New Pattern Erasure Codes

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***Abstract*—In this paper, we study binary pattern erasure codes, i.e., binary codes that are resiliant to erasures from a family P of possible erasures. We give an algorithmic proof of the existence of a binary linear code with codewords of length n that is resiliant to erasures from P when P satisfies the properties: every pattern $p \in P$ has size m and every letter in the alphabet occurs in at most c patterns.**

The density of the parity matrix is plays a important role in storage applications,so we also introduce a new low density code basing on graph theory.

I. Introduction

The classical binary erasure channel is a communications channel in which each bit will be erased independently with equal probability. To deal with this type of erasure, code schemes have been designed that provide equitable protection at each location, for example, those cyclic codes[1]. However, this is not necessarily a realistic scenario.

Unequal error protect codes (UEP codes) [2], [3], for instance, offer resilience against errors in transmissions that are not equally distributed. UEP code also assumes that the elements at different positions will fail independently, which again might not be realistic.

For some applications, we require resilience against failures in one or more groups of locations. For example, in a large storage system, data located in same disk will always fail in same time, called array erasure channel [4]. As another example, in the classical burst erasure channel, bits clustering together will more probably fail simultaneously [1], [5]. Researchers have developed optimal code schemes for those two kinds channels [4], [6].

In recent years, big data storage systems are of growing importance. In large data centers, RAID architecture utilizing array codes plays an important role in disk array systems [7]. However, disk physical errors are not the only reason of the data retrieving failure. For example, a local power off can cause a group of disks fail simultaneously.

In this paper, we research the pattern erasure channel.Pattern erasures can be considered as an extension to the array erasures and the burst erasures.

II. Pattern erasure Channel

For $n \geq 1$, let $[n] = \{1, 2, \dots, n\}$. Consider a code C with codeword length n over GF(2). Any subset $p \in [n]$ is called a *pattern*. A *pattern erasure* on p is when the elements with indices $i \in p$ of a codeword are all simultaneously erased. If pattern erasures may only occur for p belonging to a family P of subsets of $[n]$, we call P a *pattern erasure set* on $[n]$. If P is a pattern erasure set, we assume equal probability of each $p \in P$ being erased.

We also introduce the following parameters:

- If a pattern erasure set P is such that all $p \in P$ have size m, then we say P is m*-regular*.
- We call a code C a t*-pattern erasure code* on P if every t pattern erasures can be recovered.
- If $i \in [n]$ and $i \in p$ where $p \in P$ for some pattern erasure set P, i is *covered* by p. We define the *cover number*

$$c = \max_{i \in [n]} \#\{p \in P : i \text{ is covered by } p\}.$$

  If all $i \in [n]$ covered by c patterns,then we say P is *c-regular covered.*

Note:If pattern erasure set P is not regular ,then we can normalize it to a regular pattern set by introducing some imaging elements. And if P is not regular covered,we can also normalize it to a regular covered pattern set by introducing some imaging patterns.So in this paper we focus on those m*-regular* and *c-regular covered* pattern set.

We use $C = C(n, m, c, t)$ to denote a binary t-pattern erasure code on an m*-regular* and *c-regular covered* pattern erasure set P , and $C^{\text{lin}} = C^{\text{lin}}(n, m, c, t)$ to denote a linear $C(n, m, c, t)$ code.

Obviously ,there are totally $\binom{m}{n}$ possible different patterns of size m on $[n]$. If P includes all those $\binom{m}{n}$ patterns then we say P an m*-complete pattern set*. It is easy to see that single pattern erasure resilient codes on m*-complete pattern set* are same to the traditional binary block code with minimum Hamming distance $m + 1$. In this paper,we are interested in the case $\#P \ll \binom{m}{n}$.There are several well studied channels meeting the situation. E.g.

- A $C(n, m, 1, t)$ code is a t array erasure resiliant code.

- For a given pattern of size $m \le n$, and $q \subseteq [n]$, we can construct a pattern erasure set

$$P = \big\{\{i + j \bmod n : j \in q\} : i \in \{0, 1, \dots, m-1\}\big\}.$$

In this case, a code would be for the classical burst erasure channel .

Because $\#P$ is not too large, so describing P by a 2-parts pattern graph will be more intuitionistic. In a pattern graph ,the higher part vertexes are for patterns,and the lower vertexes for elements.The edges are for those covering relations. Fig. 1 show the pattern graph of a array erasure channel,and fig. 2 is for a burst erasure channel.

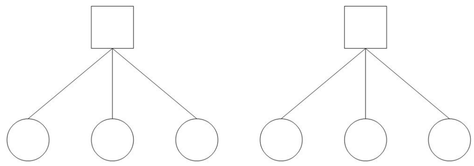


Fig. 1. Array Erasure Channel

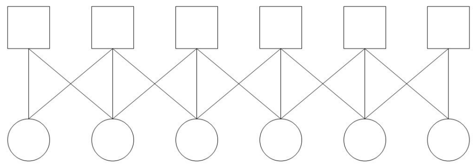


Fig. 2. Burst Erasure Channel

A. P-MDS codes

The Singleton bound for q-ary codes C, with codewords of length n, says that if d is the maximum Hamming distance between two codewords in C, then $|C| \le q^{n-d+1}$. Maximum distance separable (MDS) are codes that satisfy the Singleton bound.

Theorem 1. *Let P be an m-regular pattern erasure set on $[n]$, and let $\mathcal{T}$ be the set of t-subsets of P. If C is a binary t-pattern erasure code for P, then*

$$|C| \le 2^{n - \max_{T \in \mathcal{T}} |\cup T|}. \tag{1}$$

Proof: Let $T^* \in \mathcal{T}$ such that $|\cup T^*| = \max_{T \in \mathcal{T}} |\cup T|$. If (1) does not hold, then, by the pigeonhole principle, there are two codewords whose bits agree outside of $\cup T^*$. Thus, these codewords will not be uniquely recoverable from erasure of T^*. ∎

We say a code binary code C is a *P-MDS code* if (1) holds for C. Note that the notion of a P-MDS code genearlises the notion of a binary MDS code: when $\mathcal{T}$ is the set of all $(d-1)$-subsets of $[n]$, (1) becomes the Singleton bound with $q = 2$ [1]. The argument used to prove Theorem 1 would work for q-ary codes, but in this work, we only consider binary codes. Equation (1) also genearlises the Rieger bound [8] for burst errors.

Based on numerical evidence and partial results we will present in this paper, we make the following conjecture.

Conjecture 2. *For all m-regular pattern erasure sets P with cover number c, there are exist a P-MDS 1-pattern erasure code $C(n, m, c, 1)$ whenever $c \le m + 1$.*

There are some examples of P-MDS code on some special Ps.

1) On array channel:The cover number is 1,and the well known Reed Solomon code provides the construction of t erasure resilience P-MDS code.
2) On burst channel:The cover number is m,while simply group parity code is a 1 erasure resilience P-MDS code.
3) On simple mixed channel:In [8], Xu study the method to construct burst code from a MDS array code.By this method,we can construct new P-MDS codes of a mixed channel from a k erasure MDS code.While the mixed channel is combined by a $k-1$ array erasures and a single burst erasure of length m(the height of the array).

B. Construction method

A *parity-check matrix* of a $C^{\text{lin}}(n, m, c, t)$ code is an $m \times n$ matrix M over GF(2) such that $Mw^T = \mathbf{0}$ for $w \in \text{GF}(2)^n$ if and only if $w \in C$. We can think of C as the null space of M (or, more specifically, the transpose of the vectors in the null space of M). Further, as we describe in the following theorem, if we can find a suitable matrix M, we can construct a P-MDS $C^{\text{lin}}(n, m, c, 1)$ code.

Theorem 3. *Let $m \le n$. Let P be a m-regular pattern erasure set with cover number c. Let M be a $m \times n$ matrix over* GF(2) *such that*

1) *M has full rank and*
2) *for all $p \in P$, the $m \times m$ submatrix formed by the columns of M with indices in p has full rank.*

Then the null space of M forms a P-MDS $C^{lin}(n, m, c, 1)$ code.

Proof: Let C be the linear code formed by the null space of M. Suppose $w \in C$ and we have erased m bits of w whose indices belong to some $p \in P$. We know $Mw^T = \mathbf{0}$

Let $p \in P$. For simplicity, we assume $p = \{1, 2, \dots, m\}$, otherwise we can reorder the coordinates of C. In order to show that C is a $C^{\text{lin}}(n, m, c, 1)$ code, let $w' \in C$ such that w and w' agree at the coordinates whose indices belong to $[n] \backslash p$. We decompose M into two submatrices: A, formed by the first m columns of M, and B, formed by the last $n-m$ columns of M. Let v and v' be the vectors formed by the first m elements of w and w', respectively. We know that $Av^T = A(v')^T$, since both w^T and $(w')^T$ belong to the null space of M and agree at the last $n - m$ coordinates. Hence $(v - v')^T$ is in the null space of A. But since A has full rank, we must have $v = v'$, implying $w = w'$. Hence C is a $C^{\text{lin}}(n, m, c, 1)$ code.

Since M has rank m, we know C is a $(n-m)$-dimensional subspace. Hence $|C| = 2^{n-m}$, and hence C is a P-MDS code. ∎

The methods we present for generating P-MDS $C^{\text{lin}}(n, m, c, 1)$ codes will consist of generating matrices that satisfy the constraints of Theorem 3.

C. Generating P-MDS $C(n,m,c,1)$ codes with low redundancy

In this section, we will analyze an algorithm for generating matrices that satisfy a modified version of Theorem 3, namely the following.

Theorem 4. *Let P be a m-regular pattern erasure set with cover number c. Let M be a $m' \times n$ matrix over $\mathrm{GF}(2)$ such that for all $p \in P$, the $m' \times m$ submatrix formed by the columns of M with indices in p has full rank. Then the null space of M forms a $C^{lin}(n,m,c,1)$ code.*

Theorem 4 can be proved analogously to Theorem 3, and, provided m' is not much larger than m, gives a "near" P-MDS $C(n,m,c,1)$ code. The algorithm is given in the proof of Theorem 7 and will generate M satisfying the conditions of Theorem 4, while trying to minimize m'. For $c \le 2$ it will always achieve $m = m'$. Our analysis of this algorithm requires the study of vector space coverings.

We say a vector space V is *covered* by subspaces $\{A_i\}_{i=1}^c$ if $V = \cup_i A_i$. We also say $\{A_i\}_{i=1}^c$ is a *covering* of V.

Lemma 5. *Let $1 \le s \le k$. Any covering of $\mathrm{GF}(2)^k$ by subspaces of dimension s contains at least 2^{k-s} subspaces. Moreover, if $s < k$ then we require more than 2^{k+s} such subspaces.*

Proof: A subspace of dimension s contains $2^s - 1$ non-zero vectors. Hence we need at least

$$\frac{2^k - 1}{2^s - 1} \ge 2^{k-s} \qquad (2)$$

subspaces to cover $\mathrm{GF}(2)^k$. Equality in (2) occurs if and only if $s = k$, thus proving the second part of the lemma. ■

A weakened converse of Lemma 5 is given in the following lemma. It gives a "factor of 2" upper bound on how close Lemma 5 is to the actual minimum number of subspaces.

Lemma 6. *Let $1 \le s \le k$. The vector space $\mathrm{GF}(2)^k$ can be covered by $2^{k-s+1} - 1$ subspaces of dimension s.*

Proof: When $s = 1$, subspaces of dimension s contain 2 vectors, including the zero vector $\mathbf{0}$. Since $|\mathrm{GF}(2)^k| = 2^k$, we need at least $2^k - 1$ subspaces to cover $\mathrm{GF}(2)^k$. We can cover $\mathrm{GF}(2)^k$ by the $2^k - 1$ subspaces $\{\mathbf{0}, a\}_{a \in \mathrm{GF}(2)^k \setminus \{\mathbf{0}\}}$.

Given a covering $\{A_i\}_{i=1}^c$ of $\mathrm{GF}(2)^k$ by subspaces of dimension s, we can "blow up" this covering to obtain a covering of $\mathrm{GF}(2)^{k+r}$ by subspaces of dimension $s+r$. A covering of $\mathrm{GF}(2)^{k+r}$ is given by

$$A'_i := \{(x_j)_{j=1}^{k+r} : (x_j)_{j=1}^k \in A_i\}.$$

We see that $\dim(A'_i) = \dim(A_i) + r$.

Thus, to cover $\mathrm{GF}(2)^k$ by s-dimensional subspaces, we can blow up a covering of $\mathrm{GF}(2)^{k-s+1}$ by 1-dimensional subspaces. ■

Let C be a $C(n,m,c,t)$ code with codeword length n. We define the *redundancy* of C to be $n - m - \log_2 |C|$, i.e., the number of dimensions away C is from being a P-MDS code. Note, for binary linear codes, $\log_2 |C|$ is always a non-negative integer.

Theorem 7. *Let P be a m-regular pattern erasure set with cover number $c \ge 2$. Then there exists a $C^{lin}(n,m,c,1)$ code for P with redundancy at most $\lfloor \log_2(c-1) \rfloor$.*

Proof: We generate a matrix M using an algorithm that proceeds column-by-column. We begin with M as an $m \times 0$ matrix. When we add the i-th column to M, we ensure that the submatrix formed by the columns in $p \cap \{1,2,\ldots,i\}$ would have full rank. If it turns out that this is impossible, we add a row of zeroes to M and then continue. Eventually, we will obtain an $(m+r) \times n$ matrix that satisfies the conditions of Theorem 4.

Suppose, during the course of the algorithm, M is a $(m+j) \times (i-1)$ matrix, and it is impossible to place the i-th row. Let

$$P^* = \{p \cap \{1,2,\ldots,i-1\} : p \in P \text{ and } i \in p\}.$$

For $q \in P^*$ define $s(q)$ to be the vector space spanned by the columns of M whose indices are in q. Then

$$\{s(q) : q \in P^*\}$$

gives a cover of the vector space $\mathrm{GF}(2)^{m+j}$ by at most c subspaces of dimension at most $m-1$. Hence, by Lemma 5, $c > 2^{j+1}$, and so $c \ge 2^{j+1} + 1$ since c is a positive integer.

Hence, if the algorithm terminates with M as a $(m+r) \times n$ matrix, then $c \ge 2^{(r-1)+1} + 1$, and so $r \le \log_2(c-1)$. It follows that the rank of M is at most $m + \log_2(c-1)$, and hence $|C| \ge 2^{n-m-\log_2(c-1)}$. The result follows since the redundancy must be an integer. ■

Theorem 7 implies the existence of P-MDS $C^{\mathrm{lin}}(n,m,c,1)$ codes when $c = 2$, and e.g. that a $C^{\mathrm{lin}}(n,m,c,1)$ code exists with redundancy at most 1 for $c \in \{3,4\}$. The algorithm will also always produce a P-MDS $C^{\mathrm{lin}}(n,m,c,1)$ code when $c = 1$, but the mathematical analysis in the proof of Theorem 7 does not hold in this case.

The algorithm in Theorem 7 cannot be guaranteed to create a P-MDS $C^{\mathrm{lin}}(n,m,c,1)$ code when $c = 3$, since $\mathrm{GF}(2)^m$ can, in fact, be covered by 3 subspaces of dimension $m-1$ when $m \ge 2$, as shown in Lemma 6.

III. Constructing low density P-MDS $C^{\mathrm{lin}}(n,m,2,1)$ codes

In Section II-C, we give an algorithm that can produce a P-MDS $C^{\mathrm{lin}}(n,m,c,1)$ code for $c \ge 2$. However, it is difficult to analyze the *density* of the code produced by this algorithm (i.e., the number of 1's in the parity-check matrix). The density of the parity matrix is plays a important role in applications [11]: lower density codes require less IO. Researchers therefore tend to prefer the low-density codes; e.g. [12]. In this section, we will give a construction of certain P-MDS $C^{\mathrm{lin}}(n,m,2,1)$ codes that tend to have low density.

We begin with a construction of P-MDS $C^{\mathrm{lin}}(n,2,2,1)$ codes, which can be "combined" to give P-MDS $C^{\mathrm{lin}}(n,m,2,1)$ codes for even m. We will assume $|P| = n$, otherwise we can extend n and P so that this is true. Hence, P can be regarded as a collection of disjoint cycles, where a *cycle* is a subset of the form

$$\{a_1a_2, a_2a_3, \ldots, a_{k-1}a_k, a_ka_1\}; \qquad (3)$$

for brevity, we use e.g. a_1a_2 to denote $\{a_1, a_2\}$.

Theorem 8. *Let P be a 2-regular pattern erasure set with cover number 2, satisfying $|P| = n$. There exists a P-MDS $C^{\text{lin}}(n, 2, 2, 1)$ code containing $n + z$ 1's, where z is the number of odd-length cycles in P. Moreover, this is the minimum density possible for a P-MDS $C^{\text{lin}}(n, 2, 2, 1)$ code.*

Proof: We will construct a parity check matrix that satisfies the conditions of Theorem 3. We begin with M as an empty $2 \times n$ matrix, and add columns in the following way. We proceed cycle-by-cycle. If P contains a cycle of the form (3), we assign $(0, 1)^T$ to column a_1, then $(1, 0)^T$ to column a_2, and so on, alternating, until we reach column a_k. If k is odd, then we assign $(1, 1)^T$ to column a_k, otherwise we assign $(0, 1)^T$ to column a_k.

The 2×2 submatrices of M with indices in some $p \in P$ are of the form

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

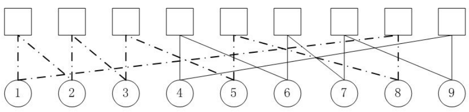
or can have its columns permuted to have this form. Hence, they all have full rank. Hence, the conditions of Theorem 3 are satisfied, and so M produces a P-MDS $C^{\text{lin}}(n, 2, 2, 1)$ code.

This is the minimum density possible: since $|P| = n$, each index $i \in [n]$ belongs to some $p \in P$, so each column must contain a 1, and odd cycles must contain at least one column containing two 1's. ■

For example, if $|P| = n = 9$ and

$$P = \{12, 23, 35, 58, 81, 46, 67, 79, 94\},$$

we can decompose P into 2 disjoint cycles $\{12, 23, 35, 58, 81\}$ and $\{46, 67, 79, 94\}$. The algorithm in the proof of Theorem 8 gives a parity check matrix as depicted in Figure 3.



$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Fig. 3. The parity check matrix of a P-MDS $C^{\text{lin}}(9, 2, 2, 1)$ code

Theorem 9. *Let P be a 2-regular pattern erasure set with cover number 2, such that m is even and $n = \frac{m|P|}{2}$. Then there exists a P-MDS $C^{\text{lin}}(n, m, 2, 1)$ code.*

Proof: We begin with a direct product-like construction. If M_1 and M_2 are parity-check matrices for a P_1-MDS $C^{\text{lin}}(n_1, m_1, 2, 1)$ code and a P_2-MDS $C^{\text{lin}}(n_2, m_2, 2, 1)$ code, respectively. Then the $(m_1 + m_2) \times (n_1 + n_2)$ matrix with block structure

$$\begin{array}{|c|c|} \hline M_1 & \cdot \\ \hline \cdot & M_2 \\ \hline \end{array},$$

where $\cdot$ denotes an all-0 submatrix, will be a $P_1 \cup P_2'$-MDS $C^{\text{lin}}(n_1 + n_2, m_1 + m_2, 2, 1)$ code, where $P_2' = \{i + n_1 : i \in P_2\}$. We can use this construction recursively for any number of parity-check matrices.

It is now sufficient to show that a P-MDS $C^{\text{lin}}(n, m, 2, 1)$ code, such that m is even and $n = \frac{m|P|}{2}$, can be constructed this way.

Given $P = \{p_0, p_1, \ldots, p_{2n/m-1}\}$, we construct an m-regular multigraph G with vertex set $\{0, 1, \ldots, 2n/m - 1\}$, with $|p_i \cap p_j|$ edges between i and j.

For example, with $m = 6$, $|P| = 10$, and $n = 30$, if

$$P = \{p_1, p_2, \ldots, p_9\}$$

where

$$\begin{aligned} &p_0 = \{1, 2, 3, 4, 5, 6\}, && p_1 = \{6, 7, 8, 9, 10, 11\}, \\ &p_2 = \{11, 12, 13, 14, 15, 16\}, && p_3 = \{10, 16, 17, 18, 19, 20\}, \\ &p_4 = \{18, 19, 20, 21, 22, 23\}, && p_5 = \{8, 9, 15, 23, 24, 25\}, \\ &p_6 = \{13, 14, 22, 24, 25, 26\}, && p_7 = \{5, 7, 21, 26, 27, 28\}, \\ &p_8 = \{3, 4, 27, 28, 29, 30\}, && p_9 = \{1, 2, 12, 17, 29, 30\}, \end{aligned}$$

the pattern graph is as Fig.4 ,then the multigraph is:

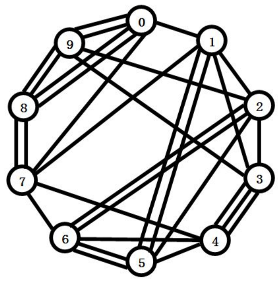


Fig. 5. multigraph

The multigraph G is indeed m-regular: each element of $[n]$ occurs in exactly two patterns (since $c = 2$ and $n = \frac{m|P|}{2}$) and each pattern has m elements.

From the multigraph G, we decompose it into $m/2$ disjoint 2-factors. This is always possible, as shown by Petersen [9], [10] , a result which became known as "Petersen's 2-Factor Theorem". In the above example, we can decompose G e.g. into the 2-factors in Figure 6.

The cycles in the 2-factorization can be used to generate pattern erasure sets $P_1, P_2, \ldots, P_{m/2}$: if $(a_1, a_2, \ldots, a_k)$ is a cycle in the i-th 2-factor, then we add $\{a_1a_2, a_2a_3, \ldots, a_{k-1}a_k, a_ka_1\}$ to P_i (here, we again use the shorthand e.g. $a_1a_2 = \{a_1, a_2\}$).

We generate $2 \times |P|$ parity check matrices $M_1, M_2, \ldots, M_{m/2}$ for $P_1, P_2, \ldots, P_{m/2}$, respectively, using the construction in the proof of Theorem 8. We then use the direct product construction mentioned at the start

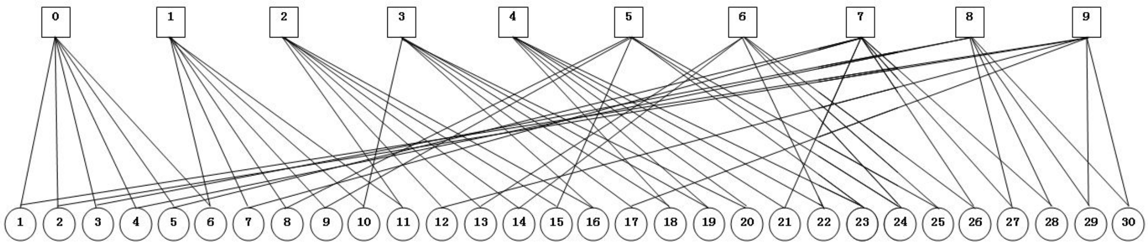


Fig. 4. Pattern Graph of P

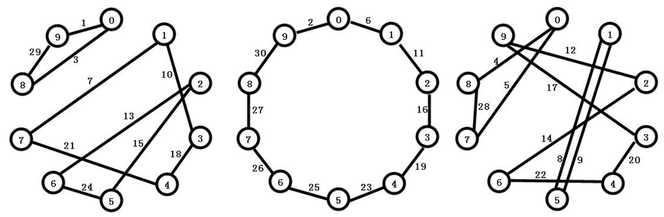


Fig. 6. A 2-factorization of G

of this proof to create a $m \times n$ parity check matrix whose columns we permute to obtain a parity check matrix M for a P-MDS $C^{\text{lin}}(n, m, 2, 1)$ code. We ensure that the columns of M are permuted such that the cycles in M_i (as in the proof of Theorem 8) correspond to the cycles in the i-th 2-factor.

Thus, for our running example, the matrix M is:

1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	1	0	0
0	0	0	0	1	0	0	0	1	0

11	12	13	14	15	16	17	18	19	20
0	0	1	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	0	0
0	0	0	1	0	0	1	0	0	1

21	22	23	24	25	26	27	28	29	30
0	0	0	1	0	0	0	0	1	0
1	0	0	1	0	0	0	0	1	0
0	0	0	0	1	0	1	0	0	0
0	0	1	0	0	1	0	0	0	1
0	1	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0	0

where the three shades of gray indicate M_1, M_2 and M_3. The cycle $(1, 3, 29)$ in the first 2-factor, for example, corresponds to columns 1, 3, and 29 in M. ■

Now let's analyse the density of the code. Obviously ,for a 1-pattern erasure code, there are at least n "1"s in its parity matrix. For our code ,on the best case the parity matrix contains exactly n "1"s ,while each 2-factor is a single circle. But on the worst case , i.e. each 2-factor will be decomposed into some 3-circles, there are about $1.3n$ "1"s in the parity matrix.

IV. Concluding Remarks

In this paper,we have introduced two ways to construct pattern erasure code. For the next,there are a lots work to do, e.g.

- Construct new P-MDS C(n,m,c,1) codes,especially for $c = 3$.
- Construct new P-MDS C(n,m,2,t) codes ,while $t > 1$.
- Improve the bound in Theorem 7.

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