SRL1: Structured Reweighted ℓ_1 Minimization for Compressive Sampling of Videos

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Abstract—**In this paper, we study compressive sampling of difference frames in videos and introduce a novel reconstruction method that exploits the structural characteristic, i.e., clustered sparsity in difference frames. Our method, referred to as structured reweighted ℓ_1 minimization (SRL1), estimates the signal support and adjusts the weights associated with the signal coefficients in a weighted ℓ_1 minimization in an iterative fashion. For the signal support estimation we propose *local exploration* and *global purification* steps to promote the clustered sparsity in difference frames. It is shown that by exploiting the clustered sparsity, isolated non-zero noise could be eliminated, and undiscovered signal coefficients could be retrieved. It should be noted that these steps are done based on the clustered sparsity, rather than the exact signal support distribution. This makes our method robust and distinct from many state-of-the-art algorithms. Experimental results show the effectiveness of our method.**

I. Introduction

Compressive sampling [1], [2] is a novel technique that exploits the sparsity/compressibility of signals to reconstruct them under sub-Nyquist sampling rates. This makes compressive sampling attractive in fields such as medical imaging, infra-red imaging, wireless multimedia sensor networks and many more where increasing sampling rate would be very costly.

Video coding employing compressive sampling is an emerging field. Since the reconstruction of under-sampled frames depends on the sparsity of the target signal, most endeavours in this area lie in exploiting the correlation, either temporally or spatially, to prompt sparsity of a video sequence. Authors in [3] studied a compressive-sampling-based video streaming technique for wireless multimedia sensor networks. It is shown that the difference frames of a video sequence coded by compressive sampling technique are more resilient to channel error compared to other coding techniques. In [4], a motion-compensation-based residual reconstruction for compressive sampling of video was proposed to explore the temporal correlation of video frames.

Aside from the temporal and spatial correlations, the structural feature of the video frames is of great importance for the compressive sampling reconstruction problem. Authors in [8] studied the compressive sampling reconstruction problem for clustered sparse signal. The clustered sparsity is modelled by the Ising model and a novel algorithm, referred to as LaMP, was proposed to explore this structural feature. However, LaMP [8] is sensitive to model parameters and the performance may degrade when a not very accurate model is selected. In order to explore the clustered sparsity, several parameters in the Ising model need to be estimated accurately, which may not be feasible for a resource limited encoder. In [11], a three-pattern model was proposed to prompt the clustered sparsity. Markov Chain Monte Carlo sampling (MCMC) is then used to infer the signal coefficients from the random samples. While no parameter needs to be estimated before the reconstruction, CluSS [11] suffers from the huge computation time inherent in MCMC sampling. Moreover, the convergence of CluSS is not guaranteed.

As in [8] and [11], the clustered sparsity is of our interest. We follow the general setting in [3]. The difference frame is calculated as the algebraic difference of the non-reference frame w.r.t. the reference frame and is then compressive sampled by projection with a random matrix. The clustered sparsity of the difference frame in a video sequence is explored by our proposed structure-aware reconstruction technique, referred to as SRL1. The proposed method reconstructs the difference frame of a video sequence and estimates its signal support in an iterative fashion. The clustered sparsity of current reconstruction is utilized to estimate the signal support with which the weights associated with signal coefficients can be estimated. The weights are then used to direct the reconstruction of the difference frame in the next iteration. It is shown that through what we call *local exploration* and *global purification*, unrecovered signal coefficients could be prompted and isolated non-zero noises can be eliminated.

Our method is distinguished from other structural-aware algorithms in two aspects. First of all, unlike LaMP [8], our method takes advantage of the connectivity of the non-zero pixels in the difference frame and there are few parameters which need to be tuned. In this sense, our method is more robust than LaMP [8] and will not suffer from selecting an inaccurate model. Secondly, since our algorithm is an ℓ_1 based method, the convergence of the algorithm is guaranteed. As one can see from the experiment results, our method provides more stable reconstruction results than CluSS [11], which is based on MCMC sampling.

The remainder of this paper is organized as follows: necessary background is provided in Section II. In Section III, we describe the system architecture. Section IV shows the details of the proposed algorithm. Experimental results are illustrated in Section V. Section VI concludes this paper.

II. Background

A. Compressive Sampling

Suppose $\underline{x} = [x_1, x_2, \ldots, x_N]^T \in \mathbb{R}^N$ is the coefficients vector of an image of size $\sqrt{N} \times \sqrt{N}$. Let $\underline{\theta} = [\theta_1, \theta_2, \ldots, \theta_N]^T \in \mathbb{R}^N$ be the projection of $\underline{x}$ into *wavelet* or DCT bases i.e., $\underline{x} = \Psi\underline{\theta}$. It is noted that after such projection, the representation of the image becomes sparse (i.e., $\underline{\theta}$ will have a large number of zero or close-to-zero coefficients). Suppose there are only $K \ll N$ large coefficients. Without loss of generality, we assume the canonical sparsity basis: $\Psi = I$ in the sequel when dealing with difference frames. In compressive sampling, the target signal $\underline{x}$ is sampled by a random matrix through

$$\underline{y} = \Phi * \underline{x}, \tag{1}$$

where Φ is *M*-by-*N* with $M \ll N$, called the measurement matrix. Since $M \ll N$, finding a solution to $\underline{y} = \Phi\underline{x}$ is an ill-posed problem and there would be infinite number of solutions that satisfy (1).

What compressive sampling states is, under certain conditions [1], [2], it is possible to recover the signal vector $\underline{x}$ from $M \ll N$ random samples. Several methods could be used. Among them, the optimal one is the ℓ_0 method which focuses on the sparsity of the signal directly and looks for the sparsest solution from the infinite number of candidates. This can be expressed as:

$$\begin{aligned} &\widehat{\underline{x}} = argmin \left\|\underline{x}\right\|_0, \\ &s.t. \;\; \underline{y} = \Phi * \underline{x}, \end{aligned} \tag{2}$$

where $\left\|\underline{x}\right\|_0$ is the number of non-zero coefficients of $\underline{x}$. However, since the ℓ_0 method is NP-hard, the ℓ_1 method, which is a convex relaxation of ℓ_0 is more favorable. It can be expressed as:

$$\begin{aligned} &\widehat{\underline{x}} = argmin \left\|\underline{x}\right\|_1, \\ &s.t. \;\; \underline{y} = \Phi * \underline{x}, \end{aligned} \tag{3}$$

where $\left\|\underline{x}\right\|_1 = \sum_i |x_i|$. Given $M = O\left(Klog(N/K)\right)$ measurements[1], the reconstruction by ℓ_1 minimization is generally very accurate. Aside from these two methods, many other reconstruction schemes including greedy algorithms, such as CoSaMP [12] and IHT [13] exist.

B. Iterative Reweighted L1 Minimization

The ℓ_0 minimization (2) finds the optimal solution of the compressive sampling problem. This is because it minimizes the number of the non-zero signal coefficients. It should be noted that in the ℓ_0, the penalization is uniform regardless of the magnitude of each coefficient. On the other hand, the ℓ_1 minimization method (3) penalizes signal coefficients according to their magnitude [9]. This difference explains why ℓ_1 is suboptimal in finding the sparsest solution that agrees with the measurements. To fill the gap between ℓ_0 norm method and ℓ_1 norm method, an iterative reweighted ℓ_1 norm minimization, IRWL1, is proposed in [9], [10]. The basic idea of IRWL1 is to penalize large signal coefficients with weights smaller than those for small coefficients. This can be summarized as follows:

$$\begin{aligned} &\widehat{\underline{x}} = argmin \left\|W^{iter}\underline{x}\right\|_1, \\ &s.t. \;\; \underline{y} = \Phi * \underline{x} \end{aligned} \tag{4}$$

where W^{iter} is a diagonal reweighting matrix with entries

$$w_{n,n}^{iter} = \left(\left|x_n^{iter-1}\right| + \epsilon\right)^{-1} \tag{5}$$

with ϵ denoting the regularization constant and $iter \geq 1$ denoting the iteration counter. Since no prior knowledge of the signal magnitude is known, all of the entries in W^1 are set to 1 in the first iteration. As long as a current reconstruction is obtained, the reweighting matrix W^{iter} can be updated.

III. System Architecture

Here we follow the setting in [3]. Let $\underline{x}_{bj}$ be the reference frame in the j^{th} group of pictures (GoPs) in a video. $\underline{x}_{bj}$ is followed by G non-reference frames, denoted as $\underline{x}_{tj}^1, \underline{x}_{tj}^2, \ldots, \underline{x}_{tj}^G$. The difference frame between the i^{th} non-reference frame in the j^{th} group of pictures and its reference frame is calculated as a pixel-by-pixel algebraic difference:

$$\underline{d}_j^i = \underline{x}_{tj}^i - \underline{x}_{bj}, \tag{6}$$

for $i = 1, 2, \ldots, G$. The difference frame is then hard thresholded and is calculated as:

$$D_j^i(n) = \begin{cases} d_j^i(n) & \text{if } d_j^i(n) \geq \tau; \\ 0 & \text{else,} \end{cases} \tag{7}$$

for $n = 1, 2, \ldots, N$, where τ is a threshold value. The thresholded difference frame $\underline{D}_j^i$ is then sampled using compressive sampling and (1) is restated as:

$$\underline{V}_j^i = \Phi * \underline{D}_j^i. \tag{8}$$

It is known that when the number of compressed samples, M, is above the weak threshold $O\left(Klog(N/K)\right)$, the reconstruction by ℓ_1 minimization is generally very accurate. However, compressive sampling reconstruction degrades a lot when M is below the weak threshold. Now, we are interested in answering the following question: if the number of compressive sensing samples (M) is less than $O\left(Klog(N/K)\right)$, given the full knowledge of the reference frame and the compressive sampling measurements of difference frames, could we reconstruct the video sequence better than the state-of-the-art methods? As we will see in the following sections, by taking advantage of the clustered sparsity, the answer is positive.

IV. SRL1: Structured Reweighted L1 Minimization

We propose a novel structured reweighted ℓ_1 optimization technique, called SRL1, to explore the clustered sparsity of the difference frames. We will first show two key components of our technique in the first two subsections and the algorithm is then summarized in the last subsection.

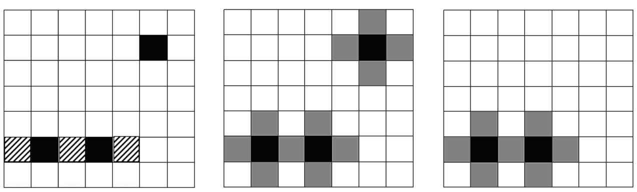
[1] $M = O\left(Klog(N/K)\right)$ is also known as the weak threshold.

A. Clustered Sparsity

Definition 1. A cluster is the set of contiguous non-zero pixels and the size of a cluster is defined as the cardinality of the set.

Definition 2. Two pixels are said to be connected in the sense of dilation by *Structural Element*[2] (*SE*) [14] if the clusters dilated by *SE* from these pixels are contiguous or intersected; similarly, a non-zero pixel is said to be isolated in the sense of dilation by *SE* if the cluster dilated from this pixel is not contiguous or intersected with other clusters.

For brevity, *connected* and *isolated* are used as *connected in the sense of dilation by SE* and *isolated in the sense of dilation by SE*, respectively. Fig. 1a is an example where each grid represents a pixel. Three out of $7 * 7$ pixels are non-zero and marked with solid black. As shown in Fig. 1b, the clusters (marked with solid gray) dilated from the two black pixels at the bottom left are intersected and thus these two pixels are connected; the non-zero black pixel at the top right is isolated.



(a) (b) (c)

Fig. 1: Local exploration and global purification steps.
(a) Initial reconstruction; (b) Local exploration by SE; (c) Global purification has removed the isolated non-zero pixel.

One of the structural characteristics of the difference frame is the clustered sparsity in which non-zero pixels tend to cluster. Here is an example. Fig. 2a is the thresholded difference frame between 3^{rd} frame and 1^{st} frame (reference frame) of “Foreman” video sequence while Fig. 2b is derived from Fig. 2a by removing isolated non-zero pixels. As one can see, most of the non-zero pixels in the thresholded difference frame are clustered, and just a small fraction of the non-zero pixels are isolated. Fig. 2c shows the ratio of the number of the isolated non-zero pixels to that of non-zero pixels. 10 video sequences each with 90 frames are tested. Clearly, in all 10 videos tested, most of the non-zero pixels are not isolated. Specifically, 7 out of the 10 videos have isolated pixel ratios smaller than 5%. Video sequences including “Hall”, “Akiyo” and “Clarie” have isolated pixel ratios between 5% to 10%.

As discussed in the previous section, when the number of compressed samples (M) is less then $O\left(K log(N/K)\right)$, the reconstruction is inaccurate. In the context of difference frame reconstruction, that is to say, some non-zero pixels may not be recovered and some zero pixels may be reconstructed as

[2]In this paper, the shape of the *SE* is a disk. As shown in Fig. 1b, the *SE* around the black pixel at the top right corner is shown by gray pixels. The size of the *SE* is 5.

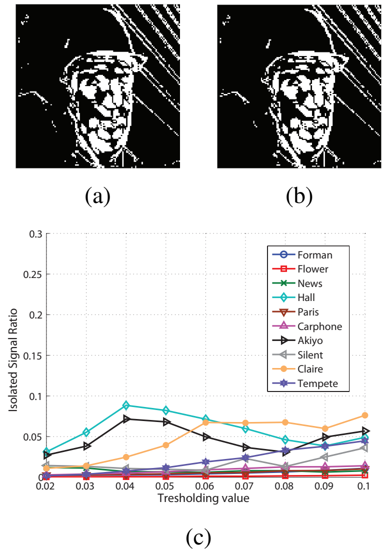


Fig. 2: Clustered Sparsity of difference frame.

non-zeros. The clustered sparsity of the difference frame gives rise to two heuristics that can be used to analyze and enhance the reconstructed difference frame $\underline{\widehat{D}}_j^i$ for videos:

1) If a pixel $\widehat{D}_j^i(n)$ is zero but is connected (in the sense of dilation) to other non-zero pixels, there is a high probability that this pixel is actually non-zero rather than zero;
2) If a pixel $\widehat{D}_j^i(n)$ is non-zero and is isolated (in the sense of dilation), there is a high probability that this pixel is actually zero rather than non-zero.

In the following subsections, we will see these two heuristics give rise to two key steps, Local Exploration and Global Purification, in signal support estimation. With these two steps, even when the number of compressed samples is below the weak threshold, difference frame reconstruction could be improved where unrecovered non-zero pixels can be prompted and non-zero errors can be eliminated.

B. Signal Support of Difference Frame and Weights Allocation

Signal support estimation is of great importance in the compressive sampling reconstruction step. Our signal support estimation starts from the initial reconstruction $\underline{\widehat{D}}_j^i$ and is followed by two steps, *local exploration* and *global purification*.

The local exploration stage is inspired by the first heuristic and is to find the unrecovered signal pixels. In the Local Exploration stage, we first make $\underline{\widehat{D}}_j^i$ a binary frame, and the pixel of the resulting binary frame is expressed as:

$$B_j^i(n) = \begin{cases} 1 & \text{if } \widehat{D}_j^i(n) > \tau; \\ 0 & \text{else}, \end{cases} \qquad (9)$$

This give rise to the binary frame $\underline{B}_j^i$ and is illustrated in Fig. 1a. In Fig. 1a, two black pixels at the bottom left represent recovered non-zero pixels. The black pixel at the top right represents a non-zero error. The three shaded pixels at the bottom left represent unrecovered non-zero pixels. Then, each non-zero pixel in $\underline{B}_j^i$ (see Fig. 1a) serves as an *anchor* and is morphologically dilated by the *Structuring Element* (*SE*) [14], and the dilated frame is denoted as $\underline{L}_j^i$. As a result, after the Local Exploration stage, (see Fig. 1b), each non-zero pixel (*anchor*) is dilated to a cluster.

The global purification stage is inspired by the second heuristic and is to eliminate non-zero errors. In the global purification stage, depending on the size, certain clusters and their corresponding *anchors* will be deleted from $\underline{L}_j^i$. There are two cases for the size of the cluster. If two *anchors* in $\underline{B}_j^i$ are connected (in the sense of dilation by *SE*), the clusters dilated from these two *anchors* will intersect and form a larger cluster. As a result, these two connected *anchors* will locate in the same larger cluster with size greater than the size of *SE*. On the other hand, if a *anchor* in $\underline{B}_j^i$ is isolated, the size of the cluster dilated from this *anchor* will be equal to the size of *SE*. To eliminate non-zero errors, the clusters with size smaller than a predefined threshold[3], N_{conn}, are deleted from $\underline{L}_j^i$ and the resulting binary object (see Fig. 1c), is our estimation of the signal support of $\underline{D}_j^i$, denoted as $\underline{S}_j^i$.
Then the weights vector is calculated based on the signal support $\underline{S}_j^i$ and (5) is restated as:

$$w^{iter}(n,n) = \begin{cases} 1/(w_1+\epsilon) & \text{if } S_j^i(n)=1; \\ 1/(w_0+\epsilon) & \text{if } S_j^i(n)=0, \end{cases} \quad (10)$$

where w_1, w_0 and ϵ are set to 1, 0 and 0.1 respectively in our tests, and $w^{iter}(n,n)$ is the $n \times n$ element of the re-weighting matrix W^{iter}.

C. The Proposed Algorithm Summary

The proposed structured reweighted algorithm, SRL1, is summarized in Algorithm 1. In Step 1, an initial reconstruction of difference frame can be obtained through a variety of solvers, for example, SPGL1 [6], [7]. This initial reconstruction is served as side information for further refinement and will be updated in each iteration. In Step 2, the reconstructed difference frame is then thresholded and converted to a binary frame based on (9). Local Exploration and Global Purification are implemented in Step 3 and Step 4. Signal support and weights vector are then updated in Step 5 and Step 6 correspondingly.

V. Simulation Results

Experiments are taken to show the effectiveness of the proposed algorithm. In all of the experiments, each frame is gray scale with size $128*128$ pixels, and the value of each pixel has been scaled to [0,1]. As in [15] and [16], to relieve the computational burden at the video encoder which is resource-constraint, instead of using dense matrices, we use sparse random Φ with row weight 16. It should be noted that non-zero elements of Φ are drawn from normalized Gaussian distribution and are uniformly distributed across the columns.

[3] The threshold value is calculated based on the size of *SE* and is set to 6 in this paper.

Algorithm 1: SRL1-Structured Reweighted L1 Minimization.

Input: $\underline{V}_j^i, \Phi, \underline{x}_{bj}$
Output: $\widehat{\underline{x}}_{tj}^i$
Algorithm:
Initialize: $W^1 = I, iter = 1$;
while $iter \leq ITER$ **do**
Step 1: Solve (4):

$$\widehat{\underline{D}}_j^i = argmin \left\|W^{iter}\underline{D}_j^i\right\|_1, s.t.\ \underline{V}_j^i = \Phi * \underline{D}_j^i$$

Step 2: Convert $\underline{D}_j^i$ to binary frame $\underline{B}_j^i$ as in (9);
Step 3: Perform local exploration using SE;
Step 4: Perform global purification by removing clusters with size small small than N_{conn};
Step 5: Estimate signal support: $\underline{S}_j^i$;
Step 6: Update Weight using (10), $iter = iter + 1$;
end while
Return: $\underline{x}_{tj}^i = \underline{x}_{bj} + \widehat{D}_j^i$

In the first experiment, we test the number of iterations $iter$ on reconstruction quality. The 1^{st} (reference frame) and 3^{rd} frames of "Foreman" are picked. The difference frame is calculated and then sampled at 40% $(M/N = 0.4)$. Then, reconstructions by our proposed algorithm SRL1 and IRWL1 [10] are compared and the PSNRs of the reconstructed 3^{rd} frame are shown in Fig. 3. Clearly, our proposed technique outperforms IRWL1 considerably. Compared to IRWL1, SRL1 increases the PSNR of the reconstructed video frame by $3.01\ dB$ in the second iteration and the gain is $3.85\ dB$ after five iterations. Similar results are also observed on other pairs of frames.

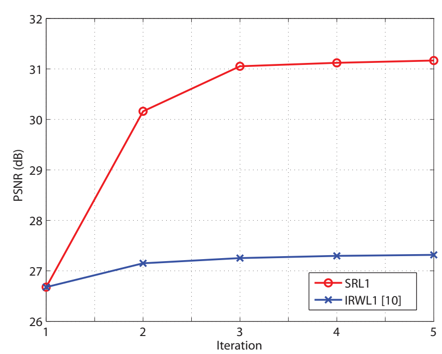
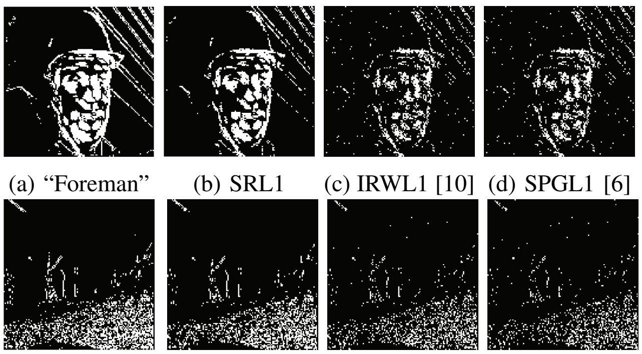


Fig. 3: PSNR comparison between SRL1 and IRWL1 as a function of reconstruction iterations for the 3^{rd} frame of Foreman video.

In the second experiment, 1^{st} and 3^{rd} frames of "Foreman" and 1^{st} and 2^{nd} frames of "Flower" are selected. The 1^{st} frame is set as the reference frame and the difference frame is calculated as (6). The sampling rate (M/N) is set as 40% for both of these two tests. ITER is set 5. Threshold τ is set 0.08. The comparision of the performance of different reconstruction techniques applied for “Foreman” and “Flower” are shown in Fig. 4. The reconstructed difference frames have been converted to binary frames with threshold value 0.08 for illustration purpose here. The PSNRs of the reconstructed non-reference frame for “Foreman” are 31.35 dB (SRL1), 27.31 dB (IRWL1) and 26.84 dB (SPGL1). The PSNRs of the reconstructed non-reference frame for “Flower” are 31.32 dB (SRL1), 29.22 dB (IRWL1) and 28.28 dB (SPGL1). Comparing these results, we can see that SRL1 can eliminate non-zero errors. Moreover, unrecovered non-zero pixels could be prompted.



(a) “Foreman” (b) SRL1 (c) IRWL1 [10] (d) SPGL1 [6]

(e) “Flower” (f) SRL1 (g) IRWL1 [10] (h) SPGL1 [6]

Fig. 4: Demonstration of difference frame reconstruction using different techniques.

In the last experiment, more frames from “Flower” are tested. The size of GoP is set 5 and maximum iteration (ITER) is set 5. The sampling rate (M/N) is set based on the sparsity (K/N) of each difference frame and is set below the weak threshold. Specifically, the sampling rate (M/N) for the $1^{st}, 2^{nd}, 3^{rd}, 4^{th}$ non-reference frame in each GoP is set $40\%, 55\%, 60\%, 65\%$ for “Flower”[4]. Threshold τ is set 0.08 for each 1^{st} non-reference frame and 0.09 for $2^{nd}, 3^{rd}, 4^{th}$ non-reference frame. The PSNR of 15 reconstructed non-reference frames in the video sequence using our proposed method, SRL1, and Block-CoSaMP [5], CluSS [11], IRWL1 [10], SPGL1 [6] are shown in Fig. 5. It can be seen that even though the sampling rate is set below the weak threshold, taking advantage of the clustered sparsity, SRL1 still gives decent reconstruction and outperforms other schemes.

VI. Conclusion

In this paper, a novel structured reweighted ℓ_1 minimization algorithm, referred to as SRL1, is proposed to reconstruct difference frames in the video sequences. It is shown that by exploiting the clustered non-zero coefficients, isolated non-zero noises could be eliminated and unrecovered signal coefficients could be prompted. We showed that SRL1 can reconstruct the difference frame much better than many other state-of-the-art algorithms.

[4]The sparsity (K/N) increases as the frame distance becomes larger. As a results, we gradually increase the sampling rates (M/N) within each GoP.

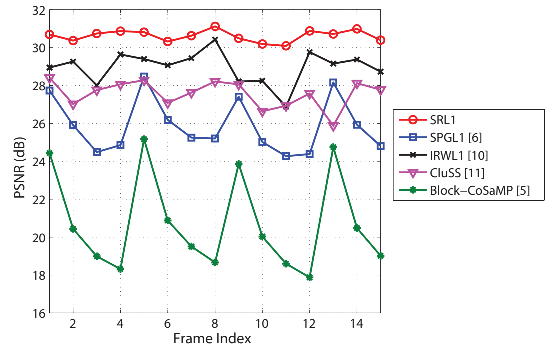


Fig. 5: Comparison of SRL1 with other techniques.

VII. Acknowledgement

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