Interference Canceling Power Control Games in Gaussian Interference Channels

Liang Zhou, Kalle Ruttik, Olav Tirkkonen
Dep.of Communications & Networking
Aalto University
Espoo, Finland
Email: {liang.zhou,kalle.ruttik,olav.tirkkonen}@aalto.fi

Randall Berry
Dep. of Electrical Eng. & Computer Science
Northwestern University
Evanston, IL 60208 USA
Email: rberry@eecs.northwestern.edu

***Abstract*—We investigate a non-cooperative interference canceling and power control (IC-PC) game in a Gaussian interference channel, where the receivers are able to process at most two codewords at a time. We characterize the equilibria of this game. As opposed to the pure power control game and the rate splitting game between the same players, we find that in the IC-PC game, there exist Nash equilibria where a player voluntarily reduces his power in order to enable interference canceling and achieve a higher rate.**

I. Introduction

Game theoretic approaches have been widely used in analyzing the behavior of wireless systems [1]–[10], and have shown to provide a powerful tool to address power control [1]–[4], resource allocation [5], [6] and multiantenna precoding [7]. In all of these problems, the interaction between players is determined by interference. The Gaussian Interference Channel (GIC) is a particularly simple and powerful model where elementary interactions between multiple interference-coupled systems can be analyzed [11]–[16].

The best known coding strategy in a GIC is based on Han-Kobayashi rate splitting [12], [13], where the transmitters split their messages into two parts, one (the public part) intended to be decodable at both receivers, the other (private part) intended to be decodable only at the intended receiver. The receivers perform Serial Interference Cancelation (SIC), first potentially jointly decoding the public codewords [13], then canceling them before decoding their respective private codeword.

Game theoretic analyses of GIC have been performed in [8]–[10]. In [8] a power control game in which each receiver treats interference as noise was studied. It was shown that in a Nash Equilibrium (NE), both transmitters apply full power. Limiting to a single codeword, prohibits the use of Han-Kobayashi rate-splitting. In [10], transmitters were allowed to use arbitrary rate-splitting strategies. Also here, in a NE, both transmitters apply full transmit power. If there were a transmitter not using all the power, the transmitter uses that power to transmit to a virtual user, who would achieve whatever rate it can treating all other users as noise. The receiver can continue achieving whatever rate it had before the virtual user was added, plus the additional rate of the virtual user. In addition to [10], interference canceling games have been addressed in the context of multiple access channels [4].

In this paper, we address the case that a receiver is capable of processing at most two codewords using successive interference canceling. In [17], similar setting is employed as a part of *opportunistic multiuser detection*, in which the corresponding game is not studied.

This is a logical possibility different from legacy non-interference-canceling receivers, where only the intended codeword may be processed, considering all other transmissions as noise, and from receivers capable of receiving Han-Kobayashi rate-split messages, which would need to be able to deal with three codewords. In an evolution of wireless communication receivers, loosening of the hardware constraint from dealing with one codeword to dealing with two in a SIC manner may be a first step. At the moment, such receivers are under consideration for Multiple Input-Multiple Output (MIMO) receivers in LTE. Here, we do not investigate the MIMO-domain—the results give a hint to what may be done in a cellular system when serving cell-edge users with a two-codeword SIC-receiver, but would not reflect the benefit from using multistream MIMO transmission.

We call the strategic game played by the two transmitter-receiver pairs in a GIC without rate splitting an Interference Canceling Power Control (IC-PC) game. To the best of our knowledge, this game has not been addressed in the literature. The IC-PC game seeks the trade-off between the capacity with full power and no IC ability and reduced power and IC. We observe that the game has eleven different NE domains, nine of which have a single pure NE. In some domains and in contrast to the previous work discussed above, one of the transmitters will voluntarily reduce the transmission power in a NE.

II. System Model

We consider a Gaussian interference channel, with two transmitters (Tx) and two receivers (Rx). The transmitters send unicast messages from Gaussian codebooks, targeted at one of the receivers, and each receiver is interested only in the transmission from one transmitter. There is no transmitter or receiver collaboration. We assume perfect interference canceling (IC), if the interference victim is able to decode the interfering message, and a *hardware constraint*: each receiver is capable of handling at most two codewords.

A. Configuration Space

The channel gain from transmitter i to receiver j is G_{ij}, and the Signal-to-Interference-plus-Noise Ratios (SINR) of signal i at receiver i are

$$\gamma_{ii}^{\mathrm{noIC}} = \frac{P_i G_{ii}}{P_j G_{ji} + N_0} \quad \text{if no IC at Rx } i \tag{1}$$

$$\gamma_{ii}^{\mathrm{IC}} = \frac{P_i G_{ii}}{N_0} \quad \text{if IC at Rx } i \tag{2}$$

for $(i, j) = (1, 2)$ or $(2, 1)$. Any Gaussian interference channel may be transformed to an equivalent *standard form* [12], where $G_{ii} = G_{jj} = N_0 = 1$. In this paper we use standard-form channels. The configuration space of the network is thus four-dimensional, characterized by G_{12}, G_{21} and the maximum transmit power of the two transmitters, $P_i^{\max}$.

B. Strategy Space

Each of the transmitter-receiver (Tx-Rx) pairs in the system are considered as a player in a game. The utility of each player pair is the achievable rate at the receiver.

The strategy space of each Tx-Rx player i consists of the transmit power P_i, the receiver IC state(IC or noIC), and the rate of the used codebook. The transmit power is a positive real continuous variable. At an equilibrium of a strategic game, a rate maximizing encoding at transmitter of

$$r_i = \log_2(1 + \gamma_{ii}) \ , \tag{3}$$

would be the maximum achievable given the transmit power, the receiver operation, the network configuration and the strategy of the opponent. We shall assume that the rational transmitter always transmits at the maximum achievable rate, and accordingly we shall not explicitly denote the rate in the strategy of a player assuming it is always maximized. Thus the strategy of a player will be denoted by a combination of power, and receiver IC state. The strategy space of player i has four logical parts: $(P_{\max}, \mathrm{noIC}), (P_{\max}, \mathrm{IC}), (P_{\lim}, \mathrm{noIC}), (P_{\lim}, \mathrm{IC})$.

III. Single-player Strategy in IC-PC Game

A. Capability to perform IC

Receiver i may perform perfect IC without joint decoding if the SINR of transmission from Tx j to the interference victim Rx i is not worse than the SINR of transmission from Tx j to Rx j

$$\gamma_{ji} \equiv \frac{P_j G_{ji}}{P_i G_{ii} + N_0} \geq \gamma_{jj} \tag{4}$$

Depending on the receiver state of the opponent, and the corresponding rate selection strategy, this leads to different conditions on the power applied by the player. For example, if the opponent j is not applying IC, this leads to

$$\begin{aligned} \frac{P_j G_{ji}}{P_i G_{ii} + N_0} &\geq \frac{P_j G_{jj}}{P_i G_{ij} + N_0} \\ \Leftrightarrow (G_{ji} - G_{jj}) &\geq (G_{ii} G_{jj} - G_{ji} G_{ij}) P_i \ . \end{aligned} \tag{5}$$

Thus the configuration space splits into two parts, strong and weak *product interference*, according to

$$G_{11} G_{22} - G_{12} G_{21} \lesseqgtr 0 \ . \tag{6}$$

Note that if interference is strong or weak in conventional GIC-terminology [14], this implies strong or weak product interference, but the converse does not hold. Conventional mixed interference cases are also classified as strong/weak product interference, with the demarcation line being the degraded GIC case, when $G_{11} G_{22} = G_{12} G_{21}$. As a consequence, for strong product interference, we get a lower limit $P_i^{\lim}$ to allow IC:

$$P_i \geq P_i^{\lim} = \frac{G_{ji} - G_{jj}}{G_{ii} G_{jj} - G_{ij} G_{ji}} = \frac{G_{ji} - 1}{1 - G_{ij} G_{ji}} \ , \tag{7}$$

where the second equality holds due to assuming the standard form. Thus in strong product interference, a player always uses the maximum power, if the opponent does not use IC. If the maximum power is larger than the limit, the player may use IC. Note that the lower limit may be negative. For weak product interference we get an upper limit, which reads

$$P_i \leq P_i^{\lim} = \frac{G_{ji} - 1}{1 - G_{ij} G_{ji}} \tag{8}$$

in the standard form. Thus in this case it is possible to use IC if $P_i^{\lim}$ is positive. If the opponent uses IC, a similar analysis can be performed. Then there is always an upper limit on power for enabling IC at receiver i:

$$P_i \leq P_i^{\lim} = G_{ji} - 1 \ . \tag{9}$$

B. Benefit of Performing IC

Even if IC is possible, it may not be beneficial. If with a given strategy of the opponent j, IC is possible for player i, it is beneficial for i only if the achievable rate with IC is better than without. From (1,2) we see that IC should be used if

$$\gamma_{ii}^{\mathrm{IC}} > \gamma_{ii}^{\mathrm{noIC}} \Leftrightarrow P_j > \frac{P_i^{\max} - P_i^{IC}}{G_{ji} P_i^{IC}} \ , \tag{10}$$

where P_i^{IC} is maximum usable power of player i with IC.

C. Shaping the MAC Region with IC

The benefits of IC with limited transmission power can be illustrated by considering the MAC region of one user, receiving two signals, illustrated in Figure 1. The data rate that receiver 1 can reliable decode from transmitter 2 is denoted by $R_{2,\text{rec }1}$. The blue line illustrates the boundary of the MAC region for user 1 for full power usage at both transmitters. In the IC-PC game, however, R_2 is determined by the transmission to receiver 2, and it may or may not be larger than $R_{2,\text{rec1}}$. When receiver 2 does not use IC, the rate R_2 is fixed to the thick horizontal line in the figure. The green star on the plot illustrates the data rates achievable at receiver 1 while both transmitters apply full power and the other transmitter's signal is treated as interference. Receiver 1 can decode the interference signal from transmitter 2 if

$$R_{2,\text{rec1}} = \log_2 \left(1 + \frac{P_2^{\max} G_{21}}{P_1^{\max} G_{11} + N_0} \right) < R_2. \tag{11}$$

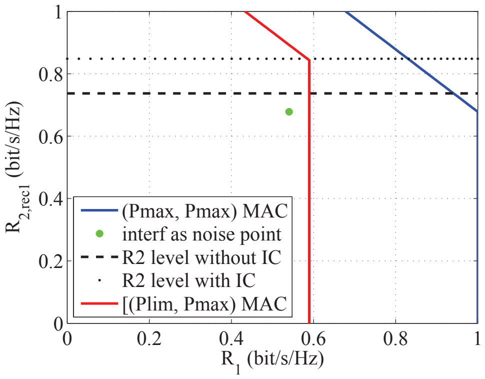

Fig. 1. MAC region for receiver 1, domain with NE $\{(\mathrm{lim}, \mathrm{IC}), (\mathrm{max}, \mathrm{noIC})\}$ while $G_{12} = 0.5, G_{21} = 1.2$.

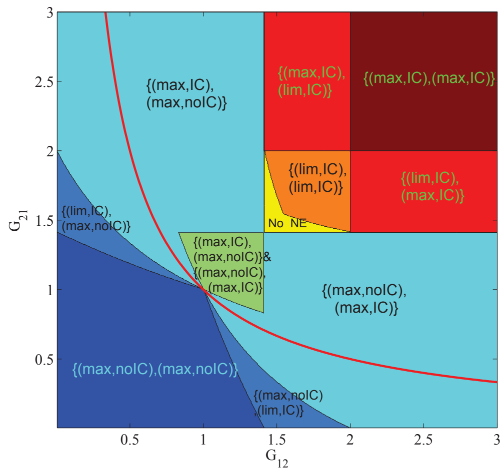

Fig. 2. NE regions of IC-PC game in GIC while $P_1^{\max} = P_2^{\max} = 1$. The red curve depicts strong/weak product interference border.

Since (5) is not satisfied the receiver 1 is not able to decode the opponent's data and IC is not possible.

The achievable MAC region at the receiver 1 is encompassing points that support data rate R_2 as well as a higher data rate for the player 1. These points would, however, require time sharing, which is not a stable strategy for a selfish player.

Reduction of P_1 to P_1^{lim} has multiple consequences. First, the opponent will have less interference and he can increase his data rate to the dotted line in Figure 1. A smaller P_1 also reduces interference to the reception of signal 2 at receiver 1, and the signal becomes decodable. A change of power levels shifts the achievable MAC region at receiver 1. The optimal new power level P_1^{lim} is the one where the new rate pair (R_1', R_2') is located at the corner point of the achievable MAC region. The analysis in the next section shows that such a change can indeed be an equilibrium of this IC-PC game.

IV. Characteristics of Nash Equilibria

A. Nash Equilibrium Types

We consider pure strategy equilibria. In total, nine types of NEs exist, in ten domains of configuration space. There is also one domain in which no Nash equilibrium exists. The different type of pure strategy NEs are

1) $\{(\mathrm{max}, \mathrm{noIC}), (\mathrm{max}, \mathrm{noIC})\}$
2) $\{(\mathrm{lim}, \mathrm{IC}), (\mathrm{max}, \mathrm{noIC})\}$ and vice versa
3) $\{(\mathrm{max}, \mathrm{IC}), (\mathrm{max}, \mathrm{noIC})\}$ and vice versa
4) $\{(\mathrm{lim}, \mathrm{IC}), (\mathrm{lim}, \mathrm{IC})\}$
5) $\{(\mathrm{max}, \mathrm{IC}), (\mathrm{max}, \mathrm{IC})\}$
6) $\{(\mathrm{max}, \mathrm{IC}), (\mathrm{lim}, \mathrm{IC})\}$ and vice versa

For weak product interference, we have five domains in configuration space. In each domain, a unique pure strategy NE exists, of the type 1, 2 or 3. Thus sometimes *it is beneficial for a greedy player to reduce its transmit power.*

For strong product interference, there are eight domains in configuration space. In two of these domains, unique NEs of type 3 exist. These domains are neighbors to the type 3 NE domains in weak product interference and thus can be merged with these. In one domain, there are two pure strategy NEs, and in part of configuration space, no pure strategy NE exists. In the remaining four domains of configuration space, a unique pure strategy NE exists, of type 4, 5 or 6. Thus for some cases with strong product interference, there exists a NE where *both* players reduce their power. Next we will completely characterize the boundary between these regions.

B. Division of NE domains

Figure 2 shows the Nash equilibrium domains of IC-PC game when we assume $P_i^{max} = 1$ for $i = 1, 2$ in the standard-form GIC.

a) No IC domain: Shown in deep blue in Figure 2. The best response functions of both players are $\{\mathrm{max}, \mathrm{noIC}\}$. The border between this domain and *limited power single IC domain* is :

$$G_{ij} = -\frac{P_j^{\max} G_{ji}^2 + (1 - P_j^{\max}) G_{ji} - 1 - P_i^{\max}}{G_{ji} P_i^{\max}} \qquad (12)$$

with $G_{ji} > 1$ where $(i, j) = (1, 2)$ for upper bound and $(i, j) = (2, 1)$ for right bound. This follows from (10). This region always includes the region $G_{12} + G_{21} < 1$ identified in [16] where treating interference as noise is sum-rate optimal.

b) Limited power single IC domain: As G_{ji} increases while $G_{ij} < 1$, it is better for player i to use IC while the opponent is not able to apply IC. From the IC power upper bound $P_i^{\mathrm{lim}} < P_i^{\max}$, player i has to limit its power to enable IC. Thus the boundary between *limited power single IC domain* and *maximum power single IC domain* is given by equating P_i^{lim} in (8) with $P_i^{\max}$ yielding:

$$G_{ji} = B_{ji}^{\mathrm{FP(noIC)}}, G_{ji} > 1, G_{ij} < 1 \qquad (13)$$

where the *full power bound*

$$B_{ij}^{\mathrm{FP(noIC)}} = \frac{1 + P_j^{\max}}{1 + P_j^{\max} G_{ji}}. \qquad (14)$$

for $(i,j) = (1,2)$ and $(i,j) = (2,1)$ which specify two borders. The borders shifts towards strong/weak product interference border(red line in Figure 2) as $P_i^{\max}$ or $P_j^{\max}$ increases.

c) Maximum power single IC domain: In this domain power limits of IC disappear since the lower bound on power is smaller than zero or the upper bound of power is larger than $P_i^{\max}$. The red curve in Figure 2 indicates strong/weak product interference border $G_{12}G_{21} = 1$. This divides the each *Maximum power single IC domains* into two parts - one part has weak product interference, with the upper limit power for IC larger than the maximum power, the other has strong product interference with the lower limit power for IC less than the maximum power. If inequalities (15) is true only for either $(i,j) = (1,2)$ or $(2,1)$, but not for both, we have *maximum power single IC domain.*

$$G_{ji} \geq B_{ji}^{\mathrm{FP(noIC)}} \text{ and } G_{ij} < B_{ij}^{\mathrm{BIC}} \tag{15}$$

where the *Both IC bound*

$$B_{ij}^{\mathrm{BIC}} = \frac{P_i^{\max} - 1 + \sqrt{(P_i^{\max} - 1)^2 + 4P_i^{\max}(1 + P_j^{\max})}}{2P_i^{\max}} \tag{16}$$

d) Domain with two NEs: When (15) hold for both $(i,j) = (1,2)$ and $(2,1)$, two pure strategy NEs exist. This domain is shown as green in Figure 2. Each player is only able to apply IC while the opponent is not using IC. This is the only domain with double pure strategy NE.

e) Limited power double-IC domain: The orange area in Figure 2 represents this domain. In this domain, both players can and prefer to limit the transmit power in order to apply IC, while the opponent applies IC. There is a unique NE $\{(\mathrm{lim}, \mathrm{IC}), (\mathrm{lim}, \mathrm{IC})\}$. The boundary of this domain is:

$$B_{ij}^{\mathrm{FP(IC)}} > G_{ij} \geq \max\{B_{ij}^{\mathrm{BIC}}, B_{ij}^{\mathrm{NE(BIC)}}\} \tag{17}$$

where

$$B_{ij}^{\mathrm{FP(IC)}} = 1 + P_j^{\max} \tag{18}$$

$$B_{ij}^{\mathrm{NE(BIC)}} = 1 + \frac{P_i^{\max} + 1 - G_{ji}}{G_{ji}(G_{ji} - 1)} \tag{19}$$

for both $(i,j) = (1,2)$ and $(2,1)$.

f) Mixed power double-IC domain: If each player can and prefers to apply IC while the opponent is operating with IC, and only one of players observes so much interference that it may apply IC with full transmit power, there is a NE $\{(\mathrm{lim}, \mathrm{IC}), (\max, \mathrm{IC})\}$ or $\{(\max, \mathrm{IC}), (\mathrm{lim}, \mathrm{IC})\}$. These domains are depicted in red in Figure 2. The boundaries of these domains are described as follows:

$$B_{ji}^{\mathrm{FP(IC)}} > G_{ji} \geq B_{ji}^{\mathrm{BIC}} \tag{20}$$

$$G_{ij} \geq B_{ij}^{\mathrm{FP(IC)}} \tag{21}$$

for $(i,j) = (1,2)$ or $(2,1)$.

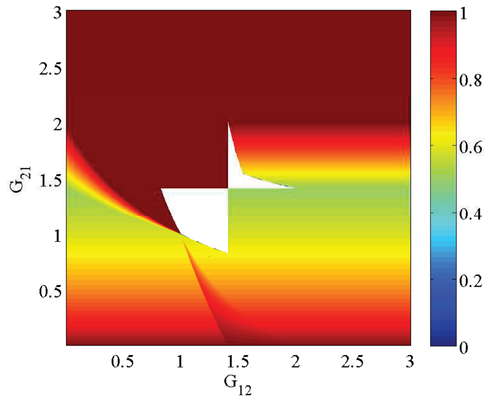

Fig. 3. Achievable data rates (bit/s/Hz) for user 1 in unique pure NE regions of IC-PC game. $P_1^{\max} = P_2^{\max} = 1$.

g) Maximum power double IC domain: When both players receive so much interference that both players may apply IC with maximum transmit power, the NE is $\{(\max, \mathrm{IC}), (\max, \mathrm{IC})\}$. The domain is simply defined as:

$$G_{ij} \geq B_{ij}^{\mathrm{FP(IC)}} \tag{22}$$

for $(i,j) = (1,2)$ and $(2,1)$. This corresponds to the "very strong interference" regime studied by Carleial [12].

h) No NE domain: In this domain, for fixed IC strategy for both players, there would exist an equilibrium, but each of these equilibria is unstable against the change of IC strategy of at least one of the players. Each player could apply IC while the opponent is operating with IC, but at least one player prefers to use the $\{(\max, \mathrm{noIC})\}$ strategy while the opponent is applying IC. Thus this domain is defined as:

$$B_{ij}^{\mathrm{BIC}} \leq G_{ij} < B_{ij}^{\mathrm{NE(BIC)}} \tag{23}$$

$$G_{ji} \geq B_{ji}^{\mathrm{BIC}} \tag{24}$$

for $(i,j) = (1,2)$ or $(2,1)$.

V. Achievable Rate Regions and NE Efficiency

Next we look at the rates achieved at the pure NE identified above and the consider whether these rates are efficient. Figure 3 shows the achievable data rate at a pure strategy NE as function of G_{ij}. It shows that user 1 prefers the interference factor G_{21} to be either very small, or large enough to apply IC. This differs from the conventional PC game.

We are interested in Pareto efficiency of the NE, i.e. whether the NE is at a boundary of the rate region achievable by cooperative players that may apply time sharing in addition to PC and IC. It is known that in a pure Power Control (PC) game, a Nash Equilibrium is efficient in the weak interference case, but inefficient in the strong interference case [18]. Here we treat the *Limited power double-IC* domain, as an example of a strong interference case.

The achievable rate region is a convexified union of the four achievable rate regions with fixed IC-strategies. If the opponent is using IC, the requirement for using IC is characterized

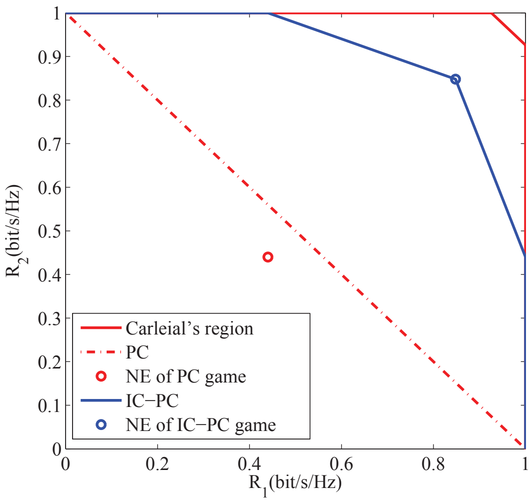

Fig. 4. Achievable rate regions. $G_{12} = G_{21} = 1.8$, and $P_1^{\max} = P_2^{\max} = 1$.

by (9), so that

$$P_i \leq G_{ji} - 1 \tag{25}$$

The rate for player i applying IC then becomes

$$R_i = \log_2(1 + P_i) \leq \log_2(G_{ji}) \ , \tag{26}$$

When both players apply IC, the rate region is thus characterized by (26) for $(i, j) = (1, 2)$ and $(2, 1)$.

If neither receiver uses IC, we have $R_j \leq \log_2(1+\frac{P_j}{G_{ij}P_i+1})$, and the region is characterized by one of the transmitters using the maximum power, and the other varying it between 0 and the maximum.

If receiver i uses IC and j not, the rate of receiver j is as above, and $R_i = \log_2(1 + P_i)$. This is always possible in the domain analyzed. The boundary of the region is characterized by the non-IC player using full transmit power.

In the domain of interest, the rate region is a union of the regions for $(\mathrm{IC}, \mathrm{IC})$, $(\mathrm{noIC}, \mathrm{IC})$ and $(\mathrm{IC}, \mathrm{noIC})$ receivers. If the NE corner point of the $(\mathrm{IC}, \mathrm{IC})$ region is higher than the line connecting the corner points of the two other regions, the rate region has three corner points, one in each of the component regions, and the Nash equilibrium is efficient. Otherwise, the NE is not efficient.

The rate region of the PC-IC game with $G_{21} = G_{12} = 1.8$ and $P_1^{\max} = P_2^{\max} = 1$ is depicted in Figure 4. The NE with these parameters is $\{(\lim, \mathrm{IC}), (\lim, \mathrm{IC})\}$. In this case, both selfish players actively reduce their transmit power in order to apply IC, and reach a point on the Pareto boundary of the system. The rate region for the PC-IC capable system is compared to the ones of a pure PC system, and to Carleial's rate splitting [12].

VI. Conclusion

We have analyzed an Interference Canceling Power Control game in a Gaussian Interference Channel, where a receiver is able to cancel the interference from at most one codeword. In this case, a transmitter cannot apply rate splitting. We have found that in this setting, selfish players sometimes voluntarily reduce their transmit power to enable interference canceling, and that in some strong interference scenarios a limited power Nash Equilibrium is efficient.

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