problem are inseparable (for more on separability, see [22]–[24]), so that the

DoF of the alternating CSIT setting can be strictly larger than a proportionally weighted combination of the DoF values of the constituent fixed-CSIT settings. We call this the *synergistic DoF gain* of alternating CSIT. As we will show in this work, the benefits of alternating CSIT over non-alternating CSIT can be quite substantial.

Related work: In terms of the constituent fixed-CSIT schemes, this work is related to most prior studies of the MISO BC DoF. While several recent works on mixed CSIT models, such as [15]–[17], also jointly consider multiple forms of CSIT, it is noteworthy that these works are fundamentally distinct as in [15]–[17], the multiple forms of CSIT are assumed to be simultaneously present in what ultimately amounts to a fixed-CSIT setting, as opposed to the alternating CSIT setting considered in this work. More closely related to our setting, are the recent works in [25] and [26] which involve alternating perfect and delayed CSIT models. In particular, the three receiver MISO BC with two transmit antennas is studied in [26], leading to an interesting observation that the presence of a third user, even with only two transmit antennas, can strictly increase the DoF.

Organization: Our model of the MISO broadcast channel with alternating CSIT is described in Section II. In Section III, we present the DoF region of the MISO BC under alternating CSIT and highlight several aspects and interpretations of the results. Due to space limitations, the proofs of the main results are not presented here and can be found in [28].

II. System Model

A two user MISO BC is considered, in which a transmitter (denoted as Tx) equipped with two transmit antennas wishes to send independent messages W_1 and W_2, to two receivers (denoted as Rx_1, and Rx_2, respectively), and each receiver is equipped with a single antenna. The input-output relationship is given as

$$Y(t) = H(t)X(t) + N_y(t) \tag{1}$$
$$Z(t) = G(t)X(t) + N_z(t), \tag{2}$$

where $Y(t)$ (resp. $Z(t)$) is the channel output at Rx_1 (resp. Rx_2) at time t, $X(t) = [x_1(t) \quad x_2(t)]^T$ is the 2×1 channel input which satisfies the power constraint $E[||X(t)||^2] \leq \mathbf{P}$, and $N_y(t), N_z(t) \sim \mathcal{CN}(0,1)$ are circularly symmetric complex additive white Gaussian noises at receivers 1 and 2 respectively. The 1×2 channel vectors $H(t)$ (to receiver 1) and $G(t)$ (to receiver 2) are independent and identically distributed (i.i.d.) with continuous distributions, and are also i.i.d. over time. The rate pair (R_1, R_2), with $R_i = \log(|W_i|)/n$, where n is the number of channel uses, is achievable if the probability of decoding error for $i = 1, 2$ can be made arbitrarily small for sufficiently large n. We are interested in the degrees of freedom region $\mathcal{D}$, defined as the set of all achievable pairs (d_1, d_2) with $d_i = \lim_{\mathbf{P}\to\infty} \frac{R_i}{\log(\mathbf{P})}$.

While a variety of CSIT models are conceivable, here we identify the two most important characteristics of CSIT as — 1) precision, and 2) delay. Based on these two characteristics we identify three forms of CSIT to be considered in this work.

1) *Perfect CSIT (P)*: Perfect CSIT, or P, denotes those instances in which CSIT is available instantaneously and with infinite precision.
2) *Delayed CSIT (D)*: Delayed CSIT, or D, denotes those instances in which CSIT is available with infinite precision but only after such delay that it is independent of the current channel state.
3) *No CSIT (N)*: No CSIT, or N, denotes those instances in which no CSIT is available. The users' channels are statistically indistinguishable in this case.

The CSIT state of user 1, I_1, and the CSIT state of user 2, I_2, can each belong to any of these three cases,

$$I_1, I_2 \in \{P, D, N\},$$

giving us a total of 9 CSIT states $I_1 I_2 \in \{PP, PD, DP, PN, NP, DD, DN, ND, NN\}$ for the two user MISO BC. Further, let us denote by $\lambda_{I_1 I_2}$ the fraction of time that the state $I_1 I_2$ occurs, so that

$$\sum_{I_1 I_2} \lambda_{I_1 I_2} = 1. \tag{3}$$

We assume that $\lambda_{I_1 I_2} = \lambda_{I_2 I_1}$. Specifically,

$$\lambda_{PD} = \lambda_{DP} \tag{4}$$
$$\lambda_{PN} = \lambda_{NP} \tag{5}$$
$$\lambda_{DN} = \lambda_{ND}. \tag{6}$$

This assumption is justified by the inherent symmetry of the problem, e.g., it is easy to see that if DoF were to be optimized subject to a symmetric CSIT cost constraint (the cost for acquiring CSIT state $I_1 I_2$ equals the cost of $I_2 I_1$) then the optimal choice of CSIT states will always satisfy the property $\lambda_{I_1 I_2} = \lambda_{I_2 I_1}$. Furthermore, we assume that both the receivers have perfect *global* channel state information.

Problem Statement: Given the probability mass function (pmf) $\lambda_{I_1 I_2}$, the problem is to characterize the degrees-of-freedom region $\mathcal{D}(\lambda_{I_1 I_2})$.

III. Main Results and Insights

Starting with the 9 parameters $\lambda_{I_1 I_2}$, even if we use the 4 constraints (3)-(6) to eliminate 4 parameters (say, $\lambda_{DP}, \lambda_{NP}, \lambda_{ND}, \lambda_{NN}$), we are still left with 5 free parameters ($\lambda_{PP}, \lambda_{PD}, \lambda_{DD}, \lambda_{PN}, \lambda_{DN}$), and a challenging task of characterizing the DoF region which is a function of these 5 remaining parameters, i.e., a mapping from a region in $\mathbb{R}^5$ to a region in $\mathbb{R}^2$. While such a problem can easily become intractable or at least extremely cumbersome, it turns out — rather serendipitously — to be not only completely solvable but also surprisingly easy to describe.

A. Main Result

We start with the main result, stated in the following theorem.

Theorem 1: The DoF region $\mathcal{D}(\lambda_{I_1 I_2})$, for the two user MISO BC with alternating CSIT is given by the set of non-negative pairs (d_1, d_2) that satisfy

$$d_1 \leq 1 \quad (7)$$

$$d_2 \leq 1 \quad (8)$$

$$d_1 + 2d_2 \leq 2 + \lambda_{PP} + \lambda_{PD} + \lambda_{PN} \quad (9)$$

$$2d_1 + d_2 \leq 2 + \lambda_{PP} + \lambda_{PD} + \lambda_{PN} \quad (10)$$

$$d_1 + d_2 \leq 1 + \lambda_{PP} + 2\lambda_{PD} + \lambda_{DD} + \lambda_{PN} + \lambda_{DN}. \quad (11)$$

A proof of Theorem 1 can be found in [28].

Note the dependence of the DoF region in Theorem 1 on the 5 remaining parameters λ_{PP}, λ_{PD}, λ_{DD}, λ_{PN}, λ_{DN}. As remarkable as the simplicity of the DoF region description in Theorem 1 may be, it is possible to simplify it even further, in terms of only two *marginal* parameters – λ_P and λ_D. This simplification and associated insights are presented next through a set of remarks.

Remark 1: **[Representation in Terms of Marginals]** The DoF region in Theorem 1 can also be expressed as follows:

$$d_1 \leq 1 \quad (12)$$

$$d_2 \leq 1 \quad (13)$$

$$d_1 + 2d_2 \leq 2 + \lambda_P \quad (14)$$

$$2d_1 + d_2 \leq 2 + \lambda_P \quad (15)$$

$$d_1 + d_2 \leq 1 + \lambda_P + \lambda_D, \quad (16)$$

where λ_P and λ_D defined below denote the **total** fraction of time that perfect and delayed CSIT, respectively, are associated with a user:

$$\lambda_P \triangleq \lambda_{PP} + \lambda_{PD} + \lambda_{PN} \quad (17)$$

$$\lambda_D \triangleq \lambda_{DD} + \lambda_{PD} + \lambda_{DN}. \quad (18)$$

Note that these two marginal fractions satisfy $\lambda_P + \lambda_D + \lambda_N = 1$, where $\lambda_N = \lambda_{NN} + \lambda_{PN} + \lambda_{DN}$ is the total fraction of time that no CSIT is associated with a user.

Remark 2: **[Same-Marginals Property]** From Remark 1, we make a surprising observation. Given any alternating CSIT setting considered in this work, i.e., given any λ_{I_1,I_2}, there exists an **equivalent** alternating CSIT problem, having only three states: PP, DD and NN, with fractions λ_P, λ_D, and λ_N as defined above. The two are equivalent in the sense that they have the same DoF regions; thus, all alternating CSIT settings considered in this work can be reduced to only symmetric CSIT states with the same marginals, without any change in the DoF region.

This equivalence, which greatly simplifies the representation of the DoF region, remains rather mysterious because we have not found an argument that could establish this equivalence *a priori*. The equivalence is only evident after Theorem 1 is obtained, which allows us to simplify the statement of the theorem, but does not simplify the proof of the theorem. Nevertheless, the possibility of a general relationship along these lines is intriguing.

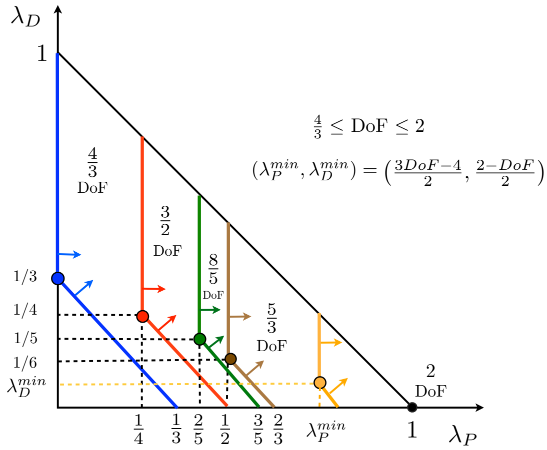

Fig. 1. Tradeoff between Delayed and Perfect CSIT.

Remark 3: **[Sum-DoF]** From (12)-(16), we can write the sum DoF as follows:

$$d_1 + d_2 = \min\left(\frac{4 + 2\lambda_P}{3}, 1 + \lambda_P + \lambda_D\right) \quad (19)$$

$$= 2 - \frac{2\lambda_N}{3} - \frac{\max(\lambda_N, 2\lambda_D)}{3}, \quad (20)$$

where we used the fact that $\lambda_P + \lambda_D + \lambda_N = 1$.

Remark 4: **[Cost of Delay]** It is interesting to contrast the two different forms of CSIT, delayed versus perfect. From (19) and (20) we notice that, depending on the following condition:

$$\lambda_D \geq \lambda_N/2, \quad (21)$$

we have two very distinct observations. We note that in the region where (21) is true, delayed CSIT is interchangeable with no CSIT, because the DoF region depends only on λ_P. Here, delay makes CSIT useless. On the other hand, in the region where $\lambda_D < \frac{\lambda_N}{2}$, delayed CSIT is as good as perfect CSIT.

Remark 5: **[Minimum Required CSIT for a DoF Value]** This tradeoff between marginal λ_P and λ_D is illustrated in Fig. 1. The most efficient point, in terms of marginal CSIT required to achieve any given value of DoF, is uniquely identified to be the bottom corner of the left most edge (highlighted corner in Fig. 1) of the corresponding trapezoid. Note that any other feasible CSIT point involves either redundant CSIT or unnecessary "instantaneous" CSIT requirements when delayed CSIT would have sufficed just as well.

B. Synergistic Benefits

As mentioned previously, the most interesting aspects of the alternating CSIT problem are the synergistic DoF gains, for which we next present some representative examples:

Example 1: Consider the non-alternating CSIT setting, PD, in which perfect CSIT is available for one user and delayed CSIT is available for the other user. It has been shown in [19] that this setting has $3/2$ DoF. Now, let us make this an alternating CSIT setting. Suppose that half of the time the CSIT is of the form PD and the remaining half of the time, the CSIT is of the form DP. From the main result

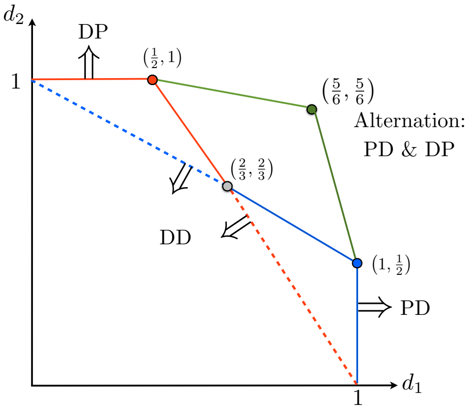

Fig. 2. DoF Gain via Alternating CSIT.

stated in Theorem 1, it is easy to see that the optimal DoF value is now increased to $5/3$. This is an example of a synergistic DoF gain from alternating CSIT. Figure 2 shows the DoF regions corresponding to the three fixed-CSIT states – DD, PD and DP; and the DoF region resulting by permitting alternation between states PD and DP in which each state occurs for half of the total communication period. This result also highlights the inseparability of operating over such CSIT states and shows that by jointly coding across these states, thereby collaboratively using the CSIT distributed over time, significant gains in DoF can be achieved.

Example 2: Another interesting example for which alternating CSIT provides provable DoF gains over non-alternating CSIT is the case in which states DD, PN and NP are present. Individually, the optimal DoF for DD state is $4/3$ as shown in [7]. For the PN and NP states, the optimal DoF value is not known; however an upper bound of $3/2$ can be readily established. In contrast, if alternation is permitted among DD, PN and NP, according to $(\lambda_{DD}, \lambda_{PN}, \lambda_{NP}) = (\frac{1}{5}, \frac{2}{5}, \frac{2}{5})$, then the optimal DoF value is $8/5$, which is larger than both $4/3$ and $3/2$, thereby showing strict synergistic gains made possible by alternating CSIT.

Example 3: As mentioned above, the DoF value is not known individually for fixed-CSIT state PN. In fact, it is our conjecture that for fixed-CSIT state PN, the optimal DoF value is only 1. However, in the alternating CSIT setting, if the states PN and NP are present for equal fractions of the time, then $3/2$ is the optimal DoF value.

Example 4: Interestingly enough, the Maddah-Ali and Tse (henceforth referred as MAT) scheme [7], or rather the alternative version of it presented in [16], may also be seen as an alternating CSIT scheme that achieves $4/3$ DoF with $(\lambda_{DD}, \lambda_{NN}) = (\frac{1}{3}, \frac{2}{3})$. Since the DoF of the DD setting by itself is $4/3$ and the DoF of the NN setting is 1, and $4/3 > 1/3(4/3) + 2/3(1)$, synergistic gains are evident here as well.

We conclude this section by highlighting some of key aspects of the achievability and converse proofs. The converse proofs are inspired by the techniques developed for mixed CSIT configurations in [15] but also include some novel elements. A simple setting that highlights the novel aspects of the converse proof may be the case in which $(\lambda_{PN}, \lambda_{NP}) = (\frac{1}{2}, \frac{1}{2})$. For the achievability proof, the main challenge lies in identifying the core constituent schemes. In particular, core constituent schemes achieving DoF values of $3/2, 5/3$ and $8/5$ by using *minimal* CSIT under various CSIT states are fundamental to the achievability of the DoF region.

In the next section, we highlight one such constituent scheme that achieves $5/3$ DoF by using the states PD, PN and NP for equal fractions one-third of the time each. In contrast the optimal DoF for the PD state is known to be $3/2$ [19].

IV. Achieving 5/3 DoF using PD, PN, NP

In this scheme, we show that it is possible to reliably transmit three symbols (u_1, u_2, u_3) to receiver 1 and two symbols (v_1, v_2) to receiver 2 in a total of three channel uses. The CSIT states are chosen as PD at $t = 1$, PN at $t = 2$, and NP at $t = 3$. At $t = 1$, the encoder sends

$$X(1) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B(1)v_1, \tag{22}$$

where the 2×1 precoding vector $B(1)$ is chosen to satisfy $H(1)B(1) = 0$. The channel outputs are given as[1]

$$Y(1) = H(1) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \triangleq L_1(u_1, u_2), \tag{23}$$

$$Z(1) = G(1) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + G(1)B(1)v_1 \tag{24}$$

$$\triangleq L_2(u_1, u_2) + \alpha_1 v_1. \tag{25}$$

Due to delayed CSIT, transmitter has access to $G(1)$ after $t = 1$. It can reconstruct the interference $L_2(u_1, u_2)$ seen at receiver 2. Hence, at $t = 2$, it sends

$$X(2) = \begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} + B(2)v_2, \tag{26}$$

where the 2×1 precoding vector $B(2)$ is chosen to satisfy $H(2)B(2) = 0$. The channel outputs are given as[2]

$$Y(2) = H(2) \begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} = H_1(2)L_2(u_1, u_2) \tag{27}$$

$$Z(2) = G(2) \begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} + B(2)v_2 \tag{28}$$

$$\triangleq G_1(2)L_2(u_1, u_2) + \alpha_2 v_2. \tag{29}$$

The key consequence of this encoding step is that receiver 2 still faces the *same* interference (up to a known scaling factor) as it encountered at $t = 1$. However, to successfully decode (v_1, v_2), it still requires this interference cleanly, i.e., it requires $L_2(u_1, u_2)$. The transmitter now uses the freedom provided under the *alternating* CSIT model and switches from CSIT state PN at $t = 2$ to the state NP at $t = 3$. Having access to $G(3)$, it sends

$$X(3) = \begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} + B(3)u_3, \tag{30}$$

[1]Here, $L_1(u_1, u_2)$ is used to indicate a linear combination of (u_1, u_2).
[2]We write the 1×2 channel vector $H(t)$ as $H(t) = [H_1(t)\, H_2(t)]$.

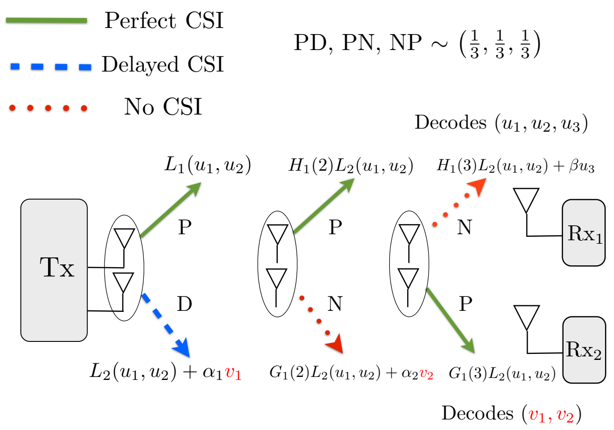

Fig. 3. Achieving 5/3 DoF with (PD, PN, NP) $\sim (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

where the 2×1 precoding vector $B(3)$ is chosen such that $G(3)B(3) = 0$. The outputs are given as

$$Y(3) = H(3)\begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} + H(3)B(3)u_3 \tag{31}$$

$$\triangleq H_1(3)L_2(u_1, u_2) + \beta u_3, \tag{32}$$

$$Z(3) = G(3)\begin{bmatrix} L_2(u_1, u_2) \\ 0 \end{bmatrix} = G_1(3)L_2(u_1, u_2). \tag{33}$$

Having access to $(Y(1), Y(2), Y(3))$, the symbols (u_1, u_2, u_3) can be decoded. Finally, upon receiving $Z(3)$, receiver 2 successfully decodes (v_1, v_2) (see Figure 3).

V. Conclusion

In practice, the channel availability at the transmitter can vary dynamically over time and a new model of alternating CSIT has been introduced and studied in the context of the MISO broadcast channel. As our results illustrate in several cases, a complete understanding of the dynamic settings can be easier than of the corresponding fixed CSIT settings. The results highlight the benefits of configurable channel state information; and also reveal the inseparability of these channel states.

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