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Non-homogeneous Two-Rack Model for Distributed Storage Systems

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***Abstract*—In the traditional two-rack distributed storage system (DSS) model, due to the assumption that the storage capacity of each node is the same, the minimum bandwidth regenerating (MBR) point becomes infeasible. In this paper, we design a new non-homogeneous two-rack model by proposing a generalization of the threshold function used to compute the tradeoff curve. We prove that by having the nodes in the rack with higher regenerating bandwidth stores more information, all the points on the tradeoff curve, including the MBR point, become feasible. Finally, we show how the non-homogeneous two-rack model outperforms the traditional model in the tradeoff curve between the storage per node and the repair bandwidth.**

I. Introduction

Cloud storage has been consolidated as a growing paradigm, as it provides a convenience solution for online storage that is accessible with any device at anywhere and anytime.

To ensure reliability, in practice, cloud storage is implemented in terms of distributed storage system (DSS), where several geographically distributed storage nodes collaboratively to provide storage or backup services. Such distributed system provides diversity and achieves fault-tolerance against catastrophic failure, it also minimizes the probability of losing the stored data and maximizes the data availability.

Erasure coding has been proven in [1], [2] as an effective technique for such DSS. Through the use of erasure coding, fault tolerance level is improved and the size of stored data is minimized. Moreover, [3] shows that with the use of regenerating codes, not only achieves most of the improvements of erasure coding, but also minimizes the amount of data needed to regenerate a failed node. Since then, the theoretical and fundamental tradeoffs among the system resources, e.g. storage capacity and repair bandwidth, has been discovered. Several novel coding schemes, e.g. [4], [5], have been constructed to achieve the tradeoff curve in certain special points, e.g. minimum storage regenerating (MSR) and minimum bandwidth regenerating (MBR).

The previous theoretical results were assuming a symmetric and homogeneous model in terms of data storage and repair bandwidth. However, in a realistic implementation, not all nodes are equal in terms of storage size, repair bandwidth, or even reliability. By considering the difference in terms of repair bandwidth, [6] proposes a DSS model where there is a static classification of storage nodes based on their repair bandwidth, storage nodes are divided into two groups, one is “cheap bandwidth” and another “expensive bandwidth”.

To generalize the above static model, [9] considers that the storage nodes are organized in two racks. The repair bandwidth cost between nodes within the same rack is much lower than between nodes across different racks. This situation introduces a dynamic model, where the classification of “cheap/expensive bandwidth” falls on the relation between two nodes. The bandwidth between two nodes is “cheap” if both are from the same rack and “expensive” otherwise. Using this two-rack model, the authors in [9] have shown the tradeoff between bandwidth and storage with repair cost. In this paper, our focus is on such two-rack model due to its practical implication, for example, consider a DSS that spans across two countries, it can be easily modeled with two-rack model where the storage nodes within the same country enjoy “cheap bandwidth”, while the storage nodes across different countries have “expensive bandwidth”. Unfortunately, the authors in [9] show that it is infeasible to achieve the MBR point for such two-rack model.

While the previous models, e.g. the static model in [6] and the two-rack model in [9], have considered a DSS with different repair bandwidth among the storage nodes, all of them assume the storage nodes have the same storage capacity. Recent development have included the emergence of non-homogeneous DSS that pool together nodes with truly different characteristics, including the storage size. The capacity of such non-homogeneous DSS with different storage size and repair bandwidth has been studied in [8]. Coding scheme for a non-homogeneous storage system with one super-node that is more reliable and has more storage capacity is studied in [7].

In this paper, we show that by considering a non-homogeneous model, where all the nodes have different storage size and repair bandwidth, not only such model is closer to practical system, it also provides a solution to the problem of infeasible MBR point in the two-rack model mentioned above. We design a two racks DSS such that storage size at each node is depending on the repair bandwidth of each rack, and prove that such design can achieve the MBR point.

Our paper is organized as follows. In Section II we describe various DSS models. We start with the symmetric model used to explain the information flow graph. Then, we explain the static cost model because it is the first model presenting storage nodes with different repair bandwidth. Then, we introduce the two-rack model as a generalization of the static cost model. We start Section III by presenting the problem of the two-rack model on infeasible MBR point, and then propose our solution of creating a non-homogeneous two-rack model. Finally, we conclude the paper in Section IV.

II. Previous Models of DSS

In this section, we present three different models of DSS. In Subsection II-A we show the symmetric model, where the repair bandwidth and the storage size is the same for all the nodes. We will use this model to explain the information flow graph, which is essential for the readers to better understand our contribution at a later time. In Subsection II-B, we present a static cost DSS model, where the nodes are divided into two groups, namely cheap and expensive, based on their repair bandwidth. In this case, since the nodes are always cheap or expensive, no matter who is connecting to them, the repair bandwidth is always the same. This static cost model is a particular case of the two-rack model that will be presented in Subsection II-C. In the two-rack model, the cheap or expensive connection depends on the helper nodes and the newcomer. Hence, there are two different repair bandwidths. Figure 2 shows the differences between the three models. We will discuss each model in great details, as understanding them is the key to understand our contribution.

A. Symmetric Model

In [3], Dimakis et. al. first introduced a symmetric distributed storage model, where every storage node has the same storage size and the same repair bandwidth. As such the repair cost for every storage node is the same. Moreover, the fundamental tradeoff between the amount of stored data per node and the repair bandwidth can be obtained by analyzing the mincut of the information flow graph.

The information flow is a directed acyclic graph including three types of nodes: (i) A single source node (S), (ii) Some intermediate nodes and (iii) Data collectors (DC). The source node is the source of original data file, intermediate nodes are storage nodes and each data collector corresponds to a request to reconstructing the original file. Each storage node is represented by pairs of incoming and outgoing nodes connected by a directional edge whose capacity is the corresponding storage capacity α of this storage node. Moreover, it is assumed edges departing the storage nodes and arriving to a DC node have an infinite capacity. This reflects the fact that DC nodes have access to all stored data of the surviving nodes they are connected to.

The graph evolves constantly across time to capture any changes happening throughout the network. This graph starts from the source node. It is the only active node at the first step. The total number of storage nodes is n and the source node divides the original data file of size M into k pieces. These k pieces are encoded to n data fragments each to be stored in one of existing storage nodes through direct edges of infinite capacity. In the case that a storage node leaves the system or a failure occurs, this node is replaced by a new one, called the newcomer node. The newcomer connects to d active nodes out of $n-1$ existing nodes and downloads β bits from

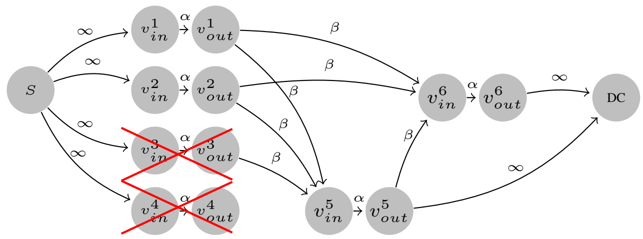


Fig. 1. Information flow graph corresponding to a $[4, 2, 3]$ regenerating code.

each. Accordingly, the corresponding information flow graph is updated through establishing d directed edges of capacity β, starting from outgoing nodes affiliated to the selected storage nodes and terminating to the corresponding incoming node of the newcomer (See Figure 1). In this case, the total information received by the newcomer node, $d\beta$, is called the repair bandwidth (γ). Finally, the data is reconstructed at each DC node through connecting to any arbitrary set of k nodes, including the newcommer nodes.

The use of a $[n, k, d]$ regenerating code having an access to the data of k storage nodes out of existing n nodes is adequate to reconstruct the original data file. Thus, the newcomer needs to connect to exactly $d = k$ nodes and downloads all of stored data ($\alpha = M/k$), thus $\beta = \alpha = M/k$. So the repair bandwidth is the same as the size of data file, i.e., $\gamma = d\beta = M$. On the other hand, Dimakis et al. in [3] show that if a newcomer could connect to more than k surviving nodes and downloads a certain fraction of their stored information, a lower repair bandwidth would be achieved.

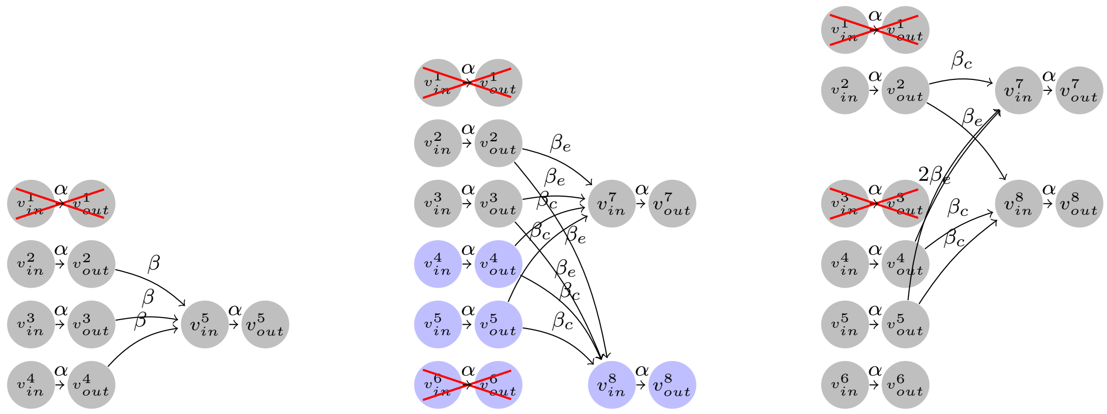
To this end, it is shown the task of computing the repair bandwidth can be translated to a multicast problem over the corresponding information flow graph for which an optimal trade-off between the storage per node, α, and the repair bandwidth, γ, is identified. This optimal trade-off curve includes two extremal points corresponding to the minimum storage capacity (MSR) per node and minimum repair bandwidth (MBR), respectively.

Consider any given finite information flow graph G, with a finite set of data collectors. In [3], it is argued that if $\min(mincut(S, DC)) \geq M$, then there exists a linear network code such that all data collectors can recover the data object.

From this symmetric model, the mincut is computed and lower bounds on the parameters α and γ are given. Let $\alpha^*(d, \gamma)$ be the threshold function, which is the function that minimizes α.

Figure 1 illustrates an information flow graph G associated to a $[4, 2, 3]$ regenerating code. Note that $mincut(S, DC) = min(3\beta, \alpha) + min(2\beta, \alpha)$. In general, it can be claimed that $mincut(S, DC) \geq \sum_{i=0}^{k-1} min((d-i)\beta, \alpha) \geq M$, which after an optimization process leads to the threshold function shown in [3].

To find the mincut equation, the k terms in the summation are computed as the minimum between two parameters: the sum of the weights of the arcs that we have to cut to isolate the corresponding v_{in}^j from S, and the weight of the arc that



(a) Symmetric model ($\gamma = 3\beta$).

(b) Static cost model ($\gamma = 2\beta_c + 2\beta_e$).

(c) Traditional two-rack model ($\gamma^1 = \beta_c + 2\beta_e$, $\gamma^2 = 2\beta_c + \beta_e$).

Fig. 2. Different models of DSS.

we have to cut to isolate the corresponding v_{out}^j from S. Let's call the first parameter as the income of the corresponding newcomer s_j. Note that the income of the newcomer s_j depends on the previous newcomers. The newcomers can be ordered according to their income from the highest to the lowest. Then, the MSR point corresponds to the lowest income, which is given by the last newcomer added to the information flow graph; and the MBR point corresponds to the highest, which is given by the first newcomer.

B. Static Cost Model

In [6], Akhlaghi et al. presented another DSS model, where the storage nodes V_S are partitioned into two sets V^1 and V^2 with different repair bandwidth. Let $V^1 \subset V_S$ be the "cheap bandwidth" nodes, where each data unit has a sending cost C_c, and $V^2 \subset V_S$ be the "expensive bandwidth" nodes, where each data unit has a sending cost C_e with $C_e > C_c$. When a newcomer enter the system, the cost of downloading data from a node in V^1 will be lower than the cost of downloading data from a node in V^2.

Consider the same situation as in the model described in Subsection II-A. When a storage node fails, the newcomer node $s_j, j = n+1, \ldots, \infty$, connects to d_1 existing storage nodes from V^1 and receives from each one of them β_c data units; it also connects to d_2 existing storage nodes from V^2 and receives from each one of them β_e data units. Let $d = d_1 + d_2$ be the number of helper nodes. Assume that d, d_1, and d_2 are fixed, that is, they do not depend on the storage node $s_j, j = n+1, \ldots, \infty$. In terms of the information flow graph G, there is one arc from v_{out}^i to v_{in}^j of weight β_c or β_e respectively (depending on whether s_i sends β_c or β_e data units) in the regenerating process. The new vertex v_{in}^j, is also connected to its associated v_{out}^j with an arc of weight α.

Let the repair cost be $C_T = d_1 C_c \beta_c + d_2 C_e \beta_e$ and the repair bandwidth $\gamma = d_1\beta_c + d_2\beta_e$. To simplify the model, we can assume, without loss of generality, that $\beta_c = \tau\beta_e$ for some real number $\tau \geq 1$. This means that we minimize the repair cost C_T by downloading more data units from the "cheap bandwidth" set of nodes V^1 than from the "expensive bandwidth" set of nodes V^2. Note that if τ is increased, the repair cost is decreased and vice-versa.

C. Two-Rack Model

In [9], a new DSS model - two-rack model is presented. In this case, the repair cost between nodes that are in the same rack is much lower than between nodes that are in the other rack. Consider the same situation as in Subsection II-B, but now the sets of "cheap bandwidth" and "expensive bandwidth" nodes are not static or predefined, they depend on the specific replaced node.

Let the newcomers be $s_j, j = n+1, \ldots, \infty$, d_c^i be the number of helper nodes providing cheap bandwidth, and d_e^i be the number of helper nodes providing expensive bandwidth to the newcomer in the i-th rack, $i = 1, 2$. The system must satisfy $d = d_c^i + d_e^i$ for all i. Without lost of generality, assume $d_c^1 \leq d_c^2$. There is a different repair bandwidth for both racks, i.e. $\gamma^1 = \beta_e(d_c^1\tau + d_e^1) \leq \gamma^2 = \beta_e(d_c^2\tau + d_e^2)$. Recall that $\beta_c = \tau\beta_e$, where $\tau \geq 1$. If the $\gamma^1 \geq \alpha$ is not satisfied then the file cannot be restored.

In this model, it is not straightforward to determine which is the set of newcomers that minimize the mincut. This set may change according to the parameters of the system. The authors of [9] show how to find the mincut set as follows: let I be the indexed multiset containing the incomes of k newcomers that minimizing the mincut.

- Define $I_1 = \{((d_c^1 - i)\tau + d_e^1)\beta_e | i = 0, \ldots, min(d_c^1, k-1)\}$ as the indexed multiset where $I_1[i], i = 0, \ldots, min(d_c^1, k-1)$, are the incomes of this set of $d_c^1 + 1$ newcomers from rack 1.
- Define $I_2 = \{d_e^1\beta_e | i = 1, \ldots, min(k - d_c^1 - 1, n_1 - d_c^1 - 1)\} \cup \{(d_c^2 - i)\tau\beta_e | i = 0, \ldots, min(d_c^2, k - n_1 - 1)\}$ as the indexed multiset where $I_2[i], i = 0, \ldots, k - d_c^1 - 2$, are the incomes of a set of $k - d_c^1 - 1$ newcomers, including the remaining newcomers from rack 1 and newcomers

from rack 2.

- Define $I_3 = \{(d_c^2 - i)\tau\beta_e | i = 0, \ldots, min(d_c^2, k - d_c^1 - 2)\}$ as the indexed multiset where $I_3[i], i = 0, \ldots, k - d_c^1 - 2$, are the incomes of a set of $k - d_c^1 - 1$ newcomers from rack 2.
- Then, either $I = I_1 \cup I_2$ or $I = I_1 \cup I_3$.

Let L be the increasing ordered list of values such that for all $i, i = 0, \ldots, k-1, I[i]/\beta_e \in L$ and $|I| = |L|$. Note that any of the information flow graphs representing any model from this two-rack model can be described in terms of I, so they can be represented by L. Therefore, once L is found, it is possible to find the parameters α and β_e (and then γ or $\gamma^i, i = 1, 2$) using the following threshold function.

$$\alpha^*(\beta_e) = \begin{cases} \frac{M - g(i)\beta_e}{k-i}, & \text{if } \beta_e \in [f(i), f(i-1)), \\ & i = 0, \ldots, k-1, \end{cases}$$

subject to $\gamma^1 = (d_c^1\tau + d_e^1)\beta_e \geq \alpha$, where

$$f(i) = \frac{M}{L[i](k-i) + g(i)} \text{ and } g(i) = \sum_{j=0}^{i-1} L[j].$$

Note that, $f(-1) = +\infty$ and $g(0) = 0$ must be defined.

III. Achieving MBR for Two-Rack Model

In this section, we first show that the two-rack model in [9] has an issue to achieve MBR point. A solution based on non-homogenous distributed storage model is proposed, and then a generalization of the threshold function is given. Finally, there is an example comparing the traditional and non-homogeneous two-rack models where the improvement is presented.

A. Feasibility of MBR point

We show that in the two-rack model presented in [9] there are some situations where the MBR point is not feasible, this is because the condition $\gamma^1 = (d_c^1\tau + d_e^1)\beta_e \geq \alpha$ is not satisfied.

From [9], the value of α at the MBR point is $\alpha_{MBR} = \max(I)$. It is clear that $\max(I) = \max(I_1)$, or $\max(I) = \max(I_2)$, or $\max(I) = \max(I_3)$, depending on the situation. It is easy to see that $\max(I_1) = ((d_c^1 - i)\tau + d_e^1)\beta_e$ for $i = 0$, and $\max(I_1) = \gamma^1$. Hence, if $\max(I) = \max(I_1) = \gamma^1$, then $\alpha_{MBR} = \gamma^1$, and $\gamma^1 \geq \alpha_{MBR}$ holds.

However, if $\max(I) = \max(I_2)$ or $\max(I) = \max(I_3)$, then $\alpha_{MBR} = \max(I) > \max(I_1) = \gamma^1$, which breaks the required condition of $\gamma^1 \geq \alpha_{MBR}$. This implies that some nodes receive less information than the information required for storing during the regenerating process, and this leads to contradiction.

The authors of [9] avoid such situation by deleting as much elements of multisets I_2 or I_3 as possible until $\max(I) = \max(I_1)$. Such solution avoids the impossible points, but at the same time, it also ignores better bounds in the tradeoff curve between α and β_e. In other words, this is not an efficient solution.

In fact, it is not difficult to find a case where $\max(I) = \max(I_3)$. This happens when $\max(I_3) > \max(I_1)$, i.e $d_c^2\tau > d_c^1\tau + d_e^1$. For example, two in Figure 2(c) with $\tau = 3$, $d_c^1 = 1, d_e^1 = 2, d_c^2 = 2, d_e^2 = 1$. Hence, $3 \cdot 2 > 1 \cdot 3 + 2$. In fact, the greater the difference between the two racks, the greater the likelihood of this situation will happen.

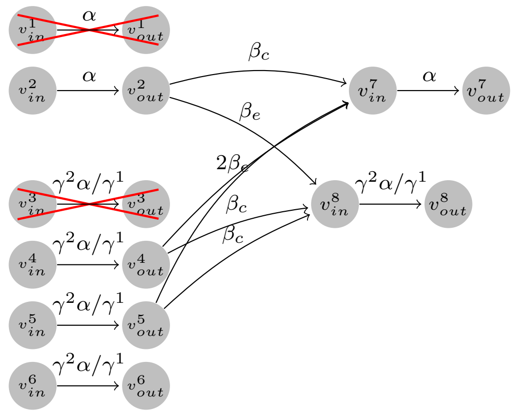


Fig. 3. Non-homogeneous two-rack DSS model. Rack 1 with two nodes and rack 2 with four nodes. Note that, $\gamma^1 = \beta_c + 2\beta_e$ and $\gamma^2 = 2\beta_c + \beta_e$.

B. Non-homogeneous two-rack model

In this subsection we design a non-homogeneous two-rack DSS model, and we prove that this design can achieve the MBR point that is not feasible previously.

In the traditional two-rack model, the storage capacity of every node is considered to be the same, say α. Even though, the system has two different repair bandwidths (γ^1, γ^2) for each rack. The fixed α and different γ are causing the non-feasible points described above.

Assuming that $\gamma^2 \geq \gamma^1$, the nodes of the rack 2 are receiving γ^2/γ^1 more information than the nodes of rack 1.

Our approach is to design a non-homogeneous two-rack model where the nodes of rack 1 stores α information and the nodes of rack 2 stores $\frac{\gamma^2}{\gamma^1}\alpha$ information. Recall $\gamma^1 = \beta_e(d_c^1\tau + d_e^1) \leq \gamma^2 = \beta_e(d_c^2\tau + d_e^2)$. Figure 3 shows this new model.

In the proposed non-homogeneous two-rack model, the mincut equation, which is not constant in terms of α (as it was in the original two-rack model), becomes:

$$C = \min\{I[i], \alpha\} + \min\{I[j], \frac{\gamma^2}{\gamma^1}\alpha\},$$

where $I[i]$ are the incomes of the rack 1, and $I[j]$ are the incomes of the rack 2.

Note that, the mincut set for the newly proposed non-homogeneous two-rack model is still the same as the traditional two-rack model. Hence, the set of incomes I is exactly the same. The main difference arises in L. In the traditional two-rack model, the list L is created in ascendant order by picking the elements of I. Let's define the following multiset of tuples:

$$L^n = \{(\frac{I[i]}{\beta_e}, 1)\} \cup \{(\frac{I[j]}{\beta_e}, \frac{\gamma^2}{\gamma^1})\}$$

where $I[i]$ are the incomes of the rack 1 and $I[j]$ are the incomes of the rack 2. Moreover, L^n is ordered by the following total order:

$$L^n[i] \geq L^n[j] \iff L^n[i][1]L^n[i][2]^{-1} \geq L^n[j][1]L^n[j][2]^{-1}.$$

Next, we can generalize the threshold function for the non-homogeneous two-rack model:

$$\alpha^*(\beta_e) = \left\{ \frac{M-g'(0,i,1)\beta_e}{g'(i,k-1,2)}, \quad \text{if } \beta_e \in [f(i), f(i-1)), \right.$$

for $i = 0, \dots, k-1$, where

$$f(i) = \frac{M}{g'(0,i,1) + g'(i,k-1,2)L^n[i][1]L^n[i][2]^{-1}}$$

and

$$g'(a,b,c) = \sum_{j=a}^{b} L^n[j][c].$$

Note that, $f(-1) = +\infty$.

The next theorem shows how all the points on the tradeoff curve are feasible in the newly proposed non-homogeneous two-rack model.

Theorem 1: Given a non-homogeneous two-rack model with repair bandwidths $\gamma^1 \leq \gamma^2$ and the nodes of the rack 1 stores α information and the nodes of rack 2 stores $\frac{\gamma^2}{\gamma^1}\alpha$ information. Then, all the points of the tradeoff curve are feasible.

Proof: As in the traditional two-rack model, α_{MBR} is defined by the maximum income. But now, the "maximum income" is taken from the multiset L^n and depending on the total order defined above (definitely it depends on the storage too). Thus, we need to prove that the "maximum income" is always γ^1. The problem can be translated to the multiset I. Since it is constructed by I_1, I_2 from the rack 1 and $\frac{\gamma^1}{\gamma^2}I_3$ from the rack 2, we need to show that $\alpha_{MBR} = \max(I) = \max(I_1) = \gamma^1$.

Since $\max(I) = \max(I_1 \cup I_2)$ or $\max(I) = \max(I_1 \cup \frac{\gamma^1}{\gamma^2}I_3)$. And $\max(I_1) = \gamma^1 = (d_c^1\tau + d_e^1)\beta_e$, $\max(I_2) = d_e^1\beta_e \leq \gamma^1$. Then, in this case, $\max(I) = \max(I_1 \cup I_2) = \max(I_1) = \gamma^1$. On the other hand, if we consider $\frac{\gamma^1}{\gamma^2}I_3$, we can see that $\max(I_3) = d_c^2\tau\beta_e \leq \gamma^2 = d_c^2\tau\beta_e + d_e\beta_e$. Hence, $\max(\frac{\gamma^1}{\gamma^2}I_3) = \gamma^1\frac{d_c^2\tau\beta_e}{\gamma^2} \leq \gamma^1$. And it holds too that $\max(I_1 \cup \frac{\gamma^1}{\gamma^2}I_3) = \max(I_1) = \gamma^1$

Finally, the MBR point becomes feasible without the need of deleting any element of the list I. Since $\gamma^1 \geq \alpha$ then $\gamma^2 \geq \frac{\gamma^2}{\gamma^1}\alpha$. □

C. Example

A comparison between the traditional two-rack model and the newly proposed non-homogeneous two-rack model is shown in Figure 4. We consider a two-rack model with 3 nodes in the first rack, 7 nodes in the second rack, and with $\tau = 4$. Three points has been deleted in the traditional model. The non-homogeneous case not only achieves the MBR, the performance on the MSR is also better, even this is not due to any deleted point in the traditional model.

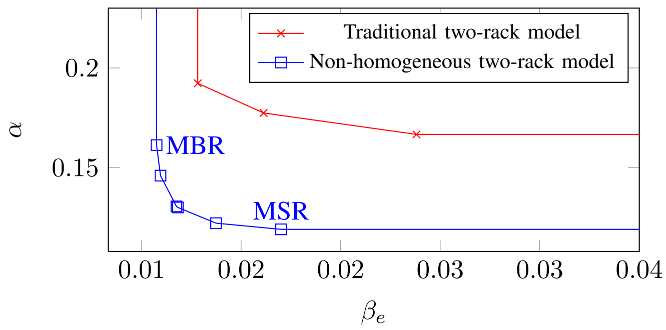


Fig. 4. Chart comparing the traditional and the non-homogeneous two-rack models. $M = 1, k = 6, d_e^1 = 7, d_e^2 = 3, d = 9, n_1 = 3, n_2 = 7, \tau = 4$.

IV. Conclusion

In this paper, we show that a traditional two-rack DSS model that considering only different repair bandwidth across the rack but same storage size for all the nodes cannot achieve the MBR point. We propose a non-homogenous model by having a different storage size for the storage nodes in each rack, and prove that this non-homogenous model makes MBR point becomes feasible. Moreover, we show how much information should be stored on each node and derive a generalized threshold function. The generalization of this non-homogeneous model to any number of racks is straightforward after the traditional two-rack model is generalized.

V. Acknowledgment

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