Compute-and-Forward: Multiple Bi-directional Sessions on the Line Network

Zhijie Ren*, Jasper Goseling†*, Jos H. Weber* and Michael Gastpar‡*
* Department of Intelligent Systems,
Delft University of Technology, The Netherlands
† Stochastic Operations Research,
University of Twente, The Netherlands
‡ Laboratory for Information in Networked Systems,
Ecole Polytechnique Fédérale de Lausanne, Switzerland
z.ren@tudelft.nl, j.goseling@utwente.nl, j.h.weber@tudelft.nl, michael.gastpar@epfl.ch

***Abstract*—Signal superposition and broadcast are important features of the wireless medium. Compute-and-Forward, also known as Physical Layer Network Coding, is a technique exploiting these features in order to improve performance of wireless networks. In this paper, the possible benefits for the line network with multiple bi-directional sessions and local interference are investigated. Four different modes, indicating whether or not broadcast and/or superposition are exploited, are considered. In particular, expressions for the maximum achievable common rate are derived for each of the four different modes. Scheduling and coding schemes achieving these rates are presented. From the results it follows that, in most cases, the common rate is improved by a factor close to two by using Compute-and-Forward. However, it is also found that the benefit may be smaller for particular session configurations.**

I. Introduction

Compute-and-Forward, also known as Reliable Physical Layer Network Coding (PLNC) [1], is a novel Network Coding (NC) technique for wireless networks. Uncoded versions of PLNC have been considered in the literature, see e.g. [2], [3], suffering from noise accumulation along the stages of the network. By constrast, Compute-and-Forward works with codes in such a way as to allow nodes to efficiently and reliably recover a function of the messages from multiple senders. A natural question concerns the performance gains enabled by Compute-and-Forward in larger networks. The goal of the present paper is to shed light on this question by studying a simple line network under arbitrary configurations of bi-directional unicast sessions.

The line network scenario has been widely studied to understand the benefits of network coding, starting with [4] which applies NC in the presence of broadcast, and succeeding by [1] [2] [5] which apply PLNC with superposition. Some successive studies are [6], which obtains tight bounds for the throughput on the local interference line network model in terms of Transport Capacity (Rate $\times$ Distance), and [7], which obtains the bounds for the throughput on the line network with a more general interference configuration. Comparisons between the throughput for multiple unicast with and without NC are presented in [8]. Other related papers are [9], which presents general bounds for NC on the ring network, and [10], which gives a general upper bound for the improvement of the throughput of multiple unicasts by PLNC.

In this paper, we study the capacity of line networks with arbitrary configurations of bi-directional unicast sessions. To obtain compact results, we specifically consider the *common rate* scenario, i.e., the case where all sessions have the same rate. We seek to characterize the maximum common rate for four different transmission modes. These four modes correspond to whether or not the features of broadcast and/or superposition are exploited. We characterize the common rate in terms of so-called “bottle-neck” nodes, more precisely, the number of sessions that these “bottle-neck” nodes involve and whether these nodes are terminal nodes for sessions. Our main contributions are expressions for the maximum common rates, together with the schemes achieving these rates, under four different modes of the physical layer behavior.

Our results extend several existing studies. For example, if we specialize to the case of a single bi-directional session that spans the entire line network, we recover [6]; when the number of sessions is approaching infinity, we recover the improvements that were found in [11]; and our results are also consistent with [7].

This paper is organized as follows. In Section II, we set up our model and give important definitions. Then, in Section III, we give our main results, i.e., expressions for the maximum achievable common rates, as well as two examples with illustrations. Next, we prove our results by providing an upper bound on the common rate in Section IV and feasible schemes achieving this bound in Section V. We conclude with a discussion in Section VI.

II. Model Set-up and Notations

A. Local Interference Line Network Model

We consider a line topology modeled as a directed graph $(\mathcal{V}, \mathcal{E})$, with nodes $\mathcal{V} = \{1, 2, \ldots, M\}$ with unit distance, and edges $\mathcal{E} = \{(u, v)|u, v \in \mathcal{V}, |u - v| = 1\}$. Then, we build a communication model upon this topology, by considering the nodes in $\mathcal{V}$ as wireless devices. We assume time is slotted and transmitted messages are symbols from $\mathbb{F}(q) \cup \sigma$, where σ denotes an empty transmission. Let $X_t(u)$ and $Y_t(u)$ denote the transmitted and received messages, respectively, for node u in time slot t, and $A_t(u,v)$ the transmitted message on the directed edge (u,v) in time slot t. The capacity of each edge is one symbol per time slot. We assume half-duplex constraints, i.e., a node cannot both transmit and receive in the same time slot. If node u is not transmitting in time slot t then $X_t(u) = \sigma$. For notational convenience, we sometimes use the symbol τ to denote a uniformly distributed random variable from $\mathbb{F}(q)$ which is useless to the receiving node. With the assumption that the reliable NC and PLNC schemes are applied, we consider a local interference model in which the broadcast and superposition properties are now characterized as follows. For convenience, we define the nodes $\{m | m \le 0 \vee m \ge M+1\}$ as virtual nodes which are always silent.

No Broadcast: For any u and t it holds that

$$\begin{array}{l} A_t(u,u-1) = A_t(u,u+1) = X_t(u) = \sigma \text{ or} \\ A_t(u,u-1) = X_t(u) \neq \sigma \text{ and } A_t(u,u+1) = \tau \text{ or} \\ A_t(u,u+1) = X_t(u) \neq \sigma \text{ and } A_t(u,u-1) = \tau. \end{array} \quad (1)$$

Broadcast: For any u and t it holds that

$$A_t(u,u-1) = A_t(u,u+1) = X_t(u). \quad (2)$$

No Superposition: For any u and t it holds that

$$Y_t(u) = \begin{cases} A_t(u-1,u) & \text{if } A_t(u+1,u) = \sigma, \\ A_t(u+1,u) & \text{if } A_t(u-1,u) = \sigma, \\ \tau & \text{if } X_t(u-1) \neq \sigma \\ & \text{and } X_t(u+1) \neq \sigma. \end{cases} \quad (3)$$

Superposition: For any u and t it holds that

$$Y_t(u) = A_t(u-1,u) + A_t(u+1,u), \quad (4)$$

where the addition is in $\mathbb{F}(q)$, with the additional rules that $X + \sigma = X$ for any symbol X, and that $X + \tau = \tau'$, for any X and τ, where τ' is, like τ, a uniformly distributed random variable from $\mathbb{F}(q)$ which is useless to the receiver.

As in [6], we consider the following four modes of operation:

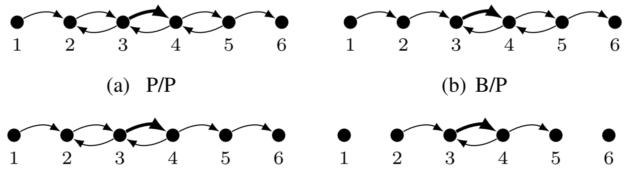
P/P (Point-to-Point):	neither broadcast nor superposition,
B(roadcast)/P:	broadcast, but no superposition,
P/M(ulti-access):	superposition, but no broadcast,
B/M:	both broadcast and superposition.

(5)

From (1)–(4), we obtain the following properties, which give negative implications of useful communication over the link $(u, u+1)$ on the communication over other links, for these four modes. By symmetry, similar properties also hold for the link $(u+1, u)$.

P/P mode:

$$\begin{array}{l} \text{If } A_t(u,u+1) \notin \{\tau,\sigma\}, \text{ and } Y_t(u+1) \neq \tau, \text{ then} \\ \quad A_t(u-1 \pm 1, u-1) \in \{\tau,\sigma\}, A_t(u-1,u) = \{\tau,\sigma\}, \\ \quad A_t(u+1,u+1 \pm 1) = \sigma, A_t(u+2,u+2 \pm 1) = \sigma. \end{array} \quad (6)$$



(a) P/P (b) B/P

(c) P/M (d) B/M

Fig. 1. Illustration of constraints for the four modes: useful communication on the thick edges implies that no useful communication is possible on the thin edges.

B/P mode:

$$\begin{array}{l} \text{If } A_t(u,u+1) \notin \{\tau,\sigma\}, \text{ and } Y_t(u+1) \neq \tau, \text{ then} \\ \quad A_t(u-2, u-1) \in \{\tau,\sigma\}, A_t(u-1,u) \in \{\tau,\sigma\}, \\ \quad A_t(u+1,u+1 \pm 1) = \sigma, A_t(u+2,u+2 \pm 1) = \sigma. \end{array} \quad (7)$$

P/M mode:

$$\begin{array}{l} \text{If } A_t(u,u+1) \notin \{\tau,\sigma\}, \text{ and } Y_t(u+1) \neq \tau, \text{ then} \\ \quad A_t(u-1 \pm 1, u-1) \in \{\tau,\sigma\}, A_t(u-1,u) \in \{\tau,\sigma\}, \\ \quad A_t(u+1,u+1 \pm 1) = \sigma, A_t(u+2,u+3) \in \{\tau,\sigma\}. \end{array} \quad (8)$$

B/M mode:

$$\begin{array}{l} \text{If } A_t(u,u+1) \notin \{\tau,\sigma\}, \text{ and } Y_t(u+1) \neq \tau, \text{ then} \\ \quad A_t(u-1,u) \in \{\tau,\sigma\}, A_t(u+1,u+1 \pm 1) = \sigma. \end{array} \quad (9)$$

The modes are illustrated in Figure 1.

B. Multiple Bi-directional Sessions and Common Rate

For $n = 1, 2, \ldots, N$, let b_n denote a bi-directional session, defined by $\{T_n^L, T_n^R\}$, where T_n^L and $T_n^R > T_n^L$ are its left and right terminal nodes, respectively, which exchange messages. The nodes in between the terminals act as relays for this session. We will use the following notation:

$$\begin{array}{l} \mathcal{B} = \{b_1, b_2, \cdots, b_N\}, \\ \mathcal{T}^L = \{T_1^L, T_2^L, \cdots, T_N^L\}, \\ \mathcal{T}^R = \{T_1^R, T_2^R, \cdots, T_N^R\}, \\ \mathcal{T} = \mathcal{T}^L \cup \mathcal{T}^R, \\ S_m = |\{b_n \in \mathcal{B} | T_n^L \le m \le T_n^R\}|. \end{array}$$

We assume any two nodes in $\mathcal{T}$ are distinct. Note that S_m represents the number of sessions in which node m is involved. A local interference line network model with M nodes and bi-directional sessions $\mathcal{B}$ is denoted as $L^{mode}(M, \mathcal{B})$, where $mode \in \{pp, bp, pm, bm\}$ as defined in (5).

The *rate* of a session in one direction is the long-term ratio of the number of successfully retrieved symbols at the receiving terminal and the number of time slots used. If all sessions communicate with the same rate R in both directions, we call R the *common rate* of $L^{mode}(M, \mathcal{B})$.

III. Main Results

The main results of this paper are expressions for the maximum achievable common rate on $L^{mode}(M, \mathcal{B})$ for all modes.

Theorem 1: Let

$$C^{pp} = \min\left\{ \min_{m \notin \mathcal{T} : S_m \neq 0} \frac{1}{4S_m}, \min_{m \in \mathcal{T}} \frac{1}{4S_m - 2} \right\}, \quad (10)$$

$$C^{bp} = C^{pm} = \min\left\{\min_{m \notin \mathcal{T}: S_m \neq 0} \frac{1}{3S_m}, \min_{m \in \mathcal{T}} \frac{1}{3S_m - 1}\right\}, \tag{11}$$

and

$$C^{bm} = \min_{m \in \mathcal{V}: S_m \neq 0} \frac{1}{2S_m}. \tag{12}$$

Then, for any $mode \in \{pp, bp, pm, bm\}$, M, and $\mathcal{B}$, the rate C^{mode} specified by (10)–(12) is the maximum achievable common rate on $L^{mode}(M, \mathcal{B})$.

Note that it follows from Theorem 1 that the maximum common rate is determined by so-called "bottle-neck" nodes, i.e., nodes m for which S_m is maximum, and that for the P/P, B/P, and P/M modes, its value does not only depend on this maximum S_m, but also whether or not it is achieved for a non-terminal node.

This result will be proved in the next two sections, by first providing an upper bound on the common rate of a line network (Lemma 1) and then providing a matching lower bound (Lemma 2) by presenting a scheduling and coding scheme achieving the upper bound.

Define the improvement factor $I_{m'}^{m''}$ of mode m'' over mode m' as $C^{m''}/C^{m'}$. For example, I_{pp}^{bm} is the common rate improvement obtained by applying PLNC, i.e., by exploiting both broadcast and superposition in comparison to exploiting neither feature. Note that it follows from (10)–(12) that

$$\begin{aligned} I_{pp}^{bm} &\leq 2, && (13)\\ I_{pp}^{bp} = I_{pp}^{pm} &\leq 4/3, && (14)\\ I_{bp}^{bm} = I_{pm}^{bm} &\leq 3/2, && (15) \end{aligned}$$

where the improvement factors are close to the upper bounds in case the network contains nodes m with large S_m values. However, the improvement factors may be considerably smaller in case of small maximum S_m values which are only attained for terminal nodes. Next, we give examples providing some numerical results and illustrations.

Example 1: Consider $L^{mode}(4, \{\{1, 3\}, \{2, 4\}\})$, i.e., the line network with four nodes and two bi-directional sessions: one between nodes 1 and 3 and the other between nodes 2 and 4. By (10)–(12), it is easy to calculate that $C^{pp} = 1/6$, $C^{bp} = C^{bm} = 1/5$ and $C^{bm} = 1/4$. It can be concluded that for this network exploiting both broadcast and superposition gives an improvement by a factor $I_{pp}^{bm} = 1.5$, while exploiting either of these features only gives a factor $I_{pp}^{bp} = I_{pp}^{pm} = 1.2$.

Example 2: Next, we consider, for fixed N and $M \geq 2N$, the average improvement factors over all possible configurations of N bi-directional sessions on a line network with M nodes. We focus on I_{pp}^{bm}, but results for other cases could be obtained in a similar way. It follows from (10) and (12) that I_{pp}^{bm} equals 2 except in the case that the "bottle-neck" nodes are terminals. By combinatorial arguments, we find that the average improvement factor is

$$\bar{I}_{pp}^{bm} = \begin{cases} 2 - \frac{2}{M}, & \text{if } N = 1, \\ 2 - \frac{4}{3M} - \frac{4}{M(M-1)}, & \text{if } N = 2. \end{cases} \tag{16}$$

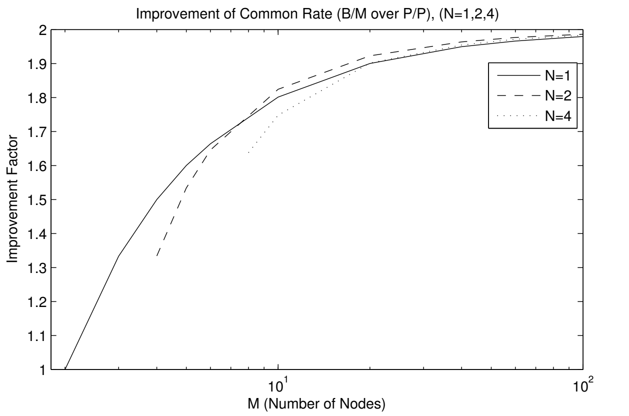


Fig. 2. Average improvement factor I_{pp}^{bm} of the common rate by applying PLNC, in case of $N = 1, 2, 4$ sessions, $2N \leq M \leq 100$ nodes, and uniformly distributed terminal nodes.

For larger values of N, the calculations become cumbersome. We run simulations for $N = 4$ and $8 \leq M \leq 100$, the results of which are depicted in Figure 2, together with the results of (16). Note that $\bar{I}_{pp}^{bm}$ is smallest if $M = 2N$, i.e., in case all nodes are terminals. For large values of M, the average improvement factor approaches 2, which can be observed from both (16) and Figure 2. Other simulations, with $M = 100$ and $1 \leq N \leq 50$, also show average improvement factors very close to 2.

IV. Upper Bounds

In this section, we present a general upper bound on the common rate by using the constraints listed in (6)–(9).

Lemma 1: For any $mode \in \{pp, bp, pm, bm\}$, M, and $\mathcal{B}$, the common rate on $L^{mode}(M, \mathcal{B})$ is upper bounded by the rate C^{mode} specified by (10)-(12).

Proof: For any scheduling scheme, define

$$R(u, v) = \frac{|\{t \in \{1, 2, \ldots, U\} : A_t(u, v) \notin \{\tau, \sigma\}\}|}{U}, \tag{17}$$

where U is the total number of time slots. Furthermore, we use the notation $\mathcal{E}_m$ for the set of all edges starting or ending in node m.

Assume the network $L^{mode}(M, \mathcal{B})$ has a common rate of R, then for all nodes $m \notin \mathcal{T}$, it holds that

$$S_m R \leq R(u, v) \tag{18}$$

for all edges $(u, v) \in \mathcal{E}_m$. This inequality also holds for the edges $(u, v) \in \{(m-1, m), (m, m-1)\}$ with $m \in \mathcal{T}^R$, and for the edges $(u, v) \in \{(m+1, m), (m, m+1)\}$ with $m \in \mathcal{T}^L$. However, for the edges $(u, v) \in \{(m+1, m), (m, m+1)\}$ with $m \in \mathcal{T}^R$ and $m < M$, and for the edges $(u, v) \in \{(m-1, m), (m, m-1)\}$ with $m \in \mathcal{T}^L$ and $m > 1$, it holds that

$$(S_m - 1)R \leq R(u, v). \tag{19}$$

Since the following reasoning is similar for all four modes, we only consider the B/P mode. For any node m, we obtain new inequalities by summing up the inequalities for one edge starting from node m and all edges ending in node m. Due to the B/P mode constraint indicated in (7), the RHS of the new inequality cannot be larger than 1. Hence, for any node $m \notin \mathcal{T}$ and $S_m \neq 0$, we have an upper bound on the common rate $R \leq 1/(3S_m)$. For any node $m \in \mathcal{T}$, in which case two inequalities are obtained depending on which edge starting from m is chosen for summation. We choose the tighter upper bound $R \leq 1/(3S_m - 1)$. ■

V. Lower Bounds

In this section, we prove that the upper bound from Lemma 1 can be achieved.

Lemma 2: For any $mode \in \{pp, bp, pm, bm\}$, M, and $\mathcal{B}$, the common rate R on $L^{mode}(M, \mathcal{B})$ is achievable when $R \leq C^{mode}$ specified by (10)–(12).

This lemma can be proved by giving feasible scheduling and coding schemes. In Subsection V-A, the definition and properties of the *load factor*, which will be used for scheduling purposes, are given. Then, an optimal scheduling scheme and an intra-session coding scheme will be given in Subsections V-B and V-C, respectively. It will be shown that the common rates achieved by our scheduling and coding schemes reach the upper bounds. Hence, we also prove that inter-session coding will not be beneficial in this scenario.

A. Load Factor

We define the load factor of node m for the various modes as

$$F_m^{pp} = \begin{cases} 4S_m & m \notin \mathcal{T} \\ 4S_m - 2 & m \in \mathcal{T} \end{cases}, \qquad (20)$$

$$F_m^{bp} = F_m^{pm} = \begin{cases} 3S_m & m \notin \mathcal{T} \\ 3S_m - 1 & m \in \mathcal{T} \end{cases}, \qquad (21)$$

$$F_m^{bm} = 2S_m. \qquad (22)$$

Also, we define the maximum load factor

$$F_{max}^{mode} = \max_{m=1}^{M} F_m^{mode} \qquad (23)$$

Furthermore, we use

$$\mathcal{K}_n = \{m \in \mathcal{V} | T_n^L \leq m \leq T_n^R\},$$

and

$$\mathcal{J}_m = \{b_n \in B | m \in K_n\},$$

for the set of all nodes that are involved in session b_n and the set of all sessions that involve node m, respectively.

The following lemma shows important properties of the load factor for the three modes P/P, B/P, and P/M, in which cases the maximum load factor does not only depend on the largest S_m value, but also on whether or not it is achieved for a non-terminal node.

Lemma 3: For any $mode \in \{pp, bp, pm\}$, the following conditions hold.

(a) If node $m \in \mathcal{T}^L$ and $F_m^{mode} = F_{max}^{mode}$, then node $m+1 \in \mathcal{T}^R$ and $F_{m+1}^{mode} = F_{max}^{mode}$.

(b) Under Condition (a), if $F_{m-1}^{mode} = F_{max}^{mode}$, then node $m-1 \in \mathcal{T}^R$.

(c) If node $m \in \mathcal{T}^R$ and $F_m^{mode} = F_{max}^{mode}$, then node $m-1 \in \mathcal{T}^L$ and $F_{m-1}^{mode} = F_{max}^{mode}$.

(d) Under Condition (c), if $F_{m+1}^{mode} = F_{max}^{mode}$, then node $m+1 \in \mathcal{T}^L$.

This lemma can easily be induced by the definition of the load factor (20)–(22) and the assumption that any node can only be the terminal of at most one session. We omit the proof due to space constraints.

B. Scheduling Schemes

We reconsider the notation $A_t(u, v)$ for the transmitted message on the link from u to v in time slot t, defined in Subsection II-A. In this subsection, we frequently use the phrase "assign transmission $A_t(u, v)$ to session b_n", which means that the link from u to v will be used for communication related to session b_n in time slot t.

Because of the similarities of the scheduling scheme for P/P, B/P and P/M modes (due to the same properties claimed by Lemma 3), we only show the scheduling schemes for B/P mode and B/M mode for the sake of space. By applying the scheme, schedules with the following properties will be obtained.

Property 1: The schedules are specified by a round with F_{max}^{mode} time slots which can be repetitively proceeded.

Property 2: The chosen transmissions in each time slot will not violate the constraints in (6)–(9).

Property 3: The transmissions are assigned to sessions in such a way that for all bi-directional sessions, all involved relay nodes will have 2 transmissions to relay the messages for both directions, and the terminal nodes will have 1 transmission for transmitting the original message.

1) B/P Mode: We distinguish two cases.

Case I: F_{max}^{bp} is achieved for node $m \notin \mathcal{T}$.

Assume b_n is the k-th element of $\mathcal{J}_m$. We assign the transmissions $A_{3k-2+(m-1) \pmod 3}(m, m-1)$ and $A_{3k-2+(m-1) \pmod 3}(m, m+1)$ to session b_n. Clearly, the maximum value of t for any $A_t(u, v)$ assigned this way will be $3S_{max}$.

Case II: F_{max}^{bp} is achieved for node $m \in \mathcal{T}$.

We first schedule the transmissions for those terminal nodes which achieve F_{max}^{bp}. For the nodes $m = T_n^L$, if $m \equiv 0$ or $1(4)$, assign $A_{3S_{max}-2}(m, m+1)$ to session b_n, else assign $A_{3S_{max}-1}(m, m-1)$ to session b_n. Then, for the nodes $m = T_n^R$, if $m \equiv 0$ or $3(4)$, assign $A_{3S_{max}-2}(m, m-1)$ to session b_n, else assign $A_{3S_{max}-1}(m, m-1)$ to session b_n. These assignments will not violate (7), due to the features listed out in Lemma 3. Then, for all of these nodes $m \in \mathcal{T}, F_m^{bp} = F_{max}^{bp}$, we remove b_n from $\mathcal{J}_m$, where $m = T_n^L$ or T_n^R.

Next, for all the nodes, we do the same assignment as in Case I, with the exception of those transmissions which have already been scheduled. Since all the nodes with $F_m^{bp} < F_{max}^{bp}$ will have $S_m \leq S_{max} - 1$, and we have already scheduled two time slots for the terminal nodes with $S_m = S_{max}$, the maximum value of t for any $A_t(u, v)$ assigned by the same

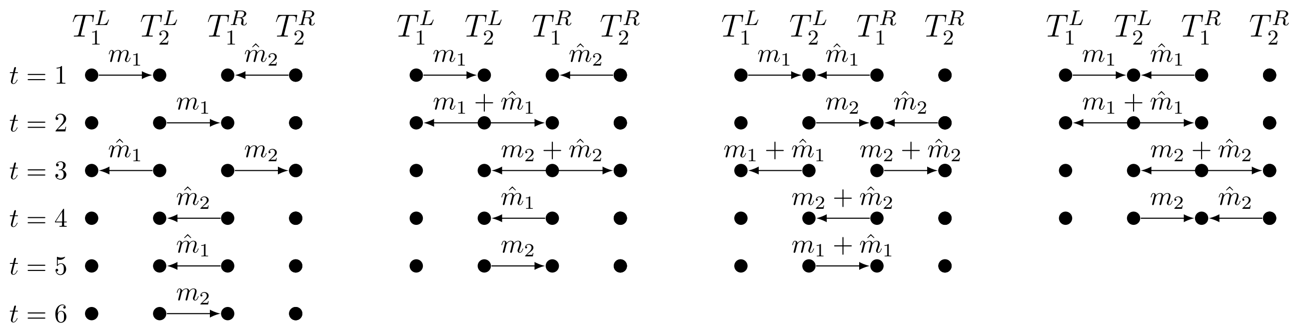


Fig. 3. Scheduling and coding schemes within one round for the line network $L^{mode}(4, \{\{1,3\},\{2,4\}\})$, where m_n and $\hat{m}_n$ represent the messages from left and right terminals, respectively, for session n. All nodes transmit the messages that are obtained from the previous round.

method as Case I will be $3S_{max} - 3$, which make the number of total time slots $3S_{max} - 1$.

2) B/M Mode: We first define two types of schedules:

Type A: assign transmissions $A_{t+(m-1) \pmod 2}(m, m-1)$ and $A_{t+(m-1) \pmod 2}(m, m+1)$ to session b_n;

Type B: assign transmissions $A_{t+m \pmod 2}(m, m-1)$ and $A_{t+m \pmod 2}(m, m+1)$ to session b_n.

We start the scheduling in B/M mode with any session b_n, find the smallest t for which $X_t(m) = \sigma, \forall m \in \mathcal{K}_n$, and schedule these nodes according to Type A. Then, find the neighboring sessions b_l (which $T_l^R+1 = T_n^L$ or $T_l^L = T_n^R+1$) for session b_n, and schedule these nodes in $\mathcal{K}_l$ according to Type B. Then, we iteratively find the neighboring unscheduled sessions of all the scheduled ones, and switch scheduling type (Type A and Type B) so that there will not be any pair of neighboring sessions scheduled with the same type. Hence, there will be no collision at the terminal nodes.

Then we repeat this process until all sessions have been scheduled. It is a simple coloring problem and the total number of time slots being scheduled for any node will be $2S_{max}$.

C. Coding Schemes

Since we have the schedule with Property 3, the piggy-backing based classical Linear Network Coding (LNC) and PLNC schemes can be applied in the same fashion as those for a single bi-directional session. Hence, we present no further details on coding here due to space issues, details can be found in, e.g., [6]. In Figure 3, we show the scheduling and coding results for Example 1.

VI. Conclusions and Recommendations

In this paper, we have shown that the common rate for multiple sessions on the line network is determined by the so-called "bottle-neck" nodes, as well as whether these nodes are terminal nodes for sessions. We have given explicit expressions for the maximum achievable common rate under four modes which indicate whether or not broadcast and/or superposition are exploited. Scheduling schemes to achieve this common rate have been provided. These results can easily be extended to noisy line network configurations. A related study in the context of transport capacity can be found in [6].

A further study on the local interference line network could consider scenarios in which sessions may have asymmetric demands (e.g., uni-directional sessions), in which the intra-session network coding may no longer be optimal. Hence, inter-session coding scheme should be investigated. We conjecture that the improvement by applying Compute-and-Forward techniques will be smaller than the result for the symmetric case.

Acknowledgment

This work was supported by ERC Starting Grant 259530-ComCom and by NWO Grant 612.001.107.

References

[1] B. Nazer and M. Gastpar, "Reliable physical layer network coding," *Proc. IEEE*, vol 99, pp. 438-460, 20

[2] S. Zhang, S. Kiew, and P. Lam, "Hot topic: Physical-layer network coding," *Proc. 12th Annu. Int. Conf. Mobile Computing and Networking*, pp. 358-365, 2006.

[3] S. Katti, S. Gollakota and D. Katabi, "Embracing wireless interference: Analog network coding," *Proc. ACM SIGCOMM*, pp. 397-408, 2007.

[4] Y. Wu, P. A. Chou, and S. Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," Technical Report MSR-TR-2004-78, Microsoft Research, 2004.

[5] M. P. Wilson, K. Narayanan, H. D. Pfister and A. Sprintson, "Joint physical layer coding and network coding for bidirectional relaying," *IEEE Trans. Inf. Theory*, vol. 56, pp. 5641-54, 2010.

[6] J. Goseling, M. Gastpar, and J. H. Weber, "Line and lattice networks under deterministic interference models," *IEEE Trans. Inf. Theory*, vol. 57, pp. 3080-3099, 2011.

[7] K. Lu, S. Fu and Y. Qian, "Capacity of random wireless networks: impact of physical-layer network coding," *Proc. IEEE ICC*, pp. 3903-07, May 2008.

[8] Y. E. Sagduyu, A. Ephremides, "Cross-layer optimization of MAC and network coding in wireless queueing tandem networks," *IEEE Trans. Inf. Theory*, vol. 54, pp 554-571, 2008.

[9] S. M. S. T. Yazdi, S. A. Savari, G. Kramer, K. Carlson and F. Farnoud, "On the multimessage capacity region for undirected ring networks," *IEEE Trans. Inf. Theory*, vol. 56, pp. 1930-47, 2010

[10] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the throughput gain of network coding in unicast and multicast wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 582-592, 2009.

[11] S. Zhang and S. Liew, "Applying physical-layer network coding in wireless networks," *Eurasip J. Wirel. Commun. Netw.*, vol. 2012, Article No.1, 2010.