The Index Coding Problem: A Game-Theoretical Perspective

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***Abstract*—The Index Coding problem has recently attracted a significant interest from the research community. In this problem, a server needs to deliver a set of packets to a group of wireless clients over a noiseless broadcast channel. Each client requests a subset of packets and has another subset given to it as side information. The objective is to satisfy the demands of all clients with the minimum number of transmissions.**

In this paper, we study the Index Coding problem from the game-theoretic perspective. We assume that each client is selfish and has a hidden private value for each packet it requests. The objective of the server is to maximize the value of social welfare that captures the trade-off between values of the transmitted packets and the transmission cost incurred by the server. The transmission process is decided through an auction in which the clients are required to submit bids to the server.

Our goal is to design a *truthful* auction scheme that provides an incentive for each client to bid the true value of the packets and maximizes the value of the social welfare. The key challenge in this context is to determine the encoding functions of the transmitted packets. Since finding an optimal encoding function is an $\mathcal{NP}$-hard problem, we propose efficient algorithms that identify the encoding functions as well as a payment scheme that provide an approximate solution and guarantee truthfulness.

I. Introduction

The Index Coding problem is one of the basic problems in wireless network coding. Recently, this problem has attracted a significant interest from the research community (see e.g., [1, 2]). An instance of the Index Coding problem includes a server, a set of wireless clients, and a set $P = \{p_1, ..., p_m\}$ of m packets that need to be delivered to clients. Each client is interested in a certain subset of packets in P and has a (different) subset of packets given to it as side information. The server can transmit the original packets or their combinations to clients via a noiseless broadcast channel. The goal is to find a transmission scheme that requires the minimum number of transmissions to satisfy the requests of all clients.

Fig. 1 depicts an instance of the Index Coding problem, where a server needs to deliver four packets $P = \{p_1, \cdots, p_4\}$ to four clients. It can be verified that the demands of all clients can be satisfied by broadcasting three packets: p_1+p_2, p_2+p_3, and p_4 (all operations are over $GF(2)$). Note that the traditional approach (without coding) requires the transmissions of all four packets $p_1, \cdots, p_4$.

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The prior works on the Index Coding problem have focused on developing algorithms, establishing rate bounds, and analyzing the computational complexity of the problem [3–8]. In contrast to the original Index Coding problem where the goal of the server is to satisfy the demands of all clients, we focus on settings where the server's decisions are driven by the values of the packets transmitted to the clients and by the transmission cost. This is a new problem in the intersection of the coding theory and game theory and is related to the framework of *mechanism design* with *social choices* [9].

Intuitively, since each transmission at the server incurs a certain cost, the server may decide to transmit a packet only when the packet is important enough for the clients. Thus, our goal is to maximize the *social welfare*, i.e., the total value of the packets the clients are able to decode minus the total cost of transmitting the packets by the server. The social welfare reflects the *positive externalities* in terms of clients' valuations of packets and the *negative externalities* related to the transmission costs of server.

To calculate the social welfare, the server needs to know, for each client in the system, the client's valuation of all packets it requests. This information is usually private and selfish clients could not be expected to reveal this information to the server. Accordingly, we aim to design a *tractable* auction mechanism that provides an incentive for each client to reveal the true

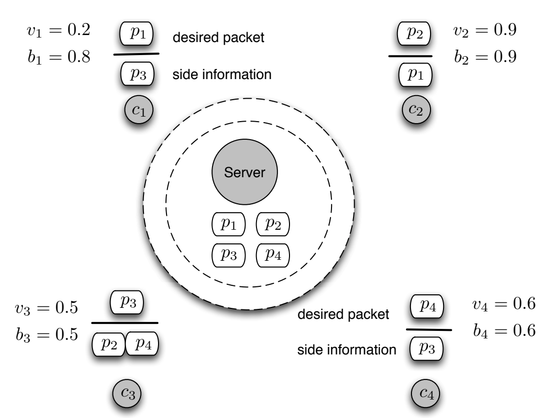

Fig. 1. An instance of the Index Coding problem. Client $c_i, i = 1, \ldots, 4$ requests packet p_i. The side information sets of clients $c_1, \ldots, c_4$ are $\{p_3\}$, $\{p_1\}$, $\{p_2, p_4\}$, and $\{p_3\}$, respectively; v_i and b_i denote the valuation and bid of packet p_i, respectively.

valuation of its packets.

As part of the auction, the clients submit *bids* to the server, each bid specifies the amount of money the client is willing to pay in order to obtain a packet it wants. Based on the auction mechanism, the server decides whether to accept or reject the bids. The auction mechanism includes two main components, namely the *coding algorithm* and *payment function*. The coding algorithm determines which packets are transmitted over the channel and how they are encoded. The payment function determines the amount of money the client will pay to the server for each packet it is able to decode.

Contributions. A classical solution for the problem of maximizing social welfare in the presence of selfish clients is to use the *Vickery-Clarke-Groves* (VCG) mechanism [10–12]. Such mechanism was shown to be *truthful* in the sense that each client maximizes its own utility by bidding its true valuations of packets. Unfortunately, finding VCG based encoding functions for our problem is an intractable ($\mathcal{NP}$-hard) optimization problem. Accordingly, we present an alternative auction mechanism which is both truthful and computationally efficient. Our mechanism includes a non-VCG based payment scheme that achieves the truthfulness property in the presence of the approximation solution for the problem.

II. Model

We begin with the definition of the Index Coding problem. The input to the problem includes a server S, a set of n wireless clients $\Lambda = \{c_1, \cdots, c_n\}$, and a noiseless wireless broadcast channel. The server has a set of m packets $P = \{p_1, \cdots, p_m\}$ that need to be distributed to clients in Λ. We assume, without loss of generality, that each client requests a single packet in P (i.e., $n = m$) and has access to a subset of packets in P as side information. Indeed, a client that wants more than one packet can be substituted by multiple clients that share the same side information set. In particular, each client $c_i \in \Lambda$ is characterized by the ordered pair (w_i, H_i), where $w_i \in P$ is the packet requested by c_i and $H_i \subseteq P$ is the set of packets available to c_i as side information. We assume that each packet p_i represents an element of the Galois field $GF(q)$.

We assume that each client $c_i \in \Lambda$ has the internal (private) valuation v_i for the packet w_i it requests. The transmission process includes an auction, in which each client submits a bid b_i for packet w_i. We denote by $V = \{v_1, \cdots, v_n\}$ and $B = \{b_1, \cdots, b_n\}$ the arrays that include the internal valuations and bids of the clients. Based on the bid vectors, the server identifies linear combinations that will be transmitted over the channel. In this paper, we focus on the *scalar-linear coding* schemes in which each packet

$$q_i = \sum_{j=1}^{m} g_{i,j} p_j$$

transmitted by server at the iteration i, $1 \leq i \leq \eta$, is a linear combination of packets in P. Here, η is the total number of transmissions made by the server and

$$g_i = [g_{i,1}, \cdots, g_{i,m}] \in GF(q)^m$$

is the *encoding vector* for the iteration i. We denote by $\Gamma = [g_i]$ the *encoding matrix* whose rows correspond to encoding vectors of the packets transmitted over the channel. A *sparse code* [13] is the linear code in which each transmitted packet from the server is a linear combination of *at most two* packets in P. Without loss of generality, we assume that the cost of transmitting a packet by the server is equal to one, hence the total cost incurred by the server is equal to η.

The goal of the server is to maximize the *social welfare*, defined as

$$\sum_{i=1}^{n} v_i \cdot \theta_i - \eta,$$

where θ_i is the indicator function that specifies whether client c_i is able to decode the required packet w_i. In particular, $\theta_i = 1$ if there exists a linear decoding function r_i such that $w_i = r_i(q_1, \cdots, q_\eta, H_i)$; otherwise, $\theta_i = 0$. We denote by $\Theta = \{\theta_1, \cdots, \theta_n\}$ the vector of indicator functions.

For example, in the setting depicted in Fig. 1, the social welfare of broadcasting the solution to the Index Coding problem, i.e. $\{p_1+p_2; p_2+p_3; p_4\}$, is $0.2+0.9+0.5+0.6-3 = -0.8$, while the social welfare of the sequence consisting of a single combination $\{p_3+p_4\}$ has a higher value of $0.5+0.6-1 = 0.1$. Thus, transmitting a single combination p_3+p_4 would be more desirable than transmitting three packets $\{p_1+p_2; p_2+p_3; p_4\}$ from the server's point of view.

The payment function is an important part of the auction mechanism. Server determines the amount of payment ϕ_i that each client needs to pay for the obtained packet. We denote by $\Phi = \{\phi_1, \cdots, \phi_n\}$ the payment vector for all clients. The value of ϕ_i is a function of B and the encoding matrix Γ (the latter determines the vector of indicator functions Θ). We assume that the auction mechanism, including the algorithm for determining the encoding function and the payment function is known to all the parties.

We also assume that every client c_i is selfish and chooses its best bidding policy b_i that maximizes its utility defined by $u_i(b_i, B_{-i}) = v_i \cdot \theta_i - \phi_i$, where $B_{-i} = B \setminus \{b_i\}$. Note that $u_i(b_i, B_{-i})$ depends on the bids of all clients as well as the server's mechanism of choosing the encoding matrix and the payment function. We say that a mechanism is *truthful* if $u_i\big(v_i, B_{-i}\big) \geq u_i\big(\hat{b}_i, B_{-i}\big)$ for all $\hat{b}_i$ and B_{-i}. That is, regardless of the bids submitted by other clients, the utility of client c_i is maximized when $b_i = v_i$.

In this paper, we design the encoding matrix and the payment function that satisfies the following two conditions: (i) every client is incentivized to report its true valuation of the desired packet (i.e., truthful property) and (ii) the social welfare is maximized.

III. VCG-based mechanism design

In this section, we present a truthful mechanism based on the celebrated Vickrey-Clarke-Groves (VCG) approach [10–12].

Our scheme uses the following objective function

$$w(B, \Gamma) = \sum_{i=1}^{n} b_i \cdot \theta_i - \eta.$$

Note that this function is similar to the social welfare, but instead of vector of bids B we use a vector of packet valuations V. In addition, we define $w_{-i}(B,\Gamma) = \sum_{j \neq i} b_j \cdot \theta_j - \eta$. In our mechanism, the server chooses the encoding matrix that maximizes the value of $w(B,\Gamma)$ for the given bids B.

VCG-coding scheme: Choose the encoding matrix Γ^* that maximizes the value of $w(B,\Gamma)$, i.e.,

$$\Gamma^* = \arg\max_{\Gamma} w(B,\Gamma). \tag{1}$$

If there are multiple encoding functions that maximize $w(B,\Gamma)$, we choose one that satisfies the larger number of clients. Note that without loss of generality, we can assume that $v_i \in [0,1]$, since client c_i is assured to get the desired packet when submitting the bid $b_i = 1$.

Our scheme uses the following payment function ϕ_i:

VCG-payment function: The client c_i is charged as follows.

$$\phi_i = \max_{\Gamma} w_{-i}(B,\Gamma) - w_{-i}(B,\Gamma^*). \tag{2}$$

The first term in Equation (2) corresponds to the maximum value of the objective function when client c_i is removed, while the second term represents the objective function for all clients excluding c_i when the optimal encoding matrix Γ^* determined by Equation (1) is employed. The VCG-payment function charges the "externality" of client c_i, i.e., the decrease in optimal objective function when client c_i is included in the system.

Proposition 1.

1) *The VCG-coding scheme associated with the VCG-payment function is a truthful mechanism, i.e. $B = V$. Moreover, the pricing function in Equation (2) is non-negative, and the utilities of all clients are non-negative, i.e., $\phi_i \leq v_i$.*
2) *The VCG-coding scheme associated with the VCG-payment function maximize the value of social welfare.*

The proof of Proposition 1 follows from the properties of the VCG mechanism [9] and is omitted due to space constraints.

The problem of finding an optimal VCG encoding matrix Γ^* is an interesting combinatorial problem by itself. Unfortunately, this problem is computationally intractable. More specifically, for the instance of the problem in which the bid of each client b_i is equal to one, there exists an optimal solution Γ^* that satisfies all clients, since the client's bid is equal to the cost of transmitting a single packet. Then, the problem of maximizing the value of $w(B,\Gamma)$ (where $w(B,\Gamma) = n - \eta$) is equivalent to the problem of minimizing the required number of transmissions to satisfy all clients, which is $\mathcal{NP}$-hard.

IV. Approximation Algorithm

In this section, we present an approximation algorithm for the special *multiple unicast* case, in which every packet is required by exactly one client. We also focus on finding a solution that uses *sparse codes*. With sparse coding, both encoders and decoders can be implemented in a simpler manner which has a significant advantage for practical applications.

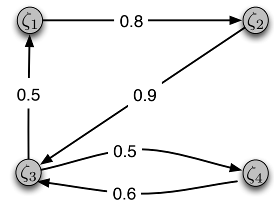

Fig. 2. Weighted dependency graph for the instance of the multiple unicast problem in Fig. 1

Our scheme uses the notion of a *weighted dependency graph* described below.

Definition 2. The weighted dependency graph is a directed graph $G(V,A)$ defined as follows:

- For each client $c_i \in \Lambda$, there is a corresponding vertex ζ_i in V.
- For any two clients c_i and c_j such that $w_i \in H_j$, there is a directed arc $(\zeta_i, \zeta_j) \in A$.
- The weight γ_a of arc $a = (\zeta_i, \zeta_j)$ is equal to b_i.

Fig. 2 illustrates the weighted dependency graph $G(V,A)$ corresponding to the instance of the problem depicted in Fig. 1. Note that for each cycle $C \in G(V,A)$, the server can save one transmission. That is, for a cycle C of length $|C|$, all of the clients that correspond to its vertices can be satisfied using $|C|-1$ transmissions. In particular, the clients that correspond to the vertices of C share the transmission cost of $|C|-1$. In the example depicted in Fig. 2 we have a cycle $(\zeta_1,\zeta_2,\zeta_3)$ that involves three clients. It is easy to verify that these three clients can be satisfied by two transmissions: $p_1 + p_2$ and $p_2 + p_3$. Note that if there is no cycle in $G(V,A)$ then a sparse code cannot improve the value of $w(B,\Gamma)$.

Note that reduced transmission cost results in a higher value of the social welfare. Unfortunately, even with sparse coding, the problem of finding an optimum encoding function under the VCG scheme is intractable.

Lemma 3. *In multiple unicast scenario the problem of finding the a sparse code Γ that maximizes the value of $w(B,\Gamma)$ is $\mathcal{NP}$-hard.*

The proof of Lemma 3 uses the reduction from the cycle packing problem and follows the same lines as the complexity proof in [13]. The details are omitted due to the space constraints.

A. Approximation Algorithm

In this section, we present an alternative mechanism that uses Algorithm 1 to determine encoding functions and Algorithm 2 to determine clients payments. The main idea of our approach is to find an approximation solution to the problem of maximizing $w(B,\Gamma)$ and then use a pricing scheme that uses this approximation solution to motivate all clients to reveal the true valuations of the required packets.

We define the weight of cycle C in $G(V,A)$ as

$$\gamma(C) = \sum_{a \in C} \gamma_a - (|C| - 1),$$

Algorithm 1:

```
   input  : Bids vector B and side information H_i for all clients
   output: Encoding matrix Γ*
1  Construct the weighted dependency graph G(V, A);
2  Define the cost λ_a of arc a ∈ A by λ_a = 1 − γ_a;
3  S_1, S_2 ← ∅
4  while There is a cycle in G(V, A), with the cost less than or equal to
   one do
5      Find the cycle C in G(V, A) with the smallest cost;
6      Find |C| − 1 linear combinations that satisfy clients corresponding
       to vertices in C;
7      Add |C| − 1 corresponding vectors as new rows in Γ*;
8      S_1 ← S_1 ∪ {c_i : ζ_i ∈ C};
9      Remove all vertices of C, along with incident arcs, from the
       graph.
10 end
11 for i ← 1 to n do
12     if b_i = 1 but c_i ∉ S_1 then
13         Add a singleton vector that corresponds to w_i as a new row
           in Γ*;
14         S_2 ← S_2 ∪ {c_i};
15     end
16 end
```

where the first term is the sum of all bids and the second term is the total transmission cost of all packets that correspond to the arcs of this cycle.

Let $\mathbf{C}$ be a set of vertex-disjoint cycles in $G(V,A)$. Then, let Γ be the encoding function that includes $\sum_{C\in\mathbf{C}}(|C|-1)$ linear combinations that correspond to the cycles in $\mathbf{C}$ ($|C|-1$ linear combinations for each cycle $C \in \mathbf{C}$). Note that Γ satisfies all clients that correspond to the vertices of $\mathbf{C}$. Note also that

$$w(B,\Gamma) = \sum_{C\in\mathbf{C}} \gamma(C).$$

The idea of Algorithm 1 is to iteratively identify and remove a maximum weight cycle. We note that $\gamma(C)$ can be written as

$$\gamma(C) = 1 - \sum_{a\in C}(1-\gamma_a).$$

Thus, in this case, a maximum weight cycle with respect to weight $\gamma(C)$ corresponds to the minimum cost cycle with respect to cost $\sum_{a\in C}(1-\gamma_a)$. Therefore, we define the *cost* of arc $a \in A$ by $\lambda_a = 1-\gamma_a$ and the cost of cycle C by

$$\lambda(C) = \sum_{a\in C}\lambda_a.$$

The algorithm finds the minimum cost cycle in Line 5. To this end, we can use well-known polynomial time algorithms such as Floyd-Warshall algorithm. After such a cycle is identified, it is removed from the graph in Line 9. The condition in Line 4 guarantees that the maximum weight cycle in the remaining graph $G(V,A)$ (i.e., the minimum cost cycle) has a non-negative weight. Line 13 ensures that the clients whose the bid is equal to the transmission cost are served. Such clients are then included in set $\mathbf{S}_2$. Set $\mathbf{S}_1$ contains the clients that correspond to the cycles chosen in Line 5. Both set $\mathbf{S}_1$ and $\mathbf{S}_2$ will be used by Algorithm 2 in order to determine the payments of the clients. Note that all procedures in Algorithm 1 require a polynomial number of steps, hence Algorithm 1 can be executed in polynomial time.

Unfortunately, the approximate solution Γ^* identified by Algorithm 1 cannot be used for the VCG-payment functions specified by Equation (2). In particular, the following example shows that such combination might not satisfy the truthfulness property.

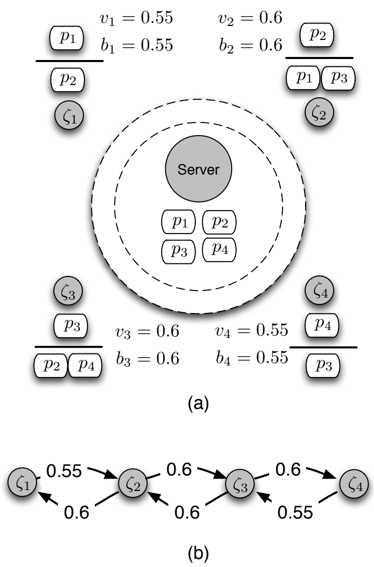

Fig. 3. (a) Counter-example of the truthful property for the greedy-VCG-coding scheme accompanied with the VCG-payment function (b) The weighted dependency graph

Example 4. Consider the instance of the problem depicted in Fig. 3. We show that client c_1 does not have an incentive to reveal the true valuation of its packets. Suppose that client c_1 submits the bid $b_1 = 0.55$ equal to its internal value of packet p_1. In this case, Algorithm 1 will identify the cycle (ζ_2, ζ_3) and output the corresponding solution $p_2 + p_3$. In this case, the utility u_1 of c_i is equal to zero.

Now, suppose that c_1 submits bid $b_1 = 0.7$. In this case, Algorithm 1 will transmit the sequence of packets $\{p_1 + p_2; p_3 + p_4\}$ that corresponds to cycles (ζ_1, ζ_2) and (ζ_3, ζ_4), respectively. According to Equation (2), the payment of client c_1 is equal to $\phi_1 = (0.6+0.6-1)-(0.6+0.6+0.55-2) = 0.45$. In this case, the utility of client c_1 is equal to 0.1, which is bigger than the case in which the client is truthful. ◀

B. Payment scheme

In this section, we propose a payment scheme for our algorithm. We start with a definition. We say that a coding algorithm is *monotone* if, for any B_{-i}, there exists a single *critical value* $\bar{b}_i$ such that c_i gets the desired packet (i.e. $\theta_i = 1$) when $b_i \geq \bar{b}_i$; otherwise, c_i does not get the packet it needs.

Using the results of *Mualem* and *Nisan* [14] it can be shown that if the coding mechanism is monotone and the payment scheme is based on the critical value (i.e., each client is required to pay the critical value $\bar{b}_i$), then the resulting mechanism is truthful. We prove in Lemma 5 below that Algorithm 1 is monotone. Accordingly, we use Algorithm 2 (presented below) to compute the critical value for each client

c_i. The intuition of Algorithm 2 is, given B_{-i}, to find the minimum bid (i.e. critical vale) of client c_i to get the desired packet w_i.

Algorithm 2:

input : Bids vector B; side information H_i for all clients; sets $\mathbf{S}_1$ and $\mathbf{S}_2$ returned by Algorithm 1; client c_i
output: Payment ϕ_i for client c_i

1 If $c_i \notin \mathbf{S}_1 \cup \mathbf{S}_2$ **return** $\phi_i = 0$;
2 If $c_i \in \mathbf{S}_2$ **return** $\phi_i = 1$;
3 Construct the weighted dependency graph $G(V, A)$ with the following costs:
- for arc $a = (\zeta_i, \zeta_j) \in A$, $\lambda_a = 1$
- for arc $a \neq (\zeta_i, \zeta_j)$, which is not outgoing from vertex ζ_i, $\lambda_a = 1 - \gamma_a$

4 $\beta \leftarrow \infty$
5 **while** *there exists a cycle C in $G(V, A)$ such that $\lambda(C)$ is less than or equal to one, as well as there exists a cycle that includes vertex ζ_i* **do**
  Find the cycle C_1 in $G(V, A)$ with the smallest cost;
  Find the cycle C_2 in $G(V, A)$ that traverses vertex ζ_i and has the smallest cost among those that go through vertex ζ_i;
  if $\lambda(C_2) - \lambda(C_1) = 0$ **then**
    return $\phi_i = 0$
  end
  else
    $\beta \leftarrow \min\{\beta, \lambda(C_2) - \lambda(C_1)\}$
    Remove all vertices of C_1 along with the arcs incident to them from $G(V, A)$
  end
end
6 Find a cycle C_2 in $G(V, A)$ (if any) that traverses vertex ζ_i and has the smallest cost among these that go through vertex ζ_i;
7 $\beta \leftarrow \min\{\beta, \lambda(C_2) - 1\}$
8 **return** $\phi_i = \beta$;

Algorithm 2 is similar to Algorithm 1, but has a different cost function. To compute the payment for client $c_i \in \mathbf{S}_1$, we assign the unit cost for every outgoing arc (ζ_i, ζ_j) of vertex ζ_i (i.e., set $\gamma_{i,j} = 0$ and $b_i = 0$). For each iteration of Line 5, we calculate the difference of the cost of cycles C_1 and C_2 in $G(V, A)$, where C_1 has the minimum global cost while C_2 is the local optimal cycle that traverses vertex ζ_i. In other words, given B_{-i}, we are looking for the lowest bid of client c_i such that a cycle containing vertex ζ_i in the remaining $G(V, A)$ would be selected by Line 5 of Algorithm 1.

The value β is equal to the minimum value of $\lambda(C_2) - \lambda(C_1)$ among all iterations. The payment of each client in $\mathbf{S}_1$ will be equal to β.

C. Analysis

In this section we prove that Algorithms 1 and 2 result in a truthful mechanism. We begin by showing that Algorithm 1 is monotone.

Lemma 5. *Algorithm 1 is monotone.*

Proof: (Sketch) Suppose client c_i is able to recover the desired packet. Then, a cycle containing vertex ζ_i is chosen by Algorithm 1. If c_i increases its bid, all cycles that contain ζ_i will have lower cost. Therefore, the algorithm will select a cycle containing vertex ζ_i and the lemma follows. ∎

Lemma 6. *Given B_{-i}, for each node c_i that is able to decode the required packet, the payment function ϕ_i is equal to the critical value.*

Proof: (Sketch) At each iteration of Algorithm 2, we calculate the lowest bid b_i for client c_i such that a cycle containing vertex ζ_i would be selected by Algorithm 1, i.e., client c_i can get the desired packet w_i. ∎

We summarize our results in the following theorem.

Theorem 7. *The Algorithms 1 and 2 run in polynomial time and result in a truthful mechanism.*

Proof: It is easy to verify that the both algorithms require a polynomial number of steps. The truthfulness property follows from lemmas 5, 6, and [14, Theorem 1]. ∎

V. Conclusion

In this paper, we focus on a new challenging problem that lies in the intersection of game theory and coding theory. Based on the Vickrey-Clarke-Groves (VCG) mechanism, we propose the VCG-coding scheme and the VCG-payment function that maximize the social welfare and provide incentives for the clients to submit truthful bids. Unfortunately, in many cases of practical interest it is intractable to implement the exact VCG-coding scheme. Accordingly, we proposed an approximation algorithm as well as the corresponding payment scheme that guarantee trustfulness.

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