Hybrid Digital-Analog Coding for Interference Broadcast Channels

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***Abstract*—We consider the transmission of bivariate Gaussian sources $(\mathbf{V_1}, \mathbf{V_2})$ over the two-user Gaussian broadcast channel in the presence of interference that is correlated to the source and known to the transmitter. Each user i is interested in estimating $\mathbf{V_i}$. We study hybrid digital-analog (HDA) schemes and analyze the achievable (square-error) distortion region under matched and expansion bandwidth regimes. These schemes require proper combinations of power splitting, bandwidth splitting, rate splitting, Wyner-Ziv and HDA Costa coding. An outer bound on the distortion region is also derived by assuming knowledge of $\mathbf{V_1}$ at the second user and full/partial knowledge of the interference at both users. Numerical results show that the HDA schemes outperform tandem and linear schemes and perform close to the derived bound for certain system settings.**

I. Introduction

The traditional approach for analog source transmission over noisy channels is to use separate source and channel coders. This approach is well known to be optimal for point-to-point communications. For multi-terminal systems, tandem coding is no longer optimal; a joint source-channel coding (JSCC) scheme may be required to achieve optimality. One simple scenario where tandem scheme is suboptimal concerns the broadcast of Gaussian sources over Gaussian channels [1].

For a single Gaussian source sent over a Gaussian broadcast channel with matched source-channel bandwidth, the distortion region is known, and can be realized by a linear scheme [1]. For mismatched source-channel bandwidth, the best known coding schemes are based on JSCC with hybrid signalling [2]–[5]. One extension to this problem is the broadcasting of two correlated sources to two users, each of which is interested in recovering one of the two sources; in [6], it was proven that the linear scheme is optimal when the system's signal-to-noise ratio is below a certain threshold under matched bandwidth. In [7], a hybrid digital-analog (HDA) scheme is proposed for the same matched bandwidth system and is shown to be optimal whenever the linear scheme of [6] is not; hence providing a complete characterization of the distortion region. Under mismatched bandwidth, various HDA schemes are proposed in [8], consisting of different combinations of several known schemes using either superposition or dirty paper coding. Recently, in [9], a tandem scheme based on successive coding is studied and shown to outperform the HDA schemes of [8]. In [10], the authors investigate the transmission of a Gaussian source over a correlated interference channel and propose a hybrid layered scheme for point-to-point communications.

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In this work, we consider the transmission of two correlated sources over broadcast interference channels, where the interference is assumed to be correlated with the sources. This broadcast system with correlated source-interference can model situations where two nearby nodes are transmitting correlated information simultaneously. One node sends directly its signal; the other, however, has knowledge about its neighbour signal and treats it as correlated interference. We propose and analyze HDA schemes for this system based on Wyner-Ziv [11], Costa [12] and HDA Costa coding [13]. The rest of the paper is organized as follows. Section II presents the problem formulation. Section III introduces an outer bound on the system's distortion region and some reference schemes. In Section IV, inner bounds on the distortion region under matched and expansion bandwidth are studied by proposing HDA schemes. Numerical results are included in Section V. Finally, conclusions are drawn in Section VI.

II. Problem Formulation

We consider the transmission (Fig. 1) of a pair of correlated Gaussian sources (V_1, V_2) over a two-user Gaussian broadcast channel in the presence of Gaussian interference S known to the transmitter. User i $(i = 1, 2)$ receives the transmitted signal corrupted by additive white Gaussian noise W_i and interference S with variances $\sigma^2_{W_i}$ and σ^2_S, respectively. Each user i aims to estimate $V_i^K = (V_i(1), V_i(2), ..., V_i(K))$, where each sample $V_i(j), j = 1, ..., K$, is drawn from an independent and identically distributed (i.i.d.) Gaussian. In this work, we assume that $(V_1(i), V_2(i), S(i)),\ i = 1, ..., K$, are correlated via the following covariance matrix

$$\Sigma_{V_1V_2S} = \begin{bmatrix} \sigma^2_{V_1} & \rho\sigma_{V_1}\sigma_{V_2} & \rho_1\sigma_{V_1}\sigma_S \\ \rho\sigma_{V_1}\sigma_{V_2} & \sigma^2_{V_2} & \rho_2\sigma_{V_2}\sigma_S \\ \rho_1\sigma_{V_1}\sigma_S & \rho_2\sigma_{V_2}\sigma_S & \sigma^2_S \end{bmatrix} \quad (1)$$

where $\sigma^2_{V_1}$ and $\sigma^2_{V_2}$ are the variances of V_1 and V_2, respectively, ρ, ρ_1 and ρ_2 are the correlation coefficients between V_1 and V_2, S and V_1 and S and V_2, respectively. The covariance matrix in (1) being positive definite restricts the possible values of ρ, ρ_1 and ρ_2. As shown in Fig. 1, the source pair vector (V_1^K, V_2^K) is transformed into an N dimensional channel input $X^N \in \mathbb{R}^N$ via $\alpha(\cdot)$, a mapping from $(\mathbb{R}^K \times \mathbb{R}^K \times \mathbb{R}^N) \to \mathbb{R}^N$. The received vector at user i is $Y_i^N = X^N + S^N + W_i^N$, where addition is component-wise, $X^N = \alpha(V_1^K, V_2^K, S^N)$, S^N is the i.i.d. Gaussian interference $(S \sim \mathcal{N}(0, \sigma^2_S))$ known to the transmitter, and each sample in the additive noise W_i^N is drawn from an i.i.d. Gaussian distribution $(W_i \sim \mathcal{N}(0, \sigma^2_{W_i}))$

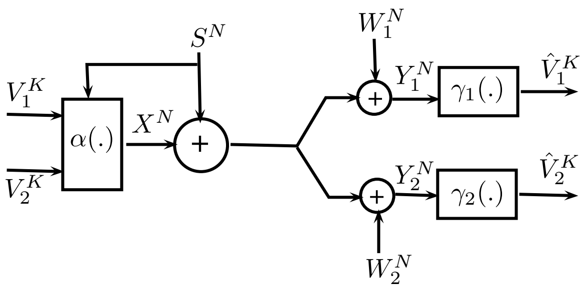

Fig. 1. System model structure.

independently from both sources and interference. The system operates under an average power constraint P given by

$$\mathbb{E}[||\alpha(V_1^K, V_2^K, S^N)||^2]/N \leq P \quad (2)$$

where $\mathbb{E}[(\cdot)]$ denotes the expectation operator. The reconstructed signal is given by $\hat{V}_i^K = \gamma_i(Y_i^N)$, where the decoder functions $\gamma_i(.)$ are mappings from $\mathbb{R}^N \to \mathbb{R}^K$. The system's rate is given by $\lambda = \frac{N}{K}$ channel use/source symbol and the reconstruction quality at each user is the mean square error (MSE) $D_i = \mathbb{E}[||V_i^K - \hat{V}_i^K||^2]/K$ for $i = 1, 2$. We assume that $\sigma^2_{W_1} > \sigma^2_{W_2}$, and hence user 1 is the weak user and user 2 is the strong one. For a given power constraint P and rate λ the distortion region is defined as the closure of all distortion pairs $(\tilde{D}_1, \tilde{D}_2)$ for which $(P, \tilde{D}_1, \tilde{D}_2)$ is achievable, where a power-distortion triple is achievable if for any $\delta > 0$, there exist sufficiently large integers K and N with $N/K = \lambda$, encoding and decoding functions $(\alpha, \gamma_1, \gamma_2)$ satisfying (2), such that $D_i < \tilde{D}_i + \delta, i = 1, 2$. In this work, we are interested in analyzing the distortion region of this system under matched $(\lambda = 1)$ and expansion bandwidth modes $(\lambda > 1)$. Note that for $\lambda > 1$, V_i^K and the first K interference samples S^K in $S^N = [S^K,\ S^{N-K}]$ are correlated via the covariance matrix in (1), while V_i^K and S^{N-K} are independent.

III. Outer Bound and Reference Schemes

A. Outer Bound

In [14] and [8], an outer bound on the distortion region for sending correlated sources over the broadcast channel without interference was obtained for $\lambda = 1$ and $\lambda \neq 1$, respectively, by assuming knowledge of the source V_1^K at the strong user. In [10], [15], several bounds are derived for point-to-point communications under correlated interference. In this section, we derive an outer bound on the distortion region for the interference broadcast channel for $\lambda \geq 1$. Since $S(i)$ and $V_1(i)$ are correlated for $i = 1, ..., K$, we have $S(i) = S_I(i) + S_D(i)$, with $S_D(i) = \frac{\rho_1 \sigma_S}{\sigma_{V_1}} V_1(i)$ and $S_I \sim \mathcal{N}(0, (1-\rho_1^2)\sigma_S^2)$. To derive an outer bound, we assume knowledge of V_1^K at the strong user (this is a reasonable assumption for small correlation coefficients; this bound, however, might not be tight for high correlation values) and $(\tilde{S}^K, S^{N-K})$ at both users, where $\tilde{S}^K = \beta_1 S_I^K + \beta_2 S_D^K$. The knowledge of the linear combination $\tilde{S}$ is motivated by [15]. The outer bound on the distortion region can be expressed as follows

$$D_1 \geq \frac{\text{Var}(V_1|\tilde{S})(\eta P + \sigma^2_{W_1})^\lambda}{(\text{MSE}(Y_1;\tilde{S}))(P + \sigma^2_{W_1})^{\lambda-1}},\ D_2 \geq \frac{\text{Var}(V_2|V_1, S)}{\left(1 + \frac{\eta P}{\sigma^2_{W_2}}\right)^\lambda} \quad (3)$$

where $\eta \in [0, 1]$, $\text{Var}(V_2|V_1, S) = \sigma^2_{V_2}\big(1 - \frac{\rho^2 - 2\rho\rho_1\rho_2 + \rho_2^2}{1-\rho_1^2}\big)$, $\text{Var}(V_1|\tilde{S}) = \sigma^2_{V_1}\big(1 - \frac{\beta_2^2\rho_1^2}{\beta_1^2(1-\rho_1^2)+\beta_2^2\rho_1^2}\big)$ and $\text{MSE}(Y_1;\tilde{S})$ is the distortion from estimating Y_1 based on $\tilde{S}$ using a linear minimum MSE estimator (LMMSE). This distortion is a function of β_1, β_2, $\mathbb{E}[XS_I]$ and $\mathbb{E}[XS_D]$. By Cauchy-Schwartz, we have $|\mathbb{E}[XS_I]| \leq \sqrt{\mathbb{E}[X^2]\mathbb{E}[S_I^2]}$ and $|\mathbb{E}[XS_D]| \leq \sqrt{\mathbb{E}[X^2]\mathbb{E}[S_D^2]}$. For a given β_1 and β_2, the maximum value of $\text{MSE}(Y_1;\tilde{S})$ has to be used in (3). Note that we need to maximize D_1 over the parameters β_1 and β_2.

Proof: For a $K:N$ system with $N \geq K$, we have

$$\frac{K}{2}\log\frac{\sigma^2_{V_2}}{D_2} \leq I(V_2^K;\hat{V}_2^K) \leq I(V_2^K; Y_2^N, V_1^K, \tilde{S}^K, S^{N-K})$$
$$= I(V_2^K; V_1^K, \tilde{S}^K, S^{N-K}) + I(V_2^K; Y_2^N|V_1^K, \tilde{S}^K, S^{N-K})$$

where the first and the second terms are

$$h(V_2^K) - h(V_2^K|V_1^K, S^N) = \frac{K}{2}\log\frac{\sigma^2_{V_2}}{\text{Var}(V_2|V_1, S)},$$

and

$$h(Y_2^N|V_1^K, S^N) - h(Y_2^N|V_1^K, V_2^K, S^N)$$
$$= \frac{N}{2}\log 2\pi e(\eta P + \sigma^2_{W_2}) - \frac{N}{2}\log 2\pi e(\sigma^2_{W_2})$$
$$= \frac{N}{2}\log(1 + \frac{\eta P}{\sigma^2_{W_2}}), \quad (4)$$

respectively. Note that we used $\frac{N}{2}\log 2\pi e(\sigma^2_{W_2}) \leq h(Y_2^N|V_1^K, S^N) \leq \frac{N}{2}\log 2\pi e(P + \sigma^2_{W_2})$; hence there exists an $\eta \in [0 \quad 1]$ such that $h(Y_2^N|V_1^K, S^N) = \frac{N}{2}\log 2\pi e(\eta P + \sigma^2_{W_2})$. To get a bound on estimating V_1^K, we can write the following

$$\frac{K}{2}\log\frac{\sigma^2_{V_1}}{D_1} \leq I(V_1^K;\hat{V}_1^K) \leq I(V_1^K; Y_1^N, \tilde{S}^K, S^{N-K})$$
$$= I(V_1^K;\tilde{S}^K, S^{N-K}) + I(V_1^K; Y_1^N|\tilde{S}^K, S^{N-K}) \quad (5)$$

with the first and the second terms satisfy

$$h(V_1^K) - h(V_1^K|\tilde{S}^K, S^{N-K}) = \frac{K}{2}\log\frac{\sigma^2_{V_1}}{\text{Var}(V_1|\tilde{S})},$$

and

$$h(Y_1^N|\tilde{S}^K, S^{N-K}) - h(Y_1^N|V_1^K, S^N)$$
$$\leq h(Y_1^K|\tilde{S}^K) + h(Y_1^{N-K}|S^{N-K}) - h(Y_1^N|V_1^K, S^N)$$
$$\leq \frac{K}{2}\log 2\pi e(\text{MSE}(Y_1;\tilde{S})) - \frac{N}{2}\log 2\pi e(\eta P + \sigma^2_{W_1})$$
$$+ \frac{N-K}{2}\log 2\pi e(P + \sigma^2_{W_1}),$$

respectively, where $h(Y_1^N|V_1^K, S^N) \geq \frac{N}{2}\log 2\pi e(\eta P + \sigma^2_{W_1})$ due to the entropy power inequality and since $Y_1^N = Y_2^N + Z^N$ with $Z \sim \mathcal{N}(0, \sigma^2_{W_1} - \sigma^2_{W_2})$. Moreover, we used $h(Y_1^K|\tilde{S}^K) \leq h(Y_1^K - \gamma_{\text{lmse}}(\tilde{S}^K))$, where $\gamma_{\text{lmse}}(\tilde{S}^K)$ is the LMMSE estimator of Y_1 based on $\tilde{S}$. Note that most inequalities follow from rate-distortion theory, the data processing inequality, the non-negativity of mutual information, conditioning reduces differential entropy and the fact that the Gaussian distribution maximizes differential entropy. □

Remark: The bound in (3) reduces to the one in [14] when there is no interference. This can be seen by setting $\rho_1 = \rho_2 = 0$ and $\beta_1 = \beta_2 = 0$. Neglecting the strong user (i.e., reducing the broadcast problem to point-to-point communications), the bound on V_1 in (3) reduces to the bounds derived in [10], [15] for point-to-point communications over the interference channel under equal bandwidth. This can be seen by setting $\eta = 0$ and $\lambda = 1$ for different values of β_1 and β_2.

B. Linear Scheme

In this section, we assume that the encoder transforms the K dimensional sources $(V_1^K,\ V_2^K)$ into an N dimensional channel input X^N using a linear transformation according to

$$X^N = \mathbf{A}V_1^K + \mathbf{B}V_2^K + \mathbf{C}S^N \tag{6}$$

where $\mathbf{A}$, $\mathbf{B}$ are $N \times K$ matrices and $\mathbf{C}$ is a $N \times N$ matrix. At the receiver side, we use a linear decoder that minimizes the MSE distortion. The estimated source is $\hat{V}_i^K = F_i G_i^{-1} Y_i^N$, where F_i is the correlation matrix between V_i^K and Y_i^N, and G_i is the covariance matrix of Y_i^N, for $i = 1, 2$.

C. Tandem Digital Scheme

This strategy is based on successive coding where the sources are encoded jointly at both the common and the refinement layers. Using [9], the achievable source coding rate (R_1, R_2) for any distortion (D_1, D_2) is given by

$$\begin{aligned} R_1(\nu) &= \frac{1}{2}\log\frac{1-\rho^2}{D_1(1-\nu^2\delta)-(\rho-\nu\delta)^2} \\ R_2(\nu) &= \left[\frac{1}{2}\log\frac{1-\nu^2\delta}{D_2}\right]^+ \end{aligned} \tag{7}$$

where $\nu \in \left[\rho, \min\left(\frac{1}{\rho}, \frac{\rho}{\delta}, \sqrt{(1-D_2)/\delta}\right)\right]$, $[x]^+ = \max(x, 0)$, and $\delta = 1 - D_1$. For a Gaussian interference broadcast channel, the rate (R_1, R_2) can be achieved if and only if there exists $0 \le \eta \le 1$ such that $R_1 \le \frac{\lambda}{2}\log\left(1 + \frac{\eta P}{(1-\eta)P + \sigma_{W_1}^2}\right)$ and $R_2 \le \frac{\lambda}{2}\log\left(1 + \frac{(1-\eta)P}{\sigma_{W_2}^2}\right)$. By plugging these rates into (7), we get the achievable distortions for D_1 and D_2. The above rates can be achieved via Costa coding. Note that this is the best tandem scheme for uncorrelated interference in terms of achievable distortion region.

IV. HDA Coding Schemes

A. HDA Scheme 1 for Matched Bandwidth

As shown from the encoder structure in Fig. 2, this scheme has four layers that are merged to output X^N. The first layer, which uses an average power of P_u, outputs $X_u^N = \sqrt{u}(\beta_1 V_1^N + \beta_2 V_2^N + \beta_3 S^N)$, a linear combination of the sources and the interference, where $\beta_1, \beta_2, \beta_3 \in [-1,\ 1]$, and $u = P_u/(\beta_1^2\sigma_{V_1}^2 + \beta_2^2\sigma_{V_2}^2 + \beta_3^2\sigma_S^2 + 2\beta_1\beta_2\rho\sigma_{V_1}\sigma_{V_2} + 2\beta_1\beta_3\rho_1\sigma_{V_1}\sigma_S + 2\beta_2\beta_3\rho_2\sigma_{V_2}\sigma_S)$ is a gain factor related to power constraint P_u. The second layer, which outputs X_a^N with power P_a, uses HDA Costa coding on the linear combination $X_a'^N = \sqrt{a}X_u^N$, where $a = P_a/P_u$ is a gain factor related to power constraint P_a. This layer is meant for both users and treats X_u^N and S^N as known interference. The auxiliary random variable of the HDA Costa encoder is $U_a^N = X_a^N + \alpha_a(S^N + X_u^N) + \kappa_a X_a'^N$, where $X_a \sim \mathcal{N}(0, P_a)$, $\alpha_a = \frac{P_a}{P - P_u + \sigma_{W_1}^2}$, $\kappa_a^2 = \frac{P_a^2}{(P - P_u + \sigma_{W_1}^2)D_a}$, and D_a is defined in (8) below. The HDA Costa encoder forms a codebook $\mathcal{U}_a$ with codeword length N and 2^{NR_a} codewords (R_a is defined later). Every codeword is generated following the random variable U_a^N. The codebook is revealed to both the encoder and decoder. The encoder searches for a $U_a^N \in \mathcal{U}_a$ such that $(X_a'^N, (S^N + X_u^N), U_a^N)$ are jointly typical. The third layer encodes the source V_1^K using Wyner-Ziv coding at a rate $R_1 = \frac{1}{2}\log(1 + \frac{P_1}{P - P_u - P_a - P_1 + \sigma_{W_1}^2})$. The Wyner-Ziv index m_1 is then encoded using Costa coding that treats S^N, X_u^N, and X_a^N as interference and uses an average power of P_1; the output of this layer is denoted by X_1^N. Similarly, in the fourth layer the source V_2^K is first encoded using a Wyner-Ziv at rate $R_1' = \frac{1}{2}\log(1 + \frac{P_2}{\sigma_{W_2}^2})$, where $P_2 = P - P_u - P_a - P_1$, followed by a Costa coder that treats S^N as well as the outputs of the first three layers as interference and outputs X_2^N.

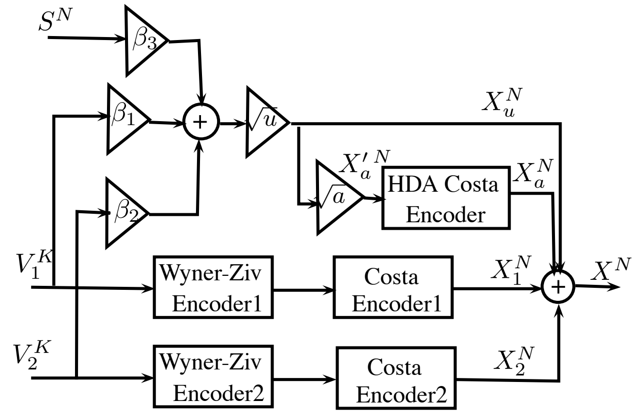

Fig. 2. HDA Scheme 1 encoder structure for rate $\lambda = \frac{N}{K} = 1$.

At the weak user, from the noisy received signal Y_1^N, a LMMSE estimator is used to get an estimate of $X_a'^N$ denoted by $\hat{X}_a'^N$. The distortion $D_a = \mathbb{E}[(X_a' - \hat{X}_a')^2]$, used in the parameter κ_a, is given by

$$D_a = P_a - \frac{\mathbb{E}[X_a' Y_1]^2}{\mathbb{E}[Y_1^2]} = P_a - \frac{(\sqrt{a}P_u + \mathbb{E}[X_a' S])^2}{P + \sigma_S^2 + \sigma_{W_1}^2 + 2\mathbb{E}[SX_u]}. \tag{8}$$

With our choice of parameters α_a and κ_a, the HDA Costa decoder can estimate U_a^N with low probability of error (for K sufficiently large) by searching for a U_a^N such that $(U_a^N, Y_1^N, \hat{X}_a'^N)$ is jointly typical. Note that the HDA codeword U_a^N can be decoded at both users. This can be proved by noting that the HDA Costa rate R_a satisfies $I(U_a; (S + X_u), X_a') \le R_a \le I(U_a; Y_i, \hat{X}_a')$, for $i = 1, 2$. The HDA Costa decoder then forms a LMMSE estimate of V_1^K, denoted by $\hat{V}_{1_a}^K$, based on Y_1^N and the decoded codeword U_a^N. The resulting distortion is given by

$$D_1^* = \sigma_{V_1}^2 - \Gamma_1^T \Lambda_1^{-1} \Gamma_1 \tag{9}$$

where Λ_1 is the covariance matrix of $[U_a\ \ Y_1]$, and Γ_1 is the correlation vector between V_1 and $[U_a\ \ Y_1]$. Note that after some manipulations, the distortion in (9) can be written as

$$D_1^* = \sigma_{V_1}^2\left(1 - \rho_{v_1 x_a'}^2\left(1 - \frac{D_{x_a'}}{P_a}\right)\right) \tag{10}$$

where $\rho_{v_1 x_a'}$ is the correlation coefficient between X_a' and V_1, and $D_{x_a'} = \frac{D_a}{1 + P_a/(P - P_u - P_a + \sigma_{W_1}^2)}$. A better estimate of V_1^K is obtained from the third layer by using the decoded Wyner-Ziv 1 codeword T_1^K and the previous estimate $\hat{V}_{1_a}^K$. Note that $T_1^K = \alpha_{wz} V_1^K + H_1$, where $\alpha_{wz} = \sqrt{1 - \frac{P_2 + \sigma_{W_1}^2}{P_1 + P_2 + \sigma_{W_1}^2}}$, and $H_1 \sim \mathcal{N}(0, D_1^*/(1 + \frac{P_1}{P_2 + \sigma_{W_1}^2}))$. The overall distortion in reproducing V_1^K is then given by

$$D_1 = \frac{D_1^*}{1 + \frac{P_1}{P_2 + \sigma_{W_1}^2}}. \tag{11}$$

The above distortion is found by equating $\frac{1}{2}\log\left(\frac{D_1^*}{D_1}\right)$ to λR_1. The strong user, that is able to decode all codewords used by the weak user, estimates the source V_2^K by first finding a linear MMSE estimate of V_2^K, denoted by $\hat{V}_{2_a}^K$, based on the HDA Costa codeword U_a^N, the Wyner-Ziv codeword T_1^K, and Y_2^N. The distortion in reproducing V_2^K is

$$D_2^* = \sigma_{V_2}^2 - \Gamma_2^T \Lambda_2^{-1} \Gamma_2 \tag{12}$$

where Λ_2 is the covariance matrix of $[U_a \quad Y_2 \quad T_1]$, and Γ_2 is the correlation vector between V_2 and $[U_a \quad Y_2 \quad T_1]$. A better estimate $\hat{V}_2^K$ is then found using the decoded Wyner-Ziv 2 codeword T_2^K and $\hat{V}_{2_a}^K$. The resulting overall distortion in estimating V_2^K is given by equating $\frac{1}{2}\log\left(\frac{D_2^*}{D_2}\right)$ to $\lambda R_1'$ as follows

$$D_2 = \frac{D_2^*}{1 + \frac{P_2}{\sigma_{W_2}^2}}. \tag{13}$$

The inner bound for HDA Scheme 1 is given by (11) and (13).

B. HDA Scheme 2 for Bandwidth Expansion

This scheme comprises two layers that are concatenated to output the transmitted signal as shown in Fig. 3. The first layer, which outputs $\tilde{X}_1^K$, consists of the HDA Scheme 1 encoder for $\lambda = 1$ (composed of four sublayers) as described in the previous section. The second layer is composed of two sublayers. The first sublayer encodes V_1^K using a Wyner-Ziv at a rate $R_2 = \frac{1}{2}\log\left(1 + \frac{P_1'}{P - P_1' + \sigma_{W_1}^2}\right)$ followed by a Costa coder with an average power P_1'. Note that the Costa coder treats S^{N-K} as interference and outputs X_{21}^{N-K} that is meant for both users. The second sublayer encodes V_2^K using a Wyner-Ziv at a rate $R_2' = \frac{1}{2}\log\left(1 + \frac{P_2'}{\sigma_{W_2}^2}\right)$ followed by a Costa coder with an average power $P_2' = P - P_1'$ that treats X_{21}^{N-K} and S^{N-K} as interference and outputs X_{22}^{N-K}. The output of the second layer is then given by $\tilde{X}_2^{N-K} = X_{21}^{N-K} + X_{22}^{N-K}$. Note that X^N is the concatenation of $\tilde{X}_1^K$ and $\tilde{X}_2^{N-K}$.

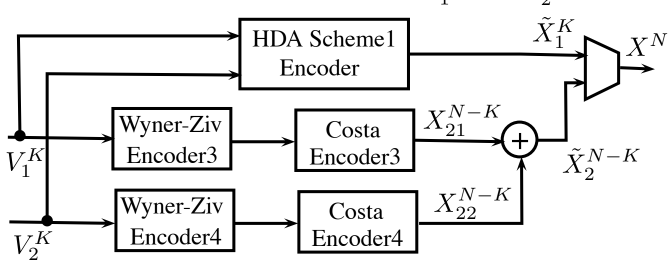

Fig. 3. HDA Scheme 2 encoder structure for rate $\lambda = \frac{N}{K} > 1$.

At the weak user, a LMMSE decoder based on the decoded HDA Costa codeword U_a^K and the first K received samples Y_1^K is used to get an estimate of V_1^K denoted by V_{1a}^K. The distortion in estimating V_1^K can be expressed in a similar way as given in (10). A better estimate, $V_1'^K$, is then achieved using the Wyner-Ziv decoder 1 (of the HDA scheme 1). The resulting distortion is then $D_1' = \frac{D_1^*}{1 + \frac{P_1}{P_2 + \sigma_{W_1}^2}}$. Using the last $N-K$ samples of the received signal Y_1^{N-K}, a better refinement of V_1^K can be obtained using the Wyner-Ziv decoder 3. The overall distortion in reconstructing V_1^K is then

$$D_1 = \frac{D_1'}{\left(1 + \frac{P_1'}{P_2' + \sigma_{W_1}^2}\right)^{\lambda - 1}}. \tag{14}$$

At the strong user, using the received signal Y_2^K, the HDA Costa codeword U_a^K, the decoded codeword T_1^K and T_3^K of the Wyner-Ziv encoder 1 (of HDA scheme 1) and 3, we can obtain an estimate of V_2^K using a LMMSE estimator. The resulting distortion is $D_2^* = \sigma_{V_2}^2 - \Gamma_2^T \Lambda_2^{-1} \Gamma_2$, where Λ_2 is the covariance matrix of $[U_a \quad Y_2 \quad T_1 \quad T_3]$, and Γ_2 is the correlation vector between V_2 and $[U_a \;\; Y_2 \;\; T_1 \;\; T_3]$. Note that $T_3 = \alpha_{wz3} V_1 + H_3$, where $\alpha_{wz3} = \sqrt{1 - \left(\frac{P_2' + \sigma_{W_1}^2}{P_1' + P_2' + \sigma_{W_1}^2}\right)^{\lambda - 1}}$ and $H_3 \sim \mathcal{N}(0, D_1)$. A refinement of this estimate can be obtained using the Wyner-Ziv decoder 2 (of HDA scheme 1) and 4. The resulting distortion in estimating V_2^K is then

$$D_2 = \frac{D_2^*}{\left(1 + \frac{P_2}{\sigma_{W_2}^2}\right)\left(1 + \frac{P_2'}{\sigma_{W_2}^2}\right)^{\lambda - 1}}. \tag{15}$$

The inner bound for HDA Scheme 2 is given by (14) and (15).

Remark: The presented layered schemes use a purely analog layer that consists of a linear combination of the sources and the interference. The use of this layer is to benefit from the interference when the source-interference correlation is high.

V. Numerical Results

In this section, we assume that the source pairs, with variance $\sigma_{V_1}^2 = \sigma_{V_2}^2 = 1$, are broadcasted to two users with interference variance $\sigma_S^2 = 0$ dB, and observation noise variance $\sigma_{W_1}^2 = 0$ dB and $\sigma_{W_2}^2 = -5$ dB, respectively. The system's average power is set to $P = 1$. To evaluate the performance, we plot the inner and outer bounds derived in the previous sections for $\lambda = 1$ and 2. Fig. 4 focuses on the

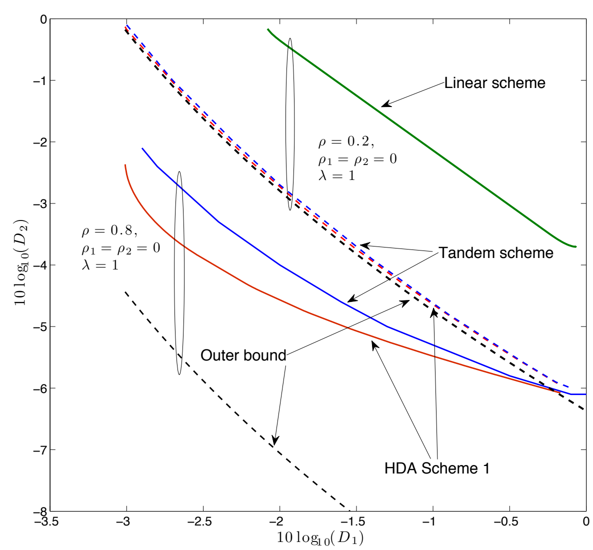

Fig. 4. Distortion regions for HDA Scheme 1 for $\lambda = 1$.

uncorrelated source-interference case ($\rho_1 = \rho_2 = 0$) under matched bandwidth ($\lambda = 1$). For low correlation between the source pairs ($\rho = 0.2$), the proposed scheme gives some improvement over the tandem scheme and considerably performs very close to the outer bound (derived in Section III); for high source correlation levels ($\rho = 0.8$), however, the HDA scheme outperforms the tandem system but has a larger gap with respect to the outer bound. Note that for uncorrelated source-interference, the linear scheme gives a poor performance. Fig. 5 shows the performance of the HDA scheme for $\rho = 0.5$,

$\rho_1 = \rho_2 = 0.2$ and $\lambda = 1$. We can notice that the purely analog scheme outperforms slightly the tandem scheme without being able to approach the HDA scheme. This can be explained from the fact that we operate at high noise levels, and since the linear scheme can benefit from the source-interference correlation. For the tandem scheme, which uses Costa coding, the transmitted signal is designed to be orthogonal to the interference; hence it cannot exploit the source-interference correlation and no performance improvement can be detected. Note that for moderate to low noise levels, the linear scheme does not outperform the tandem scheme for low source-interference correlations. Moreover, from other simulations, we noticed that for $\lambda = 1$, $\rho = 0.8$, and $\rho_1 = \rho_2 = 0.5$, the linear scheme gives the best performance (our HDA scheme reduces to a linear scheme in this case). This can be explained by noting that in [6], the authors proved that under some conditions on the noise power and source correlation (which are in accordance with the conditions for the last simulation), the linear scheme is optimal for broadcasting bivariate Gaussians under no interference. As a result, under similar conditions, the linear scheme is expected to give good performance for our problem when the source-interference correlation gets high as in [10] for point-to-point system. Fig. 6 shows that the HDA scheme outperforms the tandem scheme under bandwidth expansion ($\lambda = 2$). Note that for $\rho = 0.2$ and $\rho_1 = \rho_2 = 0$, it is hard to notice (from Fig. 6) the gain of the HDA scheme over the tandem system on the plotted scale; the outer bound for this case is not shown, since both schemes perform very closely to it. Moreover, the tandem scheme cannot benefit from the source-interference correlation and its performance depends solely on ρ in Fig. 6.

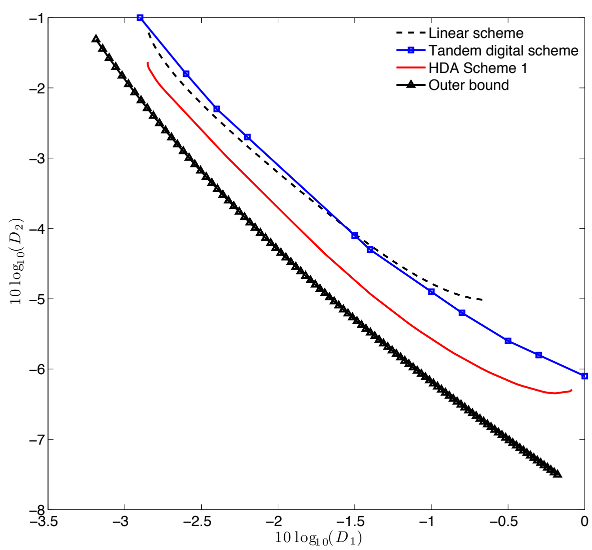

Fig. 5. Distortion regions for HDA Scheme 1 for $\lambda = 1$.

VI. Summary and Conclusions

In this paper, we consider the transmission of a pair of correlated Gaussian sources over the two-user Gaussian broadcast channel in the presence of interference that is correlated to the source. We propose layered HDA schemes under matched and expansion bandwidth scenarios based on Wyner-Ziv and HDA Costa coding and analyze their inner bounds. An outer bound on the system's distortion region is also derived. Numerical results indicate that the HDA schemes outperform the 'best' tandem scheme and perform close to the derived outer bound under some system settings.

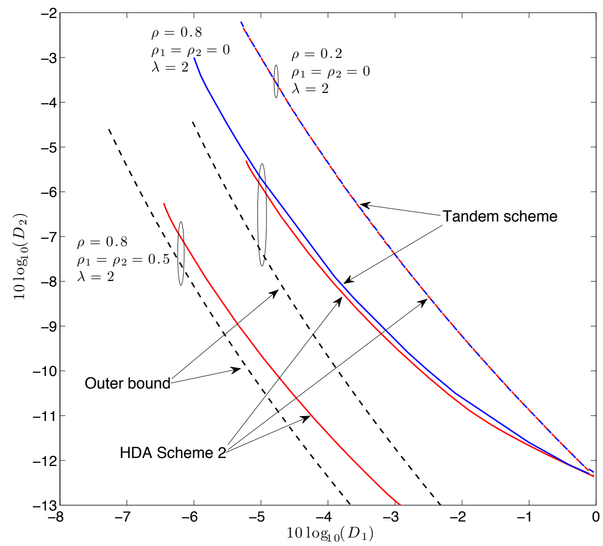

Fig. 6. Distortion regions for HDA Scheme 2 for $\lambda = 2$.

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