Cooperative Unicasting in Large Wireless Networks

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***Abstract*—This paper investigates the potential gains of half-duplex unicast strategies in a large wireless network consisting of clusters in which a source node attempts to transmit a message with the aid of other inactive –potential relays– nodes within the cluster. The network is modeled as an independently marked Poisson point process and a slow-fading scenario is considered. A two-phase transmission protocol is studied and an achievable (upper) bound on the asymptotic error probability of any cluster in the network is derived in terms of the outage probability (OP). The proposed scheme takes advantage of the spatial distribution of the cooperating nodes to create a distributed virtual antenna array, thus improving the OP over direct transmission (DT) through cooperative diversity. Finally, we obtain a converse (lower) bound on the OP of any unicast strategy when restricting all admissible protocols to belong to the same –still quite general– class of half-duplex codes that includes the one studied.**

I. Introduction

Large wireless networks and their ultimate limits of performance have been a widely studied subject [1]–[3] with enormous relevance in the field of information theory and communications. As present and future networks will have to cope with increasing traffic demands, advanced strategies which provide an optimal use of resources such as bandwidth and power should be employed. Over the past decade there has been a great interest in cooperative networks [4], [5] where relay nodes can be exploited as a mean to increase throughput and reliability, through the use of "cooperative diversity".

The information-theoretic research on this topic was mainly focused on simple networks with few nodes or fixed topologies where perfect channel state information (CSI) is available to all terminal nodes. Finding explicit capacity regions of large networks may be –if feasible– very hard. To tackle this limitation, spatial models employing tools from stochastic geometry and graphs provide a comprehensive framework for the analysis of large wireless networks with little interference management [6]–[8]. In this setup, the interference between users is treated as noise whose statistical properties depend on the particular spatial distribution of the nodes and fading realizations of the wireless paths. The tools provided by stochastic geometry build up in a natural framework which permits to combine all the above mentioned effects in order to obtain a comprehensive bound on the asymptotic error probability experienced by a typical user in the network. This is done through the computation in closed form of the "outage probability" (OP) [9] experienced by a typical user.

In this paper, we focus on the set of half-duplex unicast strategies which have been first proposed in [10] and then further investigated in [11] assuming full-duplex networks. That is, strategies which apply to a network in which a source node attempts to transmit a message to a destination with the aid in a half-duplex fashion of several nearby inactive transmitters. More specifically, we study achievable (upper) and converse (lower) bounds –in terms of the OP– on the asymptotic error probability of a large network formed by clusters in which a source attempts to transmit a message with the aid of K randomly deployed nearby inactive nodes.

Our network is modeled as an independently marked Poisson point process, limited by the signal-to-interference ratio (SIR) where signal attenuation occurs both through path loss and Rayleigh fading. We assume that CSI is only available at the receiver-side of each link, which is a reasonable assumption in many wireless scenarios without feedback. An outage event is declared whenever the distribution of nodes or fading causes the target transmission rate R to be higher than the achievable rate for the unicast strategy. The probability of these events (OP) becomes an achievable (upper) bound on the asymptotic error probability of any cluster in the network.

This paper has two main contributions. First, the OP for the proposed unicast strategy is derived in a closed form in terms of the Laplace transforms [12] of the interference random variables at the receivers. The expressions derived allow to compare the performance of this strategy with respect to the baseline scheme of direct transmission (DT), showing interesting gains even when only a few cooperating nodes are considered. Finally, a converse (lower) bound on the OP of any unicast strategy is derived when restricting all admissible protocols to belong to the same –still quite general– class of half-duplex codes that includes the one studied.

II. System model and protocols

We consider a large planar random wireless network formed by clusters, each consisting of a source node and $K+1$ receiver nodes labelled with $\{1,\dots,K+1\}$. Each source node attempts to transmit a message to the $(K+1)$-th node inside its cluster, with the aid of the others $\{1,\dots,K\}$ receivers acting as relays.

We assume that each cluster uses the same protocol and operates independently of the others, without any kind of cooperation or coordination. We restrict ourselves to protocols functioning in a half-duplex fashion. Basically the source node selects one of $M \equiv 2^{nR}$ possible messages and starts transmitting a codeword associated with it over n channel uses. The K receiving nodes listen to this transmission and, depending on the signals they receive and the CSI, can decide to start transmitting for the remainder $(n - n_k)$ channel uses with $k = \{1, \ldots, K\}$, where n_k is the time instant in which the k-th node decides to start its transmission. The source node as well as all nodes that decide to enter in the transmitting state do not revert to a listening one, that is, once a node decides to transmit, it remains in that state until the end with n.

We will consider a narrowband slow-fading model where the channel gains between each pair of nodes remain fixed during the whole transmission interval and that perfect CSI is available only at the receiving nodes. For the signal attenuation model we consider that the power received at y by a transmitter at x transmitting with power P is $P|h_{xy}|^2 l(x, y)$ where $l(x, y) = \|x - y\|^{-\alpha}$ ($\alpha > 2$) is the usual path loss function and $|h_{xy}|^2$ is the power gain of Rayleigh fading with unit mean. For shortness we will write $l(y) \equiv l(0, y)$.

Similarly to [10], the hypotheses over the cluster dynamics lead to the following formal definition of the universe of admissible protocols $\mathcal{P}$ that are allowed.

Definition 2.1: An admissible protocol $p \in \mathcal{P}$ consists on:

- A set of indicator functions $\{A_k(l)\}_{k=0}^{K} \in \{0, 1\}$, $l = 1, \ldots, n$ which determines if node k is transmitting or receiving and such that $A_k(m) = 1$ for $l \leq m \leq n$ if $A_k(l) = 1$ for $k = 0, \ldots, K$. We will assume that $A_0(l) = 1$, $l = 1, \ldots, n$.
- A set of encoding functions $\left\{f_k(l) : \mathcal{Y}_k^{l-1} \to \mathbb{C}\right\}_{k=1}^{K}$, $l = 1, \ldots, n$ at the relay nodes which give the symbol that node k transmits at channel use l and where $\mathcal{Y}_k^{l-1}$ denotes the channel outputs at node k until time $l-1$. Associated with the source node we have an encoding function $f_0(l) : [1 : 2^{nR}] \to \mathbb{C}$, $l = 1, \ldots, n$ which maps the source messages to appropriate codewords.
- A decoding function at the destination $g : \mathcal{Y}_{K+1}^{n} \to [1 : 2^{nR}]$.

We will consider that the source nodes transmit with power P and that the receiving nodes $k = 1, \ldots, K$ transmit with a joint power constraint:

$$\mathbb{E}\left[\frac{1}{n}\sum_{l=1}^{n}\sum_{k:A_k(l)=1}^{K}|X_k(l)|^2\right] \leq P, \tag{1}$$

where $X_k(l)$ is the symbol transmitted by node k at time l and where the expectation is with respect to the ensemble of channel codes and realizations.

The network is modeled as an independently marked Poisson point process (PP) [13]:

$$\tilde{\Phi} = \{(x_i, b_{x,1}, \ldots, b_{x,K}, \{|h_{i,n}|^2\}_n)\}, \tag{2}$$

where:

- The set of sources constitutes a homogeneous Poisson point process $\Phi = \{x_i\}$ of intensity λ on $\mathbb{R}^2$.
- $\{b_{x,1}, \ldots, b_{x,K}\}$ represent the positions of the K receivers associated to the user at x, which are independently and identically distributed (i.i.d.) around this user. Hence, the sources act as the centers of each cluster.
- An additional source with the same marks as the others, independent of the point process $\tilde{\Phi}$ is added at $r_0 = 0$ with its receivers located at $\{r_1, \ldots, r_K\}$ with the same distribution as other clusters from the process. The destination of this "typical" cluster is located at r_{K+1}.
- The set of marks $\{|h_{i,n}|^2\}_n$ model the power fading coefficients from each source and each receiver in each cluster, to the receivers $\{r_1, \ldots, r_{K+1}\}$ in the typical cluster. In addition for the typical cluster, we define the power fading coefficients $\{|h_{0,1}|^2, \ldots, |h_{0,K+1}|^2\}$ from the source to each receiver and $\{|h_{1,K+1}|^2, \ldots, |h_{K,K+1}|^2\}$ from each receiver to the destination of the message.

III. Achievability: A Two-Phase Cooperative Protocol for Unicasting

In this section we introduce a transmission scheme based on the one presented in [10] and analyse its performance in terms of the OP for the network we introduced in the previous section. The protocol employs a decode-and-forward strategy and has two phases: in phase 1, the source broadcasts its message over n_1 channel uses at a rate R_1, and all nodes, including the destination, attempt to decode the message. In the second phase, the users which were able to decode the message in the first phase, together with the source, jointly transmit the message to the destination over $n_2 = n - n_1$ channel uses at rate R_2. An outage is declared if the destination is not able to decode its message after the first and the second phase.

Codebook Generation: the source generates M codewords of length n_1 in an i.i.d. fashion drawn as $\mathcal{CN}(0, P)$, and M codewords of length n_2 in the same fashion. The other nodes independently generate M codewords of length n_2 drawn in an i.i.d fashion with distribution $\mathcal{CN}(0, P_2)$. The codes are revealed to all nodes and to the destination.

Phase 1: The source selects a message and transmits its corresponding codeword during the first n_1 channel uses. This is done using a rate R_1 and average power P. All nodes attempt to decode the message from the source (including the intended destination) using typicality decoding.

Phase 2: In the following n_2 channel uses, the subset of the $\{1, \ldots, K\}$ users which could decode the message in the first phase jointly transmit the message to the destination using a rate R_2. The source repeats the same message using its corresponding codeword of length n_2. If the destination is able to decode the message in the first phase, it remains inactive in the second phase. Otherwise it continues to listen to attempt to decode the message in the second phase. The destination attempts to decode the message using typicality decoding in order to determine the message jointly with the subset of nodes that were able to decode the message in the first phase, as pointed out in [10].

If the overall rate of the protocol is R so there are $M = 2^{nR}$ messages, then the pairs (n_1, R_1) and (n_2, R_2) must satisfy

$n_1R_1 = n_2R_2 = \log M = nR$, and since $n = n_1 + n_2$ we have that the attempted rate R satisfies:

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}. \tag{3}$$

The value of P_2 is chosen such that:

$$P_2 \leq \frac{P}{L}, \tag{4}$$

where $L = \mathbb{E}\left[\sum_{k=1}^{K} A_k(n_2)\right]$ is the average number of nodes that successfully decode the message after the first phase. It is easily seen that the constraint (1) is satisfied with this choice.

A. Interference Powers

In the first phase only the sources transmit, so the interference at the i-th receiver during this phase reads as:

$$I_1(r_i) = \sum_{x\in\Phi} P|h_{x,r_i}|^2 l(x, r_i). \tag{5}$$

An outage event for the i-th receiver will take place in the first phase if:

$$\text{SIR}_1(r_i) = \frac{P|h_{0,i}|^2 l(r_i)}{I_1(r_i)} < T_1 \tag{6}$$

where $T_1 = 2^{R_1} - 1$. To evaluate the OP of the protocol we introduce the assumption that the nodes of the typical cluster see the interference coming from the other clusters as a point source of interference. In the second phase the nodes that have already decoded in the first phase transmit together with the source, so the interference at the i-th receiver would be:

$$I_2(r_i) = \sum_{x\in\Phi}\left(P_2\sum_{i=1}^{K} \mathbb{1}\{\text{SIR}_1(r_i) < T_1\}|h_{x,b_i,r_i}|^2 + P|h_{x,r_i}|^2\right) l(x, r_i). \tag{7}$$

To simplify the evaluation of the OP in (7) we bound $\mathbb{1}\{\text{SIR}_1(r_i) < T_1\} \leq 1$ which leads to an upper bound on the interference power for the second phase. We still write $I_2(r_i)$ but bearing in mind that we are considering its upper bound. Thus for the second phase the outage event at the destination is $\{\text{SIR}(r_{K+1}) < T_2\}$ where $T_2 = 2^{R_2} - 1$,

$$\text{SIR}_2(r_{K+1}) = \frac{P|h_{0,K+1}|^2 l(r_{K+1}) + \sum_{i=1}^{K_1} P_2|h_{i,K+1}|^2 l(r_i, r_{K+1})}{I_2(r_{K+1})} \tag{8}$$

and K_1 is the number of users which decode the message at the end of the first phase.

B. Outage probability

Let $\mathcal{F} = \{\text{SIR}_2(r_{K+1}) < T_2\}$ be the event that the destination does not decode the message at the end of the second phase and for $k_1 \in \{0, \ldots, K\}$ let $\mathcal{A}_{k_1} = \{K_1 = k_1\} \cap \mathcal{F}$. The message is not properly received by the destination if $K_1 \leq K$ and $\mathcal{F}$ takes place. The outage event is therefore:

$$\mathcal{E} = \bigcup_{k_1=0}^{K} \mathcal{A}_{k_1}. \tag{9}$$

To evaluate the OP we introduce the single and joint Laplace transforms of the interference random variables at the receivers:

$$\mathcal{L}_{I_1(\mathbf{r})}(\boldsymbol{\omega}) = \mathbb{E}\left[e^{-\sum_{i=1}^{K+1}\omega_i I_1(r_i)}\right]$$

$$\mathcal{L}_{I_1(\mathbf{r}),I_2(r_{K+1})}(\boldsymbol{\omega}, \omega_{K+2}) = \mathbb{E}\left[e^{-\sum_{i=1}^{K+1}\omega_i I_1(r_i) + \omega_{K+2} I_2(r_{K+1})}\right]$$

where $\mathbf{r} = (r_1, \ldots, r_{K+1})$ are the locations of the receivers and $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_{K+1})$. To simplify the notation we will write:

$$\mathcal{L}_{I_1(\mathbf{r})}(\boldsymbol{\omega}) \equiv \mathcal{L}_{I_1}(\omega_i) \tag{10}$$

$$\mathcal{L}_{I_1(\mathbf{r}),I_2(r_{K+1})}(\boldsymbol{\omega}, \omega_{K+2}) \equiv \mathcal{L}_{I_1,I_2}(\omega_i, \omega_{K+2}). \tag{11}$$

Theorem 3.1 (OP of the considered protocol): The OP of the two-phase above described protocol (considering the hypotheses of section III-A) is:

$$\begin{aligned}\mathbb{P}(\mathcal{E}) = \sum_{k_1=0}^{K}\binom{K}{k_1}\mathbb{E}_{\mathbf{r}}\Bigg[&\mathcal{L}_{I_1}\left(\frac{T_1\mathbb{1}\{i<k_1\}}{P_1 l(r_i)}\right) \\ &- \sum_{m=1}^{k_1} A_m \mathcal{L}_{I_1,I_2}\left(\frac{T_1\mathbb{1}\{i<k_1\}}{P_1 l(r_i)}, \frac{T_2}{P_2 l(r_m, r_{K+1})}\right) + \\ &\sum_{\mathcal{I}\in\hat{\mathcal{G}}(k_1,K)} (-1)^{\#\mathcal{I}-k_1}\Bigg(\mathcal{L}_{I_1}\left(\frac{T_1\mathbb{1}\{i\in\mathcal{I}\}}{P_1 l(r_i)}\right) \\ &- \sum_{m=1}^{k_1} A_m \mathcal{L}_{I_1,I_2}\left(\frac{T_1\mathbb{1}\{i\in\mathcal{I}\}}{P_1 l(r_i)}, \frac{T_2}{P_2 l(r_m, r_{K+1})}\right)\Bigg)\Bigg],\end{aligned} \tag{12}$$

where $\mathcal{G}(k_1, K)$ is the set of all non-empty subsets of $\{k_1 + 1, \ldots, K + 1\}$ and:

$$\hat{\mathcal{G}}(k_1, K) = \{\{1, \ldots, k_1\} \cup \mathcal{D} : \mathcal{D} \in \mathcal{G}(k_1, K)\}, \tag{13}$$

and:

$$A_m = \prod_{\substack{l=0\\l\neq m}}^{k_1} \frac{l(r_m, r_{K+1})}{l(r_m, r_{K+1}) - l(r_l, r_{K+1})}. \tag{14}$$

$\#$ denotes the cardinal of the set $\mathcal{I}$.

Proof: See appendix A for a sketch of proof. ■

The Laplace transforms can be evaluated as [12]:

$$\mathcal{L}_{I_1,I_2}(\omega_i, \omega_{K+2}) = \exp\left\{-\lambda\int_{\mathbb{R}^2}\left(1 - \mathbb{E}\left[e^{-\sum_{i=1}^{K+1}\omega_i f_i(x,r_i) + \omega_{K+2} g(x, r_{K+1})}\right]\right) dx\right\}$$

where $f(x, r_i) = P|h_{x,r_i}|^2 l(x, r_i)$,

$$g(x, r_i) = \left(P_2\sum_{i=1}^{K}|h_i|^2 + P|h_{K+1}|^2\right) l(x, r_i), \tag{15}$$

and the expectation, which can be evaluated in closed form, is with respect to the exponential fading coefficients.

IV. CONVERSE: OUTAGE PROBABILITY LOWER BOUND

In the following we present a converse result on the minimum OP attainable with a protocol belonging to the family $\mathcal{P}$. As pointed out in [10] a natural lower bound is provided by the outage probability given by a K-MISO (multiple input-single output) system in the cluster at the origin, assuming that a genie provides the true message to all the nodes with the exception of the destination node. In this way, the receiving nodes jointly with the source form a virtual antenna array that transmits the message coherently to the destination, which has full CSI. In the mean time, in other clusters only the sources are assumed to be transmitting which gives a minimum level of interference observed by the destination of the cluster at the origin. The SIR at the destination node r_{K+1} when $1 \leq k_2 \leq K$ relay nodes are active is:

$$\text{SIR}(r_{K+1}) = \frac{P|h_{0,K+1}|^2 l(r_{K+1}) + \sum_{i=1}^{k_2} \hat{P}|h_{i,K+1}|^2 l(r_i, r_{K+1})}{I_{lb}(r_{K+1})},$$

with $I_{lb}(r_{K+1}) = \sum_{x\in\Phi} P|h_{x,r_{K+1}}|^2 l(x, r_{K+1})$. Due to the symmetry in the receiver distributions and to the fact that when they are acting as transmitters they do not have any CSI, the transmission power $\hat{P}$ has to be the same for all the nodes although the number of active relays n has to be determined so as to minimize the bound:

Theorem 4.1 (OP lower bound): An OP lower bound for any admissible protocol $\mathcal{P}$ is:

$$\mathbb{P}_{out,lb} = \min_{1\leq k_2 \leq K} \left\{ 1 - \sum_{m=0}^{k_2} \mathbb{E}_{\mathbf{r}} \left[B_m \mathcal{L}_{I_{lb}} \left(\frac{T}{\hat{P}_m l(r_m, r_{K+1})} \right) \right] \right\}$$

with:

$$B_m = \prod_{\substack{l=0 \\ l\neq m}}^{k_2} \frac{\hat{P}_m l(r_m, r_{K+1})}{\hat{P}_m l(r_m, r_{K+1}) - \hat{P}_l l(r_l, r_{K+1})}. \tag{16}$$

and $\hat{P}_0 = P$ and $\hat{P}_m = P/k_2$ for $1 \leq m \leq k_2$.

Proof: It follows along the lines of the proof of theorem 3.1 so it is skipped due to space limitations. ■

V. NUMERICAL RESULTS

We place the destination at $r_{K+1} = [10\ 0]$ and the K cooperating users are uniformly and independently distributed on a disc of radius $||r_{K+1}||/2$ centered at the midpoint between the source and the destination. This is done so that when obtaining the average OP over the receivers' positions we consider situations in which the receivers are closer to the source or to the destination. The sources transmit with power $P = 1$ and the path loss exponent is $\alpha = 3$. The power in the second phase for the relays is set to achieve equality in (4).

Fig. 1 shows the OP as a function of the density of clusters λ for different values of K. We set the attempted rate at $R = 0.5$ b/use, and optimize the rate of the first phase numerically ($R_1 = 0.9$ b/use). We compare the OP with respect to DT, whose OP is [7]:

$$\mathbb{P}_{out,DT} = 1 - e^{-\lambda C(2^R-1)^{2/\alpha} D^2}, \tag{17}$$

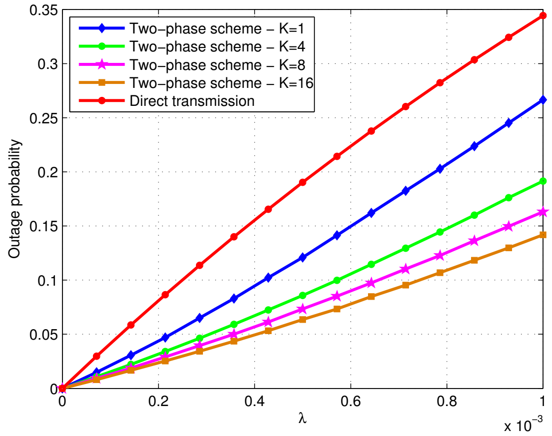

Fig. 1. OP as a function of the density of clusters λ for different values of K vs DT. $R = 0.5$ b/use, $R_1 = 0.9$ b/s, $\alpha = 3$.

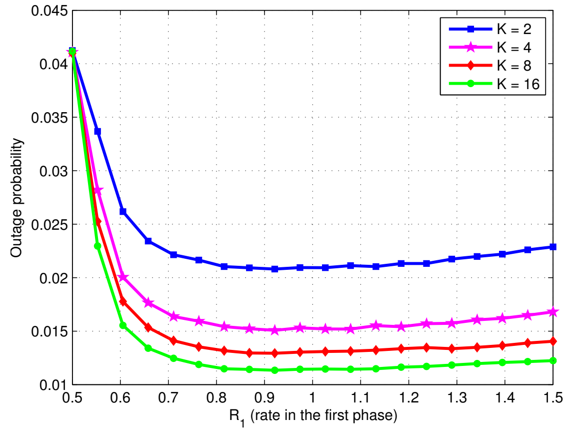

Fig. 2. OP as a function of the rate of the first phase R_1 for an attempted rate $R = 0.5$ b/use. $\lambda = 10^{-4}$, $\alpha = 3$.

where $C = 2\pi\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\alpha^{-1}$ and $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function. We observe that the protocol shows interesting gains (a reduction of more than 60%) with respect to DT even for small values of K, even though we are working with an upper bound on the interference and the receivers are distributed over a large region.

In Fig. 2 we fix $\lambda = 10^{-4}$ and plot the value of the OP as a function of the rate of the first phase R_1 for an attempted rate $R = 0.5$ b/use and for different values of K. We observe that for a given set-up the scheme is not very sensitive to errors in the optimization of the rate of the first phase even for different values of K.

VI. SUMMARY AND DISCUSSION

In this paper we focused on a family of half-duplex protocols and studied the performance of a two-phase cooperative protocol belonging to this family, in the context of a large wireless network formed by clusters. In each cluster, a source node attempts to transmit a message to their destinations, with the aid of other nodes. The protocol works by first broadcasting the message and then creating a distributed antenna array to jointly transmit the message to the destination. Through the use of cooperative diversity, the protocol showed substantial reductions in OP with respect to DT. In addition, we also derived a converse (lower) bound on the OP attainable by any unicast strategy when restricting all admissible protocols to belong to the same class of half-duplex codes that includes the one studied.

As future work, it would be of interest to extend the results in the present work to the multicast scenario in which the goal is to transmit the message to all $K+1$ nodes of each cluster. Also, developing more advanced strategies for selecting the relays, as it is proposed in [11], could improve the behavior of the protocol in both the unicast and multicast scenario.

Appendix

A. Proof of Theorem 3.1

Starting from (9) we see that the events $\{\mathcal{A}_{k_1}\}$ are disjoint. Let $\mathcal{B}_i$ be the event that the i-th receiver is not able to decode during phase 1, that is:

$$\mathcal{B}_i = \{\mathrm{SIR}_1(r_i) < T_1 I_1(r_i)\} \quad 1 \le i \le K+1. \tag{18}$$

The event $\{K_1 = k_1\}$ can be written as the union of all events $\mathcal{B}_i$ and $\mathcal{B}_i^c$ for $1 \le i \le K$ such that there are k_1 successes (complemented $\mathcal{B}_i$ events). By the i.i.d. assumption of the distribution of the receivers, the probability of any combination having k_1 successes has the same probability so we can write:

$$\mathbb{P}(\mathcal{A}_{k_1}) = \binom{K-1}{k_1} \mathbb{P}\left(\left(\bigcap_{i=1}^{k_1} \mathcal{B}_i^c \right) \cap \left(\bigcap_{j=k_1+1}^{K} \mathcal{B}_j \right) \cap \mathcal{F} \right). \tag{19}$$

To evaluate the OP we condition on $\tilde{\Phi}$ and the position of the receivers $\tilde{\mathbf{r}} = (r_1, \dots, r_K)$ and use that the fading coefficients are independent:

$$\mathbb{P}\left(\left(\bigcap_{i=1}^{k_1} \mathcal{B}_i^c \right) \cap \left(\bigcap_{j=k_1+1}^{K} \mathcal{B}_j \right) \cap \mathcal{F} \right) = \mathbb{E}\left[\prod_{i=1}^{k_1} \mathbb{P}(\mathcal{B}_i^c | \tilde{\Phi}, \tilde{\mathbf{r}}) \prod_{j=k_1+1}^{K} \mathbb{P}(\mathcal{B}_j | \tilde{\Phi}, \tilde{\mathbf{r}}) \mathbb{P}(\mathcal{F} | \tilde{\Phi}, \tilde{\mathbf{r}}) \right]. \tag{20}$$

Now we rewrite the terms in the following way:

$$\prod_{j=k_1+1}^{K} \mathbb{P}(\mathcal{B}_j | \tilde{\Phi}, \tilde{\mathbf{r}}) = 1 + \sum_{\mathcal{I} \in \mathcal{G}(k_1, K)} (-1)^{\#\mathcal{I}} \prod_{i \in \mathcal{I}} \mathbb{P}(\mathcal{B}_j^c | \tilde{\Phi}, \tilde{\mathbf{r}}) \tag{21}$$

so that:

$$\prod_{i=1}^{k_1} \mathbb{P}(\mathcal{B}_i^c | \tilde{\Phi}, \tilde{\mathbf{r}}) \prod_{j=k_1+1}^{K} \mathbb{P}(\mathcal{B}_j | \tilde{\Phi}, \tilde{\mathbf{r}}) = \prod_{i=1}^{k_1} \mathbb{P}(\mathcal{B}_i^c | \tilde{\Phi}, \tilde{\mathbf{r}}) + \sum_{\mathcal{I} \in \mathcal{G}(k_1, K)} (-1)^{\#\mathcal{I}} \prod_{i=1}^{k_1} \mathbb{P}(\mathcal{B}_i^c | \tilde{\Phi}, \tilde{\mathbf{r}}) \prod_{i \in \mathcal{I}} \mathbb{P}(\mathcal{B}_j^c | \tilde{\Phi}, \tilde{\mathbf{r}}) = \prod_{i=1}^{k_1} \mathbb{P}(\mathcal{B}_i^c | \tilde{\Phi}, \tilde{\mathbf{r}}) + \sum_{\mathcal{I} \in \hat{\mathcal{G}}(k_1, K)} (-1)^{\#\mathcal{I} - k_1} \prod_{i \in \mathcal{I}} \mathbb{P}(\mathcal{B}_j^c | \tilde{\Phi}, \tilde{\mathbf{r}}). \tag{22}$$

Since the power fading coefficients are exponential with unit mean we have:

$$\mathbb{P}(\mathcal{B}_i | \tilde{\Phi}, \tilde{\mathbf{r}}) = 1 - \exp\left\{ -\frac{T_1 I_1(r_i)}{P_1 l(r_i)} \right\}. \tag{23}$$

For the event $\mathcal{F}$ we have:

$$\mathbb{P}(\mathcal{F} | \tilde{\Phi}, \tilde{\mathbf{r}}) = F_{Z|\tilde{\Phi}, \tilde{\mathbf{r}}}\left(\frac{T_2 I_2(r_{K+1})}{P_2} \right), \tag{24}$$

where:

$$Z = |h_{0,K+1}|^2 l(r_{K+1}) + \sum_{m=1}^{k_1} |h_{m,K}|^2 l(r_m, r_{K+1}). \tag{25}$$

The distribution of Z is a split function depending on the location of the receivers $\{r_1, \dots, r_K\}$. When all the distances $\{||r_1 - r_{K+1}||, \dots, ||r_{K-1} - r_{K+1}||, ||r_{K+1}||\}$ are pairwise different, the distribution of Z is:

$$F_Z(z) = 1 - \sum_{m=0}^{k_1} A_m \exp\left\{ -\frac{z}{l(r_m, r_{K+1})} \right\} \tag{26}$$

with A_m given by (14) and $r_0 = 0$ (the position of the source of the message). If any of the distances are the same, (26) has an avoidable singularity and can be continuously extended. For most reasonable receiver distributions the event that two receiver are at the same distance from the source will have 0 probability so the expectation will be defined by (26). Using these equations, taking the average with respect to the point process and using the definition of the Laplace transforms we conclude the proof.

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