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Diversity-Multiplexing Tradeoff for the Interference Channel With a Relay

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***Abstract*—We study the diversity-multiplexing tradeoff (DMT) for the slow fading interference channel with a relay (ICR). We first derive an outer bound on the DMT based on the cut-set bound. We then derive two inner bounds on the DMT: One is based on the compress-and-forward relaying scheme and the other is based on the decode-and-forward relaying scheme. We find conditions on the channel parameters and the multiplexing gains under which the proposed inner bounds achieve the optimal DMT. We also identify cases in which the DMT of the ICR is the same as two parallel fading relay channels, implying that interference does not decrease the DMT for each pair, and that a single relay can be DMT-optimal for two pairs *simultaneously*. Lastly, we identify conditions under which adding a relay strictly improves the DMT relative to the interference channel without a relay.**

I. Introduction

The interference channel with a relay (ICR) models the scenario in which a relay helps several independent transmitters in sending messages to their corresponding receivers simultaneously over a shared channel. The ICR was first studied in [1] and has since gained considerable interest as an extension of the canonical relay and interference channel models. Inner and outer bounds on the capacity region of the two-pair ICR with additive white Gaussian noise (AWGN) were characterized in [1], [2], and [3]. Specifically, in [1] an achievable rate region was obtained by employing a rate splitting scheme at the transmitters, decode-and-forward (DF) strategy at the relay, and a backward decoding scheme at the receivers. The achievable rate region in [2] was obtained using the compress-and-forward (CF) strategy at the relay. Outer bounds were obtained by applying the cut-set bound [2], [3], and by using a potent relay [2]. For the ergodic phase fading and the ergodic Rayleigh fading channels, the capacity region for the strong interference regime was characterized in [4] for the case in which the links from the sources to the relay are good, i.e., when DF is capacity optimal. In this paper we study the diversity-multiplexing tradeoff (DMT) for the slow Rayleigh fading ICR. We derive inner and outer bounds on the DMT and identify situations, in which the optimal DMT can be characterized.

In [5], DMT characteristics of several cooperation strategies were obtained. The work [5] showed that the DF scheme with receive channel state information (Rx-CSI) at the relay and destination is DMT-optimal for full duplex single-antenna single-relay channels; but it is suboptimal for multiple-antenna relay channels at high multiplexing gains. CF with Rx and Tx CSI at the relay and destination, on the other hand, was shown to be DMT-optimal for the multiple-antenna relay channel over all multiplexing gains. In [6], it was shown that quantize-map-and-forward (QMF) achieves the optimal DMT of certain configurations of the half-duplex relay channel without CSI at the relay node. The DMT of single-antenna block Rayleigh fading interference channels (ICs) was studied in [7] for the scenario in which there is Rx-CSI but no Tx-CSI, and studied in [8] for the case in which both Rx-CSI and Tx-CSI are available. In [7] it was shown that in the very strong interference regime, successive decoding with interference cancelation is DMT optimal. Note that for the ergodic case, using the same approach achieves the capacity region of the IC. Hence, the same strategy is optimal from both DMT and capacity perspectives. For the general interference regime, [7] proposed a transmission scheme using Han-Kobayashi (HK) type superposition encoding where each receiver jointly decodes both the common messages (from both transmitters) and the private message from its intended transmitter. This scheme was shown to be DMT optimal under some conditions on the strength of the interference and over a certain range of multiplexing gains. The DMT of the Gaussian MIMO ICR was studied in [9] for the case where all links have the same exponential behaviour over the signal-to-noise ratio (SNR). In [9], an outer bound on the DMT was derived using the cut-set theorem, and an achievable DMT was characterized by employing CF at the relay node. The generalized degrees of freedom (GDoF) of the IC with a relay was studied in [10] in which it was shown that the relay helps to achieve a higher GDoF compared to the IC.

Main Contributions

In this paper, we study the DMT of the single-antenna ICR with a full-duplex relay. All links are subject to slow Rayleigh fading. We consider the scenario in which the receivers have perfect Rx-CSI, but there is no Tx-CSI at the sources. We allow the direct links gains, interfering links gains, and the relay-to-destinations links gains to scale differently as exponential functions of the SNR, while the channel is symmetric in the sense that the scaling of the corresponding links in both pairs is identical. The main contributions of this work are:

1) An outer bound on the DMT is presented using the cut-set bound. Note that while [9] studied the scenario in which all links scale identically over the SNR, our outer bound allows different scalings.

2) Two achievable DMT regions are derived based on the

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CF and DF schemes. We analyze how the gains of the cross-links and of the relay-to-destination links affect the achievable DMT.
3) We provide sufficient conditions under which the *optimal* DMT can be achieved by either the CF or the DF scheme. We then present sufficient conditions under which the ICR has the same DMT as two parallel single-relay channels. Thus, the relay assistance to one pair does not degrade the DMT performance at the other pair, and a single relay is simultaneously DMT-optimal for two separate pairs.
4) We compare the DMT of the ICR with that of the IC, and provide sufficient conditions under which adding a relay to the IC improves the DMT.

The rest of the paper is organized as follows: The channel model and notations are presented in Section II. The case in which the relay has Tx-CSI and Rx-CSI is studied in Section III, in which an outer bound on the achievable DMT is obtained and a CF-based DMT is derived. The case in which the relay has only Rx-CSI is studied in Section IV where an outer bound and a DF-based achievable DMT are presented. Concluding remarks are presented in Section V.

II. System Model and Notation

We denote random variables (RVs) with capital letters, e.g., X, Y, and their realizations with lower case letters, e.g., x, y. $\mathbb{E}\{X\}$ denotes the stochastic expectation of X. Boldface letters, e.g., $\mathbf{x}$, denote column vectors (unless otherwise specified), and the i'th element of a vector $\mathbf{x}$ is denoted by x_i. We use x^j to denote the vector $(x_1, x_2, ..., x_{j-1}, x_j)$, and X^* to denote the conjugate of X. We denote the circularly symmetric, complex Normal distribution with mean μ and variance σ^2 as $\mathcal{CN}(\mu, \sigma^2)$. All logarithms are of base 2. We also define $(x)^+ \triangleq \max\{x, 0\}$, and denote $f(\text{SNR}) \doteq \text{SNR}^c$ if $\lim_{\text{SNR}\to\infty} \frac{\log f(\text{SNR})}{\log \text{SNR}} = c$.

The ICR consists of two transmitters, two receivers, and a full-duplex relay that assists communication from the transmitters to their respective receivers, as shown in Fig. 1. Tx_k sends messages to Rx_k, $k = 1, 2$. The received signals at Rx_1, Rx_2 and at the relay at time i are denoted by $Y_{1,i}$, $Y_{2,i}$, and $Y_{3,i}$ respectively. The channel inputs from Tx_1, Tx_2 and the relay are denoted by $X_{1,i}$, $X_{2,i}$, and $X_{3,i}$, respectively.

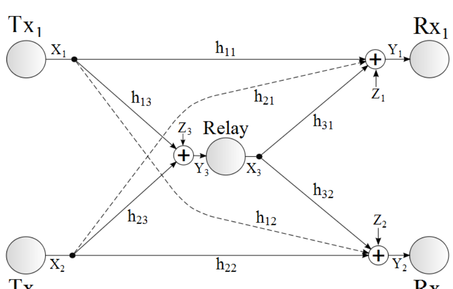

Fig. 1: The ICR model.

The relationship between the channel inputs and outputs is:

$$Y_1 = \sqrt{\text{SNR}} H_{11} X_1 + \sqrt{\text{SNR}^\alpha} H_{21} X_2 + \sqrt{\text{SNR}^\beta} H_{31} X_3 + Z_1$$
$$Y_2 = \sqrt{\text{SNR}^\alpha} H_{12} X_1 + \sqrt{\text{SNR}} H_{22} X_2 + \sqrt{\text{SNR}^\beta} H_{32} X_3 + Z_2$$
$$Y_3 = \sqrt{\text{SNR}} H_{13} X_1 + \sqrt{\text{SNR}} H_{23} X_2 + Z_3.$$

Here, Z_1, Z_2, and Z_3 are mutually independent RVs, distributed according to $\mathcal{CN}(0, 1)$, independent over time and independent of the channel inputs and channel coefficients. Each channel input has a unit power constraint: $\mathbb{E}\{|X_k|^2\} \le 1, k \in \{1, 2, 3\}$. Note that in the above equations, SNR denotes the average received signal-to-noise ratio over the direct link for both pairs. Observe that the cross-link gains scale as $\sqrt{\text{SNR}^\alpha}$, and the relay-to-destination link gains scale as $\sqrt{\text{SNR}^\beta}$, while the remaining link gains scale as $\sqrt{\text{SNR}}$. Thus, this work studies the impact of the scaling of interference and relay-destination links on the DMT. The different SNR exponents (see also [7]) represent different pathloss scaling behaviour due to different propagation scenarios. For example, when a receiver is located closer to the opposite transmitter, the fading pathloss exponent from the interferer to the receiver may be smaller than the pathloss exponent from the desired transmitter, as there is less scattering on the path from the interferer than on the path from the desired transmitter. This is translated into a larger SNR exponent on the interfering link than on the direct link.

H_{kl} denotes the channel coefficient from node k to node l. All channel coefficients are i.i.d., each distributed as $\mathcal{CN}(0, 1)$. The coefficients change only between messages which corresponds to slow fading. It is assumed that receiver k has Rx-CSI represented by $\tilde{H}_k = (H_{1k}, H_{2k}, H_{3k}) \in \mathfrak{C}^3 \triangleq \tilde{\mathfrak{H}}_k, k \in \{1, 2\}$, and that the relay has Rx-CSI represented by $\tilde{H}_{3,R} = (H_{13}, H_{23}) \in \mathfrak{C}^2 \triangleq \mathfrak{H}_3$. In some scenarios, the relay has also Tx-CSI represented by $\tilde{H}_{3,T} = (H_{31}, H_{32}) \in \mathfrak{H}_3$. However, we do not assume Tx-CSI at Tx_1 or Tx_2. Let $\underline{\tilde{H}} = (\tilde{H}_1, \tilde{H}_2, \tilde{H}_3)$ be the vector of all channel coefficients.

Definition 1. An (R_1, R_2, n) code for the ICR consists of two message sets $\mathcal{M}_k \triangleq \{1, 2, ..., 2^{nR_k}\}$, $k = 1, 2$, two encoders at the sources, $e_k : \mathcal{M}_k \mapsto \mathfrak{C}^n$, $k = 1, 2$, and two decoders at the destinations, $g_k : \tilde{\mathfrak{H}}_k \times \mathfrak{C}^n \mapsto \mathcal{M}_k$, $k = 1, 2$. With only Rx-CSI, the transmitted signal at the relay at time i is $x_{3,i} = t_i(y_3^{i-1}, \underline{\tilde{h}}_{3,R}) \in \mathfrak{C}$, $i = 1, 2, ..., n$. With both Rx and Tx-CSI $x_{3,i} = t_i(y_3^{i-1}, \underline{\tilde{h}}_{3,R}, \underline{\tilde{h}}_{3,T}) \in \mathfrak{C}$. We denote a coding scheme by $\mathcal{S}_c$.

Definition 2. The average probability of error is defined as $\text{P}_e^{(n)} \triangleq \Pr\left(g_1(\tilde{H}_1, Y_1^n) \neq M_1 \text{ or } g_2(\tilde{H}_2, Y_2^n) \neq M_2\right)$, where each sender selects its message independently and uniformly from its message set.

Definition 3. A rate pair (R_1, R_2) is called achievable if for any $\epsilon > 0$ and $\delta > 0$ there exists some blocklength $n_0(\epsilon, \delta)$ such that (s.t.) for every $n > n_0(\epsilon, \delta)$ there exists an $(R_1 - \delta, R_2 - \delta, n)$ code with $\text{P}_e^{(n)} < \epsilon$. Let $\mathcal{R}(\underline{\tilde{H}}, \mathcal{S}_c, \text{SNR})$ denote the maximum achievable rate region achieved by a coding scheme $\mathcal{S}_c$ for the ICR whose channel coefficients are $\underline{\tilde{H}}$, and the direct-link signal to noise ratio is SNR.

Definition 4. The probability of an outage event in the ICR, for the scheme $\mathcal{S}_c$ and target rates $R_{T,1}$ for pairs 1, and $R_{T,2}$ for pair 2, is defined as: $P_{\mathcal{O}}(R_{T,1}, R_{T,2}, \text{SNR}, \mathcal{S}_c) \triangleq \Pr\left((R_{T,1}, R_{T,2}) \notin \mathcal{R}(\underline{\tilde{H}}, \mathcal{S}_c, \text{SNR})\right)$.

Definition 5. We say that a coding scheme $\mathcal{S}_c$ for the ICR achieves multiplexing gains (MGs) of (r_1, r_2), if there exist rates $\big(R_1(\text{SNR}), R_2(\text{SNR})\big) \in \mathcal{R}(\underline{\tilde{H}}, \mathcal{S}_c, \text{SNR})$ that scale as $\lim_{\text{SNR}\to\infty} \frac{R_k(\text{SNR})}{\log(\text{SNR})} = r_k, k = 1, 2$.

Definition 6. We say that a scheme $\mathcal{S}_c$ achieves a diversity gain of $d(r_1, r_2)$ for multiplexing gains r_1 ,r_2, if

$$-\lim_{\text{SNR}\to\infty} \frac{\log P_{\mathcal{O}}\big(r_1 \log(\text{SNR}), r_2 \log(\text{SNR})\big)}{\log(\text{SNR})} = d(r_1, r_2).$$

Note that while the diversity gain is a function of $\text{P}_e^{(n)}$, we can follow the arguments in [5, Section III] and characterize the diversity gain via the outage probability. This can be done since $\lim_{\text{SNR}\to\infty} \frac{\log P_{\mathcal{O}}}{\log(\text{SNR})} = \lim_{\text{SNR}\to\infty} \frac{\log \text{P}_e^{(n)}}{\log(\text{SNR})}$.

III. DMT WITH RX-CSI AND TX-CSI AT THE RELAY

A. DMT Outer Bound

We now present an outer bound on the achievable DMT of the ICR, based on the cut-set theorem:

Theorem 1. *For the symmetric ICR with Rx-CSI at the receivers and with Rx and Tx-CSI at the relay, as defined in Section II, an outer bound on the DMT is given by:*

$$d^+(r_1, r_2) = \min_{k\in\{1,2,\cdots,10\}} \big\{d_k^+(r_1, r_2)\big\}, \quad (1)$$

where

$$d_1^+(r_1, r_2) = 2(1 - r_1)^+ \quad (2a)$$
$$d_2^+(r_1, r_2) = (1 - r_1)^+ + (\beta - r_1)^+ \quad (2b)$$
$$d_3^+(r_1, r_2) = 2(1 - r_2)^+ \quad (2c)$$
$$d_4^+(r_1, r_2) = (1 - r_2)^+ + (\beta - r_2)^+ \quad (2d)$$
$$d_5^+(r_1, r_2) = 2(1 - r_1 - r_2)^+ + 2(\alpha - r_1 - r_2)^+ + 2(\beta - r_1 - r_2)^+ \quad (2e)$$
$$d_6^+(r_1, r_2) = (2 - r_1 - r_2)^+ + 2(\alpha + \beta - r_1 - r_2)^+ \quad (2f)$$
$$d_7^+(r_1, r_2) = (2\alpha - r_1 - r_2)^+ + 2(1 + \beta - r_1 - r_2)^+ \quad (2g)$$
$$d_8^+(r_1, r_2) = 4(1 - r_1 - r_2)^+ + 2(\alpha - r_1 - r_2)^+ \quad (2h)$$
$$d_9^+(r_1, r_2) = 2(2 - r_1 - r_2)^+ + (2\alpha - r_1 - r_2)^+ \quad (2i)$$
$$d_{10}^+(r_1, r_2) = (2 - r_1 - r_2)^+ + 2(1 + \alpha - r_1 - r_2)^+. \quad (2j)$$

Proof: The cut-set bound for the ICR was evaluated in [9, Proof of Thm. 1] for the case of $\alpha = \beta = 1$. The proof of Thm. 1 is obtained by following similar arguments as in that proof while taking into account the exponents α and β. Details can be found in [11]. ■

Remark 1. Observe that (2a) and (2b) are the DMT upper bound for the single-relay channel with Tx_1 as the source and Rx_1 as the destination, while (2c) and (2d) are the DMT upper bound for the Tx_2-relay-Rx_2 relay channel. The remaining DMT bounds correspond to the cut-set bounds for the IC, see [9, Proof of Thm. 1].

B. An Achievable DMT Region via CF

We now derive an achievable DMT region using the CF scheme. In the next section we shall provide sufficient conditions under which this DMT region coincides with the outer bound of (1), leading to the characterization of the optimal DMT of the ICR. The achievable DMT with the CF scheme is characterized in the following theorem:

Theorem 2. *For the symmetric ICR with Rx-CSI at the receivers and with Rx and Tx-CSI at the relay, as defined in Section II, an achievable DMT region is given by:*

$$d_{CF}^-(r_1, r_2) = \min_{k\in\{1,2,3\}} \Big\{d_{k,CF}^-(r_1, r_2)\Big\}, \quad (3)$$

where

$$d_{1,CF}^-(r_1, r_2) = \quad (4a)$$
$$\begin{cases} (1 - r_1)^+ + \big(1 - (1 + \alpha - \beta)^+ - r_1\big)^+ & \alpha > 1 \\ (1 - r_1)^+ + \big(1 - (2 - \beta)^+ - r_1\big)^+ & \alpha \le 1 \end{cases}$$

$$d_{2,CF}^-(r_1, r_2) = \quad (4b)$$
$$\begin{cases} (1 - r_2)^+ + \big(1 - (1 + \alpha - \beta)^+ - r_2\big)^+ & \alpha > 1 \\ (1 - r_2)^+ + \big(1 - (2 - \beta)^+ - r_2\big)^+ & \alpha \le 1 \end{cases}$$

$$d_{3,CF}^-(r_1, r_2) = \quad (4c)$$
$$\begin{cases} (1 - r_1 - r_2)^+ + (\alpha - r_1 - r_2)^+ \\ \quad +2\big(1 - (1 + \alpha - \beta)^+ - r_1 - r_2\big)^+ & \alpha > 1 \\ (1 - r_1 - r_2)^+ + (\alpha - r_1 - r_2)^+ \\ \quad +2\big(1 - (2 - \beta)^+ - r_1 - r_2\big)^+ & \alpha \le 1 \end{cases}$$

Proof: The proof is omitted due to space limitations. A detailed proof is provided in [11]. ■

C. Discussion

Corollary 1. *Consider the ICR defined in Section II. If* $\max\{1, \alpha\} \le \beta - 1$ *and* $\min\big\{2(1 - r_1)^+, 2(1 - r_2)^+\big\} \le 3(1 - r_1 - r_2)^+ + (\alpha - r_1 - r_2)^+$*, the optimal DMT region is*

$$d_{\textit{Opt-CF}}(r_1, r_2) = \min\big\{2(1 - r_1)^+, 2(1 - r_2)^+\big\}, \quad (5)$$

and it is achieved with CF at the relay.

Proof: The proof is based on Theorem 2. Note that if $\max\{1, \alpha\} \le \beta - 1$ and $\min\big\{2(1 - r_1)^+, 2(1 - r_2)^+\big\} \le 3(1-r_1-r_2)^+ + (\alpha-r_1-r_2)^+$, the achievable DMT region of the CF scheme is $\min\big\{2(1 - r_1)^+, 2(1 - r_2)^+\big\}$. This region coincides with the DMT outer bound derived in Theorem 1, leading to the characterization of the optimal DMT region. ■

Remark 2. Observe that the optimality holds for large multiplexing gains, i.e., for any $0 \le r \le 1$.

Remark 3. Note that if $\beta < \max\{\alpha, 1\} + 1$, CF is not DMT-optimal. Hence, the achievable DMT region of CF does not coincide with the DMT of the cut-set bound over *all* ranges of channel coefficients, *contrary* to the situation in the single-relay channel observed in [5]. In such scenarios, the assistance of the relay is not enough to support decoding both messages at each receiver.

Remark 4. Note that under the conditions of Corollary 1, (5) corresponds to the optimal DMT of two interference-free parallel relay channels. This can be seen by inspecting the cut-set bound for the relay channel: consider the relay channel in which the relationship between the channel inputs and its outputs is given by: $Y_1 = \sqrt{\text{SNR}}H_{11}X_1 + \sqrt{\text{SNR}^\beta}H_{31}X_3 + Z_1$, $Y_3 = \sqrt{\text{SNR}}H_{13}X_1 + Z_3$, where X_1 and X_3 denote the transmitted signals of the source and the relay, respectively, and Y_1 and Y_3 denote the received signals at the destination and at the relay node, respectively. The rest of the definitions remain the same as given in Section II. Let $\underline{H}_R = (H_{11}, H_{13}, H_{31})$. From [5, Eqns. (4), (5)], for a given realization $\underline{h}_r$ the capacity of the relay channel is upper-bounded by

$$C_{\text{Relay}} \le \max_{f(x_1,x_2)} \min\big\{I(X_1, X_3; Y_1|\underline{h}_r), I(X_1; Y_1, Y_3|X_3, \underline{h}_r)\big\}.$$

Following derivations similar to [9, Proof of Thm. 1], we further bound each expression. Define $|h_{kl}|^2 = \text{SNR}^{-\theta_{kl}}$, then

$$I(X_1; Y_1, Y_3|X_3, \underline{h}_r) \le \log\Big(1 + \text{SNR}|h_{11}|^2 + \text{SNR}|h_{13}|^2\Big) = \log\Big(1 + \text{SNR}^{1-\theta_{11}} + \text{SNR}^{1-\theta_{13}}\Big). \quad (6)$$

$$I(X_1, X_3; Y_1|\underline{h}_r) \le \log\big(\text{SNR}^{1-\theta_{11}} + 2\text{SNR}^{\frac{1-\theta_{11}+\beta-\theta_{31}}{2}} + \text{SNR}^{\beta-\theta_{31}} + 1\big). \quad (7)$$

Hence, the DMT of the relay channel is upper-bounded by

$$d^+_{\text{Relay}}(r) = \min\big\{2(1-r)^+, (1-r)^+ + (\beta - r)\big\}. \quad (8)$$

Next, recall that in Corollary 1 we have $\beta \ge 2$. Comparing $d^+_{\text{Relay}}(r)$ and $d_{\text{Opt-CF}}(r,r)$, we conclude that for $\beta \ge 2$, $d^+_{\text{Relay}}(r) = d_{\text{Opt-CF}}(r,r)$. Thus, under the conditions of Corollary 1, the optimal DMT of the ICR coincides with the outer bound on the optimal DMT of two interference-free parallel relay channels. Hence, *a single relay* employing the CF strategy in this situation is DMT optimal for *both* communicating pairs simultaneously. Furthermore, interference does not degrade the performance in this case.

Remark 5. From Theorem 2 it follows that when $\alpha \le 1$ the DMT achievable region using CF (when each receiver decodes both messages) is an increasing function of α, that is, increasing the interference between the communicating pairs enlarges the DMT region. On the other hand, if $\alpha > 1$, there are two cases:

- If $\beta \ge \alpha+1$, (4a) and (4b) do not depend on α, while (4c) increases with respect to α. We conclude that the DMT performance of CF improves as the interference becomes stronger.
- If $\beta < \alpha+1$ and $r_1+r_2 \le \beta-\alpha$, (4a)-(4c) decrease when α increases. Thus, increasing the interference decreases the achievable DMT of the CF strategy.

Note that for the regimes where $\alpha \le 1$ or $1 \le \alpha \le \beta - 1$, increasing α improves decoding the interference at the receivers and, hence, enhances the DMT performance. Similarly, it can be observed that the achievable DMT of CF is a non-decreasing function of β, which represents the strength of the links from the relay and the destinations. Thus, better relay-to-destinations links improve the DMT performance for the ICR.

Remark 6. Consider the Gaussian ICR defined in Section II. The maximum achievable diversity gain with CF relaying is:

$$D_{\text{CF}} = \begin{cases} 1 + \min\big\{(\beta-\alpha)^+, 1\big\} & \alpha > 1 \\ 1 + \min\big\{(\beta-1)^+, 1\big\} & \alpha \le 1. \end{cases}$$

In [5] it is shown that if $\beta = 1$, the maximum achievable diversity gain of the single-relay channel is 2. Note that in the ICR the same diversity gain is achieved if $\max\{2, \alpha+1\} \le \beta$. This is due to the fact that the achievable DMT in the ICR is affected by the interfering links, while in the single-relay channel there is no interference. Therefore, in order to achieve the same diversity gain, the relay-destination links in the ICR should be stronger than that in the single-relay channel.

Remark 7. The DMT region of the IC without a relay is studied in [7], where it is shown that an outer bound on the achievable DMT region of the IC is given by $\min\big\{(1 - r_1)^+, (1 - r_2)^+\big\}$. The optimal DMT can be achieved in certain regimes, e.g., in the very strong interference regime, characterized by $\alpha \ge 2$. Note that in the scenario considered in Corollary 1, the achievable DMT of ICR is twice the maximum achievable DMT for the IC. This observation supports employing relay nodes in wireless networks. Additionally, from Theorem 2 it follows that the DMT performance of the ICR is better than that of the IC also in the scenarios where the CF is not DMT optimal: for example, if $\max\{2, 1+\alpha\} \le \beta$ and $\min\big\{(1-r_1)^+, (1-r_2)^+\big\} \le 3(1-r_1-r_2)^+ + (\alpha-r_1-r_2)^+ \le \min\big\{2(1-r_1)^+, 2(1-r_2)^+\big\}$. This follows from the fact that, under these conditions, the optimal DMT region of the IC [7] is $d_{\text{Opt-IC}}(r_1, r_2) = \min\big\{(1-r_1)^+, (1-r_2)^+\big\}$, while for the ICR $d^-_{\text{CF}}(r_1, r_2) = 3(1 - r_1 - r_2)^+ + (\alpha - r_1 - r_2)^+$, and hence, $d_{\text{Opt-IC}}(r_1, r_2) \le d^-_{\text{CF}}(r_1, r_2)$.

IV. DMT with Only Rx-CSI at the Relay

A. DMT Outer Bound

Note that since lack of Tx-CSI can only decrease the performance, then for the ICR with only Rx-CSI at the relay as defined in Section II, the region $d^+(r_1, r_2)$ defined in Eqns. (1)-(2) is an outer bound on the DMT region.

B. An Achievable DMT Region via DF

We now present an achievable DMT for the ICR based on the DF strategy at the relay:

Theorem 3. *Consider the ICR with only Rx-CSI at the relay, as defined in Section II. An achievable DMT region is given by*

$$d^-_{DF}(r_1, r_2) = \begin{cases} \min\Big\{d^{\mathcal{E}}(r_1, r_2) + d^{Relay}, d^{\bar{\mathcal{E}}}(r_1, r_2)\Big\} & , r_1 + r_2 < 1 \\ d^{\mathcal{E}}(r_1, r_2) & , r_1 + r_2 \ge 1, \end{cases} \quad (9)$$

where

$$d^{Relay} = \min\Big\{(1-r_1)^+, (1-r_2)^+, 2(1-r_1-r_2)^+\Big\}$$

$$d^{\mathcal{E}}(r_1, r_2) = \min\Big\{(1-r_1)^+, (1-r_2)^+, (1-r_1-r_2)^+ + (\alpha - r_1 - r_2)^+\Big\}$$

$$d^{\bar{\mathcal{E}}}(r_1, r_2) = \min\Big\{(1-r_1)^+ + (\beta-r_1)^+, (1-r_2)^+ + (\beta-r_2)^+, (1-r_1-r_2)^+ + (\alpha-r_1-r_2)^+ + (\beta-r_1-r_2)^+\Big\}.$$

Proof: The proof is omitted due to space limitations. A detailed proof is provided in [11]. ■

C. Discussion

Corollary 2. *Consider the Gaussian ICR as defined in Section II. If the following inequalities are satisfied:*

$$r_1 + r_2 \le 1 \quad (10a)$$

$$\min\big\{(1-r_1), (1-r_2)\big\} \le \min\big\{2(1-r_1-r_2), (1-r_1-r_2) + (\alpha-r_1-r_2)^+\big\} \quad (10b)$$

$$\min\big\{(1-r_1) + (\beta-r_1)^+, (1-r_2) + (\beta-r_2)^+\big\} \le (1-r_1-r_2) + (\alpha-r_1-r_2)^+ + (\beta-r_1-r_2)^+ \quad (10c)$$

then the optimal DMT for the Gaussian ICR is

$$d_{Opt\text{-}DF}(r_1, r_2) = \min\big\{2(1-r_1)^+, 2(1-r_2)^+, (1-r_1)^+ + (\beta-r_1)^+, (1-r_2)^+ + (\beta-r_2)^+\big\}, \quad (11)$$

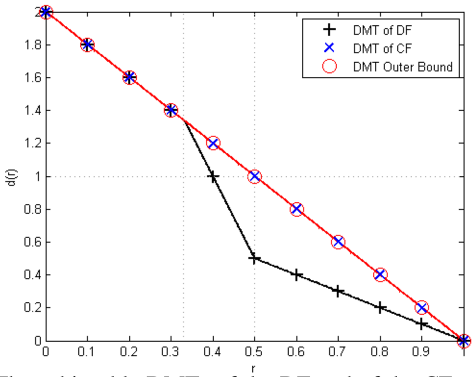

Fig. 2: The achievable DMTs of the DF and of the CF strategies for the ICR, for the scenario in which $\alpha = 4$ and $\beta = 5$.

and it is achieved using DF at the relay.

Proof: The proof is based on Theorem 3. Note that if (10) is satisfied, (9) coincides with (1), leading to the characterization of the optimal DMT for the ICR. ■

Remark 8. From Theorem 3 we observe that the achievable DMT of DF (when each receiver decodes both messages) increases as α and β increase, i.e., the DMT performance of the DF scheme improves as the relay-destination links and the interference become stronger. This is contrary to CF for which there are regimes of α and β in which increasing the interference decreases the rate.

Remark 9. Recall the upper bound on the DMT of the relay channel given in (8). Setting $r_1 = r_2 = r$, we note that under conditions (10), the optimal DMT (11) corresponds to the optimal DMT of two interference-free parallel relay channels, as each pair achieves a DMT of $\min\left\{2(1-r)^+, (1-r)^+ + (\beta - r)^+\right\}$, which coincides with the upper bound (8). We conclude that the DF strategy can also be DMT optimal for both communicating pairs simultaneously. Observe that optimality holds also for large multiplexing gains. Note that under CF this optimality was shown only for $\beta \geq \max\{1, \alpha\} + 1$; but with DF it applies for any value of $\beta \geq 0$ as long as (10) is satisfied. It is important to note that for DF and for CF this optimality is achieved over different multiplexing gains.

Remark 10. The maximum diversity gain achieved by the DF scheme is $D_{\text{DF}} = \min\{2, 1 + \beta\}$. Compared with the relay channel whose maximum diversity gain is 2, we conclude that the DF scheme achieves for each pair the maximum diversity gain of the relay channel as long as $\beta \geq 1$. Observe that this diversity gain is obtained for both pairs simultaneously, using only *a single* relay.

Remark 11. When DF is optimal, its DMT region outer bounds the optimal DMT region of the IC (for the same set of multiplexing gains). Note that this conclusion holds for any value of $\beta > 0$. Moreover, there are scenarios in which the DF achievable DMT for the ICR outperforms the optimal DMT of the IC even when DF is not optimal. One such example is when (11a) and (11c) are satisfied, while (11b) is not satisfied. For example, if we set $r_1 = r_2 = 0.4$, $\alpha = 1.8$, and $\beta = 1$, for the ICR we achieve a diversity gain of 1, while for the IC the achievable diversity gain is *upper-bounded* by 0.6.

Remark 12. Fig. 2 demonstrates the achievable DMT of the CF and the DF strategies for a symmetric scenario with $r_1 = r_2 = r$. The achievable DMT of the CF strategy is presented for the scenario in which the conditions of Corollary 1 are satisfied, i.e., when CF is optimal. Observe that indeed the DMT of CF coincides with the outer bound on the DMT of Theorem 1. The achievable DMT of DF is presented for both the case where the conditions for Corollary 2 are satisfied ($r \leq \frac{1}{3}$), and for the case in which DF is suboptimal ($r > \frac{1}{3}$). Note that for the values of $r > 0.5$ the outage event at the relay dominates the outage probability. For these values of r, we have $d^-_{DF}(r) = (1-r)^+$, corresponding to the optimal DMT of the symmetric IC.

V. Summary

We have studied the DMT region of single-antenna Gaussian ICRs. We derived an outer bound on the DMT region of the ICR using the cut-set theorem, and two achievable DMT regions based on DF and CF. Additionally, we derived conditions on the channel coefficients to achieve optimal DMTs with CF and with DF. We also identified situations in which the maximum diversity gain of the ICR, obtained using either CF or DF, is equal to the diversity gain of two parallel interference-free relay channels. These results give a strong motivation for employing relay nodes in multi-user wireless networks that have to deal with interference.

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