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On the Capacity of Interference Channels with Partial Codebook Knowledge

Alex Dytso, Natasha Devroye, and Daniela Tuninetti
University of Illinois at Chicago, Chicago IL 60607, USA,
Email: odytso2, devroye, danielat @ uic.edu

***Abstract*—Shannon theoretic multi-user capacity problems are traditionally formulated under the assumption that all decoding nodes possess all codebooks. However, for certain networks such as cognitive ones, this may be an unrealistic assumption. We work towards understanding the impact of lack of codebook knowledge at some decoding nodes in the network. We do so by considering a two-user interference channel in which one of the receivers has no information about the codebook of the interfering transmitter, while the other receiver has both codebooks. We derive a novel outer bound for the special class of injective semi-deterministic interference channels which incorporates this codebook knowledge explicitly. For the linear deterministic channel, which models the Gaussian channel at high SNR, we demonstrate the surprising fact that non i.i.d. Bernoulli(1/2) points achieve points on the outer bound not achievable by Bernoulli(1/2) inputs. We then show that this is achievable to within a constant gap by a modified Han-Kobayashi scheme. We characterize the capacity region of the Gaussian noise channel to within $1/2$ bit, even though we could not determine the set of optimal input distributions. Numerical evaluations suggest that if the non-oblivious transmitter uses a discrete input a larger sum-rate is achievable compared to the case where both users employ Gaussian codebooks or use time division in strong interference regime at high SNR.**

I. Introduction

While most multi-user information theoretic capacity results assume that all decoding nodes are aware of the codebooks of all transmitting nodes, this may not be practically relevant in certain networks. For example, in large wireless networks, it may be impractical to share all codebooks. In cognitive networks, primary nodes may be oblivious to overlain cognitive nodes. In networks in which nodes join and leave over time, it may be unrealistic to assume that all existing users would learn the codebook of a new user, and vice-versa. Such networks motivate the study of networks where users have *partial codebook knowledge*, by which we mean that any node in the network knows only a subset of all codebooks (and not actually portions of codebooks).

Past Work. To the best of our knowledge, networks with partial codebook information were first introduced in [1], where partial codebook information was modeled by lack of knowledge of the index of a random encoding function that maps messages to codewords at some decoding nodes. In [2], [3] partial codebook knowledge was similarly modeled at the so-called *oblivious* relays (i.e., relay nodes that lack codebook knowledge). We will use a similar model here. The authors in [2], [3] derived multi-letter capacity expressions which are generally not computable. In particular, the optimal input distribution for the practically relevant Gaussian noise channel remains unknown, see [2, Section III.A] and [3, Remark 5].

Contribution and Paper Outline. We focus on a two-user interference channel where one receiver knows both codebooks, while the other only knows one. This differs from [2], where capacity results were shown for the case where both receivers are oblivious. Our main contributions, after formally introducing the channel model in Section II, are:

1) the derivation of a novel outer bound which incorporates this partial codebook knowledge explicitly (Section III);
2) capacity to within a constant gap for the injective semi-deterministic interference channel (Section IV);
3) for the Linear Deterministic Interference Channel (LD-IC), which models the Gaussian noise channel at high SNR, we demonstrate examples of sum-rate optimal input distributions (Section V). Surprisingly, the sum-capacity achieving distributions are no longer i.i.d. Bernoulli(1/2) (as it is usually the case for the LD-IC), even though they achieve the same sum-capacity as uniform i.i.d. Bernoulli(1/2) distributions in the IC with full codebook knowledge. This might suggest that there is no loss of optimality in lack of codebook knowledge as long as the oblivious receiver can remove the interfering signal, regardless of whether or not it can decode the message carried by the interference.
4) we show capacity to with $1/2$ bit for the real-valued Gaussian noise channel, even though we could not determine the set of optimal input distributions (Section V). Interestingly, inspired by the LD-IC, if the non-oblivious transmitter uses a discrete input we numerically show that a larger sum-capacity is attainable in strong interference than by selecting Gaussian inputs, using time-division, or treating interference as noise, at high SNR. Here the oblivious receiver cannot decode the interfering message but can do soft-estimation of the interfering codeword symbols.

Notation. Lower case variables are instances of upper case random variables which take values in calligraphic alphabets. We let $\delta(\cdot)$ denote the Dirac delta function. The vector $x^n := (x_1, x_2, \cdots x_n)$. The cardinality of a set A is denoted by $|A|$.

II. Channel Model

General Memoryless Model. We consider the Interference Channel with an Oblivious Receiver (IC-OR). It consists of the two-user memoryless interference channel

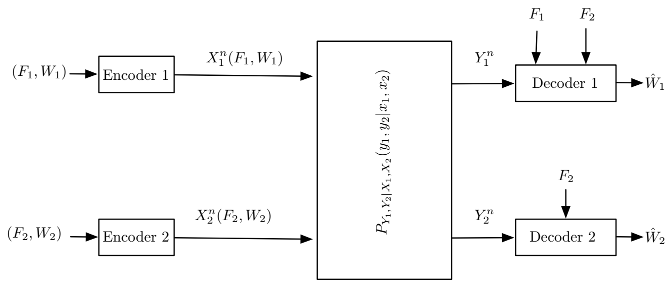

Fig. 1. An Interference Channel with an Oblivious Receiver (IC-OR). F_1 and F_2 represent codebook indices known to one or both receivers.

$(\mathcal{X}_1, \mathcal{X}_2, P_{Y_1Y_2|X_1X_2}, \mathcal{Y}_1, \mathcal{Y}_2)$ where receiver 2 is oblivious of the transmitter 1's codebook. We model this lack of codebook knowledge as in [1], where transmitters use randomized encoding function, which are indexed by a message index and a "codebook index" (F_1 and F_2 in Fig. 1). An oblivious receiver is unaware of the "codebook index" (F_1 is not given to decoder 2 in Fig. 1). The basic modeling assumption is that without the knowledge of the codebook index a codeword looks unstructured. More formally, by extending [2, Definition 2], a $(2^{nR_1}, 2^{nR_2}, n)$ code for the IC-OR with enabled time sharing is a six-tuple $(P_{F_1|Q^n}, \sigma_1^n, \phi_1^n, P_{F_2|Q^n}, \sigma_2^n, \phi_2^n)$, where the distribution $P_{F_i|Q^n}$, $i \in \{1, 2\}$, is over a finite alphabet $\mathcal{F}_i$ conditioned on the time-sharing sequences q^n from some finite alphabet $\mathcal{Q}$, and where the encoders σ_i^n and the decoders ϕ_i^n, $i \in \{1, 2\}$, are mappings

$$\begin{aligned}
&\sigma_1^n : [1 : 2^{nR_1}] \times [1 : |\mathcal{F}_1|] \to \mathcal{X}_1^n, \\
&\sigma_2^n : [1 : 2^{nR_2}] \times [1 : |\mathcal{F}_2|] \to \mathcal{X}_2^n, \\
&\phi_1^n : [1 : |\mathcal{F}_1|] \times [1 : |\mathcal{F}_2|] \times \mathcal{Y}_1^n \to [1 : 2^{nR_1}], \\
&\phi_2^n : [1 : |\mathcal{F}_2|] \times \mathcal{Y}_2^n \to [1 : 2^{nR_2}].
\end{aligned}$$

Moreover, when user 1's codebook index is unknown at decoder 2, the encoder σ_1^n and distribution $P_{F_1|Q^n}$ must satisfy

$$\begin{aligned}
&\sum_{w_1=1}^{2^{nR_1}} \sum_{f_1=1}^{|\mathcal{F}_1|} P_{F_1|Q^n}(f_1|q^n)\ 2^{-nR_1}\ \delta\big(x_1^n - \sigma_1^n(w_1, f_1)\big) \\
&=: \mathbb{P}[X_1^n = x_1^n|Q^n = q^n] = \prod_{t\in[1:n]} P_{X_1|Q}(x_{it}|q_t), \qquad (1)
\end{aligned}$$

according to some distribution $P_{X_1|Q}$. In other words, when averaged over the probability of selecting a given codebook and over a uniform distribution on the message set, the transmitted codeword conditioned on any time sharing sequence has a product distribution. Besides the restriction in (1) on the allowed class of codes, the probability of error, achievable rates and capacity region are defined in the usual way [4].

Injective Semi-Deterministic Memoryless Model. For a general memoryless IC, no restrictions are imposed on the transition probability $P_{Y_1Y_2|X_1X_2}$. The *injective semi-deterministic* interference channel (ISD-IC) is a special interference channel with transition probability

$$\begin{aligned}
&P_{Y_1Y_2|X_1X_2}(y_1, y_2|x_1, x_2) = \sum_{t_1,t_2} P_{T_1|X_1}(t_1|x_1) P_{T_2|X_2}(t_2|x_2) \\
&\cdot \delta\big(y_1 - f_1(x_1, t_2)\big)\ \delta\big(y_2 - f_2(x_2, t_1)\big),
\end{aligned}$$

for some memoryless transition probabilities $P_{T_1|X_1}$ and $P_{T_2|X_2}$, and some deterministic functions f_1 and f_2 that are injective when their first argument is held fixed [5], that is, for all $P_{X_1X_2} = P_{X_1}P_{X_2}$ one has $H(Y_1|X_1) = H(T_2|X_1)$ and $H(Y_2|X_2) = H(T_1|X_2)$, see [5, Fig. 1].

III. Outer Bound for the Injective Semi-Deterministic Interference Channel

We now present a new outer bound for the ISD-IC-OR. We begin by proving a property of the output distributions that is key to the converse, i.e., that the distribution $P_{Y_2^n|X_2^n,F_2}(Y_2^n|X_2^n, F_2)$ may be written as a product distribution. This will enable the outer bound to be single letterized.

Lemma 1. *The output of the oblivious decoder has a product distribution conditioned on the parameters of the known codebook, that is,*

$$P_{Y_2^n|X_2^nF_2Q^n}(y_2^n|x_2^n, f_2, q^n) = \prod_{i\in[1:n]} P_{Y_2|X_2,Q}(y_{2i}|x_{2i}, q_i). \qquad (2)$$

Proof: Starting with

$$\begin{aligned}
&P_{Y_2^nX_1^n|X_2^nF_2Q^n}(y_2^n, x_1^n|x_2^n, f_2, q^n) \\
&\overset{\text{(i)}}{=} P_{X_1^n|Q^n}(x_1^n|q^n) \prod_{i\in[1:n]} P_{Y_2|X_1,X_2}(y_{1i}|x_{1i}, x_{2i}) \\
&\overset{\text{(ii)}}{=} \prod_{i\in[1:n]} P_{X_1|Q}(x_{1i}|q_i) P_{Y_2|X_1,X_2}(y_{1i}|x_{1i}, x_{2i}),
\end{aligned}$$

where (i) follows since X_1^n and (X_2^n, F_2) are conditionally independent given the time sharing sequence and the channel is memoryless, (ii) uses the fact that X_1^n has a product distribution if not conditioned on F_1 as in (1). Marginalization of the above over X_1^n implies (2) for

$$P_{Y_2|X_2,Q}(y|x, q) := \sum_{x'\in\mathcal{X}_1} P_{X_1|Q}(x'|q) P_{Y_2|X_1,X_2}(y|x', x).$$

as claimed. ■

We are now ready to prove a converse for the ISD-IC-OR. In the following we shall derive rate bounds for R_1 and R_2 individually that hold for any memoryless channel. The ISD condition will only be needed in the sum-rate bound.

Theorem 2 (Region $\mathcal{R}_O$)**.** *An achievable rate pair (R_1, R_2) for the ISD-IC-OR must satisfy*

$$\begin{aligned}
R_1 &\le I(Y_1; X_1|X_2, Q) &(3)\\
R_2 &\le I(Y_2; X_2|Q) &(4)\\
R_1 + R_2 &\le H(Y_1|Q) + H(Y_2|U_2, Q) \\
&\quad - H(T_2|X_2, Q) - H(T_1|Q) &(5)
\end{aligned}$$

for some input distribution

$$P_{Q,X_1,X_2,U_2} = P_Q P_{X_1|Q} P_{X_2|Q} P_{U_2|X_2}, \tag{6}$$

where U_2 is a conditionally independent copy of T_2 given X_2

$$P_{U_2,T_2|X_2,Q}(u,t|x,q) = P_{T_2|X_2}(t|x) P_{T_2|X_2}(u|x), \tag{7}$$

and where $|\mathcal{Q}| \le 2$. We denote this region as $\mathcal{R}_O$.

Proof: In the following the inequalities marked with (a) follow from Fano's inequalities

$$H(W_1|Y_1^n, F_1, F_2) \le n\epsilon_n,\ H(W_2|Y_2^n, F_2) \le n\epsilon_n,$$

with $\epsilon_n \to 0$ as $n \to \infty$, those with (b) since F_1, F_2, W_1 and W_2 are mutually independent, those with (c) from the data processing $(F_i, W_i) \to (X_1^n, X_2^n) \to Y_i^n$, $i \in \{1, 2\}$, and those with (d) from the chain rule, conditioning reduces entropy, and from the memoryless property of the channel (for R_1) or from Lemma 1 (for R_2). We have

$$\begin{aligned}
n(R_1 - \epsilon_n) &\overset{(a)}{\le} I(W_1; Y_1^n, F_1, F_2) \overset{(b)}{=} I(W_1; Y_1^n | F_2, F_1, W_2) \\
&\overset{(c)}{\le} I(X_1^n; Y_1^n | F_2, F_1, X_2^n) \overset{(d)}{\le} \sum_{i\in[1:n]} I(X_{1i}; Y_{1i} | X_{2i}),
\end{aligned}$$

$$\begin{aligned}
n(R_2 - \epsilon_n) &\overset{(a)}{\le} I(W_2; Y_2^n, F_2) \overset{(b)}{=} I(W_2; Y_2^n | F_2) \\
&\overset{(c)}{\le} I(X_2^n; Y_2^n | F_2) \overset{(d)}{\le} \sum_{i\in[1:n]} I(X_{2i}; Y_{2i}).
\end{aligned}$$

For the sum-rate, we proceed as above until step (c) (except that we do not give side information W_2 to receiver 1) and then we provide receiver 2 with a genie side information as in [5], that is, we give a U_2^n such that U_{2i} is jointly distributed with X_{2i} according to (7); by doing do we have

$$\begin{aligned}
&n(R_1 + R_2 - 2\epsilon_n) \\
&\le I(X_1^n; Y_1^n | F_1, F_2) + I(X_2^n; Y_2^n, U_2^n | F_2) \\
&= H(Y_1^n | F_1, F_2) - H(Y_1^n | F_1, F_2, X_1^n) \\
&+ H(U_2^n | F_2) - H(U_2^n | F_2, X_2^n) \\
&+ H(Y_2^n | F_2, U_2^n) - H(Y_2^n | F_2, X_2^n, U_2^n) \\
&\overset{(e)}{=} H(Y_1^n | F_1, F_2) - H(T_2^n | F_1, F_2) \\
&+ H(U_2^n | F_2) - H(U_2^n | F_2, X_2^n) \\
&+ H(Y_2^n | F_2, U_2^n) - H(T_1^n) \\
&\overset{(f)}{=} H(Y_1^n | F_1, F_2) - H(T_2^n | F_1, F_2) \\
&+ H(T_2^n | F_2) - H(T_2^n | F_2, X_2^n) \\
&+ H(Y_2^n | F_2, U_2^n) - H(T_1^n) \\
&\overset{(g)}{=} H(Y_1^n | F_1, F_2) + H(Y_2^n | F_2, U_2^n) \\
&- H(T_2^n | F_2, X_2^n) - H(T_1^n) \\
&\overset{(h)}{\le} \sum_{i\in[1:n]} H(Y_{1i}) + H(Y_{2i} | U_{2i}) - H(T_{2i} | X_{2i}) - H(T_{1i}),
\end{aligned}$$

where the inequalities follow since: (e) ISD property and independence of (X_1^n, T_1^n) and X_2^n, (f) by definition of U_2^n, (g) by independence of X_1^n and X_2^n, $H(T_2^n|F_1, F_2) - H(T_2^n|F_2) = 0$, (h) the single letterization of the entropy terms with negative sign follows from the memoryless property of the channel (for $H(T_2^n|F_2, X_2^n)$) or by using Lemma 1 (for $H(T_1^n)$).

Finally, the introduction of a time-sharing random variable $Q \sim \mathrm{Unif}[1:n]$ yields the claimed bounds. By using the Fenchel-Eggleston-Caratheodory theorem and arguments as in [4, Appendix A] one can show that $|Q| \le 2$ suffices. ■

IV. Capacity Region to Within a Constant Gap

For achievability for the ISD-IC-OR, we consider the following simplified Han-Kobayashi scheme [6]: encoder 1 transmits private messages only, while encoder 2 (corresponding to the oblivious receiver) does rate-splitting and sends both a common and a private message. We have:

Lemma 3 (Region $\mathcal{R}_i$)**.** *The following rate region is achievable for the ISD-IC-OR*

$$\begin{aligned}
R_1 &\le I(X_1; Y_1 | U_2, Q) &(8)\\
R_2 &\le I(X_2; Y_2 | Q) &(9)\\
R_1 + R_2 &\le I(X_1, U_2; Y_1 | Q) + I(X_2; Y_2 | U_2, Q) &(10)
\end{aligned}$$

for all distributions in (6)-(7).

Compared to [6], we restricted the distribution of encoder 2's common codebook to be that of U_2 in the outer bound and we set $U_1 = \emptyset$. We denote this region as $\mathcal{R}_i$. We further note that this region is indeed achievable without knowledge of F_1 at receiver 2 since the first transmitter only sends a private message, i.e., receiver 2 does not attempt to jointly decode transmitter 1's private signal in the Han-Kobayashi scheme.

Comparing inner and outer bounds gives:

Theorem 4 (Constant Gap)**.** *For all distributions in* (6)-(7)

$$(R_1, R_2) \in \mathcal{R}_O \Longrightarrow (R_1 - I(X_2; T_2 | U_2, Q), R_2) \in \mathcal{R}_i.$$

Proof: First, we loosen the upper bound to $\bar{\mathcal{R}}_O$ by replacing X_2 with U_2 in all *positive* entropy terms of region $\mathcal{R}_O$, which is permitted since $H(Y_1|X_1, X_2) = H(T_2|X_2) = H(T_2|X_2, U_2) \le H(T_2|U_2)$ by injectivity, independence of inputs, data processing inequality, and conditioning reduces entropy. We conclude that $\mathcal{R}_O \subseteq \bar{\mathcal{R}}_O$. Comparing $\bar{\mathcal{R}}_O$ and $\mathcal{R}_i$ inequality by inequality yields the claim. ■

V. Gaussian and linear deterministic models

We consider here two ISD-IC-ORs: the Gaussian Interference Channel (G-IC) and the Linear Deterministic Interference Channel (LD-IC), which models the G-IC at high SNR [7].

A. Linear Deterministic IC-OR

The LD-IC-OR has input/output relationship

$$Y_1 = \mathbf{S}^{q-n_{11}} X_1 + \mathbf{S}^{q-n_{12}} X_2, \tag{11}$$

$$Y_2 = \mathbf{S}^{q-n_{21}} X_1 + \mathbf{S}^{q-n_{22}} X_2, \tag{12}$$

where summations and multiplications are in GF(2), $\mathbf{S}$ is the $q \times q$ shift matrix [7], and $q := \max\{n_{11}, n_{12}, n_{21}, n_{22}\}$ for some non-negative integers $(n_{11}, n_{12}, n_{21}, n_{22})$. The LD-IC is an approximation at high SNR of the G-IC with the parameters of the two models related as $n_{ij} = \lfloor \log(1 + |h_{ij}|^2) \rfloor$, $(i, j) \in \{1, 2\}^2$ [7], where h_{ij} is defined in (13).

For the LD-IC-OR, by specializing Theorem. 4, we have the following lemma:

Lemma 5. *For the Linear Deterministic IC-OR $\mathcal{R}_O = \mathcal{R}_i$.*

Proof: For the LD-IC-OR we have $T_2 = U_2 = \mathbf{S}^{q-n_{12}} X_2$ and therefore $I(X_2; T_2|U_2, Q) = 0$. ∎

We next evaluate the capacity region $\mathcal{R}_O$ for the case of symmetric channel gains, that is $n_{11} = n_{22} = n_{\rm S}$ and $n_{12} = n_{21} = n_{\rm I} := n_{\rm S}\ \alpha$ for some non-negative α. The normalized sum-capacity of the classical LD-IC with full codebook knowledge is denoted as

$$d^{(\rm W)}(\alpha) := \frac{\max\{R_1 + R_2\}}{2 n_{\rm S}},$$

i.e., $d^{(\rm W)}(\alpha)$ is the so-called W-curve. In [7] it was shown that i.i.d. Bernoulli(1/2) bits yield $d^{(\rm W)}(\alpha)$. Clearly, since we may provide the LD-IC-OR with additional codebook index F_1 at receiver 2 to obtain the LD-IC with full codebook knowledge, we have that the normalized sum-capacity of the LD-IC-OR is upper bounded by $d^{(\rm W)}(\alpha)$. By extensive computer evaluations we could observe the surprising result that even with $|Q| = 1$, i.e., without time sharing, that the normalized sum-capacity of the LD-IC-OR equals $d^{(\rm W)}(\alpha)$. This implies that partial codebook knowledge at one receiver does not impact the performance of the LD-IC. This might suggest a more general principle: there is no loss of optimality in lack of codebook knowledge as long as the oblivious receiver can remove the interfering signal, regardless of whether or not it can decode the message carried by the interference.

Another interesting observation is that i.i.d. Bernoulli(1/2) inputs bits are no longer optimal. In Table I we report for some values of α the input distributions to be used in $\mathcal{R}_O$. We notice that inputs are now correlated and not uniform. For example, Table I shows that for $\alpha = 4/3$ the inputs X_1 and X_2 are binary vectors of length $\log_2(16) = 4$ bits; out of the 16 different possible bit sequences, only 4 are actually used at each transmitter with strictly positive probability to achieve $d^{(\rm W)}(4/3) = 4/6$; with i.i.d. Bernoulli(1/2) input bits we would obtain a normalized sum-rate of $1/2 = 3/6$, as for time division [7]. In other words, we demonstrate the surprising fact that non i.i.d. Bernoulli(1/2) points achieve points on the outer bound of the LD-IC-OR that are not achievable by Bernoulli(1/2) input bits.

B. Gaussian IC-OR

We now consider a single-antenna real-valued G-IC-OR, whose input/output relationship is

$$Y_1 = h_{11} X_1 + h_{12} X_2 + Z_1 \quad (13a)$$
$$Y_2 = h_{21} X_1 + h_{22} X_2 + Z_2 \quad (13b)$$

where h_{ij}, are constant channel coefficients, the inputs $X_i \in \mathbb{R}$ are subject to power constraints $\mathbb{E}[X_i^2] \le 1$, $i \in \{1, 2\}$, and the noise processes $Z_i \sim \mathcal{N}(0, 1)$ and are i.i.d.

TABLE I
EXAMPLES OF SUM-RATE OPTIMAL DISTRIBUTIONS FOR THE LD-IC-OR.

α	Probability mass function
$\frac{1}{2}$	$P_{X_1} = [0.5, 0, 0.5, 0]$ $P_{X_2} = [0, 0.5, 0, 0.5]$
$\frac{2}{3}$	$P_{X_1} = [0, 0, 0.25, 0.25, 0, 0, 0, 0.25, 0.25]$ $P_{X_2} = [0, 0, 0.25, 0.25, 0, 0, 0, 0.25, 0.25]$
1	$P_{X_1} = [0, 0, 0.5, 0.5]$ $P_{X_2} = [0, 0.5, 0, 0.5]$
$\frac{4}{3}$	$P_{X_1} = [0, 0, 0, 0, 0.25, 0, 0.25, 0, 0, 0, 0, 0, 0.25, 0, 0.25]$ $P_{X_2} = [0, 0, 0, 0.25, 0, 0.25, 0, 0, 0, 0, 0, 0, 0, 0.25, 0, 0.25]$
2	$P_{X_1} = [0, 0.5, 0, 0.5]$ $P_{X_2} = [0, 0.5, 0, 0.5]$

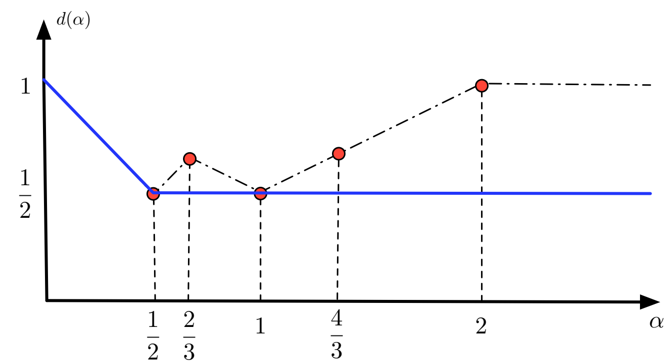

Fig. 2. The dash-dotted black line: normalized sum-capacity of the LD-IC-OR, i.e., the W-curve (also an outer bound to G-IC-OR); red dots: points achieved by input distributions in Table I; solid blue line: achievable gDoF with Gaussian inputs for the G-IC-OR.

By specializing Theorem. 4, we have:

Lemma 6 (Half Bit Gap for G-IC-OR)**.** *For the G-IC-OR, if $(R_1, R_2) \in \mathcal{R}_O$, then $(R_1 - 1/2, R_2)$ is achievable.*

Proof: In Gaussian noise, $T_2 := h_{12} X_2 + Z_1$ and thus U_2 is chosen to be $U_2 := h_{12} X_2 + Z_1'$ where X_2, Z_1, Z_1' are mutually independent and $Z_1' \sim Z_1$. Then,

$$\begin{aligned} I(X_2; T_2|U_2, Q) &= h(T_2 - U_2|U_2, Q) - h(Z_1) \\ &\le h(Z_1 - Z_1') - h(Z_1) \le 1/2 \log(2). \end{aligned}$$

as claimed. ∎

An interesting question is whether Gaussian inputs are optimal for $\mathcal{R}_O$. The following discussion shows that in general the answer is in the negative. For simplicity we focus on the achievable generalized Degrees of Freedom (gDoF) for the symmetric G-IC-OR ($|h_{11}| = |h_{22}| = \sqrt{S}$ and $|h_{12}| = |h_{21}| = \sqrt{I}$ with $I = S^\alpha$ for some non-negative α) without time sharing, i.e., $|Q| = 1$, defined as

$$d(\alpha) := \lim_{S \to +\infty} \frac{R_1 + R_2}{2 \cdot \frac{1}{2} \log(1 + S)}$$

By Evaluating Theorem 2 for independent Gaussian inputs (which we do *not* claim is the optimal) we obtain

$$(R_1 + R_2)^{(\rm GG)} = \min\left\{ \frac{1}{2} \log(1 + S) + \frac{1}{2} \log\left(1 + \frac{S}{1 + I}\right), \right.$$

$$\frac{1}{2}\log\left(1+\frac{S}{I+1}\right)+\frac{1}{2}\log\left(\frac{(I+1)^2+S}{1+I}\right)\Bigg\}.$$

resulting in

$$d^{(\mathrm{GG})}(\alpha)=\frac{1}{2}+\left[\frac{1}{2}-\alpha\right]^+.$$

For future reference, with Time Division (TD) and Gaussian codebooks we can achieve

$$(R_1+R_2)^{(\mathrm{TD})}=\frac{1}{2}\log(1+S)\Longleftrightarrow d^{(\mathrm{TD})}(\alpha)=\frac{1}{2}.$$

We plot the achievable gDoF vs. α in Fig. 2, together with the gDof of the classical IC given by $d^{(\mathrm{W})}(\alpha)$ [8], which forms an outer bound to the gDoF of the G-IC-OR. We note that Gaussian inputs are indeed optimal for $0 \le \alpha \le 1/2$, i.e., $d^{(\mathrm{GG})}(\alpha) = d^{(\mathrm{W})}(\alpha)$, where interference is treated as noise even for the classical IC (which is also achievable by the G-IC-OR). For $\alpha \ge 1/2$ we have $d^{(\mathrm{GG})}(\alpha) = d^{(\mathrm{TD})}(\alpha)$, that is, Gaussian inputs perform as time division. Gaussian inputs are sub-optimal in general as we show next.

Consider $\alpha = 4/3$: with Gaussian inputs or with time division we only achieve $d^{(\mathrm{GG})}(4/3) = d^{(\mathrm{TD})}(4/3) = 1/2$. Notice the similarity with the LD-IC-OR: the input distribution that is optimal for the non-oblivious IC performs as time division for the G-IC-OR. Inspired by the LD-IC-OR we explore now the possibility of using a non-Gaussian input. In particular, we choose an input distribution that allows the obvious receiver to soft-estimate the interfering codeword symbols (even though it is not able in general to decode the interfering message). By following [1, Section VI.A], which demonstrated that binary signaling outperforms Gaussian signaling for a fixed finite SNR, we consider a uniform PAM constellation with N points. Fig. 3 shows the achievable normalized sum-rate $\frac{R_1+R_2}{2\cdot\frac{1}{2}\log(1+S)}$ as a function of S for the case where X_1 (the input of the non-oblivious pair) is a PAM constellation with $N = \lfloor S^{1/6}\rfloor$ points and X_2 (the input of the oblivious pair) is Gaussian. Notice that the number of points in the discrete input is a function of the direct link channel gain S. We also report the achievable normalized sum-rate with time division and Gaussian inputs. Fig. 3 shows that for sufficiently large S using a discrete input outperforms time division; moreover, for the range of simulated S, it seems that the proposed discrete input achieves a gDoF of $d^{(\mathrm{DG})}(\alpha) = \alpha/2 = 4/6$ as for the classical IC with full codebook knowledge.

We conjecture that a strategy with one discrete input outperforms Gaussian signaling for all $\alpha > 1$, which appears to be the case from extensive numerical evaluations and is the subject of ongoing work. Proving the validity of our conjecture could also help the settle the open question whether Gaussian inputs exhaust the outer bound in related oblivious channel models – see [2, Section III.A] and [3, Remark 5].

VI. Conclusion

We focused on an IC in which one of the decoders only possesses one of the two transmitting codebooks (in contrast to classical ICs where all nodes are aware of all codebooks). We

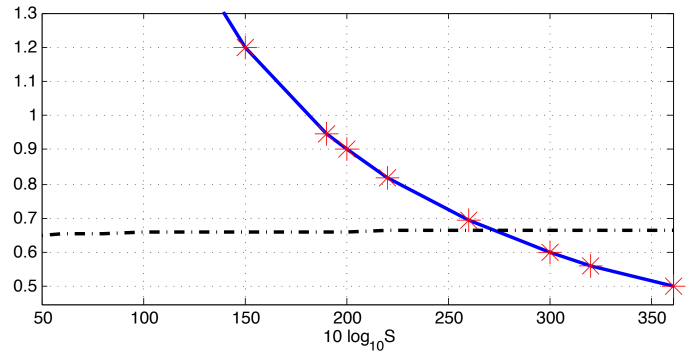

Fig. 3. Achievable normalized sum-rate for the symmetric G-IC-OR with $\alpha = 4/3$: (1) time division: solid blue line; (ii) Gaussian inputs at both transmitters: red stars; (3) X_1 is a uniform PAM with $N = \lfloor S^{1/6}\rfloor$ points and X_2 is Gaussian: dash-dotted black line.

characterized the capacity of the injective semi deterministic IC to within a constant gap and specialized it to the Gaussian channel and to the Linear Deterministic approximation of the Gaussian channel at high SNR; in the former case we established capacity to within 1/2 bit, even though we could not determine the optimal input distribution; in the latter, we showed the exact capacity region and that the sum-capacity with partial codebook knowledge is the same as that of the classical IC with full codebook knowledge. An important next step is to identify optimal input distributions for the Gaussian noise channel. In this direction, we are currently investigating the usage of discrete inputs for the non-oblivious user and of Gaussian input for the oblivious transmitter, which numerically seems to outperform Gaussian signaling and time division.

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