Multiple-Access Relay Channels with Non-Causal Side Information at the Relay

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***Abstract*—In this paper, we derive an inner bound for two-user state-dependent multiple-access relay channel (MARC) via decode and forward strategy (DF) in which the states of channel are known non-causally at the relay. The inner bound is obtained by using a combination of binning scheme and codeword splitting. Codeword splitting at the relay is applied to generating two codebooks, one for utilizing a joint codebook between the relay and the transmitters and another for utilizing the channel state information. Furthermore, we obtain an outer bound for this model. Actually, our work is a generalization of Zaidi et al. work on the relay channel with informed relay to the MARC.**

***Index Terms*—Multiple-access relay channel; Non-causal side information; decode and forward strategy.**

I. Introduction

The relay channel was first introduced by Van der Meulen [1]. Relay channel has been extensively studied in [2], where for some special cases such as degraded relay channel, reversely degraded relay channel and the relay channel with feedback capacity was derived and also upper and lower bounds on the capacity of the general relay channel were established.

Multiple-access relay channel was first introduced in [3]. This channel is considered as a combination of the multiple-access channel (MAC) and the relay channel where some sources communicate with one single destination with the help of a relay node.Coding strategies such as decode and forward (DF), compress and forward (CF) and amplify and forward have been extensively investigated for MARC in [4], [5], [6].

For the first time, Shannon obtained the capacity of single user discrete memoryless state dependent channel with causal side information at the transmitter (CSIT) [7]. In [8], side information non-causally available at the encoder for single user discrete memoryless channel was investigated and its capacity was found. In [9], Heegard and El Gamal studied non-causal state information available at the decoder. Also, Costa derived the capacity of a Gaussian single user channel with non-causally CSI at the transmitter [10].

Many researches have been done on multi-terminal channels with side information. Multiple-access channels with states known causally or non-causally at the encoders have been studied recently in [11], [12], [13], [14], [15], and [16]. In [17], an achievable rate for the discrete memoryless relay channel with channel state information non-causally available at the transmitter and the relay is obtained. In [18], by using *code splitting* and Gel'fand-Pinsker method (over-generating codewords) and binning scheme, a lower bound for capacity of relay channel with side information only available at the relay is derived.

Our Work

In this paper, first, we introduce two-user state-dependent multiple-access relay channel with non-causal channel state information at the relay. Using binning scheme, decode and forward strategy, backward decoding and codeword splitting, an inner bound for this channel is derived. Also, we obtain an outer bound for this channel with applying Fano's inequality. These bounds subsume the state dependent discrete memoryless relay channel with non-causal channel state information at the relay.

Paper Organization

The rest of paper is organized as follows: In Section II, system model and definitions are described. In Section III, we express the main result and prove it. Finally, the paper is concluded in Section IV.

Notation

Throughout the paper, we use upper-case letters to denote random variables and lower-case letters to denote deterministic realizations. The probability mass function (p.m.f) of a r.v. X over alphabet set $\mathcal{X}$ is denoted by $p_X(x)$. For brevity, we write $p_X(x)$ as $p(x)$. A sequence of r.v.'s $(X_1, \cdots, X_n)$ over the same alphabet set $\mathcal{X}$ is denoted by X^n. $A_\epsilon^n(X)$ denotes the set of all ϵ-typical n-sequences x^n with respect to the p.m.f $p_X(x)$.

II. System Model and Definition

In this subsection, our communication model and definitions related to it are described. The discrete memoryless two-user MARC with side information consists of input alphabets $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_R$, relay side information alphabet $\mathcal{S}$, and output alphabets $\mathcal{Y}_D$, $\mathcal{Y}_R$ (all alphabets are finite), and a probability transition function $p(y_D, y_R|x_1, x_2, x_R, s)$, where $X_k\,(k = 1, 2)$, and X_R are the channel inputs from the sources and the relay, and Y_R and Y_D are the channel outputs at the relay and destination, respectively. Fig. 1 shows the multiple-access relay channel with side information non-causally available at the relay.

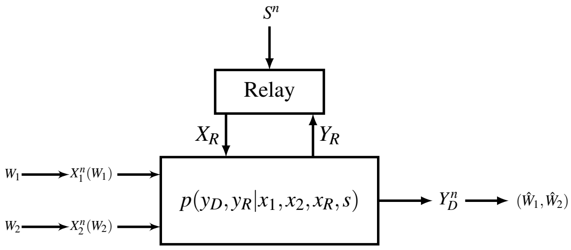

Fig. 1. Multiple-access relay channel with state information non-causally available at the relay.

A $((2^{nR_1}, 2^{nR_2}), n)$ code for the two-user multiple-access relay channel with side information available to the relay consists of

1) two encoding functions, $X_k : \mathcal{W}_k \to \mathcal{X}_k^n, \quad k = 1, 2$ where $\mathcal{W}_k = [1 : 2^{nR_k}]$, $(k = 1, 2)$ called the message sets;
2) a set of relay functions $\{f_t\}_{t=1}^n$ such that
$$X_{R,i} = f_i(Y_R^{i-1}, S^n), \qquad 1 \le i \le n \tag{1}$$
3) and a decoding function, $g : \mathcal{Y}_D^n \to \mathcal{W}_1 \times \mathcal{W}_2$

The average probability of error is defined as follows.

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{(w_1,w_2)} P_r\{g(Y_D^n) \neq (w_1, w_2) | (w_1, w_2) \text{ sent}\}$$

A rate pair (R_1, R_2) is said to be achievable for the multiple-access relay channel with side information non causally available at the relay if there exists a sequence of codes $((2^{nR_1}, 2^{nR_2}), n)$ with $P_e^{(n)} \to 0$ when $n \to \infty$. The joint probability mass function on $\mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{S}^n \times \mathcal{X}_1^n \times \mathcal{X}_2^n \times \mathcal{X}_R^n \times \mathcal{Y}_R^n \times \mathcal{Y}_D^n$ is given by:

$$p(w_1, w_2, s^n, x_1^n, x_2^n, x_R^n, y_R^n, y_D^n) = p(w_1)p(w_2)p(s^n)p(x_1^n|w_1) \cdot p(x_2^n|w_2) \cdot \prod_{i=1}^n p(x_{R,i}|s^n, y_R^{i-1}) p(y_{D,i}, y_{R,i}|x_{1,i}, x_{2,i}, x_{R,i}, s_i)$$

III. Main Theorem

The following theorem provides an inner bound on the capacity region of state-dependent discrete memoryless multiple-access relay channel with side information only at relay.

Theorem 1. *An achievable rate region for state-dependent two-user multiple-access relay channel with informed relay is given by* $\bigcup (R_1, R_2)$:

$$R_1 \le \min\{I(X_1; Y_R|V_1, V_2, X_2, X_R, S), I(X_1, V_1, U_R; Y_D|V_2, X_2) - I(U_R; S|V_1, V_2)\} \tag{2a}$$
$$R_2 \le \min\{I(X_2; Y_R|V_1, V_2, X_1, X_R, S), I(X_2, V_2, U_R; Y_D|V_1, X_1) - I(U_R; S|V_1, V_2)\} \tag{2b}$$
$$R_1 + R_2 \le \min\{I(X_1, X_2; Y_R|V_1, V_2, X_R, S), I(X_1, X_2, V_1, V_2, U_R; Y_D) - I(U_R; S|V_1, V_2)\} \tag{2c}$$

where the union is taken over

$$p(s, v_1, v_2, u_R, x_1, x_2, x_R, y_R, y_D) = p(v_1)p(v_2)p(s)p(x_1|v_1)p(x_2|v_2)p(x_R, u_R|v_1, v_2, s) \cdot p(y_R, y_D|x_1, x_2, x_R, s) \tag{3}$$

Corollary 1. *By setting $V_2 = X_2 = \varnothing$, the achievable rate region for state dependent relay channel with informed relay [18] is obtained.*

Outline of Proof of Theorem 1: To prove the theorem, we apply the achievability proof presented by Ahlswede [19] and Liao [20] for the state-independent multiple-access channel, and *code splitting* mehod proposed by A. Zaidi et al. [18] that in which Gel'fand-Pinsker method has been used by over-generating codewords and binning scheme.

Now, we proceed with proof of achievability using a random coding technique. We consider B blocks, each of n symbols. We use superposition block Markov coding.

Random codebook generation: First, fix a choice of $p(v_1)p(v_2)$ $p(s)p(x_1|v_1)p(x_2|v_2)p(x_R, u_R|v_1, v_2, s)$ and $n \ge 1$.

1) Generate 2^{nR_k}, $(k = 1, 2)$ independent and identically distributed (i.i.d) n-sequence v_k^n, each drawn according to $p(v_k^n) = \prod_{i=1}^n p(v_{k,i})$. Index them as $v_k^n(j_k)$, $j_k \in [1 : 2^{nR_k}]$, $k = 1, 2$.
2) For each $v_k^n(j_k)$ generate 2^{nR_k} conditionally independent n-sequence x_k^n, each drawn according to $p(x_k^n|v_k^n(j_k)) = \prod_{i=1}^n p(x_{k,i}|v_{k,i}(j_k))$ and index them as $x_k^n(m_k, j_k)$, $m_k \in [1 : 2^{nR_k}]$, $k = 1, 2$.
3) For each $(v_1^n(j_1), v_2^n(j_2))$ generate $2^{nI(U_R;S|V_1,V_2)}$ conditionally independent auxiliary n-sequence u_R^n, each drawn according to $p(u_R^n|v_1^n, v_2^n) = \prod_{i=1}^n p(u_{R,i}|v_{1,i}(j_1), v_{2,i}(j_2))$ and index them as $u_R^n(j_1, j_2, l)$, $l \in [1 : 2^{nI(U_R;S|V_1,V_2)}]$.
4) For each $(v_1^n(j_1), v_2^n(j_2), u_R^n(j_1, j_2, l))$ choose a conditionally independent n-sequence x_R^n, drawn according to $p(x_R^n|v_1^n(j_1), v_2^n(j_2), u_R^n(j_1, j_2, l)) = \prod_{i=1}^n p(x_{R,i}|v_{1,i}(j_1), v_{2,i}(j_2), u_{R,i}(j_1, j_2, l))$ and index it as $x_R^n(j_1, j_2, l)$.

In each n-block, $b = 1, 2, \cdots, B$, we use the same set of codebooks:

$$\mathcal{C} = \{v_1^n(j_1), v_2^n(j_2), x_1^n(m_1, j_1), x_2^n(m_2, j_2), x_R^n(j_1, j_2, l), u_R^n(j_1, j_2, l), j_k \in [1 : 2^{nR_k}], m_k \in [1 : 2^{R_k}], k = 1, 2, l \in [1 : 2^{nI(U_R;S|V_1,V_2)}]\}$$

The channel state at relay in block b, $b = 1, \cdots, B + 1$ is denoted by s_b^n.

Encoding: Let $w_{k,b} \in \{1, \cdots, 2^{nR_k}\}$ be the new index to be sent in block b by transmitter k. The transmitter k sends $x_{k,b}^n(w_{k,b}, w_{k,b-1})$, where $w_{k,0} = w_{k,B+1} = 1$.

The relay will have an estimate $\hat{w}_{1,b-1}$ and $\hat{w}_{2,b-1}$ of the previous indices $w_{1,b-1}$ and $w_{2,b-1}$. The relay, by knowing the $(\hat{w}_{1,b-1}, \hat{w}_{2,b-1})$, looks forward to smallest $l_b \in [1 : 2^{n(I(U_R;S|V_1,V_2))}]$ such that $v_{1,b}^n(\hat{j}_1(b) = \hat{w}_{1,b-1})$, $v_{2,b}^n(\hat{j}_{2,b} = \hat{w}_{2,b-1})$, $u_{R,b}^n(w_{1,b-1}, w_{2,b-1}, l_b)$ and s_b^n are jointly typical. Denote this l_b by $l_b^\star = l_b(s_b^n, w_{1,b-1}, w_{2,b-1})$. Then the relay sends $x_{R,b}^n \in A_\epsilon^n(X_{R,b}^n|v_{1,b}^n, v_{2,b}^n, u_{R,b}^n, s_b^n)$. The encoding strategy is shown in Table I.

Decoding and error Analysis: The relay declares that messages

TABLE I
ENCODING STRATEGY

Block 1	Block 2	$\cdots$	Block B+1
$v_{1,1}^n(1)$	$v_{1,2}^n(w_{1,1})$	$\cdots$	$v_{1,B+1}^n(w_{1,B})$
$v_{2,1}^n(1)$	$v_{2,2}^n(w_{2,1})$	$\cdots$	$v_{2,B+1}^n(w_{2,B})$
$x_{1,1}^n(w_{1,1},1)$	$x_{1,2}^n(w_{1,2},w_{1,1})$	$\cdots$	$x_{1,B+1}^n(w_{1,B+1}=1,w_{1,B})$
$x_{2,1}^n(w_{2,1},1)$	$x_{2,2}^n(w_{2,2},w_{2,1})$	$\cdots$	$x_{2,B+1}^n(w_{2,B+1}=1,w_{2,B})$
$u_{R,1}^n(1,1,l_1(s_1^n,1,1))$	$u_{R,2}^n(w_{1,1},w_{2,1},l_2(s_2^n,w_{1,1},w_{2,1}))$	$\cdots$	$u_{R,B+1}^n(w_{1,B},w_{2,B},l_{B+1}(s_{B+1}^n,w_{1,B},w_{2,B}))$
$x_{R,1}^n(1,1,l_1(s_1^n,1,1))$	$x_{R,2}^n(w_{1,1},w_{2,1},l_2(s_2^n,w_{1,1},w_{2,1}))$	$\cdots$	$x_{R,B+1}^n(w_{1,B},w_{2,B},l_{B+1}(s_{B+1}^n,w_{1,B},w_{2,B}))$

of $\hat{w}_{1,b}$ and $\hat{w}_{2,b}$ are transmitted if there is a unique $\hat{w}_{1,b},\hat{w}_{2,b}$ such that

$$\Big(v_{1,b}^n(w_{1,b-1}),v_{2,b}^n(w_{1,b-1}),x_{1,b}^n(\hat{w}_{1,b},w_{1,b-1}),x_{2,b}^n(\hat{w}_{2,b},w_{2,b-1}),\\ x_{R,b}^n(w_{1,b-1},w_{2,b-1}),u_{R,b}^n(w_{1,b-1},w_{2,b-1},l_b^\star),s_b^n,y_{R,b}^n\Big)\\ \in A_\epsilon^n(V_1,V_2,X_1,X_2,X_R,U_R,S,Y_R)$$

It can be shown (see Appendix) that $\hat{w}_{1,b}$=$w_{1,b}$ and $\hat{w}_{2,b}$=$w_{2,b}$ with arbitrarily small probability of error, if n is sufficiently large and

$$\begin{aligned} R_1 &\le I(X_1;Y_R|V_1,V_2,X_2,X_R,S)\\ R_2 &\le I(X_2;Y_R|V_1,V_2,X_1,X_R,S)\\ R_1+R_2 &\le I(X_1,X_2;Y_R|V_1,V_2,X_R,S)\end{aligned}$$

At the destination, the decoder uses the backward decoding strategy [21]. It waits until all transmission are completed. In block $B+1$, the decoder jointly decodes $w_{1,B}$, $w_{2,B}$ from $y_{D,B+1}$. Note that $y_{D,B+1}$ depends on $x_{1,B+1}^n(1,w_{1,B})$, $x_{2,B+1}^n(1,w_{2,B})$, $v_{1,B+1}^n(w_{1,B})$, $v_{2,B+1}^n(w_{2,B})$, $u_{R,B+1}^n(w_{1,B},w_{2,B},l_{B+1}(s_{B+1},w_{1,B},w_{2,B}))$. The receiver decoder declares that $\hat{w}_{1,B}$ and $\hat{w}_{2,B}$ are sent if there exists exactly one $(\hat{w}_{1,B},\hat{w}_{2,B})$ such that

$$\Big(v_{1,B+1}^n(\hat{w}_{1,B}),v_{2,B+1}^n(\hat{w}_{2,B}),x_{1,B+1}^n(1,\hat{w}_{1,B}),\\ x_{2,B+1}^n(1,\hat{w}_{2,B}),u_{R,B+1}^n(\hat{w}_{1,B},\hat{w}_{2,B},\hat{l}_{B+1}),\\ y_{D,B+1}^n\Big)\in A_\epsilon^n(V_1,V_2,X_1,X_2,U_R,Y_D)$$

It can be shown that destination can decode reliably, if n is sufficiently large and

$$\begin{aligned} R_1&\le I(X_1,V_1,U_R;Y_D|V_2,X_2)-I(U_R;S|V_1,V_2)\\ R_2&\le I(X_2,V_2,U_R;Y_D|V_1,X_1)-I(U_R;S|V_1,V_2)\\ R_1+R_2&\le I(X_1,X_2,V_1,V_2,U_R;Y_D)-I(U_R;S|V_1,V_2)\end{aligned}\tag{4}$$

By supposing that the receiver correctly decodes $w_{1,B}$, $w_{2,B}$, it will be able to decode $w_{1,B-1}$ and $w_{2,B-1}$ from $y_{D,B}$ if n is large and (4) is satisfied. This fashion will be continued until all message blocks have been decoded (See Appendix for error analysis). ■

Theorem 2. *The capacity region of a state-dependent two-user multiple-access relay channel with informed relay is a subset of the union of all rate pairs (R_1,R_2) satisfying*

$$\begin{aligned} R_1 \le \min\{&I(X_1;Y_D,Y_R|X_2,X_R,S),\\ &I(X_1,X_R;Y_D|X_2,S)-I(S;X_1|X_2,Y_D)\} \end{aligned}\tag{5a}$$

$$\begin{aligned} R_2 \le \min\{&I(X_2;Y_D,Y_R|X_1,X_R,S),\\ &I(X_2,X_R;Y_D|X_1,S)-I(S;X_2|X_1,Y_D)\} \end{aligned}\tag{5b}$$

$$\begin{aligned} R_1+R_2 \le \min\{&I(X_1,X_2;Y_D,Y_R|X_R,S),\\ &I(X_1,X_2,X_R;Y_D|S)-I(S;X_1,X_2|Y_D)\} \end{aligned}\tag{5c}$$

where the union is taken over all distribution of the form

$$p(s)p(x_1)p(x_2)p(x_R|x_1,x_2,s)p(y_D,y_R|x_1,x_2,x_R,s)\tag{6}$$

Remark 1. *Not knowing the side information at the sources is considered as the minus terms in the second terms of minimization (5).*

Corollary 2. *By setting $X_2=\varnothing$, the outer bound region for state dependent relay channel with informed relay [18] is obtained.*

Proof: The proof of Theorem 2 is omitted for lack of space. ■

IV. CONCLUSION

In this paper, we considered a state-dependent MARC with the channel state information available non-causally at only the relay. We investigated this problem in discrete memoryless case and obtained inner and outer bounds on the channel capacity region. These bounds subsume the relay channel lower bound with non-causal side information at the relay.

APPENDIX

The average probability of error is such that

$$\Pr(\text{Error})\le \sum_{(s^n,v_1^n,v_2^n)\notin A_\epsilon^n(S,V_1,V_2)} \Pr(s^n)\Pr(v_1^n)\Pr(v_2^n)\\ +\sum_{(s^n,v_1^n,v_2^n)\in A_\epsilon^n(S,V_1,V_2)} \Pr(s^n)\Pr(v_1^n)\Pr(v_2^n)\Pr(\text{error}|s^n,v_1^n,v_2^n)$$

There are 14 sources of potential error. Define the following events:

$$E_{0,b}=\Big\{\big(v_{1,b}^n(w_{1,b-1}),v_{2,b}^n(w_{2,b-1}),s_b^n\big)\notin A_\epsilon^n\Big\}$$

$$E_{1,b}=\Big\{\nexists l\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}:\\ \big(v_{1,b}^n(w_{1,b-1}),v_{2,b}^n(w_{2,b-1}),u_{R,b}^n(w_{1,b-1},w_{2,b-1},l_b),s_b^n\big)\in A_\epsilon^n\Big\}$$

$$E_{2,b}=\Big\{\big(v_{1,b}^n(w_{1,b-1}),v_{2,b}^n(w_{2,b-1}),u_{R,b}^n(w_{1,b-1},w_{2,b-1},l_b^\star),\\ x_{1,b}(w_{1,b},w_{1,b-1}),x_{2,b}(w_{2,b},w_{2,b-1}),\\ x_{R,b}^n(w_{1,b-1},w_{2,b-1},l_b^\star),s_b^n,y_{R,b}^n\big)\notin A_\epsilon^n\Big\}$$

$$E_{3,b}=\Big\{\exists w'_{1,b}\in\{1,\cdots,2^{nR_1}\}\text{ s.t. } w'_{1,b}\neq w_{1,b}:\\ \big(v^n_{1,b}(w_{1,b-1}),v^n_{2,b}(w_{2,b-1}),u^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),\\ x_{1,b}(w'_{1,b},w_{1,b-1}),x_{2,b}(w_{2,b},w_{2,b-1}),\\ x^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),s^n_b,y^n_{R,b}\big)\in A^n_\epsilon\Big\}$$

$$E_{4,b}=\Big\{\exists w'_{2,b}\in\{1,\cdots,2^{nR_2}\}\text{ s.t. } w'_{2,b}\neq w_{2,b}:\\ \big(v^n_{1,b}(w_{1,b-1}),v^n_{2,b}(w_{2,b-1}),u^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),\\ x_{1,b}(w_{1,b},w_{1,b-1}),x_{2,b}(w'_{2,b},w_{2,b-1}),\\ x^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),s^n_b,y^n_{R,b}\big)\in A^n_\epsilon\Big\}$$

$$E_{5,b}=\Big\{\exists w'_{1,b}\in\{1,\cdots,2^{nR_1}\}\text{ and }\exists w'_{2,b}\in\{1,\cdots,2^{nR_2}\}\\ \text{s.t. } w'_{1,b}\neq w_{1,b}\text{ and } w'_{2,b}\neq w_{2,b}:\\ \big(v^n_{1,b}(w_{1,b-1}),v^n_{2,b}(w_{2,b-1}),u^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),\\ x_{1,b}(w'_{1,b},w_{1,b-1}),x_{2,b}(w'_{2,b},w_{2,b-1}),\\ x^n_{R,b}(w_{1,b-1},w_{2,b-1},l^\star_b),s^n_b,y^n_{R,b}\big)\in A^n_\epsilon\Big\}$$

$$E_{6,B}=\Big\{\big(v^n_{1,B+1}(w_{1,B}),v^n_{2,B+1}(w_{2,B}),\\ u^n_{R,B+1}(w_{1,B},w_{2,B},l_{B+1}(s^n_{B+1},w_{1,B},w_{2,B})),\\ x_{1,B+1}(1,w_{1,B}),x_{2,B+1}(1,w_{2,B}),y^n_{D,B+1}\big)\notin A^n_\epsilon\Big\}$$

$$E_{7,B}=\Big\{\exists w'_{1,B}\in\{1,\cdots,2^{nR_1}\}\text{ and}\\ l'_{B+1}\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\text{ s.t. } w'_{1,B}\neq w_{1,B}:\\ \big(v^n_{1,B+1}(w'_{1,B}),v^n_{2,B+1}(w_{2,B}),\\ u^n_{R,B+1}(w'_{1,B},w_{2,B},l'_{B+1}),x_{1,B+1}(1,w'_{1,B}),\\ x_{2,B+1}(1,w_{2,B}),y^n_{D,B+1}\big)\in A^n_\epsilon\Big\}$$

$$E_{8,B}=\Big\{\exists w'_{2,B}\in\{1,\cdots,2^{nR_2}\}\text{ and}\\ l'_{B+1}\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\text{ s.t. } w'_{2,B}\neq w_{2,B}:\\ \big(v^n_{1,B+1}(w_{1,B}),v^n_{2,B+1}(w'_{2,B}),\\ u^n_{R,B+1}(w_{1,B},w'_{2,B},l'_{B+1}),x_{1,B+1}(1,w_{1,B}),\\ x_{2,B+1}(1,w'_{2,B}),y^n_{D,B+1}\big)\in A^n_\epsilon\Big\}$$

$$E_{9,B}=\Big\{\exists w'_{1,B}\in\{1,\cdots,2^{nR_1}\}\text{ and }\exists w'_{2,B}\in\{1,\cdots,2^{nR_2}\}\\ \text{and } l'_{B+1}\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\\ \text{s.t. } w'_{2,B}\neq w_{2,B}\text{ and } w'_{2,B}\neq w_{2,B}:\\ \big(v^n_{1,B+1}(w'_{1,B}),v^n_{2,B+1}(w'_{2,B}),u^n_{R,B+1}(w'_{1,B},w'_{2,B},l'_{B+1}),\\ x_{1,B+1}(1,w'_{1,B}),x_{2,B+1}(1,w'_{2,B}),y^n_{D,B+1}\big)\in A^n_\epsilon\Big\}$$

$$E_{10,b-1}=\Big\{\big(v^n_{1,b}(w_{1,b-1}),v^n_{2,b}(w_{2,b-1}),\\ u^n_{R,b}(w_{1,b-1},w_{2,b-1},l_b(s^n_b,w_{1,b-1},w_{2,b-1})),x_{1,b}(1,w_{1,b-1}),\\ x_{2,b}(1,w_{2,b-1}),y^n_{D,b}\big)\notin A^n_\epsilon\Big\}$$

$$E_{11,b-1}=\Big\{\exists w'_{1,b-1}\in\{1,\cdots,2^{nR_1}\}\text{ and}\\ l'_b\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\text{ s.t. } w'_{1,b-1}\neq w_{1,b-1}:\\ \big(v^n_{1,b}(w'_{1,b-1}),v^n_{2,b}(w_{2,b-1}),u^n_{R,b}(w'_{1,b-1},w_{2,b-1},l'_b),\\ x_{1,b}(1,w'_{1,b-1}),x_{2,b}(1,w_{2,b-1}),y^n_{D,b}\big)\in A^n_\epsilon\Big\}$$

$$E_{12,b-1}=\Big\{\exists w'_{2,b-1}\in\{1,\cdots,2^{nR_2}\}\text{ and}\\ l'_b\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\text{ s.t. } w'_{2,b-1}\neq w_{2,b-1}:\\ \big(v^n_{1,b}(w_{1,b-1}),v^n_{2,b}(w'_{2,b-1}),u^n_{R,b}(w_{1,b-1},w'_{2,b-1},l'_b),\\ x_{1,b}(1,w_{1,b-1}),x_{2,b}(1,w'_{2,b-1}),y^n_{D,b}\big)\in A^n_\epsilon\Big\}$$

$$E_{13,b-1}=\Big\{\exists w'_{1,b-1}\in\{1,\cdots,2^{nR_1}\}\text{ and}\\ \exists w'_{2,b-1}\in\{1,\cdots,2^{nR_2}\}\text{ and } l'_b\in\{1,\cdots,2^{nI(U_R;S|V_1,V_2)}\}\\ \text{s.t. } w'_{2,b-1}\neq w_{2,b-1}\text{ and } w'_{2,b-1}\neq w_{2,b-1}:\\ \big(v^n_{1,b}(w'_{1,b-1}),v^n_{2,b}(w'_{2,b-1}),u^n_{R,b}(w'_{1,b-1},w'_{2,b-1},l'_b),\\ x_{1,b}(1,w'_{1,b-1}),x_{2,b}(1,w'_{2,b-1}),y^n_{D,b}\big)\in A^n_\epsilon\Big\}$$

Error Analysis

- By the asymptotic equipartition property (AEP), $\Pr(E_{0,b})\to 0$ as $n\to\infty$.
- To bound $\Pr(E_{1,b})$, we use a standard argument. The probability that $u^n_{R,b}(w_{1,b-1},w_{2,b-1},l_b)$ is jointly typical with s^n_b given $v_{1,b}(w_{1,b-1})$ and $v_{2,b}(w_{2,b-1})$ is greater than $(1-\epsilon)2^{-n(I(U_R;S|V_1,V_2)+\epsilon)}$ for n sufficiently large. There are a total of $2^{nI(U_R;S|V_1,V_2)}$ such u^n_R in each bin. The probability of event $E_{1,b}$, bounded as

  $$\Pr(E_{1,b})\leq[1-(1-\epsilon)2^{-n(I(U_R;S|V_1,V_2)+\epsilon)}]^{2^{nI(U_R;S|V_1,V_2)}}$$

  which tends to zero as $n\to\infty$.
- For the probability of $E_{2,b}$, we note by the Markov lemma ([22, p. 579]), that if $(V^n_1,V^n_2,U^n_R,X^n_1,X^n_2,X^n_R,S^n)$ is strongly jointly typical and $(U_R,V_1,V_2)\to(X_1,X_2,X_R,S)\to Y_R$ forms a Markov chain, then $(V^n_1,V^n_2,U^n_R,X^n_1,X^n_2,X^n_R,S^n,Y^n_R)$ will be strongly jointly typical with high probability. This shows that $\Pr(E_{2,b}|\cap^1_{m=0}E^c_{m,b})\to 0$, as $n\to\infty$.
- The probability of $E_{3,b}$ conditioned on $E^c_{0,b}$, $E^c_{1,b}$, and $E^c_{2,b}$ can be easily bounded using union bound and standard lemma as

  $$\begin{aligned}\Pr(E_{3,b}|E^c_{0,b},E^c_{1,b},E^c_{2,b})&\\ \leq 2^{nR_1}\cdot 2^{-nI(X_1;Y_R,S|V_1,V_2,U_R,X_2,X_R)}&\\ =2^{-n(I(X_1;Y_R,S|V_1,V_2,U_R,X_2,X_R)-R_1)}&\\ \overset{(a)}{=}2^{-n(I(X_1;Y_R|V_1,V_2,U_R,X_2,X_R,S)-R_1)}&\end{aligned}$$

  where (a) follows from the fact that $I(X_1;S|V_1,V_2,U_R,X_2,X_R)=0$ according to (3). The $\Pr(E_{3,b}|E^c_{0,b},E^c_{1,b},E^c_{2,b})$ goes to zero as $n\to\infty$, if $R_1<I(X_1;Y_R|V_1,V_2,U_R,X_2,X_R,S)$. This constraint can be written as

  $$\begin{aligned}R_1&<I(X_1;Y_R|V_1,V_2,U_R,X_2,X_R,S)\\ &=H(Y_R|V_1,V_2,U_R,X_2,X_R,S)\\ &\quad-H(Y_R|V_1,V_2,U_R,X_1,X_2,X_R,S)\\ &\overset{(b)}{=}H(Y_R|V_1,V_2,X_2,X_R,S)\\ &\quad-H(Y_R|V_1,V_2,X_1,X_2,X_R,S)\\ &=I(X_1;Y_R|V_1,V_2,X_2,X_R,S)\end{aligned}$$

  where (b) follows from the fact that $(U_R,V_1,V_2)\to(X_1,X_2,X_R,S)\to Y_R$ forms a Markov chain, and

$p(y_R|v_1,v_2,u_R,x_2,x_R,s) = p(y_R|v_1,v_2,x_2,x_R,s)$ according to (3), because:
It is clear that $(U_R,V_1,V_2) \to (X_1,X_2,X_R,S) \to Y_R$ forms a Markov chain. We can conclude that $U_R \to (V_1,V_2,X_1,X_2,X_R,S) \to Y_R$ forms a Markov chain too, so we have:

$$I(Y_R;U_R|V_1,V_2,X_1,X_2,X_R,S) = 0 \tag{7}$$

On the other hand, according to probability mass function distribution (3), $(U_R,X_R,S) \to (V_1,V_2) \to (X_1,X_2)$ forms a Markov chain, i.e.,

$$I(X_1,X_2;U_R,X_R,S|V_1,V_2) = 0 \Rightarrow$$
$$I(X_1,X_2;X_R,S|V_1,V_2) + I(X_1,X_2;U_R|V_1,V_2,X_R,S) = 0$$

Since sum of two non-negative terms will be zero, if and only if each term is zero, we have

$$I(X_1,X_2;U_R|V_1,V_2,X_R,S) = 0 \tag{8}$$

By adding (7) and (8), and rewriting the result, we have

$$I(X_1,X_2,Y_R;U_R|V_1,V_2,X_R,S) = 0$$

By applying chain role and non-negativity of mutual information, we have

$$I(Y_R;U_R|V_1,V_2,X_2,X_R,S) = 0$$
$$\Longrightarrow p(y_R|v_1,v_2,u_R,x_2,x_R,s) = p(y_R|v_1,v_2,x_2,x_R,s)$$

So, R_1 is bounded as follows,

$$R_1 \leq I(X_1;Y_R|V_1,V_2,X_2,X_R,S)$$

- Proceeding like for event $E_{3,b}$, it can be shown that $\Pr(E_{4,b}|\cap_{m=0}^{2} E_{m,b}^c) \overset{n\to\infty}{\longrightarrow} 0$ if $R_2 < I(X_2;Y_R|V_1,V_2,X_1,X_R,S)$ and $\Pr(E_{5,b}|\cap_{m=0}^{2} E_{m,b}^c) \overset{n\to\infty}{\longrightarrow} 0$ if $R_1 + R_2 < I(X_1,X_2;Y_R|V_1,V_2,X_R,S)$.
- $\Pr(E_{6,B}|\cap_{m=0}^{5} E_{m,b}^c) \overset{n\to\infty}{\longrightarrow} 0$, by the Markov lemma.
- By union bound, the probability of $E_{7,B}$ can be bounded as

$$\begin{aligned}&\Pr(E_{7,B}|\cap_{m=0}^{5} E_{m,b}^c)\\ &\leq 2^{nR_1}\cdot 2^{nI(U_R;S|V_1,V_2)}\cdot 2^{-nI(X_1,V_1,U_R;Y_D|V_2X_2)}\\ &= 2^{-n(I(X_1;Y_R,S|V_1,V_2,U_R,X_2,X_R)-R_1-I(U_R;S|V_1,V_2))}\end{aligned}$$

Thus $\Pr(E_{7,B}|\cap_{m=0}^{6} E_{m,b}^c) \to 0$ as $n \to \infty$, if

$$R_1 \leq I(X_1,V_1,U_R;Y_D|V_2,X_2) - I(U_R;S|V_1,V_2) \tag{9}$$

Similarly, it can be shown that

$$R_2 \leq I(X_2,V_2,U_R;Y_D|V_1,X_1) - I(U_R;S|V_1,V_2)$$
$$R_1+R_2 \leq I(X_1,X_2,V_1,V_2,U_R;Y_D) - I(U_R;S|V_1,V_2)$$

- $\Pr(E_{10,B}|\cap_{m=0}^{9} E_{m,b}^c) \overset{n\to\infty}{\longrightarrow} 0$, by the Markov lemma.
- Proceeding like for event $E_{7,B}$, it can be shown that $\Pr(E_{11,b}|\cap_{m=0}^{6} E_{m,b}^c \bigcap \cap_{m=7}^{9} E_{m,B}^c \bigcap E_{10,b}^c) \overset{n\to\infty}{\longrightarrow} 0$, if

$$R_1 \leq I(X_1,V_1,U_R;Y_D|V_2,X_2) - I(U_R;S|V_1,V_2)$$

Similarly, it can be shown that

$$R_2 \leq I(X_2,V_2,U_R;Y_D|V_1,X_1) - I(U_R;S|V_1,V_2)$$
$$R_1+R_2 \leq I(X_1,X_2,V_1,V_2,U_R;Y_D) - I(U_R;S|V_1,V_2)$$

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