Approaching Multiple-Access Channel Capacity by Nonbinary Coding-Spreading

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***Abstract*—As a generalization of the binary coding-spreading scheme, nonbinary coding-spreading scheme is proposed for a synchronous binary-input multiple-access channel (MAC) with Gaussian noise, equal-power, and equal-rate users. In this scheme, each user employs the same nonbinary low-density parity-check code serially concatenated with a nonbinary low-rate mapping, referred to as nonbinary spreading. A user-specific interleaving is employed to make the transmitted data of each user random-like. It is shown that the iterative multi-user decoding threshold of nonbinary coding-spreading scheme is less than 0.5 dB away from the MAC capacity at many sum rates.**

I. Introduction

In a multiple-access channel (MAC), multiple users communicate with a single receiver over a common channel. The signal of each user suffers both the noise and interference from signals of other users. The MAC capacity region was found by Ahlswede [1] and Liao [2], while practical coding scheme to approach a general point of this region is still not presented.

Many works consider the MAC with equal-power and equal-rate users, where only a sum rate of all users is concerned. For the MAC with very few number of users, such as two users, works of Amraoui *et al*. [3], Kudekar *et al*. [4], and Yedla *et al*. [5] showed that the sum rate approaches the MAC capacity when each user employs a low-density parity-check (LDPC) code. A common coding scheme, that is used in the MAC with large number of users, is a channel code serially concatenated with a spreading. In this scheme, the channel code is mainly to combat noise and the spreading is mainly to combat user interference, such as in the coded code-division multiple-access (CDMA) and coded interleave-division multiple-access (IDMA). Shi and Schlegel [6] analyzed the performance of serially concatenated coded CDMA, and in their results the required signal-to-noise ratio (SNR) is more than 3 dB away from that of the MAC capacity for the given sum rates. The IDMA, which uses different interleaving as the only way for user separation, has been shown to outperform the conventional CDMA in many aspects [7] [8] [9]. Li *et al.* [10] considered low-rate Turbo-Hadamard coded IDMA, and the required SNR at bit-error-rate (BER) 10^{-5}, by simulation for 35 users at sum rate 0.5, is 1.4 dB away from that of the MAC capacity. Our previous work [11] shown that the iterative decoding threshold of regular repeat-accumulate (RA) coded IDMA is about 2 dB away from the MAC capacity for the sum rate less than 2 with arbitrary number of users. Note that above works only considered the binary coding and binary spreading scheme.

Nonbinary code, such as the nonbinary LDPC code proposed by Mackey and Davey [12], has been shown to outperform the binary code in single-user communication. Kasai *et al*. [13] proposed a low-complexity low-rate multiplicatively repeated nonbinary LDPC code with decoding threshold about 0.5 dB away from the capacity of single-user AWGN channel. In our previous work [14], we proposed a nonbinary spreading scheme of finite field spreading for the MAC and showed its signal-to-interference-and-noise ratio (SINR) amplification advantage over the binary spreading scheme under iterative detection by an extrinsic information transfer (EXIT) analysis. In fact, the multiplicatively repeated nonbinary LDPC code in [13] is a serial concatenation between a nonbinary LDPC code and the finite field spreading for single-user case. Works [13] and [14] motivates us to extend nonbinary coding and spreading to the Gaussian MAC.

In this paper, we propose a *nonbinary* coding-spreading scheme for the K-user synchronous binary-input MAC with Gaussian noise, equal-power, and equal-rate users. In our scheme, each user employ a nonbinary LDPC code serially concatenated with a nonbinary spreading, realized by a low-rate mapping. User-specific interleaving is employed to make the transmitted data of each user random-like. At the receiver, the message of each user is jointly recovered by an iterative multi-user decoding. We give an EXIT chart analysis to obtain the multi-user decoding threshold. We show that even with a random spreading, the decoding threshold of the nonbinary coding-spreading scheme is less than 0.5 dB away from the MAC capacity at many sum rates. Moreover, we solve a nonbinary coding-spreading trade-off problem, where the optimal nonbinary LDPC code rate and the nonbinary spreading length is obtained by our EXIT chart analysis.

II. Nonbinary Coding-Spreading Scheme

In this section, we give the nonbinary coding-spreading scheme for the K-user MAC.

Fig. 1 illustrates the transmitter of our K-user nonbinary coding-spreading scheme. Let $\{\alpha_i^k, i = 1, ..., N\}, \alpha_i^k \in \mathrm{GF}(q), k = 1, ..., K$, over finite field $\mathrm{GF}(q), q = 2^s$, be the length-N information vector of user k. This information vector is first encoded by a rate-R q-ary LDPC encoder, and the output codeword is denoted as $\{\beta_i^k, i = 1, ..., n\}, n = \frac{N}{R}, \beta_i^k \in$

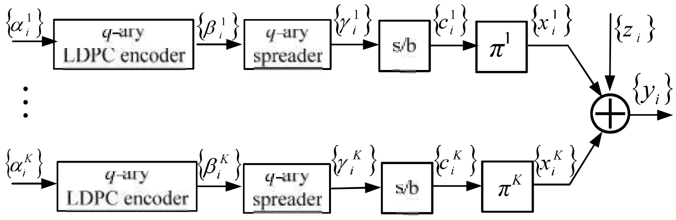

Fig. 1. K-user q-ary coding-spreading transmitter.

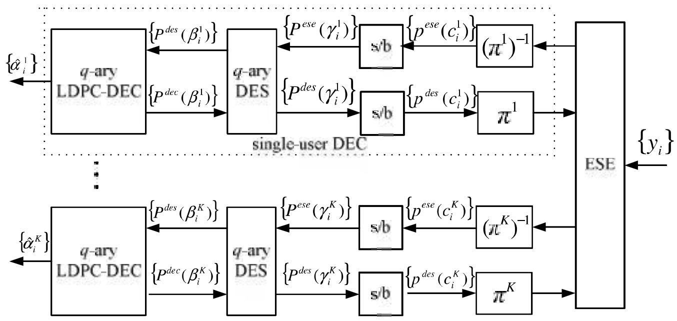

Fig. 2. Iterative K-user decoder.

GF(q). A q-ary spreading with length L is then performed on codeword $\{\beta_i^k, i = 1, ..., n\}$ to generate the field chip vector $\{\gamma_i^k, i = 1, ..., nL\}$ with length nL. This nonbinary spreading is a low-rate mapping $\mathcal{M} : \mathrm{GF}(q) \to \mathrm{GF}(q)^L$, where each field symbol is exclusively mapped into a length-L vector over the same field. Note that the multiplicative repetition [13] and finite field spreading [14] is a special case of the nonbinary spreading. Field vector $\{\gamma_i^k, i = 1, ..., nL\}$ is transformed into length-(snL) bit vector $\{x_i^k, i = 1, ..., snL\}$ by mapping $\lambda : \mathrm{GF}(q) \to \mathcal{X}^s, \mathcal{X} \triangleq \{1, -1\}$, where each field symbol over GF(q) is mapped into s bits. User-specific interleaving π^k is performed to generate the transmitted vector $\{x_i^k, i = 1, ..., snL\}$ to the Gaussian MAC. The user-specific interleaving is to make the transmitted data of each user random-like or to make the data of each user separable at the receiver as in the IDMA system [7] [8]. Note that the nonbinary LDPC encoding and the nonbinary spreading can be the same for each user while the user-specific interleaving should be different for each user.

III. Iterative Multi-User Decoding and EXIT Chart Analysis

In this section, we first give an iterative multi-user decoding scheme of the nonbinary coding-spreading, then we give an EXIT chart analysis and solve a nonbinary coding-spreading trade-off problem.

A. Iterative Multi-User Decoding

The receiver receives a superimposed signal vector $\{y_i,\ i = 1, 2, ..., snL\}$ of the K users with

$$y_i = \sum_{k=1}^{K} x_i^k + z_i \tag{1}$$

where $x_i^k \in \mathcal{X}$ and z_j is a zero-mean Gaussian variable with a variance of σ^2. Iterative multi-user decoding is performed to recover information vectors $\{\alpha_i^k, i = 1, ..., snL\}, k = 1, ..., K$.

Fig. 2 illustrates our iterative K-user decoder. This K-user decoder consists of $2K + 1$ component decoders, i.e., an elementary signal estimation (ESE), K q-ary despreaders (DES), and K q-ary LDPC decoders (DEC). The whole decoding is accomplished by these $2K + 1$ component decoding and interactions. Note that interleavers π^k, deinterleavers $(\pi^k)^{-1}, k = 1, ..., K$, and bit-probability/symbol-probability convertor s/b are not regarded as component decoders here. At each decoding iteration, each component decoder acts once and outputs an extrinsic message, which is as a priori message for its neighbouring component decoder at next iteration. This extrinsic message is usually a probability message about the bit or field symbol associated to the component decoder. In the following, we briefly introduce the operation of the ESE, q-ary DES, q-ary DEC, and bit-probability/symbol-probability convertor s/b.

1) ESE: Based on the deinterleaved extrinsic message from the DES and the received signal, the ESE gives an estimation for each transmitted bit of each user. This ESE method is proposed in [7] and [8], where the superimposed signal of the other $K - 1$ users is regarded as Gaussian variable during the the signal estimation of one user.

2) q-ary DES: Based on the extrinsic message from the ESE and DEC, the DES performs a component decoding to feed back a new message to the DEC and ESE, respectively.

Let $P^{dec}(\beta_1^k = \varpi), \varpi \in \mathrm{GF}(q)$, be the symbol probability of $\beta_1^k = \varpi$ from the DEC and $P^{ese}(\gamma_i^k = \varpi), \varpi \in \mathrm{GF}(q), i = 1, ..., L$, be the symbol probability about γ_1^k from the ESE with $\mathcal{M}(\beta_1^k) = (\gamma_1^k, ..., \gamma_L^k)$. The extrinsic probability output of the DES about β_1^k to the DEC is

$$P^{des}(\beta_1^k = \varpi) = \prod_{i=1}^{L} P^{ese}(\gamma_i^k = \mathcal{M}_i(\varpi)), \varpi \in \mathrm{GF}(q) \tag{2}$$

where $\mathcal{M}_i(\varpi)$ denotes the i-th element of $\mathcal{M}(\varpi)$.

The extrinsic probability output of the DES about γ_ℓ^k to the ESE is calculated as

$$P^{des}(\gamma_\ell^k = \varpi) = \sum_{\varpi' \in \mathrm{GF}(q), \mathcal{M}_\ell(\varpi') = \varpi} P^{dec}(\beta_1^k = \varpi') \prod_{i=1, i \neq \ell}^{L} P^{ese}(\gamma_i^k = \mathcal{M}_i(\varpi')), \quad \varpi \in \mathrm{GF}(q). \tag{3}$$

3) q-ary DEC: Based on the extrinsic probability from the DES, a q-ary LDPC decoding is performed and outputs a new probability message to the DES. We use the fast Fourier transform based q-ary sum-product algorithm (FFT-QSPA) proposed in [15] to decode the q-ary LDPC code.

4) s/b: Since extrinsic probability from the ESE after the deinterleaving is a bit probability, this message should first be converted to field symbol probability, which is used in the q-ary DES. As an example, let $p^{ese}(c_i^k = a), i = 1, ..., s, a \in \mathcal{X}$, be the bit probability of $c_i^k = a$ from ESE after deinterleaving. The corresponding field symbol probability with $\lambda(\gamma_1^k) = (c_1^k, ..., c_s^k)$

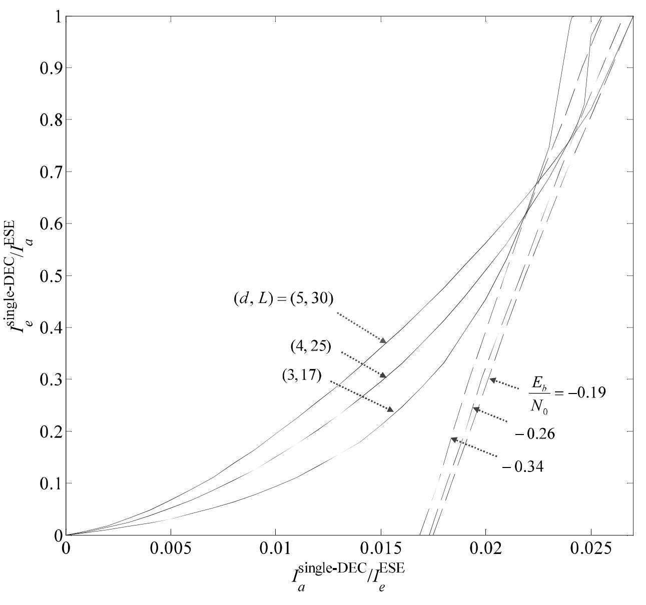

Fig. 3. EXIT chart of 15-user 64-ary coding-spreading with sum rate 0.3, i.e., $(d, L) = (3, 17), (4, 25)$, and $(5, 30)$.

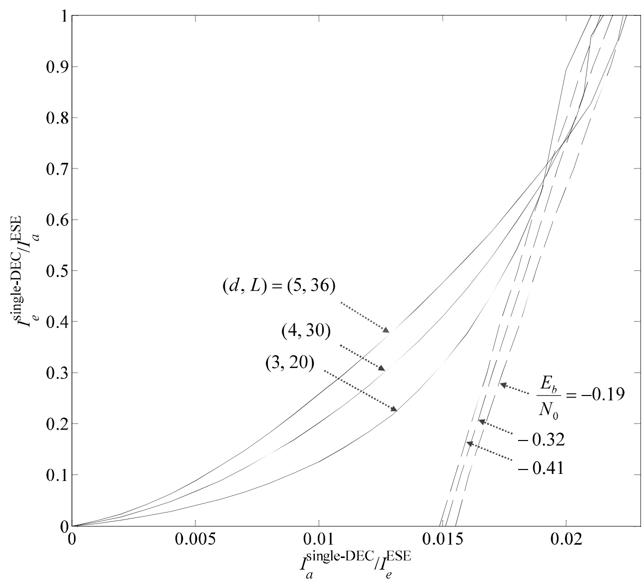

Fig. 4. EXIT chart of 15-user 64-ary coding-spreading with sum rate 0.25, i.e., $(d, L) = (3, 20), (4, 30)$, and $(5, 36)$.

are

$$P^{ese}(\gamma_1^k = \varpi) = \prod_{i=1}^{s} p^{ese}(c_i^k = \lambda_i(\varpi)), \varpi \in \mathrm{GF}(q) \tag{4}$$

where $\lambda_i(\varpi)$ is the i-th bit of $\lambda(\varpi)$.

With the same reason, the field symbol probability from the DES should be converted to bit probabilities, which are interleaved as priori message of the ESE, i.e., the probability of γ_1^k is converted to bit probability as

$$p^{des}(c_i^k = a) = \sum_{\varpi \in \mathrm{GF}(q), \lambda_i(\varpi) = a} P^{des}(\gamma_1^k = \varpi), i = 1, ..., s, a \in \mathcal{X}. \tag{5}$$

Note that there should be a normalization processing for each of the probability calculated in (2-5) in the practical decoding.

B. EXIT Chart Analysis

The EXIT chart describes the relation between the statistical priori input and the extrinsic output of a decoder, both of the input and output are measured by a mutual information. Moreover, the EXIT chart analysis is based on assumptions of infinite code length and the Gaussian approximation [16].

The original EXIT chart method is used to judge the convergence of iterative decoding between two decoders. By plotting their EXIT charts in the same graph, if there exists a tunnel between these two EXIT charts, the iterative decoding is error free, otherwise, a decoding error occurs. To use this method, we consider the K-user decoder as concatenation between the ESE and K single-user decoders. Since the decoding for each user is the same, we only need to track the mutual information of the decoder for user k. We plot the EXIT charts for both the ESE and the single-user decoder (single-DEC) in the same graph and judge the convergence of the whole iterative K-user decoding.

An EXIT chart of the ESE based on Monte Carlo simulation is given in our previous work [11]. Since there is still no tractable theoretical analysis for the nonbinary decoder over large finite field, we obtain the EXIT chart of single-DEC by Monte Carlo simulation. To obtain the EXIT chart of the single-DEC, we consider the $(2, d)$-regular nonbinary LDPC codes followed by length-L random spreading, i.e., mapping $\mathcal{M}$ for spreading is random generated. Note that the $(2, d)$-regular q-ary LDPC codes are empirically known as the best performing codes for $q \geq 64$. We perform practical decoding for single-DEC with information bit length 12000. It should be clarified that the location of non-zero elements in the parity check matrices of the LDPC code used here are generated by the progressive edge-growth algorithm proposed by Hu *et al.* [17]. Each non-zero element is selected randomly and uniformly from GF(q). The number of decoding iteration between the q-ary LDPC DEC and the q-ary DES is 100. Note that we need not to employ the user-specific interleaving during obtaining the EXIT chart of single-DEC.

Figs. 3 and 4 illustrate the EXIT charts of (K = 15)-user (q = 64)-ary coding-spreading scheme with sum rate of $\frac{KR}{L} = \frac{K}{L}(1 - \frac{2}{d}) \approx 0.3$ and 0.25, respectively. In Fig. 3, we first plot the EXIT chart of single-DEC for three combinations of $(d, L) = (3, 17), (4, 25)$, and $(5, 30)$, all of which (approximately) give the sum rate of 0.3 in the 15-user MAC. Then we plot the EXIT chart of the ESE under the critical bit-energy-to-noise ratio $\frac{E_b}{N_0} = \frac{L}{2R\sigma^2}$ that have no intersection with the three EXIT charts of single-DEC, respectively. We see that in Fig. 3, the critical values of $\frac{E_b}{N_0}$ are $-0.34, -0.26$, and -0.19 dB, which are in fact the corresponding multi-user decoding thresholds, i.e., the minimum $\frac{E_b}{N_0}$ under which the decoding is error free. We emphasize that they are only $0.31, 0.39$ and 0.46 dB away from Shannon bound -0.65 dB for the sum rate of

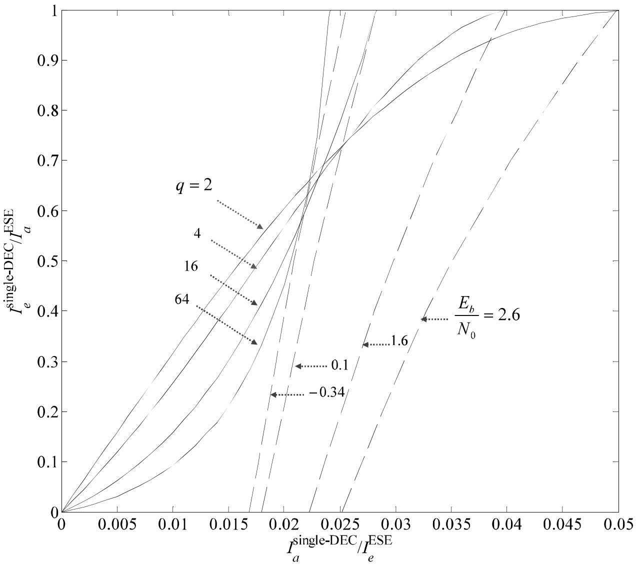

Fig. 5. EXIT chart of 15-user q-ary coding-spreading of $(d, L) = (3, 17)$ with $q = 2, 4, 16,$ and 64.

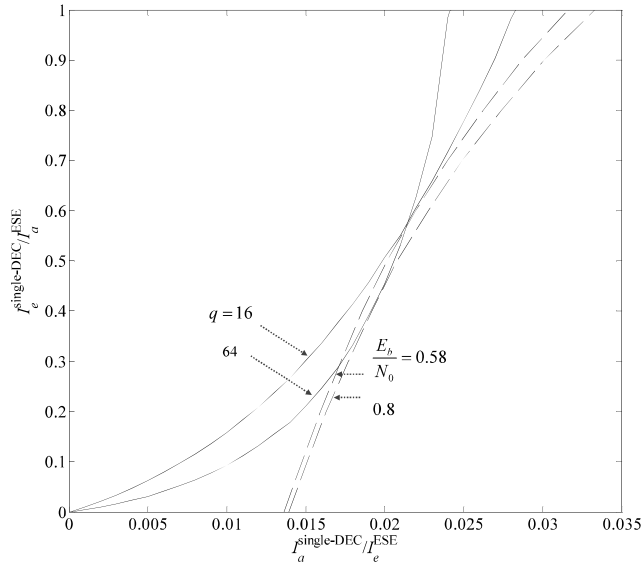

Fig. 6. EXIT chart of 30-user q-ary coding-spreading of $(d, L) = (3, 17)$ with $q = 16$ and 64.

0.3. Moreover, $(d, L) = (3, 17)$ gives the optimal combination of the LDPC code rate and the spreading length. Similarly, in Fig. 4, $(d, L) = (3, 20), (4, 30)$, and $(5, 36)$ hold decoding thresholds $-0.41, -0.32$, and -0.19 dB, which are $0.41, 0.5$, and 0.63 dB away from Shannon bound -0.82 dB for the sum rate of 0.25. Pair $(d, L) = (3, 20)$ gives the optimal combination of the LDPC code rate and the spreading length.

To show the effect of the finite field order on the multi-user decoding threshold. Fig. 5 illustrates the EXIT charts for 15-user q-ary coding-spreading scheme of $(d, L) = (3, 17)$ with $q = 2, 4, 16,$ and 64. We see that the decoding threshold improves as q increases. Note that for $q = 2$ our scheme deduces to the conventional binary coding-spreading scheme. Another example is for 30-user case with $q = 16$ and 64 in Fig. 6. It is interesting that $q = 16$ provides better threshold of 0.58 dB, which is only 0.28 dB away from the Shannon bound of 0.3 dB for the sum rate of 0.588.

IV. BER Simulation

In this section, we verify our EXIT chart analysis above by BER Monte Carlo simulations of multi-user nonbinary coding-spreading systems.

We consider the 15-user 64-ary coding-spreading with $(d, L) = (3, 17)$ and $(3, 20)$ that give the optimal coding-spreading trade-off. The information bit length is also fixed to 12000. The parity matrix of LDPC code and mapping $\mathcal{M}$ for nonbinary spreading are generated using the same method as mentioned above. All the user-specific interleavings are random generated. The overall multi-user decoding iteration number is 50. We see that in Fig. 7 the gap between the BER curve and the decoding threshold obtained by our EXIT chart analysis is about 0.6 and 0.77 dB at BER$= 10^{-5}$ for the given information bit length, respectively. Note that due to the lower

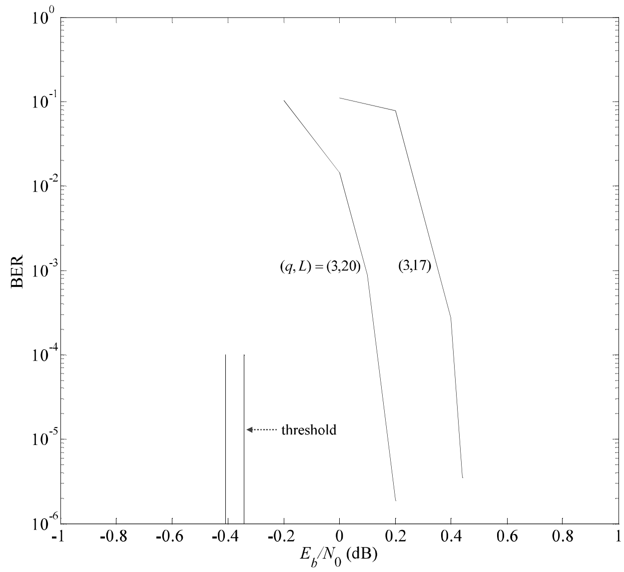

Fig. 7. BER simulation for 15-user 64-ary coding-spreading with $(d, L) = (3, 17)$ and $(3, 20)$.

code rate, the total code length of $(3, 20)$ is larger than that $(3, 17)$ even for the same information bit length.

V. Conclusion

In this paper, we proposed a nonbinary coding-spreading scheme for the MAC with Gaussian noise. We give an EXIT chart analysis to obtain the multi-user decoding threshold. Although we only considered a random spreading, the iterative multi-user decoding threshold of our scheme is within less than 0.5 dB away from the MAC capacity at many sum rates.

There still remains some interesting topics for future investigation, such as design of the nonbinary spreading and design of the field order for a given number of users and sum rate.

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