On Spatial Capacity in Ad-Hoc Networks with Threshold Based Scheduling

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***Abstract*—This paper studies the spatial capacity of wireless *ad hoc* networks. We propose a transmission scheme with threshold-based scheduling, where each transmitter decides to transmit in the data transmission phase if the signal-to-interference-ratio (SIR) at its receiver in the preceding pilot phase is no smaller than a predefined threshold. For comparison, we also consider a reference scheme, where all transmitters transmit independently in both the pilot and data transmission phases. For both schemes, we assume a homogeneous Poisson Point Process (PPP) to model the locations of transmitters that have the intention to transmit. However, for the proposed scheme, the point process formed by the retained transmitters in the data transmission phase is generally not a PPP due to the SIR-based scheduling. First, we show how to set the SIR threshold in the proposed scheme to assure that it outperforms the reference scheme in terms of network spatial capacity. Then, we present exact/approximate spatial capacity expressions for the proposed scheme with different SIR-threshold values. Finally, we provide simulation results to validate our analysis.**

I. Introduction

Capacity analysis is challenging in wireless *ad hoc* networks, mainly due to the difficulty in quantifying the multi-transmitter interference. Recently, tools from stochastic geometry [1] have been applied to characterize the multi-transmitter interference. According to [2], based on stochastic geometry, we are able to obtain, if not completely exact, asymptotically exact dependence between the system performance (e.g., system capacity, outage probability) and the system designing parameters (e.g., transmitter density, transmitted power level). In this paper, we also resort to stochastic geometry to analyze the ad hoc network capacity.

In the existing work based on stochastic geometry, Poisson point process (PPP) [3] has been widely adopted to model the transmitter locations. Under the PPP model, transmitters in a network are assumed to be independently distributed over the space in $\mathbb{R}^2$. Due to the availability of rich analytical tools for PPP, such as probability generating functional (PGFL) and reduced Palm distribution, existing work based on PPP has obtained tractable analytical results on characterizing the ad hoc network performance. For example, the authors in [4] derived a closed-form expression for the transmission capacity, which measures the single-hop communication performance in an ad-hoc network. In both [5] and [6], the authors considered the case where each transmitter decides whether to transmit based on its direct channel strength to its intended receiver, and studied the transmission capacity for this case.

It came to our attention that most of the existing work, including the aforementioned ones, only studied the system performance under some simple communication schemes, i.e., each transmitter independently transmits based upon its own willing [4] or its own channel strength [5], [6]. There has been very limited work on investigating the system performance for more practical transmission schemes, such as the carrier sensing multiple access (CSMA) and signal-to-interference-ratio (SIR) based scheduling, which involve more complex transmitter interactions. It is noted that based on Matern point process, the authors in [7] studied the system performance of a dense IEEE 802.11 network with CSMA.

In this paper, we propose a transmission scheme with SIR-threshold based scheduling and characterize the spatial capacity of ad-hoc networks by this scheme. Specifically, we consider a "probe-then-send" protocol, which includes a pilot phase, a feedback phase, and a data transmission phase. In the pilot phase, all transmitters transmit pilot signals independently to their intended receivers, which estimate the SIR values of their respective links. In the feedback phase, each receiver sends back to its transmitter the estimated SIR. In the data transmission phase, each transmitter starts data transmission if the feedback SIR is no smaller than a predefined threshold, denoted by $\gamma \geq 0$, or otherwise stay idle. For comparison, we also consider a reference transmission scheme in the conventional setup (see e.g. [4]), where all transmitters transmit independently in both the pilot and data transmission phases, without the SIR feedback from their receivers. For both schemes, a link communicates data successfully if the SIR at its receiver in the data transmission phase is no smaller than a required SIR level, denoted by $\beta > 0$. We assume a homogeneous PPP to model the locations of transmitters that have intention to transmit in the pilot phase for both schemes. However, unlike the reference scheme in which all transmitters are retained in the data transmission phase, in the proposed scheme, due to the SIR-based scheduling, the point process formed by the retained transmitters with a given SIR threshold $\gamma > 0$ is generally not a PPP in the data transmission phase. First, we compare the spatial capacity under these two schemes, and show that if $0 < \gamma < \beta$, the spatial capacity by the proposed scheme is strictly larger than that by the reference scheme. Next, we study closed-form characterizations of the spatial capacity for both schemes. In particular, for the proposed scheme, we find the exact spatial capacity expressions for the cases of $\gamma = 0$ and $\gamma \geq \beta$, and an approximate expression for the case of $0 < \gamma < \beta$. Finally, simulation results are provided to validate our analysis. It is shown that for the case of $0 < \gamma < \beta$, the approximate spatial capacity expression for the proposed scheme is indeed very tight.

II. System Model and Performance Metric

In this section, we first describe the two transmission schemes considered in this paper. We then develop the network model based on stochastic geometry. At last, we define the spatial capacity as our performance metric, and compare it for the two considered schemes.

A. Transmission Schemes

We consider two transmission schemes in this paper. One is the scheme with SIR-threshold based scheduling, and the other is a reference scheme without any transmission scheduling. In both schemes, we assume that all transmitters transmit in a synchronized time-slotted manner. We also assume that all transmitters transmit at the same power level, which is assumed to be unity for convenience.

1) SIR-Threshold Based Scheme: In each time slot, three phases, namely the pilot phase, the feedback phase, and the data transmission phase, are implemented sequentially, for which the details are given as follows:[1]

- In the pilot phase, to initialize the communication between each transmitter and receiver pair, all transmitters independently transmit pilot signals to their intended receivers. Each receiver then estimates the channel amplitude and phase (for possible coherent communication in the subsequent data transmission phase), as well as the received signal power over the total interference power, i.e., the SIR.
- In the feedback phase, each receiver sends back the measured SIR to its associated transmitter. For simplicity, we assume that the feedback is perfect and the SIR value is exactly known at the transmitter.
- In the data transmission phase, if the feedback SIR is above a predefined threshold, denoted by $\gamma \geq 0$, the transmitter sends data to its intended receiver; otherwise, the transmitter remains silent in the rest of the time slot. The data transmission is successful if the SIR at the receiver is larger than or equal to a required SIR level, denoted by $\beta > 0$.

2) Reference Scheme: In the reference scheme, we assume that there is no SIR feedback in each time slot, and thus each slot consists of only the pilot and data transmission phases. Due to the lack of SIR feedback, each transmitter is unaware of its link quality and thus transmits in both the pilot and data transmission phases continuously. The data transmission is successful if the SIR at the receiver is larger than or equal to the the required SIR level β as the proposed scheme.

[1] In general, there may exist multiple transmission and feedback iterations, such that each transmitters can iteratively adjust power levels based on the feedback information, as in [8], [9]. In this paper, for simplicity, we assume that there is only one feedback phase followed by the data transmission phase, and the transmit power adaptation is restricted to be binary.

B. Network Model

Next, we develop the network model based on stochastic geometry. For both considered transmission schemes, we focus on single-hop communication in one particular time slot.

For both schemes, we assume that initially all transmitters are independently uniformly distributed in the unbounded two-dimensional plane $\mathbb{R}^2$. We thus model the locations of all transmitters by a homogeneous PPP, denoted by Φ_0, of density λ_0. Each transmitter has one intended receiver, which is assumed to be uniformly distributed on a circle of radius d meters (m) centered at the transmitter. We denote the locations of the i-th transmitter and its intended receiver as x_i, with $x_i \in \Phi_0$, and r_i (not included in Φ_0), respectively. The path loss between the i-th transmitter and the j-th receiver is given by $l_{ij} = |x_i - r_j|^{-\alpha}$, where $\alpha > 2$ is the path-loss exponent. We use h_{ij} to denote the distance-independent channel fading coefficient from transmitter i to receiver j. We assume flat Rayleigh fading, where all h_{ij}'s are independent and exponentially distributed random variables with unit mean. We denote the SIR at the i-th receiver in the pilot phase as $\text{SIR}_i^{(0)}$, which is given by

$$\text{SIR}_i^{(0)} = \frac{h_{ii} d^{-\alpha}}{\sum_{x_j \in \Phi_0, j \neq i} h_{ji} l_{ji}}. \quad (1)$$

For the proposed scheme with SIR-threshold based scheduling, in the data transmission phase, only transmitters with $\text{SIR}_i^{(0)} \geq \gamma$ are retained to transmit. Denote $\Phi_1 = \{x_i \in \Phi_0 : \text{SIR}_i^{(0)} \geq \gamma\}$, which is the point process formed by all the retained transmitters in the data transmission phase. Due to the stationarity of the homogeneous PPP Φ_0, the resulting point process Φ_1 is also stationary [6]. However, since each $\text{SIR}_i^{(0)}$ is dependent on all transmitters' locations, the transmitters are *not* retained independently to transmit in the data transmission phase. As a result, the point process Φ_1 is generally *not* a PPP [1]. Supposing that transmitter i is retained to transmit, we denote the SIR at receiver i in the data transmission phase as

$$\text{SIR}_i^{(1)} = \frac{h_{ii} d^{-\alpha}}{\sum_{x_j \in \Phi_1, j \neq i} h_{ji} l_{ji}}. \quad (2)$$

Due to the reduced number of transmitters in the data transmission phase as compared to that in the pilot phase, it is easy to verify that $\text{SIR}_i^{(1)} \geq \text{SIR}_i^{(0)}$, with any given $\gamma > 0$. For the proposed scheme, the data transmission of transmitter i is successful if both $\text{SIR}_i^{(0)} \geq \gamma$ and $\text{SIR}_i^{(1)} \geq \beta$ are satisfied.

On the other hand, for the reference scheme without transmission scheduling, since all transmitters transmit in the data transmission phase, the point process formed by all transmitters in the data transmission phase is still Φ_0, the same as that in the pilot phase. As a result, the received SIR at receiver i in the data transmission phase remains unchanged as $\text{SIR}_i^{(0)}$. For the reference scheme, the data transmission of transmitter i is successful if $\text{SIR}_i^{(0)} \geq \beta$ is satisfied.

C. Spatial Capacity

We consider a typical (reference) pair of transmitter and receiver. Without loss of generality, we assume that the typical receiver is located at the origin. The typical pair of transmitter and receiver is named pair 0, i.e., $i=0$. Denote the *successful transmission probability* of the reference pair in the data transmission phase of the proposed scheme or the reference scheme as $\mathcal{P}_0^p$ or $\mathcal{P}_0^r$. We thus have

$$\mathcal{P}_0^p = \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma, \mathrm{SIR}_0^{(1)} \geq \beta) \tag{3}$$

$$= \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)\mathbb{P}(\mathrm{SIR}_0^{(1)} \geq \beta | \mathrm{SIR}_0^{(0)} \geq \gamma). \tag{4}$$

$$\mathcal{P}_0^r = \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \beta). \tag{5}$$

In this paper, we adopt the *spatial capacity* of ad hoc networks as our performance metric, which is defined as the spatial density of successful transmissions, or more specifically the average number of transmitters with successful data transmission per unit area.[2] We thus define the spatial capacity in the proposed scheme and the reference scheme as $\mathcal{C}^p$ and $\mathcal{C}^r$, respectively, given by

$$\mathcal{C}^p \triangleq \lambda_0 \mathcal{P}_0^p, \tag{6}$$

$$\mathcal{C}^r \triangleq \lambda_0 \mathcal{P}_0^r. \tag{7}$$

From (4) and (6), we can express $\mathcal{C}^p$ alternatively as

$$\mathcal{C}^p = \lambda_0 \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)\mathbb{P}(\mathrm{SIR}_0^{(1)} \geq \beta | \mathrm{SIR}_0^{(0)} \geq \gamma)$$

$$= \lambda_1 \mathbb{P}(\mathrm{SIR}_0^{(1)} \geq \beta | \mathrm{SIR}_0^{(0)} \geq \gamma) \tag{8}$$

where $\lambda_1 = \lambda_0 \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)$ is the density of Φ_1, with $\lambda_1 \leq \lambda_0$.

Clearly, to obtain $\mathcal{C}^p$ and $\mathcal{C}^r$, the key is to find the successful transmission probabilities $\mathcal{P}_0^p$ and $\mathcal{P}_0^r$. In the reference scheme, denote the total interference power received at the typical receiver in both pilot and data transmission phases as $I_0 = \sum_{x_i \in \Phi_0, i \neq 0} h_{i0} l_{i0}$. In the proposed scheme, the received total interference power at the typical receiver in the pilot phase is thus I_0, while that in the data transmission phase is given by $I_1 = \sum_{x_i \in \Phi_1, i \neq 0} h_{i0} l_{i0}$. Clearly, $I_0 \geq I_1$ since Φ_1 is a subset of Φ_0. In the following proposition, by comparing $\mathcal{P}_0^p$ and $\mathcal{P}_0^r$, we show the relationship between $\mathcal{C}^p$ and $\mathcal{C}^r$.

Proposition 1: Given the required SIR level $\beta > 0$ for reliable data transmission, for any $\gamma \in [0, \infty)$, we have

$$\begin{cases} \mathcal{C}^p > \mathcal{C}^r, & \text{if } 0 < \gamma < \beta \\ \mathcal{C}^p = \mathcal{C}^r, & \text{if } \gamma = 0 \text{ or } \gamma = \beta \\ \mathcal{C}^p < \mathcal{C}^r, & \text{if } \gamma > \beta. \end{cases} \tag{9}$$

Proof: By expressing $\mathrm{SIR}_0^{(0)} = h_{00} d^{-\alpha}/I_0$ and $\mathrm{SIR}_0^{(1)} = h_{00} d^{-\alpha}/I_1$, based on (3)-(7), we have

$$\frac{\mathcal{C}^p}{\mathcal{C}^r} = \frac{\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0) \times \mathbb{P}(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma d^\alpha I_0)}{\mathbb{P}(h_{00} \geq \beta d^\alpha I_0)}. \tag{10}$$

In the following, we compare $\frac{\mathcal{C}^p}{\mathcal{C}^r}$ with 1 by varying $\gamma \in [0, \infty)$. Clearly, when $\gamma = 0$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} = 1$. Next, we consider the case of $\gamma \geq \beta$. Since $I_0 \geq I_1$, if $\gamma \geq \beta$, we obtain $\mathbb{P}(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma d^\alpha I_0) = 1$. Moreover, for the non-negative and continuous random variables h_{00} and I_0, it is easy to find that if $\gamma > \beta$, $\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0) < \mathbb{P}(h_{00} \geq \beta d^\alpha I_0)$, and if $\gamma = \beta$, $\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0) = \mathbb{P}(h_{00} \geq \beta d^\alpha I_0)$. As a result, from (10), if $\gamma > \beta$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} < 1$, and if $\gamma = \beta$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} = 1$. At last, we consider the case of $0 < \gamma < \beta$. In this case, we have $\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0 | h_{00} \geq \beta d^\alpha I_0) = 1$, or equivalently,

$$\frac{\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0) \times \mathbb{P}(h_{00} \geq \beta d^\alpha I_0 | h_{00} \geq \gamma d^\alpha I_0)}{\mathbb{P}(h_{00} \geq \beta d^\alpha I_0)} = 1. \tag{11}$$

Moreover, since $\gamma \neq 0$ in this case, we have $F_{I_1}(x) > F_{I_0}(x)$, $\forall x > 0$, where $F_{I_0}(\cdot)$ and $F_{I_1}(\cdot)$ denote the cumulative distribution functions (CDFs) of I_0 and I_1, respectively. It is then easy to verify that $\mathbb{P}(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma d^\alpha I_0) > \mathbb{P}(h_{00} \geq \beta d^\alpha I_0 | h_{00} \geq \gamma d^\alpha I_0)$, for which, by multiplying $\frac{\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0)}{\mathbb{P}(h_{00} \geq \beta d^\alpha I_0)}$ on both sides and based on (11), we have

$$\frac{\mathbb{P}(h_{00} \geq \gamma d^\alpha I_0) \times \mathbb{P}(h_{00} \geq \beta d^\alpha I_1 | h_{00} > \gamma d^\alpha I_0)}{\mathbb{P}(h_{00} \geq \beta d^\alpha I_0)} > 1.$$

That is, $\frac{\mathcal{C}^p}{\mathcal{C}^r} > 1$. Proposition 1 thus follows. ■

Remark 1: Proposition 1 shows that, compared to the spatial capacity in the reference scheme, for the proposed scheme with SIR-threshold based scheduling, if the transmission decision is *aggressive* with $\gamma > \beta$, the spatial capacity is reduced; however, if the transmission decision is *conservative* with $0 < \gamma < \beta$, the spatial capacity can be improved. It is also noted that if the transmission decision is too conservative with $\gamma = 0$, the proposed scheme is reduced to the reference scheme; as a result, the spatial capacity is identical for the two schemes. Moreover, if the transmission decision is neither conservative nor aggressive, i.e., $\gamma = \beta$, the spatial capacity is also identical for the two schemes. At last, it is worth noting that Proposition 1 holds regardless of the specific channel fading distribution and/or transmitter location distribution.

In the next two sections, we will characterize the exact/approximate expressions of $\mathcal{C}^p$ and $\mathcal{C}^r$ for the reference scheme and the proposed scheme, respectively.

III. Spatial Capacity of Reference Scheme

In this section, we characterize the spatial capacity of the reference scheme. We first derive $\mathcal{P}^r$, given in (5), which is the complementary cumulative distribution function (CCDF) of $\mathrm{SIR}_0^{(0)}$ taken at the value of β.

Proposition 2: The successful transmission probability in the reference scheme is

$$\mathcal{P}_0^r = \exp(-\pi \lambda_0 d^2 \beta^{\frac{2}{\alpha}} \rho), \tag{12}$$

where $\rho = \int_0^\infty \frac{1}{1+v^{\alpha/2}} dv$. When the path loss exponent is $\alpha = 4$, we have $\rho = \frac{\pi}{2}$.

The proof of Proposition 2 is similar to that of [10, Theorem 2] and thus is omitted here.

Based on (7) and (12), we can express the spatial capacity of the reference scheme as $\mathcal{C}^r = \lambda_0 \exp(-\pi \lambda_0 d^2 \beta^{\frac{2}{\alpha}} \rho)$.

[2] In general, the time overhead for pilot transmissions in both considered schemes as well as the feedback delay in the proposed scheme should be considered since they reduce the capacity; however, in this paper, for the purpose of exposition, we ignore such factors in the capacity analysis.

IV. Spatial Capacity of Proposed Scheme

In this section, we characterize the spatial capacity of the proposed scheme with SIR-threshold based scheduling. Similar to the case of the reference scheme, we first derive the successful transmission probability $\mathcal{P}_0^p$ given in (3). Different from $\mathcal{P}_0^r$ in the reference scheme case, $\mathcal{P}_0^p$ is given by the *joint* CCDF of $\mathrm{SIR}_0^{(0)}$ and $\mathrm{SIR}_0^{(1)}$ taken at values (γ, β). Furthermore, the point process Φ_1 that determines $\mathrm{SIR}_0^{(1)}$ is generally not a PPP. We also note that $\mathrm{SIR}_0^{(0)}$ and $\mathrm{SIR}_0^{(1)}$ are not independent of each other. Therefore, it is challenging to find $\mathcal{P}_0^p$ and thus $\mathcal{C}^p$ in the case of proposed scheme.

In the following, we consider three cases of γ, which are: $\gamma = 0$, $\gamma \geq \beta$, and $0 < \gamma < \beta$. As will be shown in the next two subsections, for the case of $\gamma = 0$ or $\gamma \geq \beta$, we find a simple way to express $\mathcal{P}_0^p$ and thus an exact expression for $\mathcal{C}^p$; however, for the case of $\gamma \geq \beta$, $\mathcal{P}_0^p$ cannot be exactly expressed and thus we find an approximate expression for it.

A. The Case of $\gamma = 0$ or $\gamma \geq \beta$

We first consider the simple case with $\gamma = 0$, for which we can infer from Proposition 1 directly that $\mathcal{C}^p = \mathcal{C}^r$.

Next, consider the case of $\gamma \geq \beta$. Since $\mathrm{SIR}_0^{(0)} \leq \mathrm{SIR}_0^{(1)}$, if $\gamma \geq \beta$, we have $\mathbb{P}(\mathrm{SIR}_0^{(1)} \geq \beta | \mathrm{SIR}_0^{(0)} \geq \gamma) = 1$. Thus, based on (3) and (4), we can reduce the joint CCDF of $\mathrm{SIR}_0^{(0)}$ and $\mathrm{SIR}_0^{(1)}$ at (γ, β) to the CCDF of $\mathrm{SIR}_0^{(0)}$ at γ, i.e.,

$$\begin{aligned}\mathcal{P}_0^p &= \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)\mathbb{P}(\mathrm{SIR}_0^{(1)} \geq \beta | \mathrm{SIR}_0^{(0)} \geq \gamma) \\ &= \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma) \\ &\overset{(a)}{=} \exp(-\pi\lambda_0 d^2 \gamma^{\frac{2}{\alpha}} \rho)\end{aligned}$$

where (a) is obtained by replacing β with γ in Proposition 2. Therefore, when $\gamma \geq \beta$, we have $\mathcal{C}^p = \lambda_0 \exp(-\pi\lambda_0 d^2 \gamma^{\frac{2}{\alpha}} \rho)$.

B. The Case of $0 < \gamma < \beta$

For the case of $0 < \gamma < \beta$, $\mathcal{P}_0^p$ cannot be simply expressed as in the cases of $\gamma = 0$ and $\gamma \geq \beta$. Moreover, due to the dependence of $\mathrm{SIR}_0^{(0)}$ and $\mathrm{SIR}_0^{(1)}$ as well as the non-Poisson retained transmitter distribution of Φ_1, it is very difficult, if not impossible, to find an exact expression of $\mathcal{P}_0^p$ in this case. In this subsection, we propose an approximation method to express $\mathcal{P}_0^p$.

Denote the point process formed by the transmitters which are not retained in the data transmission phase as Φ_1^c, of density $\lambda_1^c = \lambda_0 - \lambda_1$. Similar to Φ_1, due to the location dependence of transmitters, Φ_1^c is generally not a PPP. Note that $\Phi_1 \cup \Phi_1^c = \Phi_0$ and $\Phi_1 \cap \Phi_1^c = \emptyset$, where Φ_1 and Φ_1^c are *not* independent of each other in general. Next, we state one assumption, based on which we will obtain two independent homogeneous PPPs of the same density as that of Φ_1 and Φ_1^c, respectively.

Assumption 1: In the proposed scheme with SIR-threshold based scheduling, the transmitters are retained independently in the data transmission phase, with probability $\mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)$.

Under Assumption 1, the resulting point process formed by the retained transmitters in the data transmission phase is a homogeneous PPP, denoted by $\hat{\Phi}_1$, of density $\lambda_1 = \lambda_0 \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma)$. Similarly, the point process formed by the transmitters which are not retained in the data transmission phase is also a homogeneous PPP under Assumption 1, denoted by $\hat{\Phi}_1^c$, of density $\lambda_1^c = \lambda_0 - \lambda_1$. Since the two homogeneous PPPs $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$ are disjoint, they are *independent* of each other [1]. Denote $\hat{I}_1 = \sum_{i \in \hat{\Phi}_1} h_{i0} l_{i0}$ and $\hat{I}_1^c = \sum_{i \in \hat{\Phi}_1^c} h_{i0} l_{i0}$ as the received interference power at the typical receiver in $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively. Next, we approximate Φ_1 and Φ_1^c by $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively, and thereby obtain an approximate expression of $\mathcal{P}_0^p$ in the following proposition.

Proposition 3: With $0 < \gamma < \beta$, under Assumption 1, the successful transmission probability in the proposed scheme is approximated as

$$\mathcal{P}_0^p \approx \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2)\, dx_2\, dx_1\, dh_{00} \quad (13)$$

where $f_{\hat{I}_1}(x_1)$ is the probability density function (pdf) of $\hat{I}_1$ and $f_{\hat{I}_1^c}(x_2)$ is the pdf of $\hat{I}_1^c$.

Proof: Under Assumption 1, we obtain two independent PPPs $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, with $\hat{\Phi}_1 \cup \hat{\Phi}_1^c = \Phi_0$ and $\hat{\Phi}_1 \cap \hat{\Phi}_1^c = \emptyset$. Since from (3), it follows that

$$\begin{aligned}\mathcal{P}_0^p &= \mathbb{P}(\mathrm{SIR}_0^{(0)} \geq \gamma, \mathrm{SIR}_0^{(1)} \geq \beta) \\ &= \mathbb{P}\Big(\sum_{i \in \Phi_0} h_{i0} l_{i0} \leq \frac{h_{00}}{\gamma d^\alpha}, \sum_{i \in \Phi_1} h_{i0} l_{i0} \leq \frac{h_{00}}{\beta d^\alpha}\Big),\end{aligned}$$

we have

$$\begin{aligned}\mathcal{P}_0^p &\approx \mathbb{P}\Big(\Big(\sum_{i \in \hat{\Phi}_1} h_{i0} l_{i0} + \sum_{i \in \hat{\Phi}_1^c} h_{i0} l_{i0}\Big) \leq \frac{h_{00}}{\gamma d^\alpha}, \sum_{i \in \hat{\Phi}_1} h_{i0} l_{i0} \leq \frac{h_{00}}{\beta d^\alpha}\Big) \\ &= \mathbb{P}\Big(\hat{I}_1 + \hat{I}_1^c \leq \frac{h_{00}}{\gamma d^\alpha}, \hat{I}_1 \leq \frac{h_{00}}{\beta d^\alpha}\Big).\end{aligned}$$

Due to the independence of $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, $\hat{I}_1$ is independent of $\hat{I}_1^c$. Given h_{00}, we thus have

$$\begin{aligned}&\mathbb{P}\Big(\hat{I}_1 + \hat{I}_1^c \leq \frac{h_{00}}{\gamma d^\alpha}, \hat{I}_1 \leq \frac{h_{00}}{\beta d^\alpha} \Big| h_{00}\Big) \\ &= \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2)\, dx_2\, dx_1. \quad (14)\end{aligned}$$

By integrating (14) over the (exponential) distribution of h_{00}, we obtain (13). Proposition 3 thus follows. ■

For a homogeneous PPP of density λ, when the channel fading is Rayleigh distributed, the pdf of the received interference I at the typical receiver is given by [2]

$$f_I(x) = \frac{1}{\pi x} \sum_{i=1}^\infty \frac{(-1)^{i+1}\Gamma(1+i\delta)\sin(\pi i\delta)}{i!} \Big(\frac{\lambda\pi^2\delta}{x^\delta \sin(\pi\delta)}\Big)^i \quad (15)$$

where $\delta = 2/\alpha$. Note that when $\alpha = 4$, (15) can be further expressed in a simpler closed-form as

$$f_I(x) = \frac{\lambda}{4}\Big(\frac{\pi}{x}\Big)^{3/2} \exp\Big(-\frac{\pi^4\lambda^2}{16x}\Big). \quad (16)$$

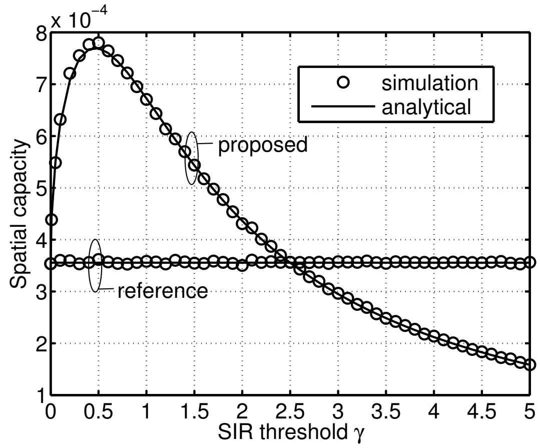

Fig. 1: Spatial capacity against γ.

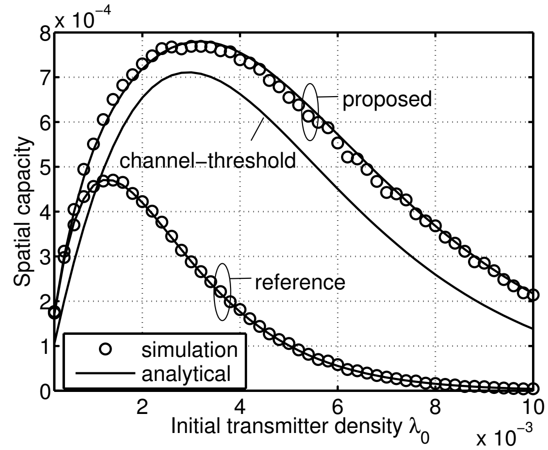

Fig. 2: Spatial capacity against λ_0.

By substituting $\lambda = \lambda_1$ to (15) and (16), we obtain $f_{\hat{I}_1}(x_1)$ for the cases of general α and $\alpha = 4$, respectively. Similarly, with $\lambda = \lambda_1^c$, from (15) and (16) we obtain $f_{\hat{I}_1^c}(x_2)$ for general α and $\alpha = 4$, respectively.

From (6), for the case of $0 < \gamma < \beta$, an approximate expression of $\mathcal{C}^p$ is obtained by multiplying λ_0 with the right-hand side of (13).

V. Simulation results

Numerical results are obtained according to the simulation method described in [1], of which the details are omitted due to the space limitation. We set $\beta = 2.5$, $\alpha = 4$, and $d = 10$m.

Fig. 1 shows the spatial capacity versus the SIR threshold γ, for both the reference scheme without transmission scheduling and the proposed scheme with SIR-based scheduling. We set the initial transmitter density as $\lambda_0 = 0.0025/\text{m}^2$ in both schemes. From the simulation results, we observe that $\mathcal{C}^r$ is constant over γ as expected. We also observe that 1) when $\gamma < \beta$, $\mathcal{C}^p > \mathcal{C}^r$; 2) when $\gamma = 0$ or $\gamma = \beta$, $\mathcal{C}^p = \mathcal{C}^r$; and 3) when $\gamma > \beta$, $\mathcal{C}^p < \mathcal{C}^r$. This is in accordance with our analytical results in Proposition 1. Furthermore, it is observed that for both schemes, the simulation results of the spatial capacity fit well to the analytical counterparts, even when $0 < \gamma < \beta$, for which only approximate expressions for the spatial capacity of the proposed scheme are available.

Fig. 2 shows the spatial capacity versus the initial transmitter density λ_0. We set the SIR threshold as $\gamma = 0.6$. It is observed that the spatial capacity of the proposed scheme is always larger than that of the reference scheme for all values of λ_0, which is as expected from Proposition 1 since $\gamma < \beta$ in this example. For another comparison, Fig. 2 also shows the spatial capacity by the scheme proposed in [5] and [6] with a channel-threshold based scheduling, where transmitter i decides to transmit in the data transmission phase if its direct channel power is no smaller than a predefined threshold γ', i.e., $h_{ii} \geq \gamma'$. In this example, we set $\gamma' = \gamma$. We observe that the spatial capacity of this scheme is smaller than that of the reference scheme when λ_0 is small, and becomes larger when λ_0 is sufficiently large. This is in sharp contrast to the case of proposed scheme, which always guarantees a capacity improvement over the reference scheme, provided that $0 < \gamma < \beta$. Furthermore, for all three schemes considered, we observe that the spatial capacity first increases with λ_0 due to more available transmitters, but as λ_0 exceeds a certain threshold, it starts to decrease with increasing λ_0, due to the more dominant interference effect.

VI. Conclusion

In this paper, we studied the spatial capacity of *ad hoc* networks based on stochastic geometry. We proposed a transmission scheme with SIR-threshold based scheduling as compared to a reference scheme with no transmission scheduling. We showed the conditions under which the spatial capacity of the proposed scheme performs strictly better than that of the reference scheme. Furthermore, we characterize the spatial capacity of the proposed scheme in closed-form. In particular, we propose a new method to approximate the spatial capacity, which is useful for analyzing performance of wireless networks with interacted transmitters.

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