

Optimal Wireless Scheduling with Interference Cancellation

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Abstract—Interference cancellation (IC) can provide significant gains in wireless networks with strong interference, that arise, for example, in emerging femto- and picocellular deployments. This paper consider the problem of optimal downlink rate selection in networks where each mobile can perform IC on up to one interferer. When mobiles are capable of IC, it is argued that rate selection can play an analogous roles as power control by permitting a tradeoff between rates on the desired link with “cancellability” on interfering links. A utility maximizing scheduler based on loopy belief propagation is presented that enables computationally-efficient local processing and low communication overhead. It is shown that the fixed points of the method are provably globally optimal for arbitrary (potentially non-convex) rate and utility functions. In addition, the result applies to an arbitrary networks where the interference is determined by a single dominant interferer, for which the IC problem is a special case. Simulations are presented in industry standard femtocellular network models.

I. INTRODUCTION

A central challenge in next-generation cellular networks is the presence of strong interference. This feature is particularly prominent in emerging femto- and pico-cellular networks where, due to *ad hoc* and unplanned deployments, restricted association and mobility in small cell geometries, mobiles may be exposed to much higher levels of interference than in traditional planned macrocellular networks [1]–[3]. Methods for advanced intercellular interference coordination (ICIC) have thus been a key focus of 3GPP LTE-Advanced standardization efforts [4] and other cellular standards organizations [5].

Most current mobile receiver implementations treat interference as noise, which is both computationally simple and presents little loss in traditional cellular networks where the interference is weak or aggregated from many sources. However, in emerging networks with strong, localized interference, more advanced receivers with interference cancellation (IC) may provide significant gains. When an interfering signal is sufficiently strong, it can be decoded and cancelled prior to or jointly with the decoding of the desired signal, thus eliminating the interference entirely [6], [7]. IC, particularly when used in conjunction with techniques such as rate splitting [8], is known to provide large increases in rate in scenarios with strong interference, where it may even be optimal or near optimal [9]. IC may also be useful for mitigating cross-tier interference between short-range (e.g. femtocellular) links operating below

larger macrocells [10]. However, while improved computational resources have recently made IC implementable in practical receiver circuits [11]–[13], it is an open problem of how rates should be selected in larger networks when IC is available at the link-layer.

In this paper, we consider a network where the receiver in each link can jointly detect the desired signal along with at most one interfering link. The limitation of joint detection with at most one interfering link is desirable since the computational complexity at the receiver grows significantly with each additional link to perform joint detection with. In addition, cancelling higher links may often have diminishing returns, particularly with limited receiver dynamic range. For networks with IC, we perform interference coordination based on *rate control* rather than traditional power control that has been the dominant method in cellular systems without IC [14]. Specifically, each transmitter operates at a fixed power, and the system utility is controlled by rate: increasing the rate improves the utility to the desired user, while decreasing the rate makes the transmission more “decodable” and hence “cancelable” at the receiver in any victim link. Hence, we suggest that in networks with IC, rate control, as opposed to power control, can be a viable method for interference coordination.

In this paper, we propose a novel ICIC algorithm for optimal rate selection based on graphical models and message passing. Graphical models [15] are a widely-used tool for high-dimensional optimization and Bayesian inference problems applicable whenever the objective function or posterior distribution factors into terms with small numbers of variables. The loopy BP methodology is particularly well-suited to wireless scheduling problems since the resulting algorithms are inherently distributed and require only local computations. Indeed, [16] showed that many widely-used network routing, congestion control and power control algorithms can be interpreted as instances of the sum-product variant of loopy BP. More recently, [17], [18] used BP techniques for networks with contention graphs, [19] proposed BP for MIMO systems, and [20], [21] considered a general class of ICIC problems with weak interference exploiting an approximate BP technique in [22]. Here, we apply loopy BP methods to a general network utility maximization problem where the goal is to maximize a sum of utilities across a system with n links.

We establish rigorously (Theorem 1) that any fixed point of the loop BP algorithm is guaranteed to be *globally* optimal. Remarkably, this result applies to arbitrary utility and rate functions. In particular, this optimality holds even for non-convex problems. Optimality results for loop BP are generally difficult to establish: Aside from graphs that are cycle free, loop BP generally only provides approximate solutions that may not be globally optimal. Our proof of global optimality in this case rests on showing that graphs with single dominant interferers have at most one cycle in each connected component. The result then follows from a well-known optimality property in [23].

Moreover, our analysis applies to arbitrary networks under a “dominant” interferer assumption where the received rate on any one link is a function of the scheduling decisions by the transmitter for that link as well as at most one interfering transmitter. The IC problem where each mobile can cancel up to one transmitter is precisely of this form. However, the result may also be useful in other models where the dominant interferer assumption is a reasonable approximation, although this requires further study.

A full paper [24] provides more simulation details and proofs.

II. SYSTEM MODEL

To understand the IC problem, we first consider a general network utility maximization problem under what we will call a dominant interferer assumption. It will be shown that IC rate allocation is a special case of this problem.

The dominant interferer network is a system with n links, each link i having one transmitter, TX i and one receiver, RX i . Each link i is to make some *scheduling decision*, meaning a selection of a variable $x_i \in \mathcal{X}_i$ for some set \mathcal{X}_i . For the IC problem, the variable will be the rate, although the variable could also represent power, beamforming directions, or vectors of such quantities in systems with subband scheduling – the problem is general. Associated with each RX i is a *utility function* representing some value or quality of service obtained by RX i . We assume that the utility function has the form

$$f_i(x_i, x_{\sigma(i)}),$$

where $\sigma(i) \in \{1, \dots, n\} \setminus \{i\}$ represents the index of a *dominant* interferer to RX i . Thus, the assumption is that utility on each link i is a function of the scheduling decisions of the serving transmitter, TX i , and one dominant interfering transmitter, TX j for $j = \sigma(i)$. The problem is to find the optimal solution

$$\hat{\mathbf{x}} := \arg \max_{\mathbf{x}} F(\mathbf{x}), \quad (1)$$

where $F(\mathbf{x})$, the objective function, is the sum utility,

$$F(\mathbf{x}) := \sum_{i=1}^n f_i(x_i, x_{\sigma(i)}). \quad (2)$$

A general treatment of utility functions for scheduling problems can be found in [25], [26]. Scheduling vectors x_i are

selected once for a long time period and the utility function is typically of the form

$$f_i(x_i, x_{\sigma(i)}) = U_i(R_i(x_i, x_{\sigma(i)})), \quad (3)$$

where $R_i(x_i, x_{\sigma(i)})$ is the long-term rate as a function of the TX scheduling decision x_i and decision $x_{\sigma(i)}$ on the dominant interferer, while $U_i(R)$ is the utility as any function of the rate. Penalties can also be added if there is a cost associated with the selection of the TX vector x_i such as power.

A. Systems with IC

To apply the dominant interferer framework to systems with IC, we consider the following simple model: Assume the power of each transmitter, TX j , is fixed to some level P_j , and let G_{ij} denote the instantaneous gain from TX j to RX i as let N_i denote the thermal noise at RX i . The channel fading is assumed to be slow and flat, although fast fading can also be accounted for in this framework provided that the statistics of the fading were known. Without IC, the receiver RX i would experience a signal-to-interference-and-noise ratio (SINR) given by

$$\rho_i := \frac{G_{ii}P_i}{\sum_{k \neq i} G_{ik}P_k + N_i}. \quad (4)$$

If the interference is treated as Gaussian noise, and the system were to operate at the Shannon capacity, the set of rates R_i attainable at RX i would be limited to

$$R_i \leq \log_2(1 + \rho_i). \quad (5)$$

We call this set of achievable rates the *reuse 1 region*, since this set is precisely the rates achievable in a cellular system operating with frequency reuse 1 (i.e. all transmitters transmitting across the entire bandwidth) and treating interference as noise. We denote the reuse 1 region by $\mathcal{C}_{\text{reuse1}}(i)$.

The addition of IC can be seen as a method to expand this rate region. Specifically, suppose each RX i can potentially jointly detect and cancel the signals from at most one interfering transmitter TX j for some index $j = \sigma(i) \neq i$. The limitation to a single interferer will enable the system to fit within the dominant interferer model described above.

Now, to compute the region achievable with joint detection and IC consider Fig. 1 which shows the receiver RX i being served by the transmitter TX i while receiving interference from TX j for $j = \sigma(i)$. In this model, assuming again that RX i can operate at the Shannon capacity, it can jointly detect the signals from both the serving transmitter TX i and interferer TX j , if and only if the rates R_i and R_j satisfy the multiple access channel (MAC) conditions [7]:

$$R_i \leq \log_2(1 + \tilde{\rho}_{i,i}) \quad (6a)$$

$$R_{\sigma(i)} \leq \log_2(1 + \tilde{\rho}_{i,\sigma(i)}) \quad (6b)$$

$$R_i + R_{\sigma(i)} \leq \log_2(1 + \tilde{\rho}_{i,i} + \tilde{\rho}_{i,\sigma(i)}), \quad (6c)$$

where, for $\ell = i$ or $\ell = \sigma(i)$, $\tilde{\rho}_{i,\ell}$ is the SINR,

$$\tilde{\rho}_{i,\ell} = \frac{G_{i\ell}P_\ell}{\sum_{k \neq i, \sigma(i)} G_{ik}P_k + N_i}. \quad (7)$$

We denote this region by $\mathcal{C}_{\text{IC}}(i)$ which is plotted in the bottom panel of Fig. 1 along with reuse 1 region $\mathcal{C}_{\text{reuse1}}(i)$. We can see that the addition of IC expands the set of achievable in the region where the interfering rate is low. We let $\mathcal{C}(i)$ denote the total set of feasible rates, which is the union of the reuse 1 and IC regions:

$$\mathcal{C}(i) = \mathcal{C}_{\text{IC}}(i) \cup \mathcal{C}_{\text{reuse1}}(i).$$

Note that this region is *not* necessarily convex.

With these observations, we can now pose the IC problem in the dominant interferer model as follows: The decision variable at each transmitter TX i is an *attempted* rate, denoted x_i . The *achieved* rate at the receiver RX i is then simply

$$R_i = R_i(x_i, x_{\sigma(i)}) := \begin{cases} x_i, & \text{if } (x_i, x_{\sigma(i)}) \in \mathcal{C}(i) \\ 0, & \text{if } (x_i, x_{\sigma(i)}) \notin \mathcal{C}(i) \end{cases}$$

That is, the rate is achieved rate is equal to the attempted rate on the serving link if and only if the serving and interfering rates are feasible; otherwise, the achieved rate is zero. Then, the optimization can be formulated as (3) for any utility function $U_i(R_i)$.

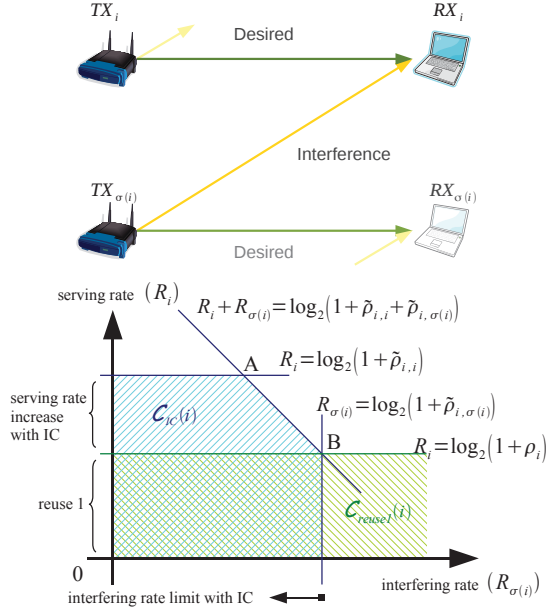


Fig. 1: Rate region with IC: When a receiver RX i is capable of IC, its achievable rate depends on both the serving rate rate R_i as well as the rate $R_{\sigma(i)}$ on the interfering link. The total rate region, which includes the options of both reuse 1 and treating interference as noise as well as joint detection, is non-convex.

More sophisticated methods, such as power control, time sharing and rate splitting as used in the well-known Han-Kobayashi (HK) method [8], can also be incorporated into this methodology.

III. MESSAGE PASSING ALGORITHMS

A. Graphical Model Formulation

As mentioned in the Introduction, graphical models provide a general and systematic approach for distributed optimization

problems where the objective function admits a factorization into terms each with small numbers of variables [15]. The optimization (1) with objective function (2) is ideally suited for this methodology.

To place the optimization problem into the graphical model formalism, we define a *factor* graph $G = (V, E)$, which is an undirected bipartite graph whose vertices consists of n variable nodes associated with the decision variables x_i , $i = 1, \dots, n$, and n factor nodes associated with factors f_i , $i = 1, \dots, n$. There is an edge between x_ℓ and f_i if and only x_ℓ appears as an argument in f_i – namely if $\ell = i$ or $\ell = \sigma(i)$.

As an example of the factor graph consider the network in the left panel of Fig. 2 with $n = 6$ links.

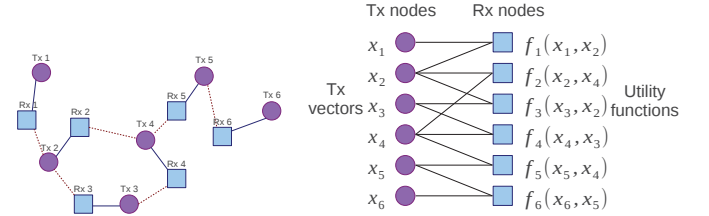


Fig. 2: Example factor graph representation of a network: The left panel shows an example network with $n = 6$ links, each with a transmitter and receiver. Solid lines indicate serving links and dotted lines indicate interfering links. The right panel shows its factor graph representation.

B. Max-Sum Loopy BP

Once the optimization problem has been formulated as a graphical model, we can apply the standard max-sum loopy BP algorithm. A general description of this algorithm can be found in any standard graphical models text such as [15]. For the dominant interferer optimization problem, the max-sum algorithm can be implemented as shown in Algorithm 1.

As shown in Algorithm 1, the messages at the receiver are initialized at zero and then iteratively updated in a set of *rounds*. In the first half of each round, the receivers send messages to the transmitters and, in the second half, the transmitters send messages back to the receivers. The process is a repeated for a fixed number of iterations, and the final messages are used to compute the scheduling decision at the transmitters. At the end, decisions can be projected on feasible region if they are infeasible due to limited number of iterations. Note that the algorithm that uses channel state information only at the RX side only; its use at the transmitter is implicit through the messages.

C. Optimality under the Dominant Interferer Assumption

The full paper [24] discusses many of the practical implementation issues. Here, we just prove the algorithm optimality:

Theorem 1. Consider the message passing max-sum algorithm, Algorithm 1, applied to the optimization (1) with the objective function (2) for any utility functions $f_i(x_i, x_{\sigma(i)})$ and dominant interferer selection function $\sigma(i)$. Then, if all messages $\mu_{i \rightarrow j}(x_i)$ and $\mu_{i \leftarrow j}(x_i)$ are fixed-points of the

Algorithm 1 Max-sum loopy BP for scheduling

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{ Initialization at the RX }
for all RXi do
   $\mu_{i \rightarrow i}(x_i) \leftarrow 0$ 
   $\mu_{j \rightarrow i}(x_j) \leftarrow 0$  for  $j = \sigma(i)$ 
end for
repeat
  { Receiver half round }
  for all RXi do
     $j \leftarrow \sigma(i)$ 
     $\mu_{i \leftarrow i}(x_i) \leftarrow \max_{x_j} f_i(x_i, x_j) + \mu_{j \rightarrow i}(x_j)$ 
     $\mu_{j \leftarrow i}(x_j) \leftarrow \max_{x_i} f_i(x_i, x_j) + \mu_{i \rightarrow i}(x_i)$ 
  end for
  { Transmitter half round }
  for all TXj do
     $H_j(x_j) \leftarrow \mu_{j \leftarrow j}(x_j) + \sum_{i: j = \sigma(i)} \mu_{j \leftarrow i}(x_j)$ 
     $\mu_{j \rightarrow j}(x_j) \leftarrow H_j(x_j) - \mu_{j \leftarrow j}(x_j)$ 
    for all  $i$  s.t.  $j = \sigma(i)$  do
       $\mu_{j \rightarrow i}(x_j) \leftarrow H_j(x_j) - \mu_{j \leftarrow i}(x_j)$ 
    end for
  end for
end for
until max number of iterations
{ Final scheduling decision }
for all TXj do
   $H_j(x_j) \leftarrow \mu_{j \leftarrow j}(x_j) + \sum_{i: j = \sigma(i)} \mu_{j \leftarrow i}(x_j)$ 
   $\hat{x}_j \leftarrow \arg \max_{x_j} H_j(x_j)$ 
end for

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algorithm, the resulting scheduling decisions \hat{x}_j are globally optimal solutions to (1) in that

$$F(\hat{\mathbf{x}}) \geq F(\mathbf{x}),$$

for all other scheduling decision vectors \mathbf{x} .

Proof: The result is proven by showing that the resulting graph for the network with a dominant interferer assumption has at most one cycle in each connected component. The optimality then follows from [23] which shows that the fixed-points of max-sum loopy BP is optimal on graphs where the connected components are either acyclic or have at most one cycle. See the full paper [24] for more details. ■

The theorem shows that if the max-sum algorithm converges to a fixed point, the solution will be optimal. Remarkably, this result applies to arbitrary utility functions as long as the dominant interferer assumption is valid. In particular, the result applies to even non-convex utilities such as the ones arising in the IC problem.

Of course, the theorem does *not* imply the convergence of the algorithm. Indeed, since the graph has cycles, the algorithm may not converge. However, as we will see in the simulations, convergence does not appear to be an issue in the cases we examine.

IV. SIMULATION RESULTS

We validate the max-sum loopy BP algorithm for interference cancellation in two scenarios. In both cases, we consider optimizations of log utility, $U(R_i) = \log R_i$, which is proportion fair metric [25]. For the link-layer model, we assume that the rate region is described by the Shannon capacity with a loss of 3dB and there is a maximum spectral efficiency of 5 bps/Hz.

A. 3GPP Femtocellular Apartment Model

In our first simulation, we use a simplified version of an industry standard model [5] for validating ICIC algorithms of femtocells in densely packed apartments. We assume there is an active link with a mobile and the femto base station in its apartment. This restricted association is a major source of strong interference in femtocellular networks and thus a good test case for IC. The complete list of simulation parameters are given on the Table I.

Parameter	Value
Network topology	4×4 apartment model, with active links in 10 of the 16 apartments.
Bandwidth	5 MHz
Wall loss	10 dB
Lognormal shadowing	10 dB std. dev.
Path loss	$38.46 + 20 \log_{10}(R) + 0.7R$ dB, R distance in meters.
Femto BS TX power	0 dBm
Femto UE noise figure	4 dB

TABLE I: Simulation parameters for apartment model.

Fig. 3 shows the cumulative probability distribution of resulting spectral efficiencies (bits/sec/Hz) based on 100 random drops of the network. In this figure, IC is compared against reuse 1. We see that for cell edge users (defined as users in the lowest 10% of rates), there is a gain of almost 4 times in the rate by employing IC.

Also shown in Fig. 3 is a comparison of the optimal rate selection against the selection from max-sum loopy BP run for 4 iterations. We see there is an exact match. In fact, even after only 1 iteration there is a minimal difference.

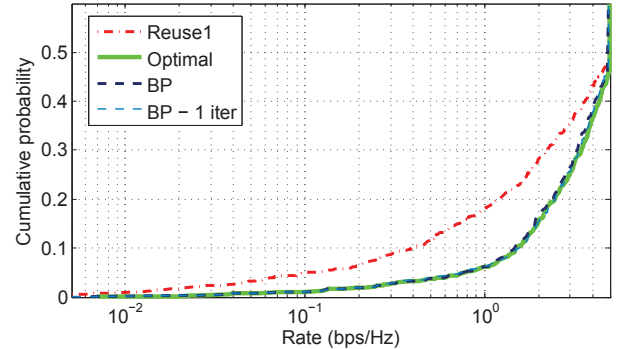


Fig. 3: **Simulation with 3GPP apartment model:** Plotted are the distribution of rates (bps/Hz) over 100 drops. We see that IC provides significant gains particularly at the cell edge. Moreover, even one iteration of max-sum loopy BP finds near-optimal rates for the non-convex optimization problem.

B. Road Network with Mobility

A second case of strong interference in femto- or pico-cellular networks is handover delays. When cells are small, handovers are frequent. Also, since femtocells are not be directly into the operator's core network [27], the handovers between femtocells or from the femtocells to the macrocell

may be significantly delayed [3]. Due to the delays, the mobile may drag significantly into cells other than the serving cell, exposing the mobile to strong interference.

To evaluate the ability of IC to mitigate this strong interference, we considered a road network model similar to the one in [28]. In this model, the transmitters are femtocells placed in apartments at two sides of the road. The receivers are mobile on the road with a random velocity. Each mobile is initially connected to the strongest serving cell. After the connection, we let mobiles to move for a time uniformly distributed between 0 and 1 second to model the effect of delayed handover – details are provided in the full paper [24].

Fig. 4 shows the distribution of rates based on 100 random drops. Again, we see that IC improves cell edge rate significantly: the 10% rate is increased by more than a factor of 3. In fact, even the median rate is increased almost twofold. In addition, max-sum looppy BP finds near optimal rates.

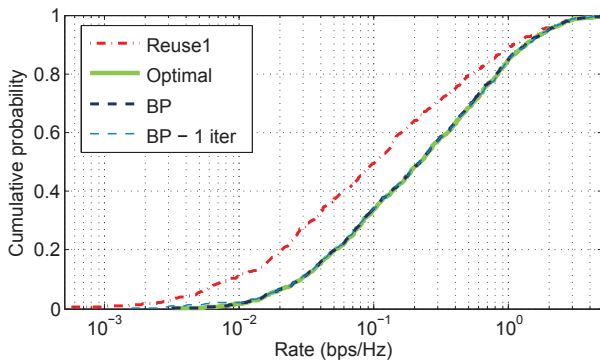


Fig. 4: Simulation for the road network model: Plotted are the distribution of rates (bps/Hz) over 100 drops. Again, we see that IC provides significant gains in rates throughout the rates, but particularly at the cell edge. Moreover, max-sum looppy BP finds near-optimal selections after only 1 iteration.

V. CONCLUSIONS

A general methodology based on max-sum looppy BP is presented for optimal downlink rate allocation for systems where each mobile is capable of performing IC on the signals from up to one interfering base station. The proposed method is completely distributed, general, requires low communication overhead and minimal computation. In addition, although we did not establish the convergence of the algorithm, it we have shown that if it converges, the result is guaranteed to be globally optimal. Remarkably, this result holds for arbitrary problems, even when the problems are non-convex, the only requirement being the dominant interferer assumption.

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