Joint Opportunistic Scheduling and Network Coding for Bidirectional Relay Channel

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***Abstract*—In this paper, we consider a two-way communication system in which two users communicate with each other through an intermediate relay over block-fading channels. We investigate the optimal opportunistic scheduling scheme in order to maximize the long-term average transmission rate in the system assuming symmetric information flow between the two users. Based on the channel state information, the scheduler decides that either one of the users transmits to the relay, or the relay transmits to a single user or broadcasts to both users a combined version of the two users' transmitted information by using linear network coding. We obtain the optimal scheduling scheme by using the Lagrangian dual problem. Furthermore, in order to characterize the gains of network coding and opportunistic scheduling, we compare the achievable rate of the system versus suboptimal schemes in which the gains of network coding and opportunistic scheduling are partially exploited.**

***Index Terms*—Bidirectional relay, block-fading, linear network coding, multi-user diversity, opportunistic scheduling.**

I. Introduction

Network coding (NC) [1]–[3] can considerably enhance the utilization efficiency of the channels' resources in multi-direction information flow over multi-hop wireless networks due to the exploitation of the broadcast nature of the wireless channel as well as the available side information at the users about their own transmitted information. Consequently, NC is considered for application in next generation wireless networks [4]. Also, channel-aware opportunistic scheduling (OS) is a useful way to exploit multiuser diversity in wireless networks, e.g. [5], by allocating the channel resources to the users dynamically such that each user transmits when its channel is in a good state. By combining NC and OS together, we can obtain the gains of both schemes jointly. This is the main objective of this paper.

In [6], [7], the authors considered optimizing the transmission from a central relay in a multiuser network that has a star topology, and they discussed the optimal number of information flows to be combined together using network coding. However, they did not consider the uplink from the users to the relay in the scheduler design as we do in this paper. In [8], the authors considered a large wireless cell with many users and relays and discussed when to apply network coding. However, they divided each channel block into three orthogonal phases making the users and relay transmit on every channel block. However, in our paper, we let a single node transmit per channel block and we demonstrate its gains over the other scheme.

In our paper, we characterize the optimal scheduling scheme for a two-user system that is described in details in Section II in order to maximize the symmetric long-term average transmission rate in the system. We assume that the system applies linear network coding using the conventional xor ($\oplus$) operation on the source information bits[1]. The optimal scheduler, which is derived in Section III, decides which node[2] (i.e. user or relay) should transmit in a given channel block based on the channel state information. We characterize the performance of the optimal scheme analytically. Furthermore, we compare the system performance with suboptimal schemes that apply either network coding or opportunistic scheduling, but not both of them, leading to partial exploitation of the multiuser diversity and side information gains. We provide numerical results in Section V and we summarize the main conclusions in Section VI.

II. System Model and Problem Formulation

System Setup: We consider the three-node system that is shown in Fig. 1. Two users, denoted N_1 and N_2, want to communicate with each other with the assistance of a third node, which is the relay (R). There is no direct link between the two users, and the three nodes operate in half-duplex mode. We assume that there are no delay constraints on the two-way communication, and hence opportunistic communication can be applied.

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[1]We know from information theory that xor-based network coding is suboptimal and it can be enhanced by using a special channel coding technique [9], which is out of the scope of this paper.

[2]We know from information theory that for the uplink (i.e. multiple access channel), single user selection is in general suboptimal and the optimal scheme involves non-orthogonal transmission from multiple users. However, this is out of the scope of this paper.

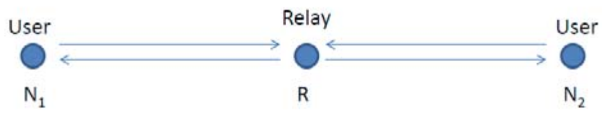

Fig. 1. System model: Bidirectional relay.

Channel Model: We assume a block-fading channel model, e.g. [10], for the channels between each user and the relay. We assume that all channel blocks (CBs) have the same duration, denoted T_{CB}, and bandwidth, denoted W_{CB}. Moreover, the channel gains for the two users' channels with the relay are independent of each other. Furthermore, we assume that all nodes use the same constant transmit power. For mathematical notation, we use $R_1[k]$, $R_2[k]$ to indicate the channel achievable rate (in bits/sec/Hz) of N_1 and N_2 respectively in the kth CB. Furthermore, we assume uplink/downlink reciprocity for the two users' channels with the relay. Therefore, $R_1[k]$ represents the achievable rate for the uplink of N_1 and at the same time it represents the achievable rate for the downlink from R to N_1. In the numerical examples in Section V, we assume that all nodes are equipped with a single antenna and the channels are Rayleigh block-faded. However, the analysis in this paper can be extended to any other fading model or number of antennas by using the appropriate model for the achievable rates' statistics, where the probability density function (PDF) and cumulative distribution function (CDF) are continuous functions over $[0, \infty)$ and denoted (e.g. for the first user's channel) $f_1(r)$ and $F_1(r)$, respectively.

Communication Scheme: We assume orthogonal channel access for the two users and the relay. Therefore, non-orthogonal transmission techniques are not supported. We assume perfect channel knowledge of the instantaneous (i.e. in the current CB) achievable rates ($R_1[k]$ and $R_2[k]$), which is utilized in the OS decisions to select which node should transmit in a given CB. We could assume that the scheduling decisions are taken at the relay since it is the central node in the system. When a user transmits, it adjusts its rate according to its channel achievable rate. We denote the actual transmitted rate in the kth CB as $\mathcal{R}_{\mathsf{N1}}[k]$ and $\mathcal{R}_{\mathsf{N2}}[k]$ for the uplink transmissions from N_1 and N_2, respectively. Therefore, when N_1 transmits over the kth CB, $\mathcal{R}_{\mathsf{N1}}[k] = R_1[k]$, and hence a total number of $T_{\text{CB}} W_{\text{CB}} \mathcal{R}_{N1}[k]$ information bits get transmitted. The relay decodes the user's codeword and stores the information bits in a designated buffer. When the relay transmits, it can either transmit to one user ($\mathcal{R}_{\mathsf{R1}}[k] = R_1[k]$ or $\mathcal{R}_{\mathsf{R2}}[k] = R_2[k]$), or it can broadcast to both users a combined version (via NC) of the two users' transmitted information bits. In the latter case, the relay broadcast channel is bounded by the worst channel achievable rate since both users have to decode the codeword reliably.

$$\mathcal{R}_{\mathsf{RNC}}[k] = \min(R_1[k], R_2[k]) \tag{1}$$

Furthermore, when the relay transmits using NC, it combines a total number of $T_{\text{CB}} W_{\text{CB}} \mathcal{R}_{\mathsf{RNC}}[k]$ source information bits from both buffers using a bit-by-bit xor ($\oplus$) operation. Each user decodes the relay's codeword, and performs an xor operation for the received information bits from the relay with its own information bits in order to get the information bits of the other user.

Opportunistic Scheduling: We aim to find the optimal scheduling scheme for our system. The scheduler task is to select in every CB the optimal transmission mode; $\xi[k] \in \{\mathsf{N1}, \mathsf{N2}, \mathsf{R1}, \mathsf{R2}, \mathsf{RNC}\}$ to distinguish between uplink from N_1, uplink from N_2, downlink to N_1, downlink to N_2 and downlink to both users using NC.[3]

The main optimization problem is

$$\max_{\{\xi[k]\}} \quad \bar{\mathcal{R}}_{\mathsf{N1}} + \bar{\mathcal{R}}_{\mathsf{N2}}, \qquad \text{subject to} \tag{2a}$$

$$\bar{\mathcal{R}}_{\mathsf{N1}} \le \bar{\mathcal{R}}_{\mathsf{RNC}} + \bar{\mathcal{R}}_{\mathsf{R2}}, \quad \bar{\mathcal{R}}_{\mathsf{N2}} \le \bar{\mathcal{R}}_{\mathsf{RNC}} + \bar{\mathcal{R}}_{\mathsf{R1}}, \tag{2b}$$

where $\bar{\mathcal{R}}_{\mathsf{N1}}$, $\bar{\mathcal{R}}_{\mathsf{N2}}$, $\bar{\mathcal{R}}_{\mathsf{R1}}$, $\bar{\mathcal{R}}_{\mathsf{R2}}$, and $\bar{\mathcal{R}}_{\mathsf{RNC}}$ are the long-term average transmission rates for the corresponding scheduling cases, and the averaging is over K, the total number of CBs (very large number approaching ∞). It is clear that the optimal solution must satisfy the constraints (2b) at equality.

III. Solution Structure

To solve (2), we use the Lagrangian dual problem, e.g. [11]:

$$\min_{\lambda} \quad Q(\lambda), \tag{3}$$

where $\lambda = [\lambda_1, \lambda_2]$ are the Lagrangian dual variables, and

$$\begin{aligned} Q(\lambda) = \max_{\{\xi[k]\}} \quad & \alpha\left(\bar{\mathcal{R}}_{\mathsf{N1}} + \bar{\mathcal{R}}_{\mathsf{N2}}\right) - \lambda_1\left(\bar{\mathcal{R}}_{\mathsf{N1}} - \bar{\mathcal{R}}_{\mathsf{RNC}}\right. \\ & \left. - \bar{\mathcal{R}}_{\mathsf{R2}}\right) - \lambda_2\left(\bar{\mathcal{R}}_{\mathsf{N2}} - \bar{\mathcal{R}}_{\mathsf{RNC}} - \bar{\mathcal{R}}_{\mathsf{R1}}\right), \end{aligned} \tag{4}$$

where α can be any arbitrary positive number. We define $\mu_1 = \alpha - \lambda_1$ and $\mu_2 = \alpha - \lambda_2$, and we use $\alpha = \frac{1}{2}$. The equivalent problem formulation becomes

$$\min_{\mu} \quad Q(\mu), \tag{5}$$

where $\mu = [\mu_1, \mu_2]$, and

$$\begin{aligned} Q(\mu) = \max_{\{\xi[k]\}} \quad & \mu_1 \bar{\mathcal{R}}_{\mathsf{N1}} + \mu_2 \bar{\mathcal{R}}_{\mathsf{N2}} + (1 - \mu_1 - \mu_2)\bar{\mathcal{R}}_{\mathsf{RNC}} \\ & + \left(\frac{1}{2} - \mu_2\right)\bar{\mathcal{R}}_{\mathsf{R1}} + \left(\frac{1}{2} - \mu_1\right)\bar{\mathcal{R}}_{\mathsf{R2}}. \end{aligned} \tag{6}$$

[3] The problem formulation in (2) guarantees the stability of the relay's buffers. However, since the scheduling process is opportunistic, it may happen occasionally that when the relay is scheduled to transmit, it does not have as many information bits to transmit as the channels' achievable rate can support. However, for simplicity of our analysis, we assume that this case does not happen. We could assume, for example, that there is an initial state of uplink transmissions only such that when we consider the optimal opportunistic scheduling, we can always assume that the relay buffers have sufficient number of information bits from both users and the probability of getting empty buffers becomes very negligible.

By using $\bar{\mathcal{R}} = \frac{1}{K}\sum_{k=1}^{K}\mathcal{R}[k]$ we can write

$$Q(\mu) = \max_{\{\xi[k]\}} \quad \frac{1}{K}\sum_{i=1}^{K}\Bigg(\mu_1\mathcal{R}_{\mathsf{N1}}[k] + \mu_2\mathcal{R}_{\mathsf{N2}}[k] + (1-\mu_1 -\mu_2)\mathcal{R}_{\mathsf{RNC}}[k] + \left(\frac{1}{2}-\mu_2\right)\mathcal{R}_{\mathsf{R1}}[k] + \left(\frac{1}{2}-\mu_1\right)\mathcal{R}_{\mathsf{R2}}[k]\Bigg). \tag{7}$$

The problem can be divided into independent set of problems. So, we maximize for each channel block independently,

$$\max_{\{\xi[k]\}} \quad \Big(\mu_{\mathsf{N1}}\mathcal{R}_{\mathsf{N1}}[k] + \mu_{\mathsf{N2}}\mathcal{R}_{\mathsf{N2}}[k] + \mu_{\mathsf{RNC}}\mathcal{R}_{\mathsf{RNC}}[k] + \mu_{\mathsf{R1}}\mathcal{R}_{\mathsf{R1}}[k] + \mu_{\mathsf{R2}}\mathcal{R}_{\mathsf{R2}}[k]\Big), \tag{8}$$

where $\mu_{\mathsf{N1}} = \mu_1$, $\mu_{\mathsf{N2}} = \mu_2$, $\mu_{\mathsf{RNC}} = 1-\mu_1-\mu_2$, $\mu_{\mathsf{R1}} = \frac{1}{2}-\mu_2$, and $\mu_{\mathsf{R2}} = \frac{1}{2}-\mu_1$. The solution (i.e. scheduling scheme) of (8), for a given μ_1 and μ_2, is

$$\xi[k] = \arg\max_{i\in\{\mathsf{N1,N2,RNC,R1,R2}\}} \mu_i\mathcal{R}_i[k] \tag{9}$$

However, to solve the dual problem (5), we need to find the optimal values for μ_1 and μ_2. The optimal solution for the dual problem (5) is at μ that satisfies the constraints in (2b), where the long-term average rates are obtained given that the scheduling policy (9) is used. Furthermore, μ that satisfies (2b) does always exist and it is unique. Therefore, we have strong duality and the optimal solution of the primal problem (2) is the same as the solution of the dual problem (5).

We can obtain μ that satisfies (2b) numerically by a two dimensional bisection search over μ_1 and μ_2, with the aid of the knowledge of the channels' achievable rates statistics. This is sufficient for our objective of characterizing the maximum average achievable rates. Furthermore, in a practical implementation, the solution could also be obtained by real-time adaptation of the weights, similar to the method adopted in [12, Section V-C], or by incorporating the queue lengths into the problem. We assume here that $f_1(r)$ and $f_2(r)$ are known, and we use them to obtain the optimal μ_1 and μ_2. We start first be characterizing the feasible region to search for μ_1 and μ_2.

A. Feasible Region for μ_1 and μ_2

From (9), it is clear that in order to have uplink transmission from N1, we must have $\mu_{\mathsf{N1}} \geq \mu_{\mathsf{R1}}$, since $\mathcal{R}_{\mathsf{N1}}[k] = \mathcal{R}_{\mathsf{R1}}[k] = R_1[k]$. Similarly, in order to have uplink from N2, we must have $\mu_{\mathsf{N2}} \geq \mu_{\mathsf{R2}}$. Therefore, the feasible region of μ must satisfy

$$\mu_1 + \mu_2 \geq \frac{1}{2} \tag{10}$$

Furthermore, when $\mu_1+\mu_2 > \frac{1}{2}$, we do not have downlink from the relay to a single node, and only NC is used when the relay is scheduled. Therefore, the optimal solution in this case has symmetric transmitted rates from the two users ($\bar{\mathcal{R}}_{\mathsf{N1}} = \bar{\mathcal{R}}_{\mathsf{N2}} = \bar{\mathcal{R}}_{\mathsf{RNC}}$). However, in the case $\mu_1+\mu_2 = \frac{1}{2}$, we have an infinite set of possible solutions for $\bar{\mathcal{R}}_{\mathsf{N1}}$ and $\bar{\mathcal{R}}_{\mathsf{N2}}$ with the same optimal value of μ and the corresponding optimal value

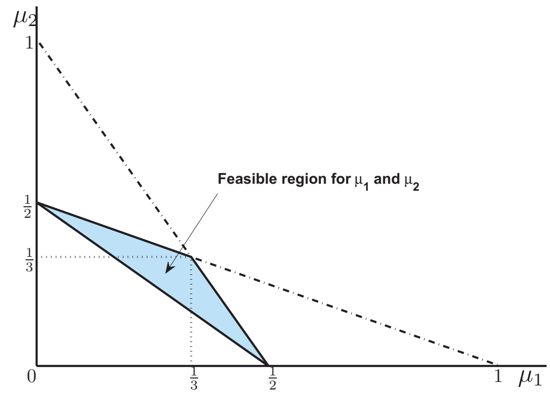

Fig. 2. The optimal values for μ_1 and μ_2 must be within the shaded region or on its boundaries.

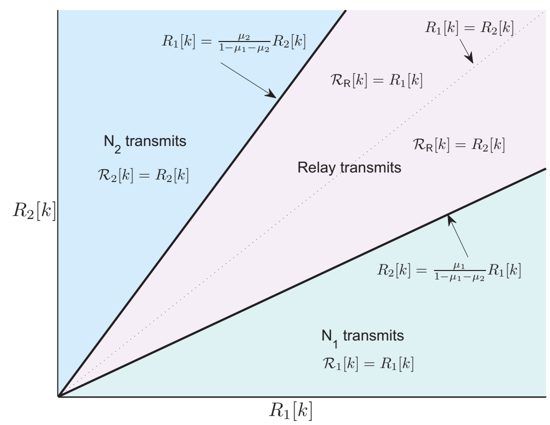

Fig. 3. The scheduling regions for the two users and the relay.

of $\bar{\mathcal{R}}_{\mathsf{N1}} + \bar{\mathcal{R}}_{\mathsf{N2}}$. In this case, we search for the solution with symmetric transmitted rates from the two users.

Lemma 1 (Feasible values for μ): In order to satisfy (2b), μ must satisfy

$$1-\mu_1-\mu_2 \;\geq\; \mu_1, \quad \text{and} \quad 1-\mu_1-\mu_2 \;\geq\; \mu_2 \tag{11}$$

The proof is shown in Appendix A. Fig. 2 shows the region defined by (10) and (11).

B. Solution in the Interior of the Feasible Region

In the interior region of (10) and (11), $\xi[k] \in \{\mathsf{N1,N2,RNC}\}$. Fig. 3 shows the scheduling regions defined by (9) in this case. With the aid of Fig. 3, we can find the long-term average transmitted rate by each node for given μ.

$$\bar{\mathcal{R}}_1(\mu) = \int_0^{\infty} r\; f_1(r)\; F_2\left(\frac{\mu_1}{1-\mu_1-\mu_2}\,r\right)dr \tag{12}$$

$$\bar{\mathcal{R}}_2(\mu) = \int_0^{\infty} r\; f_2(r)\; F_1\left(\frac{\mu_2}{1-\mu_1-\mu_2}\,r\right)dr \tag{13}$$

$$\bar{\mathcal{R}}_{\mathsf{R}}(\mu) = \int_0^{\infty} r\ f_1(r)\ \left[F_2\left(\frac{1-\mu_1-\mu_2}{\mu_2}\ r\right) - F_2(r)\right] dr + \int_0^{\infty} r\ f_2(r)\ \left[F_1\left(\frac{1-\mu_1-\mu_2}{\mu_1}\ r\right) - F_1(r)\right] dr \tag{14}$$

The optimal weights, denoted μ^*, must satisfy $\bar{\mathcal{R}}_1(\mu^*) = \bar{\mathcal{R}}_2(\mu^*) = \bar{\mathcal{R}}_{\mathsf{R}}(\mu^*)$.

C. Solution on the Boundaries of the Feasible Region

If the solution is on the boundary $\mu_1 = (1-\mu_2)/2$, which could happen when the average achievable rate of $f_1(r)$ is much less than the average achievable rate of $f_2(r)$, then in the relay transmission region (refer to Fig. 3) we must have orthogonal time multiplexing of two scheduling modes (RNC and N1) in order to enable satisfying the constraint $\bar{\mathcal{R}}_{\mathsf{N1}} = \bar{\mathcal{R}}_{\mathsf{RNC}} + \bar{\mathcal{R}}_{\mathsf{R2}}$. Therefore, the time sharing ratio for RNC equals $\tau_{\mathsf{RNC}} = (\bar{\mathcal{R}}_{\mathsf{R}} + \bar{\mathcal{R}}_1)/2\bar{\mathcal{R}}_{\mathsf{R}}$, and $\tau_{\mathsf{N1}} = 1 - \tau_{\mathsf{RNC}}$, where $\bar{\mathcal{R}}_{\mathsf{R}}$ and $\bar{\mathcal{R}}_1$ are obtained using (14) and (12), respectively. Notice that in this particular case, the relay transmission region (in Fig. 3) has one border at $R_2[k] = R_1[k]$ and the other border in the region $R_2[k] > R_1[k]$. A similar analysis can be used if the solution is on the boundary $\mu_2 = (1-\mu_1)/2$, where the time multiplexing in the relay scheduling region (in Fig. 3) is between RNC and N2.

In the other case when the optimal solution is at the boundary $\mu_1 + \mu_2 = \frac{1}{2}$, which could happen when the average achievable rates of $f_1(r)$ and $f_2(r)$ are low, then downlink from the relay using network coding alone is not optimal. We should additionally use the scheduling modes R1 and R2 in this case. With reference to Fig. 3, we should have orthogonal multiplexing of N1 and R1 in the N1 region. Additionally, we should have orthogonal multiplexing of N2 and R2 in the N2 region. We should search for μ_1 and μ_2 that make $\bar{\mathcal{R}}_1 = \bar{\mathcal{R}}_2$, where $\bar{\mathcal{R}}_1$ and $\bar{\mathcal{R}}_2$ are given by (12) and (13), respectively. There are infinite possible selections of the time sharing ratios in these two regions in order to satisfy the two constraints in (2b). For the solution that makes the transmissions from the two users equal, we should have $\tau_{\mathsf{N1}} = \tau_{\mathsf{N2}} = (\bar{\mathcal{R}}_{\mathsf{R}} + \bar{\mathcal{R}}_1)/2\bar{\mathcal{R}}_1$, and $\tau_{\mathsf{R1}} = \tau_{\mathsf{R2}} = 1 - \tau_{\mathsf{N1}}$ for the ratio of time sharing given to N1, R1 and N2, R2, respectively, in the regions N1 and N2 (in Fig. 3). In this case, the actual transmitted rate by user N_1 equals $\tau_{\mathsf{N1}}\bar{\mathcal{R}}_1$. Similarly, the actual transmitted rate by user N_2 equals $\tau_{\mathsf{N2}}\bar{\mathcal{R}}_2$. The total rate transmitted from the relay to user N_1 equals $\bar{\mathcal{R}}_{\mathsf{R}} + \tau_{\mathsf{R1}}\bar{\mathcal{R}}_1$, where $\bar{\mathcal{R}}_1$, $\bar{\mathcal{R}}_2$ and $\bar{\mathcal{R}}_{\mathsf{R}}$ are given by (12), (13) and (14), respectively.

IV. Partial Exploitation of Multiuser Diversity and Side Information Gains

We characterize the long-term average rates (for symmetric flow from N_1 and N_2) for suboptimal schemes that exploit OS and NC gains partially. Due to space limitation, we give the final closed-form expressions to characterize these schemes and we omit the derivations.

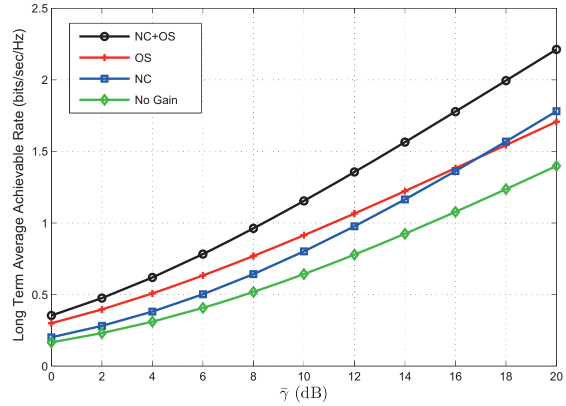

Fig. 4. Achievable rate for symmetric channels (bits/sec/Hz).

A. Opportunistic Scheduling without Network Coding

When the relay does not use network coding, the scheduler has to select from $\xi[k] \in \{\mathsf{N1}, \mathsf{N2}, \mathsf{R1}, \mathsf{R2}\}$. In this case, we have

$$\bar{\mathcal{R}}_{\mathsf{N1}} = \frac{1}{2}\int_0^{\infty} r\ f_1(r)\ F_2\left(\frac{1-\lambda}{\lambda}\ r\right) dr, \tag{15}$$

$$\bar{\mathcal{R}}_{\mathsf{N2}} = \frac{1}{2}\int_0^{\infty} r\ f_2(r)\ F_1\left(\frac{\lambda}{1-\lambda}\ r\right) dr, \tag{16}$$

where λ is selected such that $\bar{\mathcal{R}}_{\mathsf{N1}} = \bar{\mathcal{R}}_{\mathsf{N2}}$.

B. Network coding without Opportunistic Scheduling

In this scheme, all three nodes transmit in every CB, where each CB is divided into three orthogonal sub-blocks, with optimal time sharing ratios between them. The long-term average achievable rate in the system is given by

$$\bar{\mathcal{R}}_{\mathsf{N1}} = \bar{\mathcal{R}}_{\mathsf{N2}} = \int_0^{\infty}\int_0^{r_1} \frac{r_1 r_2}{2r_1 + r_2} f_1(r_1) f_2(r_2) dr_2 dr_1 + \int_0^{\infty}\int_0^{r_2} \frac{r_2 r_1}{2r_2 + r_1} f_2(r_2) f_1(r_1) dr_1 dr_2. \tag{17}$$

C. Basic Scheme with No Gains

In this scheme, each CB is divided orthogonally into four slots for N1, N2, R1 and R2. The long-term average transmission rate in this scheme is given by

$$\bar{\mathcal{R}}_{\mathsf{N1}} = \bar{\mathcal{R}}_{\mathsf{N2}} = \frac{1}{2}\int_0^{\infty}\int_0^{\infty} \frac{r_1 r_2}{r_1 + r_2} f_1(r_1) f_2(r_2) dr_1 dr_2. \tag{18}$$

V. Numerical Results

In this Section, we provide numerical results for the comparison between the optimal solution of problem (2) presented in Section III and the suboptimal solutions in Section IV. Fig. 4 shows the comparison when the two users' channels are Rayleigh block-faded and has the same average Signal-to-Noise-Ratio (SNR), denoted $\bar{\gamma}$. The achievable rates are plotted versus $\bar{\gamma}$. Moreover, Fig 5 and Fig. 6 show the results when

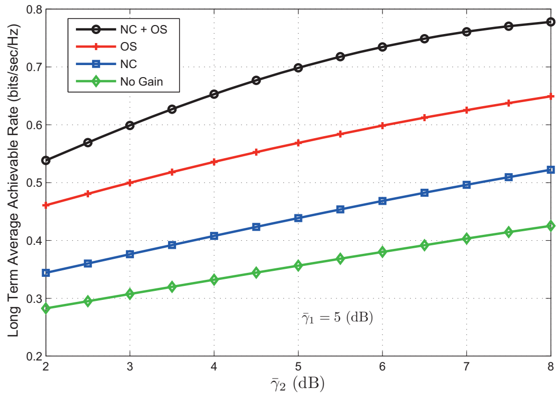

Fig. 5. Achievable rate for non-symmetric channels (bits/sec/Hz).

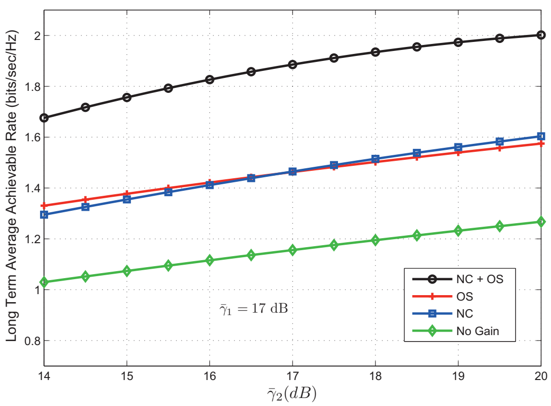

Fig. 6. Achievable rate for non-symmetric channels (bits/sec/Hz).

$\bar{\gamma}_1$ is fixed and the achievable rates are plotted versus $\bar{\gamma}_2$. All results are for the achievable rates for the transmission from one node to another. The total rate is twice this value since we have symmetric transmission flaw.

The numerical results demonstrate the gains of applying network coding (NC) and opportunistic scheduling (OS) jointly. Furthermore, we observe that for the suboptimal solutions, the gains of OS are higher than the gains of NC at low SNR values, while the opposite is true at high SNR values.

VI. Conclusions

Network coding and opportunistic scheduling have been jointly applied in order to enhance the resources' utilization efficiency and to maximize the transmission rate over a bidirectional relay channel. The optimal solution to maximize the long-term average achievable rates of the channel was obtained by using the Lagrangian dual problem. The optimal scheduling scheme has been compared with suboptimal schemes that apply either network coding or opportunistic scheduling, but not both of them. The numerical results demonstrate the gains of the optimal scheme.

Appendix A
Proof of Lemma 1

In compact form, we can write (11) as

$$1 - \mu_1 - \mu_2 \; > \; \max\left(\mu_1 \; , \; \mu_2\right) \tag{19}$$

Let's assume arbitrary that $\mu_1 > \mu_2$ (we can do the proof with the other way around), and $\mu_1 > 1 - \mu_1 - \mu_2$. We can distinguish three cases (i.e. we divide the space of $R_1[k]$ and $R_2[k]$ into three non-overlapping regions spanning the whole two-dimensional space).

In the first case, $R_1[k] > R_2[k]$. Therefore, $\mathcal{R}_{\mathsf{RNC}}[k] = R_2[k]$. We can show that $\mu_1 R_1[k] > \mu_2 R_2[k]$ and $\mu_1 R_1[k] > (1-\mu_1-\mu_2)R_1[k] > (1-\mu_1-\mu_2)R_2[k]$. So, N1 is scheduled in this case.

In the second case, $R_1[k] < R_2[k]$ and $\mu_1 R_1[k] > \mu_2 R_2[k]$. Therefore $\mathcal{R}_{\mathsf{RNC}}[k] = R_1[k]$. We have that $\mu_1 R_1[k] > (1 - \mu_1 - \mu_2)R_1[k]$. Therefore, N1 is scheduled as well.

In the third case, $\mu_1 R_1[k] < \mu_2 R_2[k]$, which necessitates that $R_1[k] < R_2[k]$ and hence $\mathcal{R}_{\mathsf{RNC}}[k] = R_1[k]$. We have in this case, $\mu_2 R_2[k] > \mu_1 R_1[k] > (1-\mu_1-\mu_2)R_1[k]$. Therefore, N2 is scheduled in this case.

From the three cases, we find that the relay is never scheduled if $\mu_1 > 1-\mu_1-\mu_2$. Consequently, the constraints of the primal problem cannot be maintained. Thus, we conclude that the optimal values of μ_1 and μ_2 that satisfy the constraints must be in the region defined by (19).

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