On The Capacity of the MIMO Cognitive Interference Channel

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***Abstract*—The cognitive interference channel is a variation of the classical interference channel in which one of the transmitters, the cognitive transmitter, has *full* and *a priori knowledge* of the message of the other user, the primary user. This additional knowledge is termed *cognition* and idealizes the cognitive transmitter learning the messages of the primary user by overhearing its transmissions over a wireless channel. This paper studies the multiple-input multiple-output cognitive interference channel and derives inner and outer bounds for the capacity of this channel model as well as approximate characterizations of the capacity region. In particular, it is shown that capacity can be achieved to within an additive gap which depends on the number of antennas at the cognitive decoder and to within a constant multiplicative factor of two.**

I. Introduction

The Cognitive InterFerence Channel (CIFC) is a well studied communication channel which models the ability of smart and agile devices to acquire information regarding neighboring users by overhearing the transmissions taking place over the network. This capability can be employed for the scenario in which a licensed user, the *primary user*, allows a second user, the *cognitive user*, to access its proprietary spectrum in exchange for cooperation in transmitting the primary message. The CIFC idealizes this situation in two ways: (i) it assumes that the extra knowledge of the primary message is available a priori at the cognitive transmitter, instead of causally learned through successive transmissions and (ii) it considers the case in which the cognitive transmitter is able to acquire the primary message in its entirety. Although more realistic channels have been considered in the literature, only for this idealized model it is possible to derive capacity results which precisely characterize the fundamental benefits offered by this form of unidirectional cooperation.

The CIFC was originally introduced by Devroye et al. [1]. General outer [2] and inner bounds [3] on the capacity region are available in the literature but capacity remains unknown in general case. For the discrete memoryless case capacity is known in the "better cognitive decoding" regime, in which capacity is achieved with rate splitting and superposition coding [3]. For the Gaussian case capacity is known in three different regimes. In the "weak interference" regime [4] capacity is achieved by having the encoders cooperate in transmitting the primary message and by having the primary receiver treat the interference as noise while the cognitive transmitter pre-codes its message against the known interference. Capacity is also known for channels in the "very strong interference"[2] regime and is achieved by superimposing the cognitive message over the primary message and having both receivers decode both messages. The last regime in which capacity is known for the Gaussian case is the "primary decodes cognitive" regime [5]. Here capacity is achieved by pre-coding the cognitive codeword against the interference created by the primary transmission and having the primary receiver decode both the primary and the cognitive codeword. The primary decoder gains insight over its own message by decoding the cognitive codeword, since the interference against which the cognitive codeword is pre-coded is indeed the primary codeword. Capacity for the Gaussian case is also known to within 1/2 bit/s/Hz and a factor two [6], that is, a bounded difference and a bounded ratio between the inner and the outer bounds has been shown for any channel parameter.

In the following we study the Multiple-Input Multiple-Output (MIMO) CIFC, a generalization of the Gaussian CIFC in which transmitters and receivers posses multiple antennas. The capacity of this channel model is still largely unknown since the capacity results for the "weak interference" and the "primary decodes cognitive" regimes do not extend from the single antenna to the MIMO scenario. The authors of [7] are the first to specifically study the capacity of the MIMO CIFC and propose an outer bound and an achievable region based on dirty paper coding [8] in the attempt of extending the "weak interference" capacity result to the MIMO CIFC. The sum Degrees Of Freedom (DOF) of the MIMO CIFC are studied in [9] where it is shown that the MIMO CIFC has a larger sum DOF than the classical InterFerence Channel (IFC).

Contributions: In the following we

• **Present outer and inner and bounds** to the capacity region of the MIMO CIFC. In particular we show that for the outer bound under consideration one needs only to consider jointly Gaussian random variables.

• **Obtain capacity to within a constant additive gap** which depends on the number of antennas at the cognitive receiver. This result is akin to the result in [10] for the MIMO IFC.

• **Derive the capacity to within a multiplicative factor of two** by having the cognitive transmitter pre-cancel the interference created by the primary transmitter at the cognitive receiver while the primary decoder treats the interference as noise.

Organization: The remainder of the paper is organized as follows: The channel model is introduced in Section II. An outer bound to the capacity region presented in Section III while an inner bound can be found in Sec. IV. Capacity to within a constant additive gap and a constant multiplicative factor is shown in Section V. Section VI concludes the paper.

II. Channel Model

A two user InterFerence Channel (IFC) is a multi-terminal network with two senders and two receivers. Each transmitter i wishes to communicate a message W_i to receiver i, $i \in [1, 2]$. In the classical IFC the two transmitters operate independently and have no knowledge of each others' messages. In this paper we consider a variation of this setup and instead assume that transmitter 1 (the *cognitive* transmitter), in addition to its own message W_1, also knows the message W_2 of transmitter 2 (the *primary* transmitter). We refer to transmitter/receiver 1 as the cognitive pair and to transmitter/receiver 2 as the primary pair. This model is termed the Cognitive InterFerence Channel (CIFC) and is an idealized model for unilateral transmitter cooperation.

More precisely, transmitter $i \in [1, 2]$ wishes to communicate a message W_i, uniformly distributed on $[1 : 2^{NR_i}]$, to receiver i in N channel uses. The two messages are independent. Transmitter 1 knows both messages while transmitter 2 knows only W_2. The channel is assumed to be memoryless and the input/output relationship in each channel use is described by the conditional probability

$$P_{Y_1,Y_2|X_1,X_1} \tag{1}$$

A rate pair (R_1, R_2) is said to be achievable if there exists a sequence of encoding functions

$$X_1^N = X_1^N(W_1, W_2) \tag{2a}$$
$$X_2^N = X_2^N(W_2), \tag{2b}$$

and a sequence of decoding functions

$$\widehat{W}_i = \widehat{W}_i(Y_i^N), \quad i \in [1, 2], \tag{3}$$

such that

$$\lim_{N\to\infty} \max_{i\in[1,2]} \mathbb{P}\left[\hat{W}_i \neq W_i\right] \to 0. \tag{4}$$

The capacity region is defined as the closure of the region of all achievable (R_1, R_2) pairs.

The Multiple-Input Multiple-Output (MIMO) CIFC is a special case of the general CIFC in which (1) is obtained as

$$Y_1 = H_{11}X_1 + H_{12}X_2 + Z_1 \tag{5a}$$
$$Y_2 = H_{21}X_1 + H_{22}X_2 + Z_2, \tag{5b}$$

where H_{ij}, $i, j \in \{1, 2\}$ are real matrices of size $m_i \times n_j$, Y_i's and X_j's are column vectors of size m_i and n_j respectively and Z_i are iid, zero mean and unitary variance Gaussian random column vectors of size m_i. Additionally, the channel inputs X_j are subject to the covariance constraint

$$\mathbb{E}[X_j X_j^H] = \Sigma_j \preceq \mathbf{S}_j \quad j \in \{1, 2\}, \tag{6}$$

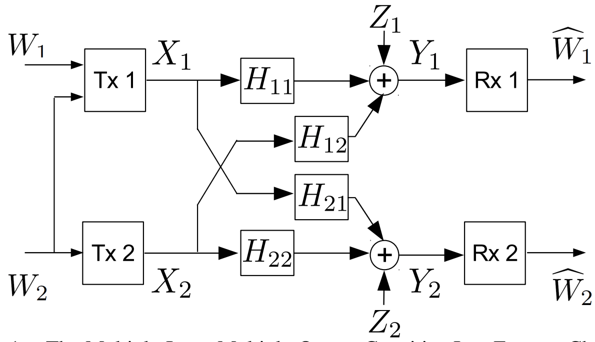

Fig. 1. The Multiple-Input Multiple-Output Cognitive InterFerence Channel (MIMO CIFC)

for some $\mathbf{S} \succeq 0$ where $\preceq$ and $\succeq$ denote partial ordering between symmetric matrices for which $\mathbf{B} \succeq \mathbf{A}$ means that $\mathbf{B}-\mathbf{A}$ is a positive semidefinite matrix. A graphical representation of the MIMO CIFC is provided in Fig. 1.

III. Outer Bounds

This section introduces an outer bound to the capacity region of the general CIFC originally devised in [4] and inspired by an outer bound for the Broadcast Channel (BC) in [11]. This outer bound is capacity in the "better cognitive decoding" regime of [3] and reduces to the "strong interference" outer bound [2]. The outer bound is expressed as a function of the channel inputs and an additional auxiliary random variable (RV): this implies that the evaluation of the outer bound requires to calculate the union over a large set of distributions for which no cardinality bounds have been derived so far. For the MIMO CIFC we prove that this outer bound can be evaluated considering only jointly Gaussian channel inputs and auxiliary RV, which simplifies the evaluation of the outer bound and the derivation of closed-form expressions for the boundary points.

Theorem 1. Wu et al. Outer Bound [4, Thm. 3.2]: *Let $\mathcal{R}^{\rm out}$ be defined as the region*

$$R_1 \leq I(Y_1; X_1|X_2) \tag{7a}$$
$$R_2 \leq I(Y_2; U, X_2) \tag{7b}$$
$$R_1 + R_2 \leq I(Y_2; U, X_2) + I(Y_1; X_1|X_2, U), \tag{7c}$$

union over all the distributions P_{U,X_1,X_2}, then $\mathcal{R}^{\rm out}$ is an outer bound to the capacity region of a general CIFC.

Let in the following $(\cdot)_{\rm R}$ indicate the right hand side of $(\cdot)$. Since $(7a)_{\rm R} + (7b)_{\rm R} \geq (7c)_{\rm R}$, the region in (7) has two Pareto-optimal corner points which we define as

$$A^{\rm out} = (R_1^{A-{\rm out}}, R_2^{A-{\rm out}}) = ((7a)_{\rm R}, (7c)_{\rm R} - (7a)_{\rm R}) \tag{8a}$$
$$B^{\rm out} = (R_2^{B-{\rm out}}, R_1^{B-{\rm out}}) = ((7b)_{\rm R}, (7c)_{\rm R} - (7b)_{\rm R}), \tag{8b}$$

In order to evaluate the outer bound of Thm. 1 one needs to take the union over all the possible distributions P_{U,X_1,X_2}, which is not feasible in general. We next show that for the MIMO CIFC one only needs to consider the case in which $[U \ X_1 \ X_2]$ are zero mean jointly Gaussian RVs. Since the covariance matrix of $[U\ X_1\ X_2]$ is parameterized by six variables, this result implies that one can efficiently evaluate the outer bound by considering all the possible covariance matrices for which the channel inputs satisfy the constraint in (6).

Theorem 2. MIMO CIFC Outer Bound: *The outer bound in Thm. 1 for the MIMO CIFC can be equivalently obtained by considering the region in* (7) *and taking the union over all the zero mean jointly Gaussian* $[U\ X_1\ X_2]$ *for which* (6) *is satisfied.*

Proof: The proof involves showing that the union over all possible distributions P_{U,X_1,X_2} is equivalent to the union over jointly Gaussian $[U\ X_1\ X_2]$.

First of all notice that we can express $\mathcal{R}^{\rm out}$ as

$$\mathcal{R}^{\rm out} = \bigcup_{P_{X_1,X_2}} Conv \left\{ \mathcal{R}^{\rm out-A} \cup \left(\bigcup_{P_{U|X_1,X_2}} \mathcal{R}^{\rm out-B} \right) \right\}, \tag{9}$$

where $Conv(A)$ indicates the convex closure of A, $\mathcal{R}^{\rm out-A}$ is defined as

$$R_1 \le I(Y_1; X_1|X_2) \tag{10a}$$
$$R_2 \le I(Y_2; X_2, U^{\rm sum}) \tag{10b}$$
$$R_1 + R_2 \le I(Y_2; X_2, U^{\rm sum}) - I(Y_1; X_1|X_2, U^{\rm sum}), \tag{10c}$$

for

$$U^{\rm sum} \sim \underset{P_{U|X_1,X_2}}{\rm argmax} \quad I(Y_2; U, X_2) + I(Y_1; X_1|X_2, U), \tag{11}$$

and $\mathcal{R}^{\rm out-B}$ is defined as

$$R_1 \le I(Y_1; X_1|X_2, U) \tag{12a}$$
$$R_2 \le I(Y_2; U, X_2). \tag{12b}$$

The argument of the convex closure in the RHS of (9) contains all the points $A^{\rm out}$ and $B^{\rm out}$ which are obtained from (7) when considering the union over $P_{U|X_1,X_2}$ for a fixed P_{X_1,X_2}. Since the bound in (10a) does not depend on U, the largest $R_2^{A-\rm out}$ is obtained by maximizing the sum rate bound (10c). On the other hand, the coordinates of the point $B^{\rm out}$ depend on $P_{U|X_1,X_2}$ and the region $\mathcal{R}^{\rm out-B}$ corresponds to all the points $B^{\rm out}$ generated by varying $P_{U|X_1,X_2}$. Since $\mathcal{R}^{\rm out-A}$ contains all the points $A^{\rm out}$ and $\mathcal{R}^{\rm out-B}$ contains all the points $B^{\rm out}$, the equivalence in (9) is shown by considering the convex closure of these two regions.

Consider now the the region $\bigcup_{P_{U|X_1,X_2}} \mathcal{R}^{\rm out-B}$: the points on the convex hull of this region can be expressed as

$$\mu R_1 + \overline{\mu} R_2 = \max_{P_{U|X_1,X_2}} \mu I(Y_1; X_1|X_2, U) + \overline{\mu} I(Y_2; X_2, U) \tag{13a}$$
$$\le \overline{\mu} H(Y_{2G}) - \mu H(Z_1) + \mu \left(\max_{P_{U|X_1,X_2}} H(Y_1|X_2, U) - \rho H(Y_2|X_2, U) \right) \tag{13b}$$

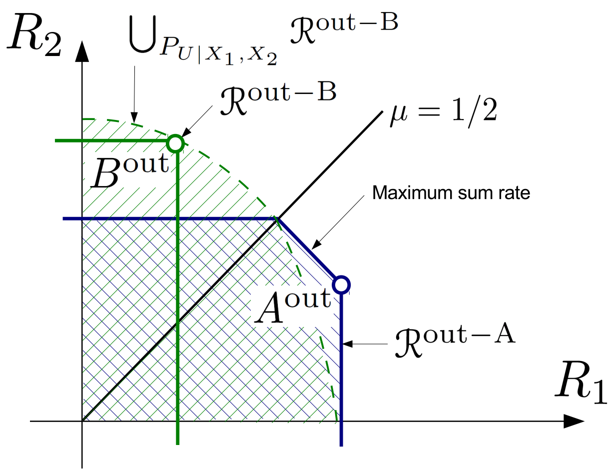

Fig. 2. A graphical representation of the proof of Thm. 2 which shows the region (10) union over $P_{U|X_1,X_2}$ for a fixed P_{X_1,X_2}.

for $\mu \in [0, 1]$ and $\rho = \overline{\mu}/\mu$ and where Y_{2G} indicates the zero mean Gaussian vector having the same covariance as Y_2. Equality is achieved in (13b) by showing that choosing jointly Gaussian $[X_1\ X_2\ U]$ is optimal. Note that the optimization in (13) for $\mu = 1/2$ is attained by $U^{\rm sum}$ in (11). Values of $\mu \in (1/2 \ldots 1]$ need not be considered for (13): these points are already contained in the region $\mathcal{R}^{\rm out-A}$ given that $(10c)_{\rm R} \ge (12a)_{\rm R} + (12b)_{\rm R}$ and $(10a)_{\rm R} \ge (12a)_{\rm R}$. For this reason we can write the argument of the convex closure in the RHS of (9) as

$$\mu R_1 + \overline{\mu} R_2 = \begin{cases} \max_{P_{U|X_1,X_2}} \mu(12a)_{\rm R} + \overline{\mu}(12b)_{\rm R} \\ \qquad\qquad \text{for} \quad 0 \le \mu \le 1/2 \\ (10c)_{\rm R} \\ \qquad\qquad \text{for} \quad o1/2 < \mu \le 1 \end{cases} \tag{14}$$

For the range $0 \le \mu \le 1/2$, we have $\rho \ge 1$ and thus we can apply the extremal inequality of [12, Thm. 8] to conclude that maximum of (13b) is attained by Gaussian X_1, X_2 and U and that (13b) holds with equality. By the same token, (11) is also maximized by Gaussian inputs and U, since it corresponds to (13b) for $\rho = 1$. This shows that all the point on the boundary of the region $\mathcal{R}^{\rm out-A} \cup \left(\bigcup_{P_{U|X_1,X_2}} \mathcal{R}^{\rm out-B} \right)$ are maximized by jointly Gaussian $[X_1\ X_2\ U]$.

A graphical representation of the proof is provided in Fig. 2: the region $\mathcal{R}^{\rm out-A}$ and the region $\bigcup_{P_{U|X_1,X_2}} \mathcal{R}^{\rm out-B}$ intersect for $\mu = 1/2$ in (13b). For $\mu \le 1/2$, points of $\bigcup_{P_{U|X_1,X_2}} \mathcal{R}^{\rm out-B}$ are on the boundary of the region while for $\mu > 1/2$ points of $\mathcal{R}^{\rm out-A}$ are. In this point the maximum sum rate is achieved by the assignment in (11) and for Gaussian $[X_1\ X_2\ U]$ according to the extremal inequality of [12]. ∎

IV. Inner Bound

In this section we present an achievable scheme first considered in [13], which achieves the capacity for the semi-deterministic CIFC [6], that it the CIFC in which the channel output at the cognitive receiver is a deterministic function of the inputs, while the output at the primary decoder is any random function. This scheme also approaches the capacity of the Gaussian CIFC to within 1/2 bit/s/Hz for real channel

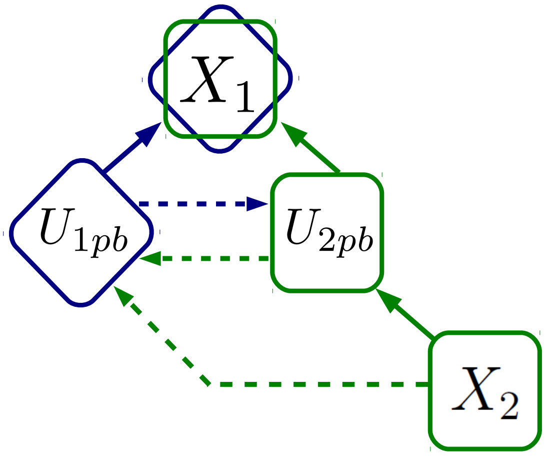

Fig. 3. A graphical representation of the achievable scheme in Thm. 3. The RVs for message 1 are in blue diamond boxes while the RVs for message 2 are in green square boxes. A solid line among RVs indicates that the RVs are superimposed while a dashed line that the RVs are binned against each other.

parameters [6]. In Marton's scheme, the transmitter pre-codes the codeword for each user against the interference created by the other user. The situation is similar in this scheme, but it considers also the interference created by the primary transmitter.

Theorem 3. Inner Bound [14, Scheme (C)]: *Let $\mathcal{R}^{\rm in}$ be defined as*

$$\begin{aligned} R_1 &\leq I(Y_1; U_{1pb}) - I(U_{1pb}; X_2) && \text{(15a)}\\ R_2 &\leq I(Y_2; U_{2pb}, X_2) && \text{(15b)}\\ R_1 + R_2 &\leq I(Y_2; U_{2pb}, X_2) + I(Y_1; U_{1pb}) \\ &\quad - I(U_{1pb}; X_2, U_{2pb}), && \text{(15c)} \end{aligned}$$

for any distribution $P_{U_{1pb},U_{2pb},X_1,X_2}$, then $\mathcal{R}^{\rm in}$ is an inner bound to the capacity region of the general CIFC.

A graphical representation of the inner bound in Thm. 3 is provided in Fig. 3. Each box represents a codebook associated with an auxiliary RV and lines connecting boxes represent coding operations: dashed lines represent interference pre-cancelation while solid lines superposition coding. Encoder 2 transmit W_2 through the RV X_2 while encoder 1 sends W_1 through U_{1pb}. The RV U_{2pb} is superimposed over X_2 and pre-coded at transmitter 1 against the interference created by U_{1pb} at the primary receiver. Similarly, the RV U_{1pb} is pre-coded at transmitter 1 against the interference created by X_2 and U_{2pb} at the cognitive receiver.

Since $(15a)_{\rm R} + (15b)_{\rm R} \geq (15c)_{\rm R}$, the region in (15) has two Pareto optimal corner points which we define as

$$A^{\rm in} = (R_1^{A-{\rm in}}, R_2^{A-{\rm in}}) = ((15a)_{\rm R}, (15c)_{\rm R} - (15a)_{\rm R}) \quad \text{(16a)}$$
$$B^{\rm in} = (R_1^{B-{\rm in}}, R_2^{B-{\rm in}}) = ((15c)_{\rm R} - (15b)_{\rm R}, (15b)_{\rm R}) \,. \quad \text{(16b)}$$

V. Approximate Capacity

After having introduced outer and inner bounds, we now establish two approximate characterizations of the capacity region in the spirit of [10] and [15]. We begin by deriving the capacity of the general CIFC to within a constant additive gap, a result that generalizes the capacity result for the semi-deterministic CIFC in [6]. We then specialize this result to the MIMO CIFC to obtain the capacity to within a constant additive factor which depends on the number of antennas at the cognitive receiver. We also show capacity to within a multiplicative factor of two.

Theorem 4. Additive Gap Between Inner and Outer Bound for the general CIFC: *If $(R_1, R_2) \in \mathcal{R}^{\rm out}$, then*

$$(R_1 - I(Y_1; X_1, X_2 | \widetilde{Y}_1), R_2), \quad \text{(17)}$$

is achievable for $\widetilde{Y}_1 \sim P_{Y_1|X_1,X_2}$.

Proof: This result is shown by considering the corner points of the outer bound in (8) for a fixed distribution P_{U,X_1,X_2} and showing an assignment of U_{1pb}, U_{2pb}, X_1 and X_2 for the corner points of the inner bound in (16) for which a constant gap between the two bounds can be established.

Consider in particular the assignment in which P_{X_1,X_2} is the same for the inner and outer bound while $U_{2pb} = U$. Also let $U_{1pb} = \widetilde{Y}_1$ which is obtained by passing X_1 and X_2 through a virtual channel (1) at the cognitive transmitter. With this assignment we have $R_2^{A-{\rm out}} - R_2^{A-{\rm in}} = R_2^{B-{\rm out}} - R_2^{B-{\rm in}} = 0$ and $R_1^{B-{\rm out}} - R_1^{B-{\rm in}} = R_1^{A-{\rm out}} - R_1^{A-{\rm in}}$. ■

In [6] it is shown that the choice $U_{1pb} = Y_1$ yields capacity for the semi-deterministic CIFC as well as achieving the capacity of the Gaussian CIFC to within 1/2 bit/s/Hz. Thm. 4 generalizes this approach to the general CIFC and reduces to the results in [6] when the channel is a semi-deterministic CIFC. Intuitively, the gap between inner and outer bound depends on the ability of the cognitive transmitter to predict the channel output Y_1. For the semi-deterministic channel, the gap is zero since the channel output at the cognitive receiver is a deterministic function of the channel input. For the Gaussian CIFC, the channel output can be predicted up to the additive Gaussian noise which results in an additive gap of 1/2 bit/s/Hz.

We now specialize the above result to the MIMO CIFC to obtain a gap between inner and outer bounds which depends on the number of antennas at the cognitive receiver.

Lemma V.1. Capacity of the MIMO CIFC to within a constant additive gap: *The outer bound of Thm. 2 can achieved to within $m_1/2$ bit/s/Hz.*

Proof: This is shown by evaluating the term $I(Y_1; X_1, X_2 | \widetilde{Y}_1)$ in for the Gaussian case. In this case we have $\widetilde{Y}_1 = H_{11} X_1 + H_{12} X_2 + \widetilde{Z}_1$ with $\widetilde{Z}_1 \sim Z_1$ and thus we can write

$$\begin{aligned} I(Y_1; X_1, X_2 | \widetilde{Y}_1) &= H(Z_1 - \widetilde{Z}_1 | \widetilde{Y}_1) - H(Z_1) && \text{(18a)}\\ &\leq H(Z_1 - \widetilde{Z}_1) - H(Z_1) && \text{(18b)}\\ &\leq m_1/2 \log(2) = m_1/2. && \text{(18c)} \end{aligned}$$

■

Note that, similarly to the result in [10] for the MIMO IFC, the result in Lem. V.1 does not depend on the covariance constraints on channel inputs (6).

Theorem 5. Capacity of the MIMO CIFC to within a factor two: *If* $(R_1, R_2) \in \mathcal{R}^{\rm out}$*, then* $(2R_1, 2R_2)$ *is achievable.*

Proof: Consider first the outer bound in (7): by taking the maximum of each bound, we obtain the looser outer bound

$$R_1 \leq I(Y_1; X_1 | X_2) \tag{19a}$$
$$R_2 \leq I(Y_2; X_1, X_2). \tag{19b}$$

union over all the distributions p_{X_1,X_2}. Since Gaussian maximizes entropy, it is necessary to consider only jointly Gaussian channel inputs.

Consider then the following two achievable points

$$A^{\rm in,DPC} = (I(Y_1; X_1|X_2); I(Y_2; X_2)) \tag{20a}$$
$$B^{\rm in,MISO} = (0, I(Y_2; X_1, X_2)), \tag{20b}$$

the point $A^{\rm in,DPC}$ is achievable using the inner bound in Thm. 3 by setting $U_{2pb} = \emptyset$ and

$$U_{1pb} = X_1 + AX_2 \tag{21a}$$
$$A = \mathbb{V}{\rm ar}[X_1] H_{11}^H (H_{11} \mathbb{V}{\rm ar}[X_1] H_{11}^H + I)^{-1} \tag{21b}$$

which completely cancels the effect of the interference at receiver 1. The point $B^{\rm in,MISO}$ is achieved with the choice $U_{1pb} = U_{2pb} = \emptyset$ which corresponds of having both encoders transmit to Rx 2 as in a MISO channel.

We now show that simplified outer bound of (19) is to within a factor of two from the convex closure of $A^{\rm in,DPC}$ and $B^{\rm in,MISO}$. Since $A^{\rm in,DPC}$ and $B^{\rm in,MISO}$ are achievable, with time sharing we can achieve any point (R_1, R_2) such that

$$R_2 = -\frac{I(Y_2; X_1|X_2)}{I(Y_1; X_1|X_2)} R_1 + I(Y_2; X_2). \tag{22}$$

for $R_1 \in [0 \ldots I(Y_1; X_1|X_2)]$. In particular the following rate point is achievable

$$\begin{aligned} C^{\rm in} &= (R_1^{C-{\rm in}}, R_2^{C-{\rm in}}) \qquad (23a)\\ &= (1/2 I(Y_1; X_1|X_2), 1/2(I(Y_2; X_1, X_2) + I(Y_2; X_2))) \end{aligned}$$

For this point we have that $2R_1^{C-{\rm in}} =$ (19a) while

$$\begin{aligned} 2R_2^{C-{\rm in}} - \text{(19b)} &= I(Y_2; X_1, X_2) + I(Y_2; X_2) - I(Y_2; X_1, X_2)\\ &= I(Y_2; X_2) \geq 0. \qquad (24a) \end{aligned}$$

■

VI. Conclusion

This paper studies the MIMO cognitive interference channel, a variation of the classical MIMO interference channel in which one of the transmitters, the cognitive transmitter, has also knowledge of the message of the other user, the primary user. For this channel model, inner and outer bounds are presented and the approximate characterizations of the capacity are shown. In particular we derive the capacity to within a constant additive gap and a constant multiplicative factor. The constant additive gap depends on the number of antennas at the receiver cognitive receiver and well characterizes the capacity region in the high SNR regime. For the low SNR regime, instead, we show that the ratio between inner and outer bound is at most two. Our results show how cognition benefits both the primary and the cognitive user: the primary user is able to attain much larger rates thanks to the cooperation with the cognitive transmitter. On the other hand, the knowledge of the primary message at the cognitive encoder, allows it to remove the effect of the interference at the cognitive receiver.

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