On Layered Transmission in Clustered Cooperative Cellular Architectures

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***Abstract*—"Layered" rate-splitting based transmission strategies are investigated for the uplink of cellular communication systems employing clustered cooperative processing. Accordingly, partial decoding of some received out-of-cluster "layers" is employed, while undecoded "layers" are treated as noise. A two-dimensional Wyner-type system model is considered, by which only adjacent cell interference is present and characterized by a single parameter $\alpha \in (0, 1]$. Focusing on the average throughput per cell, the setting is shown to be equivalent to a certain multiple-input multiple-output (MIMO) multiple access channel (MAC). An achievable average throughput is then obtained by efficiently solving an appropriately formalized *Complementary Geometric Programming (CGP)* problem. *Significant* performance enhancement is demonstrated compared to non-cooperative single-cell processing, as well as to "naive" cooperation, where out-of-cluster interference is treated as noise.**

I. Introduction

Cooperative multicell processing is well recognized by now as a crucial means for enhancing the performance of future cellular communication networks (see, e.g., [1] and references therein). Early work on cooperative processing often considered architectures by which signals received, or transmitted, by *all* of the system's cell sites are being jointly processed (e.g., [2], [3]). Such full cooperation becomes prohibitively complex in practical systems comprising hundreds and thousands of cells. Clustered cooperative processing is therefore investigated in recent years as a practical form of cooperation in cellular networks. The idea is to divide the network into clusters, each comprising a relatively small number of cell sites, where the received, or transmitted, signals are jointly processed. See [1] and [4] for recent summaries of the state of the art.

Signals received at, or transmitted from, neighboring clusters induce interference, which puts the setting in the framework of the multiple-input multiple-output (MIMO) interference channel model (see, e.g., [5], [6]). Treating out-of-cluster interference as noise is a straightforward approach, but essentially leads to *interference limited* behavior. Focusing on the *uplink*, we propose here instead a rate-splitting ("layering") strategy, which is inspired by Han and Kobayashi [7]. Accordingly, the users transmit a superposition of codewords ("layers"), part of which are to be decoded only locally by the cluster's joint processor, while others are to be decoded in neighboring clusters as well (which enables interference cancellation). Optimum power allocation among the layers can then be exercised in order to optimize the average throughput per cell.

For purposes of analytical tractability, we restrict here the discussion to a simple two-dimensional (2-D) hexagonal cellular network model of the Wyner type. Accordingly, only adjacent cell interference is present and characterized by a single parameter $\alpha \in (0, 1]$. The impact of rate-splitting is demonstrated by considering 7-cell clusters as shown in Fig. 1. We show that the per-cell average throughput optimization problem, which is *nonconvex*, is equivalent to throughput optimization in a corresponding MIMO multiple access channel (MAC). Furthermore, the problem can be formalized as a *Complementary Geometric Programming (CGP)* problem [8] (see also [9] for a recent relevant literature review), which can be solved efficiently to provide a lower bound on the achievable throughput. Numerical results provide valuable insights on the potential performance gains of the proposed rate-splitting approach.

The remainder of this paper is organized as follows. Section II presents the system model and the proposed rate-splitting strategy. Section III presents the underlying optimization problem, yielding the achievable throughput of the proposed scheme. Section IV reformulates the achievable throughput as a CGP problem. Section V includes some numerical results. Finally, Section VI ends the paper with some concluding remarks.

II. System Model and Preliminaries

A. Notation

We use upper-case letters to denote random variables (RV) and bold upper-case letters to denote random vectors. Matrices are denoted by bold italicized upper-case letters,

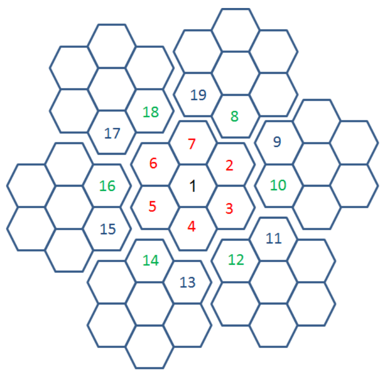

Fig. 1: A simple two-dimensional cellular network model.

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while lower-case letters are used to denote deterministic arguments of the underlying optimization problem.

B. Two-Dimesional Hexagonal Array Model

We focus here on the uplink of a two-dimensional hexagonal Wyner-type model for a cellular network [2], as depicted in Fig. 1. We restrict the discussion to nonfading additive white Gaussian noise (AWGN) channels, while assuming a single active user per cell. This corresponds to employing time-division multiple access (TDMA), which was shown to be sum-rate optimal in this setting in the absence of fading [2]. It is assumed that each cell site receives the signal of its own user in addition to the signals transmitted by the users in adjacent cells, multiplied by an interference factor $\alpha \in (0, 1]$. The received signals are embedded in white Gaussian noises, assumed independent and identically distributed at all cell-sites. Referring to Fig. 1, and considering without loss of generality the receiver at cell 1, the received signal at the cell site (at some arbitrary time instance) is given by

$$Y_1 = X_1 + \alpha \sum_{j=2}^{7} X_j + Z_1 \ , \tag{1}$$

where X_j denotes the signal transmitted by the user in cell j, and $Z_j \sim \mathcal{N}(0, 1)$ denotes the zero-mean AWGN at the receiver of the jth cell site.

The maximum achievable rates for the above system model with full joint processing were derived in [2]. Here we consider a simple clustering scheme, by which the system cells are divided into identical *nonoverlapping* clusters comprising 7 cells, as shown in Fig. 1 (thus fully exploiting the natural symmetry of the hexagonal system model). Each cluster is assumed to employ a central processor that jointly processes the signals received at the cluster's cell sites. No cooperation between clusters is assumed. Note that the achievable throughputs by the above clustering scheme provide lower bounds to the achievable throughputs with more general clustering approaches, including in particular overlapping clusters and various approaches for sharing information between different clusters (see, e.g., [4], [10], [11]).

In view of the full symmetry of the system model, we focus henceforth without loss of generality on the central cluster in Fig. 1, comprising cells $1, 2, \ldots, 7$. Note that the system cells from which signals are received by the central cluster's joint processor can be divided into the following four sets, while referring to the cell numbers in Fig. 1. The set $\mathcal{I} \triangleq \{2, 3, 4, 5, 6, 7\}$ (shown in red in the figure) comprises all *intracluster* cells, except for the central cell (cell 1), which we consider as a separate set. The set $\mathcal{J} \triangleq \{8, 10, 12, 14, 16, 18\}$ (shown in green) comprises out-of-cluster cells from which each of the signals is received by *two* of the cluster's cell sites. Finally, the set $\mathcal{K} \triangleq \{9, 11, 13, 15, 17, 19\}$ (shown in blue) comprises the out-of-cluster cells from which each of the signals is received by only one of the cluster's cell sites. For simplicity, as well as by symmetry considerations, we assume that users in cells $\{2, 3, \ldots, 19\}$ (referred to henceforth as the "noncentral" cells) employ the same transmission strategy and transmit at the same rate, which we denote by R_{nc}. The user in the central cell (cell 1) is assumed to transmit at a rate R_c.

C. Layered Transmission Scheme

We propose here a rate-splitting (layering) transmission strategy for all users in noncentral cells. Accordingly, each user splits its messages into three *independently encoded* layers. The transmitted signal is formed as a superposition of the three codewords corresponding to each of the three layers. Layer 1, transmitted at rate R_1, is intended to be decoded in all clusters in which the corresponding user's signal is received (i.e., both in the local cluster and in neighboring clusters). Layer 2, transmitted at rate R_2, is intended to be decoded only in clusters in which the corresponding user's signal is received by *at least two cell sites*. Finally, Layer 3, transmitted at rate R_3, is intended to be decoded only by the local cluster's joint processor. The target performance measure considered here is the average throughput per cell, defined as

$$R_{\text{avg}} \triangleq \frac{1}{7}[R_c + 6R_{nc}] = \frac{1}{7}[R_c + 6(R_1 + R_2 + R_3)] \ . \tag{2}$$

Note that the above transmission strategy lets us treat the signal of each noncentral transmitter as a superposition of the signals of (up to) three independent virtual transmitters. The signals corresponding to layers that are not decoded by the cluster's joint processor are treated as AWGN. Thus, out of the signals transmitted by users in the cells forming the set $\mathcal{K}$, the cluster's processor needs only decode the messages corresponding to Layer 1, while the signals corresponding to Layers 2 and 3 are treated as noise. Similarly, out of the signals transmitted by users in cells forming the set $\mathcal{J}$ only the messages corresponding to Layers 1 and 2 are decoded, while the signals corresponding to Layer 3 are treated as noise. Finally, all transmissions from users in cells forming the set $\mathcal{I}$ are decoded by the cluster's processor. Assuming an identical power constraint P for all users in all cells (including cell 1), it follows that the signals corresponding to the three layers in each noncentral cell must satisfy

$$P_1 + P_2 + P_3 \leq P \ , \tag{3}$$

where P_i denotes the power allocated to the ith layer. We assume throughout that Gaussian random codebooks are employed for all layers, and hence the symbols transmitted by each of the virtual transmitters can be treated as independent zero mean Gaussian random variables, identically distributed within a layer, and with variances corresponding to the chosen power allocation scheme. This approach, combined with treating undecoded interference as AWGN, is justified by its robustness, and the fact that Gaussian signaling achieves the sum-rate capacity of the Gaussian MIMO interference channel in certain regimes, as shown, e.g., in [5], [6].

We denote henceforth by $X_k^{(i)}$ the signal transmitted by the virtual transmitter corresponding to Layer i in cell k. Similarly, $\mathbf{X}_{\mathcal{G}}^{(i)}$, respectively $\mathbf{X}_{\mathcal{G}}^{(\mathcal{S})}$, denote the column vector of signals transmitted by the virtual transmitters corresponding to Layer i, respectively the layers in the set $\mathcal{S}$, from the cells included in the set $\mathcal{G}$ (with vector entries numerically ordered by cell numbers). The signal observed by the central cluster's joint processor can thus be written as (see Fig. 1)

$$\begin{aligned}\mathbf{Y} &= \begin{bmatrix}1 & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha\\ \alpha & 1 & \alpha & 0 & 0 & 0 & \alpha\\ \alpha & \alpha & 1 & \alpha & 0 & 0 & 0\\ \alpha & 0 & \alpha & 1 & \alpha & 0 & 0\\ \alpha & 0 & 0 & \alpha & 1 & \alpha & 0\\ \alpha & 0 & 0 & 0 & \alpha & 1 & \alpha\\ \alpha & \alpha & 0 & 0 & 0 & \alpha & 1\end{bmatrix}\mathbf{X} + \begin{bmatrix}0 & \mathbf{0}^T\\ \mathbf{0} & \alpha \boldsymbol{I}_{6\times 6}\end{bmatrix}\begin{bmatrix}0\\ \mathbf{X}_{\mathcal{K}}^{(1)}\end{bmatrix}\\ &+ \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \alpha & \alpha & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha & \alpha & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha & \alpha & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha & \alpha & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha & \alpha\\ 0 & \alpha & 0 & 0 & 0 & 0 & \alpha\end{bmatrix}\left(\begin{bmatrix}0\\ \mathbf{X}_{\mathcal{J}}^{(1)}\end{bmatrix} + \begin{bmatrix}0\\ \mathbf{X}_{\mathcal{J}}^{(2)}\end{bmatrix}\right)\\ &+ \mathbf{N},\end{aligned} \tag{4}$$

where $\mathbf{Y} = [Y_1, \ldots, Y_7]^T$, $\mathbf{X} = [X_1, \ldots, X_7]^T$ (note that X_i denotes the *overall* signal transmitted by the user in cell i), $\boldsymbol{I}_{n\times n}$ denotes the $n \times n$ identity matrix, and $\mathbf{N} = [N_1, \ldots, N_7]^T$ denotes the *effective* noise vector. The latter is given by

$$\mathbf{N} = \begin{bmatrix} 0\\ \alpha\left(X_8^{(3)} + X_9^{(2)} + X_9^{(3)} + X_{10}^{(3)}\right)\\ \alpha\left(X_{10}^{(3)} + X_{11}^{(2)} + X_{11}^{(3)} + X_{12}^{(3)}\right)\\ \alpha\left(X_{12}^{(3)} + X_{13}^{(2)} + X_{13}^{(3)} + X_{14}^{(3)}\right)\\ \alpha\left(X_{14}^{(3)} + X_{15}^{(2)} + X_{15}^{(3)} + X_{16}^{(3)}\right)\\ \alpha\left(X_{16}^{(3)} + X_{17}^{(2)} + X_{17}^{(3)} + X_{18}^{(3)}\right)\\ \alpha\left(X_{18}^{(3)} + X_{19}^{(2)} + X_{19}^{(3)} + X_{8}^{(3)}\right)\end{bmatrix} + \mathbf{Z}\ , \tag{5}$$

where $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I}_{7\times 7})$ is the vector of AWGNs at the cell-site receivers. With our underlying assumption on the codebook construction, the covariance matrix of the effective noise is given by

$$\begin{aligned}\boldsymbol{\Lambda}_N &= E\left\{\boldsymbol{N}\boldsymbol{N}^T\right\}\\ &= \boldsymbol{I}_{7\times 7} + \alpha^2(P_2+P_3)\begin{bmatrix}0 & \mathbf{0}^T\\ \mathbf{0} & \boldsymbol{I}_{6\times 6}\end{bmatrix}\\ &+ \alpha^2 P_3 \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0 & 0 & 0 & 1\\ 0 & 1 & 2 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 2 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 & 2 & 1\\ 0 & 1 & 0 & 0 & 0 & 1 & 2\end{bmatrix}.\end{aligned} \tag{6}$$

The first term in (6) is due to the AWGN at the receivers, the second term is due to undecoded signals received from cells forming the set $\mathcal{K}$, and the third term corresponds to undecoded signals received from cells forming the set $\mathcal{J}$.

III. MIMO MAC Interpretation

With the proposed rate-splitting transmission scheme, the underlying channel experienced by the cluster's joint processor is a MIMO MAC, where the effective noise incorporates all undecoded out-of-cluster signals (in addition to AWGN). Interpreting each of the layers transmitted by the users as an independent virtual transmitter, the total number of virtual transmitters sums up to 37. Hence, ignoring symmetry considerations, per each possible power allocation among the different layers, the achievable region of the underlying MAC is determined by $2^{37}-1$ partial sum-rate inequality constraints (see, e.g., [12]). The overall capacity region is then the convex hull of the union of all achievable regions over all possible power allocations that satisfy (3). The average throughput per cell should then be optimized over the entire region to produce the maximum achievable throughput of the rate-splitting scheme.

Fortunately, the inherent symmetry properties of the system model, combined with our underlying assumptions, imply that all virtual transmitters corresponding to the same layer within any of the sets of cells $\mathcal{I}$, $\mathcal{J}$, or $\mathcal{K}$, are in fact completely equivalent, and employ the same rates. This observation dramatically simplifies the task of determining the maximum average achievable throughput per cell, as summarized by the following proposition.

Proposition 1. *The maximum achievable average throughput per cell with the proposed rate-splitting scheme is given by the solution of the following optimization problem (omitting the obvious constraint of nonnegative rates):*

$$\begin{aligned}&\underset{R_c, R_i, P_i}{\textit{maximize}} \quad \tfrac{1}{7}[R_c + 6(R_1+R_2+R_3)]\\ &\textit{subject to:}\\ &R_c + \textstyle\sum_{\mathcal{S}_\mathcal{K},\mathcal{S}_\mathcal{J},\mathcal{S}_\mathcal{I}} R_i\\ &\quad \le I\left(X_1, \mathbf{X}_\mathcal{K}^{(\mathcal{S}_\mathcal{K})}, \mathbf{X}_\mathcal{J}^{(\mathcal{S}_\mathcal{J})}, \mathbf{X}_\mathcal{I}^{(\mathcal{S}_\mathcal{I})}; \mathbf{Y} | \mathbf{X}_\mathcal{K}^{(\mathcal{S}_\mathcal{K}^c)}, \mathbf{X}_\mathcal{J}^{(\mathcal{S}_\mathcal{J}^c)}, \mathbf{X}_\mathcal{I}^{(\mathcal{S}_\mathcal{I}^c)}\right)\\ &\textstyle\sum_{\mathcal{S}_\mathcal{K},\mathcal{S}_\mathcal{J},\mathcal{S}_\mathcal{I}} R_i\\ &\quad \le I\left(\mathbf{X}_\mathcal{K}^{(\mathcal{S}_\mathcal{K})}, \mathbf{X}_\mathcal{J}^{(\mathcal{S}_\mathcal{J})}, \mathbf{X}_\mathcal{I}^{(\mathcal{S}_\mathcal{I})}; \mathbf{Y} | X_1, \mathbf{X}_\mathcal{K}^{(\mathcal{S}_\mathcal{K}^c)}, \mathbf{X}_\mathcal{J}^{(\mathcal{S}_\mathcal{J}^c)}, \mathbf{X}_\mathcal{I}^{(\mathcal{S}_\mathcal{I}^c)}\right)\\ &\textit{for all } \mathcal{S}_\mathcal{I} \subseteq \{1,2,3\}, \mathcal{S}_\mathcal{J} \subseteq \{1,2\}, \mathcal{S}_\mathcal{K} \subseteq \{1\}\end{aligned} \tag{7}$$

where $\sum_{\mathcal{S}_\mathcal{K},\mathcal{S}_\mathcal{J},\mathcal{S}_\mathcal{I}} R_i$ represents the sum-rate of all virtual transmitters corresponding to the constraining mutual information expression, $\{P_1, P_2, P_3\}$ satisfy (3)*, and entries corresponding to empty sets should be ignored.*

Note that Proposition 1 implies that the underlying MAC can be viewed as an equivalent MIMO MAC with 7 users. 6 of the users employ 6 transmit antennas, and correspond to the virtual transmitters of each of the decoded layers in the sets of cells $\mathcal{K}$, $\mathcal{J}$ and $\mathcal{I}$. The 7th user is the user in cell 1, who employs a single transmit antenna. By this interpretation the transmit covariance matrices must be restricted to be multiples of the identity, to facilitate the symmetry properties and the fact that virtual transmitters in different cells do not cooperate.

Proof Outline: The proof of Proposition 1 can be divided into two stages. First, constraints that only involve virtual transmitters corresponding to a single layer, and a single set of cells (namely, either $\mathcal{I}$, $\mathcal{J}$, or $\mathcal{K}$) are examined. It can then be shown through repeated use of the chain rule for mutual information, that each constraint is given by an average of mutual information terms. The more virtual transmitters are incorporated into the constraint, it turns out that the average of mutual information terms *decreases*. Thus, the strictest constraint is the one that includes *all* transmitters within the set.

Next, constraints that involve virtual users from dif-

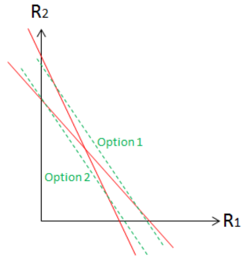

Fig. 2: Rate constraints example.

ferent layers and sets of cells are examined. Again, it can be shown that the dominating constraints are only the ones that involve *all* virtual users corresponding to a certain layer in a given set of cells. To demonstrate the approach, Fig. 2 illustrates constraints relating only to virtual transmitters corresponding to Layers 1 and 2. Recall that all constraints of this type are of the form $aR_1 + bR_2 \le \mathcal{R}$, where a and b are integers, and $\mathcal{R}$ represents a corresponding mutual information term. The two solid lines in the figure represent the constraints involving either *all* virtual transmitters corresponding to the two layers (the line with the steeper slope), or the corresponding virtual transmitters *but* the ones in the cells forming the set $\mathcal{K}$ (the second solid line). Now any other constraint involving only part of the virtual users from the cells forming the set $\mathcal{K}$ can be in general represented by either of the two dashed lines in Fig. 2 (denoted as "Option 1" and "Option 2"). Relying on the properties of the system model, as well as the underlying assumptions, it can then be shown that any such "partial" constraint can only be represented by the line denoted as "Option 1". This then immediately implies that the dominating constraints are the ones represented by the solid lines in the figure, corresponding to the full sets of users, as explained above. The same procedure can be exercised with respect to all combinations of layers and sets of cells. It is also important to emphasize at this point that the above proof does not depend on the assumed Gaussianity of either transmitted signals or noise. ∎

IV. Achievable Throughput via CGP

For random Gaussian codebooks (see Section II-C) the constraints in (7) can be shown to take the following form

$$aR_1 + bR_2 + cR_3 + dR_c \le \le \frac{1}{2}\log_2\left(\frac{A(P_1,P_2,P_3) - B(P_1,P_2,P_3)}{C(P_1,P_2,P_3) - D(P_1,P_2,P_3)}\right) , \tag{8}$$

where a, b, c, d are integers, and $A(P_1, P_2, P_3)$, $\ldots, D(P_1, P_2, P_3)$ are posynomials[1]. The optimization problem specified by Proposition 1 is hence a *nonconvex* optimization problem. However, it is still possible to efficiently obtain an approximate solution to the problem, by applying the CGP algorithm [8] (see also [9] for additional examples for the use of CGP in related problems). In order to formulate (7) as a CGP, we first transform it to the following equivalent optimization problem

$$\begin{aligned} &\underset{R_c,R_i,P_i}{\text{minimize}} && (2^{R_1})^{-\frac{6}{7}}(2^{R_2})^{-\frac{6}{7}}(2^{R_3})^{-\frac{6}{7}}(2^{R_c})^{-\frac{1}{7}} \\ &\text{subject to} && \left\{(2^{R_1})^{2a}(2^{R_2})^{2b}(2^{R_3})^{2c}(2^{R_c})^{2d} \le \frac{A(P_1,P_2,P_3) - B(P_1,P_2,P_3)}{C(P_1,P_2,P_3) - D(P_1,P_2,P_3)}\right\} , \end{aligned} \tag{9}$$

where we use the bracelets to compactly denote the set of all 127 inequalities of the above form implied by Proposition 1 and (8), as well as the power constraint (3). With a change of optimization variables to $\{w_1, w_2, w_3, w_4\} \triangleq \{2^{R_1}, 2^{R_2}, 2^{R_3}, 2^{R_c}\}$, and $\{w_5, w_6, w_7\} \triangleq \{P_1, P_2, P_3\}$, the problem obtains the form

$$\begin{aligned} &\underset{w}{\text{minimize}} && w_1^{-\frac{6}{7}} w_2^{-\frac{6}{7}} w_3^{-\frac{6}{7}} w_4^{-\frac{1}{7}} \\ &\text{subject to} && \left\{w_1^{2a} w_2^{2b} w_3^{2c} w_4^{2d} \le \frac{A(w_5,w_6,w_7) - B(w_5,w_6,w_7)}{C(w_5,w_6,w_7) - D(w_5,w_6,w_7)}\right\} . \end{aligned} \tag{10}$$

Since here $C(\cdot,\cdot,\cdot) - D(\cdot,\cdot,\cdot) > 0$, then further defining

$$E(\mathbf{w}) \triangleq w_1^{2a} w_2^{2b} w_3^{2c} w_4^{2d} C(w_5, w_6, w_7) , \tag{11}$$

$$F(\mathbf{w}) \triangleq w_1^{2a} w_2^{2b} w_3^{2c} w_4^{2d} D(w_5, w_6, w_7) , \tag{12}$$

each constraint can finally take the standard form [8]

$$\frac{E(\mathbf{w}) + B(\mathbf{w})}{A(\mathbf{w}) + F(\mathbf{w})} \le 1. \tag{13}$$

Note that (10) resembles the form of a standard *geometric programming (GP)* problem [13] (which is equivalent to a *convex* optimization problem), except that here the constraints cannot be expressed as posynomials in terms of the optimization variables (a ratio of two posynomials is not necessarily a posynomial). The CGP algorithm guarantees a quasi-minimum in the above setting, which is a point that satisfies certain *necessary* conditions for a local minimum (see [8] for details). The CGP algorithm reaches this quasi-minimum through an iterative process, by which, in each iteration, the denominator of each constraint (in this case $A(\mathbf{w}) + F(\mathbf{w})$) is approximated by a *posynomial term*. The division of a posynomial by a posynomial term yields a posynomial, thus generating a GP problem in each iteration. The solutions for these GP problems are shown in [8] to converge to a quasi-minimum of the original CGP problem. The resulting throughput can therefore be considered as a lower bound for the best achievable throughput using our rate-splitting scheme.

V. Numerical Results

In this section we present some numerical results that demonstrate the impact of the proposed rate splitting scheme. Fig. 3 shows the corresponding spectral efficiency in [bits/sec/Hz] as a function of the average transmit $\frac{E_b}{N_0}$, as derived using the CGP algorithm (and the relation $P = 2R_{\text{avg}}\frac{E_b}{N_0}$)[2]. The results are shown for an interference

[1] A *posynomial* is a function $f : \mathbb{R}^n \to \mathbb{R}$ of the form $f(\mathbf{x}) = \sum_{k=1}^{K} \gamma_k x_1^{\beta_{1k}} x_2^{\beta_{2k}} \cdots x_n^{\beta_{nk}}$ where $\gamma_k > 0$, and $\beta_{jk} \in \mathbb{R}$, $k = 1, \ldots, K$, $j = 1, \ldots, n$. Each term in the sum is referred to as a posynomial term or *monomial* [8], [13].

[2] Numerical examination indicates that CGP produces results that are very close to the global optimum, as obtained via "brute force" optimization.

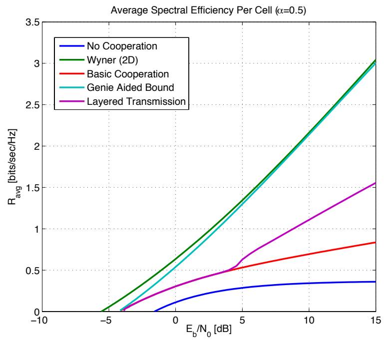

Fig. 3: Spectral efficiency comparison for $\alpha = 0.5$.

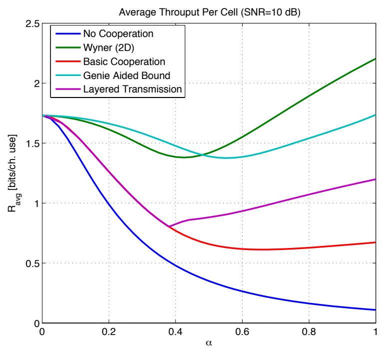

Fig. 4: The impact of α for SNR $= 10$dB.

factor of $\alpha = 0.5$. Two lower bounds, as well as two upper bounds for the average spectral efficiency were also included for comparison. The lower bounds correspond to the case of *no cooperation* (which boils down to single-cell processing), and to basic "naive" cooperation by which joint processing of intracluster transmissions is employed, while *all* out-of-cluster interference is treated as noise. The upper bounds correspond to full joint multicell processing [2, Eq. (2.15a)], and to a "genie aided" achievable rate, where the genie provides as side information all out-of-cluster interfering signals (the setting thus corresponds to a *single isolated cluster*). The results clearly indicate that the rate splitting scheme significantly enhances performance as one moves away from the low-interference regime. In the latter case treating interference as noise is known to be optimal [5], [6], and the spectral efficiency coincides with the basic cooperation lower bound. The performance enhancement over single cell processing is clearly apparent for the whole $\frac{E_b}{N_0}$ range. Note, however, that the clustering scheme induces a loss in the minimum $\frac{E_b}{N_0}$ that enables reliable communications when compared to full joint multicell processing [2], as the latter achieves the full array diversity gain factor of $1 + 6\alpha^2$ in the low signal-to-noise ratio (SNR) regime (the loss amounts to 1.29dB for $\alpha = 0.5$).

Fig. 4 shows the achievable throughput as a function of α for a *fixed* SNR of 10dB. Again the results indicate significant performance enhancement as one moves away from the low-interference regime (corresponding here to increasing α). It is also interesting to note that the throughput is *not* monotonically decreasing with the interference factor, which resembles analogous observations for the classical two users interference channel (see references in [5]). Such behavior was already reported in [2] for full joint multicell processing. Another insightful observation is that the optimal power allocation among the layers is nontrivial. In fact, beyond the low interference regime (where silencing Layers 1 and 2 is optimal), one can identify interference level regions where power allocation to different subsets of the layers is optimal.

VI. Concluding Remarks

We considered here a rate splitting ("layering") approach for clustered multicell processing architectures, inspired by [7]. Focusing on a simple 2-D Wyner-type system model, the proposed scheme was shown to exhibit a significant impact on system performance for medium to high interference levels. Our results provide valuable insights into the appropriate treatment of interference in cellular networks, emphasizing the benefits of partial decoding of interfering signals, particularly in dense networks where out-of-cell interference is a dominant factor. The recent study [14] supports the intuition that employing more sophisticated decoding rules not treating undecoded "layers" as noise, will have at best a limited effect. Tighter upper bounds for the achievable rates, as well as extensions to Rayleigh fading channels, are currently investigated.

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