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On Optimal Online Power Policies for Energy Harvesting with Finite-State Markov Channels

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***Abstract*—We investigate the problem of continuous-time energy harvesting in communication systems operating over fading wireless channels. We model the fading as a finite-state continuous-time Markov process, and the battery dynamics as a storage dam process with reflecting boundary conditions. We describe a set of necessary conditions for the ergodicity of the dam process. Followed by these conditions, we establish an upper bound on the ergodic channel throughput. We further determine some structure for good transmission power policies based on a throughput maximization problem. Specifically, using calculus of variations techniques, we derive Euler-Lagrange equations as a necessary condition for optimal power policies. In the case of a Markov channel with two channel states (i.e. Gilbert-Elliot channel), we characterize power policies by solving these equations numerically.**

***Index Terms*—Finite-state Markov channel; Energy harvesting.**

I. Introduction

Recent improvements in dynamic energy harvesting technologies have provided an alternative energy resource for data transmission in many low-power applications. The utilization of energy harvesters creates the potential for service-free, wireless sensor solutions in industrial automation, and significantly reduces the cost of sensor maintenance.

A key element that impacts the performance of energy harvesting systems is the power policy whose role is to efficiently allocate transmission power and manage fluctuations in real-time communication parameters. In particular, fluctuations in wireless channels due to small and large scale fading have adverse effect on performance metrics like channel throughput. To mitigate these effects, dynamic power allocation based on channel state information (CSI) at the transmitter is widely used in communication networks.

For example, in [1], the problem of power allocation over a finite time horizon in a time varying wireless channel is considered where offline and online energy harvesting in the discrete-time regime is treated separately. Moreover, in [2], throughput maximization in a fading channel subject to energy constraints and a deadline is considered. Specifically, using dynamic programming, discrete-time power policies for both non-causal and causal channel state information (CSI) at the transmitter are studied. In [3], the transmission of a fixed amount of data over a time varying channel subject to a time deadline is studied. Furthermore, based on continuous dynamic programming, the Hamilton-Jacobi-Bellman (HJB) equation for the structure of optimal rate control is formulated.

Among more recent results for energy harvesting are [4] and [5]. In [4], the problem of energy harvesting communication over fading channels subject to minimum delay is considered. Therein, it is shown that for data arrival rates bigger than a given threshold, the average delay incurred by data in the queue is infinite. Along the same lines, in [5], various scenarios for communication over a fading channel in both offline and online settings are studied. For the online problem, some heuristic solutions as well as the dynamic programming solution are considered.

In this paper, we study continuous-time power policies for communication over a fading wireless channel with causal knowledge of energy arrivals. Motivated by [6], we use a finite-state Markov channel (FSMC) approach to model the channel fading. For modeling the battery with finite storage capacity, we leverage a storage dam process with a reflecting boundary condition (see [7]). Generalizing the results of [8], we determine necessary conditions for ergodicity of this process. Based on these conditions, we establish an upper bound on the ergodic channel throughput. We then consider a throughput maximization problem that creates a general framework for deriving achievable power policies. By applying calculus of variations to this maximization problem, we subsequently obtain necessary conditions for optimal transmission power policies in the form of second-order differential equations. In the special case of the Gilbert-Elliot channel, we show structural results for optimal power policies by solving these equations numerically.

The rest of this paper is organized as follows. We define the channel and energy harvesting models in Section II. In Section III, we study necessary conditions for the storage process to be positive recurrent. From these conditions, we then find an upper bound on the average throughput in Section IV. There, we also obtain the structure of achievable power policies by a system of autonomous ordinary differential equations (ODEs). Finally, in Section V, we numerically compute solutions of these ODEs for a Gilbert-Elliot channel.

II. Preliminaries

A. Communication Model

We consider a single user Rayleigh fading wireless channel modeled as a finite-state continuous-time Markov process. In particular, let $\{H(t) : t \geq 0\}$ be a stationary, time-homogenous, and irreducible Markov process with state space $\mathcal{H} = \{h_1, h_2, \cdots, h_M\}$ that specifies the fading gain at time t. The channel state information (CSI) is assumed to be known at the transmitter through causal feedback. Furthermore, $P(t) \triangleq [P_{ij}(t)]_{M\times M}$ denotes the probability matrix of the process where $P_{ij}(t) = \mathbb{P}\{H(s+t) = h_j | H(s) = h_i\}$, for all $s \geq 0$.

The stationary distribution $\pi \triangleq (\pi_0, \pi_1, \cdots, \pi_M)$ with elements $\pi_i \triangleq \mathbb{P}\{H(t) = h_i\}$ is in turn defined as the left eigenvector of $P(t)$, *i.e.*, $\pi P(t) = \pi$. Moreover, the transition rate matrix Q can be determined through $P(t)$ as follows,

$$Q \triangleq \lim_{t\to 0}\ (P(t)-I)/t, \tag{1}$$

where I is the identity matrix, and $\{q_{ij}\}_{ij\in[M]\times[M]}([M] \triangleq \{1,2,\cdots,M\})$ are such that $\sum_{j\in[M]} q_{ij}=0$, and the global balance equation is satisfied, i.e.,

$$\sum\nolimits_{j\in[M]\setminus\{i\}} q_{ij}\pi_i = \sum\nolimits_{j\in[M]\setminus\{i\}} q_{ji}\pi_j. \tag{2}$$

To specify the relationship between instantaneous transmission rate (bit/sec) and instantaneous transmission power (Joule/sec) in state h_i, we use a rate function $r(p|h), p, h \neq 0$ that satisfies the regular properties

[R1] $r(p|h) > 0$ for all $p > 0, h \neq 0$ and $r(p|h) = 0$ otherwise.
[R2] $\forall p \geq 0, h \neq 0 : \partial r(p|h)/\partial p > 0$.
[R3] $\forall p \geq 0, h \neq 0 : \partial^2 r(p|h)/\partial p^2 < 0$.

For instance, Shannon's rate function $r(p|h) = \log_2(1 + |h^2|\frac{p}{\sigma^2})$ satisfies these properties for any $\sigma^2 > 0$.

B. Energy Harvesting and Storage Model

We consider a continuous-time model for energy arrivals as well as energy consumption. In particular, the energy arrivals $\{E_n\}_{n=1}^{\infty}$ are independent identically distributed (i.i.d) according to the distribution $\mathbb{P}\{E \leq x\} = 1 - e^{-\zeta x}$, and occur at random arrival times $\{T_n^E\}_{n=1}^{\infty}$. The interarrival times $\Delta T_n^E \triangleq T_{n+1}^E - T_n^E$ are i.i.d, exponentially distributed with parameter λ, and independent of the fading process. Furthermore, the total energy flow $A(0,t]$ into the transmission node up to time t is a compound Poisson process

$$A(0,t] = \sum\nolimits_{n=1}^{N(t)} E_n. \tag{3}$$

The energy is replenished into a battery that has a finite storage capacity, say L, and at time t contains $X(t)$ units of energy. The battery charge is also consumed at instantaneous rate $p(X(t),H(t))$, i.e., it is modulated by both the fading state and the battery state. Therefore, the interaction between energy arrivals, energy consumption, and the battery charge is

$$X(t) = X(0) + A(0,t] - \int_0^t p\left(X(s),H(s)\right) ds - Z(t), \tag{4}$$

where $X(0)$ denotes the energy content of the battery at time $t=0$, and $Z(t)$ is a $\mathbb{R}^+$ valued reflection process that is null at zero, continuous almost everywhere, and such that $\int_{\mathbb{R}^+}(X(s)-L)dZ(s) = 0$ [9]. Specifically, the reflection process accounts for battery overflow, and ensures that for all energy arrival values, the energy $X(t)$ of the battery does not exceed the storage capacity, i.e., $X(t)\in[0,L]$.

Notation: In the rest of the paper, we use $r_i(p(x,h_i))$ to denote $r(p(x,h_i)|h_i)$ when there is no confusion. Also $x^+ \triangleq \max(0,x)$.

III. Ergodicity of the Storage Process

We assume that the state space of the battery is irreducible in the sense that at each specific state, all other states are accessible. This is implied by power policies that satisfy the following two conditions for all $h_i \in \mathcal{H}$ (see [10]),

1) $\forall x \in (0,L] : p(x,h_i) > 0,$ and $p(0,h_i) = 0$.
2) $\sup_{0<x\leq L} p(x,h_i) < \infty$.

Proposition 1: For all finite battery capacity $L < \infty$, the storage process in (4) is positive recurrent, and there exist conditional probability measures $\nu(x|h_i), i \in [M]$, absolutely continuous on $(0,L]$ with possible atoms at zero $\nu_0(h_i) \triangleq \nu(0|h_i)$, i.e.,

$$\nu(x|h_i) = \nu_0(h_i) + \int_{0^+}^x f(u|h_i)\ du, \tag{5}$$

such that for $x > 0$,

$$\begin{aligned} p(x,h_i)f(x|h_i)\pi_i = &\int_0^x \lambda e^{-\zeta(x-u)}\nu(du|h_i)\pi_i \\ &+\sum_{j\in[M]\setminus\{i\}} q_{ij}\nu(x|h_i)\pi_i - \sum_{j\in[M]\setminus\{i\}} q_{ji}\nu(x|h_j)\pi_j. \end{aligned} \tag{6}$$

Furthermore, for a given channel state h_i, $\nu(x|h_i)$ is the unique conditional probability measure of $X(t)$.

Proof Outline: The proof generalizes Theorem 3 of [8] by extending the kernel function to include negative jumps. Specifically, $(X(t),H(t))$ is a two-dimensional process that obeys the two-dimensional rate conservation law (RCL) in the stationary regime.

Remark 1: The forward equation in (6) can be viewed as an equilibrium condition between inward flux and outward flux from the interval $(0,x]$ at a given channel state $h_i \in \mathcal{H}$.

The condition (6) can equivalently restated as

$$p(x,h_i) = G_i(x)/f(x|h_i), \tag{7}$$

with $G_i(x)$ defined as

$$\begin{aligned} G_i(x) \triangleq &\int_0^x \lambda e^{-\zeta(x-u)}\nu(du|h_i) - q_{ii}\nu(x|h_i) \\ &-\sum\nolimits_{j\in[M]\setminus\{i\}} q_{ji}\frac{\pi_j}{\pi_i}\nu(x|h_j), \end{aligned} \tag{8}$$

where we used the fact that $-q_{ii} = \sum_{j\in[M]\setminus\{i\}} q_{ij}$ must hold for any transition rate matrix.

IV. Bounds on Long-term Average Channel Throughput

To characterize an upper bound and a lower bound on the average channel throughput, we use the ergodicity of the storage process in (4). More precisely, we define the functional R as follows

$$\begin{aligned} R[p] &\triangleq \lim_{T\to\infty}\frac{1}{T}\int_0^T r\big(p\left(X(t),H(t)\right)\big|H(t)\big)\ dt \\ &\overset{\text{a.s}}{=} \sum_{i=0}^M \pi_i \int_0^L r_i\left(p(x,h_i)\right)\nu(dx|h_i) \qquad (9)\\ &= \sum_{i=0}^M \pi_i \int_{0^+}^L r_i\left(p(x,h_i)\right) f(x|h_i)\ dx, \qquad (10) \end{aligned}$$

where the last term is true as $p(0,h_i)=0$ by the first constraint on the power policy, and $r_i(0)=r(0|h_i)=0$ by property [R1].

A. Upper Bound

We first bound the average transmission power $P_{\text{avg}} \triangleq \mathbb{E}_{X,H}[p(X,H)]$ using (6),

$$P_{\text{avg}} = \sum_{i=1}^{M} \pi_i \int_{0^+}^{L} p(x,h_i) f(x|h_i)\ dx \tag{11}$$

$$= \sum_{i=1}^{M} \pi_i \int_{0^+}^{L} \int_{0}^{x} \lambda e^{-\zeta(x-v)} \nu(du|h_i), \tag{12}$$

where in writing (12) we used the fact that the sum of inward and outward rates over all state transitions must be zero. Using integration by parts, we proceed

$$\begin{aligned}
&\lambda \int_{0^+}^{L} e^{-\zeta x} \int_{0}^{x} e^{\zeta u} \nu(du|h_i)\ dx = -\frac{\lambda}{\zeta} e^{-\zeta x} \int_{0}^{x} e^{\zeta u} \nu(du|h_i)\Big|_{0^+}^{L} \\
&+ \frac{\lambda}{\zeta} \int_{0^+}^{L} e^{-\zeta x} \frac{d}{dx}\Big(\int_{0}^{x} e^{\zeta v} \nu(du|h_i)\Big) dx \\
&\le -\frac{\lambda}{\zeta}\Big(e^{-\zeta L} \int_{0}^{L} \nu(du|h_i) - \nu_0(h_i) - \int_{0^+}^{L} f(x|h_i) dx\Big) \\
&= -\frac{\lambda}{\zeta}\Big(e^{-\zeta L} - \nu_0(h_i) - \int_{0^+}^{L} f(x|h_i) dx\Big) \qquad (13) \\
&= \frac{\lambda}{\zeta}(1-e^{-\zeta L}). \qquad (14)
\end{aligned}$$

where (13) and (14) comes from

$$\int_{0}^{L} \nu(du|h_i) = \nu_0(h_i) + \int_{0^+}^{L} f(u|h_i) dv = 1. \tag{15}$$

Hence,

$$P_{\text{avg}} \le \frac{\lambda}{\zeta}(1-e^{-\zeta L}) \triangleq P_{\text{upper}}. \tag{16}$$

Now consider a communication system that is constrained by an average transmission power equal to the upper bound in (16), and that has the same transition rate matrix Q. For such a communication system, standard water-filling power allocation is optimal. Therefore,

$$R[p] \le \sum_{i=1}^{M} \pi_i r_i(P_W(h_i)) \triangleq R_{\text{water-filling}}, \tag{17}$$

where $P_W(h_i)$ denotes the assigned power to the fading state $h_i \in \mathcal{H}$ by the water-filling solution with an average power $\sum_{i=1}^{M} \pi_i P_W(h_i) = P_{\text{upper}}$.

B. Power Allocation Scheme

To achieve a good performance for average channel throughput, we determine the structure of power policies based on the following maximization problem

$$\sup_{\{\nu_0(h_i), f(x|h_i)\}} \sum_{i=0}^{M} \pi_i \int_{0^+}^{L} r_i\left(p(x,h_i)\right) f(x|h_i)\ dx \tag{18}$$

$$\text{subject to: } \nu_0(h_i) + \int_{0^+}^{L} f(u|h_i)\ du = 1, \tag{19}$$

$$p(x,h_i) f(x|h_i) = G_i(x), \tag{20}$$

$$\nu_0(h_i) \ge 0, f(x|h_i) \ge 0, \tag{21}$$

for all $h_i \in \mathcal{H}$. In particular, we apply a calculus of variations approach to (18)-(21), and compute the variation of conditional densities $f^{\epsilon_i}(x|h_i) = f(x|h_i) + \epsilon_i \psi_i(x)$ where the perturbation functions $\{\psi_i(x)\}$ are continuous and bounded on $(0,L]$ with $\psi_i(0) = \psi_i(L) = 0$, and satisfy the following condition

$$\int_{0^+}^{L} \psi_i(x)\ dx = 0. \tag{22}$$

We now assume that only $f(x|h_i)$ is perturbed while $f(x|h_j), \forall j \ne i$ remain fixed ($\epsilon_j = 0, \forall j \ne i$). Then for sufficiently small ϵ_i, the perturbed density $f^{\epsilon_i}(x|h_i)$ satisfies (19)-(21) with the same atom $\nu_0(h_i)$ (due to (22)) and thus belongs to the feasible region. In conjunction with $f^{\epsilon_i}(x|h_i)$, we denote the perturbed probability measure by

$$\nu^{\epsilon_i}(dx|h_i) = [\nu_0(h_i)\delta(x) + f(x|h_i) + \epsilon_i \psi_i(x)]dx \tag{23}$$

$$= \nu(dx|h_i) + \epsilon_i \psi_i(x)\ dx. \tag{24}$$

Using (24) as well as the relations in (7) and (8) yields the perturbed power policy

$$p^{\epsilon_i}(x,h_i) = \frac{G_i(x) + \epsilon_i \Psi_{ii}(x)}{f(x|h_i) + \epsilon_i \psi_i(x)}, \tag{25}$$

$$p^{\epsilon_i}(x,h_j) = \frac{G_j(x) + \epsilon_i \Psi_{ij}(x)}{f(x|h_j)}, \quad \forall j \ne i, \tag{26}$$

where

$$\Psi_{ii}(x) = \int_{0^+}^{x} (\lambda e^{-\zeta(x-u)} - q_{ii}) \psi_i(u)\ du, \tag{27}$$

$$\Psi_{ij}(x) = -\frac{\pi_i}{\pi_j} q_{ij} \int_{0^+}^{x} \psi_i(u)\ du. \tag{28}$$

As a necessary condition for a local (and thus a global) maximum solution of (18)-(21), the following inequality must hold

$$R[p^{\epsilon_i}] \le R[p], \tag{29}$$

for all sufficiently small ϵ_i, where

$$\begin{aligned}
R[p^{\epsilon_i}] = &\sum_{j \in [M]\setminus\{i\}} \pi_j \int_{0^+}^{L} r_j\left(p^{\epsilon_i}(x,h_j)\right) f(x|h_j)\ dx \\
&+ \pi_i \int_{0^+}^{L} r_i\left(p^{\epsilon_i}(x,h_i)\right) f^{\epsilon_i}(x|h_i)\ dx. \qquad (30)
\end{aligned}$$

Expanding (30) up to the first order results in

$$\begin{aligned}
R[p^{\epsilon_i}] = &R[p] + \epsilon_i \pi_i \int_{0^+}^{L} r_i\left(p(x,h_i)\right) \psi_i(x)\ dx \qquad (31) \\
&+ \epsilon_i \sum_{j=0}^{M} \pi_j \int_{0^+}^{L} r_j'\left(p(x,h_j)\right) \frac{dp^{\epsilon_i}(x,h_j)}{d\epsilon_i}\Big|_{\epsilon_i=0} f(x|h_j)\ dx.
\end{aligned}$$

Combining (29) with (31), the necessary condition is then recast into

$$\pi_i \int_{0^+}^{L} r_i\left(p(x,h_i)\right)\psi_i(x)\ dx \tag{32}$$
$$+\sum_{j=0}^{M}\pi_j\int_{0^+}^{L} r_j'\left(p(x,h_j)\right)\frac{dp^{\epsilon_i}(x,h_j)}{d\epsilon_i}\Big|_{\epsilon_i=0} f(x|h_j)\ dx = 0.$$

On the other hand, from (25) and (26) we compute

$$\frac{dp^{\epsilon_i}(x,h_i)}{d\epsilon_i}\Big|_{\epsilon_i=0} f(x|h_i) = \Psi_{ii}(x) - \frac{\psi_i(x)G_i(x)}{f(x|h_i)} \tag{33}$$
$$= \Psi_{ii}(x) - \psi_i(x)p(x,h_i) \tag{34}$$
$$\frac{dp^{\epsilon_i}(x,h_j)}{d\epsilon_i}\Big|_{\epsilon_i=0} f(x|h_j) = \Psi_{ij}(x), \quad j\neq i. \tag{35}$$

Substituting (34) and (35) into (32) subsequently results

$$\pi_i\int_{0^+}^{L}\psi_i(x)\big(r_i\left(p(x,h_i)\right) - p(x,h_i)r_i'\left(p(x,h_i)\right)\big)\ dx$$
$$+\pi_i\int_{0^+}^{L}\Psi_{ii}(x)r_i'\left(p(x,h_i)\right)\ dx$$
$$+\sum_{j\in[M]\setminus\{i\}}\pi_j\int_{0^+}^{L}\Psi_{ij}(x)r_j'\left(p(x,h_j)\right)\ dx = 0. \tag{36}$$

For the last term of (36), we write from (28),

$$\int_{0^+}^{L}\Psi_{ij}(x)r_j'\left(p(x,h_j)\right)\ dx$$
$$= -\frac{\pi_i}{\pi_j}q_{ij}\int_{0^+}^{L}\int_{0}^{x}\psi_i(u)r_j'\left(p(x,h_j)\right)\ dudx \tag{37}$$
$$= -\frac{\pi_i}{\pi_j}q_{ij}\int_{0^+}^{L}\int_{u}^{L}\psi_i(u)r_j'\left(p(x,h_j)\right)\ dxdu \tag{38}$$
$$= -\frac{\pi_i}{\pi_j}q_{ij}\int_{0^+}^{L}\psi_i(x)\Big[\int_{x}^{L} r_j'\left(p(u,h_j)\right)\ du\Big]\ dx, \tag{39}$$

where in (38) we changed the order of the double integral, and in (39) we interchanged the roles of dummy variables. Following the same approach, we reformulate the second integral of (36) as below

$$\int_{0^+}^{L}\Psi_{ii}(x)r_i'\left(p(x,h_i)\right)\ dx$$
$$= \int_{0^+}^{L}\psi_i(x)\Big[\int_{x}^{L} -q_{ii}r_i'\left(p(u,h_i)\right)\ du\Big]\ dx$$
$$+\int_{0^+}^{L}\psi_i(x)\Big[\int_{x}^{L}\lambda e^{-\zeta(u-x)}r_i'\left(p(u,h_i)\right)\ du\Big]\ dx. \tag{40}$$

Replacing (39) and (40) in (36) leaves us

$$\int_{0^+}^{L}\psi_i(x)\Bigg[\lambda\int_{x}^{L} e^{-\zeta(u-x)}r_i'\left(p(u,h_i)\right)\ du \tag{41}$$
$$-\int_{x}^{L}\Big(q_{ii}r_i'\left(p(u,h_i)\right) + \sum_{j\in[M]/\{i\}} q_{ij}r_j'\left(p(u,h_j)\right)\Big)\ du$$
$$+ r_i\left(p(x,h_i)\right) - p(x,h_i)r_i'\left(h_i, p(x,h_i)\right)\ du\Bigg]\ dx = 0,$$

where we dropped the common term π_i. Equation (41) must be valid for all perturbation functions $\psi_i(x)$ that conform to the boundary conditions as well as the relations in (22). Therefore, based on the fundamental lemma of the calculus of variations, (41) holds only if terms inside the bracket is a constant, say $-K_i$. The Euler-Lagrange differential equation then is given by

$$\lambda\int_{x}^{L} e^{-\zeta(u-x)}r_i'\left(p(u,h_i)\right)\ du$$
$$+\int_{x}^{L}\Big(q_{ii}r_i'\left(p(u,h_i)\right) + \sum_{j\in[M]/\{i\}} q_{ij}r_j'\left(p(u,h_j)\right)\Big)\ du$$
$$+ r_j\left(p(x,h_j)\right) - p(x,h_i)r_i'\left(p(x,h_i)\right) + K_i = 0. \tag{42}$$

We multiply all terms of (42) by $e^{-\zeta x}$, and then take derivative with respect to x. After some simplifications we derive

$$p'(x,h_i)p(x,h_i)r_i''\left(p(x,h_i)\right) + (\lambda-\zeta p(x,h_i))r_i'\left(p(x,h_i)\right)$$
$$+\zeta r_i(p(x,h_i)) + \sum_{j\in[M]\setminus\{i\}} q_{ij}\Big(r_i'\left(p(x,h_i)\right) - r_j'\left(p(x,h_j)\right)\Big)$$
$$-\sum_{j\in[M]\setminus\{i\}}\zeta q_{ij}\int_{0}^{x}\Big(r_i'\left(p(u,h_i)\right) - r_j'\left(p(u,h_j)\right)\Big)\ du + \hat{K}_i = 0, \tag{43}$$

for some new constant $\hat{K}_i$. We accordingly obtain a system of autonomous integro-differential equation with $2M$ degree of freedoms, namely $\{\hat{K}_i\}_{i=1}^{M}$, and $\{p(0,h_i)\}_{i=1}^{M}$.[1] In the sequel, we provide numerical solutions in the case of $M=2$.

V. Numerical Solutions

We now consider a Gilbert-Elliot channel with two channel states: a bad channel state (b) with corresponding square gain of $|h_{\mathsf{b}}|^2 = 0.5$, and a good channel state (g) with $|h_{\mathsf{g}}|^2 = 1$. We also consider the storage capacity of $L = 3$, energy arrival rate of $\lambda = 1$, the energy distribution parameter of $\zeta = 1$, and the transition rates $-q_{\mathsf{bb}} = -q_{\mathsf{gg}} = q_{\mathsf{bg}} = q_{\mathsf{gb}} = 1$. Furthermore, we use Shannon's rate function $r(p|h) = \log_2(1+|h|^2 p)$.

To compute the average rate from (10), we need to determine the densities $f(x|h_{\mathsf{g}})$ and $f(x|h_{\mathsf{b}})$ as functions of the power policies $p(x,h_{\mathsf{g}})$ and $p(x,h_{\mathsf{b}})$. To do so, we notice from (7) that $f(x|h_{\mathsf{g}}) = G_{\mathsf{g}}(x)/p(x,h_{\mathsf{g}})$, and $f(x|h_{\mathsf{b}}) = G_{\mathsf{b}}(x)/p(x,h_{\mathsf{g}})$ where $G_{\mathsf{g}}(x)$ and $G_{\mathsf{b}}(x)$ are both defined according to (8). Taking the derivative with respect to x, we obtain a system of ODEs for $f(x|h_{\mathsf{g}})$ and $f(x|h_{\mathsf{b}})$ that can be solved numerically.

Fig. 1 shows the transmission power policies for good and bad channel states based on the set of ODEs in (43), and for a choice of (non-optimized) initial conditions $p(0^+,h_{\mathsf{g}}) = p(0^+,h_{\mathsf{b}}) = 0.001$, $p'(0^+,h_{\mathsf{g}}) = 1.7\times 10^3$, and $p(0^+,h_{\mathsf{b}}) = 0$. From the figure it can be observed that both transmission power policies have an increasing behavior in terms of available charge in the battery. This is due to the fact that as the battery charge becomes large, there is insufficient room

[1] Alternatively, by taking another derivative from (43), we obtain a system of second order ODEs where $p(x,h_i)|_{x=0^+}$ and $p'(x,h_i)|_{x=0^+}, i = 1,2,\cdots,M$ are the degree of freedoms.

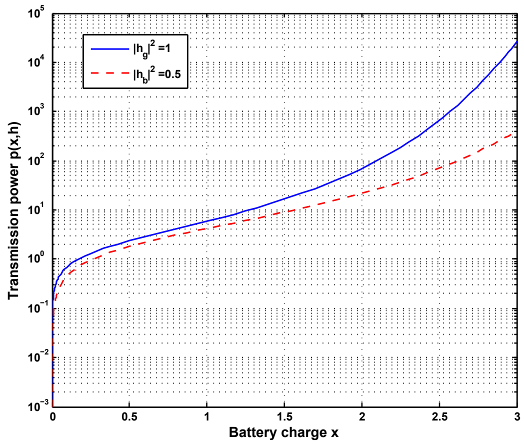

Fig. 1. Transmission power policies $p(x,h_\mathrm{g})$, $p(x,h_\mathrm{b})$ with initial values $p(x,h_\mathrm{g})|_{x=0^+} = p(x,h_\mathrm{b})|_{x=0^+} = 0.001$ and $p'(x,h_\mathrm{b})|_{x=0^+} = 0$, and $p'(x,h_\mathrm{g})|_{x=0^+} = 1.7 \times 10^3$.

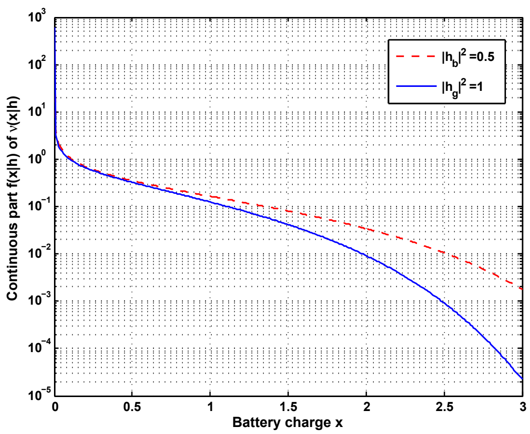

Fig. 2. Absolutely continuous part $f(x|h)$ of probability measure $\nu(x|h)$ for $h = h_\mathrm{g}, h_\mathrm{b}$, where $p(x,h_\mathrm{g})|_{x=0^+} = p(x,h_\mathrm{b})|_{x=0^+} = 0.001$ and $p'(x,h_\mathrm{b})|_{x=0^+} = 0$, and $p'(x,h_\mathrm{g})|_{x=0^+} = 1.7 \times 10^3$.

for energy arrivals and thus they may be lost. Under these circumstances, by allocating a large transmission power, the potential for overflow in the battery is partly mitigated.

Corresponding to power policies in Fig. 1 and for the same set of initial conditions, Fig. 2 shows the absolutely continuous parts of the probability measures for both the good and the bad channel states. Clearly, as the transmission power is made larger, densities decreases, i.e., the battery spends little time with large stored energy.

Table I shows the average channel throughput in conjunction with different values of $p(x,h_\mathrm{b})|_{x=0^+}$, and fixed initial values of $p(x,h_\mathrm{g})|_{x=0^+} = 10^{-3}$, $p'(x,h_\mathrm{g})|_{x=0^+} = 1.2 \times 10^3$, $p'(x,h_\mathrm{b})|_{x=0^+} = 0$. The upper bound is also calculated from (17) where $\pi = (1/2, 1/2)$ is the stationary measure for the channel states, and the allotted powers to the bad channel and good channels states by the water-filling strategy are

$$P_W(h_\mathrm{b}) = \frac{1}{2}\left(P_{\text{upper}} + (|h_\mathrm{b}|^2 - |h_\mathrm{g}|^2)/|h_\mathrm{g}h_\mathrm{b}|^2\right)^+, \tag{44}$$

and $P_W(h_\mathrm{g}) = P_{\text{upper}} - P_W(h_\mathrm{b})$, respectively. Decreasing

TABLE I

TOTAL AVERAGE THROUGHPUT FOR FIXED $p(x,h_\mathrm{g})|_{x=0^+} = 0.001$, $p'(x,h_\mathrm{g})|_{x=0^+} = 1.2 \times 10^3$, $p'(x,h_\mathrm{b})|_{x=0^+} = 0$, AND DIFFERENT VALUES OF $p(x,h_\mathrm{b})|_{x=0^+} = p_\mathrm{b}$. THE UPPER BOUND IS $R_{\text{WATER-FILLING}} = 0.4818$.

	Initial Value p_b			
	0.001	0.01	0.1	1
R_g	0.4908	0.4968	0.4754	0.5622
R_b	0.2303	0.2240	0.2110	0.1082
R_{total}	0.3605	0.3604	0.3432	0.3352

$p(x,h_\mathrm{b})|_{x=0^+}$ (while fixing other parameters) increases the overall channel throughput as well as the throughput in the bad channel state. A similar effect can be shown for the case of fixed $p(x,h_\mathrm{b})|_{x=0^+}$ and changing $p(x,h_\mathrm{g})|_{x=0^+}$. This observation can be justified by the fact that decreasing the transmission power when the battery happens to have small stored charge extends the transmission time. By concavity of the rate function, this in turn increases the channel throughput.

VI. Conclusion

In this paper, we have studied transmission power policies for online energy harvesting in fading wireless channels. We adapted a finite-state Markov process to model the channel fading state, as well as a storage dam process to model the dynamics of the battery. Using a water-filling scheme, we established an upper bound on the average channel throughput. We also derived structure on good power policies in the form of a system of autonomous ODEs. For a Gilbert-Elliot channel model, we solved these ODEs using standard numerical methods.

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References

[1] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications powered by energy harvesters," in *Proc. IEEE International Symposium on Information Theory (ISIT), Texas*, 2010.

[2] A. Fu, E. Modiano, and J. Tsitsiklis, "Optimal transmission scheduling over a fading channel with energy and deadline constraints," *IEEE Tran. Wireless Commun.*, vol. 5, no. 3, pp. 630–641, 2006.

[3] M. Zafer and E. Modiano, "Optimal rate control for delay-constrained data transmission over a wireless channel," *IEEE Trans. Inform. Theory*, no. 9, pp. 4020–4039, 2008.

[4] M. Khoshnevisan and J. Laneman, "Minimum delay communication in energy harvesting systems over fading channels," in *Proc. 45th Annual Conference on Information Sciences and Systems (CISS), Baltimore*, 2011.

[5] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Select. Areas Commun.*, vol. 29, pp. 1732 – 1742, Sept. 2011.

[6] H. S. Wang and N. Moayeri, "Finite-state markov channel-a useful model for radio communication channels," *IEEE Transaction on Vehicular Technology*, vol. 44, pp. 163–171, Feb. 1995.

[7] S. Asmussen, *Applied Probability and Queues*. Wiley, 1987.

[8] F. Guillemin and R. Mazumdar, "Rate conservation laws for multidimensional processes of bounded variation with applications to priority queuing systems," *Methodology and Computing in Applied Probability*, pp. 135–159, Oct. 2004.

[9] F. J. Piera, R. Mazumdar, and F. M. Guillemin, "Boundary behavior and product-form stationary distributions of jump diffusions in the orthant with state-dependent reflections," *Advances in Applied Probability*, vol. 40, no. 2, pp. 529–547, 2008.

[10] M. Badiei and P. Mitran, "On online energy harvesting in multiple access communication systems," Oct. 2012. (arXiv:1301.1027).