Euclidean Information Theory of Networks

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***Abstract*—In this paper, we extend the information theoretical framework that was developed in [1] to multi-hop communication networks. For a given network, we construct a deterministic model that models the ability of the channels in transmitting private and common messages between users in this network. Based on this model, we formulate a linear optimization problem to study the network throughput, where the solution indicates what kind of common messages should be generated in a network to optimize the throughput. Our results provide fundamental guidelines of how users in a network should cooperate with each other to communicate efficiently.**

I. Introduction

In this paper, we utilize the information theoretical framework that was developed in [1][2] to investigate the issue of how nodes in a network should cooperate with each other to increase the network throughput. Specifically, we want to study the following problem. For a communication network such that the input and output distributions of each terminal in this network are given, how a small amount of information can be conveyed through this network to certain destinations the most efficiently, by slightly varying the given input distributions of some terminals.

This class of information theory problems are called *linear information coupling problems* in [1], and the authors therein studied the linear information coupling problems for several single-hop multi-terminal communication channels. The main result of [1] is that for a multi-terminal communication channel, under certain local assumptions, the transmission of different types of messages, such as private and common messages, can be viewed as transmitted through separated deterministic links with channel parameters that can be computed as linear algebra problems. As a consequence, for a multi-terminal channel, we can quantify the difficulty of broadcasting common messages than sending private messages, and compute the gain of transmitting common messages by the cooperation between transmitters. This development is particularly useful when studying the multi-hop networks, because it quantifies the trade-off between the gain of sending a common message and the cost to create this common message from the previous layer, and hence evaluates whether or not a certain common message should be created.

Our goal in this paper is to study the linear information coupling problems of multi-hop layered networks with this information theoretical framework. It turns out that with this approach, we can construct a deterministic network model that captures the channel parameters, which model the channels in the ability of transmitting private and common messages. Then, the linear information coupling problems become linear optimization problems of the network throughputs, and the solutions indicate what kind of common messages should be generated to optimize the throughputs. We also consider the large scale layered networks with identical layers. In these cases, the optimal communication schemes are composed of some fundamental transmission modes, and we specify these transmission modes in section IV-A. Our results in general provide the insights of how users in a communication network should cooperate with each other to increase the network throughput.

The rest of this paper is organized as follows. We review the linear information coupling problems of several multi-terminal communication channels in section II. In section III, we study the linear information coupling problems of interference channels, and develop deterministic models that capture the channel parameters. With this deterministic model, we investigate the multi-hop layered networks in section IV, where a Viterbi algorithm is proposed to search for the optimal communication scheme. We also present the optimal transmission modes for large scale layered networks with identical layers in section IV-A.

II. Linear Information Coupling Problems

In this section, we briefly review the linear information coupling problems of point-to-point channels, general broadcast channels, and multiple access channels. For more details, we refer the readers to [1].

A. Point-to-point Channels

The linear information coupling problem for a point-to-point channel with the transmitter X and receiver Y is formulated as, for some given input distribution P_X:

$$\max_{U \to X \to Y : I(U;X) \leq \frac{1}{2}\epsilon^2} I(U;Y), \tag{1}$$

where, ϵ is assumed to be small. The goal of (1) is to design $P_{X|U=u}$ for different u, such that the marginal distribution is fixed as P_X, and (1) is optimized. The key idea here is that, by assuming ϵ to be small, the objective distributions $P_{X|U=u}$ for different u are close to each other. Therefore, we can "linearize" the space of the input distributions $P_{X|U=u}$ approximately as an Euclidean space, and the mutual information (or equivalently, the KL divergence) serves as the Euclidean metric in this space. In addition, the corresponding output distribution space can also be approximated as an Euclidean space, and the channel becomes a linear map between these two Euclidean spaces. So, (1) is reduced to designing $P_{X|U=u}$ according to the channel map with large singular value σ^2, and the optimized value of (1) is $\frac{1}{2}\epsilon^2\sigma^2$. The technique of computing the channel parameter σ^2 is referred to [1].

B. General Broadcast Channels

For a general broadcast channel with transmitters X and receivers Y_1 and Y_2, the linear information coupling problems for communicating private messages to Y_1 and Y_2 are the same as the point-to-point channel case, and the channel parameters $\sigma^2_{1,\mathsf{BC}}$, and $\sigma^2_{2,\mathsf{BC}}$ can be similarly obtained.

The linear information coupling problem for the common message U_0 is more interesting, which is formulated as:

$$\max_{I(U_0;X)\leq \frac{1}{2}\epsilon_0^2} \min\{I(U_0;Y_1), I(U_0;Y_2)\}.$$

The optimized value of this problem is $\frac{1}{2}\epsilon_0^2\sigma^2_{0,\mathsf{BC}}$, where $\sigma^2_{0,\mathsf{BC}}$ is the channel parameter for transmitting common messages that can be computed as in [1]. In particular, it can be shown that $\sigma^2_{0,\mathsf{BC}} \leq \sigma^2_{i,\mathsf{BC}}$, for $i = 1, 2$, which evaluates the difficulty of generating common messages for receivers.

C. Multiple Access Channels

For a multiple access channel with transmitters X_1, X_2, and a receiver Y, we are interested in the linear information coupling problem of the common source U_0 that is accessible for both transmitters:

$$\max_{I(U_0;X_1,X_2)\leq \frac{1}{2}\epsilon_0^2} I(U_0;Y)$$

where $X_1 \to U_0 \to X_2$ forms a Markov relation. The optimized value of this problem is $\frac{1}{2}\epsilon^2\sigma^2_{0,\mathsf{MAC}}$, where the channel parameter $\sigma^2_{0,\mathsf{MAC}}$ can be computed by a similar approach as [1]. Moreover, it can be shown that $\sigma^2_{0,\mathsf{MAC}} \geq \sigma^2_{i,\mathsf{MAC}}$, for $i = 1, 2$, where $\sigma^2_{i,\mathsf{MAC}}$ is the channel parameter for the private source U_i of the transmitter X_i in the corresponding linear information coupling problem, and $\sigma^2_{i,\mathsf{MAC}}$ can be computed similarly as the point-to-point channel case. This evaluates the gain of the cooperation between transmitters due to the common knowledge.

III. Interference Channels

The results in the previous section provides some insights in studying the network throughput. First, we know that transmitting sources that are known by multiple transmitters is more advantageous as the transmitters can cooperate with each other to create coherent combining gains. Therefore, in order to increase the throughput of a network, it is motivated to create common messages between transmitters. On the other hand, it costs more network resources, such as time, frequency, or power, to create such common messages than private messages. Hence, there is a trade-off relation between the cost of generating the common messages and the coherent combining gain in transmitting common sources. With our framework in the previous section, we want to investigate the structure of this trade-off relation and obtain the optimal throughputs for networks. As the first step, we study the interference channels in this section, since the interference channel is the simplest channel model that includes the notion of both common sources and common messages.

For an interference channel with transmitters X_1, X_2, and receivers Y_1, Y_2, we model the common sources for transmitters and the common messages to receivers by considering the transmission of nine types of messages U_{ij}, for $i, j = 0, 1, 2$. Here, the first index i represents that the message is private to the transmitter X_1, X_2, and common to both transmitters, for $i = 1$, 2, and 0, and the next index is similarly defined for the receivers. The linear information coupling problem for the interference channel is:

$$\begin{aligned} &R_{ij} \leq I(U_{ij};Y_j),\ j \neq 0,\ \forall i, \\ &R_{i0} \leq \min\{I(U_{i0};Y_1), I(U_{i0};Y_2)\},\ j = 0,\ \forall i, \end{aligned} \tag{2}$$

subject to the constraints:

$$\begin{aligned} &I(U_{ij};X_i) \leq \delta_{ij},\ i \neq 0,\ \forall j, \\ &I(U_{0j};X_1,X_2) \leq \delta_{0j},\ \forall j, \\ &\sum_{i,j=0,1,2} \delta_{ij} = \delta. \end{aligned}$$

Here, we employ δ and δ_{ij} to indicate $\frac{1}{2}\epsilon^2$ and $\frac{1}{2}\epsilon_{ij}^2$ for the convenience of notation. Then, following the same procedure, (2) can be reduced to

$$R_{ij} = \delta_{ij}\lambda_{ij},\ \text{for } i,j = 0,1,2,\ \sum_{i,j=0,1,2} \delta_{ij} \leq \delta, \tag{3}$$

where λ_{ij}'s are the channel parameters that model the ability of the channel in transmitting different kinds of messages, and can be computed in a similar manner as the σ^2's in section II. Since δ is only a scaling factor in our objective rate region (3), in the following, we will simply assume $\delta = 1$.

The optimized rate region (3) leads us to model an interference channel as a deterministic model with nine bit-pipes, each having the capacity of $\delta_{ij}\lambda_{ij}$. Unlike traditional wired networks, the capacities of these bit-pipes are flexible: $\delta_{ij}\lambda_{ij}$ can change depending on different allocations of $\{\delta_{ij}\}$ subject to $\sum_{i,j}\delta_{ij} \leq 1$.

Fig. 1 shows a pictorial representation of our deterministic model for an interference channel. The idea of this deterministic model is that, since for $k = 1, 2$, a physical transmitter X_k serves two purposes: transmitting its privates sources, and cooperating with the other transmitter to transmit the common sources. Therefore, we can model X_k by two *virtual* transmitters Tx k and Tx 0, such that Tx k intends to send the private sources U_{kj}, and Tx 0 intends to send the common sources U_{0j}. Similarly, the physical receiver Y_k can be modeled by two virtual receivers Rx k and Rx 0, such that Rx k wishes to decode the private messages U_{ik}, and Rx 0 wishes to decode the common messages U_{i0}. By presenting the virtual transmitters and receivers, the interference channel is modeled with three transmitters and receivers, where each

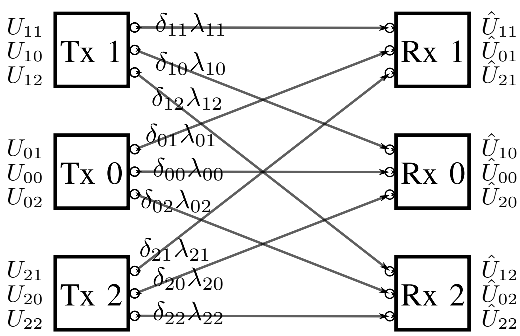

Fig. 1. A deterministic model for interference channels. The transmitter Tx k transmits the messages U_{kj}, and the receiver Rx k decodes the messages U_{ik}.

transmitter transmits its individual type of sources, and each receiver decodes its individual type of messages. Then, with this model, the message U_{ij} is transmitted from the Tx i to the Rx j. In addition, the circles here indicate bit-pipes intended for the transmission of different messages. For instance, the top circle indicates a bit-pipe for transmitting the private message w.r.t Rx 1. Note that the circles also denote that the messages are transmitted through parallel channels without interferring with each other. Therefore, the circles should be viewed as nine pairs, where each pair represents one of the parallel links.

In this deterministic model, it is easy to see that the largest among all λ_{ij}'s is either λ_{01} or λ_{02}. So, to optimize the total throughput through this interference channel, we will just let either δ_{01} or δ_{02} be 1, and deactivate other links. In other words, the optimal strategy to convey information through this interference channel is to transmit it as the common source for both transmitters to the receiver ends as the private message. However, this model is less interesting in the single-hop case, because the single-hop interference channel does not capture the difficulty of generating this common source before transmitting it to the receiver ends cooperatively. On the other hand, in general multi-hop layered networks, this kind of cost has to be taken into account, since the common source of transmitters in one layer is generated from the previous layer as the common message. This creates an issue of planning which kinds of common messages should be generated in a given network to optimize the throughput. It turns out that the deterministic model we developed in this section becomes a powerful tool in studying this issue for multi-hop layered networks, which will be demonstrated in the next section.

IV. Multi-hop Layered Networks

In this section, for the simplicity of presentation, we consider the general layered networks with two users in each layer, while our approach can extend to more general cases without difficulty. To simplify the problem, we assume a decode-and-forward scheme [3], such that the received signals of each layer are decoded as a part of messages, which are then forwarded to the next layer. Then, an L-layered network

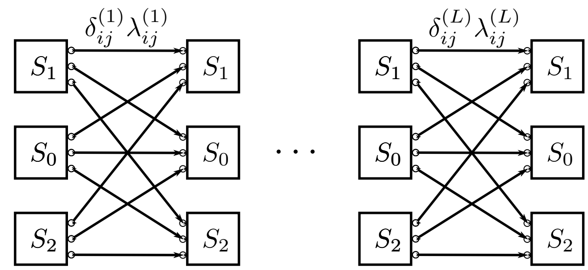

Fig. 2. A deterministic model for multi-hop interference networks. Here, $\lambda_{ij}^{(\ell)}$, for $i,j=0,1,2$ represent the channel parameters of the ℓ-th layer. We use S_i to represent both the transmitter and receiver for every layer, for $i=0,1,2$.

can be viewed as cascading L interference channels, and the deterministic model can be constructed as Fig. 2. Here, we use node S_i to represent both the transmitter i and receiver i for all $L+1$ layers of nodes, for $i=0,1,2$. In addition, the channel of layer ℓ consists of 9 bit-pipes, each having the capacity of $\delta_{ij}^{(\ell)}\lambda_{ij}^{(\ell)}$, for $i,j=0,1,2$, and $\ell\in[1:L]$. The constraint for δ_{ij}'s is simply extended to

$$\sum_{\ell=1}^{L}\sum_{i=0}^{2}\sum_{j=0}^{2}\delta_{ij}^{(\ell)}\le L. \tag{4}$$

Then, the linear information coupling problem is reduced to finding the optimal throughput (or sum rate) of the network in Fig. 2, under the constraint (4). Here, the sum rate is defined as the amount of information that can be communicated from the first layer to the last layer through the network such that each link has their individual capacity constraint $\delta_{ij}^{(\ell)}\lambda_{ij}^{(\ell)}$, and for each node, the outflow of information is no more than the inflow. In this paper, we focus on the routing capacity, that is, we do not allow for network coding [4]. The following theorem characterizes the optimal sum rate.

Theorem 1 (Viterbi algorithm)**.** *Consider a layered network illustrated in Fig. 2, the optimal sum rate under the constraint* (4) *is*

$$C_{\mathsf{sum}}=\max_{i_1,i_2,\ldots,i_{L+1}\in[0:2]}M(\lambda_{i_1i_2}^{(1)},\lambda_{i_2i_3}^{(2)},\ldots,\lambda_{i_Li_{L+1}}^{(L)}), \tag{5}$$

where $M(x_1,\cdots,x_n)$ denotes the harmonic mean

$$M(x_1,\cdots,x_n)\triangleq\frac{n}{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}.$$

Moreover, the optimal $i_1,i_2,\ldots,i_{L+1}$ of (1) *can be obtained by a Viterbi algorithm [5] with complexity $O(L)$.*

Proof. Note that for a path $i_1,i_2,\ldots,i_{L+1}$ from the node S_{i_1} of the first layer to the node $S_{i_{L+1}}$ of the last layer, if we assign all available $\delta_{ij}^{(\ell)}$'s to this path by the assignment $\delta_{i_ki_{k+1}}^{(k)}=M(\lambda_{i_1i_2}^{(1)},\ldots,\lambda_{i_Li_{L+1}}^{(L)})/\lambda_{i_ki_{k+1}}^{(k)}$, then we can achieve (5). Therefore, theorem 1 is achievable by assigning all $\delta_{ij}^{(\ell)}$'s to the optimal path in (5).

The key idea of proving the converse of theorem 1 is to observe that in order to transmit one unit of information through the path $i_1, i_2, \ldots, i_{L+1}$, we need to assign $1/M(\lambda^{(1)}_{i_1i_2}, \lambda^{(2)}_{i_2i_3}, \ldots, \lambda^{(L)}_{i_Li_{L+1}})$ of the δ's to this path. Therefore, $M(\lambda^{(1)}_{i_1i_2}, \lambda^{(2)}_{i_2i_3}, \ldots, \lambda^{(L)}_{i_Li_{L+1}})$ measures the efficiency of transmitting information. If there are two pieces of messages that are transmitted through two different paths in the network, since $\delta^{(\ell)}_{ij}$'s are linearly assigned to the links, we can compute the harmonic mean of both paths, and reassign the $\delta^{(\ell)}_{ij}$'s in the worse path to the better one. This increases the sum rate while keeping the sum of all $\delta^{(\ell)}_{ij}$'s unchanged. Therefore, the optimal scheme is simply to pick the optimal path. $\square$

Theorem 1 leads to a Viterbi-type algorithm to search for the optimal path. Instead of searching for all possible paths with complexity $O(3^L)$, note that (1) is equivalent to finding a path from the first layer to the last layer, where the inverse sum of $\lambda^{(k)}_{i_ki_{k+1}}$ is minimized. Thus, we can take $1/\lambda^{(k)}_{i_ki_{k+1}}$ as the cost, and run the Viterbi algorithm [5] to find the path with minimal total cost, and the complexity is reduced to $O(L)$.

A. Multi-hop Networks with Identical Layers

While theorem 1 indicates how to find the optimal communication strategy in polynomial time for general layered networks, it is sometimes more useful to understand the "patterns" or structures of the optimal communication schemes for large scale networks. For example, if the channel parameters are only available locally, then the communication patterns can be helpful in the designs. In this subsection, we investigate this issue by considering the L-layered networks, for $L \to \infty$, with identical layers, such that $\lambda^{(\ell)}_{ij} = \lambda_{ij}$ for all ℓ in Fig. 2. The following theorem specifies the fundamental transmission modes that consists the optimal communication strategy for the layered networks with identical layers.

Theorem 2 (Identical layers)**.** *Consider a layered network illustrated in Fig. 2, where $\lambda^{(\ell)}_{ij} = \lambda_{ij}, \forall \ell$, and $L \to \infty$. Then, the optimal sum rate is*

$$\begin{aligned} C_{\mathsf{sum}} = \max\{&\lambda_{11}, \lambda_{00}, \lambda_{22}, \\ &M(\lambda_{10}, \lambda_{01}), M(\lambda_{20}, \lambda_{02}), M(\lambda_{12}, \lambda_{21}) \\ &M(\lambda_{10}, \lambda_{02}, \lambda_{21}), M(\lambda_{20}, \lambda_{01}, \lambda_{12})\}, \end{aligned} \tag{6}$$

where $M(x_1, \cdots, x_n)$ denotes the harmonic mean.

Proof. For the converse part, first observe that for any optimal communication scheme, we have the inflow equals to outflow for every node in the intermedia layers, i.e. $\sum_{i=0}^2 \delta^{(\ell)}_{ik}\lambda_{ik} = \sum_{i=0}^2 \delta^{(\ell+1)}_{ki}\lambda_{ki}$, for all k and ℓ. Thus, we can define $R^{(\ell)}_k$ as the rate of the information that flow in and out of node S_k in the ℓ-th layer of nodes, for $k = 0, 1, 2$, and $\ell \in [1 : L+1]$. Now, for any optimal communication scheme $\mathcal{A}^{(L)} = \{\delta^{(\ell)}_{ij}\}$, we can "average" each $\delta^{(\ell)}_{ij}$ over all L layers, and get a communication scheme $\mathcal{A}' = \{\delta_{ij} = \sum_{\ell=1}^L \delta^{(\ell)}_{ij}/L\}$ for a single-layered network with channel parameters λ_{ij}. Moreover, for the scheme $\mathcal{A}'$ of this single-layered network, the rate of the

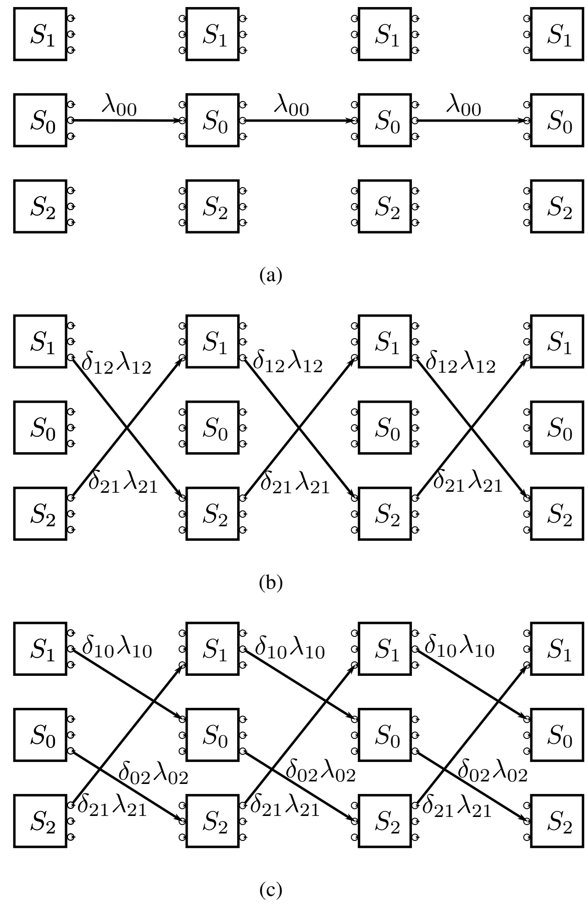

Fig. 3. The communication schemes that achieve (a) λ_{00}, (b) $M(\lambda_{12}, \lambda_{21})$, where $\delta_{ij} = M(\lambda_{12}, \lambda_{21})/2\lambda_{ij}$, and (c) $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$, where $\delta_{ij} = M(\lambda_{10}, \lambda_{02}, \lambda_{21})/3\lambda_{ij}$.

outflow of node S_k in the first layer is $R_{1k} = \sum_{\ell=1}^L R^{(\ell)}_k/L$, and the rate of the inflow of node S_k in the second (last) layer is $R_{2k} = \sum_{\ell=2}^{L+1} R^{(\ell)}_k/L$, for $k = 0, 1, 2$. Since we consider $L \to \infty$, the rate tuples $(R_{10}, R_{11}, R_{12}) = (R_{20}, R_{21}, R_{22})$. This says that an optimal scheme for the L-layered network can be reduced to a scheme of a single-layered network with the same sum rate such that the rate tuple of both layers of nodes in this single-layered network are equal. On the contrary, we can repeat the scheme $\mathcal{A}'$ for L times in the L-layered network to come up with a scheme for the L-layered network with the same sum rate. Therefore, we only need to consider the following linear optimization problem of the sum rate for a single-layered network with channel parameters λ_{ij}:

$$\max.\ R_0 + R_1 + R_2 \tag{7}$$

$$\text{subject to: } R_k = \sum_{i=0}^2 \delta_{ki}\lambda_{ki} = \sum_{i=0}^2 \delta_{ik}\lambda_{ik},\ \sum_{ij=0}^2 \delta_{ij} = 1.$$

Solving (7) (e.g. see [6]), we get the optimal sum rate (6).

For the achievability, note that $\lambda_{ii} = M(\lambda_{ii})$, so all 8 modes in (6) can be written in the form $M(\lambda_{i_1i_2}, \lambda_{i_2i_3}, \ldots, \lambda_{i_ki_1})$, for $k = 1, 2, 3$, and $i_1, \ldots, i_k$ are mutually different. Then,

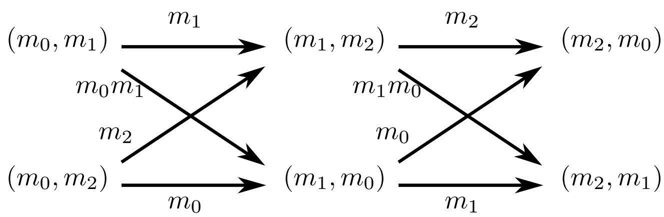

Fig. 4. The rolling of different pieces of information between users layer by layer for the optimal communication scheme that achieves $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$.

for $k = 1, 2, 3$, $n \in [1 : k]$, and $\ell \in [1 : L]$, the $M(\lambda_{i_1 i_2}, \lambda_{i_2 i_3}, \ldots, \lambda_{i_k i_1})$ can be achieved by setting

$$\delta^{(\ell)}_{i_n i_{n+1}} = \delta_{i_n i_{n+1}} = \frac{M(\lambda_{i_1 i_2}, \lambda_{i_2 i_3}, \ldots, \lambda_{i_k i_1})}{k \lambda_{i_n i_{n+1}}}, \tag{8}$$

and deactivating all other links by setting their δ_{ij}'s to zero. We assume that in (8), when $n = k$, $\delta_{i_k i_{k+1}}$ denotes $\delta_{i_k i_1}$. It is easy to verify that the assignment of (8) satisfies the constraint (4), thus we prove the achievability. □

Theorem 2 implies that the optimal communication scheme is from one of the eight modes in (6). Fig. 3 illustrates the communication schemes that achieves modes λ_{00}, $M(\lambda_{12}, \lambda_{21})$, and $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$, where other modes can be achieved similarly. For example, the mode $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$ is achieved by using links S_1S_0, S_0S_2, and S_2S_1, such that

$$\delta_{10}\lambda_{10} = \delta_{02}\lambda_{02} = \delta_{21}\lambda_{21} = \frac{M(\lambda_{10}, \lambda_{02}, \lambda_{21})}{3},$$

and other $\delta_{ij} = 0$. Then, the information flow for each layer and the sum rate are both $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$.

More interestingly, in order to achieve (6), it requires the cooperation between users, and rolling the knowledges of different part of messages between users layer by layer. We demonstrate this by considering the communication scheme that achieves $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$ as an example. Suppose that at the first layer, the node S_i has the knowledge of information m_i, for $i = 0, 1, 2$. Since S_0 is the virtual node that represents the common message of both users, user 1 knows messages (m_0, m_1), and user 2 knows (m_0, m_2). Then, to achieve $M(\lambda_{10}, \lambda_{02}, \lambda_{21})$, user 1 broadcasts its private message m_1 to both users in the next layer, and both users in the first layer cooperate to transmit their common message to user 2 in the next layer as the private message. Thus, in the second layer user 1 decodes messages (m_1, m_2) and user 2 decodes (m_1, m_0). Similarly, in the third layer, user 1 decodes (m_2, m_0) and user 2 decodes (m_2, m_1), and then loop back. This effect is shown by Fig. 4. Therefore, according to the values of channel parameters, theorem 2 illustrates the optimal communication mode, and hence indicates what kind of common messages should be generated to achieve the optimal sum rate.

B. Discussions

It has been shown in recent literatures (e.g. [7]) that the interference between messages has important impacts in studying the communication systems. However, in our deterministic network model, since the δ_{ij}'s are linearly assigned to communication links, different pieces of messages can be viewed as transmitted through separated paths in the network without interfering even when these paths share some common nodes. Therefore, our model to some level ignores the correlation of messages within networks. This effect comes from our assumption that we only transmit tiny pieces of messages through networks, so the interference caused by our messages in our model becomes negligible. On the other hand, our model indeed captures the effect that physically how transmitters can cooperate and how the interference can be handled in transmitting a certain type of message. Thus, one can say that our framework models how efficient a message can be transmitted through a network by the cooperation between users, and on top of that, ignoring some interference between messages.

In addition, in this paper, we consider the routing only scheme, which also ignores the possible correlation of messages that can potentially increase the throughput. It can be an interesting to future research direction to study the network throughput of our model with network coding [4].

V. Conclusion

In this paper, we extend the information theoretical framework in [1] to multi-hop layered networks. We construct deterministic models for layered network to model the ability of generating common messages, and study the optimal throughputs of the deterministic models. For general layered networks, we propose a Viterbi algorithm to search for the communication scheme that optimize the throughputs. For layered networks with identical layers, we specify the fundamental communication modes that consists the optimal communication strategies.

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