A Reduced-Complexity Multilevel Coded Modulation for APSK Signaling

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Abstract— **The amplitude and phase shift keying (APSK) signal has been adopted in the recent satellite communication standards such as DVB-S2 due to its peak-to-average power ratio (PAPR) property lower than the quadrature amplitude modulation (QAM). Unlike square QAM constellations that allow separate detection of in-phase and quadrature components (i.e., I-Q decomposition), the detection process for APSK is generally complex. This paper investigates the use of multilevel coding (MLC) together with multistage decoding (MSD) for APSK with particular emphasis on an introduction of a novel labeling that allows I-Q decomposition at the highest level, thereby significantly reducing the decoding complexity at the cost of slight performance degradation.**

I. Introduction

In order to achieve higher bandwidth efficiency with higher reliability, the use of multilevel modulations such as amplitude phase shift keying (APSK) and quadrature amplitude modulation (QAM) together with the capacity approaching coded modulation techniques is essential. Compared to QAM, APSK has a salient practical advantage in that it has a signal with lower peak-to-average power ratio (PAPR) [1, 2]. Since higher PAPR signal causes either higher nonlinear distortion or significant loss of power conversion efficiency of high power amplifier (HPA), lower PAPR property is of particular importance for communication systems with severe power constraint such as satellite communications. For this reason, APSK has been recently adopted in the second generation standard of digital video broadcasting via satellite (DVB-S2). The APSK constellations that are adopted in these standards consist of several concentric PSKs where each PSK has distinct number of points. For example, the 16-APSK adopted by the standard is usually composed of 4-PSK and 12-PSK (denoted by 4+12-APSK), rather than the two 8-PSK rings (8+8-APSK), due to its lower PAPR property. (See Fig. 3 shown in later sections for its constellation example.)

The major advantage of a square QAM is its separate detection capability of in-phase and quadrature components (i.e., so-called I-Q decomposition). Unlike such QAM, a general APSK constellation does not allow one to separate I and Q components and thus the overall detection process becomes complicated even with bit-interleaved coded modulation (BICM) [1, 2].

Several complexity mitigation techniques have been recently studied for APSK constellations. For example, a simplified demapper has been proposed in [3], but the main assumption is that all the rings of APSK have an identical number of points (such as 8+8-APSK), thus yielding higher PAPR.

In this paper, we designed a new 4+12-APSK coded modulation based on the multilevel coding (MLC) with multistage decoding (MSD) [4] where the orthogonality principle is introduced for the highest *equivalent channel* and thus allows the use of I-Q decomposition at the decoder. The MLC/MSD system can achieve near capacity performance by using capacity-approaching codes such as low-density parity-check (LDPC) codes and turbo codes with appropriate designing of coding rates for each level based on the capacity rule [5] usually with lower complexity compared to that based on BICM. The use of the proposed labeling together with an appropriate partitioning further facilitates the implementation of MLC/MSD with 4+12-APSK at the cost of slight degradation in terms of achievable performance.

II. System model

In this section, we describe a general MLC/MSD system considered in this paper, followed by the analysis of average mutual information (AMI) over a constellation-constrained AWGN channel.

A. Coded Modulation Systems

Figure 1 shows a general model of transmitter and receiver of MLC/MSD system. A sequence of binary information bits $\boldsymbol{q}$ is partitioned into m blocks of sequences denoted by $\boldsymbol{q}^{(i)}, i = 1, 2, \cdots, m$, and each of them is encoded to $\boldsymbol{c}^{(i)}$ by the ith binary regular LDPC encoder with a coding rate R_i. For the MLC system, it is well known that the overall error performance considerably depends on the coding rate of each level, which should be designed carefully [5]. It is also required that all the codewords $\boldsymbol{c}^{(i)}$ have an identical codeword length, denoted by N in what follows. The transmitting APSK symbol sequence of length N, denoted by $\boldsymbol{x}$, consists of the combination of the codewords $\boldsymbol{c}^{(i)}, i = 1, 2, \cdots, m$, and each APSK symbol is taken from the complex signal constellation $\mathcal{X}$. Therefore, the number of blocks m should be given by $m = \log_2 |\mathcal{X}|$, where $|\mathcal{X}|$ is the cardinality of $\mathcal{X}$. We refer to the set of length N symbol sequence as a *frame* in this paper. Note that each partitioned information sequence $\boldsymbol{q}^{(i)}$ should have a different number of bits given by $R_i N$ bits. The total information rate R, given in *bits per symbol*, is then described as the sum total of coding rate R_i. In this paper, MSD is

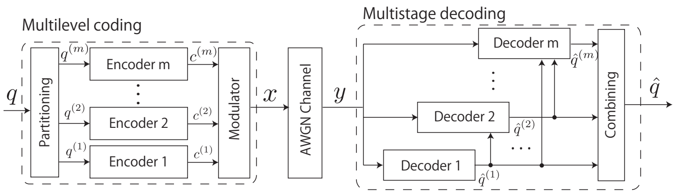

Fig. 1. MLC/MSD system model.

adopted for decoding of MLC. In MSD, each decoder makes use of the decoded bits of its lower levels. In this manner, by introducing an appropriate set partitioning [4, 5] to signal constellation, the Euclidean distance among equivalent signal constellations increases compared to that of the lower levels.

B. The AMI in Constellation-Constrained AWGN Channel

In this paper, we assume that the channel is modeled as a discrete time complex AWGN. The output y can be modeled as $y = x + n$, where $x \in \mathcal{X}$ and $n \in \mathbb{C}$ is the complex Gaussian noise with zero mean and variance $N_0/2$. Assuming that a complex input signal point x is chosen with equal probability from a set of signal constellation $\mathcal{X}$, the AMI can be described as

$$C = m - \mathrm{E}_{x,y}\left[\log_2 \frac{\sum_{z\in\mathcal{X}} p(y|z)}{p(y|x)}\right], \tag{1}$$

where E denotes expectation and $p(y|x)$ denotes a conditional probability density function (pdf) of y given x. The AMI C defined in (1) is referred to as a *coded modulation (CM) capacity* [6] since it is the maximum transmission rate determined exclusively from the constraint of a set of signal constellation $\mathcal{X}$.

III. APSK Signaling

The performance of APSK in terms of AMI and PAPR is determined by the so-called ring ratio denoted by ρ. Furthermore, the error performance of the coded modulation system with APSK is also affected by the bit partitioning strategy. In this section, we first describe the construction of APSK signal constellation considering PAPR and then introduce a design guideline for optimal ring ratio and set partitioned labeling of our APSK.

A. APSK Constellations

The constellation of M-ary APSK consists of I concentric rings, each of which is a uniformly spaced PSK. The signal set can be described by

$$\mathcal{X} = \Big\{ r_i e^{j(\phi_i + k\frac{2\pi}{M_i})} : i = 0, 1, \cdots, I-1; \quad k = 0, 1, \cdots, M_i - 1 \Big\}, \tag{2}$$

where r_i, ϕ_i, and M_i denote radius, phase shift, and the number of points of the ith ring, respectively. The radius r_i, where $r_{i-1} < r_i < r_{i+1}$, is defined by

$$r_i = \rho_i r_0, \tag{3}$$

$$r_0 = \sqrt{\frac{M}{M_0 + \sum_{i=1}^{I-1} {\rho_i}^2 M_i}}, \tag{4}$$

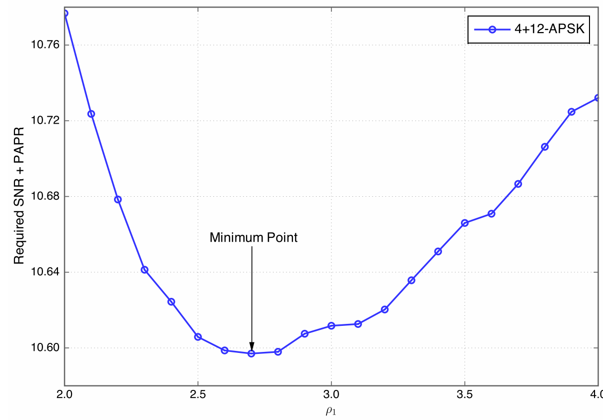

Fig. 2. Required SNR + PAPR as a function of ρ_1 with the target AMI set at 3 bits per symbol.

where ρ_i is the ring ratio of the ith ring defined as $\rho_i = r_i/r_0$ and r_0 is chosen such that the average energy is normalized to unity. This APSK is referred to as $M_0 + M_1 + \cdots + M_{I-1}$-APSK. Throughout this paper, we only consider the case of $I = 2$ with $M_0 = 4$ and $M_1 = 12$ (4+12-APSK), which is adopted in the DVB-S2 standard. However, extension of our proposed approach to higher-order APSK is complicated but straightforward.

B. PAPR

Since the signal that has high PAPR considerably decreases the power efficiency of the HPA in the satellite communications, lower PAPR is always preferable. In this paper, we define PAPR $= P_{\mathrm{max}}/P_{\mathrm{av}}$, where P_{max} and P_{av} are the maximum and average energies, respectively, of a given modulation constellation. In practice, the effect of pulse-shaping filter should be also taken into account, but for simplicity of analysis we only focus on the PAPR defined above. The PAPR of PSK and square 16-QAM are calculated as 1 (0 dB) and 9/5 (2.55 dB), respectively. On the other hand, the PAPR of M-ary APSK depends on the ring ratio ρ_i. As an example, since the maximum PAPR of 4+12-APSK is 3/2 (1.76 dB) even in the case of $\rho_1 \to \infty$, the power efficiency of 4+12-APSK in terms of HPA is considered higher than that of square 16-QAM.

C. Optimal Ring Ratio

As a figure of merit, we define the optimal ring ratio as the one that minimizes the summation of PAPR and required SNR for a given target information rate. Figure 2 shows the relationship between the summation mentioned above and the ring ratio in the case of 4+12-APSK. The required SNR is defined as the minimum SNR that achieves AMI of 3 bits per symbol for CM, which can be determined from (1). Based on the result of Fig. 2, the optimal ring ratio for 4+12-APSK is determined as $\rho_1 = 2.7$, and this value will be adopted throughout this paper.

D. Bit Labeling for APSK with General MLC/MSD

In this paper, similar to trellis coded modulation (TCM) [4], we first consider a set partitioning that maximizes the

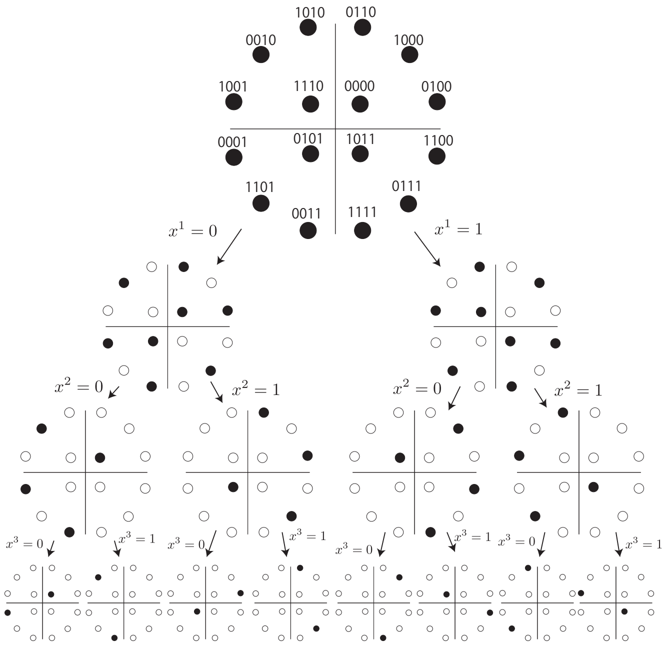

Fig. 3. Equivalent channels of 4+12-APSK (Type A set partitioning).

minimum intra-subset Euclidean distance for each equivalent channel [4, 7] in the framework of MLC/MSD with 4+12-APSK constellation. Figure 3 shows an equivalent channel for 4+12-APSK and we refer to this straightforward set partitioning approach as *type A* in what follows. In Fig. 3, the bit labeling $x^1x^2x^3x^4$ attached to each constellation point is defined such that $x^i \in \{0,1\}$ corresponds to the bit of the ith level. This labeling is suitable for MLC/MSD but not for BICM (without iterative decoding) due to the non-Gray labeling property.

IV. A New Reduced Complexity MLC/MSD for APSK

Since the general APSK constellation is not capable of I-Q decomposition, their detection complexity for coded modulation is generally high. This fact motivates us to propose our new labeling that leads to significant simplification of metric calculation in the framework of MLC/MSD. We also show the effect of finite codeword length by comparison of different coding rate design rules.

A. A Bit Labeling with Proposed Set Partitioning

Figure 4 illustrates a new labeling together with the proposed partitioning which we refer to as *type B* in this paper. Once the first and second levels are determined, all the four constellations of the equivalent channel in the third level, denoted by A, B, C, and D as shown in the figure, are the form of QPSK. Therefore, the two remaining bits can be decoded separately given that the phase rotation as well as the amplitude difference is taken into account upon metric calculation.

In practice, the log-likelihood ratio (LLR) of the ith bit of

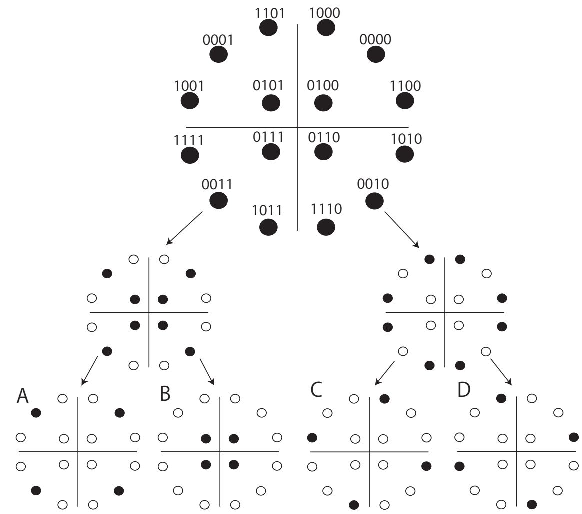

Fig. 4. The new labeling for APSK where the third level has a phase-rotated QPSK constellation with different radii (Type B set partitioning).

the kth received APSK symbol is approximated by [6]

$$\lambda_{k,i} = \min_{x \in \mathcal{X}_1^i} \frac{|y_k - x|^2}{N_0} - \min_{x \in \mathcal{X}_0^i} \frac{|y_k - x|^2}{N_0}, \tag{5}$$

where $y_k \in \mathbb{C}$ is the kth received symbol and $\mathcal{X}_b^i$ is the subset of $\mathcal{X}$ with its ith bit given by $b \in \{0,1\}$. Calculation of (5) requires comparisons between the received and estimated signal points for all the candidate signal points in the subset $\mathcal{X}_b^i$. On the other hand, in the case of the third and forth equivalent channels in our new labeling with the proposed partitioning, the bit-metric can be calculated by

$$\lambda_{k,i} = 4\alpha y_k' / N_0 \tag{6}$$

where y_k' is the kth received signal component separated based on I-Q decomposition (with possible phase rotation) and α is a coefficient that takes into account the amplitude of the equivalent channel. It is easy to observe that

$$\alpha = \begin{cases} r_0/\sqrt{2}, & \text{for equivalent channel B,} \\ r_1/\sqrt{2}, & \text{otherwise,} \end{cases} \tag{7}$$

where r_0 and r_1 are the radii of the inner and outer rings, respectively. Note that (6) is equivalent to the LLR calculation of BPSK with amplitude α. It is obvious that the proposed LLR (6) is much simpler to calculate than (5) since it does not require any minimum search operation. Since this metric calculation is applicable to the third and fourth levels only, the complexity of the first two levels is identical to that of the conventional approach.

B. Improved MSD System for Type B Labeling

Upon applying our type B labeling, conditioned that the lower levels are correct, the third and fourth levels can be considered independent due to I-Q decomposability. Therefore, we can apply the parallel independent decoding (PID) [5] to these levels. Figure 5 shows the MSD-PID system for 4+12-APSK with such a strategy. It can be observed that comparing with the original MSD shown in Fig. 1, the proposed system

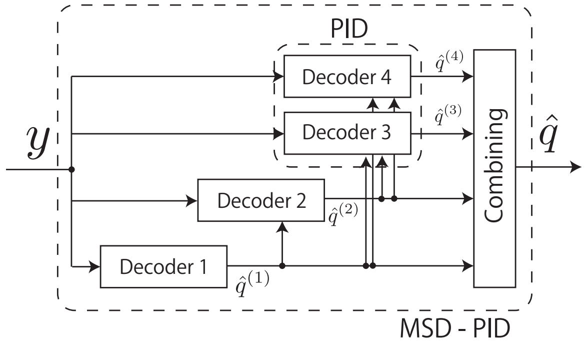

Fig. 5. MSD-PID system for 4+12-APSK with type B set partitioning.

can reduce the delay associated with the decoding of the forth level.

C. Coding Rate Design Rule

For the MLC/MSD with near optimal component codes, *capacity rule* [5] is often adopted. For a 2^m-ary modulation, the coding rate R_i should be chosen equal to the corresponding capacity C_i of equivalent channel:

$$R_i = C_i \qquad i = 1, 2, \cdots, m. \tag{8}$$

The capacity C_i of each level can be easily calculated numerically [5].

On the other hand, in practical systems, the codeword length is not necessarily large enough and thus there is a performance gap from the corresponding capacity. In this case, the capacity rule may not be strictly optimal and we should take account of the effect of finite codeword length. *Coding exponent rule* is another coding rate design rule for MLC/MSD [5]. This design rule is based on the well-known random coding bound which gives a relation between finite codeword length N and word error probability p_w [8]. The relation is described as

$$p_w \leq 2^{-NE_r(R, E_s/N_0)} \tag{9}$$

where $E_r(R, E_s/N_0)$ is a random coding exponent defined as a function of coding rate R and signal to noise ratio E_s/N_0. In other words, the relation between R and E_s/N_0 is given by (9) when N and p_w are given. For the coding rate design of MLC/MSD system, we should consider the random coding exponent for each level, $E_r^i(R^i, E_s/N_0)$, instead of $E_r(R, E_s/N_0)$.

As an example, we show an equivalent capacity and the corresponding coding rate calculated by (9) in Fig. 6. In the latter case, we set $N = 18\,000$ and $p_w = 2.5 \times 10^{-3}$. The vertical lines show the SNR where each sum of equivalent capacities or coding rates exceeds 3.25. It is obvious that the curves suggested by the random coding exponent are similar to the corresponding equivalent capacity curves. Table I shows a set of coding rates designed by the two rules for type A and B in the case of transmission rate $R \approx 3.25$. From Table I, in the case of type A, no noticeable difference is observed between the two design rules. Consequently, the FER performance may not be affected by these rules. On the other hand, in the case of type B, we observe some difference of coding rate for the

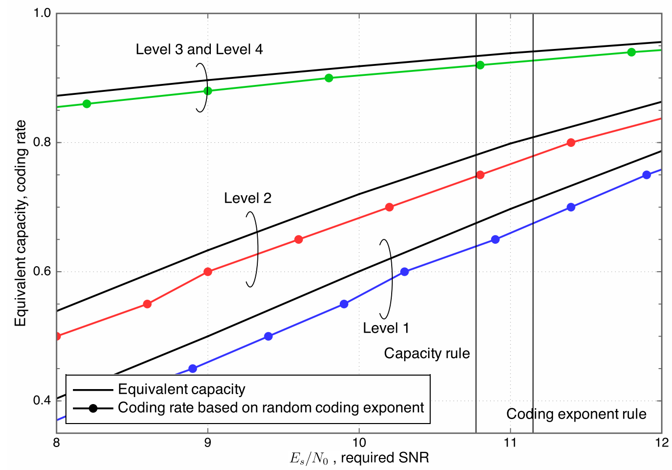

Fig. 6. Equivalent capacity and coding rate suggested by random coding exponent for type B.

TABLE I
CODING RATE OF EACH SYSTEM.

Set partitioning	Design rule	Level1	Level2	Level3	Level4
Type A	Capacity	0.458	0.873	0.984	1.00
	Coding Exp.	0.459	0.869	0.980	1.00
Type B	Capacity	0.675	0.781	0.934	0.934
	Coding Exp.	0.678	0.780	0.927	0.927

third and fourth levels, which should be taken into account in a practical system with finite-length codeword.

V. NUMERICAL RESULTS

In this section, we first investigate the FER performance of type B approach based on the two design rules developed in Section IV-C, followed by the comparison between MLC/MSD (with type A and type B) and the conventional BICM (with pseudo Gray mapping) in terms of performance and complexity.

A. Effect of Finite Codeword Length

Figure 7 compares the FER performance of the proposed MLC/MSD with type B based on the two rate design rules with $N = 18\,000$ over an AWGN channel. It can be observed that the performance of the coding exponent rule is better than that of the capacity rule. It should be also noted that the information rate of the coding exponent rule is $R = 3.286$, which is slightly higher than that of the capacity rule ($R = 3.236$). From the result, the FER performance of type B system is sensitive to the finiteness of the codeword length. The reason for this is inferred from the fact that in the proposed system with type B the amplitude of the equivalent channel labeled as B in Fig. 4 is lower than that of the others and this variance of amplitude yields a negative influence on the achievable performance of the LDPC in the third and forth levels, yielding a gap in achievable FER performance from the capacity.

B. FER Performance of BICM and MLC/MSD Systems

Figure 8 shows the FER performance of BICM with the pseudo-Gray labeling and MLC systems. The parameters of these systems are shown in Table II. The transmission rates

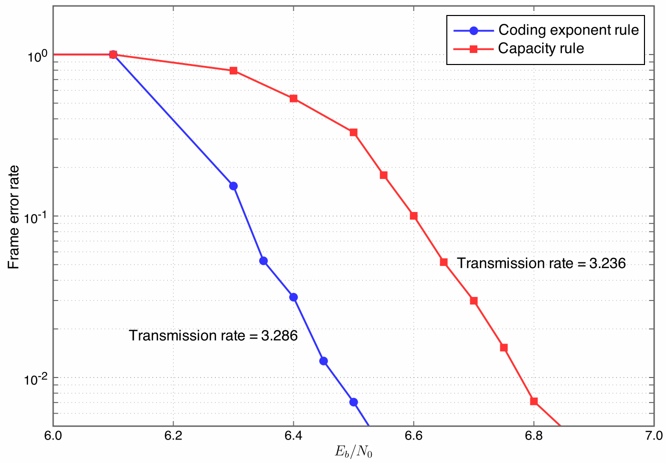

Fig. 7. The FER performance of type B labeling for each coding rate design rule.

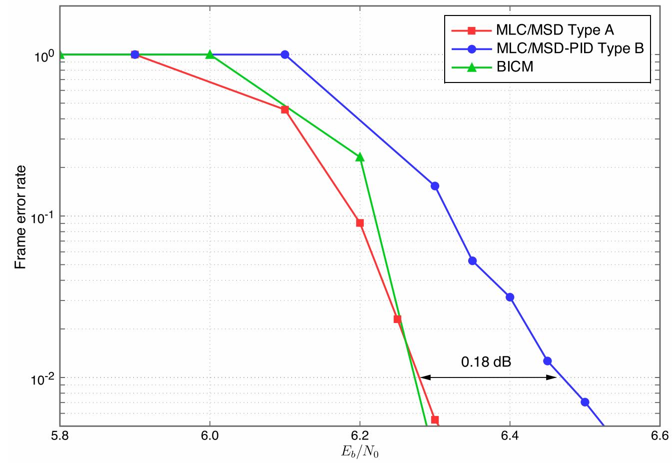

Fig. 8. The FER performance of each MLC and BICM.

TABLE II
THE PARAMETERS OF EACH MLC/MSD AND BICM.

System model	MLC/MSD type A	MLC/MSD-PID type B	BICM
Channel coding	Binary regular LDPC code		
Symbol length	18 480	18 780	18 500
Rate design rule	Capacity	Coding Exp.	-
Transmission rate	3.238	3.286	3.248
Modulation	4+12-APSK		
Set partitioning / Labeling	Type A	Type B	Pseud-Gray
Channel model	AWGN		

TABLE III
THE NUMBER OF COMPARISON OPERATIONS FOR EACH SYSTEM.

System	Decoding complexity	$M = 16$
Type A	$\sum_{i=0}^{\log_2 M-2}(M/2^i-2)$	22
Type B	$\sum_{i=0}^{\log_2 M-3}(M/2^i-2)$	20
BICM	$\log_2 M \times (M-2)$	66

are slightly different due to some difficulty in designing good precisely rate-adjusted high-rate LDPC codes.

It is observed from Fig. 8 that the FER performance of MLC/MSD type A and BICM are almost the same. Especially, in the low SNR region, the type A outperforms the others. The type B has slight FER performance degradation of about 0.18 dB at FER = 10^{-2} compared to the other systems. We note that there is a slight difference in terms of information rate, where type A is lowest and type B is highest. If this rate gap is compensated for, the performance gap may be expected to become even smaller.

C. Comparison of Complexity

Finally, we compare the three systems in terms of decoding complexity. Since the computation is dominated by the number of comparisons required for LLR calculation, the required number of comparison operations is listed in Table III, where M is the number of signal constellation. As a reference, the specific values in the case of $M = 16$ are also shown. We observe that BICM has higher complexity than the others as the number of signal constellations per each level of MLC/MSD systems is lower than that of BICM. In addition, we emphasize that since the type B uses (6) instead of (5) for higher levels, its overall complexity is much lower than that of type A.

VI. CONCLUSION

In this paper, we have proposed a low complexity MLC/MSD approach for 4+12-APSK signaling considered in the recent satellite communication standard. The comparisons are made with BICM and it has been shown that the proposed approach outperforms BICM in terms of decoding complexity required for LLR calculation. As our future work, more detailed comparison and analysis in terms of complexity and latency as well as its extension and generalization to higher-order APSK signaling will be investigated.

ACKNOWLEDGEMENT

This work was supported in part by MEXT KAKENHI 23686058.

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