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A New Achievable Scheme for Interference Relay Channels

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***Abstract*—We establish an achievable rate region for discrete memoryless interference relay channels that consist of two source-destination pairs and one or more relays. We develop an achievable scheme combining Han-Kobayashi and noisy network coding. We apply our achievability to two cases. First, we characterize the capacity region of some classes of discrete memoryless interference relay channels. These classes naturally generalize the injective deterministic discrete memoryless interference channel by El Gamal and Costa and the discrete memoryless relay channel. Moreover, for the Gaussian interference relay channel with orthogonal receiver components, we show that our scheme achieves a better sum rate than that of noisy network coding.**

I. Introduction

Discrete memoryless interference channel (DM-IC) was introduced by Ahlswede [1]. Discrete memoryless relay channel (DM-RC) was first studied by van der Meulen [2]. Neither the capacity region of the DM-IC nor the capacity of the DM-RC has been characterized yet except for some special cases. First, for DM-IC, the best known inner bound was obtained by Han and Kobayashi [3]. This inner bound was shown to be tight for the injective deterministic DM-IC by El Gamal and Costa [4]. On the other hand, one relaying strategy for DM-RC is compress-and-forward (CF) due to Cover and El Gamal [5] where the relay compresses its observation and forwards it to the destination. CF was shown to be optimal for the deterministic DM-RC with orthogonal receiver components [6] and the modulo-2 sum relay channel [7]. Recently, noisy network coding [8] generalized CF for general discrete memoryless relay networks.

A natural next step is to extend these results to more general channel scenarios in which there are more than two transmitter-receiver pairs and/or relays. As one such model, we consider a discrete memoryless interference multi-relay channel (DM-IMRC) that consists of two source-destination pairs and an arbitrary number of relays. For this channel, we combine Han-Kobayashi and noisy network coding schemes to establish an achievable rate region.

We apply our achievability to two cases. First, we characterize the capacity region of some classes of discrete memoryless interference relay channels (DM-IRCs) which naturally generalizes the injective deterministic DM-IC by El Gamal and Costa [4] and the DM-RC. Here, depending on the links between the relay and the transmitters/receivers, there are four types of the interference relay channels [9]. As a special case, we consider channels with in-band transmission/reception relay and out-of-band transmission/in-band reception relay to get the capacity result. For the converse, a cut-set bound and a genie-aided proof technique are used. Furthermore, for a Gaussian interference relay channel (GIRC) with orthogonal receiver components, we show we can obtain a better sum rate than that in [8].

The rest of the paper is organized as follows. In Section II, we introduce the DM-IMRC model. Section III presents the achievable rate region for the DM-IMRC. Section IV characterizes the capacity region of some classes of DM-IRC. Section V focuses on the GIRC with orthogonal receiver components.

II. Model

We consider a DM-IMRC as depicted in Fig. 1. A $\left(2^{nR_1}, 2^{nR_2}, n\right)$ code consists of two message sets $\mathcal{M}_1 = \{1, \dots, 2^{nR_1}\}$ and $\mathcal{M}_2 = \{1, \dots, 2^{nR_2}\}$, two encoding functions at the sources where the first source (node 1) maps its message $m_1 \in \mathcal{M}_1$ to a codeword $x_1^n(m_1) \in \mathcal{X}_1^n$ and the second source (node 2) maps its message $m_2 \in \mathcal{M}_2$ to a codeword $x_2^n(m_2) \in \mathcal{X}_2^n$, N processing functions at the relays (node 3,k where $k \in [1:N]$) that map each past received symbols $y_{3,k}^{i-1} \in \mathcal{Y}_{3,k}^{i-1}$ to a symbol $x_{3,k,i}(y_{3,k}^{i-1}) \in \mathcal{X}_{3,k}$, and two decoding functions at the destinations where the first destination (node 4) maps each received sequence $y_4^n \in \mathcal{Y}_4^n$ to a message estimate $\hat{m}_1$ and the second destination (node 5) maps each received sequence $y_5^n \in \mathcal{Y}_5^n$ to a message estimate $\hat{m}_2$. The first source (node 1) sends $x_1^n(m_1)$ and the second source (node 2) sends $x_2^n(m_2)$. The average probability of error for a $(2^{nR_1}, 2^{nR_2}, n)$ code is given as $P_e^{(n)} \triangleq \Pr\left((\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\right)$. A rate pair (R_1, R_2) is said to be *achievable* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $P_e^{(n)} \to 0$ as $n \to \infty$. The capacity region C is the closure of the set of achievable rate pairs (R_1, R_2).

III. Main results

An achievable rate region for the DM-IMRC is established in the following theorem.

Theorem 1. *A rate pair (R_1, R_2) is achievable for the DM-IMRC if there exists some probability mass function (pmf)*

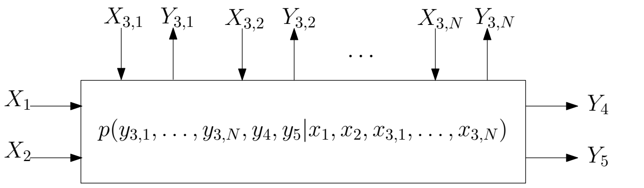

Fig. 1. A discrete memoryless interference multi-relay channel (DM-IMRC)

$p(q)p(u_1, x_1|q)p(u_2, x_2|q) \prod_{k=1}^{N} p(x_{3,k}|q)p(\hat{y}_{3,k}|y_{3,k}, x_{3,k}, q)$ *such that*

$$
\begin{aligned}
&R_1 < \min_S \{I(X_1, X_3(S); \hat{Y}_3(S^c), Y_4 | U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&R_2 < \min_S \{I(X_2, X_3(S); \hat{Y}_3(S^c), Y_5 | U_1, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&R_1 + R_2 \\
&< \min_S \{I(X_1, X_3(S); \hat{Y}_3(S^c), Y_4 | U_1, U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&\quad + \min_S \{I(X_2, U_1, X_3(S); \hat{Y}_3(S^c), Y_5 | X_3(S^c), Q) \\
&- I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&R_1 + R_2 \\
&< \min_S \{I(X_2, X_3(S); \hat{Y}_3(S^c), Y_5 | U_1, U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&\quad + \min_S \{I(X_1, U_2, X_3(S); \hat{Y}_3(S^c), Y_4 | X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&R_1 + R_2 \\
&< \min_S \{I(X_1, U_2, X_3(S); \hat{Y}_3(S^c), Y_4 | U_1, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&\quad + \min_S \{I(X_2, U_1, X_3(S); \hat{Y}_3(S^c), Y_5 | U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&2R_1 + R_2 \\
&< \min_S \{I(X_1, X_3(S); \hat{Y}_3(S^c), Y_4 | U_1, U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&\quad + \min_S \{I(X_1, U_2, X_3(S); \hat{Y}_3(S^c), Y_4 | X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\} \\
&\quad + \min_S \{I(X_2, U_1, X_3(S); \hat{Y}_3(S^c), Y_5 | U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&R_1 + 2R_2 \\
&< \min_S \{I(X_2, X_3(S); \hat{Y}_3(S^c), Y_5 | U_1, U_2, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&\quad + \min_S \{I(X_2, U_1, X_3(S); \hat{Y}_3(S^c), Y_5 | X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_2, U_1, X_3^N, \hat{Y}_3(S^c), Y_5, Q)\} \\
&\quad + \min_S \{I(X_1, U_2, X_3(S); \hat{Y}_3(S^c), Y_4 | U_1, X_3(S^c), Q) \\
&\quad - I(\hat{Y}_3(S); Y_3(S) | X_1, U_2, X_3^N, \hat{Y}_3(S^c), Y_4, Q)\}
\end{aligned}
$$

for all subsets $S \subset [1 : N]$ *such that* $X_3(S) \subset \{X_{3,1}, \cdots, X_{3,N}\}$ *which are relay nodes.*

Proof: We leave the proof in the full version of this paper [10]. ■

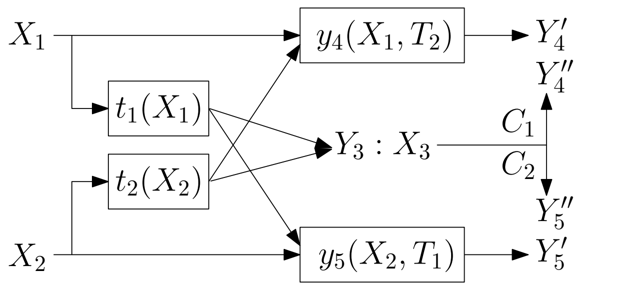

Fig. 2. A class of injective DM-IRCs with orthogonal receiver components

IV. Capacity of a class of injective interference relay channels

In this section, we characterize the capacity region of some classes of injective DM-IRCs.

A. Injective DM-IRC with orthogonal receiver components

First, we consider a class of injective DM-IRCs with orthogonal receiver components. In this class, the channel outputs are given as follows:

$$
\begin{aligned}
Y_4 &= (Y_4', Y_4'') \\
Y_5 &= (Y_5', Y_5'') \\
Y_4' &= y_4(X_1, T_2) \\
Y_5' &= y_5(X_2, T_1)
\end{aligned}
$$

where $T_1 = t_1(X_1)$ and $T_2 = t_2(X_2)$ are functions of X_1 and X_2, respectively. The functions y_4 and y_5 are injective in t_1 and t_2, respectively, i.e., for every $x_1 \in \mathcal{X}_1$, $y_4(x_1, t_2)$ is a one-to-one function of t_2 and similarly for y_5. The relay sends information over common rate-limited noiseless links of rate $C_1 \triangleq \max_{p(x_3)} I(X_3; Y_4'')$ to the first destination, and rate $C_2 \triangleq \max_{p(x_3)} I(X_3; Y_5'')$ to the second destination. This class of DM-IRCs is illustrated in Fig. 2. For the class of injective DM-IRCs illustrated in Fig. 2, the following propositions give inner and outer bounds.

Proposition 1. *The following set of rate pairs* (R_1, R_2) *constitutes an inner bound of the class of injective DM-IRCs in Fig. 2.*

$$
\begin{aligned}
R_1 \le \min\{&H(Y_4'|T_2, Q) + H(Y_3|Y_4', T_2, Q), \\
&H(Y_4'|T_2, Q) + C_1\} - H(Y_3|T_1, T_2, Q)\} \qquad (1) \\
R_2 \le \min\{&H(Y_5'|T_1, Q) + H(Y_3|Y_5', T_1, Q), \\
&H(Y_5'|T_1, Q) + C_2\} - H(Y_3|T_1, T_2, Q)\} \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&R_1+R_2\\
&\le \min\{H(Y_3,Y_4'|T_1,T_2,Q), \qquad (3)\\
&\qquad H(Y_4'|T_1,T_2,Q)+C_1\}-H(Y_3|T_1,T_2,Q)\\
&+\min\{H(Y_3,Y_5'|Q),H(Y_5'|Q)+C_2\}-H(Y_3|T_1,T_2,Q)\\
&R_1+R_2\\
&\le \min\{H(Y_3,Y_5'|T_1,T_2,Q), \qquad (4)\\
&\qquad H(Y_5'|T_1,T_2,Q)+C_2\}-H(Y_3|T_1,T_2,Q)\\
&+\min\{H(Y_3,Y_4'|Q),H(Y_4'|Q)+C_1\}-H(Y_3|T_1,T_2,Q)\\
&R_1+R_2\\
&\le \min\{H(Y_3,Y_4'|T_1,Q),H(Y_4'|T_1,Q)+C_1\} \qquad (5)\\
&+\min\{H(Y_3,Y_5'|T_2,Q),H(Y_5'|T_2,Q)+C_2\}\\
&-2H(Y_3|T_1,T_2,Q)\\
&2R_1+R_2\\
&\le \min\{H(Y_3,Y_4'|T_1,T_2,Q),H(Y_4'|T_1,T_2,Q)+C_1\} \qquad (6)\\
&+\min\{H(Y_3,Y_4'|Q),H(Y_4'|Q)+C_1\}\\
&+\min\{H(Y_3,Y_5'|T_2,Q),H(Y_5'|T_2,Q)+C_2\}\\
&-3H(Y_3|T_1,T_2,Q)\\
&R_1+2R_2\\
&\le \min\{H(Y_3,Y_5'|T_1,T_2,Q),H(Y_5'|T_1,T_2,Q)+C_2\} \qquad (7)\\
&+\min\{H(Y_3,Y_5'|Q),H(Y_5'|Q)+C_2\}\\
&+\min\{H(Y_3,Y_4'|T_1,Q),H(Y_4'|T_1,Q)+C_1\}\\
&-3H(Y_3|T_1,T_2,Q)
\end{aligned}
$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$.

Proof: The achievability is directly obtained by letting $N=1, \hat{Y}_3=Y_3, U_1=T_1$, and $U_2=T_2$ in Theorem 1. ■

Proposition 2. *The following set of rate pairs* (R_1,R_2) *constitutes an outer bound of the class of injective DM-IRCs in Fig. 2.*

$$
\begin{aligned}
R_1 &\le \min\{H(Y_4'|T_2,Q)+H(Y_3|Y_4',T_2,Q) \qquad (8)\\
&\qquad -H(Y_3|T_1,T_2,Q),H(Y_4'|T_2,Q)+C_1\}\\
R_2 &\le \min\{H(Y_5'|T_1,Q)+H(Y_3|Y_5',T_1,Q) \qquad (9)\\
&\qquad -H(Y_3|T_1,T_2,Q),H(Y_5'|T_1,Q)+C_2\}\\
R_1+R_2 &\le \min\{H(Y_4'|Y_3,T_1,T_2,Q), \qquad (10)\\
&\qquad H(Y_4'|T_1,T_2,Q)+C_1\}\\
&\quad +\min\{H(Y_3,Y_5'|Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_5'|Q)+C_2\}\\
R_1+R_2 &\le \min\{H(Y_5'|Y_3,T_1,T_2,Q), \qquad (11)\\
&\qquad H(Y_5'|T_1,T_2,Q)+C_2\}\\
&\quad +\min\{H(Y_3,Y_4'|Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_4'|Q)+C_1\}\\
R_1+R_2 &\le \min\{H(Y_3,Y_4'|T_1,Q)-H(Y_3|T_1,T_2,Q), \qquad (12)\\
&\qquad H(Y_4'|T_1,Q)+C_1\}\\
&\quad +\min\{H(Y_3,Y_5'|T_2,Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_5'|T_2,Q)+C_2\}\\
2R_1+R_2 &\le \min\{H(Y_4'|Y_3,T_1,T_2,Q), \qquad (13)\\
&\qquad H(Y_4'|T_1,T_2,Q)+C_1\}\\
&\quad +\min\{H(Y_3,Y_4'|Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_4'|Q)+C_1\}\\
&\quad +\min\{H(Y_3,Y_5'|T_2,Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_5'|T_2,Q)+C_2\}\\
R_1+2R_2 &\le \min\{H(Y_5'|Y_3,T_1,T_2,Q), \qquad (14)\\
&\qquad H(Y_5'|T_1,T_2,Q)+C_2\}\\
&\quad +\min\{H(Y_3,Y_5'|Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_5'|Q)+C_2\}\\
&\quad +\min\{H(Y_3,Y_4'|T_1,Q)-H(Y_3|T_1,T_2,Q),\\
&\qquad H(Y_4'|T_1,Q)+C_1\}
\end{aligned}
$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$.

Proof: The proof is given in Appendix A. ■

From the above two propositions, we can easily obtain the following theorem which gives the capacity region.

Theorem 2. *The capacity region of the class of injective DM-IRCs in Fig. 2, where* $C_1 \ge H(Y_3|Y_4')$ *and* $C_2 \ge H(Y_3|Y_5')$, *is the set of rate pairs* (R_1,R_2) *such that*

$$
\begin{aligned}
R_1 \le& H(Y_3,Y_4'|T_2,Q)-H(Y_3|T_1,T_2,Q)\\
R_2 \le& H(Y_3,Y_5'|T_1,Q)-H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \le& H(Y_3,Y_4'|T_1,T_2,Q)+H(Y_3,Y_5'|Q)\\
&-2H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \le& H(Y_3,Y_5'|T_1,T_2,Q)+H(Y_3,Y_4'|Q)\\
&-2H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \le& H(Y_3,Y_4'|T_1,Q)+H(Y_3,Y_5'|T_2,Q)\\
&-2H(Y_3|T_1,T_2,Q)\\
2R_1+R_2 \le& H(Y_3,Y_4'|T_1,T_2,Q)+H(Y_3,Y_4'|Q)\\
&+H(Y_3,Y_5'|T_2,Q)-3H(Y_3|T_1,T_2,Q)\\
R_1+2R_2 \le& H(Y_3,Y_5'|T_1,T_2,Q)+H(Y_3,Y_5'|Q)\\
&+H(Y_3,Y_4'|T_1,Q)-3H(Y_3|T_1,T_2,Q)
\end{aligned}
$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$.

B. Injective DM-IRC with an in-band reception/in-band transmission relay

We consider a class of injective DM-IRCs with an in-band reception/in-band transmission relay. In this class, the channel outputs are given as follows:

$$
\begin{aligned}
Y_4 &= y_4(X_1,T_2,X_3)\\
Y_5 &= y_5(X_2,T_1,X_3)
\end{aligned}
$$

where $T_1=t_1(X_1)$ and $T_2=t_2(X_2)$ are functions of X_1 and X_2, respectively. The functions y_4 and y_5 are injective in t_1 and t_2, respectively, i.e., for every $(x_1,x_3)\in\mathcal{X}_1\times\mathcal{X}_3$, $y_4(x_1,t_2,x_3)$ is a one-to-one function of t_2 and similarly for y_5. This class of DM-IRCs is illustrated in Fig. 3. For the class of injective DM-IRCs illustrated in Fig. 3, the following theorem gives the capacity region.

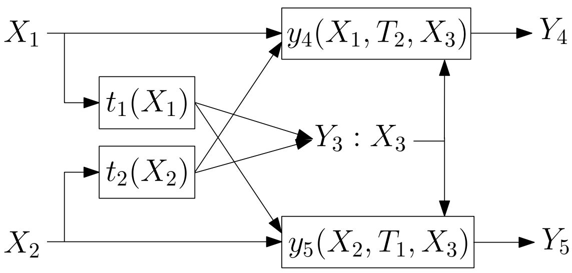

Fig. 3. A class of injective DM-IRCs with an in-band reception/in-band transmission relay

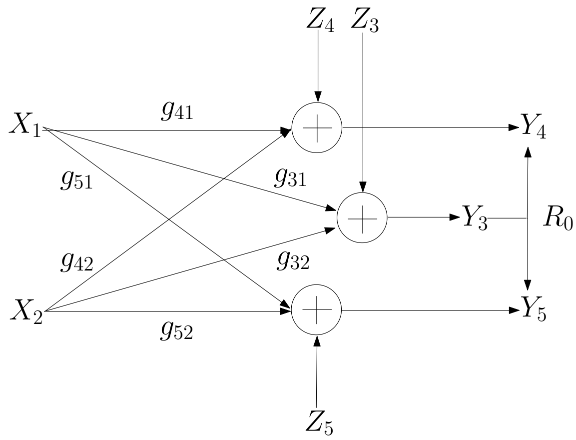

Fig. 4. GIRC with orthogonal receiver components

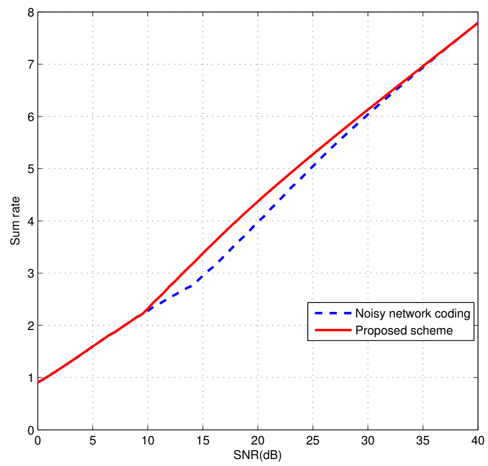

Fig. 5. Comparison of two schemes for the GIRC with orthogonal receiver components with $g_{41} = g_{52} = 1, g_{42} = g_{51} = g_{31} = 0.5, g_{32} = 0.1, R_0 = 1$.

Theorem 3. *The capacity region of the class of injective DM-IRCs in Fig. 3, where $I(X_3;Y_4) \geq H(Y_3|X_3,Y_4)$ and $I(X_3;Y_5) \geq H(Y_3|X_3,Y_5)$ for all $p(x_3)$, is the set of rate pairs (R_1,R_2) such that*

$$\begin{aligned}
R_1 \leq & H(Y_3,Y_4|T_2,X_3,Q) - H(Y_3|T_1,T_2,Q)\\
R_2 \leq & H(Y_3,Y_5|T_1,X_3,Q) - H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \leq & H(Y_3,Y_4|T_1,T_2,X_3,Q) + H(Y_3,Y_5|X_3,Q)\\
& - 2H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \leq & H(Y_3,Y_5|T_1,T_2,X_3,Q) + H(Y_3,Y_4|X_3,Q)\\
& - 2H(Y_3|T_1,T_2,Q)\\
R_1+R_2 \leq & H(Y_3,Y_4|T_1,X_3,Q) + H(Y_3,Y_5|T_2,X_3,Q)\\
& - 2H(Y_3|T_1,T_2,Q)\\
2R_1+R_2 \leq & H(Y_3,Y_4|T_1,T_2,X_3,Q) + H(Y_3,Y_4|X_3,Q)\\
& + H(Y_3,Y_5|T_2,X_3,Q) - 3H(Y_3|T_1,T_2,Q)\\
R_1+2R_2 \leq & H(Y_3,Y_5|T_1,T_2,X_3,Q) + H(Y_3,Y_5|X_3,Q)\\
& + H(Y_3,Y_4|T_1,X_3,Q) - 3H(Y_3|T_1,T_2,Q)
\end{aligned}$$

for some pmf $p(q)p(x_1|q)p(x_2|q)p(x_3|q)$.

Proof: The achievability of Theorem 3 is directly obtained by letting $N=1, \hat{Y}_3 = Y_3, U_1 = T_1$, and $U_2 = T_2$ in Theorem 1. The converse proof is similar to that in Appendix A. ■

V. Gaussian Interference Relay Channel with Orthogonal Receiver Components

Consider the GIRC with orthogonal receiver components in Fig. 4. The channel outputs are

$$\begin{aligned}
Y_3 &= g_{31}X_1 + g_{32}X_2 + Z_3\\
Y_4' &= g_{41}X_1 + g_{42}X_2 + Z_4\\
Y_5' &= g_{51}X_1 + g_{52}X_2 + Z_5.
\end{aligned}$$

where $Y_l = (Y_l', Y_l''), Y_l'$ and Y_l'' are independent for $l = 4,5$, g_{jk} is the channel gain from node k to node j and the noise $Z_i \sim \mathcal{N}(0,1)$ is independent and identically distributed (i.i.d.). Relay helps the communication of two source-destination pairs by forwarding some information about Y_3 to both destinations through a common rate-limited noiseless link of rate $R_0 \triangleq \max_{p(x_3)} I(X_3;Y_4'') = \max_{p(x_3)} I(X_3;Y_5'')$.

We consider $P_1 = P_2 = P$, $X_1 = U_1 + V_1$, $X_2 = U_2 + V_2$ and U_1, V_1, U_2 and V_2 are independent where U_i corresponds to the common message and V_i corresponds to the private message for $i = 1,2$ and power is allocated as $P_{U_i} = (1-\alpha_i)P, P_{V_i} = \alpha_i P$ for $i = 1,2$. Then, setting $\hat{Y}_3 = Y_3 + \hat{Z}$ with $\hat{Z} \sim \mathcal{N}(0,\sigma^2)$ yields the inner bound in [10]. The details are left in [10].

Remark 1. *Above inner bound is the same as that achieved by Han-Kobayashi and Generalized Hash-and-Forward schemes [11] for a GIRC with a digital relay link of rate R_0 bits per channel use.*

Remark 2. *The sum rates of the proposed scheme is compared with that of [8] in Fig. 5. The sum-rate curve for the noisy network coding from [8] is obtained by using noisy network coding via simultaneous nonunique decoding and that via treating interference as noise. Proposed scheme outperforms these two schemes since Han-Kobayashi scheme is more general and includes as special cases both simultaneous nonunique decoding and treating interference as noise.*

APPENDIX

A. Proof of Proposition 2

Let Q be a random variable uniformly distributed over $[1:n]$ and independent of $(X_1^n, X_2^n, X_3^n, Y_3^n, Y_4^n, Y_5^n)$ and let $X_1 \triangleq X_{1Q}, X_2 \triangleq X_{2Q}, X_3 \triangleq X_{3Q}, Y_3 \triangleq Y_{3Q}, Y_4 \triangleq Y_{4Q}, Y_5 \triangleq Y_{5Q}, T_1 \triangleq T_{1Q}$, and $T_2 \triangleq T_{2Q}$. The first term in the minimum in (8) in Proposition 2 is obtained as follows.

$$\begin{aligned}
nR_1 &\le I(M_1;Y_4^n) + n\epsilon_n \\
&\le I(M_1;Y_4^n, Y_3^n) + n\epsilon_n \\
&\overset{(a)}{\le} I(M_1;Y_4^n, Y_3^n|T_2^n) + n\epsilon_n \\
&= I(M_1;Y_3^n|T_2^n) + I(M_1;Y_4^n|Y_3^n, T_2^n) + n\epsilon_n \\
&= \sum_{i=1}^n I(M_1;Y_{3i}|Y_3^{i-1}, T_2^n) + H(Y_4^n|Y_3^n, T_2^n) + n\epsilon_n \\
&\overset{(b)}{=} \sum_{i=1}^n I(M_1;Y_{3i}|Y_3^{i-1}, T_2^n) + H(Y_4'^n|Y_3^n, T_2^n) + n\epsilon_n \\
&\overset{(c)}{\le} \sum_{i=1}^n I(T_{1i}, X_{1i}, M_1, Y_3^{i-1}, T_2^{i-1}, T_{2i+1}^n;Y_{3i}|T_{2i}) \\
&\quad + H(Y_{4i}|T_{2i}, Y_{3i}) + n\epsilon_n \\
&\overset{(d)}{=} \sum_{i=1}^n I(T_{1i};Y_{3i}|T_{2i}) + H(Y_{4i}'|T_{2i}, Y_{3i}, X_{3i}) + n\epsilon_n \\
&\le \sum_{i=1}^n I(T_{1i};Y_{3i}|T_{2i}) + H(Y_{4i}'|T_{2i}, Y_{3i}) + n\epsilon_n \\
&= n(I(T_1;Y_3|T_2, Q) + H(Y_4'|T_2, Y_3, Q)) + n\epsilon_n \\
&= n(H(Y_3, Y_4'|T_2, Q) - H(Y_3|T_1, T_2, Q)) + n\epsilon_n
\end{aligned}$$

where Q is the usual time-sharing random variable and (a) follows by the fact that X_2^n and X_1^n are independent. b follows since $Y_4''^n$ is a function of X_3^n and X_3^n is a function of Y_3^n. (c) follows since X_{1i} is a function of M_1, T_{1i} is a function of X_{1i}, X_{3i} is a function of Y_3^{i-1}, and conditioning reduces entropy. (d) follows by the memoryless property which implies that $(M_1, X_{1i}, Y_3^{i-1}, T_2^{i-1}, T_{2i+1}^n) - (T_{1i}, T_{2i}) - Y_{3i}$. The second term in the minimum in (8) is obtained as follows.

$$\begin{aligned}
nR_1 &\le I(M_1;Y_4^n) + n\epsilon_n \\
&\overset{(a)}{\le} I(X_1^n;Y_4^n|T_2^n) + n\epsilon_n \\
&\le I(X_1^n, X_3^n;Y_4^n|T_2^n) + n\epsilon_n \\
&= I(X_1^n, X_3^n;Y_4'^n|T_2^n) + I(X_1^n, X_3^n;Y_4''^n|T_2^n, Y_4'^n) + n\epsilon_n \\
&\le H(Y_4'^n|T_2^n) + I(X_3^n;Y_4''^n) + n\epsilon_n \\
&\le n(H(Y_4'|T_2, Q) + C_1 + \epsilon_n)
\end{aligned}$$

where (a) follows by the fact that X_1^n and T_2^n are independent. Similarly, we can obtain inequality (9). Next, we show some inequalities for rest terms.

$$\begin{aligned}
I_1 &= I(M_1;Y_4^n, Y_3^n, T_1^n|T_2^n) \\
&\le H(T_1^n) + I(X_1^n;Y_3^n|T_1^n, T_2^n) + I(X_1^n;Y_4^n|Y_3^n, T_1^n, T_2^n) \\
&\le H(T_1^n) + nH(Y_4'|Y_3, T_1, T_2, Q)
\end{aligned}$$

Next,

$$\begin{aligned}
I_2 &= I(M_2;Y_3^n, Y_5^n) \\
&\le H(Y_3^n, Y_5^n) - H(Y_3^n, Y_5^n|X_2^n) \\
&\overset{(a)}{=} H(Y_3^n, Y_5'^n) - H(T_1^n) - H(Y_3^n|Y_5^n, X_2^n) \\
&\le nH(Y_3, Y_5'|Q) - H(T_1^n) \\
&\quad - \sum_{i=1}^n H(Y_{3i}|Y_5^n, X_2^n, T_2^n, Y_3^{i-1}, T_1^n) \\
&= nH(Y_3, Y_5'|Q) - H(T_1^n) - nH(Y_3|T_1, T_2, Q)
\end{aligned}$$

where (a) follows by the channel conditions and the fact that $H(Y_5'^n|X_2^n) = H(T_1^n)$. Next,

$$\begin{aligned}
I_3 &= I(M_1;Y_4^n, T_1^n|T_2^n) \\
&\le H(T_1^n) + I(X_1^n, X_3^n;Y_4^n|T_1^n, T_2^n) \\
&= H(T_1^n) + I(X_1^n, X_3^n;Y_4''^n|T_1^n, T_2^n, Y_4'^n) \\
&\quad + I(X_1^n, X_3^n;Y_4'^n|T_1^n, T_2^n) \\
&\le H(T_1^n) + nC_1 + nH(Y_4'|T_1, T_2, Q)
\end{aligned}$$

Due to the space, we leave the rest of the proof in the full version of this paper [10].

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REFERENCES

[1] R. Ahlswede, "The capacity region of a channel with two senders and two receivers," *Ann. Prob.*, vol. 2, pp. 805–814, 1974.

[2] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob.*, vol. 3, pp. 120–154, 1971.

[3] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, pp. 49–60, Jan. 1981.

[4] A. El Gamal and M. Costa, "The capacity region of a class of deterministic interference channels," *IEEE Trans. Inf. Theory*, vol. 28, pp. 343–346, Mar. 1982.

[5] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, pp. 572–584, Sep. 1979.

[6] Y.-H. Kim, "Capacity of a class of deterministic relay channels," *IEEE Trans. Inf. Theory*, vol. 53, pp. 1328–1329, Mar. 2008.

[7] M. Aleksic, P. Razaghi, and W. Yu, "Capacity of a class of modulo-sum relay channels," *IEEE Trans. Inf. Theory*, vol. 55, pp. 921–930, Mar. 2009.

[8] S. H. Lim, Y.-H. Kim, A. El Gamal, and S.-Y. Chung, "Noisy network coding," *IEEE Trans. Inf. Theory*, vol. 57, pp. 3132–3152, May 2011.

[9] O. Sahin and E. Erkip, "Achievable rates for the Gaussian interference relay channel," in *Proc. IEEE Global Telecommun. Conf. (GLOBE-COM)*, Nov. 2007, pp. 1627–1631.

[10] B.-J. Kang, S.-H. Lee, S.-Y. Chung, and C.-H. Suh, "A new achievable scheme for interference relay channels," [Online]. Available: http://arxiv.org/abs/1303.1095.

[11] P. Razaghi, S. N. Hong, L. Zhou, W. Yu, and G. Caire, "Two birds and one stone: Gaussian interference channel with a shared out-of-band relay," *Submitted to IEEE Trans. Inf. Theory*, 2011. [Online]. Available: http://arxiv.org/abs/1104.0430.