Cooperative Broadcasting with Successive Refinement based Compression

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***Abstract*—As a new achievability scheme, successive refinement coding based compress-and-forward relaying (sCF) is proposed for both discrete memoryless and Gaussian broadcast relay channels (BRC) with one source, two destinations and a dedicated relay. A new outer bound is also provided on the capacity region of physically degraded BRC. Then, the achievability scheme is extended to K-destinations. For the Gaussian BRC, it is shown that the sCF rate region outperforms other CF schemes in the literature. Finally, it is proved that the new achievable scheme is within a half bit of the outer bound on the capacity region for a subset of channel gains.**

I. Introduction

We consider the broadcast relay channel (BRC) where the source transmits to multiple destinations with the aid of a dedicated relay terminal. The BRC contains the relay channel [1] and the broadcast channel [2] as its two special cases, each of which has been open problems in terms of their finite letter capacity expressions in general.

The prominent examples of using the relay in multiuser scenarios are discussed in [3], where decode-and-forward (DF) relaying along with a message splitting based achievability scheme is proposed for the BRC. In [4], the rate region given in [3] is improved via another DF relaying based achievable scheme. In [5], an achievability scheme based on superposition coding together with DF relaying is discussed. This result is extented to the K-destinations case in [6]. Finally, [7] derives an outer bound on the capacity region of the BRC. Another outer bound is due to [8].

The capacity region of the BRC is known for some special cases albeit it is an open problem in general. The capacity region of the physically degraded Gaussian BRC, in the sense that the destinations are physically degraded with respect to the relay and the destinations are stochastically degraded with respect to each other, is known due to [9]. Other capacity results are for the discrete memoryless physically degraded BRC [8] and semi-degraded BRC [4].

Designing channel codes for deterministic unicast networks and approximating the capacity of Gaussian unicast networks with these channel codes is known as the approximation approach [10]. It is proved that this approach is at most a constant number of bits away from the capacity. It serves as a useful tool for multiuser problems, where it is difficult to have exact capacity results. In [11], a layered wireless broadcast network, where one source broadcasts to multiple destinations with the help of possibly multiple relays is studied. It is found that quantize-and-forward relaying is within a constant gap to the capacity, in which the gap depends on the number of nodes in the network. However, when the number of relays are limited to one, the layered broadcast network studied in [11] corresponds to the case, where there are no direct links between the source and the destinations. Thus, finding robust schemes, which have a constant gap to the capacity region for the BRC remains as an interesting question.

We know from the relay channel results that when the source-relay link is worse than the relay-destination link, compress-and-forward (CF) outperforms DF in terms of achievable rates [3]. This result together with the approximate optimality of quantize-and-forward protocols [10], [11], which is a variant of CF, leads to the importance of investigating CF based schemes for the BRC.

The conventional Wyner-Ziv type CF achievable rates are derived for simultaneous relay channels in [4]. It is also studied for two-way relay channels in [12]. The successive refinement based CF (sCF) scheme is suggested for the separated two-way relay channel in [13]. In [14], noisy network coding is proposed for general multicast networks. The so-called generalized CF based achievability scheme is investigated for the interference relay channel in [15]. In [16], multiple description coding based CF (mdCF) and joint CF (jCF) are proposed for the BRC with two destinations.

In this work, our motivation is to find approximately optimal relaying for the Gaussian BRC. In that respect, we propose the successive refinement based CF (sCF) protocol for both discrete memoryless and Gaussian BRCs with two destinations. In sCF relaying, the relay compresses its received signal into two layers, where the base layer is destined to both destinations while the refinement layer is aimed for only one of them. In addition to having multiple compression layers, we adopt an encoding procedure, where there is no explicit binning at the relay as opposed to the conventional Wyner-Ziv type compression. We, then, broaden our achievability scheme to K-destinations. We also give a new outer bound on the capacity region and compare sCF with mdCF and jCF. Finally, we prove that the sCF protocol achieves within 0.5 bits from the outer bound for a set of channel gains.

The organization of the paper is as follows: Section II introduces the system model for the BRC, Section III describes

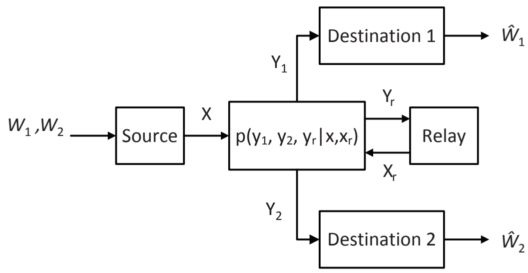

Fig. 1. The broadcast relay channel (BRC) with two destinations.

the new achievability scheme and the outer bound, Section IV extends the achievability scheme to Gaussian channels, Section V presents the numerical results and Section VI concludes the paper.

II. System Model

In this paper, we study a discrete memoryless (DM) broadcast relay channel $(\mathcal{X} \times \mathcal{X}_r, p(y_r, y_1, y_2|x, x_r), \mathcal{Y}_r \times \mathcal{Y}_1 \times \mathcal{Y}_2 \times \ldots \times \mathcal{Y}_K)$, which consists of a source with channel input $x \in \mathcal{X}$, a dedicated relay with input $x_r \in \mathcal{X}_r$ and output $y_r \in \mathcal{Y}_r$, K destinations with channel outputs $y_k \in \mathcal{Y}_k$, $k = \{1, 2, \ldots, K\}$ and a channel probability transition function $p(y_r, y_1, y_2, \ldots, y_K|x, x_r)$. In the sequel, we will use $X_{[k:l]}$ to represent the vector $\mathbf{X} = [X_k, X_{k+1}, ...X_l]$. For the memoryless BRC, the joint probability mass function is given as

$$p(y_r^n, y_{[1:K]}^n|x^n, x_r^n) = \prod_{i=1}^{n} p(y_{r,i}, y_{[1:K],i}|x_i, x_{r,i}).$$

where i is the channel use index and n is the total number of channel uses.

In the BRC, the source sends K private messages W_k to the kth destination, $k = \{1, 2, \ldots, K\}$. The channel is shown in Fig.1 for $K = 2$.

Definition 1: A $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_K}, n)$ code for a BRC consists of K message sets $\mathcal{W}_k = \{1, \ldots, 2^{nR_k}\}$, a source encoder $\mathcal{W}_1 \times \mathcal{W}_2 \times \ldots \times \mathcal{W}_K \to \mathcal{X}^n$, which assigns a codeword $x(W_1, W_2, \ldots, W_K)$ to each message tuple $(W_1, W_2, \ldots, W_K) \in (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_K)$, a set of relay functions $\{f_i\}_{i=1}^n : x_{r,i} = f(Y_{r,1}, Y_{r,2}, ..., Y_{r,i-1})$, and K decoders $\{g_k : \mathcal{Y}_k^n \to \mathcal{W}_k\}_{k=1,2,...,K}$, which map the received sequence y_k^n to a message $\hat{W}_k \in \mathcal{W}_k, k = 1, 2, \ldots, K$.

Definition 2: Assuming the transmitted message $(W_1, W_2, \ldots, W_K)$ is uniformly distributed over $(\mathcal{W}_1 \times \mathcal{W}_2 \times \ldots \times \mathcal{W}_K)$, the average probability of error for a $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_K}, n)$ code is defined as:

$$P_e^{(n)} = \frac{1}{2^{n(\sum_{k=1}^K R_k)}} \sum_{(W_1, W_2, \ldots, W_K)} \Pr\left\{ \bigcup_{k=1}^{K} (\hat{W}_k \neq W_k) \right\}.$$

The rate tuple $(R_1, R_2, \ldots, R_K)$ is said to be achievable for the broadcast relay channel if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_K}, n)$ codes with $P_e^{(n)} \to 0$ as $n \to \infty$.

Definition 3: A DM-BRC with two destinations is said to be physically degraded if $p(y_1, y_2, y_r|x, x_r) = p(y_1, y_r|x, x_r)p(y_2|y_1, y_r, x_r)$ and stochastically degraded if there is a distribution $q(y_2|y_1, y_r, x_r)$ such that $p(y_2, y_r|x, x_r) = \sum_{y_1} p(y_1, y_r|x, x_r)q(y_2|y_1, y_r, x_r)$.

Remark 1: The capacity of the BRC depends only on the conditional distributions $p(y_1, y_r|x, x_r)$ and $p(y_2, y_r|x, x_r)$. This can be proved similar to the corresponding proof for the broadcast channel [17]. Since the capacity of the BRC depends only on $p(y_1, y_r|x, x_r)$ and $p(y_2, y_r|x, x_r)$, the capacity regions of both physically and stochastically degraded BRCs are the same.

III. Main Results

In this section, we describe the new achievability scheme, sCF, and the outer bound on the capacity region for the BRC. Then, we extend our achievability scheme to K-destinations.

Theorem 1: For the DM-BRC, the rate pair (R_1, R_2)

$$R_1 \leq I(X; \hat{Y}_1, \hat{Y}_2, Y_1|U, V, X_r) \tag{1a}$$
$$R_2 \leq \min\{I(U; \hat{Y}_2, Y_1|V), I(U; \hat{Y}_2, Y_2|V)\} \tag{1b}$$

is achievable subject to the constraints

$$I(Y_r; \hat{Y}_1|\hat{Y}_2, Y_1, V, X_r) \leq I(X_r; Y_1|V) \tag{2a}$$
$$I(Y_r; \hat{Y}_2|Y_1, V) \leq I(V; Y_1) \tag{2b}$$
$$I(Y_r; \hat{Y}_2|Y_2, V) \leq I(V; Y_2) \tag{2c}$$

for some joint distribution $p(u, v, x, x_r)p(\hat{y}_1|y_r, x_r)p(\hat{y}_2|\hat{y}_1)$, such that $U - X - (Y_1, Y_2, Y_r)$, $V - X_r - (Y_1, Y_2)$ and $(Y_1, Y_2) - (Y_r, X_r) - \hat{Y}_1 - \hat{Y}_2$ form Markov chains. In addition to the above region, the rate pair $(\bar{R}_1, \bar{R}_2)$

$$\bar{R}_1 \leq I(X; \hat{Y}_2, Y_1|U, V) \tag{3a}$$
$$\bar{R}_2 \leq \min\{I(U; \hat{Y}_2, Y_1|V), I(U; \hat{Y}_1, \hat{Y}_2, Y_2|V, X_r)\} \tag{3b}$$

is also achievable subject to the constraints

$$I(Y_r; \hat{Y}_1|\hat{Y}_2, Y_2, V, X_r) \leq I(X_r; Y_2|V) \tag{4a}$$
$$I(Y_r; \hat{Y}_2|Y_1, V) \leq I(V; Y_1) \tag{4b}$$
$$I(Y_r; \hat{Y}_2|Y_2, V) \leq I(V; Y_2) \tag{4c}$$

for the same joint distribution and Markov chains given above.

Outline of the Proof: To achieve the rate region defined by (1) and (2), both the source and the relay use superposition coding, and the relay does sCF. In sCF, the relay compresses its received signal into two layers and bijectively maps each layer to a channel codeword layer, i.e., $\hat{Y}_2$ to V, and $(\hat{Y}_1, \hat{Y}_2)$ to X_r. When (2b) and (2c) are satisfied, the base layer, $\hat{Y}_2$, will be useful to both destinations. In addition to these, if (2a) is satisfied, the first destination will also benefit from the refinement layer, $\hat{Y}_1$. Using $\hat{Y}_2$ and its own observation, Y_i, both destinations are required to decode U, the cloud center of the superposition codeword the source sends. After decoding U, the first destination can further decode X, according to $\hat{Y}_1, \hat{Y}_2$ and Y_1.

The achievability of (1) is based on the superposition coding order at the relay, which inherently determines the user that resolves the refinement layer of relay compression. Instead of (2a), if $\hat{Y}_1$ is chosen such that (4a) is satisfied, then $\hat{Y}_1$ will benefit the second destination. As the superposition order at the source is the same as above, (3) becomes achievable.

Remark 2: By switching the superposition coding order at the source, two more alternative rate pairs become achievable. When this happens, all subscripts are interchanged except that of $\hat{Y}_1$ and $\hat{Y}_2$. The rate region defined by the convex hull of all possible rate pairs is also achievable by time-sharing.

We would like to emphasize here that the idea leading to the above rate region is different from the conventional Wyner-Ziv type compression and in sCF, there is no explicit binning at the relay. In sCF, the relay forms two layers of compression, where the base layer should be tailored so that it can be decoded at both destinations. As destinations observe different relay-destination links and have side information at different qualities, the relay's transmission, for the base layer, corresponds to the case of broadcasting a common source to multiple destinations with different quality side information. This problem is studied in [18], where it is suggested that there is no need for binning at the source, but the channel creates virtual bins at each destination. Adapting this source coding result to the BRC, in sCF, the relay maps each compression index to a unique channel codeword. Upon receiving X_r, the destinations form a list of possible relay channel codewords or equivalently of possible compression indices. Then, each destination reduces the uncertainty of its list according to its side information. We also note that the proposed sCF scheme simplifies to the jCF scheme of [16] when $\hat{Y}_1 = \emptyset$ and $V = X_r$.

A trivial bound, which can be applied to any network, is the cut-set bound [17]. However, the cut-set bound is not tight for the broadcast channel in general. Therefore, we need an outer bound, which better reflects the broadcast nature of the network. In the following, we give such an outer bound on the capacity region of the physically degraded BRC.

Theorem 2: For the physically degraded DM-BRC, the capacity region is included in (R_1^o, R_2^o), where

$$R_1^o \leq I(X; Y_r, Y_1 | U, X_r) \tag{5a}$$
$$R_2^o \leq I(U; Y_r, Y_2 | X_r) \tag{5b}$$

for some joint distribution of the form $p(u, x, x_r)$.

Proof: We first bound R_2:

$$\begin{aligned}
nR_2 &= H(W_2) \leq I(W_2; Y_2^n, Y_r^n) + n\epsilon_2 \\
&= \sum_{k=1}^{n} I(W_2; Y_{2,k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}) + n\epsilon_2 \\
&\leq \sum_{k=1}^{n} H(Y_{2,k}, Y_{r,k} | X_{r,k}) \\
&\quad - H(Y_{2,k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}, X_{r,k}, W_2) + n\epsilon_2 \qquad (6a) \\
&= \sum_{k=1}^{n} H(Y_{2,k}, Y_{r,k} | X_{r,k}) - H(Y_{2,k}, Y_{r,k} | U_k, X_{r,k}) + n\epsilon_2 \qquad (6b) \\
&= \sum_{k=1}^{n} I(U_k; Y_{2,k}, Y_{r,k} | X_{r,k}) + n\epsilon_2
\end{aligned}$$

where (6a) follows since $X_{r,k}$ is a function of Y_r^{k-1} and the fact that conditioning reduces entropy. (6b) is due to the definition of $U_k = (Y_2^{k-1}, Y_r^{k-1}, W_2)$. Now, we bound the rate R_1:

$$\begin{aligned}
nR_1 &= H(W_1) = H(W_1|W_2) \leq I(W_1; Y_1^n | W_2) + n\epsilon_1 \qquad (7a) \\
&\leq I(W_1; Y_1^n, Y_2^n, Y_r^n | W_2) + n\epsilon_1 \\
&= \sum_{k=1}^{n} I(W_1; Y_{[1:2],k}, Y_{r,k} | Y_{[1:2]}^{k-1}, Y_r^{k-1}, W_2) + n\epsilon_1 \\
&\leq \sum_{k=1}^{n} H(Y_{[1:2],k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}, W_2) \\
&\quad - H(Y_{[1:2],k}, Y_{r,k} | Y_{[1:2]}^{k-1}, Y_r^{k-1}, W_{[1:2]}) + n\epsilon_1 \\
&= \sum_{k=1}^{n} H(Y_{[1:2],k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}, X_{r,k}, W_2) \\
&\quad - H(Y_{[1:2],k}, Y_{r,k} | Y_{[1:2]}^{k-1}, Y_r^{k-1}, X_{r,k}, W_{[1:2]}, X_k) + n\epsilon_1 \qquad (7b) \\
&= \sum_{k=1}^{n} H(Y_{[1:2],k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}, X_{r,k}, W_2) \\
&\quad - H(Y_{[1:2],k}, Y_{r,k} | Y_2^{k-1}, Y_r^{k-1}, X_{r,k}, W_2, X_k) + n\epsilon_1 \\
&= \sum_{k=1}^{n} I(X_k; Y_{[1:2],k}, Y_{r,k} | U_k, X_{r,k}) + n\epsilon_1 \qquad (7c) \\
&= \sum_{k=1}^{n} I(X_k; Y_{1,k}, Y_{r,k} | U_k, X_{r,k}) + n\epsilon_1 \qquad (7d)
\end{aligned}$$

where (7a) is due to Fano's inequality. (7b) follows since (W_1, W_2) determines a unique X_k. (7c) is due to the same reason with (6b). Note that (7d) is due to physical degradedness. After carrying out the standard time-sharing steps, which are omitted here due to space limitations, the proof is complete.

Remark 3: The capacity region of physically and stochastically degraded BRC are the same. Therefore, the outer bound given in Theorem 2 is also an outer bound on the capacity region of the stochastically degraded BRC.

In the following theorem, we extend the sCF to the BRC with K-destinations.

Theorem 3: For the discrete memoryless BRC with K-destinations, the rate tuple $(R_1, R_2, \ldots, R_K)$

$$R_j \leq \min_{t \in [1:j]} I(U_j; Y_t, \hat{Y}_{[j+1:K]}, |U_{[j+1:K]}, V_{[j:K]}) \tag{8}$$

is achievable for $j = 1, \ldots, K$ subject to the constraint

$$I(Y_r; \hat{Y}_j | \hat{Y}_{[j+1:K]}, V_{[j:K]}, Y_t) \leq I(V_j; Y_t | V_{[j+1:K]}) \tag{9}$$

for all $t \in [1, j]$ and some joint distribution of the form $\prod_{i=1}^{K} p(u_i|u_{i+1}) \prod_{i=1}^{K} p(v_i|v_{i+1}) \prod_{i=1}^{K} p(\hat{y}_i|\hat{y}_{i-1})$ where $u_1 = x, u_1 = x_r$ and $\hat{y}_0 = \emptyset$.

We note here that, there are $(K!)^2$ alternative rate regions that can be defined by changing the encoding order at the source and at the relay and the above region gives only one of them. By sCF relaying in K layers, the convex hull of all $(K!)^2$ rate regions is also achievable by time-sharing.

IV. Gaussian Broadcast Relay Channel

In this section, we investigate the Gaussian BRC. The input-output relationships are defined as follows:

$$Y_r = h_r X + Z_r \tag{10a}$$
$$Y_i = h_i X + h_{ri} X_r + Z_i, \text{ for } i = 1, 2. \tag{10b}$$

The Gaussian noises Z_r at the relay and Z_i at the ith destination are i.i.d with zero-mean and unit variance. The channel gains of source-relay, source-ith destination and relay-ith destination links are respectively denoted by $h_r, h_i, h_{ri} \in \mathbb{R}$. The source and the relay power constraints are $E\{X^2\} = P$ and $E\{X_r^2\} = P_r$. We also define $\gamma_i \triangleq h_i^2 P, \gamma_r \triangleq h_r^2 P$ and $\gamma_{ri} \triangleq h_{ri}^2 P_r$ for $i \in \{1, 2\}$.

Let the two layers of the compressed version of Y_r be defined as $\hat{Y}_1 = Y_r + q_1$ and $\hat{Y}_2 = Y_r + q_2$ where $q_2 = q_1 + q_2'$. The compression noises q_1 and q_2' are independent zero-mean Gaussian variables with variances Q_1 and $Q_2 - Q_1$, $Q_2 \geq Q_1$, respectively. Then, for the particular superposition order in (1) and (2)

$$Q_2 \geq \max_{i\in\{1,2\}} \frac{1 + \bar{\beta}\gamma_{ri} + \gamma_i + \gamma_r + \bar{\beta}\gamma_{ri}\gamma_r}{\beta\gamma_{ri}} \tag{11a}$$
$$Q_1 \geq \frac{\sqrt{a^2 + 4(\gamma_1+1)^3(\gamma_1+\gamma_r+1)Q_2} - a}{2(\gamma_1+1)^2} \tag{11b}$$

must be satisfied for successful transmission of the compressed signal to the corresponding users, where

$$a = Q_2\bar{\beta}\gamma_{r1}(\gamma_1+1) + (\gamma_1+1)(\gamma_1+\gamma_r+1) + (\gamma_1+\bar{\beta}\gamma_{r1}+1)(\gamma_1+\gamma_r+1) \tag{12a}$$

and $\beta \in [0, 1]$. The parameter β shows the fraction of relay power allocated to transmit the base layer. Then, the achievable rate pair of (1) becomes

$$R_1 \leq \frac{1}{2}\log\left(1 + \bar{\alpha}\gamma_1 + \frac{\bar{\alpha}\gamma_r}{1+Q_1}\right) \tag{13a}$$
$$R_2 \leq \min_{i=1,2} \frac{1}{2}\log\left(1 + \frac{\frac{\alpha\gamma_i}{\bar{\beta}\gamma_{ri}+1} + \frac{\alpha\gamma_r}{1+Q_2}}{\frac{\bar{\alpha}\gamma_i}{\bar{\beta}\gamma_{ri}+1} + \frac{\bar{\alpha}\gamma_r}{1+Q_2} + 1}\right) \tag{13b}$$

where $\alpha \in [0, 1]$ is the fraction of source power allocated to the base layer.

Lemma 1: Without loss of generality, assume $h_1 \geq h_2$. Then, the rate region defined by (R_1^o, R_2^o)

$$R_1^o \leq \frac{1}{2}\log_2\left(1 + (1-\alpha)(1-\rho^2)(\gamma_1+\gamma_r)\right) \tag{14a}$$
$$R_2^o \leq \frac{1}{2}\log_2\left(1 + \frac{\alpha(1-\rho^2)(\gamma_2+\gamma_r)}{(1-\alpha)(1-\rho^2)(\gamma_2+\gamma_r)+1}\right) \tag{14b}$$

for $\alpha \in [0, 1]$ constitutes an outer bound on the capacity region of the Gaussian BRC. The term ρ defines the correlation between X and X_r in the following way: $E[XX_r] = \rho\sqrt{PP_r}$. Note that, the choice of $\rho = 0$ maximizes the above expressions. Therefore, it is sufficient to consider independent X and X_r for this upper bound.

Outline of the Proof : We omit the proof due to space limitations. Yet, the outline of the proof is as follows.

The capacity region of the Gaussian BRC with arbitrary channel gains is included in the capacity region of the Gaussian BRC with infinite relay-destination links, $h_{r1} = h_{r2} \to \infty$. When the relay-destination links are infinite, both destinations can completely decode X_r of any rate. Then, due to the information processing inequality, the optimal relay strategy is not to process the relay's output at all which means both destinations possess as much information as the relay has. Then, the channel becomes equivalent to the Gaussian SIMO broadcast channel. Since the capacity region of the Gaussian SIMO broadcast channel is known, it is an outer bound on the capacity region of the Gaussian BRC with arbitrary channel gains.

Proposition: The rate pair (R_1, R_2), which is achievable by sCF of Theorem 1, is within 0.5 bits of the capacity region of the Gaussian BRC, if $1 + \gamma_i + \gamma_r \leq \gamma_{ri}$ for $i \in \{1, 2\}$.

Proof: The proof is omitted due to limited space.

Although the approximate optimality result of quantize-and-forward for layered broadcast networks in [11] does not hold here, the authors conjecture that similar results can be derived for non-layered networks via time expansion technique at the expense of a larger gap. This emphasizes the importance of searching for inner bounds with smaller gaps.

V. Numerical Results

For the numerical results, we investigate two cases. In the first case, $h_r = 0.1, h_1 = 10.0, h_2 = 0.1, h_{r1} = 5.0$ and $h_{r2} = 5.0$. Since the channel gains satisfy $h_1 > h_2$ and $h_{r1} = h_{r2}$, the first destination is better than the second. For the second case, the channel gains are $h_r = 1, h_1 = 2, h_2 = 1, h_{r1} = 5, h_{r2} = 10$. Note that these gains satisfy $h_1 > h_2$, and $h_{r1} < h_{r2}$ which means that the destinations are not ordered in terms of their incoming channel gains. In both cases, the source and the relay transmit powers are equal and $P = P_r, N_r = N_1 = N_2$ and $\frac{P}{N_r} = 10$ dB.

In Fig. 2 and Fig. 3, we plot the rate regions achievable with sCF, mdCF and jCF of [16]. The outer bound of Lemma 1, the capacity region of the Gaussian broadcast channel, which stands for the no relay case, and DF are also included for comparison.

In Fig. 2 and Fig. 3, we observe that sCF outperforms mdCF and jCF. In the sCF protocol, the relay's observation is described in two independent layers, where the base layer is recovered at both destinations, and the refinement layer is recovered at only one of the destinations, improving upon the base layer. In mdCF, the relay creates two correlated descriptions, where each destination is required to decode its own description. Note that, in mdCF, one of the descriptions is not a refinement of the other. Moreover, decoding one of the descriptions does not guarantee any of the destinations to decode the other description either. Comparing sCF and mdCF, we conclude that layered transmission at the relay achieves a larger rate region. Unlike sCF and mdCF, in jCF, the relay creates a single description. This limits the performance

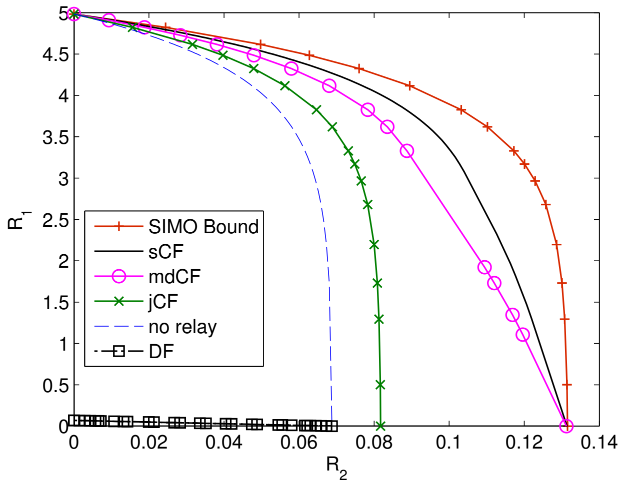

Fig. 2. The achievable rate regions for the Gaussian BRC in the first case: $h_r = 0.1, h_1 = 10.0, h_2 = 0.1, h_{r1} = 5.0, h_{r2} = 5.0$.

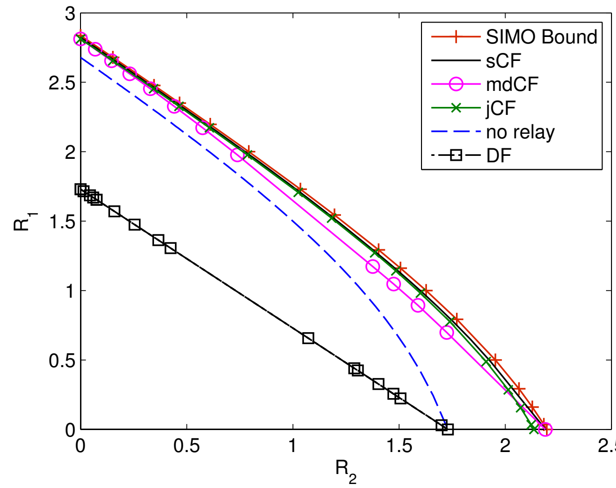

Fig. 3. The achievable rate regions for the Gaussian BRC in the second case: $h_r = 1, h_1 = 2, h_2 = 1, h_{r1} = 5, h_{r2} = 10$.

of jCF severely, and it barely improves upon the no relay case in Fig. 2. Note that for the case studied in Fig. 2, the first destination has a very strong direct link, while the second destination is very poorly connected to the source. Therefore, the destinations have unequal needs in terms of relay assistance. While sCF and mdCF can respond to this unequal demand, jCF fails to offer this adaptation. Although the fact that two layers are necessary to achieve larger rate region, in Fig. 3, sCF, mdCF and jCF, have similar rate regions that are all close the SIMO bound. This shows that when there is no trivial ordering among the destinations as in the second case, single description schemes are as good as multiple description/ layers schemes.

VI. Conclusion

In this paper the broadcast relay channel with one source, two destinations and a dedicated relay node for both discrete memoryless and Gaussian channels is studied. A new achievability scheme, called successive refinement based compress-and-forward (sCF) is proposed. In sCF, the compression is done in two layers, in which the base layer is destined to both destinations while the refinement layer is utilized by only one of them. It is observed that sCF outperforms multiple description coding based CF and joint CF of [16]. It is proved that sCF is within 0.5 bits from the outer bound for a subset of channel conditions. Future work includes expanding the constant gap result to all channel conditions and studying compression techniques in multiple antenna broadcast relay channels, where the relay's observation is no longer successively refinable in general.

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