Achievable Rate Regions of Cognitive Multiple Access Channel with Sensing Errors

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***Abstract*—In this paper, a cognitive multiple access channel in which the secondary transmitters seek to communicate with a common secondary receiver is considered. Before data transmission, it is assumed that cooperative spectrum sensing is performed with possible errors. For this channel, achievable rate regions are initially derived under two scenarios depending on how secondary transmitters access the channel. In the first scenario, secondary users can send data under both busy and idle sensing decisions by adapting the energy level of the transmitted signals according to the sensing result. In the second scenario, the secondary transmitters are not allowed to perform data transmission if the channel is sensed as busy. Subsequently, the performance in the low-power regime is analyzed by characterizing minimum energy per bit and slope regions in these scenarios. The impact of channel sensing performance (e.g., the probabilities of detection and false alarm, channel sensing duration) on achievable rate region, energy efficiency and slope region considering both scenarios are investigated.**

***Index Terms*—Achievable rate region, channel sensing, energy efficiency, multiple access channel, probability of detection, probability of false alarm, slope region.**

I. Introduction

In recent years, cognitive radio has attracted much interest as an effective method for improving the use of under-utilized licensed spectrum bands. To obtain more insights regarding the potential throughput of cognitive radio systems, information-theoretic studies on the achievable rate regions of the cognitive interference channel in [1] and multiple access channel in [2] have been conducted. Additionally, the authors in [3] investigated the optimization of the maximum sum-rate point on the achievable rate region of a cognitive multiple access channel.

The above-mentioned works do not consider possible errors during sensing, which results in false alarm and miss detection events. With this motivation, we consider in this paper imperfect spectrum sensing in a cognitive multiple access channel where secondary users initially sense the channel and then initiate communication with a common secondary receiver (e.g., a secondary base station). It is assumed that all secondary transmitters contribute to channel sensing by sending their measurements to the base station, which makes a final decision about the channel availability. In this setting, we determine the achievable rate regions under two scenarios. In the first scenario, secondary users communicate with the base station under both busy and idle sensing decisions by adapting the energy level of the transmitted signals according to the channel sensing result. In the second scenario, the secondary transmitters cannot access the channel if the primary user activity is detected. Subsequently, we analyze the performance in the low power regime. More specifically, we investigate the energy efficiency under possible channel sensing errors by analyzing the minimum energy per bit and deriving the wideband slope region as both rates vanish in a fixed ratio.

II. System Model

We consider a cognitive multiple access channel consisting of spatially distributed secondary transmitters communicating with the a common secondary receiver (e.g., a base station). Each secondary transmitter is assumed to employ frames of T symbols for channel sensing and data transmission. The first τ symbols are allocated for collaborative channel sensing by the secondary transmitters to determine the activity of a primary user. Hence, data transmission between the secondary transmitters and the base station is performed in the remaining $T-\tau$ symbols.

A. Channel Sensing

Each secondary user listens to the channel and collects τ samples. It is assumed that the secondary users do not transmit any data during spectrum sensing. Consequently, the received signal at each secondary transmitter, denoted by z_i can be expressed as

$$\begin{aligned} z_i(l) &= n_i(l) && \text{(if the primary user is absent)} \\ z_i(l) &= h_{p,i}(l)s(l) + n_i(l) && \text{(if the primary user is present)} \end{aligned} \quad (1)$$

where $i \in \{1, 2, \ldots, M\}$ and $l = 1, 2, \ldots, \tau$. In the above relationships, n represents the zero-mean Gaussian distributed noise samples with variance N_0. Also, s is the primary user signal with power P. In addition, $h_{p,i}$ represents circularly-symmetric complex fading coefficient between the primary transmitter and i-th secondary transmitter with mean zero and variance $\mathbb{E}\{|h_{p,i}|^2\} = \sigma_h^2$.

Each transmitter measures the energy of the received signal z_i. Then, the resulting measurements are sent to the base station equipped with energy detector, which combines the measurements with equal gain and compares it with a threshold λ. Hence, with the above-mentioned statistical assumptions, the test statistic is given by [4]

$$T(z) = \frac{1}{M\tau} \sum_{l=1}^{\tau} \sum_{i=1}^{M} |z_i(l)|^2 \gtrless_{\hat{\mathcal{H}}_0}^{\hat{\mathcal{H}}_1} \lambda \quad (2)$$

where $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_1$ denote the sensing decisions that the primary users are inactive and active, respectively. In the case that the number of samples τ and M are sufficiently large, the probabilities of detection and false alarm can be obtained in terms of the Gaussian Q-function by approximating $T(z)$ as a Gaussian random variable (through the Central Limit Theorem):

$$P_d = \Pr\{T(z) > \lambda|\mathcal{H}_1\} = \Pr(\hat{\mathcal{H}}_1|\mathcal{H}_1) = Q\left(\frac{\lambda - (P\sigma_h^2 + N_0)}{\sqrt{\frac{(P\sigma_h^2+N_0)^2}{M\tau}}}\right),$$
$$P_f = \Pr\{T(z) > \lambda|\mathcal{H}_0\} = \Pr(\hat{\mathcal{H}}_1|\mathcal{H}_0) = Q\left(\frac{\lambda - N_0}{\sqrt{\frac{N_0^2}{M\tau}}}\right) \tag{3}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ represent the true hypotheses corresponding to the absence and the presence, respectively, of the primary user. Hence, the base station makes a decision on the channel occupancy via energy detection, and then the sensing result is fed back to the secondary transmitters.

B. Cognitive Multiple Access Channel

After receiving the sensing decision from the base station, the secondary users start data transmission over a block flat-fading channel in which the fading coefficients stay constant in each frame duration of T symbols and change independently between the frames. Under these assumptions together with four scenarios arising as a result of different combination of channel sensing decisions and true state of the channel, the received signal at the base station in the two-user case can be described as

$$y = \begin{cases} h_1 x_{1,j}^{(0)} + h_2 x_{2,j}^{(0)} + n_j & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_0) \\ h_1 x_{1,j}^{(1)} + h_2 x_{2,j}^{(1)} + n_j & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_1) \\ h_1 x_{1,j}^{(0)} + h_2 x_{2,j}^{(0)} + n_j + s_j & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_0) \\ h_1 x_{1,j}^{(1)} + h_2 x_{2,j}^{(1)} + n_J + s_j & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_1) \end{cases} \tag{4}$$

where $j = 1, \dots, T-\tau$ and h_i, for $i \in \{1, 2\}$, is the fading coefficient from i-th secondary transmitter to the base station. As previously defined, n_j represents the circularly symmetric additive Gaussian noise samples with zero mean and variance N_0. In addition, s_j denotes the zero-mean circularly symmetric, Gaussian distributed interference from the primary user with variance $\mathbb{E}\{|s_j|^2\} = \sigma_s^2$.

Moreover, $x_{i,j}$ in (4) denotes the signal of the i-th secondary user intended for the base station. Also, the superscripts (0) and (1) indicate two different energy levels of messages depending on idle and busy sensing decisions, respectively. More specifically, i-th secondary transmitter sends the signal $x_{i,j}^{(0)}$ with energy $\mathcal{E}_i^{(0)} = \mathcal{E}_i$ if the channel is sensed as idle. On the other hand, if the channel is sensed as busy, $x_{i,j}^{(1)}$ is transmitted with energy $\mathcal{E}_i^{(1)} = \beta\mathcal{E}_i$ where $0 \le \beta \le 1$. Such a two-level transmission scheme has the goal of protecting the primary user's transmission by limiting the harmful interference caused by secondary users' transmissions. If the secondary transmitters are not allowed to communicate with the base station when primary user activity is detected in the channel, we set $\beta = 0$.

III. Achievable Rate Region

In this section, we derive an achievable rate region for the cognitive multiple access channel under imperfect spectrum sensing.

Proposition 1: When the secondary transmitters are allowed to transmit under both channel sensing decisions (denoted by $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_1$), an achievable rate region of a two-user cognitive MAC with imperfect spectrum sensing is given by the closure of the convex hull of all rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{h_1}\left\{\log\left(1+\frac{\mathcal{E}_1^{(k)}|h_1|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\}, \\ R_2 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{h_2}\left\{\log\left(1+\frac{\mathcal{E}_2^{(k)}|h_2|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\}, \\ R_1+R_2 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{\mathbf{h}}\left\{\log\left(1+\frac{\mathcal{E}_1^{(k)}|h_1|^2+\mathcal{E}_2^{(k)}|h_2|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\} \end{aligned} \tag{5}$$

where $\mathbf{h} = [h_1, h_2]$, and the scaling term $\frac{T-\tau}{T}$ indicates the rate reduction due to allocating first τ symbols of data frame to channel sensing. Above, $\Pr(\hat{\mathcal{H}}_0)$ and $\Pr(\hat{\mathcal{H}}_1)$ denote the probabilities of channel being sensed to be idle and busy.

Proof: We first express the achievable rates in terms of mutual information between the received signal y and the transmitted signals x_1, x_2 conditioned on the channel sensing decision (i.e., $\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_1$) as follows:

$$\begin{aligned} R_1 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)I(x_1^{(k)}; y|\mathbf{h}, x_2^{(k)}, \hat{\mathcal{H}}_k), \\ R_2 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)I(x_2^{(k)}; y|\mathbf{h}, x_1^{(k)}, \hat{\mathcal{H}}_k), \\ R_1+R_2 &\le \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)I(x_1^{(k)}, x_2^{(k)}; y|\mathbf{h}, \hat{\mathcal{H}}_k). \end{aligned} \tag{6}$$

We further write $I(x_1^{(0)}; y|\mathbf{h}, x_2^{(0)}, \hat{\mathcal{H}}_0)$ as [5]

$$I(x_1^{(0)}; y|\mathbf{h}, x_2^{(0)}, \hat{\mathcal{H}}_0) = \mathbb{E}_{\mathbf{h}}\left\{\log\left(\frac{f(y|x_1^{(0)}, x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0)}{f(y|x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0)}\right)\right\} \tag{7}$$

where the conditional density function $f(y|x_1^{(0)}, x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0)$ is obtained by using the input-output relation in (4)

$$\begin{aligned} f(y|x_1^{(0)}, x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0) = &\frac{\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_0\}}{\pi N_0}\mathrm{e}^{-\frac{|y-h_1x_1^{(0)}-h_2x_2^{(0)}|^2}{N_0}} \\ &+\frac{\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}}{\pi(N_0+\sigma_s^2)}\mathrm{e}^{-\frac{|y-h_1x_1^{(0)}-h_2x_2^{(0)}|^2}{N_0+\sigma_s^2}} \end{aligned} \tag{8}$$

with variance

$$\begin{aligned} \mathbb{E}\{|y|^2|x_1^{(0)}, x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0\} &= \Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_0\}N_0 + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}(N_0+\sigma_s^2) \\ &= N_0 + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}\sigma_s^2, \end{aligned} \tag{9}$$

and the other conditional density function $f(y|x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0)$ in (7) is given by

$$\begin{aligned} f(y|x_2^{(0)}, \mathbf{h}, \hat{\mathcal{H}}_0) = &\frac{\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_0\}}{\pi(N_0+\mathcal{E}_1|h_1|^2)}\mathrm{e}^{-\frac{|y-h_2x_2^{(0)}|^2}{N_0+\mathcal{E}_1|h_1|^2}} \\ &+\frac{\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}}{\pi(N_0+\mathcal{E}_1|h_1|^2+\sigma_s^2)}\mathrm{e}^{-\frac{|y-h_2x_2^{(0)}|^2}{N_0+\mathcal{E}_1|h_1|^2+\sigma_s^2}} \end{aligned} \tag{10}$$

with variance

$$\mathbb{E}\{|y|^2|x_2^{(0)},\mathbf{h},\hat{\mathcal{H}}_0\} = N_0 + \mathcal{E}_1|h_1|^2 + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}\sigma_s^2. \tag{11}$$

Above, it can be observed that these conditional densities are actually mixture of Gaussian distributions due to spectrum sensing errors. Therefore, we cannot derive a closed form expression for mutual information in (7). However, we can still find the following lower bound by replacing the above conditional density functions with pure Gaussian distributions having the same variances in (9) and (11) due to the fact that Gaussian distribution provides the worst-case noise:

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}&\left\{\log\left(\frac{f(y|x_1^{(0)},x_2^{(0)},\mathbf{h},\hat{\mathcal{H}}_0)}{f(y|x_2^{(0)},\mathbf{h},\hat{\mathcal{H}}_0)}\right)\right\}\\ &\geq \mathbb{E}_{h_1}\left\{\log(1+\frac{\mathcal{E}_1|h_1|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2})\right\}.\end{aligned} \tag{12}$$

The rest of the mutual information expressions in (6) are evaluated in a similar fashion. By combining the resulting lower bounds, we obtain the achievable rate region stated in (5). □

Remark 1: The achievable rate region of the two-user cognitive MAC in (5) can easily be extended to the general M-user case with $M \geq 2$ as

$$\sum_{i\in S} R_i \leq \frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{\mathbf{h}}\left\{\log\left(1+\frac{\sum_{i\in S}\mathcal{E}_i^{(k)}|h_i|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\} \tag{13}$$

for all subsets $S \subseteq \{1,\ldots,M\}$.

Remark 2: When the secondary users are allowed to transmit only if the channel is sensed as idle, an achievable rate region for the cognitive MAC is given by setting setting $\beta = 0$, hence $\mathcal{E}_i^{(1)} = 0$ in (13), and we have

$$\sum_{i\in S} R_i \leq \frac{T-\tau}{T}\Pr(\hat{\mathcal{H}}_0)\mathbb{E}_{\mathbf{h}}\left\{\log\left(1+\frac{\sum_{i\in S}\mathcal{E}_i|h_i|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2}\right)\right\} \tag{14}$$

for all subsets $S \subseteq \{1,\ldots,M\}$.

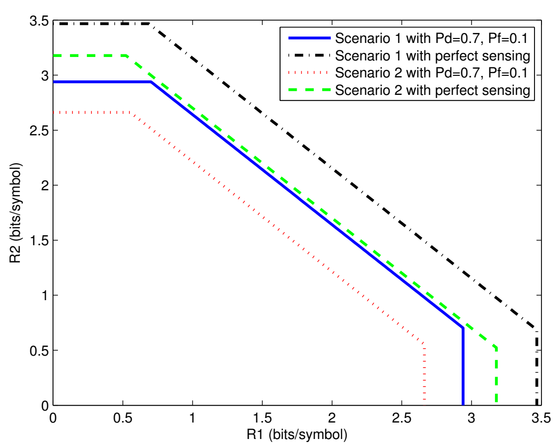

Fig. 1. Achievable rate region for a two-user cognitive multiple access channel.

In Fig. 1, we plot the achievable rate regions in scenarios 1 and 2 for both perfect sensing (in which detection probability $P_d = 1$ and false alarm probability $P_f = 0$) and imperfect sensing with $P_d = 0.7$ and $P_f = 0.1$. Scenario 1 corresponds to data transmission under both busy and idle sensing decisions whereas in scenario 2, the secondary users are allowed to perform data transmission only if the channel is sensed as idle. We observe that sensing performance significantly affects the achievable rate region. In general, imperfect sensing results in a smaller region. This is primarily due to the fact that there is either no secondary transmission or transmissions occur with lower power in the case of false alarms, and we have in all cases additional additive disturbance (i.e., additional noise variance term $\Pr(\mathcal{H}_i|\hat{\mathcal{H}}_0)\sigma_s^2$) due imperfect spectrum sensing. Note that imperfect sensing also leads to increased interference inflicted on the primary users. Finally, the achievable rate region in the scenario 1 is larger than that in scenario 2 for both perfect and imperfect sensing, as expected, since the secondary transmitters have more transmission opportunities in scenario 1.

IV. Low-Power Regime Analysis

In this section, we analyze the energy efficiency in a two-user cognitive multiple access channel in the low power regime. Energy efficiency is an important consideration in wireless systems and can be measured by the average received energy per information bit normalized by the background noise level, which is formulated as

$$\frac{E_{b,i}}{N_0} = \frac{(\Pr\{\hat{\mathcal{H}}_0\}\mathcal{E}_i + \Pr\{\hat{\mathcal{H}}_1\}\beta\mathcal{E}_i)\mathbb{E}\{|h_i|^2\}}{N_0 R_i(\mathcal{E})} \tag{15}$$

where $i \in \{1,2\}$. By using (5), rate expressions $R_1(\mathcal{E})$ and $R_2(\mathcal{E})$ can be expressed in terms of the time-sharing parameter $\alpha \in [0,1]$ as given in (16) at the top of the next page. The boundary of the achievable rate region (5) can be attained by varying α from 0 to 1.

If the secondary transmitters can access the channel only when there is no primary user activity detected, the energy efficiency can be described as

$$\frac{E_{b,i}}{N_0} = \frac{\mathcal{E}_i\mathbb{E}\{|h_i|^2\}}{N_0 R_i(\mathcal{E})} \tag{17}$$

where $R_i(\mathcal{E})$ for $i = 1,2$ can be found by setting $\beta = 0$ in (16).

The spectral and energy efficiency in the low power regime was analyzed by Verdú in [6] where linear approximation for the $R(\frac{E_b}{N_0})$ curve is provided through two low-power performance metrics, namely the minimum energy per bit and the wideband slope. $\frac{E_{b,i}}{N_0}_{\text{min}}$ is the minimum energy per bit of i-th secondary user required for reliable communication which is generally achieved as $\mathcal{E}_i \to 0$, and S_i is the slope of the $R(\frac{E_{b,i}}{N_0})$ curve as $\frac{E_{b,i}}{N_0} \to \frac{E_{b,i}}{N_0}_{\text{min}}$. These metrics are formulated as

$$\frac{E_{b,i}}{N_0}_{\text{min}} = \lim_{\mathcal{E}_i\downarrow 0}\frac{E_{b,i}}{N_0} = \frac{(\Pr\{\hat{\mathcal{H}}_0\}+\Pr\{\hat{\mathcal{H}}_1\}\beta)\mathbb{E}\{|h_i|^2\}\log 2}{N_0\dot{R}_i(0)} \tag{18}$$

$$S_i \triangleq \lim_{\frac{E_{b,i}}{N_0}\downarrow\frac{E_{b,i}}{N_0}_{\text{min}}}\frac{R(\frac{E_{b,i}}{N_0})10\log_{10}2}{10\log_{10}\frac{E_{b,i}}{N_0} - 10\log_{10}\frac{E_{b,i}}{N_0}_{\text{min}}} = \frac{2[\dot{R}_i(0)]^2}{-\ddot{R}_i(0)} \tag{19}$$

$$R_1(\mathcal{E}) = \alpha\frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{h_1}\left\{\log\left(1+\frac{\mathcal{E}_1^{(k)}|h_1|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\} + (1-\alpha)\frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{\mathbf{h}}\left\{\log\left(1+\frac{\mathcal{E}_1^{(k)}|h_1|^2}{N_0+\mathcal{E}_2^{(k)}|h_2|^2+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\},$$
$$R_2(\mathcal{E}) = \alpha\frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{\mathbf{h}}\left\{\log\left(1+\frac{\mathcal{E}_2^{(k)}|h_2|^2}{N_0+\mathcal{E}_1^{(k)}|h_1|^2+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\} + (1-\alpha)\frac{T-\tau}{T}\sum_{k=0}^{1}\Pr(\hat{\mathcal{H}}_k)\mathbb{E}_{h_2}\left\{\log\left(1+\frac{\mathcal{E}_2^{(k)}|h_2|^2}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_k)\sigma_s^2}\right)\right\}. \quad (16)$$

where $\dot{R}_i(0)$ and $\ddot{R}_i(0)$ denote the first and second derivatives of $R_i(\mathcal{E})$ with respect to $\mathcal{E}$ evaluated at $\mathcal{E}=0$

It can be seen without much difficulty that the minimum energy per bit for both users are the same and can be determined by inserting the first derivative expression in (23) on the next page into (18). The slope region $\mathbf{S}(\theta)$ introduced in [7] provides the growth of individual user's rate at minimum energy per bit as both rates vanish with a fixed proportion of θ, which is expressed as

$$\frac{\mathbb{E}\{|h_1|^2\}\mathcal{E}_1}{\mathbb{E}\{|h_2|^2\}\mathcal{E}_2} = \frac{R_1}{R_2} = \theta. \quad (20)$$

Our aim is to characterize the performance of the cognitive multiple access channel at low spectral efficiencies by deriving analytical expressions for the slope region.

Proposition 2: The slope region of the two-user cognitive MAC is the closure of the union of all wideband slope values (S_1, S_2) satisfying

$$S(\theta) = \left\{(S_1,S_2): 0 \le S_1 \le \bar{S}_1, 0 \le S_2 \le \bar{S}_2 \right.$$
$$\left. \theta\left[\frac{1}{S_1}-\frac{1}{\bar{S}_1}\right] + \frac{1}{\theta}\left[\frac{1}{S_2}-\frac{1}{\bar{S}_2}\right] = 1\right\} \quad (21)$$

where

$$\bar{S}_i = \left(\frac{T-\tau}{T}\right)\frac{2\mathbb{E}^2\{|h_i|^2\}}{\mathbb{E}\{|h_i|^4\}}\left[\frac{\left(\frac{\Pr\{\hat{\mathcal{H}}_0\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2}+\frac{\beta\Pr\{\hat{\mathcal{H}}_1\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2}\right)^2}{\frac{\Pr\{\hat{\mathcal{H}}_0\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}+\frac{\beta^2\Pr\{\hat{\mathcal{H}}_1\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2)^2}}\right]. \quad (22)$$

Proof: By using the fixed rate ratio in (20), we rewrite $R_1(\mathcal{E})$ and $R_2(\mathcal{E})$ in (16) as a function of only $\mathcal{E}_1$ and $\mathcal{E}_2$, respectively. Then, the first and second derivates of $R_1(\mathcal{E})$ and $R_2(\mathcal{E})$ are evaluated at $\mathcal{E}_1 = 0$ and $\mathcal{E}_2 = 0$, respectively. Inserting the resulting expressions in (23) given at the top of the next page into (19), we obtain the wideband slopes of each user as follows:

$$S_1 = \left(\frac{T-\tau}{T}\right)\frac{2\theta}{2-2\alpha+\theta\frac{\mathbb{E}\{|h_1|^4\}}{\mathbb{E}^2\{|h_1|^2\}}}\left[\frac{\left(\frac{\Pr\{\hat{\mathcal{H}}_0\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2}+\frac{\beta\Pr\{\hat{\mathcal{H}}_1\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2}\right)^2}{\frac{\Pr\{\hat{\mathcal{H}}_0\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}+\frac{\beta^2\Pr\{\hat{\mathcal{H}}_1\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2)^2}}\right]$$
$$S_2 = \left(\frac{T-\tau}{T}\right)\frac{2}{2\alpha\theta+\frac{\mathbb{E}\{|h_2|^4\}}{\mathbb{E}^2\{|h_2|^2\}}}\left[\frac{\left(\frac{\Pr\{\hat{\mathcal{H}}_0\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2}+\frac{\beta\Pr\{\hat{\mathcal{H}}_1\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2}\right)^2}{\frac{\Pr\{\hat{\mathcal{H}}_0\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}+\frac{\beta^2\Pr\{\hat{\mathcal{H}}_1\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2)^2}}\right]. \quad (24)$$

When the wideband slope S_1 is evaluated at $\alpha = 1$ and S_2 is evaluated at $\alpha = 0$, corresponding to the vertices of an achievable rate region in (5), the maximum wideband slope values $\bar{S}_1$ and $\bar{S}_2$ are derived. Moreover, by solving α values in terms of only S_1 and S_2 from the expressions (24) and subtracting them from each other, we obtain the equality in (22). □

Proposition 3: When the secondary users are allowed to transmit only when the channel is detected as idle, the corresponding slope region is given by the set of slope pairs (S_1, S_2) such that

$$\mathbf{S}(\theta) = \left\{(S_1,S_2): 0 \le S_1 \le \bar{S}_1, 0 \le S_2 \le \bar{S}_2 \right.$$
$$\left. \theta\left[\frac{1}{S_1}-\frac{1}{\bar{S}_1}\right] + \frac{1}{\theta}\left[\frac{1}{S_2}-\frac{1}{\bar{S}_2}\right] = 1\right\} \quad (25)$$

where $\bar{S}_i = \left(\frac{T-\tau}{T}\right)\Pr\{\hat{\mathcal{H}}_0\}\frac{2\mathbb{E}^2\{|h_i|^2\}}{\mathbb{E}\{|h_i|^4\}}$.

Proof: We adopt the same approach as in the proof of Proposition 3. First, corresponding achievable rate pairs are obtained by setting β to 0 in (16) since there is no data transmission allowed under busy sensing decision. After expressing resulting rate expressions R_1 and R_2 as a function of only $\mathcal{E}_1$ and $\mathcal{E}_2$, respectively by invoking the constraint in (20), the first and second derivates are evaluated at $\mathcal{E}_i = 0$. These derivatives can easily be found by setting $\beta = 0$ in (23). Hence, the wideband slopes are obtained by inserting the resulting derivatives in (19)

$$S_1 = \left(\frac{T-\tau}{T}\right)\Pr\{\hat{\mathcal{H}}_0\}\frac{2\theta}{2-2\alpha+\theta\frac{\mathbb{E}\{|h_1|^4\}}{\mathbb{E}^2\{|h_1|^2\}}},$$
$$S_2 = \left(\frac{T-\tau}{T}\right)\Pr\{\hat{\mathcal{H}}_0\}\frac{2}{2\alpha\theta+\frac{\mathbb{E}\{|h_2|^4\}}{\mathbb{E}^2\{|h_2|^2\}}}. \quad (26)$$

Consequently, $\bar{S}_i$ values are determined by setting α to 1 and 0 in the above S_1 and S_2 expressions, respectively. □

Next, we numerically evaluate the slope regions to investigate the performance in the wideband regime with possible channel sensing errors. In the numerical results, unless mentioned explicitly, it is assumed that the variance of primary user signal $\sigma_s^2 = 1$, and the secondary transmitters send messages with energy 10 dB or 0 dB depending on channel being detected as idle or busy, respectively. Also, the prior probabilities $\Pr\{\mathcal{H}_0\} = 0.4$ and $\Pr\{\mathcal{H}_1\} = 0.6$.

In Fig. 2, we plot slope region of the two-user cognitive multiple access channel for $\theta = 1$ and $\theta = 3$, corresponding to upper and lower subfigures, respectively. It is assumed that the noise power $N_0 = 1$. If the false alarm probability decreases while keeping the detection probability fixed, the base station less frequently declares that the channel is busy even though the channel is actually not occupied by the primary users. Therefore, channel is utilized more efficiently, yielding higher wideband slopes. If the detection probability increases while keeping the false alarm probability fixed, the base station

$$
\begin{aligned}
\dot{R}_i(0) &= \left(\frac{T-\tau}{T}\right)\left[\Pr\{\hat{\mathcal{H}}_0\}\frac{\mathbb{E}\{|h_i|^2\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2}+\Pr\{\hat{\mathcal{H}}_1\}\frac{\beta\mathbb{E}\{|h_i|^2\}}{N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_1)\sigma_s^2}\right] \quad i\in\{1,2\}\\
\ddot{R}_1(0) &= -\left(\frac{T-\tau}{T}\right)\left(\Pr\{\hat{\mathcal{H}}_0\}+\beta^2\Pr\{\hat{\mathcal{H}}_1\}\right)\left[\frac{\mathbb{E}\{|h_1|^4\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}+\frac{2(1-\alpha)\mathbb{E}\{|h_1|^2\}^2}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2\theta}\right]\\
\ddot{R}_2(0) &= -\left(\frac{T-\tau}{T}\right)\left(\Pr\{\hat{\mathcal{H}}_0\}+\beta^2\Pr\{\hat{\mathcal{H}}_1\}\right)\left[\frac{\mathbb{E}\{|h_2|^4\}}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}+\frac{2\alpha\theta\mathbb{E}\{|h_2|^2\}^2}{(N_0+\Pr(\mathcal{H}_1|\hat{\mathcal{H}}_0)\sigma_s^2)^2}\right].
\end{aligned} \tag{23}
$$

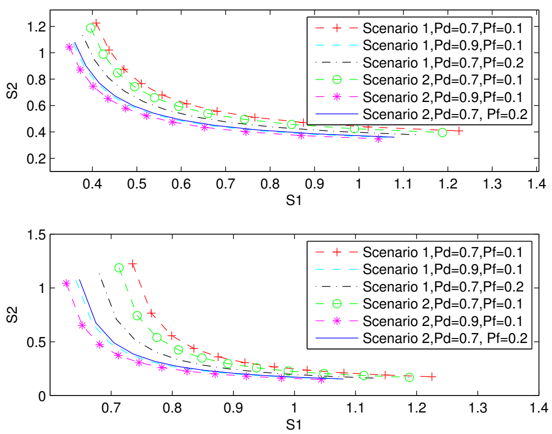

Fig. 2. Slope region of the two-user cognitive multiple access channel for $\theta = 1$ and $\theta = 3$.

correctly detects the primary user activity more often, and each secondary transmitter uses lower transmission power to limit the interference, leading to lower achievable rates, hence lower wideband slopes. In addition, in scenario 2, each users' rate grows slower in the low-power regime compared to scenario 1. This is because in scenario 2, the secondary users stop transmitting when the channel is detected as busy, and consequently achieve smaller rates.

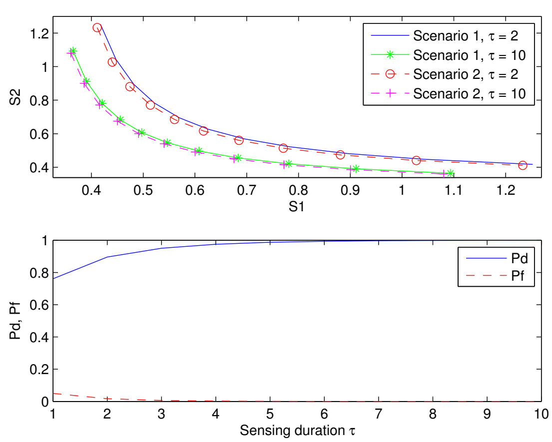

Fig. 3. Slope region and the probabilities of detection and false alarm for different values of the sensing duration τ.

In Fig. 3, we display the slope region of the two-user cognitive multiple access channel for different sensing durations, i.e., for $\tau = 2$ and $\tau = 10$, and plot the probabilities of detection and false alarm vs. sensing duration τ. The noise power is set to 0.2. In the lower subfigure, it is observed that the probabilities of detection and false alarm approach 1 and 0, respectively as the sensing duration increases. Hence, when the secondary transmitters allocate more time to channel sensing, we have better sensing performance at the expense of smaller slope region, seen in the upper subfigure. This is because as channel sensing takes more time, less time is left for data transmission, which results in lower achievable rates.

V. Conclusion

In this paper, we have initially characterized an achievable rate region for the two-user cognitive multiple access channel in the presence of sensing errors. The extensions to the case of M users is discussed briefly. Subsequently, we have analyzed the performance at low power levels by considering the energy per bit, and determined the wideband slope region, which is one of the key performance measures at low spectral efficiencies. The corresponding analytical expressions reveal that the sensing performance has significant impact on the achievable rate region and the slope region through detection and false alarm probabilities. This is also validated through numerical results. It is generally observed that imperfect channel sensing yields performance degradation.

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