# NetDoFs of the MISO Broadcast Channel with Delayed CSIT Feedback for Finite Rate of Innovation Channel Models

Yohan Lejosne and Dirk Slock†
EURECOM, Mobile Communications Dept.
Campus SophiaTech, 450 route des Chappes
06410 Biot Sophia Antipolis, FRANCE
Email: {yohan.lejosne,dirk.slock}@eurecom.fr

Yi Yuan-Wu
Orange Labs, RESA/WASA
38-40 Rue du General Leclerc
92794 Issy Moulineaux Cedex 9, FRANCE
Email: {yohan.lejosne, yi.yuan}@orange.com

***Abstract*—Channel State Information at the Transmitter (CSIT) is of utmost importance in multi-user wireless networks, in which transmission rates at high SNR are characterized by Degrees of Freedom (DoF, the rate prelog). In recent years, a number of ingenious techniques have been proposed to deal with delayed and imperfect CSIT. However, we show that the precise impact of these techniques in these scenarios depends heavily on the channel model. We introduce the use of linear Finite Rate of Information (FRoI) signals to model time-selective channel coefficients, a model which turns out to be well matched to DoF analysis. Both the block fading model and the stationary bandlimited channel model are special cases of the FRoI channel model (CM). However, the fact that FRoI CMs model stationary channel evolutions allows to exploit one more dimension: arbitrary time shifts. In this way, the FroI CM allows to maintain the DoF unaffected in the presence of CSIT feedback (FB) delay, by increasing the FB rate. We call this Foresighted Channel Feedback (FCFB). We then consider netDoF, by accounting also for the DoF consumed in training overhead and feedback. We work out the details for the MISO broadcast channel (BC), including optimization of the number of users, and exhibit unmatched netDoF performance compared to existing approaches.**

## I. Introduction

In this paper, Tx and Rx denote transmit/transmitter/ transmitting/transmission and receive/receiver/receiving/reception. Interference is undoubtedly the main limiting factor in multi-user wireless communication systems. Tx side or Rx side zero-forcing (ZF) beamforming (BF) or joint Tx/Rx ZF BF (signal space interference alignment (IA)) allow to obtain significant Degrees of Freedom (DoFs) (= multiplexing factor, or rate prelog). These technique require very good Channel State Information at Tx and Rx (CSIT/CSIR). Especially CSIT is problematic since it requires feedback (FB) which involves delay, which may be substantial if FB Tx is slot based. We shall remark here up front that these observations advocate the design of wireless systems in which the FB delay is made as short as possible. In a TDD system this may be difficult but in a FDD system the FB delay can be made as short as the roundtrip delay! These considerations are independent of the fact that we can find ways to get around FB delay, as we elaborate below, because a reduction in FB delay always leads to improvements (be it in terms of DoF, or NetDoF or at finite SNR).

† EURECOM's research is partially supported by its industrial members: ORANGE, BMW Group, Swisscom, SFR, ST Microelectronics, Symantec, SAP, Monaco Telecom, iABG, and also by the EU FP7 NoE NEWCOM#. The research of Orange Labs is partially supported by the EU FP7 project METIS.

## II. Delayed CSIT State of the Art

It therefore came as a surprise that with totally outdated Delayed CSIT (DCSIT), the MAT scheme [1] is still able to produce significant DoF gains for multi-antenna transmission compared to TDMA. In the DCSIT setting, (perfect) CSIT is available only after a FB delay $T_{fb}$ ($T_{fb}$ taken as the unit of time in number of the following schemes). The channel correlation over $T_{fb}$ can be arbitrary, possibly zero. Perfect overall CSIR is assumed (which leads to significant NetDoF reduction due to CSIR distribution overhead [2], [3]). The MISO BC (Broadcast Channel) and IC (Interference Channel) cases of [1] have been extended to some MIMO cases in [4].

Using a sophisticated variation of the MAT scheme, [5] was able to propose an improved scheme for the case where the FB delay $T_{fb}$ is less than the channel coherence time $T_c$ (define as the inverse of the Doppler bandwidth (BW)). Let's focus on the temporal correlation of one channel coefficient $h$. The channel FB leads to an estimate and estimation error: $h = \hat{\hat{h}} + \tilde{\tilde{h}}$ with FB SNR $\frac{\sigma^2_{\hat{\hat{h}}}}{\sigma^2_{\tilde{\tilde{h}}}} = \mathcal{O}(\rho)$ where $\rho$ is the system SNR. At the Tx, on the basis of $\hat{\hat{h}}$, channel prediction over a horizon $T_{fb}$ leads to a prediction with error: $h = \hat{h} + \tilde{h}$ with prediction SNR $\frac{\sigma^2_{\hat{h}}}{\sigma^2_{\tilde{h}}} = \mathcal{O}(\rho^{1-\frac{T_{fb}}{T_c}})$. The scheme of [5] attains for MISO BC or IC with $K = 2$ users a sumDoF = $2(1 - \frac{T_{fb}}{3T_c}) = 2(\frac{2}{3}\frac{T_{fb}}{T_c} + 1 - \frac{T_{fb}}{T_c})$ using a sophisticated combination of analog and digital FB. The scheme is limited to mostly MISO and to $K = 2$. They also consider: imperfect CSIT (apart from delayed) and the DoF region.

It was generally believed that any delay in the feedback necessarily causes a DoF loss. However, Lee and Heath in [6] proposed a scheme that achieves $N_t$ (sum) DoF in the block fading underdetermined MISO BC with $N_t$ transmit antennas

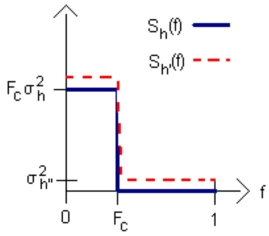


Fig. 1. A bandlimited (BL) Doppler spectrum and its noisy version.

and $K = N_t + 1$ users if the feedback delay is small enough ($\leq \frac{T_c}{K}$). We introduce FRoI channel models and exploit their approximately stationary character to propose a simple ZF scheme based on Foresighted Channel FB (FCFB). The DoF of FCFB ZF are also insensitive to FB delay.

## III. Some Channel Model State of the Art

One category of popular channel models is the (first-order) autoregressive (Gauss-Markov) channel model, see e.g. [7]. However, these models (at finite and especially low order) do not allow perfect prediction and hence do not lead to interesting DoF results. These models are called regular in [8]. The two classical (nonregular) channel models that allow permanent perfect CSIT for Doppler rate perfect channel feedback are block fading and bandlimited (BL) stationary channels. The block fading model dates back to the time of GSM where it was quite an appropriate model for the case of frequency hopping. However, though this model is very convenient for very tractable analysis (e.g. for sinngle-user MIMO [9]), it is inappropriate for DoF analysis which works at infinite SNR and requires exact channel models. Now, whereas exact channel models do not exist, channel models for DoF analysis should at least be good approximations. Indeed, mobile speeds and Doppler shifts are finite. This leads to a strictly BL Jakes Doppler spectrum. However, in the Jakes model, the mobile terminal has a certain speed without ever moving (attenuation, directions of arrival, path delays, speed vector etc. are all constant forever). In reality, the channel evolution constantly evolves from one temporarily BL Doppler spectrum to another, leading to a possibly overall stationary process but that is not BL.

Another aspect is that there is a difference between channel modeling for CSIR only and for CSIT. In the CSIR case, causality is not much of an issue and channel estimation can be done in a non-causal fashion. Hence block processing and associated channel models as in [7] and references therein are acceptable. In the CSIT case however, the CSI needs to be fed back for adaptation of the Tx. Due to the feedback delay, the channel estimation in the CSIT scenario is necessarily causal (case of prediction). Hence different channel models are required.

## IV. The Bandlimited (BL) Doppler Spectrum Case

We assume the Doppler spectrum $S_h(f)$, the spectrum of the process $h$, to be bandlimited to $F_c$, which is the total Doppler Bandwidth (as the channel coefficients are complex, the position of the Doppler spectrum w.r.t. the carrier frequency is less crucial, so we can assume the Doppler support to be $[0, F_c]$ as in Fig. 1; also, $S_h(f)$ is periodic in frequency $f$ with period 1). We denote the coherence time as $T_c = 1/F_c$. After estimation and FB, the noisy channel spectrum is $S_{\widehat{h}}(f) = S_h(f) + S_{\widetilde{\widetilde{h}}}(f) = S_h(f) + \sigma^2_{\widetilde{\widetilde{h}}}$ assuming independent white noise $\widetilde{\widetilde{h}}$. Let $T_{fb}$ be the delay with which the channel estimate $\widehat{\widehat{h}}$ arrives at the Tx for (instantaneous) adaptation of the transmitter. That means that the Tx has to perform channel prediction over a horizon of $T_{fb}$. Then we get for the $T_{fb}$ ahead (infinite order) prediction MSE

$$\widetilde{\sigma}^2 = e^{\int_0^1 \ln S(f)\, df} \sim \sigma_{\widetilde{\widetilde{h}}}^{2(1-F_c T_{fb})} \tag{1}$$

which shows the instability of prediction for BL processes: the MSE grows faster than proportional to the noise variance.

### A. No exact BL model anywhere

In [10], the behavior of (1) is exploited to show the resulting DoF of the 2 user MISO BC. However, what is not mentioned there is that these results correspond to a channel model that needs to be in a range between two extreme models. The one extreme model is block fading over blocks of length $T_{fb}$, with stationary $F_c$-BL evolution of the value of the blocks, and channel feedback every $T_{fb}$. The other extreme is a genuine $F_c$-BL stationary channel model, but then the channel needs to be fed back every sample (which is normally unacceptable in terms of NetDoF)! In [2] still another approach is taken in which block fading over some $T$ is assumed, plus BL stationary evolution between blocks (such that one of the interpretations of [10] corresponds to this with $T = T_{fb}$).

The other popular model is the block fading model of course. In [11], it was shown that the DoFs of [10] can be reproduced very simply in the case of a block fading model, by the MAT-ZF scheme, a simple combination of MAT (during $T_{fb}$, while waiting for the channel FB) and ZF for the rest of the coherence period (see further discussion below). In [12] it was shown in an alternative fashion that the channel FB rate could be reduced w.r.t. [10] by a factor $T_c/T_{fb}$ (equivalent to FB every $T_c$ instead of every $T_{fb}$, as our FRoI approach also indicates, see below). To reproduce these results for the stationary BL case is not easy though, and the scheme of [10] is quite intricate, involving, as in MAT, FB of (residual) interference (now necessarily digital, with superposition coding and sequential decoding).

The models we introduce next allow to retain the simplicity of block fading models, and even go beyond them (by exploiting stationarity).

## V. Linear Finite Rate of Innovation (FRoI) Channel Models (CM)

The linear FroI channel model, which was introduced in [13], can be considered as a filterbank with a single subband. The synthesis filter is $g_k$, and there is an analysis filter $f_k$. The analysis-synthesis cascade leads to

$$\begin{aligned} a_n &= \textstyle\sum_k f_k\, h_{nT_c-k} \\ h_{nT_c+i} &= \textstyle\sum_l a_{n-l}\, g_{lT_c+i}\ , \quad i = 0, 1, \ldots, T_c - 1\ . \end{aligned} \tag{2}$$

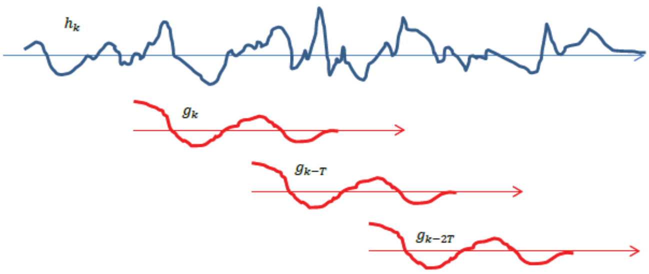


Fig. 2. Finite Rate of Innovation (FRoI) time-varying channel modeling.

Perfect reconstruction for a strictly BL process requires:

$$g_k * f_k = \mathrm{sinc}(\pi k/T)\ . \tag{3}$$

This can be satisfied with e.g. $g_k = \mathrm{sinc}(\pi k/T)\ ,\ f_k = \delta_{k0}$ (Kronecker delta). In the case of an orthogonal filterbank with causal $g_k$, this requires $f_k = g^*_{-k}$, and $(g_k * g^*_{-k})_{k=nT} = \delta_{n0}$ (the convolution $g_k * g^*_{-k}$ (correlation sequence of $g_k$) should be a Nyquist pulse). In this case, if $h_k$ is not a BL signal, the reconstructed signal resulting from applying the FRoI model in (2) would produce the least-squares projection of the signal $h_k$ on the subspace of $F_c$-BL signals [14]. However, this requires $f_k = g^*_{-k}$ (matched filter) to be non-causal! As can be seen from Fig. 2, the optimal computation of coefficient $a_n$ requires the correlation of the signal $h_k$ that follows from the time instant $k = nT_c$ onwards with the basis function $g_k$. This is impractical for the channel feedback application in which both $g_k$ and $f_k$ should be causal, and the computation of $a_n$ should be based (for optimal DoF considerations) on the first sample only of this correlation. Hence $g_0$ plays an important role (can not be small). The optimization of the filters $f_k$, $g_k$ has been considered in [13].

## VI. IA with Foresighted Channel Feedback

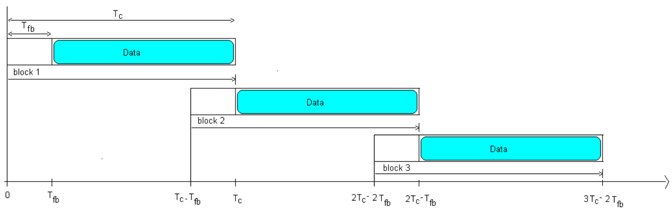


Fig. 3. Foresighted Channel Feedback (FCFB)

The main characteristic of FRoI CMs is that they closely approximate stationary (BL) signals. *This means that if a FRoI CM is a good model, so is an arbitrary time shift of the FRoI model!* This can be exploited to overcome the FB delay as explained in Fig. 3. While the current coherence period is running, as the Channel FB (CFB) is going to take a delay of $T_{fb}$, instead of waiting for the end of the current $T_c$, we start the next coherence period $T_{fb}$ samples early. This means jumping from the subsampling grid of the FRoI model to the shifted subsampling grid of another instance of the same FRoI model. This involves recalculating the (finite number of past) FRoI parameters $a_n$ for the new grid from the past channel evolution on the old grid, plus a new channel estimate at the start of the $T_c$ on the new grid. In this way the FB (sampling) "rate" increases from $\frac{1}{T_c}$ to $\frac{1}{T_c - T_{fb}}$. But the CSIT is available at the Tx all the time, with a channel prediction error SNR proportional to the general SNR.

## VII. Attainable (Sum) NetDoF of MISO BC with FRoI CMs

In order to evaluate the performances that can be expected in actual systems we now account for training overhead as well as the DoF consumption due to the feedback on the reverse link. For the $K$ Rxs to estimate their channel, a common training of length $T_{ct} \geq N_t$ is needed as explained in [2]. To maximize the DoF we take $T_{ct} = N_t$. According to [15], an additional dedicated training of 1 pilot is required when coherent reception is needed resulting in $N_t+1$ symbol periods per block devoted to training in order to perform ZF.

Since we are interested in the DoF consumed by the FB, which is the scaling of the FB rate with $\log_2(P)$ as $P \to \infty$, the noise in the fed back channel estimate can be ignored in the case of analog FB or of digital FB of equivalent rate. The FB can be considered accurate, suffering only from the delay $T_{fb}$. We consider analog output FB, the Rxs directly feed back the training signal they receive and the Tx performs the (downlink) channel estimation. The FB of $N_t$ symbols per user consumes $KN_t$ channel uses on the reverse link.

*1) $ZF_{FCFB}$:* With FB every $T_c - T_{fb}$, the netDoF by performing ZF precoding is then

$$\mathrm{netDoF}(\mathrm{ZF}_{N_t}) \quad = \quad K\left(1 - \frac{2N_t + 1}{T_c - T_{fb}}\right) \tag{4}$$

since with full CSIT, the full DoF can be achieved with ZF [16]. For sake of comparison we concisely review the netDoF yielded by other schemes in the MISO BC with delayed CSIT. When FB is done only every $T_c$, there are always two parts in each block, a first part with outdated CSIT a second part with current CSIT.

*2) Classic ZF:* Performing ZF only when CSIT is available, the netDoF is

$$\mathrm{netDoF}(\mathrm{ZF}_{N_t}) = N_t\left(1 - \frac{T_{fb}}{T_c} - \frac{2N_t + 1}{T_c}\right). \tag{5}$$

*3) TDMA-ZF:* TDMA-ZF is a direct extension of ZF. The only difference being that while the transmitter is waiting for the CSI, and not sending training symbols it performs TDMA transmission since this does not require any CSIT, thus yielding

$$\mathrm{netDoF}(\text{TDMA-ZF}_{N_t}) = \mathrm{netDoF}(\mathrm{ZF}_{N_t}) + \frac{T_{fb}}{T_c} \tag{6}$$

*4) MAT:* The MAT scheme was proposed in [1]. The authors describe an original approach that yields a DoF $\frac{N_t D}{Q}$ with no *current* CSIT at all. Here $\{D, Q\} \in \mathbb{N}^2$ are such that $\frac{1}{1+\frac{1}{2}\cdots\frac{1}{N_t}} = \frac{D}{Q}$, where $D$ is the least common multiple of $\{1, 2, \cdots, N_t\}$ and $Q = DH_{N_t}$ with $H_{N_t} = \sum_{m=1}^{N_t} \frac{1}{m}$. This scheme allows the transmission of $D$ symbols in Q time slots for each user as noted in [17]. To perform this scheme the Rxs not only need to know their channel but also that of some other Rxs (a different subset in each block), resulting in the need for a CSIR distribution.

In [3], FB and training overheads as well as the cost of the CSIR distribution are determined. Assuming $K = N_t$, we get

$$\text{netDoF}(\text{MAT}_{N_t}) = \frac{N_t(T_c - N_t) - \sum_{j=1}^{K} \frac{1}{j}(K-j)(N_t - j + 1)}{H_K T_c + \left(\left(\frac{K-1}{K}\sum_{j=1}^{K} \frac{(K-j)(N_t-j+1)}{j}\right) + H_k - K\right)} \quad (7)$$

*5) MAT-ZF:* The idea behind the MAT-ZF scheme is essentially to perform ZF and superpose MAT only during the dead times of ZF. For that purpose we consider $Q$ blocks of $T_c$ symbol periods and split each block into two parts. The first part, the dead times of ZF, spans $T_{fb}$ symbol periods and the second part, the $T_c - T_{fb}$ remaining symbols. We use the first part of each block to perform the MAT scheme $T_{fb}$ times in parallel. During the second part of each block, ZF is performed.

The sum DoF for the MAT-ZF$_K$ scheme without accounting for the overhead is

$$\text{DoF}(\text{MAT-ZF}_{N_t}) = N_t\left(1 - \frac{(Q-D)T_{fb}}{QT_c}\right).$$

Indeed, per user, in $QT_c$ channel uses, the ZF portion transmits $Q(T_c - T_{fb})$ symbols, whereas the MAT scheme transmits $DT_{fb}$ symbols.

The net DoFs yielded by this scheme is determined in [3],

$$\text{netDoF}(\text{MAT-ZF}_{N_t}) = \text{netDoF}(\text{ZF}_{N_t}) + \frac{T_{fb}}{T_c}\frac{N_t}{(H_{N_t} + \delta)} \quad (8)$$

where $\delta = \frac{\frac{K-1}{K}\sum_{j=1}^{K}\frac{D(K-j)(N_t-j+1)}{j})}{DT_{fb}}$ i.e., the netDoF of ZF plus an additional term, the DoF brought about by MAT but decreased by a factor due to the CSIR distribution.

*6) ST-ZF:* Lee and Heath [6] proposed a scheme to achieve $N_t$ DoF in the MISO BC with $K = N_t + 1$ users when $\gamma = \frac{T_{fb}}{T_c} \leq \frac{1}{K}$. For $\gamma < \frac{1}{K}$ $N_t$ DoF can also be reached by doing ZF the remaining time. We refer to this scheme as ST-ZF since it is a space-time (ST) precoding, which is combined with ZF for $\gamma < \frac{1}{K}$.

In [3] the net DoF yielded by the ST-ZF scheme is determined. Actually two values are proposed depending on how some data needed at the receiver is transmitted. The two variants, ST-ZF and ST-ZF2 being adapted for different values of the feedback delay. The net multiplexing gain of the first variant is

$$\text{netDoF}(\text{ST-ZF}_{N_t}) = N_t\left(1 - \frac{3(N_t+1)}{T_c}\right) \quad (9)$$

as long as $\frac{T_{fb}+1}{T_c - 2(N_t+1)} \leq \frac{1}{K} \Leftrightarrow T_c \geq K(T_{fb}+3)$ since ST-ZF needs a part with CSIT that is $K - 1 = N_t$ times longer than the no current CSIT part. With the second variant the netDoF is

$$\text{netDoF}(\text{ST-ZF2}_{N_t}) = \frac{N_t\left(1 - \frac{2(N_t+1)}{T_c}\right) + \text{netDoF}(\text{ZF}_{N_t})\frac{KN_t}{T_{fb}+1}}{1 + \frac{KN_t}{T_{fb}+1}} \quad (10)$$

as long as $\frac{T_{fb}+1}{T_c - (N_t+1)} \leq \frac{1}{K}$.

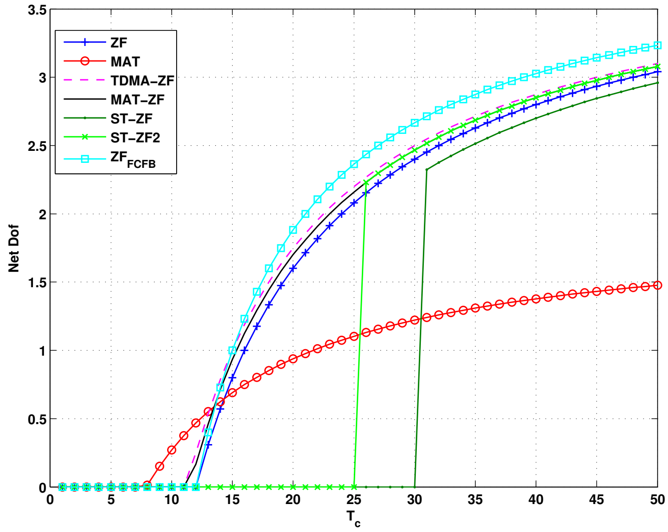


Fig. 4. NetDoF of ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA for $N_t = 4$, $T_{fb} = 3$ as a function of $T_c$.

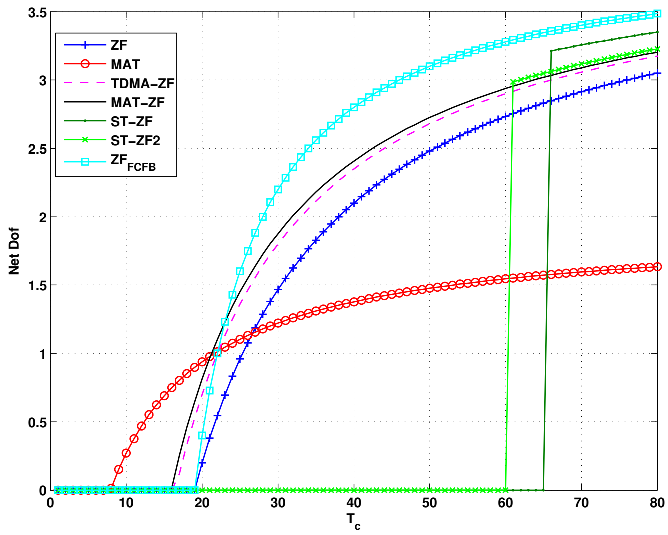


Fig. 5. NetDoF of ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA for $N_t = 4$, $T_{fb} = 10$ as a function of $T_c$.

## VIII. Numerical Results

In Fig. 4 we plot the netDoF provided by ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, TDMA and ST-ZF for $N_t = 4$, $T_{fb} = 3$ as a function of $T_c$ using (4) for ZF$_{FCFB}$, (5) for ZF, (7) for MAT, (6) for TDMA-ZF, (8) for MAT-ZF, (9) for ST-ZF and (10) for ST-ZF 2. A discussion regarding all the schemes except ZF$_{FCFB}$ is already conducted in [3], here we observe that as soon as $T_c > 15$ ZF$_{FCFB}$ outperforms all the other schemes. We also note that for $T_c < 15$ all schemes yield less than 1 net DoF meaning that simple TDMA transmission would actually be better under these conditions. In Fig. 5 the same curves are plotted for $T_{fb} = 10$, a similar behavior is observed and the gap between ZF$_{FCFB}$ and the other schemes is wider.

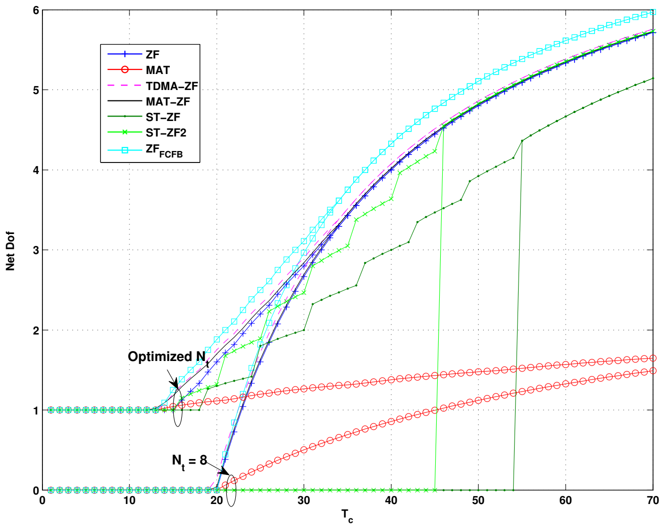

Fig. 6. NetDoF of ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA and their optimized variants for $N_t = 8$, $T_{fb} = 3$ as a fn of $T_c$.

### A. *Optimization of $K$, the number of users*

As it was noticed in [18] the number of users $K$ (and hence active antennas $N_t$) needs to be optimized to find the right channel learning/using compromise because serving more users means a larger DoF but also larger overhead. All the net DoF of the schemes we reviewed reach a single maximum as a function of the number of antennas. For the scheme we proposed, ZF$_{FCFB}$, the net DoF are a simple quadratic function in $N_t$

$$f(N_t) = -\frac{2}{T_c - T_{fb}} N_t^2 + \left(\frac{T_c - T_{fb} - 1}{T_c - T_{fb}}\right) N_t$$

which is maximized for $N_t = \frac{T_c - T_{fb} - 1}{4}$. So in Fig. 4 for $T_c = 16$ better performances could be achieved with $N_t = \frac{16-3-1}{4} = 3$. To each scheme we associate its optimized version, in which the number of active antennas is optimized, either analytically or empirically to assure the maximum net DoF. In Fig. 6 we observe the net DoF of all considered schemes and of their optimized version for $K = 8$, $T_{fb} = 3$ as a function of $T_c$. We notice that if the optimization of the number antennas results in a gain for all schemes it also confirms that ZF$_{FCFB}$ outperforms all the other schemes soon after having only one active antenna and one served user (simple TDMA) is not optimal anymore.

## IX. Beyond FRoI: Predictive Rate Distortion

The FRoI channel model is just one way to get a certain rate (DoF) for a distortion of $O(\sigma_v^2)$ (noise level). More generally, the distortion in a predicted channel at the Tx can contain a combination of approximation error and noise due to estimation and feedback. What is needed here is *predictive* rate-distortion (R-D) theory. Such theory would e.g. allow to determine which estimation and FB DoF are required to get the prediction distortion at the Tx down to the level of the noise (with the channel distortion at the Tx being reflected in an induced equivalent noise at the Rx). However, the proper evaluation of such R-D theory obviously depends heavily on the channel model, whereas all existing channel models are too approximate to allow a solid high SNR DoF analysis. Some related work appears in [19] where no causality is imposed and where the analysis filter $f$ is not optimized (fixed), and in [20] and references therein, where causal (but not predictive) R-D is developed for the application of FB in control systems.

## References


[1] M. A. Maddah-Ali and D. N. C. Tse, "Completely Stale Transmitter Channel State Information is still very Useful," *IEEE Trans. Info. Theory*, Jul. 2012.

[2] M. Kobayashi and G. Caire, " On the net DoF comparison between ZF and MAT over time-varying MISO broadcast channels," in *Proc. ISIT*, Boston, MA, USA, Jun. 2012.

[3] Y. Lejosne, D. Slock, and Y. Yuan-Wu, "Net Degrees of Freedom of Recent Schemes for the MISO BC with Delayed CSIT and Finite Coherence Time," in *Proc. WCNC*, Shanghai, China, Apr. 2013.

[4] C. Vaze and M. Varanasi, "The degrees of freedom region and interference alignment for the mimo interference channel with delayed csit," *Trans. Info Theory*, july 2012.

[5] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, "Degrees of Freedom of Time Correlated MISO Broadcast Channel with Delayed CSIT," *IEEE Trans. on Information Theory*, Feb. 2012.

[6] N. Lee and R. W. Heath Jr., "Not too delayed csit achieves the optimal degrees of freedom," in *Proc. Allerton*, Monticello, IL, USA, Oct. 2012.

[7] A. Kannu and P. Schniter, "On the Spectral Efficiency of Noncoherent Doubly Selective Block-Fading Channels," *IEEE Trans. Inf. Theory*, Jun. 2010.

[8] R. Etkin and D. Tse, "Degrees of Freedom in Some Underspread MIMO Fading Channels," *IEEE Trans. Inf. Theory*, Apr. 2006.

[9] L. Zheng and D. Tse, "Communication over the Grassmann Manifold: a Geometric Approach to the Noncoherent Multiple-Antenna Channel," *IEEE Trans. Inf. Theory*, Feb. 2002.

[10] M. Kobayashi, S. Yang, D. Gesbert, and X. Yi, "On the degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," in *IEEE Int'l Symp. Info. Theory*, Jul. 2012.

[11] Y. Lejosne, D. Slock, and Y. Yuan-Wu, "Degrees of Freedom in the MISO BC with Delayed-CSIT and Finite Coherence Time: a Simple Optimal Scheme," in *IEEE Int'l Conf. on Signal Proc., Commun. and Control (ISCCSP)*, Hong Kong, Aug. 2012.

[12] J. Chen and P. Elia, "Can Imperfect Delayed CSIT be as Useful as Perfect Delayed CSIT? DoF Analysis and Constructions for the BC," in *IEEE Allerton Conf. Comm's Control and Comput.*, Monticello, IL, USA, Sept. 2012.

[13] Y. Lejosne, D. Slock, and Y. Yuan-Wu, "Finite Rate of Innovation Channel Models and DoF of MIMO Multi-User Systems with Delayed CSIT Feedback ," in *Proc. ITA*, San Diego, CA, USA, Feb. 2013.

[14] M. Vetterli, J. Kovačević, and V. Goyal, *Foundations of Signal Processing*. http://fourierandwavelets.org/, 2013.

[15] U. Salim and D. T. M. Slock, "How much feedback is required for TDD multi-antenna broadcast channels with user selection ?" *EURASIP Journal on Advances in Signal Processing*, May 2010.

[16] N. Jindal, "Mimo broadcast channels with finite-rate feedback," *Information Theory, IEEE Transactions on*, vol. 52, no. 11, pp. 5045 –5060, nov. 2006.

[17] J. Xu, J. Andrews, and S. Jafar, "Miso broadcast channels with delayed finite-rate feedback: Predict or observe?" *Wireless Communications, IEEE Transactions*, vol. 11, no. 4, pp. 1456–1467, 2012.

[18] Y. Lejosne, D. Slock, and Y. Yuan-Wu, "Degrees of Freedom in the MISO BC with delayed-CSIT and Finite Coherence Time: Optimization of the number of users," in *Proc. NetGCooP*, Avignon, France, Nov. 2012.

[19] E. Aguerri, D. Gunduz, A. Goldsmith, and Y. Eldar, "Distortion-Rate Function for Undersampled Gaussian Processes," in *IEEE Int'l Symp. on Information Theory (ISIT)*, Istanbul, Turkey, Jul. 2013, submitted, also ITA2013 talk.

[20] E. Silva, M. Derpich, and J. Østergaard, "Rate-Distortion in Closed-Loop LTI Systems," in *IEEE Iformation Theory Workshop (ITA)*, San Diego, CA, USA, Feb. 2013.