Systematic Lossy Source Transmission over Gaussian Time-Varying Channels

Iñaki Estella Aguerri
Centre Tecnològic de Telecomunicacions de Catalunya (CTTC)
Barcelona, Spain
Email: inaki.estella@cttc.es

Deniz Gündüz
Imperial College London
London, UK
Email: d.gunduz@imperial.ac.uk

***Abstract*—Systematic lossy transmission of a Gaussian source over a time-varying Gaussian channel is considered. A noisy version of the source is transmitted in an uncoded fashion to the destination through a time-varying Gaussian "base channel", constituting the systematic part of the transmission. A second time-varying Gaussian channel, called the "enhancement channel", orthogonal to the first one, is available for coded transmission of the source sequence. A block fading model for both channels is considered, and the average end-to-end distortion is studied assuming perfect channel state information only at the receiver. It is shown that if the enhancement channel is static and the base channel gain has a discrete or continuous-quasiconcave distribution, then the separation theorem applies. However, when both channels are block fading, separation theorem does not hold anymore. A lower bound is obtained by providing the enhancement channel state to the encoder, and it is shown that uncoded transmission is exactly optimal for certain base channel fading distributions, while the enhancement channel fading has arbitrary distribution. A joint decoding scheme is also presented and is shown to outperform uncoded transmission and separate source and channel coding for other base channel distributions.**

I. Introduction

In systematic transmission the raw input data (analog or digital) is transmitted through the channel along with the encoder output. In the case of channel encoding, systematic codes are composed of source bits concatenated with the parity bits, and they allow the receiver to skip the decoding step completely if the source data is received correctly (which can be confirmed through a checksum), or simplify decoding by recovering part of the source data when channel errors are in the form of erasures. In the case of lossy source transmission, the systematic part of the transmission might correspond to an analog legacy system or it can be intentional to provide the receiver a reasonably good estimate of the source samples when the decoding process for the parity bits fails.

Consider, for example, a simple sensor node transmitting its measurements to a fusion center in an uncoded fashion. The receiver uses the received signal to estimate the source with the minimum possible distortion. We call this analog transmission as the "base channel". Occasionally, a more complex device is available in the system, which can take the same measurements as the sensor node and help its transmission over an orthogonal fading channel. This "enhancement channel" is used to further lower the final distortion. We study the expected value of the end-to-end distortion in this systematic lossy source transmission system assuming block fading base and enhancement channels under strict delay constraints.

This work is supported by the Catalan Government (grant 2009 SGR 891) and the Spanish Ministry of Science and Innovation (FPU AP2009-5007).

Systematic source-channel coding in the same setting is studied in [1] assuming static channels, and the optimal performance is shown to be achieved by separate source and channel coding; that is, by the concatenation of an optimal Wyner-Ziv source code [2] which uses the noisy uncoded signal at the destination as side information, with an optimal capacity achieving channel code. Note that, in our setting both channels are fading, which corresponds to a fading side information and a fading direct channel. In delay-limited transmission, when the transmitter does not have channel state information (CSI), it cannot generate optimal source and channel codes without being prone to outages, and the separation theorem fails. The transmitter needs to adapt to the time-varying channel and side information qualities without knowing their realizations in a way that performs well on average.

The characterization of the optimal expected distortion in the absence of the base channel in our model has received a lot of interest in recent years. While the problem remains open in the finite SNR regime [3], [4], the high SNR exponent of the expected distortion is characterized in certain regimes in [5]. The pure source coding problem in the presence of time-varying side information, i.e., when the enhancement channel is an error-free constant rate link, is studied in [6], while some initial results on the joint source-channel coding problem are presented in [7], [8] both for finite and high SNR regimes.

In this paper, we first consider a special scenario in which the enhancement channel is static. We show that the separation theorem holds if the base channel state has a discrete or continuous-quasiconcave distribution. Remarkably, most common distributions used to model wireless communication fading gains, e.g., Rayleigh, Rician, Nakagami, are continuous and quasiconcave. When both channels are time-varying, we derive a lower bound on the expected distortion by providing the current enhancement channel state to the encoder. We show that uncoded transmission meets this lower bound when the base channel follows certain distributions, including Rayleigh distribution, while separate source and channel coding is suboptimal. In the literature, much attention has been paid to settings in which simple uncoded transmission achieves the optimal performance, such as transmission of Gaussian sources over point-to-point Gaussian channels [9], or the transmission of correlated Gaussian sources [10], or noisy

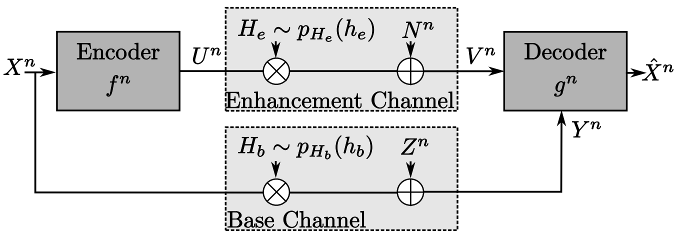

Fig. 1. Block diagram of the joint source-channel coding problem with fading base and enhancement channels.

observations of a Gaussian source [11] over Gaussian multiple access channels (MACs). Similar to these results, optimality of uncoded transmission in our setting is also sensitive to the source and channel distributions. However, to the best of our knowledge, this is the first result showing the optimality of uncoded transmission in a fading scenario while it would be suboptimal in the static case. We also study coded schemes based on separate source-channel coding as well as joint decoding. We show that the joint decoding scheme outperforms both separation and uncoded schemes in certain regimes and approaches the lower bound.

II. System Model

We consider the transmission of a sequence of independent and identically distributed (i.i.d.) zero mean, unit variance real Gaussian random variables X^n with $X_i \sim \mathcal{N}(0,1)$ over a fading channel (see Fig. 1). The "base channel" provides an uncoded noisy version of the source sequence to the destination. We model this channel as a memoryless block fading channel given by $Y^n = H_b X^n + Z^n$, where $H_b \in \mathbb{R}$ is the channel fading with probability density function (pdf) $p_{H_b}(h_b)$, X^n is the uncoded channel input and Z^n is the additive white Gaussian noise, i.e., $Z_i \sim \mathcal{N}(0,1)$, $i = 1, ..., n$.

In addition to the base channel there is an additional block fading channel connecting the source to the destination. An encoder $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ maps the source sequence X^n to the input of this channel, $U^n \in \mathbb{R}^n$, such that the average power constraint is satisfied: $\frac{1}{n}\sum_{i=1}^n E[U_i^2] \leq 1$. We call this channel the "enhancement channel", given by $V^n = H_e U^n + N^n$, where $H_e \in \mathbb{R}$ is the channel fading with pdf $p_{H_e}(h_e)$ and N^n is the additive white Gaussian noise $N_i \sim \mathcal{N}(0,1)$. We define $\Gamma_b \triangleq H_b^2 \in \mathbb{R}^+$ and $\Gamma_e \triangleq H_e^2 \in \mathbb{R}^+$ as the channel gains, with pdfs $p_{\Gamma_b}(\gamma_b)$ and $p_{\Gamma_e}(\gamma_e)$, respectively.

We assume that each source block is composed of n source samples, which need to be transmitted over one block of the channel consisting of n channel uses. Both channel states H_b and H_e are assumed to be constant, with values h_b and h_e, respectively, for the duration of one block, and independent among different blocks. The base and enhancement channel state realizations h_b and h_e are assumed to be known at the receiver, while the encoder is only aware of their distributions.

The decoder reconstructs the source sequence from both the enhancement and base channel outputs V^n and Y^n using a mapping $g : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^n$, $\hat{X}^n = g(V^n, Y^n, h_b, h_e)$.

For given channel distributions, we are interested in characterizing the minimum *expected distortion* $\mathrm{E}[D]$, denoted by ED^*, where the quadratic distortion between the source sequence and the reconstruction is given by $D \triangleq \frac{1}{n}\sum_{i=1}^n (X_i - \hat{X}_i)^2$. The expectation is taken with respect to the source, channel and noise distributions.

III. Lossy Source Coding with Fading Side Information

The source-coding version of this problem in which the fading enhancement channel is substituted by an error-free channel of rate R and the base channel output Y^n is used as a block fading time-varying side information sequence available at the destination is considered in [6]. Here we briefly review the results of [6] which will be used later in the paper.

Let the distribution $p_{\Gamma_b}(\gamma_b)$ be discrete with M states $\gamma_{b1} \leq \ldots \leq \gamma_{bM}$ with probabilities $\Pr[\Gamma_b = \gamma_{bi}] = p_i$. Let us define the side information sequence available at the decoder when the realization of the base channel is $h_{bi} = \sqrt{\gamma_{bi}}$ as $Y_{i,1}^n \triangleq \sqrt{\gamma_{bi}} X^n + Z^n$. Note that the side information has a degraded structure, given by the Markov chain $Y_{1,1}^n - \cdots - Y_{M-1,1}^n - Y_{M,1}^n - X^n$. This is equivalent to the Heegard-Berger source coding problem with degraded side information in which an encoder is connected through an error-free channel of rate R to M receivers, each one with side information $Y_{i,1}^n$. The minimum expected distortion is given by the solution to the following problem,

$$ED^*(R) = \min_{\mathbf{D}: R_{HB}(\mathbf{D}) \leq R} \mathbf{p}^T \mathbf{D}, \tag{1}$$

where $\mathbf{p} \triangleq [p_1, ..., p_M]$, $\mathbf{D} = [D_1, ..., D_M]$ with D_i defined as the achievable distortion at receiver i and $R_{HB}(\mathbf{D})$ is the Heegard-Berger rate-distortion function given by

$$R_{HB}(\mathbf{D}) = \min_{W_1^M \in \mathcal{P}(\mathbf{D})} \sum_{i=1}^M I(X; W_i | W_1^{i-1}, Y_i), \tag{2}$$

where W_1^i denotes the auxiliary random variables $W_1, ..., W_i$, and $\mathcal{P}(\mathbf{D})$ is the set of random variables W_1^M satisfying the Markov chain condition

$$W_M - \cdots - W_1 - X - Y_M - Y_{M-1} - \cdots - Y_1,$$

for which there exist source reconstructions $\hat{X}_i(Y_i, W_1^i)$ satisfying $\mathrm{E}[d_i(X, \hat{X}_i)] \leq D_i$, $i = 1, ..., M$.

The Heegard-Berger rate distortion function also extends to the set of countably many degraded fading states, $\gamma_{b1}, \gamma_{b2}, ...$ with $\sum_{i=1}^\infty p_i = 1$ [6]. The expected distortion for a Gaussian source with finite number of side information states is given by the convex optimization problem in [6, Eq. (59)-(62)] and for a countable number of states in [6, Eq. (75)-(78)], which we call $ED_C^*(R)$. When the side information distribution $p_{\Gamma_b}(\gamma_b)$ is continuous and quasiconcave[1], the optimal expected distortion is achieved by single-layer rate allocation such that all the available rate R is targeted to side information state $\bar{\gamma}_b$ [6]. Then, the optimal expected distortion is given by

$$ED_Q^*(R) = \int_0^{\bar{\gamma}_b} \frac{p_{\Gamma_b}(\gamma_b)}{1+\gamma_b} d\gamma_b + \int_{\bar{\gamma}_b}^\infty \frac{p_{\Gamma_b}(\gamma_b)}{(\bar{\gamma}_b + 1)2^{2R} + \gamma_b - \bar{\gamma}_b} d\gamma_b,$$

[1]A function $g(x)$ is quasiconcave if its super-level sets $\{x | g(x) \geq \alpha\}$ are convex for all α.

where $\bar{\gamma}_b$ is determined as follows. Let a super-level set be defined as $[\gamma_l(\alpha), \gamma_r(\alpha)] \triangleq \{\gamma_b | p_{\Gamma_b}(\gamma_b) \geq \alpha\}$. Then, $\bar{\gamma}_b = \gamma_l(\alpha^*)$ for $\alpha^* \in [0, \max p_{\Gamma_b}(\gamma_b)]$ such that $\gamma_l(\alpha^*)$ satisfies

$$\int_{\gamma_l(\alpha^*)}^{\infty} \frac{p_{\Gamma_b}(\gamma_b) - \alpha^*}{((1+\gamma_l(\alpha^*))2^{2R} + \gamma_b - \gamma_l(\alpha^*))^2} d\gamma_b = 0. \quad (3)$$

When H_b is Rayleigh distributed, it can be seen that $\bar{\gamma}_b = 0$, and the optimal expected distortion becomes

$$ED^*_{Ray}(R) = \sigma_{\Gamma_b}^{-2} 2^{\sigma_{\Gamma_b}^{-2} 2^{2R}} E_1\left(\sigma_{\Gamma_b}^{-2} 2^{2R}\right), \quad (4)$$

where $E_1(x) \triangleq \int_x^\infty t^{-1} e^{-t} dt$ is the exponential integral [6], and $\sigma^2_{\Gamma_b}$ is the variance of Γ_b.

Results in our paper are valid for finite as well as countable number of states γ_{bi} and quasiconcave pdfs $p_{\Gamma_b}(\gamma_b)$. To unify results, we define the function $E^*_s(R)$ as the minimum expected distortion for the source problem in these three setups.

IV. Static Enhancement Channel

In this section we consider a static enhancement channel, not necessarily an additive Gaussian channel, with capacity C and with CSI known at the encoder and decoder. On the contrary, the base channel is still a block-fading channel with gain following distribution $p_{\Gamma_b}(\gamma_b)$. Note that, while this is a special case of the system model introduced in Section II, it is a joint source-channel coding generalization of the source coding problem reviewed in Section III.

Optimality of separate source and channel coding can be proven when $p_{\Gamma_b}(\gamma_b)$ has finite or countable number of states or when $p_{\Gamma_b}(\gamma_b)$ is a continuous-quasiconcave distribution. This reduces the problem to the source coding problem of Section III with $R = \mathrm{C}$, as given in the next theorem.

Theorem 1. *Let Γ_b have a countable number of states γ_{bi} or a continuous-quasiconcave pdf $p_{\Gamma_b}(\gamma_B)$ and let the enhancement channel have capacity C. The minimum expected distortion, ED^*_{sta}, is achieved by separate source and channel coding, and is given by $ED^*_{sta} = ED^*_s(\mathrm{C})$.*

Proof. For Γ_b with finite number of states, the optimality of separation can be obtained as a special case of the model discussed in [12] for two base channel states. This result can be extended to M receivers (or states) by combining the converses in [12] and [13, Sec.VII] for M side information states, Y_i. The achievability follows from Heegard-Berger random source coding scheme in [13, Sec.VII] followed by channel coding at rate $R = \mathrm{C}$. The converse also applies for countably many receivers as discussed in [6] for the source problem.

To show separation when $p_{\Gamma_b}(\gamma_b)$ is a continuous-quasiconcave distribution, we construct a lower bound on the expected distortion ED^*_{sta} by discretizing the continuum of analog channel states, and showing that this bound is achievable in the limit of finer discretizations.

We divide the analog channel states γ_b into some partition $\mathbf{s}$ of the real line given by $[s_0, s_1), [s_1, s_2), \ldots$, such that $s_0 = 0 < s_1 < \ldots < s_i$ and $\gamma_b \in [s_{i-1}, s_i)$ if $s_i \leq \gamma_b < s_i$ for some $i = 1, 2, \ldots$. The length of the partition $[s_{i-1}, s_i)$ is defined by Δs_i, i.e. $\Delta s_i \triangleq s_i - s_{i-1}$. Let us define $\bar{\gamma}_b \geq 0$ as the super-level set $\bar{\gamma}_b$ satisfying (3). The partition is chosen such that for some index $\bar{i}$, we have $s_{\bar{i}} = \bar{\gamma}_b$. A fading realization belongs to the interval $[s_{i-1}, s_i)$ with probability $p_i = \int_{s_{i-1}}^{s_i} p_{\Gamma_b}(\gamma_b) d\gamma_b$.

We assume that when γ_b belongs to the interval $[s_{i-1}, s_i)$, a genie substitutes the current side information sequence $Y = h_b X + N$ with a sequence with gain s_i, i.e., $\tilde{Y} \triangleq \sqrt{s_i} X + N$. Note that this receiver has a better performance as noise can be added to $\tilde{Y}$ to recover the original side information sequence if required. Hence, the expected distortion for a given partition $\mathbf{s}$, denoted by $ED^*_{gen}(\mathbf{s})$, is a lower bound on the expected distortion of the continuous fading setup, ED^*_{sta}. The genie aided system now consists of a countable number of receivers and, due to the optimality of separation under countable number of side information states, $ED^*_{gen}(\mathbf{s})$ is achieved by the concatenation of a Heegard-Berger source encoder with side information states $u_1, u_2, \ldots$ and a capacity achieving channel code. Then, for a given partition $\mathbf{s}$, we have $ED^*_{gen}(\mathbf{s}) = ED^*_C(\mathrm{C})$. The limiting behavior of $ED^*_{gen}(\mathbf{s})$ as the partition gets finer, in the sense that $\max_i \Delta s_i \to 0$, can be obtained noting that once separation is proved for each $ED^*_{gen}(\mathbf{s})$, the problem reduces to the problem studied in [6]. Hence, by [6, Proposition 4] and [6, Proposition 5], $ED^*_{gen}(\mathbf{s})$ converges to $ED^*_Q(\mathrm{C})$, i.e., $\lim_{\max_i \Delta s_i \to 0} ED^*_{gen}(\mathbf{s}) = ED^*_Q(\mathrm{C})$.

With the enhancement channel state known, the lower bound $ED^*_Q(\mathrm{C})$ is achievable with a separate source-channel coding scheme by concatenating a single layer source encoder for base channel state $\bar{\gamma}_b$, and a channel code at a rate arbitrarily close to C. This completes the proof. ☐

V. Time-Varying Enhancement Channel

In this section we return to the problem presented in Section II in which both enhancement and base channels are fading.

A. Partially Informed Encoder Lower Bound

A trivial lower bound on ED^* is considered in [7] by providing the encoder with the instantaneous states of both the enhancement and the base channels. We call this the *informed encoder lower bound*, ED^*_{inf}. However, we can obtain a tighter lower bound by providing the encoder only with the enhancement channel realization h_e. We call this the *partially informed encoder lower bound*, and denote it by ED^*_{pi}. For a given enhancement channel state realization h_e, the setup reduces to the one considered in Section IV, and the separation theorem applies for each channel realization. Averaging over the channel states, we have the following lower bound.

Lemma 1. *The minimum expected distortion for an enhancement channel distribution $p_{H_e}(h_e)$ and a discrete or quasiconcave pdf $p_{\Gamma_b}(\gamma_b)$ is lower bounded by*

$$ED^*_{pi} \triangleq \mathrm{E}_{\Gamma_e}[ED^*_s(\mathcal{C}(\Gamma_e))], \quad (5)$$

where $\mathcal{C}(\gamma_e) \triangleq \frac{1}{2}\log(1+\gamma_e)$ is the capacity of the enhancement channel for a given realization $\gamma_e = h_e^2$.

B. Achievable Schemes

Here, we study some achievable schemes for lossy systematic joint source-channel transmission and prove the exact optimality of uncoded transmission for a certain class of base channel distributions $p_{\Gamma_b}(\gamma_b)$.

1) Separate Source-Channel Coding: We consider separate source and channel coding with a single layer based on Wyner-Ziv coding using the base channel output as a side information followed by channel coding on the enhancement channel.

At the transmitter, $2^{n(R_c+R_s)}$ Gaussian quantization codewords W^n are generated and uniformly distributed among 2^{nR_c} bins. A Gaussian channel codebook with 2^{nR_c} codewords U^n is generated independently with $U \sim \mathcal{N}(0,1)$ and each codeword assigned to a bin index. For a given source realization X^n, the codeword U^n assigned to the bin containing W^n jointly typical with X^n is transmitted over the channel. Note that R_s and R_c are fixed for all channel states due to the lack of CSI at the transmitter. At reception, the bin index is recovered using the enhancement channel output V^n if $R_c \leq I(U;V)$. Then a W^n jointly typical with the base channel output Y^n is searched within the estimated bin. If $R_c \geq I(X;W|Y)$, the correct quantization codeword is successfully decoded and $\hat{X}^n$ is reconstructed with an optimal minimum mean square error (MMSE) with distortion $D_d(R,\gamma_b) \triangleq (\gamma_b + 2^{2R})^{-1}$. If due to the randomness of the channels the quantization codebook cannot be correctly decoded, an outage is declared and only the base channel output is used to estimate the source at a distortion $D_d(0,\gamma_b)$. The expected distortion of the separation scheme is given by

$$ED_{sb}(R_s,R_c) = \mathrm{E}_{\mathcal{O}_{sb}^c}[D_d(R_s+R_c,\Gamma_b)] + \mathrm{E}_{\mathcal{O}_{sb}}[D_d(0,\Gamma_b)],$$

where $\mathcal{O}_{sb}^c$ is the complement of the outage event defined as

$$\mathcal{O}_{sb} \triangleq \{(\gamma_e,\gamma_b) : R_c > I(U;V) \text{ or } R_c < I(X;W|Y)\}.$$

For continuous-quasiconcave $p_{\Gamma_b}(\gamma_b)$, the optimal source rate minimizing the expected distortion is given in the next lemma.

Lemma 2. *For given R_c and a continuous-quasiconcave pdf $p_{\Gamma_b}(\gamma_b)$, $ED_{sb}(R_s,R_c)$ is minimized by setting $R_s = \frac{1}{2}\log(1+(1+\bar{\gamma}_b)(2^{2R_c}-1)) - R_c$ where $\bar{\gamma}_b$ is the solution to (3).*

The complexity of this scheme can be reduced by having a single codeword in each bin, i.e., $R_s = 0$, and the expected distortion is found as $ED_{nb}(R_c) \triangleq ED_{sb}(0,R_c)$. If only the base channel is available, then we have $ED_{no}^* \triangleq ED_{sb}(0,0)$.

2) No Binning with Joint Decoding (NBJD): Here, we consider a transmission scheme that does not use any explicit binning and uses joint decoding to reduce the outage event.

At the encoder, 2^{nR_j} quantization codewords W^n, are generated and each one assigned to one of the 2^{nR_j} codewords of an independent Gaussian codebook with codewords U^n with $U \sim \mathcal{N}(0,1)$. Given a source outcome X^n, the channel codeword U^n assigned to W^n is transmit over the channel. Then, the decoder looks for W^n such that U^n, V^n and Y^n, W^n are simultaneously jointly typical. The outage event is given by

$$\mathcal{O}_j \triangleq \{(\gamma_e,\gamma_b) : I(X;W|Y) > I(U;V)\}. \tag{6}$$

If the quantization codeword is successfully decoded, the source X^n is estimated using the quantization and the base channel output. On the other hand, if an outage occurs, the source X^n is reconstructed using only the base channel output. The expected distortion for the NBJD scheme is found as

$$ED_j(R_j) = \mathrm{E}_{\mathcal{O}_j^c}[D_d(R_j,\Gamma_b)] + \mathrm{E}_{\mathcal{O}_j}[D_d(0,\Gamma_b)].$$

Thanks to the joint decoding of NBJD, the outage probabilities with respect to separate source channel decoding is reduced. Indeed, it can be shown that the minimum expected distortion of NBJD is always lower than that of separate source and channel coding.

Lemma 3. *At any SNR, we have $ED_{sb}^* \geq ED_j^*$.*

3) Uncoded Transmission: Uncoded transmission is a zero delay transmission scheme in which each channel input U_i is generated by scaling the source signal X_i to satisfy the power constraint. At the receiver each component is reconstructed with an MMSE estimator using both the enhancement and base channel outputs at a distortion $D_u(\gamma_e,\gamma_b) = (1+\gamma_e+\gamma_b)^{-1}$. Averaging over the channel realizations, we have

$$ED_u = \mathrm{E}_{\Gamma_e\Gamma_b}[D_u(\Gamma_e,\Gamma_b)]. \tag{7}$$

Interestingly, for any $p_{\Gamma_b}(\gamma_b)$ satisfying (3) with $\bar{\gamma}_b = 0$, uncoded transmission achieves ED_{pi}^* in (5) characterizing the minimum expected distortion ED^*. However, the optimality of uncoded transmission does not hold in general as shown in Section V-C.

Theorem 2. *Let $p_{\Gamma_e}(\gamma_e)$ be an arbitrary pdf while $p_{\Gamma_b}(\gamma_b)$ is a continuous-quasiconcave function satisfying equation (3) for $\bar{\gamma}_b = 0$. Then, the minimum expected distortion ED^* is achieved by uncoded transmission.*

C. Performance Comparison

In this subsection we compare the proposed lower and upper bounds on the expected distortion numerically. We plot the expected distortion as the variances of H_e and H_b, or equivalently, the channel signal to noise ratios, increase.

First, we assume that H_e and H_b are both Rayleigh with equal variances, i.e., $\sigma_{H_e}^2 = \sigma_{H_b}^2 = \text{SNR}$. ED^* is achieved by uncoded transmission and coincides with the partially informed lower bound ED_{pi}^*, as shown in Theorem 2. Then, we obtain ED_{sb}^* as follows. For Rayleigh distributed H_b, since $\bar{\gamma}_b = 0$, from Lemma 2 we have $R_s = 0$, that is, the best performance is achieved by ignoring the side information (no binning) to avoid source decoding outages, and using it only in the estimation step. Optimizing over R_c we have $ED_{sb}^* = ED_{nb}^*$. Similarly, ED_j^* is obtained by minimizing (7) over the rates R_j. In Fig. 2 we observe that the informed lower bound is in general very loose compared to the partially informed transmitter bound. As shown in Lemma 3, NBJD outperforms separate source and channel coding, although it falls short of the optimal performance. Note that both schemes have the same decay rate in the high SNR regime, i.e., distortion exponent, as shown in [8].

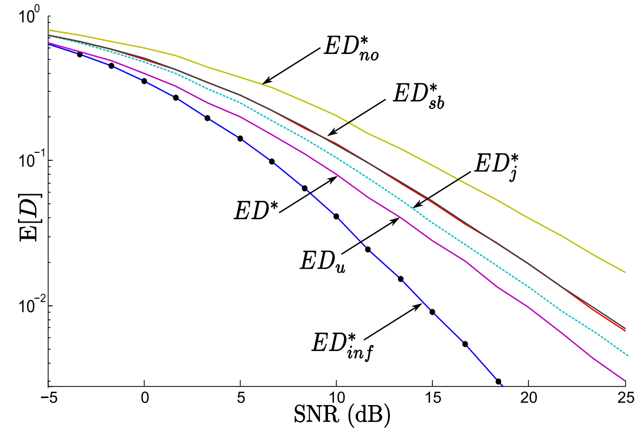

Fig. 2. Lower and upper bounds on the expected distortion versus the channel SNR σ^2_H for Rayleigh distributed base channel with $\sigma^2_{H_e} = \sigma^2_{H_b}$.

Next we assume that Γ_b follows a Gamma distribution with shape $L = 2$ and scale $\lambda = \rho\sqrt{L^{-1}}$, $\Gamma_b \sim \text{Gamma}(L, \lambda)$. This distribution arises, for example, when two independent base channels are available to the decoder. In Figure 3 we show the optimized upper and lower bounds on the minimum expected distortion. Unlike the Rayleigh fading base channel case, uncoded transmission is not optimal anymore. While uncoded transmission gets very close to the lower bound in the low SNR regime, as the SNR increases, the gap between the two increases. Moreover, both the separation and joint decoding schemes surpass uncoded transmission. Beyond a certain SNR value, NBJD achieves the lowest distortion among the proposed schemes and performs very close to the lower bound. As shown in Lemma 3, separate source and channel coding with and without binning both have worse performance than joint decoding at all SNR values, and worse than uncoded transmission in the low SNR regime. If the base channel output is used only in the estimation, and not for decoding in the enhancement channel, ED^*_{nb} is worse than that of uncoded transmission. On the contrary, when it is used as side information in decoding, i.e., binning is allowed, the achievable distortion is lower than that of uncoded transmission. While at low SNR the performances of both separate source-channel coding schemes are similar, as the SNR increases, ED^*_{sb} starts to outperform ED^*_{nb}. Again, ED^*_j has the same decay rate as ED^*_{sb}, while ED^*_{nb} decays more slowly, indicating that binning becomes important as SNR increases.

VI. Conclusions

We have studied delay-limited transmission of a Gaussian source over a block-fading enhancement channel when analog noisy versions of the source samples are available at the receiver through a block-fading base channel. We have assumed that only the receiver has full knowledge of the channel states. We have shown that if the enhancement channel is static, separate source and channel coding is optimal for discrete and continuous-quasiconcave distributions for the base channel gain. For the fading enhancement channel case, we have derived a lower bound by partially informing the transmitter with the enhancement channel state. Using this bound, we

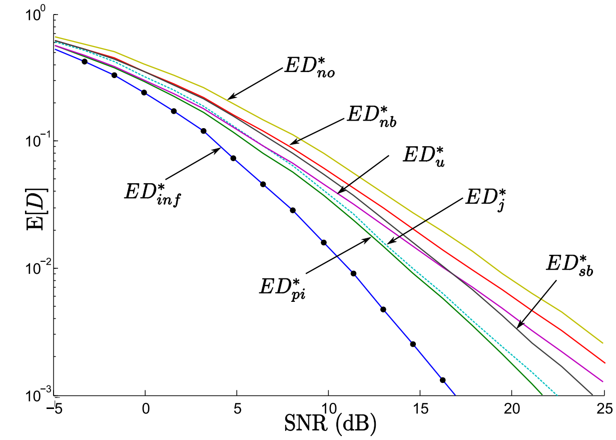

Fig. 3. Lower and upper bounds on the expected distortion versus the channel SNR for gamma distributed base channel with shape $L = 2$ and $\sigma^2_{H_e} = \sigma^2_{H_b}$.

have shown that uncoded transmission achieves the optimal performance when both channels are single-input single-output additive Gaussian and the base channel fading follows certain distributions, such as Rayleigh. We have then considered a Gamma distributed base channel, and shown that uncoded transmission is not optimal in this case. We have shown that joint decoding scheme outperforms separate source-channel coding, and its performance is very close to the partially informed encoder lower bound.

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