The GDOF of 3-user MIMO Gaussian interference channel

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***Abstract*—The paper establishes the optimal generalized degrees of freedom (GDOF) of 3-user $M \times N$ multiple-input multiple-output (MIMO) Gaussian interference channel (GIC) in which each transmitter has M antennas and each receiver has N antennas. A constraint of $2M \leq N$ is imposed so that random coding with message-splitting achieves the optimal GDOF. Unlike symmetric case, two cross channels to unintended receivers from each transmitter can have different strengths, and hence, well known Han-Kobayashi common-private message splitting would not achieve the optimal GDOF. Instead, splitting each user's message into three parts is shown to achieve the optimal GDOF as well as $\mathcal{O}(1)$ capacity approximation.**

I. Introduction

Interference plays a central role in today's wireless communication systems. In information theory, the efforts of finding the performance limit of interference channel (IC) in terms of capacity started more than 30 years ago [1]–[3]. Unfortunately, the complete capacity region for even a simple 2-user IC is known only for *strong* interference regime [1]–[3].

Although the problem of finding the exact capacity has been open for more than 30 years, the notion of *degrees of freedom* (DOF) defined for high signal-to-noise ratio (SNR) has opened a new direction of understanding IC. One surprising result was obtained by Cadambe and Jafar [4] which states that the per-user DOF of K-user IC is the same as that of 2-user IC for arbitrary K. The DOF provides valuable understanding of IC with a form of conclusive answer, but it does not capture the relationship between signal strength and interference strength which has crucial importance in understanding IC.

In [5], Etkin *et al.* came up with the notion of the generalized DOF (GDOF) which incorporates signal-to-interference ratio (SIR) in it. As the DOF does, the GDOF also assumes high SNR, and this not only makes analysis more tractable, but also provides a valuable viewpoint in understanding IC. In IC, there are two important factors of background noise and interference. Although their combined effect likely needs to be studied thoroughly for complete understanding of IC, one may want to isolate the effect of interference given that the effect of background noise has been fairly well studied through point-to-point (p2p) channel analysis. High SNR regime can essentially be considered as *interference-limited* regime, and thus provides such isolation. It turns out that the GDOF provides tremendous insight on 2-user single-input single-output (SISO) Gaussian IC (GIC) through its so called 'W' shape, and rather surprising 1-bit gap to the capacity result is also given in [5].

An important observation made in [5] is that a simple version of the Han-Kobayashi (HK) scheme [3] turns out to be the GDOF optimal. Intuition behind why the HK scheme is the GDOF optimal for 2-user SISO GIC can be found through its deterministic modeling which was originally studied by El Gamal and Costa [6]. Simply speaking, deterministic modeling assumes non-random noise or deterministic loss of transmitted signal which the transmitter is aware of. Therefore, the optimal strategy of the transmitter is easily given by not transmitting any valuable data on the part of the signal which will be lost. This strategy is indeed a special case of the HK scheme, and it is shown to achieve the capacity of this 2-user deterministic model. Directly applying the capacity-achieving strategy of the deterministic model to the Gaussian case results in the strategy which treats portion of interference message up to noise level as noise. This does not hurt GDOF achievability due to high SNR assumption, and this is the reason why the deterministic model becomes relevant for GDOF study.

An important assumption in [6] for capacity achievability is that common information of interference must be clearly observable after decoding the intended message. This assumption is discussed in Section VII of [5], and it will also be discussed later in this paper. By this assumption, a class of multiple-input multiple-output (MIMO) IC in which the HK scheme must be the GDOF optimal can be characterized. Gou and Jafar [7] found the optimal GDOF of a certain class of single-input multiple-output (SIMO) IC by extending the deterministic model of [6]. Corresponding MIMO results are obtained in [8]–[11].

For cases in which the HK scheme is not GDOF optimal, the optimal GDOF was found by using 'signal-level alignment' or 'structured coding' [12]–[14]. For these cases, we may think of a specific form of deterministic modeling for Gaussian channels which are proposed in [15]. Although such approach can possibly provide a valuable way of solving more general cases, it can only be applied for SISO symmetric cases so far. Extending this to general cases still remains to be seen.

One thing to note is that the aforementioned GDOF results only deal with symmetric IC except for 2-user results in [5], [9], [10]. Since aforementioned assumption of clearly observable interference has nothing to do with symmetric nature of the channel, it is reasonable to believe that there must be a kind of message-splitting with random coding schemes which achieves the optimal GDOF of asymmetric IC, and this is the main focus of this paper.

A simpler case than asymmetric MIMO GIC was considered in [12]. In [12], one-to-many IC was considered in which one transmitter causes interferences to all receivers, and all the other transmitters do not cause interference. In this channel, a generalization of the HK scheme which splits the message into multiple layers achieves a constant gap to the capacity. At the transmitter's view point, this one-to-many channel is equivalent to the channel considered in this paper. In this paper, therefore, we use this generalization of the HK scheme and show that it achieves the optimal GDOF of 3-user *partially asymmetric* MIMO GIC. The reason why multiple splitting is necessary is because cross channels for a given transmitter have different channel qualities unlike symmetric case.

The remainder of this paper is organized as follows. Section II defines the channel model as well as achievable rate terms. Section III discusses the deterministic model which corresponds to the GDOF of Gaussian case. Section IV establishes the optimal GDOF of the 3-user MIMO GIC. Section V concludes the paper. Due to length restriction, all proofs are omitted in this paper. For proofs, please see [16].

a) Notation: A matrix is represented with a capital letter like X, and a vector is represented as $\underline{x}$. I represents an identity matrix or mutual information, and they can be easily differentiated from the context. For a matrix X or a vector $\underline{x}$, X^H or $\underline{x}^H$ represents conjugate transpose. $Tr(X)$ represents the trace of X.

II. Channel model and preliminaries

Consider a following model with channel output $\underline{y}_i$ for the receiver i, channel input $\underline{x}_i$ for the transmitter i, and the channel H_{ij} from the transmitter i to the receiver j.

$$\begin{aligned} \underline{y}_1 &= \rho H_{11}\underline{x}_1 + \rho^{\alpha_2} H_{21}\underline{x}_2 + \rho^{\alpha_1} H_{31}\underline{x}_3 + \underline{z}_1 \\ \underline{y}_2 &= \rho^{\alpha_1} H_{12}\underline{x}_1 + \rho H_{22}\underline{x}_2 + \rho^{\alpha_2} H_{32}\underline{x}_3 + \underline{z}_2 \\ \underline{y}_3 &= \rho^{\alpha_2} H_{13}\underline{x}_1 + \rho^{\alpha_1} H_{23}\underline{x}_2 + \rho H_{33}\underline{x}_3 + \underline{z}_3, \end{aligned} \quad (1)$$

where background noise $\underline{z}_i \sim \mathcal{CN}(\underline{0}, I)$ and $\rho > 0, \alpha_1 > \alpha_2 > 0$. $\underline{x}_i$ satisfies the average power constraint $Tr(E[\underline{x}_i\underline{x}_i^H]) \leq Tr(I)$. We consider $M \times N$ MIMO channel in which each transmitter has M antennas and each receiver has N antennas. Although every result obtained in this paper with 3 user can be directly generalized into K-user case, we only consider 3 user case in this paper due to computational complexity. We call the above model *partially asymmetric* due to its symmetric nature that every transmitter sees channels with strengths $\rho, \rho^{\alpha_1}, \rho^{\alpha_2}$ and every receiver sees channels with strengths $\rho, \rho^{\alpha_1}, \rho^{\alpha_2}$. A general asymmetric case is essentially no different from this partially asymmetric case, but we do not consider a general model again due to computational complexity. Because of the symmetric nature of the channel, the achievable GDOF can be characterized by a single number as in the fully symmetric case while the asymmetric nature is enough to capture essential difference from the fully symmetric case. We assume that there is no degenerate case of channel coefficients, i.e., all H_{ij}'s are full-rank. We define the capacity region $\mathcal{C}$ of this channel in the standard Shannon sense. Because of symmetry, the maximum achievable total GDOF of the system is attained when the rate of each user is the same. Therefore, we define the symmetric capacity as

$$C_{sym} = \max_{(R_1,R_2,R_3)\in\mathcal{C}} \min\{R_1, R_2, R_3\}, \quad (2)$$

where R_i is the rate of user i. We may define C_{sym} as a function of ρ, α_1 and α_2. Then, per user GDOF $d_{sym}(\alpha_1, \alpha_2)$ is given as

$$d_{sym}(\alpha_1, \alpha_2) = \lim_{\rho\to\infty} \frac{C_{sym}(\rho, \alpha_1, \alpha_2)}{\log_2 \rho}. \quad (3)$$

To satisfy the assumption of clearly observable interference, we only consider the case where $2M \leq N < 3M$. This will be discussed in more detail in Section III.

III. Deterministic modeling

Finding the optimal GDOF involves derivation of tight-enough upper bounds. In [5], a technique of giving appropriate side information is developed to derive such upper bounds. As mentioned in [7], appropriate side information can easily be determined through deterministic modeling for certain cases. One thing to note is that the capacity region of the deterministic model given in [7] is much more complicated than the GDOF of the corresponding Gaussian model. For efficient computation, we propose more specific form of deterministic model which is closer to the corresponding Gaussian model for the GDOF analysis. By using this, the minimal number of tight upper bounds with appropriate side information can easily be determined. Due to length restriction, only description of the aforementioned simpler deterministic model is contained in this paper. For the the capacity result of this model, please see [16].

The deterministic model which corresponds to the channel defined in Section II with $\alpha_1 < 1$ can be described by the following relationship among channel input X_i from the transmitter i, channel output Y_i at the receiver i and interference V_{ij} from the transmitter i to the receiver j.

$$\begin{aligned} V_{12} &= g_1(X_1), \quad V_{13} = g_2 \circ g_1(X_1), && \text{(4a)} \\ V_{21} &= g_2 \circ g_1(X_2), \quad V_{23} = g_1(X_2), && \text{(4b)} \\ V_{31} &= g_1(X_3), \quad V_{32} = g_2 \circ g_1(X_3), && \text{(4c)} \\ Y_1 &= f_1(X_1, V_{21}, V_{31}), && \text{(4d)} \\ Y_2 &= f_2(X_2, V_{12}, V_{32}), && \text{(4e)} \\ Y_3 &= f_3(X_3, V_{13}, V_{23}), && \text{(4f)} \end{aligned}$$

where f_i and g_i are deterministic functions. There are two other possible cases of $\alpha_1 > 1 > \alpha_2$ and $\alpha_2 > 1$, and these cases are discussed in [16]. The term 'deterministic' comes from deterministic nature of channel functions especially for g_i. The above model is similar to what is defined in [7], but an important difference is that the functions representing two interference channels from one transmitter are different, which reflects asymmetric nature of the channel.

An important property which needs to be satisfied to show that the HK-like scheme achieves the capacity is given as

$$H(Y_1|X_1) = H(V_{21}, V_{31}) = H(V_{21}) + H(V_{31}), \quad (5a)$$
$$H(Y_2|X_2) = H(V_{12}, V_{32}) = H(V_{12}) + H(V_{32}), \quad (5b)$$
$$H(Y_3|X_3) = H(V_{13}, V_{23}) = H(V_{13}) + H(V_{23}). \quad (5c)$$

Note that the second equality of each line in the above equation automatically holds due to independence, and hence the assumption essentially is the first inequality of each line. When this holds, each interference is decodable given the intended message which implies that there is enough dimension to resolve interference uncertainty. In MIMO Gaussian case, it is not difficult to see that $N \geq 2M$ must be satisfied to ensure enough dimension although there is no formal proof that the HK-like scheme will not be GDOF optimal when this does not hold. We also consider only the case of $N < 3M$ since the DOF of M per user can be easily achieved by treating interference as noise if $N \geq 3M$.

If we consider the message X_1 of transmitter 1, then it is not difficult to see that the receiver 2 has to treat its degraded version V_{12} as common message and the receiver 3 need to do the same thing with further degraded version V_{13} to achieve the capacity. This essentially implies that transmitter 1 has to design its message splitting in such a way, and this results in 3-message splitting at each transmitter.

As mentioned earlier, the capacity region of the deterministic model in [7] is considerably more complicated than the GDOF of the corresponding Gaussian model. This is due to the deterministic model being more general than the corresponding Gaussian model. Although the capacity result of a general deterministic model has value by itself, we will consider a special case of the deterministic model which resembles Gaussian model more closely to reduce amount of analysis. The deterministic model described in (4) additionally satisfies the following properties. For all i and j, let

$$A_{ij} = \begin{cases} \{V_{ji}\} & \text{if } V_{ji} = g_2 \circ g_1(X_j) \\ \{V_{jk} \text{ for all } k\} & \text{if } V_{ji} = g_1(X_j) \\ \{X_j, V_{jk} \text{ for all } k\} & \text{if } V_{ji} \text{ does not exist.} \end{cases} \quad (6)$$

Simply speaking, A_{ij} is the set of messages from transmitter j which need to be decoded at receiver i. Let A be a subset of the set of messages $\{V_{12}, V_{13}, V_{21}, V_{23}, V_{31}, V_{32}\}$. Then, we have for all i and j such that $V_{ij} = g_1(X_i)$

$$I(V_{ij}; Y_i|A) = I(V_{ij}; Y_i|V_{ki} \text{ for all } k, A \cap A_{ii}) \quad (7a)$$
$$= H(V_{ij}|A \cap A_{ii}), \quad (7b)$$

if $A_{il} \subset A$ for $l \neq i$. This condition means that the maximum transmittable rate of the message V_{ij} from transmitter i to receiver i is not changed by giving one of interferer's message to receiver i as side information if another interferer's message is already given to receiver i. Since we only consider $N \geq 2M$ in Gaussian case, presence of interference whose dimension is M does not affect decodability of M dimensional message much in high SNR regime. This phenomenon is essentially described in Lemma 1, and it governs the GDOF behavior. We also assumes the following. For all i and j and $k \neq i, j$, we have

$$I(V_{ji}; Y_i|A) = I(V_{ji}; Y_i|X_i, V_{ki}, A \cap A_{ij}) \quad (8a)$$
$$= H(V_{ji}|A \cap A_{ij}), \quad (8b)$$

if $A_{ik} \subset A$. This condition is the counter part of (7) for the message from transmitter j to receiver i.

By using this simpler deterministic model, we can easily get tight upper bounds with appropriate side information which are suitable for Gaussian case as can be seen in [16].

IV. Gaussian IC

We now consider the GDOF of GIC defined in (1). To derive the GDOF, we will use $\mathcal{O}(1)$ approximation. We say that $f(x) = g(x) + \mathcal{O}(1)$ when $\lim_{x\to\infty} |f(x) - g(x)| < \infty$. Note that the GDOF optimality still allows infinite gap to capacity, but $\mathcal{O}(1)$ gap implies the finite gap. Similar to the result in [7], the result obtained in this paper is actually stronger than the GDOF because of this $\mathcal{O}(1)$ nature. To analyze behavior of MIMO IC with high SNR, we need the following lemma which is given in [9], [11].

Lemma 1. *[11] Suppose H_1, H_2, H_3 are $N \times r$ matrices with rank r. When $\alpha > \beta > \gamma$, we have*

$$\begin{aligned} &\log |I + \rho^\alpha H_1 H_1^H + \rho^\beta H_2 H_2^H + \rho^\gamma H_3 H_3^H| \\ &= r\alpha \log \rho + \min(r, (N-r)^+)\beta \log \rho \\ &\quad + \min(r, (N-2r)^+)\gamma \log \rho + \mathcal{O}(1). \end{aligned} \quad (9)$$

The above lemma essentially says that we can retain full DOF provided by exponents of ρ if there is enough dimension. This is an important property in high SNR regime, and this is why we have (7) and (8) for the deterministic model. Now we are ready to present the GDOF of GIC.

A. Case of $\alpha_1 < 1$

Theorem 1. *GDOF of Gaussian IC with $2M \leq N < 3M$ and $\alpha_1 < 1$ is given as*

$$\begin{aligned} &d_{sym}(\alpha_1, \alpha_2) = \\ &\min\Big\{ \max\Big\{M + (N-3M)\alpha_2, \\ &\qquad M + (N-3M)\alpha_1 + (3M-N)\alpha_2, \\ &\qquad (3M-N)\alpha_1 + N - 2M\Big\}, \\ &\quad \max\Big\{M + \frac{1}{2}(N-3M)\alpha_2, \\ &\qquad \frac{1}{2}(N-M) + \frac{1}{2}(3M-N)\alpha_2\Big\}, \\ &\quad M + \frac{1}{3}(N-3M)\alpha_2\Big\}. \end{aligned} \quad (10)$$

The achievability side of the above theorem closely follows standard techniques in [7]–[9], [11] while the converse side results in a new way of obtaining outer bounds to deal with asymmetric nature, and this is dicussed in [16]. Such trend is similar in other cases of α as well.

Let us now further evaluate GDOF expression in (10). By careful evaluation, we get

$$D_{sym}(\alpha_1,\alpha_2) = \begin{cases} M+(N-3M)\alpha_2 \\ \qquad\qquad \text{if } \alpha_1+\alpha_2<1, 2\alpha_2<\alpha_1, \\ \min\Big\{M+(N-3M)\alpha_1+(3M-N)\alpha_2, \\ \quad M+\frac{1}{2}(N-3M)\alpha_2\Big\} \\ \qquad\qquad \text{if } 2\alpha_1-\alpha_2<1, 2\alpha_2>\alpha_1, \alpha_2<\frac{1}{2}, \\ \min\Big\{N-2M+(3M-N)\alpha_1, \\ \quad M+\frac{1}{2}(N-3M)\alpha_2\Big\} \\ \qquad\qquad \text{if } \alpha_1+\alpha_2>1, 2\alpha_1-\alpha_2>1, \alpha_2<\frac{1}{2}, \\ \min\Big\{\frac{1}{2}(N-M)+\frac{1}{2}(3M-N)\alpha_2, \\ \quad M+\frac{1}{3}(N-3M)\alpha_2\Big\} \\ \qquad\qquad \text{if } \alpha_1+\alpha_2>1, \alpha_2>\frac{1}{2}. \end{cases} \tag{11}$$

Note that the first term $M+(N-3M)\alpha_2$ is the DOF which can be obtained by treating interference as noise (IAN). In 3-user symmetric case considered in [7], the optimal GDOF is strictly larger than the GDOF obtained by IAN for all interference regimes while 2-user SISO case in [5] also has interference regime in which IAN is GDOF optimal. It is argued in [7] that the reason why there is no interference regime in which IAN is optimal in 3-user symmetric case is because there are multiple receiver antennas. In SISO case, there is no other dimension to resolve interference common information when interference strength is weak enough to be affected by private message strength of the intended user. In SIMO case, however, there always are other dimensions to exploit when interference strength is weak. Then why do we see different behavior in asymmetric case? If we look at the condition for which IAN is GDOF optimal, then we can see that α_2 must be small with α_1 being at least as twice as larger than α_2, and both α_1 and α_2 cannot be too large. Therefore, we may think that this comes from the difficulty of decoding common message from weaker interference channel due to strong interference from stronger channel and the private message of the intended transmitter. If α_1 is large enough, then the power of the private message of the intended transmitter becomes small enough, but the condition $\alpha_1+\alpha_2<1$ prevents it. Such difficulty results in optimality of treating weaker interference as noise, and in this case, the receiver has enough dimension to resolve the intended message by treating every interference as noise.

B. Case of $\alpha_2 < 1 < \alpha_1$

Theorem 2. *GDOF of Gaussian IC with $2M \le N < 3M$ and $\alpha_2 < 1 < \alpha_1$ is given as*

$$d_{sym}(\alpha_1,\alpha_2) = \min\Bigg\{M, \frac{2M+M\alpha_1+(N-3M)\alpha_2}{3}, \max\Bigg\{\frac{M+M\alpha_1+(N-3M)\alpha_2}{2}, \frac{M+(N-2M)\alpha_1+(3M-N)\alpha_2}{2}\Bigg\}\Bigg\}. \tag{12}$$

It can be seen from (12) that the GDOF can be M which implies that the effect of interference is completely removed. In symmetric cases in [5], [7], [13], the effect of interference is removed when interference is much stronger than the desired channel. Theorem 2 shows that it can happen even when some of interferences are weaker than the desired channel for asymmetric case.

C. Case of $\alpha_2 > 1$

Theorem 3. *GDOF of Gaussian IC with $2M \le N < 3M$ and $\alpha_2 > 1$ is given as*

$$d_{sym}(\alpha_1,\alpha_2) = \min\{M, \frac{N-2M+M\alpha_1+M\alpha_2}{3}\}. \tag{13}$$

From (13), we can directly conclude that the effect of interference is removed, i.e, the GDOF is M, if $\alpha_1+\alpha_2 > \frac{5M-N}{M}$ which corresponds to very strong interference. It can be also seen that setting $\alpha_1=\alpha_2=\alpha$ exactly recovers the result in the symmetric case in [7] for $\alpha > 1$.

D. The GDOF region

Figure 1 describes the GDOF region of 3-user MIMO GIC with $M = 1$ and $N = 2$ for $0 < \alpha_1, \alpha_2 < 2$. It can be easily seen that the diagonal line which corresponds to $\alpha_1 = \alpha_2$ recovers the GDOF result of the symmetric channel obtained in [7]. Further detailed analysis on faces of the GDOF region labeled from 1 to 12 which correspond to terms in (10), (12), (13) can be found in [16].

V. Conclusion

In this paper, the GDOF of 3-user MIMO GIC is characterized. As conjectured in [7], Han-Kobayashi or Etkin-Tse-Wang-like message splitting achieves the GDOF although generalization of multiple message splitting is required. Three messages per transmitter suffice in 3-user case, and it implies that K message splitting would achieve the GDOF of any K-user MIMO GIC which satisfies the condition of $M(K-1) \le N$. Note that the GDOF result obtained in this paper essentially implies $\mathcal{O}(1)$ gap to the capacity which is finite, but the exact gap to the capacity cannot be computed. In finite SNR regime in which the exact constant gap is desirable, degraded nature does not exist in the channel anymore, and this means that more message splitting would be required to establish such result.

We define *partially asymmetric* GIC which yields valuable insights with manageable amount of computation on asymmetric GIC with more than two users which has not been well studied in literature. The form of the GDOF region gives interesting interpretation on interactions among different interferences for the first time in information theoretic study on interference channel. The methodology in this paper would achieve the GDOF of any MIMO GIC which satisfies $M(K-1) \le N$ probably through cumbersome analysis.

The deterministic model is used in this paper as in [7] to facilitate easier analysis for the Gaussian case. The most important benefit is the systematic way of determining side information for converse. In Gaussian case, however, the proof

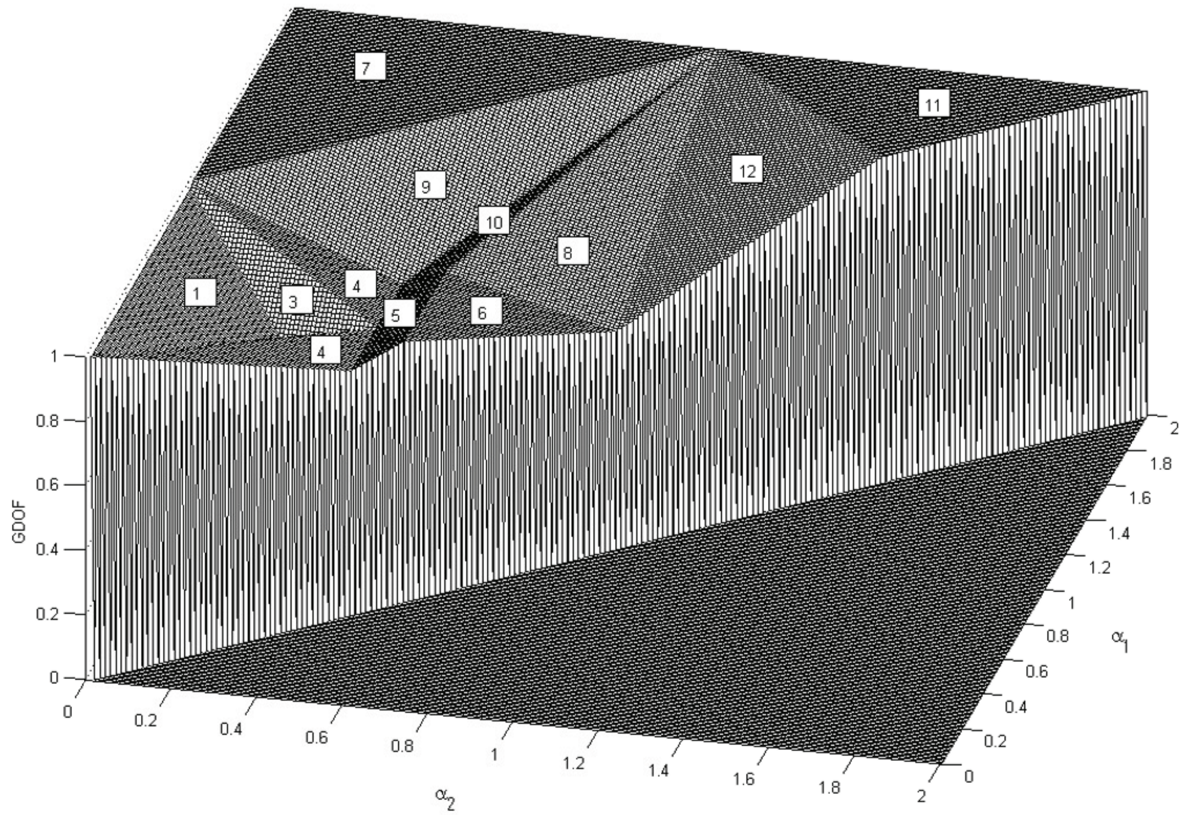

Fig. 1. The GDOF region of 3-user 1×2 GIC

of converse is not as simple as in the deterministic case due to the fact that the channel output becomes linear combination of channel inputs. As can be seen by proofs of converse and by corresponding discussions described in [16], asymmetric nature of the channel requires an approximation which implies non-trivial generalization of the symmetric case. This approximation using vector entropy inequality can possibly be used for general K-user MIMO GIC, but it relies on the fact that the GDOF is not affected by finite gap to the capacity, and hence, may not be used for analysis in finite SNR regime. This implies that a better upper bounding technique is required to obtain the exact constant cap to the capacity.

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