On the Capacity of the State-Dependent Cognitive Interference Channel

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***Abstract*—We derive the capacity region of two classes of the discrete memoryless state-dependent cognitive interference channels (SD-CICs) with noncausal channel state information known to only the cognitive transmitter: semideterministic SD-CIC and deterministic SD-CIC. We also provide new inner and outer bounds on the capacity region of the general SD-CIC. We prove that the new outer bound is the capacity region of the SD-CIC in the better cognitive decoding regime when both the cognitive transmitter and its corresponding receiver are aware of the channel state information in a noncausal manner.**

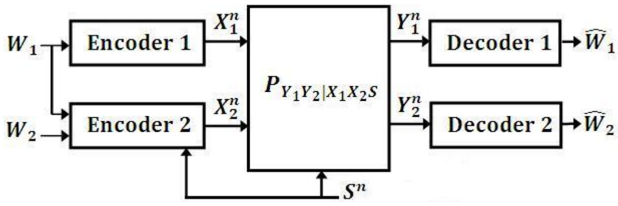

Fig. 1: The general SD-CIC

I. Introduction

The discrete memoryless state-dependent channel was the setup introduced by Shannon [1] to study the notion of channel states being observed causally by the transmitter. The idea of transmitter being aware of the channel states in a noncausal manner was studied by Gel'fand-Pinsker [2]. A review on the communication scenarios with channel state information can be found in [3], [4] and references therein.

In this paper, we study the discrete memoryless state-dependent cognitive interference channel (SD-CIC) which, as shown in Fig. 1, is a network with channel state information and two transmitting nodes which communicate two independent and uniformly distributed messages to two receiving nodes. Transmitter 2, referred to as the cognitive transmitter, knows both messages, and it also is aware of the channel states in a noncausal manner. Transmitter 1, referred to as the primary transmitter, knows only one of the messages and does not know the channel states. Each receiver is assumed to decode only its intended message. We refer to this channel model as the general SD-CIC. The general SD-CIC has been studied before in [5], where inner and outer bounds were derived on the capacity region.

We derive a new inner bound on the capacity region of the general SD-CIC. Our achievability scheme is based on Marton's inner bound for general broadcast channels [6] and Gel'fand-Pinsker coding scheme [2]. We then establish the capacity regions of two special cases of the general SD-CIC. We refer to the first channel model as the semideterministic SD-CIC, where the channel output observed by receiver 2 is a deterministic function of the channel inputs and the channel states. We refer to the second channel model as the deterministic SD-CIC, where both channel outputs are deterministic functions of the channel inputs and the channel states. Furthermore, we derive a new outer bound on the capacity region of the general SD-CIC. We prove that our outer bound is the capacity region of a variation of the SD-CIC where both cognitive transmitter (transmitter 2) and its corresponding receiver (receiver 2) are informed of the channel states in a noncausal manner and the channel model satisfies a certain condition, ensuring that the cognitive receiver can decode better than the primary receiver (receiver 1). This condition is referred to as the "better cognitive decoding" regime [7].

A variation of the SD-CIC was studied in [8], in which the message sent by the primary transmitter is decoded by *both* decoders. Therefore, in this channel model, the primary transmitter is not a source of interference on the communication of cognitive transmitter and its corresponding receiver.

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II. Channel Model

The general SD-CIC is characterized by the finite input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, the finite channel state alphabet $\mathcal{S}$ and distribution P_S, the finite output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$, and a transition probability $P_{Y_1Y_2|X_1X_2S}$. The state variable, S, is distributed over $\mathcal{S}$ according to P_S. The definition of the code, capacity and being memoryless for this channel are omitted due to the lack of space and can be found in [5].

The discrete memoryless SD-CIC is semideterministic if the transition probability distribution $P_{Y_2|X_1X_2S}$ takes on values 0 or 1 only; it is deterministic if the transition probability distribution $P_{Y_1Y_2|X_1X_2S}$ takes on values 0 or 1 only.

III. New Inner Bound for the General SD-CIC

In this section, we derive a new inner bound on the capacity region of the general SD-CIC. This new inner bound will be used later when we derive the capacity regions of the semideterministic and deterministic SD-CICs.

Theorem 1. *The capacity region of the discrete memoryless SD-CIC with the channel states known noncausally to only the cognitive transmitter contains the convex closure of the union of rate-pairs (R_1, R_2) satisfying*

$$R_1 < I(U_1, X_1; Y_1) - I(U_1, X_1; S) \tag{1a}$$
$$R_2 < I(U_2; Y_2) - I(U_2; X_1, S) \tag{1b}$$
$$R_1 + R_2 < I(U_1, X_1; Y_1) + I(U_2; Y_2) - I(U_1, X_1; S) - I(U_2; U_1, X_1, S) \tag{1c}$$

for some joint distribution

$$P_{SX_1U_1U_2X_2Y_1Y_2} = P_S P_{X_1} P_{U_1U_2|X_1,S} P_{X_2|U_1U_2X_1S} P_{Y_1Y_2|X_1X_2S}. \tag{2}$$

Proof. We fix a joint distribution for which (2) is satisfied and generate a random code by following steps outlined below.

Codebook. Generate 2^{nR_1} codewords x_1^n, where the w_1-th codeword is indexed by $x_1^n(w_1)$, $w_1 \in \{1, ..., 2^{nR_1}\}$, and is generated i.i.d. according to P_{X_1} independent of the other codewords. Next, for each $x_1^n(w_1)$, generate $2^{nR_1^{'}}$ auxiliary vectors u_1^n, where the k_1-th vector is indexed by $u_1^n(w_1, k_1)$, $k_1 \in \{1, ..., 2^{nR_1^{'}}\}$, and is generated i.i.d. according to $P_{U_1|X_1}$ independent of the other vectors. Moreover, generate 2^{nR_2} bins, each containing $2^{nR_2^{'}}$ auxiliary vectors u_2^n, where the k_2-th vector in the w_2-th bin is indexed by $u_2^n(w_2, k_2)$, $(w_2, k_2) \in \{1, ..., 2^{nR_2}\} \times \{1, ..., 2^{nR_2^{'}}\}$, and is generated according to P_{U_2} independent of the other vectors.

Encoding. Let $(w_1, w_2) \in \{1, ..., 2^{nR_1}\} \times \{1, ..., 2^{nR_2}\}$ be the messages to be transmitted. Transmitter 1 sends $x_1^n(w_1)$. Encoder 2 looks for an index-pair (k_1, k_2) such that

$$(u_1^n(w_1, k_1), u_2^n(w_2, k_2), s^n, x_1^n(w_1)) \in \mathcal{T}_\epsilon^{(n)}(P_{U_1U_2SX_1}), \tag{3}$$

where $\mathcal{T}_\epsilon^{(n)}(\cdot)$ denotes the *ϵ-strongly typical set*. If such a pair can be found, the cognitive transmitter sends x_2^n which is generated i.i.d. according to $P_{X_2|U_1U_2SX_1}$.

Decoding. Decoder 1 tries to find the unique $\hat{w}_1$ such that $(x_1^n(\hat{w}_1), u_1^n(\hat{w}_1, \hat{k}_1))$ is jointly typical with the received sequence y_1^n and outputs $\hat{w}_1$. If more than one or no such $\hat{w}_1$ can be found, it declares an error. Decoder 2 tries to find the unique $\hat{w}_2$ such that $u_2^n(\hat{w}_2, \hat{k}_2)$ is jointly typical with the received seqence y_2^n and outputs $\hat{w}_2$. If more than one or no such $\hat{w}_2$ can be found, it declares an error.

Encoding Error Analysis. If there is no pair $(k_1, k_2) \in \{1, ..., 2^{nR_1^{'}}\} \times \{1, ..., 2^{nR_2^{'}}\}$ such that (3) holds, an encoding error happens. Using the standard arguments in Multivariate Covering Lemma [9], it can be shown that the probability of this error tends to zero as n goes to infinity if

$$R_1^{'} > I(U_1, X_1; S) \tag{4a}$$
$$R_2^{'} > I(U_2; X_1, S) \tag{4b}$$
$$R_1^{'} + R_2^{'} > I(U_1, X_1; S) + I(U_2; U_1, X_1, S) \tag{4c}$$

Decoder 1 Error Analysis. If either $(x_1^n(w_1), u_1^n(w_1, k_1))$ is not jointly typical with the received sequence y_1^n, or if $(x_1^n(\hat{w}_1), u_1^n(\hat{w}_1, \hat{k}_1))$ for $\hat{w}_1 \neq w_1$ happens to be jointly typical with the received sequence y_1^n, an error occurs. It can be shown that the probability of this error tends to zero as n goes to infinity if

$$R_1 + R_1^{'} < I(U_1, X_1; Y_1) \tag{5}$$

Decoder 2 Error Analysis. If either $u_2^n(w_2, k_2)$ is not jointly typical with the received sequence y_2^n, or if $u_2^n(\hat{w}_2, \hat{k}_2)$ for $\hat{w}_2 \neq w_2$ happens to be jointly typical with the received sequence y_2^n, an error occurs. It can be shown that the probability of this error tends to zero as n goes to infinity if

$$R_2 + R_2^{'} < I(U_2; Y_2). \tag{6}$$

From (4), (5) and (6) we conclude that the error probability of our coding scheme tends to zero as n goes to infinity for all (R_1, R_2) satisfying (1). Time sharing further makes achievable the convex hull of all rate-pairs satisfying (1) for joint distributions of the form (2).

IV. Capacity Region of Semideterministic SD-CIC

In this section, we characterize the capacity region of the discrete memoryless semideterministic SD-CIC.

Theorem 2. *The capacity region of the discrete memoryless semideterministic SD-CIC with the channel states known noncausally to only the cognitive transmitter is the convex closure of the union of rate-pairs (R_1, R_2) satisfying*

$$R_1 < I(U, X_1; Y_1) - I(U, X_1; S) \tag{7a}$$
$$R_2 < H(Y_2|X_1, S) \tag{7b}$$
$$R_1 + R_2 < I(U, X_1; Y_1) - I(U, X_1; S) + H(Y_2|U, X_1, S) \tag{7c}$$

for some joint distribution

$$P_{SX_1UX_2Y_1Y_2} = P_S P_{X_1} P_{X_2U|X_1S} P_{Y_1Y_2|X_1X_2S}. \tag{8}$$

Furthermore, this is the capacity region even if the state sequence is known noncausally to receiver 2.

Proof. Achievability follows by redefining $U_1 = U$ and setting $U_2 = Y_2$ in Theorem 1; setting $U_2 = Y_2$ is possible since Y_2 is a deterministic function (X_1, X_2, S), and X_1, X_2 and S are known to the cognitive transmitter. To prove the converse, combining the ideas from Lapidoth-Wang's outer bound for the state-dependent broadcast channel [10], Nair-El Gamal's outer bound [9], and Gelfand-Pinkser's converse for the state-dependent single user channel [2], we show that any achievable rate-pair (R_1, R_2) remains in the convex closure of the union of rate-pairs satisfying (7) even if receiver 2 is noncausally aware of the channel state S^n. Considering a code with block-length n, we first bound R_1 as follows:

$$nR_1 = H(W_1) \leq I(W_1; Y_1^n) + n\epsilon_n = \sum_{i=1}^{n} I(W_1; Y_{1,i}|Y_1^{i-1}) + n\epsilon_n \tag{9}$$

$$= \sum_{i=1}^{n} [I(W_1, S_{i+1}^n; Y_{1,i}|Y_1^{i-1}) - I(Y_1^{i-1}; S_i|W_1, S_{i+1}^n)] + n\epsilon_n \tag{10}$$

$$\leq \sum_{i=1}^{n} [I(W_1, X_{1,i}, S_{i+1}^n, Y_1^{i-1}; Y_{1,i}) - I(W_1, X_{1,i}, Y_1^{i-1}, S_{i+1}^n; S_i)] + n\epsilon_n \tag{11}$$

$$= \sum_{i=1}^{n} I(V_i, X_{1,i}; Y_{1,i}) - I(V_i, X_{1,i}; S_i) + n\epsilon_n, \tag{12}$$

where ϵ_n tends to zero as n goes to infinity. Here, (9) follows from Fano's Inequality; (10) follows from Csiszár-Korner's Lemma [11]; (11) follows since conditioning does not increase entropy, S_i and (W_1, S_{i+1}^n) are independent and X_1^n is a deterministic function of W_1; finally, (12) follows by defining the auxiliary random variables $V_i \triangleq (W_1, Y_1^{i-1}, S_{i+1}^n)$ for $i \in \{1, ..., n\}$.

We next derive a bound on R_2 as follows:

$$nR_2 = H(W_2) \leq I(W_2; Y_2^n, S^n) + n\epsilon_n \tag{13}$$

$$\leq I(W_2; Y_2^n, S^n, W_1) + n\epsilon_n \tag{14}$$

$$= I(W_2; Y_2^n|W_1, S^n) + n\epsilon_n \tag{15}$$

$$= \sum_{i=1}^{n} I(W_2; Y_{2,i}|W_1, X_{1,i}, Y_2^{i-1}, S^n) + n\epsilon_n \tag{16}$$

$$\leq \sum_{i=1}^{n} H(Y_{2,i}|S_i, X_{1,i}) + n\epsilon_n. \tag{17}$$

Here, (13) follows from Fano's Inequality; (14) follows since conditioning does not increase entropy; (15) follows because (W_1, S^n) and W_2 are independent; (16) follows from the chain rule and the fact that X_1^n is a deterministic function of message W_1; finally (17) follows by dropping the negative term and considering the fact that conditioning cannot increase entropy.

We proceed to derive a bound on the sum rate $R_1 + R_2$:

$$n(R_1 + R_2) = H(W_1, W_2) \tag{18}$$

$$= H(W_1) + H(W_2|W_1) \tag{19}$$

$$\leq I(W_1; Y_1^n) + I(W_2; Y_2^n, S^n|W_1) + n\epsilon_n, \tag{20}$$

where (18) follows from Fano's Inequality. The first term on the right hand side (RHS) of (20) can be bounded as follows:

$$I(W_1; Y_1^n) = \sum_{i=1}^{n} I(W_1; Y_{1,i}|Y_1^{i-1}) \tag{21}$$

$$\leq \sum_{i=1}^{n} I(W_1, Y_1^{i-1}; Y_{1,i}) \tag{22}$$

$$= \sum_{i=1}^{n} [I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{1,i}) - I(Y_1^{i-1}; Y_{2,i}, S_i|W_1, S_{i+1}^n, Y_{2,i+1}^n)], \tag{23}$$

where (23) follows by applying Csiszár-Korner's Lemma to (S^n, Y_2^n) and Y_1^n. We now consider the sum of the second terms on the RHS of (20) and (23):

$$I(W_2; Y_2^n, S^n|W_1) - \sum_{i=1}^{n} I(Y_1^{i-1}; Y_{2,i}, S_i|W_1, S_{i+1}^n, Y_{2,i+1}^n)$$

$$= \sum_{i=1}^{n} [I(W_2; Y_{2,i}, S_i|W_1, S_{i+1}^n, Y_{2,i+1}^n) - I(Y_1^{i-1}; Y_{2,i}, S_i|W_1, S_{i+1}^n, Y_{2,i+1}^n)] \tag{24}$$

$$= \sum_{i=1}^{n} [I(W_2; Y_{2,i}, S_i|W_1, S_{i+1}^n, Y_{2,i+1}^n) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) + I(W_1, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i)] \tag{25}$$

$$= \sum_{i=1}^{n} [I(W_1, W_2, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i)] \tag{26}$$

$$= \sum_{i=1}^{n} [I(W_1, W_2, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; S_i)] - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i)], \tag{27}$$

where (24), (25), (26) and (27) follow from the chain rule. We now consider the sum of the first term and the last term on the RHS of (27):

$$\sum_{i=1}^{n} [I(W_1, W_2, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i)]$$

$$= \sum_{i=1}^{n} [I(W_1, W_2, S^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) - I(S_{i+1}^n, Y_{2,i+1}^n; S_i|W_1, W_2, S^{i-1}) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i)] \tag{28}$$

$$= \sum_{i=1}^{n} [I(W_1, W_2, S^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}, S_i) - I(W_1, W_2, S^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; S_i) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i)] \tag{29}$$

$$= \sum_{i=1}^{n} [I(W_1, W_2, S^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i) - I(W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n; Y_{2,i}|S_i)] \tag{30}$$

$$= \sum_{i=1}^{n} H(Y_{2,i}|W_1, Y_1^{i-1}, S_{i+1}^n, Y_{2,i+1}^n, S_i). \tag{31}$$

Here, (28) follows by applying Csiszár-Korner's Lemma to (S^n, Y_2^n) and S^n; (29) follows since S_i and (W_1, W_2, S^{i-1}) are independent; (30) follows from the chain rule; finally, (31) follows because, given (W_1, W_2, S^n), the channel inputs X_1^n and X_2^n are determined, and hence Y_2^n, which is a deterministic function of (X_1^n, X_2^n, S^n), is also determined, so $H(Y_{2,i}|W_1, W_2, S^n, Y_{2,i+1}^n) = 0$ for $i \in \{1, ..., n\}$.

Combining (20), (23), (27) and (31), considering the fact that $X_{1,i}$ is a deterministic function of W_1, and applying the definitions $V_i \triangleq (W_1, Y_1^{i-1}, S_{i+1}^n)$ and $T_i \triangleq Y_{2,i+1}^n$ for $i \in \{1, ..., n\}$, we obtain:

$$n(R_1 + R_2) \leq \sum_{i=1}^{n} [I(V_i, T_i, X_{1,i}; Y_{1,i}) - I(V_i, T_i, X_{1,i}; S_i) + H(Y_{2,i}|V_i, T_i, X_{1,i}, S_i)] + n\epsilon_n. \quad (32)$$

where ϵ_n tends to zero as n goes to infinity.

From (12), (17) and (32), we conclude that every achievable rate-pair (R_1, R_2) must be included in the convex closure of the union of rate-pairs (R_1, R_2) satisfying:

$$R_1 < I(V, X_1; Y_1) - I(V, X_1; S) \quad (33a)$$
$$R_2 < H(Y_2|X_1, S) \quad (33b)$$
$$R_1 + R_2 < I(V, T, X_1; Y_1) - I(V, T, X_1; S) + H(Y_2|V, T, X_1, S), \quad (33c)$$

where $(V, T) \to (X_1, X_2, S) \to (Y_1, Y_2)$ forms a Markov chain, and X_1 is independent of S. Now, to complete the converse proof of Theorem 1, we proceed to show that the outer bound (33) can be represented using only a single auxiliary random variable. To this end, we investigate the outer bound (33) in two different cases:

Case 1. $I_1 = I(T; S, Y_2|V, X_1) - I(T; Y_1|V, X_1) \geq 0$;

in this case, (33c) can be further bounded as follows:

$$\begin{aligned}R_1 + R_2 &\leq \text{RHS of } (33c)\\ &= I(V, X_1; Y_1) - I(V, X_1; S) + H(Y_2|V, S, X_1)\\ &\quad - [I(T; S, Y_2|V, X_1) - I(T; Y_1|V, X_1)]\\ &\leq I(V, X_1; Y_1) - I(V, X_1; S) + H(Y_2|V, S, X_1),\end{aligned}$$

where, the equality follows from the chain rule, and the last inequality follows from $I_1 \geq 0$. Therefore, if we choose $U = V$, the outer bound (33) can be relaxed to (7).

Case 2. $I_2 = I(T; S|V, X_1) - I(T; Y_1|V, X_1) \leq 0$;

in this case, (33a) can be further bounded as follows:

$$\begin{aligned}R_1 &\leq \text{RHS of } (33a)\\ &= I(V, T, X_1; Y_1) - I(V, T, X_1; S)\\ &\quad + [I(T; S|V, X_1) - I(T; Y_1|V, X_1)]\\ &\leq I(V, T, X_1; Y_1) - I(V, T, X_1; S),\end{aligned}$$

where the equality follows from the chain rule, and the last inequality follows from $I_2 \leq 0$. Hence, if we choose $U = (V, T)$, the outer bound (33) can be relaxed to (7).

We are left to show that at least one of the above two cases is always satisfied. Toward this end, we note that for all random variables $V, T, X_1, X_2, S, Y_1, Y_2$ we have $I_1 \geq I_2$. This implies that at least one of the above two cases must hold. This concludes the converse proof of Theorem 1.

Remark 1. *The capacity region in Theorem 1 reduces to the capacity region of the semideterministic state-dependent broadcast channel with the channel states known noncausally to the transmitter given in* [10]*. This can be seen by rewriting the RHS of* (7c) *as* $I(U, X_1; Y_1) - I(U, X_1; S, Y_2) + H(Y_2|S)$ *and then by setting* $X_1 = \varnothing$*,* $Y_1 = Z$*,* $Y_2 = Y$*,* $R_1 = R_z$ *and* $R_2 = R_y$*.*

V. Capacity Region of Deterministic SD-CIC

In this section, we characterize the capacity region of the discrete memoryless deterministic SD-CIC.

Theorem 3. *The capacity region of the deterministic SD-CIC with the channel states known noncausally to only the cognitive transmitter is the convex closure of the union of rate-pairs* (R_1, R_2) *satisfying*

$$R_1 < H(Y_1) - I(X_1, Y_1; S) \quad (34a)$$
$$R_2 < H(Y_2|X_1, S) \quad (34b)$$
$$R_1 + R_2 < H(Y_1) - I(Y_1, X_1; S) + H(Y_2|Y_1, X_1, S) \quad (34c)$$

for some joint distribution

$$P_{SX_1X_2Y_1Y_2} = P_S P_{X_1} P_{X_2|X_1S} P_{Y_1Y_2|X_1X_2S}. \quad (35)$$

Proof. Achievability follows by setting $U = Y_1$ in Theorem 2; setting $U = Y_1$ is possible since Y_1 is a deterministic function of (X_1, X_2, S), and X_1, X_2 and S are known to the cognitive transmitter. To prove the converse, we first use the outer bound in (7a) to obtain the outer bound in (34a)

$$\begin{aligned}R_1 &< I(U_1, X_1; Y_1) - I(U_1, X_1; S)\\ &= I(U_1, X_1, S; Y_1) - I(U_1, X_1, Y_1; S)\\ &= H(Y_1) - H(Y_1|U_1, X_1, S)\\ &\quad + H(S|U_1, X_1, Y_1) - H(S)\\ &< H(Y_1) + H(S|X_1, Y_1) - H(S) \quad (36a)\\ &= H(Y_1) - I(X_1, Y_1; S), \quad (36b)\end{aligned}$$

where (36a) follows from dropping a negative term and the fact that the conditioning does not increase entropy. The outer bound in (34b) is the same as the outer bound in (7b). Next, we use the outer bound in (7c) to obtain the outer bound in (34c)

$$\begin{aligned}R_1 + R_2 &< I(U, X_1; Y_1) - I(U, X_1; S) + H(Y_2|U, X_1, S)\\ &= I(U, X_1, S; Y_1) - I(U, X_1, Y_1; S)\\ &\quad + H(Y_2|Y_1, U, X_1, S) + I(Y_1; Y_2|U, X_1, S)\\ &= H(Y_1) + H(S|U, X_1, Y_1) - H(S)\\ &\quad + H(Y_2|Y_1, U, X_1, S) - H(Y_1|Y_2, U, X_1, S)\\ &< H(Y_1) + H(S|X_1, Y_1) - H(S)\\ &\quad + H(Y_2|Y_1, X_1, S) \quad (37a)\\ &= H(Y_1) - I(X_1, Y_1; S) + H(Y_2|Y_1, X_1, S), \quad (37b)\end{aligned}$$

where (37a) follows from dropping a negative term and the fact that conditioning does not increase entropy.

VI. New Outer Bound for the General SD-CIC

In this section, we first derive a new outer bound on the capacity region of the general SD-CIC. We then prove that this new outer bound is the capacity region of the discrete memoryless SD-CIC when both cognitive transmitter and cognitive receiver know the channel states in a noncausal manner, provided that the channel model satisfies the "better cognitive decoding regime" condition. Under this condition the channel model satisfies the following:

$$I(U,X_1;Y_2) \geq I(U,X_1;Y_1) \quad \text{for all} \quad P_{X_1}P_{X_2U|X_1S}. \tag{38}$$

Theorem 4. *The capacity region of the discrete memoryless SD-CIC with the states known noncausally to the cognitive transmitter and its corresponding receiver is contained in the convex closure of the rate-pairs (R_1,R_2) satisfying*

$$R_1 < I(U,X_1;Y_1) - I(U,X_1;S) \tag{39a}$$
$$R_2 < I(X_2;Y_2|X_1,S) \tag{39b}$$
$$R_1 + R_2 < \text{RHS of } (39a) + I(X_2;Y_2|U,X_1,S) \tag{39c}$$

for some joint distribution

$$P_{SX_1UX_2Y_1Y_2} = P_S P_{X_1} P_{X_2U|X_1S} P_{Y_1Y_2|X_1X_2S}. \tag{40}$$

proof. The proof is performed through generalization of converse proof of Theorem 2 to a SD-CIC that is not necessarily semideterministic; it is omitted here due to lack of space.

Theorem 5. *The outer bound in Theorem 4 is the capacity region of the discrete memoryless SD-CIC with the states known noncausally to the cognitive transmitter and its corresponding receiver if the channel satisfies condition (38).*

proof. We fix a joint distribution of the form (40) and generate a random code by following the steps outlined below.

Codebook. Generate 2^{nR_1} codewords x_1^n, where the w_1-th codeword is indexed by $x_1^n(w_1)$, $w_1 \in \{1,...,2^{nR_1}\}$, and is generated i.i.d. according to P_{X_1} independent of the other codewords. Next, for each $x_1^n(w_1)$, generate $2^{nR_{21}}$ bins, each containing $2^{nR'_{21}}$ auxiliary vectors u^n, where the k-th vector in the w_{21}-th bin is indexed by $u^n(w_1,w_{21},k)$, $w_{21} \in \{1,...,2^{nR_{21}}\}$ and $k \in \{1,...,2^{nR'_{21}}\}$, and is generated i.i.d. according to $P_{U|X_1}$ independent of the other vectors. Moreover, for each $x_1^n(w_1)$, $u^n(w_1,w_{21},k)$ and s^n, generate $2^{nR_{22}}$ codeword x_2^n, where the w_{22}-th codeword is indexed by $x_2^n(w_1,w_{21},k,w_{22})$, $w_{22} \in \{1,...,2^{nR_{22}}\}$, and is generated according to $P_{X_2|USX_1}$ independent of the other vectors.

Encoding. Let $(w_1,w_{21},w_{22}) \in \{1,...,2^{nR_1}\} \times \{1,...,2^{nR_{21}}\} \times \{1,...,2^{nR_{22}}\}$ be the messages to be transmitted. Transmitter 1 sends $x_1^n(w_1)$. Encoder 2 looks for an index k such that $u^n(w_1,w_{21},k)$ is jointly typical with $(x_1^n(w_1),s^n)$. If such an index can be found, cognitive transmitter sends $x_2^n(w_1,w_{21},k,w_{22})$.

Decoding. Since the channel is operating in the better cognitive decoding regime ((38) holds), every message which can be decoded by decoder 1 can also can be decoded by decoder 2. Therefore, w_1 is treated as a common message. Decoder 1 employs *indirect decoding* and tries to find a unique $\hat{w}_1$ such that $(x_1^n(\hat{w}_1),u^n(\hat{w}_1,\hat{\hat{w}}_{21},\hat{k}))$ is jointly typical with the received sequence y_1^n and outputs $\hat{w}_1$. If more than one or no such a $\hat{w}_1$ can be found, it declares an error. Decoder 2 tries to find the unique triple $(\hat{\hat{w}}_1,\hat{w}_{21},\hat{w}_{22})$ such that $(x_1^n(\hat{\hat{w}}_1),u^n(\hat{\hat{w}}_1,\hat{w}_{21},k),x_2^n(\hat{\hat{w}}_1,\hat{w}_{21},k,\hat{w}_{22}))$ is jointly typical with (y_2^n,s^n) and outputs $(\hat{\hat{w}}_1,\hat{w}_{21},\hat{w}_{22})$. If more than one or no such a triple can be found, it declares an error.

By upper bounding the probability of the possible error events associated with this coding scheme, we have

$$R_1 < I(U,X_1;Y_1) - I(U,X_1;S) \tag{41a}$$
$$R_2 < I(X_2;Y_2|X_1,S) \tag{41b}$$
$$R_1 + R_2 < \text{RHS of } (41a) + I(X_2;Y_2|U,X_1,S) \tag{41c}$$
$$R_1 + R_2 < I(X_1,X_2;Y_2,S). \tag{41d}$$

However, (38) implies that the RHS of (41d) is larger than the RHS of (41c), so (41d) is redundant. Therefore, we conclude that the error probability of our coding scheme tends to zero as n goes to infinity for all (R_1,R_2) satisfying (39), provided that the channel is in the better cognitive decoding regime.

VII. Conclusions

We derived the capacity of semideterministic and deterministic SD-CICs with noncausal channel state information only at the cognitive transmitter. Furthermore, We provided new inner and outer bounds on the capacity region of the general SD-CIC. We showed that the new outer bound is the capacity region of SD-CIC in the better cognitive decoding regime when both the cognitive transmitter and its corresponding receiver are aware of the channel states in a noncausal manner.

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