# Large-Scale Ad Hoc Networks With Rate-Limited Infrastructure: Information-Theoretic Operating Regimes

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Abstract—The impact and information-theoretic limits of infrastructure support with rate-limited wired links are analyzed in hybrid ad hoc networks, where multi-antenna base stations (BSs) are deployed and the rate of each BS-to-BS link scales at an arbitrary rate relative to the number of randomly located wireless nodes. For the operating regimes with respect to the number of BSs and the number of antennas per BS, we first analyze the minimum rate of each BS-to-BS link,  $C_{\rm BS}$ , required to guarantee the capacity scaling for the network using infinite-capacity backhaul links. We then identify the operating regimes in which the required rate  $C_{\rm BS}$  scales much slower than 1. We also show the achievable throughput scaling for the case where the rate of each BS-to-BS link scales lower than  $C_{\rm BS}$ .

#### I. INTRODUCTION

In [1], the sum-rate scaling was introduced and characterized in a large wireless ad hoc network. It was shown that for the network having n source-destination (S-D) pairs randomly distributed in a unit area, the total throughput scales as  $\Omega(\sqrt{n/\log n})$ . This throughput scaling is achieved by conveying packets in a multihop (MH) fashion. In [3], the throughput scaling was improved to an almost linear scaling, i.e.,  $\Omega(n^{1-\epsilon})$ , by using a hierarchical cooperation (HC) strategy for an arbitrarily small  $\epsilon > 0$ . Besides the HC scheme, there has been much research to improve the throughput of interference-limited networks up to a linear scaling by using node mobility [4], interference alignment [5], and infrastructure support [6], [7]. Especially for a hybrid network consisting of both wireless ad hoc nodes and infrastructure nodes, or equivalently base stations (BSs), where each BS is equipped with multiple antennas, the optimal capacity scaling was characterized in [7]—the achievability result is based on using one of infrastructure-supported single-hop (ISH) routing, infrastructure-supported MH (IMH) routing, pure MH transmission, and HC strategy.

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<sup>1</sup>We use the following notation: i) f(x) = O(g(x)) means that there exist constants C and c such that  $f(x) \leq Cg(x)$  for all x > c, ii) f(x) = o(g(x)) means that  $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ , iii)  $f(x) = \Omega(g(x))$  if g(x) = O(f(x)), iv) f(x) = w(g(x)) if g(x) = o(f(x)), and v)  $f(x) = \Theta(g(x))$  if f(x) = O(g(x)) and g(x) = O(f(x)) [2].

In hybrid networks [6], [7], BSs have been assumed to be interconnected by infinite-capacity wired links. However, such an assumption is hardly realistic. It is fundamentally important to analyze a new achievability of hybrid networks with *ratelimited* backhaul links. In [8], [9], finite-capacity links between BSs were taken into account in studying performance of the multi-cell processing in cooperative cellular systems using Wyner-type models. Although either the Wyner or Wyner-type model in [8], [9] provides a remarkable insight into complex and analytically intractable practical cellular environments, it is far from a realistic model. In [10], the throughput scaling was studied for a hybrid ad hoc network, where the wired link interconnecting BSs is rate-limited. However, the network model under consideration is comparatively simplified, and the form of achievable schemes is limited only to MH routings.

In this paper, we introduce a more general hybrid network where the rate of each BS-to-BS link scales at an arbitrary rate relative to n, and then generalize the capacity scaling result in [7]. Multi-antenna infrastructure nodes are deployed in the network, and the best among the aforementioned four schemes, i.e., ISH routing, IMH routing, MH transmission, and HC protocol, is used to characterize the achievability result. We first derive the minimum rate of each BS-to-BS link,  $C_{BS}$ , required to guarantee the capacity scaling for the network using infinite-capacity backhaul links. The required backhaul link rate  $C_{BS}$  is shown according to the two-dimensional operating regimes with respect to the number of BSs and the number of antennas per BS. This BS-to-BS link rate is obtained by analyzing both a matching of S-D pairs between any different two cells and the minimum BS-to-BS transmission rate for each infrastructure-supported routing, which is not straightforward since the number of S–D pairs between two cells varies according to the operating regimes. Surprisingly, it turns out that for some operating regimes, the required backhaul link rate  $C_{BS}$  is indeed sufficiently small, which scales much slower than 1, and thus does not need to be infinitely large. Second, we analyze the achievable throughput scaling for realistic hybrid networks including the case where the rate of each BS-to-BS link scales slower than  $C_{BS}$ , which is based on the derivation of the achievable transmission rate

for each infrastructure-supported routing. Our results indicate that a judicious rate scaling of the BS-to-BS link under a given operating regime leads to the order optimality of our general hybrid network along with cost-effective backhaul links.

We refer to our full paper [11] for more detailed description and all the rigorous proofs.

#### II. SYSTEM AND CHANNEL MODELS

The system and channel models closely follow that of the work [7]. Consider an extended network of unit node density, where n nodes are uniformly and independently distributed on a square of area n, except for the area covered by BSs. We randomly pick S-D pairings, so that each node acts as a source and has exactly one corresponding destination node. Assume that the BSs are neither sources nor destinations. The whole area is divided into m square cells of equal area. At the center of each cell, there is one BS with l antennas. The total number of antennas in the network is assumed to scale at most linearly with n, i.e., ml = O(n). It is assumed that BSs are interconnected by rate-limited wired links and the BS-to-BS link is not affected by the interference.

The uplink channel vector between node i and BS b is denoted by  $\mathbf{h}_{bi}^{(u)} = \left[\frac{e^{j\theta_{bi,1}^{(u)}}}{r_{bi,1}^{(u)\alpha/2}}, \frac{e^{j\theta_{bi,2}^{(u)}}}{r_{bi,2}^{(u)\alpha/2}}, \dots, \frac{e^{j\theta_{bi,l}^{(u)}}}{r_{bi,l}^{(u)\alpha/2}}\right]^T$ , where  $r_{bi,t}^{(u)}$  denotes the distance between node i and the tth antenna of

 $r_{bi,t}^{*}$  denotes the distance between node i and the tth antenna of BS b,  $\theta_{bi,t}^{(u)}$  represents the random phases uniformly distributed over  $[0,2\pi)$ , and  $\alpha>2$  denotes the path-loss exponent. The downlink channel vector and the channel between two nodes can be modeled in a similar manner.

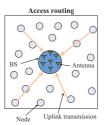
For the uplink-downlink balance, it is assumed that each BS satisfies an average transmit power constraint nP/m, while each node satisfies an average transmit power constraint P. Then, the total transmit power of all BSs are the same as the total transmit power of all wireless nodes. It is assumed that the antennas of a BS are placed as follows:

- 1) If  $l=w(\sqrt{n/m})$  and l=O(n/m), then  $\sqrt{n/m}$  antennas are regularly placed on the BS boundary and the remaining antennas are uniformly placed inside the boundary.
- 2) If  $l = O(\sqrt{n/m})$ , then l antennas are regularly placed on the BS boundary.

For analytical convenience, the parameters n, m, and l are related according to  $n=m^{1/\beta}=l^{1/\gamma}$ , where  $\beta,\gamma\in[0,1)$  with a constraint  $\beta+\gamma<1$ . The total throughput  $T_n$  of the network is defined as  $T_n=nR_n$ , where  $R_n$  is the transmission rate of each source.

# III. REVIEW ON ROUTING PROTOCOLS WITH AND WITHOUT INFRASTRUCTURE SUPPORT

In this section, routing protocols with and without infrastructure support are reviewed. Some important lemmas and theorem will be given for discussions in a later section.



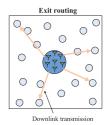
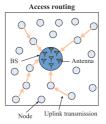


Fig. 1. The ISH protocol. Each square represents a cell in the wireless network.



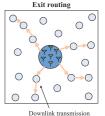


Fig. 2. The IMH protocol. Each square represents a cell in the wireless network

## A. Routing Protocols with Infrastructure Support

In infrastructure-supported routing protocols, the packet of a source is delivered to the corresponding destination of the source using three stages: access routing, BS-to-BS communication, and exit routing. We describe the following two types of infrastructure-supporting routings.

- 1) ISH Protocol: There are n/m nodes with high probability (whp) in each cell [1, Lemma 1]. The ISH protocol is first described (see Fig. 1, in which two cells are shown). For the access routing, all source nodes in each cell transmit their packets simultaneously to the home-cell BS via single-hop multiple-access. The packets of source nodes are then jointly decoded at the BS, assuming the signals transmitted from the other cells are treated as noise. In the next stage, the decoded packets are transmitted to the BS nearest to the corresponding destination of the source via BS-to-BS link. For the exit routing, the BS in each cell transmits n/m packets received from other cells to the wireless nodes in its cell.
- 2) IMH Protocol: Since the extended network is power-limited, the ISH protocol may not be effective especially when the node-BS distance is quite long, which motivates us to introduce the IMH protocol (see Fig. 2, in which two cells are shown). Each cell is further divided into smaller square cells of area  $2\log n$ , termed routing cells. Since  $\min\{l,\sqrt{n/m}\}$  antennas are regularly placed on the BS boundary,  $\min\{l,\sqrt{n/m}\}$  MH paths can be used. For the access routing, the antennas placed only on the BS boundary can receive the packet transmitted from one of the nodes in the nearest neighbor routing cell. The BS-to-BS communication is the same as the ISH protocol. For the exit routing, each antenna on the BS boundary transmits the packets to one of the nodes in the nearest neighbor routing cell.

#### B. Routing Protocols without Infrastructure Support

The protocols based only on infrastructure support may not be sufficient to achieve optimal capacity scaling with small m and l. Using one of the MH transmission [1] and the HC strategy [3] may be beneficial in terms of improving the achievable throughput scaling.

## C. The Analysis of Capacity Scaling

Capacity scaling law for an extended hybrid network was analyzed in [7] under the assumption that the rate of each BS-to-BS link is unlimited. Achievable rate scalings for ISH and IMH routings are given in the following two lemmas.

Lemma 1 ([7]): Suppose that the ISH protocol is used in the hybrid network, where the rate of each BS-to-BS link is unlimited. Then, the achievable rate is given by

$$T_{n,\rm ISH} = \Omega\left(ml\left(\frac{m}{n}\right)^{\alpha/2-1}\right). \tag{1}$$
 Lemma 2 ( [7]): Suppose that the IMH protocol is used in

Lemma 2 ([7]): Suppose that the IMH protocol is used in the hybrid network, where the rate of each BS-to-BS link is unlimited. Then, the achievable rate is given by

$$T_{n,\text{IMH}} = \Omega\left(m\min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}\right),$$
 (2)

where  $\epsilon > 0$  is an arbitrarily small constant.

From these lemmas, the total throughput can be derived in the following theorem when the ISH, IMH, MH, and HC protocols are used.

Theorem 1 ([7]): In the hybrid network, where the rate of each BS-to-BS link is unlimited, the total throughput is given by

$$T_{n} = \max \left\{ T_{n,\text{ISH}}, T_{n,\text{IMH}}, \Omega(n^{1/2 - \epsilon}), \Omega(n^{2 - \alpha/2 - \epsilon}) \right\}$$

$$= \Omega\left( \max \left\{ ml \left( \frac{m}{n} \right)^{\alpha/2 - 1}, m \min \left\{ l, \left( \frac{n}{m} \right)^{1/2 - \epsilon} \right\}, n^{1/2 - \epsilon}, n^{2 - \alpha/2 - \epsilon} \right\} \right), \tag{3}$$

where  $\epsilon > 0$  is an arbitrarily small constant.

The first to fourth terms in (3) are the achievable rate scalings of the ISH, IMH, MH, and HC protocols, respectively. In [7], it was shown that the upper bound on the total throughput matches the achievable throughput in Theorem 1 within  $n^{\epsilon}$ . That is, choosing the best one among the four schemes according to the two-dimensional operating regimes with respect to  $\beta$  and  $\gamma$ , described in Fig. 3, leads to the order optimality. The best scheme under a certain condition for each operating regime is summarized in Table I.

# IV. ROUTING PROTOCOLS WITH RATE-LIMITED INFRASTRUCTURE

In this section, routing protocols are analyzed when the infrastructure is rate-limited. Specifically, we derive the minimum rates of each BS-to-BS link required to achieve the capacity scaling in Theorem 1. We also characterize the achievable throughput scaling for the case where the rate of each BS-to-BS link scales at an arbitrary rate relative to n, which generalizes the scaling result in [7].

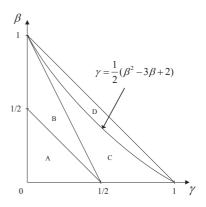


Fig. 3. Operating regimes on the achievable throughput scaling with respect to  $\beta$  and  $\gamma.$ 

TABLE I

THE BEST ACHIEVABLE SCHEMES FOR A HYBRID NETWORK WITH
INFINITE-CAPACITY INFRASTRUCTURE [7]

Regime	Condition	The best scheme
A	$2 < \alpha < 3$	HC
	$\alpha \geq 3$	MH
В	$2 < \alpha < 4 - 2\beta - 2\gamma$	HC
	$\alpha \ge 4 - 2\beta - 2\gamma$	IMH
С	$2 < \alpha < 3 - \beta$	HC
	$\alpha \geq 3 - \beta$	IMH
D	$2 < \alpha < \frac{2(1-\gamma)}{\beta}$	HC
	$\frac{2(1-\gamma)}{\beta} \le \alpha < 1 + \frac{2\gamma}{1-\beta}$ $\alpha \ge 1 + \frac{2\gamma}{1-\beta}$	ISH
	$\alpha \ge 1 + \frac{2\gamma}{1-\beta}$	IMH

### A. The Analysis of BS-to-BS Links

Let us start from the following lemma in which the number of matched S-D pairs between any different two cells is derived.

Lemma 3: Suppose that there are  $n^a$  source nodes in the ith cell. A source node in the ith cell randomly chooses its destination node in one cell among  $n^b$  cells. Then, the number of destinations in the kth cell whose source nodes are in the ith cell,  $X_{ki}$ , is given by

$$X_{ki} = \begin{cases} O(\log n) & \text{if } a \le b \\ \Theta(n^{a-b}) & \text{if } a > b, \end{cases}$$

whp as n tends to infinity.

Using Lemma 3, the required rates of the backhaul link for the ISH and IMH protocols are derived in the following two lemmas, where the ISH and IMH protocols are used in Regime D and Regimes B, C, and D, respectively. The associated two-dimensional operating regimes with respect to  $\beta$  and  $\gamma$  are illustrated in Fig. 4, where each of Regimes A, B, C, and D is divided into smaller sub-regimes.

Lemma 4: Suppose that the ISH protocol is used in Regime D of the network under consideration. Then, the number of destinations in the kth cell whose source nodes are in the ith cell,  $X_{ki}$ , is given by

$$X_{ki} = \left\{ egin{array}{ll} O\left(\log n
ight) & ext{for Regime D-1} \\ O\left(\frac{n}{m^2}\right) & ext{for Regimes D-2, D-3, and D-4,} \end{array} \right.$$

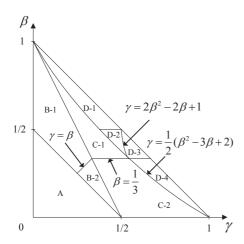


Fig. 4. Operating regimes on the required rate of each BS-to-BS link with respect to  $\beta$  and  $\gamma$ .

where  $i, k \in \{1, ..., m\}$  and  $m = n^{\beta}$ . The minimum rate of each BS-to-BS link required to achieve (3),  $C_{\text{BS,ISH}}$ , is given by

$$C_{\text{BS,ISH}} = \left\{ \begin{array}{l} \Omega \left( n \log n \left( \frac{l}{n} \right)^{\frac{\log n}{\log m}} \right) & \text{for Regime D-1} \\ \Omega \left( \frac{n^2}{m^2} \left( \frac{l}{n} \right)^{\frac{\log n}{\log m}} \right) & \text{for Regimes D-2,} \\ & & \text{D-3, and D-4.} \end{array} \right.$$

Since the achievable throughput of the IMH protocol has a different form depending on both scaling parameters  $\beta$  and  $\gamma$ , the derivation of the required rate of each BS-to-BS link for the IMH protocol is not as simple as for the ISH protocol case. In the following lemma, the required backhaul link rates for the IMH protocol, termed  $C_{\rm BS,IMH}$ , are characterized according to three operating regimes B, C, and D.

Lemma 5: Suppose that the IMH protocol is used in Regimes B, C, and D of the network under consideration. Then, the number of destinations in the kth cell whose source nodes are in the ith cell,  $X_{ki}$ , is given by

$$X_{ki} = \left\{ \begin{array}{ll} O\left(\log n\right) & \text{for Regimes B-1, C-1, D-1, D-2,} \\ & \text{and D-3} \\ O\left(\frac{l}{m}\right) & \text{for Regime B-2} \\ O\left(\sqrt{\frac{n}{m^3}}\right) & \text{for Regimes C-2 and D-4,} \end{array} \right.$$

where  $i, k \in \{1, ..., m\}$ . The minimum rate of each BS-to-BS link required to achieve (3),  $C_{\text{BS,IMH}}$ , is given by

$$C_{\text{BS,IMH}} = \begin{cases} \Omega(\log n) & \text{for Regime B-1} \\ \Omega(\frac{l}{m}) & \text{for Regime B-2} \\ \Omega\left(n^{-\epsilon}\log n\right) & \text{for Regimes C-1, D-1, D-2,} \\ & \text{and D-3} \\ \Omega\left(n^{-\epsilon}\sqrt{\frac{n}{m^3}}\right) & \text{for Regimes C-2 and D-4,} \end{cases}$$

where  $\epsilon > 0$  is an arbitrarily small constant.

Based on Lemmas 4 and 5, we are now ready to establish our first main theorem, which characterizes the minimum BSto-BS link rate guaranteeing the capacity scaling in (3) is derived when the best out of the four schemes (ISH, IMH, MH, and HC protocols) is used in our hybrid network with rate-limited infrastructure.

Theorem 2: In the hybrid network, the minimum rate of each BS-to-BS link required to achieve (3),  $C_{\rm BS}$ , is given by

$$C_{\rm BS} = \left\{ \begin{array}{ll} 0 & \text{for Regime A} \\ \Omega(\log n) & \text{for Regime B-1} \\ \Omega(\frac{l}{m}) & \text{for Regime B-2} \\ \Omega\left(n^{-\epsilon}\log n\right) & \text{for Regime C-1} \\ \Omega\left(n^{-\epsilon}\sqrt{\frac{n}{m^3}}\right) & \text{for Regime C-2} \\ \Omega\left(n^{-\epsilon}\log n\right) & \text{for Regimes D-1 and D-2} \\ \Omega\left(\frac{n^2}{m^2}\left(\frac{l}{n}\right)^{\frac{\log n}{\log m}}\right) & \text{for Regimes D-3 and D-4,} \end{array} \right.$$

where  $\epsilon>0$  is an arbitrarily small constant. The associated nine operating regimes with respect to scaling parameters  $\beta$  and  $\gamma$  are illustrated in Fig. 4.

*Proof:* Due to the page limit, a brief sketch of the proof is provided in this paper. From Table I, no infrastructure-supported protocol is needed in Regime A to achieve the capacity scaling in (3), thereby resulting in  $C_{\rm BS}=0$  in the regime. In Regimes B and C, the IMH protocol should be used to achieve the capacity scaling when path-loss exponent α is greater than or equal to a certain value. Using  $C_{\rm BS,IMH}$  in (5), one can obtain  $C_{\rm BS}$  in Regimes B-1, B-2, C-1, and C-2. In Regime D (composed of four sub-regimes D-1, D-2, D-3, and D-4), according to the value of α, either the ISH or IMH protocol should be used to achieve the capacity scaling. It is shown that  $C_{\rm BS,IMH}=\omega(C_{\rm BS,ISH})$  in Regimes D-1 and D-2, while  $C_{\rm BS,IMH}=o(C_{\rm BS,ISH})$  in Regimes D-3 and D-4, which comes from the comparison between the results in Lemmas 4 and 5. Therefore, the theorem follows.

Since the infrastructure-supported protocols are not used in Regime A, it is obvious that the required rate of BS-to-BS links is zero. In Regimes B and C, the IMH protocol should be used to achieve the capacity scaling when  $\alpha$  is larger than a certain value. The required backhaul link rate  $C_{BS}$  in Regimes B and C thus depends on the achievable rate of the IMH protocol. Since the number of antennas, l, scales slower than  $\sqrt{n/m}$  in Regime B, it is seen that the rate  $C_{BS}$  in Regime B-2 relies on l. On the other hand, in Regime C, from the fact that lscales faster than  $\sqrt{n/m}$ , the rate  $C_{\rm BS}$  is not dependent on l. In Regime D, either the ISH or IMH protocol can be used to achieve the capacity scaling in (3). As seen in (1), when  $\alpha$  is moderately small, the achievable transmission rate of the ISH protocol is larger than that of the IMH protocol, while the achievable rate of the IMH protocol does not depend on  $\alpha$ . Hence, the required rate  $C_{BS}$  of each BS-to-BS link in Regime D should be maintained when the transmission rate of the ISH protocol is maximized over  $\alpha$ .

Note that surprisingly, in Regimes A, C-1, D-1, and D-2, the required rate of each BS-to-BS link scales much slower than 1, i.e.,  $C_{\rm BS} = o(1)$ . In other words, in those four sub-regimes, the required rate of the backhaul link does not need to be infinitely high even for a large number of wireless source nodes. It

is also worth noting that the backhaul link rate required to guarantee the optimal capacity scaling in Theorem 1 regardless of network parameters m and l is given by  $\Omega(n^{1/2-\epsilon})$  in Regimes B and C and by  $\Omega(n^{1-\epsilon})$  in Regime D (the detailed derivation is omitted in this paper). This result is obtained by maximizing the rate  $C_{\rm BS}$  in (6) over scaling parameters  $\beta$  and  $\gamma$  for each operating regime. Hence, it turns out that the minimum BS-to-BS link rate  $C_{\rm BS}$  needed regardless of operating regimes (or equivalently, the values of m and l) is bound by  $\Omega(n^{1-\epsilon})$ .

#### B. The Analysis of Achievable Throughput Scaling

From Theorem 2, one can show that the optimal capacity scaling in (3) is achieved if the backhaul link rate  $C_{\rm BS}$  scales faster than a certain level given under the two-dimensional operating regimes. An important question then arises: "What is the throughput scaling if the rate of each BS-to-BS link is below  $C_{\rm BS}$ ?" In this case, the BS-to-BS link becomes a bottleneck in data transmission, and thus the achievable rate of the ISH and IMH protocols is bounded by the rate of BS-to-BS links.

In the following lemmas, the achievability results for the ISH and IMH protocols operating under infinite-capacity BS-to-BS links shown in Lemmas 1 and 2 are extended to a more general case having an arbitrary BS-to-BS link capacity,  $R_{\rm BS}$ .

*Lemma 6:* Suppose that the ISH protocol is used in the hybrid network, where the rate of each BS-to-BS link is limited by  $R_{\rm BS}$ . Then, the achievable rate is given by

$$T_{n,\text{ISH}}(R_{\text{BS}}) = \Omega\left(\min\left\{ml\left(\frac{m}{n}\right)^{\alpha/2-1}, m^2R_{\text{BS}}, \frac{n}{\log n}R_{\text{BS}}\right\}\right), \quad (7)$$

where  $m = n^{\beta}$ .

The first term of the second line in (7) corresponds to the achievable rate of the ISH protocol in (1) when the rate of the BS-to-BS link is unlimited. The remaining two terms are the throughput scalings supportable by BSs for an arbitrary  $R_{\rm BS}$ .

*Lemma 7:* Suppose that the IMH protocol is used in the hybrid network, where the rate of each BS-to-BS link is limited by  $R_{\rm BS}$ . Then, the achievable rate is given by

$$T_{n,\text{IMH}}(R_{\text{BS}}) = \Omega\left(\min\left\{ml, m\left(\frac{n}{m}\right)^{1/2-\epsilon}, m^2R_{\text{BS}}, \frac{ml}{\log n}R_{\text{BS}}, \frac{\sqrt{nm}}{\log n}R_{\text{BS}}\right\}\right), \tag{8}$$

where  $\epsilon > 0$  is an arbitrarily small constant.

The first and second terms of the second line in (8) correspond to the achievable rate of the IMH protocol in (2) when the backhaul link rate is unlimited. The remaining terms are the throughput scalings supportable by BSs for an arbitrary  $R_{\rm BS}$ . It is easy to show that the achievable transmission rate in (8) is simplified to that in (2) as  $R_{\rm BS}$  scales faster than or equal to  $C_{\rm BS}$ .

Using the two lemmas above, we finally establish the following theorem, which shows the achievable throughput along with the rate  $R_{\rm BS}$  of each BS-to-BS link.

Theorem 3: In the hybrid network with the backhaul link rate  $R_{\rm BS}$ , the throughput scales as

$$T_n(R_{\rm BS}) = \max\{T_{n,\rm ISH}(R_{\rm BS}), T_{n,\rm IMH}(R_{\rm BS}), \Omega(n^{1/2 - \epsilon}),$$
  
$$\Omega(n^{2 - \alpha/2 - \epsilon})\}, \tag{9}$$

where  $\epsilon > 0$  is an arbitrarily small constant.

Since the achievable rate of the ISH and IMH protocols can be decreased as the rate  $R_{\rm BS}$  becomes the bottleneck, either HC or MH protocol may outperform the infrastructure-supported protocols even under a certain condition such that the infrastructure-supported protocols give a better throughput scaling when the rate of each BS-to-BS link is unlimited. Furthermore, the operating regimes on the achievable throughput scaling in (9) with respect to  $\beta$  and  $\gamma$  significantly depend on the value of  $R_{\rm BS}$  (characterizing the regimes in general form is tedious).

We remark that the throughput scaling in (9) only specifies the achievability, but not the converse proof. It remains open how to derive a cut-set upper bound for the general case where  $R_{\rm BS}$  scales at an arbitrary rate relative to n (especially when  $R_{\rm BS}$  scales slower than  $C_{\rm BS}$ ). Suggestions for future research in this area include analyzing optimal capacity scaling for general hybrid networks with backhaul links scaling at an arbitrary rate by showing a tight upper bound on the total throughput.

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