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On Cooperative Multiple Access Channels with Delayed CSI

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Abstract— **We consider a two-user state-dependent multi-access channel in which the states of the channel are known, causally or only strictly causally, at both encoders, but not at the decoder. Both encoders transmit a common message and, one of the encoders also transmits an individual message. We study the capacity region of this communication model for both causal and strictly causal settings. For the model with causal states, we find an explicit characterization of the capacity region in the discrete memoryless case. For the model with strictly causal states, we establish inner and outer bounds on the capacity region. The outer bound is non-trivial, has a relatively simple form and has the advantage of incorporating *only one auxiliary random variable*. In particular, it suggests that there is none, or at best only little, to gain from having the encoder that transmits both messages also sending an individual description of the state to the receiver, in addition to the compressed version that is sent cooperatively with the other encoder. The results shed more light on the utility of delayed channel state information for increasing the capacity region of multiaccess channels; and tie with some recent progress in this framework.**

I. Introduction

A growing body of work studies multi-user state-dependent models. Relay channels with states are studied in [1]–[5]. Recent works on multiaccess channels with noncausal states at the encoders can be found in [6]–[8], among other contributions. State-controlled multiple access channels with states known causally or strictly causally at the encoders have been studied recently in [6], [9]–[15]. In [9], the authors consider a multiaccess channel with independent inputs, and the states known causally or strictly causally at both encoders. The authors show that, in both settings, the knowledge of the states can be beneficial, in the sense that it increases the capacity region. This result is reminiscent of Dueck's proof [16] that feedback can increase the capacity region of some broadcast channels. Also, in accordance with [16], the main idea of the achievability result in [9] is a block Markov coding scheme in which the two users collaborate to describe the state to the decoder by sending cooperatively a compressed version of it. As noticed in [9], although some non-zero rate that otherwise could be used to transmit pure information is spent in describing the state to the decoder, the net effect can be an increase in the capacity.

In this paper, we study a two-user state-dependent multiple access channel with the channel states known only strictly-causally or causally at the encoders. Both encoders transmit a common message and, in addition, the encoder that knows the states non-causally transmits an individual message. More precisely, let W_c and W_1 denote the common message and the individual message to be transmitted in, say, n uses of the channel; and $S^n = (S_1, \ldots, S_n)$ denote the state sequence affecting the channel during the transmission. In the causal setting, at time i both encoders know the channel states up to and including time i, i.e., the sequence $S^i = (S_1, \ldots, S_{i-1}, S_i)$. In the strictly causal setting, at time i the encoders know the channel states only up to time $i-1$, i.e., the sequence $S^{i-1} = (S_1, \ldots, S_{i-1})$.

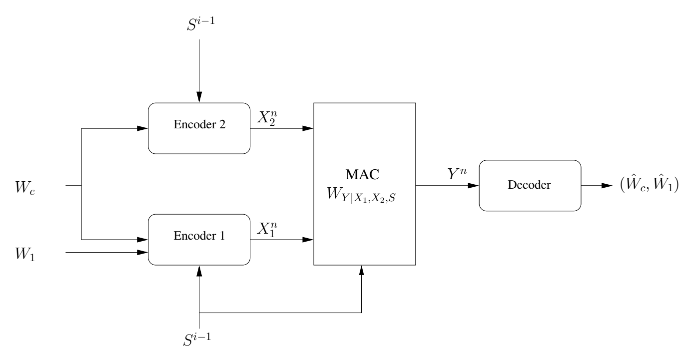

Fig. 1. State-dependent MAC with degraded messages sets and states known, strictly causally, at only the encoders.

We study the capacity region of this state-dependent MAC model under both causal and strictly causal settings. For the model with causal states, we characterize the capacity region in the discrete memoryless case. For the model with strictly causal states, building on the recent related results that we already mentioned above, and for instance the model in [6] in which, like in this paper, one of the encoders knows the other encoder's message, it can be seen that the knowledge of the states strictly causally at the encoders is helpful and increases the capacity region (this will be illustrated through an example in Section III-C). Like for the MAC with independent inputs, this is obtained by sending a compressed version of the state to the receiver. For the model that we study, it is not clear how this compression should be performed optimally, although one of the encoders knows both messages. More specifically, while it can be expected that sending a compressed version of the state cooperatively by the two encoders is beneficial, it is not clear whether sending another layer of state compression by the encoder that transmits both messages increases the transmission rates beyond what is possible with only the cooperative layer. Note that in the multiaccess channel with independent inputs of [9], it is beneficial that each encoder sends also an individual description of the state to the receiver, in addition to the description of the state that is sent cooperatively by both encoders; and this is reflected therein through the fact that the improved inner bound of [9, Theorem 2] strictly outperforms that of [9, Theorem 1] – the improvement comes precisely from the fact that, for both encoders, in each block, a part of the input is composed of an individual compression of the state and the input in the previous block.

In this paper, for the model with strictly causal states, we establish inner and outer bounds on the capacity region.The outer bound is non trivial, and has the advantage of having a relatively simple form that incorporates directly the channel inputs X_1 and X_2 and *only one auxiliary random variable*. To establish this outer bound, we first derive another outer bound on the capacity region, whose expression depends naturally on two auxiliary random variables. We then show that this outer bound can in fact be recast into a simpler form which is more insightful, and whose expression depends on only one auxiliary random variable. This is obtained by showing that the second auxiliary random variable can in fact be chosen optimally to be a constant. In addition to its simplicity, the resulting expression of the outer bound has the advantage of suggesting that, by opposition to the MAC with independent inputs, for the model that we study there is no gain, or at the best only little, to expect from having the encoder that transmits both messages also sending an individual compression of the state to the receiver, in addition to the cooperative compression. While this does not provide a formal proof of this fact, it is insightful for constructing efficient coding schemes. Next, using the insights that we gain from the obtained outer bound, i.e., in its simple form, we establish an inner bound on the capacity region. This inner bound is based on a Block-Markov coding scheme in which the two encoders collaborate in both transmitting the common message and also conveying a lossy version of the state to the decoder. Also, the encoder that transmits both messages does *not* send any individual compression of the state beyond what is done cooperatively.

The inner and outer bounds differ only through the associated joint measures; and, for instance, a Markov-chain relation that holds for the inner bound and not for the outer bound. Next, by investigating a discrete memoryless example, we show that (may be as expected) revealing the state even only strictly causally to the encoder that sends only the common message is beneficial and enlarges the capacity region in general. However, this does not increase the sum-rate capacity. Also, revealing the states to only the encoder that sends both messages is not beneficial.

II. Problem Setup

We consider a stationary memoryless state-dependent MAC $W_{Y|X_1,X_2,S}$ whose output $Y \in \mathcal{Y}$ is controlled by the channel inputs $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ from the encoders and the channel state $S \in \mathcal{S}$ which is drawn according to a memoryless probability law Q_S. We study both settings, the case in which the encoders know the states causally, and the case in which the encoders know the states only strictly causally. In the causal-states setting, at time i the encoders know the values of the state sequence up to and including time i, i.e., $S^i = (S_1, \ldots, S_{i-1}, S_i)$. In the strictly-causal states setting, at time i the encoders know the values of the state sequence up to time $i-1$, i.e., $S^{i-1} = (S_1, \ldots, S_{i-1})$.

Encoder 2 wants to send a common message W_c and Encoder 1 wants to send an independent individual message W_1 along with the common message W_c. We assume that the common message W_c and the individual message W_1 are independent random variables drawn uniformly from the sets $\mathcal{W}_c = \{1, \cdots, M_c\}$ and $\mathcal{W}_1 = \{1, \cdots, M_1\}$, respectively. The sequences X_1^n and X_2^n from the encoders are sent across a state-dependent multiple access channel modeled as a memoryless conditional probability distribution $W_{Y|X_1,X_2,S}$. The laws governing the state sequence and the output letters are given by

$$W^n_{Y|X_1,X_2,S}(y^n|x_1^n, x_2^n, s^n) = \prod_{i=1}^{n} W_{Y|X_1,X_2,S}(y_i|x_{1i}, x_{2i}, s_i) \qquad (1)$$

$$Q^n_S(s^n) = \prod_{i=1}^{n} Q_S(s_i). \qquad (2)$$

The receiver guesses the pair $(\hat{W}_c, \hat{W}_1)$ from the channel output Y^n.

***Definition** 1:* For positive integers n, M_c and M_1, an (M_c, M_1, n, ϵ) code for the multiple access channel with strictly causal states consists of a sequence of mappings

$$\phi_{1,i} : \mathcal{W}_c \times \mathcal{W}_1 \times \mathcal{S}^{i-1} \longrightarrow \mathcal{X}_1, , \quad i = 1, \ldots, n \qquad (3)$$

at Encoder 1, a sequence of mappings

$$\phi_{2,i} : \mathcal{W}_c \times \mathcal{S}^{i-1} \longrightarrow \mathcal{X}_2, \quad i = 1, \ldots, n \qquad (4)$$

at Encoder 2, and a decoder map

$$\psi : \mathcal{Y}^n \longrightarrow \mathcal{W}_c \times \mathcal{W}_1 \qquad (5)$$

such that the average probability of error is bounded by ϵ,

$$P^n_e = \mathbb{E}_S\Big[\Pr\Big(\psi(Y^n) \neq (W_c, W_1)|S^n = s^n\Big)\Big] \leq \epsilon. \qquad (6)$$

The rate of the common message and the rate of the individual message are defined as

$$R_c = \frac{1}{n}\log M_c \qquad \text{and} \qquad R_1 = \frac{1}{n}\log M_1,$$

respectively. A rate pair (R_c, R_1) is said to be achievable if for every $\epsilon > 0$ there exists an $(2^{nR_c}, 2^{nR_1}, n, \epsilon)$ code for the channel $W_{Y|X_1,X_2,S}$. The capacity region $\mathcal{C}_{\text{s-c}}$ of the state-dependent MAC with strictly causal states is defined as the closure of the set of achievable rate pairs.

***Definition** 2:* For positive integers n, M_c and M_1, an (M_c, M_1, n, ϵ) code for the multiple access channel with causal states consists of a sequence of mappings

$$\phi_{1,i} : \mathcal{W}_c \times \mathcal{W}_1 \times \mathcal{S}^{i} \longrightarrow \mathcal{X}_1, , \quad i = 1, \ldots, n \qquad (7)$$

at Encoder 1, a sequence of mappings

$$\phi_{2,i} : \mathcal{W}_c \times \mathcal{S}^{i} \longrightarrow \mathcal{X}_2, \quad i = 1, \ldots, n \qquad (8)$$

at Encoder 2, and a decoder map (5) such that the probability of error is bounded as in (6).

The definitions of a rate pair (R_c, R_1) to be achievable as well as the capacity region, which we denote by C_c in this case, are similar to those in the strictly-causal states setting in Definition 1.

Due to space limitation, the results of this paper are either outlined only or mentioned without proofs. Detailed proofs can be found in [17].

III. Strictly Causal States

A. Outer Bound on the Capacity Region

Let $\tilde{\mathcal{P}}^{\text{out}}_{\text{s-c}}$ stand for the collection of all random variables (S, U, V, X_1, X_2, Y) such that U, V, X_1 and X_2 take values in finite alphabets $\mathcal{U}$, $\mathcal{V}$, $\mathcal{X}_1$ and $\mathcal{X}_2$, respectively, and satisfy

$$P_{S,U,V,X_1,X_2,Y}(s,u,v,x_1,x_2,y) = P_{S,U,V,X_1X_2}(s,u,v,x_1,x_2)W_{Y|X_1,X_2,S}(y|x_1,x_2,s) \tag{9a}$$

$$P_{S,U,V,X_1,X_2}(s,u,v,x_1,x_2) = Q_S(s)P_{X_2}(x_2)P_{X_1|X_2}(x_1|x_2) \cdot P_{V|S,X_1,X_2}(v|s,x_1,x_2)P_{U|S,V,X_1,X_2}(u|s,v,x_1,x_2) \tag{9b}$$

$$\sum_{u,v,x_1,x_2} P_{S,U,V,X_1,X_2}(s,u,v,x_1,x_2) = Q_S(s) \tag{9c}$$

and

$$0 \leq I(V, X_2; Y) - I(V, X_2; S). \tag{10}$$

The relations in (9) imply that $(U, V) \leftrightarrow (S, X_1, X_2) \leftrightarrow Y$ is a Markov chain, and X_1 and X_2 are independent of S.

Define $\tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}$ to be the set of all rate pairs (R_c, R_1) such that

$$\begin{aligned} R_1 &\leq I(U, X_1; Y|V, X_2) - I(U, X_1; S|V, X_2) \\ R_c + R_1 &\leq I(U, V, X_1, X_2; Y) - I(U, V, X_1, X_2; S) \\ &\quad \text{for some } (S, U, V, X_1, X_2, Y) \in \tilde{\mathcal{P}}^{\text{out}}_{\text{s-c}}. \end{aligned} \tag{11}$$

As stated in the following theorem, the set $\tilde{\mathcal{R}}^{\text{out}}$ is an outer bound on the capacity region of the state-dependent discrete memoryless MAC with strictly-causal states.

***Theorem** 1:* The capacity region of the multiple access channel with degraded messages sets and strictly causal states known only at the encoders satisfies

$$C_{\text{s-c}} \subseteq \tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}. \tag{12}$$

We now recast the outer bound $\tilde{\mathcal{R}}^{\text{out}}$ into a form that will be shown to be more convenient (see Remark 1 and Remark 2 below). This is done by showing that the maximizing auxiliary random variable U in $\tilde{\mathcal{R}}^{\text{out}}$ is a constant, i.e., $U = \emptyset$; and can be formalized as follows. Define $\mathcal{R}^{\text{out}}_{\text{s-c}}$ to be the set of all rate pairs (R_c, R_1) such that

$$\begin{aligned} R_1 &\leq I(X_1; Y|V, X_2) \\ R_c + R_1 &\leq I(V, X_1, X_2; Y) - I(V, X_1, X_2; S) \end{aligned} \tag{13}$$

for some measure of the form

$$P_{S,V,X_1,X_2,Y} = Q_S P_{X_2} P_{X_1|X_2} P_{V|S,X_1,X_2} W_{Y|X_1,X_2,S} \tag{14}$$

and satisfying (10).

It is easy to see that $\mathcal{R}^{\text{out}}_{\text{s-c}} \subseteq \tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}$, as $\mathcal{R}^{\text{out}}_{\text{s-c}}$ can be obtained from $\tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}$ by setting $U = \emptyset$. As shown in the proof of the theorem that will follow, $\tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}} \subseteq \mathcal{R}^{\text{out}}_{\text{s-c}}$; and so $\mathcal{R}^{\text{out}}_{\text{s-c}} = \tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}$. Thus, by Theorem 1, $\mathcal{R}^{\text{out}}$ is an outer bound on the capacity region of the state-dependent discrete memoryless MAC model with strictly-causal states.

***Theorem** 2:* The capacity region of the multiple access channel with degraded messages sets and strictly causal states known only at the encoders satisfies

$$C_{\text{s-c}} \subseteq \mathcal{R}^{\text{out}}_{\text{s-c}}. \tag{15}$$

The outer bound can be expressed equivalently using $\tilde{\mathcal{R}}^{\text{out}}_{\text{s-c}}$ or $\mathcal{R}^{\text{out}}_{\text{s-c}}$, since the two sets coincide. However, the form $\mathcal{R}^{\text{out}}_{\text{s-c}}$ of the outer bound is more convenient and insightful. The following remarks aim at reflecting this.

Remark 1: A we already mentioned, some recent results have shown the utility of strictly causal states at the encoders in increasing the capacity region of multiaccess channels in certain settings. For example, this has been demonstrated for a MAC with independent inputs and states known strictly causally at the encoders [9], [10], and for a MAC with degraded messages sets with the states known strictly causally to the encoder that sends only the common-message and noncausally at the encoder that sends both messages [6]. Also, in these settings, the increase in the capacity region is created by having the encoders cooperate to convey a lossy version of the state of the previous block to the receiver. Furthermore, in the case of the MAC with independent inputs of [9], it is shown that additional improvement can be obtained by having each encoder also sending a compressed version of the pair (input, state) of the previous block, in addition to the cooperative transmission with the other encoder of the common compression of the state. (This is reflected in [9] through the improvement of the inner bound of Theorem 2 therein over that of Theorem 1). In our case, since one encoder knows the other encoder's message, it is not evident à-priori to see whether a similar additional improvement could be expected from having the encoder that transmits both messages also sending another compression of the state, in addition to that sent cooperatively.

Remark 2: A direct proof of the outer bound in its form $\mathcal{R}^{\text{out}}_{\text{s-c}}$ does not seem to be easy to obtain because of the necessity of introducing two auxiliary random variables in typical outer bounding approaches that are similar to that of Theorem 1. In addition to that it is simpler comparatively, the form $\mathcal{R}^{\text{out}}_{\text{s-c}}$ of the outer bound is more convenient and insightful. It involves only one auxiliary random variable, V (which, in a corresponding coding scheme, would represent intuitively the lossy version of the state that is to be sent by the two encoders cooperatively). Because the auxiliary random variable U (which, in a corresponding coding scheme, would represent intuitively the additional compression of the state that is performed by the encoder that transmits both messages) can be set maximally to be a constant, the outer bound $\mathcal{R}^{\text{out}}_{\text{s-c}}$ suggests implicitly that there is no gain to be expected from additional compression at Encoder 1. That is, by opposition to the case of the MAC with independent inputs of [9], for our model, for an efficient exploitation of the knowledge of the states strictly causally at the encoders, it seems enough to send only the cooperative compression of the state to the receiver. We should mention that, although somewhat intuitive given known results on the role of feedback and strictly causal states at the encoder in point-to-point channels, a formal proof of the aforementioned fact for the model that we study does not follow directly from these existing results.

B. Inner Bound on the Capacity Region

Let $\mathcal{P}_{\text{s-c}}^{\text{in}}$ stand for the collection of all random variables (S, V, X_1, X_2, Y) such that V, X_1 and X_2 take values in finite alphabets $\mathcal{V}$, $\mathcal{X}_1$ and $\mathcal{X}_2$, respectively, and satisfy

$$
\begin{aligned}
&P_{S,V,X_1,X_2,Y}(s, v, x_1, x_2, y) \\
&\quad = P_{S,V,X_1,X_2}(s, v, x_1, x_2) W_{Y|X_1,X_2,S}(y|x_1, x_2, s) \qquad \text{(16a)}\\
&P_{S,V,X_1,X_2}(s, v, x_1, x_2) \\
&\quad = Q_S(s) P_{X_2}(x_2) P_{X_1|X_2}(x_1|x_2) P_{V|S,X_2}(v|s, x_2) \qquad \text{(16b)}\\
&\sum_{v,x_1,x_2} P_{S,V,X_1,X_2}(s, v, x_1, x_2) = Q_S(s) \qquad \text{(16c)}
\end{aligned}
$$

and

$$0 \leq I(V, X_2; Y) - I(V, X_2; S). \tag{17}$$

The relations in (16) imply that $V \leftrightarrow (S, X_1, X_2) \leftrightarrow Y$, $X_1 \leftrightarrow X_2 \leftrightarrow V$ and $X_1 \leftrightarrow (V, X_2) \leftrightarrow S$ are Markov chains; and X_1 and X_2 are independent of S.

Define $\mathcal{R}_{\text{s-c}}^{\text{in}}$ to be the set of all rate pairs (R_c, R_1) such that

$$
\begin{aligned}
R_1 &\leq I(X_1; Y|V, X_2)\\
R_c + R_1 &\leq I(V, X_1, X_2; Y) - I(V, X_1, X_2; S)\\
&\qquad \text{for some } (S, V, X_1, X_2, Y) \in \mathcal{P}_{\text{s-c}}^{\text{in}}. \qquad (18)
\end{aligned}
$$

As stated in the following theorem, the set $\mathcal{R}_{\text{s-c}}^{\text{in}}$ is an inner bound on the capacity region of the state-dependent discrete memoryless MAC model with strictly-causal states.

Theorem 3: The capacity region of the multiple access channel with degraded messages sets and strictly causal states known only at the encoders satisfies

$$\mathcal{R}_{\text{s-c}}^{\text{in}} \subseteq \mathcal{C}_{\text{s-c}}. \tag{19}$$

Remark 3: The inner bound $\mathcal{R}_{\text{s-c}}^{\text{in}}$ differs from the outer bound $\mathcal{R}_{\text{s-c}}^{\text{out}}$ only through the Markov chain $X_1 \leftrightarrow X_2 \leftrightarrow V$. The outer bound requires arbitrary dependence of the auxiliary random variable V (which represents the joint compression of the state) on the inputs X_1 and X_2 by the encoders. While in block i the dependence of V on the input X_2 by the encoder that sends only the common message can be obtained by generating the covering codeword $\mathbf{v}$ on top of the input codeword $\mathbf{x}_2$ from the previous block $i-1$ (and performing conditional compression of the state sequence from block $i-1$, i.e., conditionally on the input $\mathbf{x}_2$ by Encoder 2 in the previous block $i-1$), the dependence of V on the input X_1 by the encoder that transmits both messages is not easy to obtain. Partly, this is because i) the codeword $\mathbf{v}$ can not be generated on top of $\mathbf{x}_1$ (since Encoder 2 does not know the individual message of Encoder 1), and ii) the input $\mathbf{x}_1$ by Encoder 1 has to be independent of the state sequence $\mathbf{s}$.

Remark 4: The proof of Theorem 3 is based on a Block-Markov coding scheme in which the encoders collaborate to convey a lossy version of the state to the receiver, in addition to the information messages. The lossy version of the state is obtained through Wyner-Ziv compression. Also, in each block, Encoder 1 also transmits an individual information. However, in accordance with the aforementioned insights that we gain from the outer bound of Theorem 2, the state is sent to the receiver *only* cooperatively. That is, by opposition to the coding scheme of [9, Theorem 2] for the MAC with independent inputs, Encoder 1 does not compress or convey the state to the receiver beyond what is done cooperatively with Encoder 2. More specifically, the encoding and transmission scheme is as follows. Let $\mathbf{s}[i]$ denote the channel state in block i, and s_i the index of the cell $\mathcal{C}_{s_i}$ containing the compression index z_i of the state $\mathbf{s}[i]$, obtained through Wyner-Ziv compression. In block i, Encoder 2, which has learned the state sequence $\mathbf{s}[i-1]$, knows s_{i-2} and looks for a compression index z_{i-1} such that $\mathbf{v}(w_{c,i-1}, s_{i-2}, z_{i-1})$ is strongly jointly typical with $\mathbf{s}[i-1]$ and $\mathbf{x}_2(w_{c,i-1}, s_{i-2})$. It then transmits a codeword $\mathbf{x}_2(w_{c,i}, s_{i-1})$ (drawn according to the appropriate marginal using (16)), where the cell index s_{i-1} is the index of the cell containing z_{i-1}, i.e., $z_{i-1} \in \mathcal{C}_{s_{i-1}}$. Encoder 1 finds $\mathbf{x}_2(w_{c,i}, s_{i-1})$ similarly. It then transmits a vector $\mathbf{x}_1(w_{c,i}, s_{i-1}, w_{1i})$ (drawn according to the appropriate marginal using (16)).

C. On the Utility of the Strictly Causal States

Example: We use $h(\alpha)$ to denote the entropy of a Bernoulli (α) source, i.e.,

$$h(\alpha) = -\alpha \log(\alpha) - (1-\alpha)\log(1-\alpha) \tag{20}$$

and $p * q$ to denote the binary convolution, i.e.,

$$p * q = p(1-q) + q(1-p). \tag{21}$$

In this example all the random variables are binary $\{0, 1\}$. The channel has two output components, i.e., $Y^n = (Y_1^n, Y_2^n)$. The component Y_2^n is deterministic, $Y_2^n = X_2^n$, and the component $Y_1^n = X_1^n + S^n + Z_1^n$, where the addition is modulo 2. Encoder 2 has no message to transmit, and Encoder 1 transmits an individual message W_1. The encoders know the states only strictly causally. The state and noise vectors are independent and memoryless, with the state process S_i, $i \geq 1$, and the noise process $Z_{1,i}$, $i \geq 1$, assumed to be Bernoulli $(\frac{1}{2})$ and Bernoulli (p) processes, respectively. The vectors X_1^n and X_2^n are the channel inputs, subjected to the constraints

$$\sum_{i=1}^{n} X_{1,i} \leq nq_1 \quad \text{and} \quad \sum_{i=1}^{n} X_{2,i} \leq nq_2, \;\; q_2 \geq 1/2. \tag{22}$$

For this example, the strictly causal knowledge of the states at Encoder 2 increases the capacity, and in fact Encoder 1 can transmit at rates that are larger than the maximum rate that would be achievable had Encoder 2 been of no help.

Claim 1: The capacity of the state-dependent binary memoryless MAC example is given by

$$C_B = \max_{p(x_1)} \; I(X_1; Y_1|S). \tag{23}$$

It is easy to see that the capacity (23) can be achieved using the coding scheme of Theorem 3. This can be obtained by setting $V = S$ and observing that this choice also satisfies the constraint (17) for the example. Also, the capacity (23) is strictly larger that the he maximum rate that would be achievable had Encoder 2 be of no help. That is,

$$C_B > C = \max_{p(x_1)} I(X_1; Y_1). \tag{24}$$

The above example shows that the knowledge of the states strictly causally at the encoders increases the capacity region of the MAC model with degraded messages sets that we study. This fact has also been shown for other related models, such as a multiaccess channel with independent inputs and strictly causal or causal states at the encoders in [9], [10], and a multiaccess channel with degraded messages sets and states known noncausally to the encoder that sends both messages and only strictly causally at the encoder that sends only the common message in [6]. Proposition 2 below shows that, for the model with degraded messages sets that we study, the increase in the capacity holds only because the encoder that sends only the common message, i.e., Encoder 2, also knows the states. That is, if the states were known strictly causally to only Encoder 1, its availability would not increase the capacity of the corresponding model. Proposition 1 shows that, like for the model with independent inputs in [9], the knowledge of the states strictly causally at the encoders does not increase the sum rate capacity, however.

***Proposition** 1:* The knowledge of the states only strictly causally at the encoders does not increase the sum capacity of the multiple access channel with degraded messages sets, i.e.,

$$R_c + R_1 \le \max_{p(x_1,x_2)} I(X_1, X_2; Y). \tag{25}$$

***Proposition** 2:* The knowledge of the states strictly causally at only the encoder that sends both messages does not increase the capacity region of the model.

IV. Causal States

Let $\mathcal{P}_c$ stand for the collection of all random variables (S, U, V, X_1, X_2, Y) such that U, V, X_1 and X_2 take values in finite alphabets $\mathcal{U}$, $\mathcal{V}$, $\mathcal{X}_1$ and $\mathcal{X}_2$, respectively, and

$$\begin{aligned}
&P_{S,U,V,X_1,X_2,Y}(s, u, v, x_1, x_2, y) \\
&\quad = P_{S,U,V,X_1X_2}(s, u, v, x_1, x_2) W_{Y|X_1,X_2,S}(y|x_1, x_2, s) && \text{(26a)}\\
&P_{S,U,V,X_1,X_2}(s, u, v, x_1, x_2) = Q_S(s) P_V(v) P_{U|V}(u|v) \\
&\cdot P_{X_2|V,S}(x_2|v, s) P_{X_1|S,V,U}(x_1|s, v, u) && \text{(26b)}\\
&\sum_{u,v,x_1,x_2} P_{S,U,V,X_1,X_2}(s, u, v, x_1, x_2) = Q_S(s). && \text{(26c)}
\end{aligned}$$

The relations in (26) imply that $(U, V) \leftrightarrow (S, X_1, X_2) \leftrightarrow Y$ is a Markov chain; and that (V, U) is independent of S.

Define $\mathcal{C}_c$ to be the set of all rate pairs (R_c, R_1) such that

$$\begin{aligned}
R_1 &\le I(U; Y|V) \\
R_c + R_1 &\le I(U, V; Y) \\
&\quad \text{for some } (S, U, V, X_1, X_2, Y) \in \mathcal{P}_c. && (27)
\end{aligned}$$

As stated in the following theorem, the set $\mathcal{C}_c$ is the capacity region of the state-dependent discrete memoryless MAC model with causal states.

***Theorem** 4:* The capacity region of the multiple access channel with degraded messages sets and states known causally at both encoders is given by $\mathcal{C}_c$.

Remark 5: For the proof of Theorem 4 , the converse part can be shown in a way that is essentially very similar to [13]. The coding scheme that we use to prove the achievability part is based on Shannon strategies. By opposition to the case of the MAC with independent inputs of [13] or that with one common message and individual messages [12], in our case one of the two encoders knows the other encoder's message, and this permits to create the desired correlation among the auxiliary codewords that is required by the outer bound. Also, we should mention that the fact that Shannon strategies are optimal for the MAC with degraded messages sets that we study is in opposition with the case of the MAC with independent inputs, for which it has been shown in [9, Section III] that Shannon strategies are suboptimal in general.

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