Large Zero Correlation Zone of Golay Pairs and QAM Golay Pairs

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***Abstract*—Sequences with desirable correlation properties have wide applications in today's communication systems. In this paper, we first extend the known results on zero autocorrelation zone of Golay sequences to 4^q-QAM Golay sequences and show three constructions of 4^q-QAM Golay sequences with a large zero periodic autocorrelation zone, where $q \geq 2$ is an arbitrary integer. We then determine the Golay pairs which have large zero periodic crosscorrelation zone.**

Keywords. Golay sequence, Golay complementary sequence, quadrature amplitude modulation (QAM), zero autocorrelation zone, and zero crosscorrelation zone.

I. Introduction

In modern communications, sequences with good correlation properties are desired for a wide variety of applications such as receiver synchronization, detection and channel estimation. In 1961, Golay proposed the idea of aperiodic complementary sequence pairs [7], for which the sum of out-of-phase aperiodic autocorrelation of two sequences equals to zero. They are called Golay complementary pairs or simply Golay pairs. We will use the term Golay pair throughout this paper. Each sequence from the Golay pair is called a Golay sequence. Later on, Davis and Jedwab formulated a method for constructing Golay pairs by using quadratic generalized boolean functions [4]. Due to this correlation property, Golay sequences have been proposed to construct Hadamard matrix for direct sequence code division multiple access (DS-CDMA) system [17], and to control the peak envelope power (PEP) in orthogonal frequency-division multiplexing (OFDM) system [20], [21], [22], [23].

The utilization of Golay sequences in the two above scenarios is based on the property that the sum of out-of-phase autocorrelation of the pair equals to zero. However, synchronization and detection of the signal is equivalent to computing its autocorrelation. In this case, the autocorrelation property of a single sequence is of our interest. This is also the case with synchronous CDMA and quasi-synchronous CDMA (QS-CDMA) systems.

QS-CDMA differs from synchronous CDMA system [8] in that it allows a small time delay in the arrival signals of different users. In this case, sequences with low or zero autocorrelations centered at the origin can reduce or eliminate the multipath interference. Sequences with low or zero crosscorrelations centered at the origin can reduce or eliminate the multiple access interference. Sequences with both zero or low autocorrelations and crosscorrelations are called low correlation zone (LCZ) and zero correlation zone (ZCZ) sequences respectively [14]. As a result, the construction of new LCZ or ZCZ sequences for QS-CDMA system has received a lot of attention [5], [6], [1], [15], [19], to just list a few.

Due to the importance of autocorrelation and crosscorrelation properties of sequences in the aforementioned applications, in this paper, we intend to examine the correlation properties of QAM Golay sequences and Golay pairs. In [10], Gong *et. al.* have presented three constructions of Golay sequences with large zero autocorrelation zone (ZACZ) of length approximately a half, a quarter or one eighth of their periods. Theorem 4 of [10] under certain conditions for the binary case have been reported [9]. The large zero odd autocorrelation zone of some Golay sequences and 4^q-QAM Golay sequences have been reported in [24]. For details on QAM Golay sequences, please refer to [3], [11], [12], [2], [13]. Those sequences can be used for signal detection, synchronization and channel estimation in the communications.

In this paper, we intend to continue this work. We will also present our findings on the three constructions of 4^q-QAM Golay sequences with a ZACZ of length approximately an half, a quarter or one eighth of their periods. Moreover, we will present on the existence of large zero crosscorrelation zones of Golay pairs.

This paper is organized as follows. In Section II, we will provide the necessary preliminary materials required for the later sections. In Section III, we will show the large ZACZ of 4^q-QAM Golay sequences, and demonstrate the ZACZ with concrete examples. In Section IV, we will introduce some Golay pairs possess large zero periodic crosscorrelation zones. Finally, we conclude our paper in Section V.

II. Definitions and Preliminaries

Let $a = (a_0, a_1, \cdots, a_{N-1})$ and $b = (b_0, b_1, \cdots, b_{N-1})$ be two complex sequences of period N. The *aperiodic cross-*

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correlation function and *periodic crosscorrelation function* of sequences a and b at some shift τ, where $0 \leq \tau \leq N-1$, are respectively given by

$$\begin{aligned} C_{a,b}(\tau) &= \sum_{i=0}^{N-1-\tau} a_i (b_{i+\tau})^*, \\ R_{a,b}(\tau) &= C_{a,b}(\tau) + (C_{a,b}(N-\tau))^*, \end{aligned}$$

where x^* denotes the complex conjugate of x. If sequences a and b are the same sequence, then we call the above equations *aperiodic autocorrelation function* and *periodic autocorrelation function* respectively. We further denote them as $C_a(\tau)$ and $R_a(\tau)$ for simplifications.

Let δ_1 and δ_2 be two integers with $0 < \delta_1 < \delta_2 < N$ and denote $L = \delta_2 - \delta_1 + 1$. If $R_a(\tau) = 0$ for any $\delta_1 \leq \tau \leq \delta_2$, then the sequence a has a *zero periodic autocorrelation zone* (ZACZ) of length L.

Let $\mathcal{S}$ be a set of M complex sequences with period N. The low correlation zone L_{cz} of $\mathcal{S}$ is defined as follows:

$$L_{cz} = \max \left\{ T : \begin{array}{l} |R_{a,b}(\tau)| \leq \delta, a, b \in \mathcal{S}, (0 < \tau < T) \\ \text{or } (\tau = 0 \text{ and } a \neq b) \end{array} \right\}.$$

Then the set $\mathcal{S}$ is called a low correlation zone (LCZ) sequence set, denoted by (N, M, L_{cz}, δ)-LCZ. When $\delta = 0$, The L_{cz} is called the zero correlation zone (ZCZ) Z_{cz}, denoted by (N, M, Z_{cz})-ZCZ.

In order to determine whether newly constructed LCZ/ZCZ sequence sets possess good parameters or not, one can compare them with the Tang-Fan-Matsufuji bound given in [18].

Fact 1: Let S be an (N, M, L, δ)-LCZ sequence set. Then

$$L \leq \left\lfloor \frac{N^2 - \delta^2}{M(N - \delta^2)} \right\rfloor. \tag{1}$$

Let $H \geq 2$ be an arbitrary integer, $\mathbf{Z}_H$ be the integer residue ring modulo H, and $\xi = \exp(2\pi\sqrt{-1}/H)$ be the primitive H-th root of unity. A sequence $a = (a_0, a_1, \cdots, a_{N-1})$ over $\mathbf{Z}_H$ can be regarded as a complex sequence $(\xi^{a_0}, \xi^{a_1}, \cdots, \xi^{a_{N-1}})$. The sequences a and b are called a *Golay pair* if $C_a(\tau) + C_b(\tau) = 0$ for any $1 \leq \tau \leq N-1$. Any one of them is called a *Golay sequence*.

In the following, we introduce some notations. We always assume that π is a permutation from $\{1, \cdots, m\}$ to itself, and $(i_1, \cdots, i_m)$ is the binary representation of the integer $i = \sum_{k=1}^{m} i_k 2^{m-k}$, where $m \geq 4$ is an integer.

Define a sequence $a = \{a_i\}_{i=0}^{2^m-1}$ over $\mathbf{Z}_H$, whose elements are given by

$$a_i = \frac{H}{2} \sum_{k=1}^{m-1} i_{\pi(k)} i_{\pi(k+1)} + \sum_{k=1}^{m} c_k i_k + c_0, \tag{2}$$

where $c_i \in \mathbf{Z}_H$, $i = 0, 1, \cdots, m$.

When $H = 2^h$, $h \geq 1$ an integer, Davis and Jedwab proved that $\{a_i\}$ and $\{a_i + 2^{h-1} i_{\pi(1)} + c'\}$ form a Golay pair for any $c' \in Z_{2^h}$ in Theorem 3 of [4]. Later on, Paterson generalized this result by replacing Z_{2^h} with $\mathbf{Z}_H$ [16], where $H \geq 2$ is an arbitrary even integer.

Fact 2: Let $a = \{a_i\}_{i=0}^{2^m-1}$ be the sequence given by (2), then the sequences a and b whose elements are given by a_i and $a_i + \frac{H}{2} i_{\pi(1)} + c'$ respectively form a Golay pair for any $c' \in \mathbf{Z}_H$.

We define

$$\begin{aligned} a_{i,0} &= 2\sum_{k=1}^{m-1} i_{\pi(k)} i_{\pi(k+1)} + \sum_{k=1}^{m} c_k i_k + c_0 \\ b_{i,0} &= a_{i,0} + \mu_i \\ a_{i,e} &= a_{i,0} + s_{i,e} \\ b_{i,e} &= b_{i,0} + s_{i,e} = a_{i,e} + \mu_i, 1 \leq e \leq q-1, \end{aligned}$$

where $c_k \in \mathbf{Z}_4$, $k = 0, 1, \cdots, m$, and $s_{i,e}$ and μ_i are defined as one of the following cases:

1. $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(m)}$, $\mu_i = 2i_{\pi(1)}$ for any element $d_{e,0}, d_{e,1} \in \mathbf{Z}_4$.
2. $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(1)}$, $\mu_i = 2i_{\pi(m)}$ for any element $d_{e,0}, d_{e,1} \in \mathbf{Z}_4$.
3. $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(w)} + d_{e,2} i_{\pi(w+1)}$, $2d_{e,0} + d_{e,1} + d_{e,2} = 0$, $\mu_i = 2i_{\pi(1)}$, or $\mu_i = 2i_{\pi(m)}$ for any element $d_{e,0}, d_{e,1}, d_{e,2} \in \mathbf{Z}_4$ and $1 \leq w \leq m-1$.

We define a pair of 4^q-QAM sequences $A = \{A_i\}_{i=0}^{2^m-1}$ and $B = \{B_i\}_{i=0}^{2^m-1}$ as follows:

$$\begin{aligned} A_i &= \gamma \sum_{e=0}^{q-1} r_e \xi^{a_{i,e}} \\ B_i &= \gamma \sum_{e=0}^{q-1} r_e \xi^{b_{i,e}}, \end{aligned} \tag{3}$$

where $\gamma = \exp(\sqrt{-1}\pi/4)$, $\xi = \sqrt{-1}$, and $r_e = \frac{2^{q-1-e}}{\sqrt{(4^q-1)/3}}$, $a_{i,e}, b_{i,e} \in \mathbf{Z}_4$, $0 \leq e \leq q-1$.

Fact 3: Sequences A and B form a 4^q-QAM Golay pair [13]. Furthermore, for $q = 2$, A and B become a 16-QAM Golay pair which is constructed by Chong, Venkataramani and Tarokh in [3]; For $q = 3$, A and B become a 64-QAM Golay pair which is presented by Lee and Golomb in [11], and Chang, Li, and Hirata [2].

III. Three Constructions of 4^q-QAM Golay Sequences with Large Zero Autocorrelation Zone

Before introducing 4^q-QAM Golay sequences with large ZACZ, we will first review the three constructions of Golay sequences with large ZACZ given in [10].

A. Known Results

For π a permutation of $S_m = \{1, \cdots, m\}$ with

$$\pi(i) = t_i, \quad 1 \leq i \leq m$$

a permutation π' of S_m satisfying

$$\pi'(i) = t_{m+1-i}, \quad 1 \leq i \leq m$$

is called the *reciprocal of* π.

Now we review the results in [10].

Theorem 4: If the Golay sequence a, defined by (2), satisfies one of the conditions listed below:

(A-1) $\pi(1) = 1$ and $\pi(2) = 2$ or the reciprocal of π ; $2c_1 = 0$.
(A-2) $\pi(2) = 2$, $\pi(3) = 1$, and $\pi(4) = 3$ or the reciprocal of π ; $2c_1 = 0$ and $c_1 = 2c_2$.
(A-3) $\pi(1) = 2$, $\pi(2) = 1$, and $\pi(3) = 3$ or the reciprocal of π ; $2c_1 = 0$ and $c_1 = 2c_2 + \frac{H}{2}$.

Then the autocorrelation of sequence a in one period has the following property:

$$R_a(\tau) = 0, \quad \tau \in (0, 2^{m-2}] \cup [3 \cdot 2^{m-2}, 2^m).$$

In other words, in one period $[0, 2^m)$, it has two zero autocorrelation zones of length 2^{m-2}, given by $(0, 2^{m-2}]$ and $[3 \cdot 2^{m-2}, 2^m)$.

Theorem 5: If the Golay sequence a, defined by (2), satisfies one of the conditions listed below:

(B) $\pi(1) = 2$, $\pi(2) = 1$, and $\pi(3) = 3$ or the reciprocal of π ; $2c_1 = 0$ and $c_1 = 2c_2$.

Then the sequence a has the following property:

$$R_a(\tau) = 0, \quad \tau \in [2^{m-2}, 3 \cdot 2^{m-2}].$$

In other words, in one period $[0, 2^m)$, it has a zero autocorrelation zone of length $2^{m-1} + 1$, given by $[2^{m-2}, 3 \cdot 2^{m-2}]$.

Theorem 6: If the Golay sequence a, defined by (2), satisfies one of the conditions listed below:

(C-1) $\pi(1) = 1$, $\pi(2) = 3$, and $\pi(3) = 2$ or the reciprocal of π ; $2c_1 = 0$.
(C-2) $\pi(1) = 1$, $\pi(2) = 3$, and $\pi(m) = 2$ or the reciprocal of π ; $2c_1 = 0$.
(C-3) $\pi(1) = 2$, $\pi(2) = 4$, $\pi(3) = 1$, and $\pi(4) = 3$ or the reciprocal of π ; $2c_1 = 0$ and $c_1 = 2c_2$.
(C-4) $\pi(1) = 2$, $\pi(2) = 3$, $\pi(3) = 1$, and $\pi(4) = 4$ or the reciprocal of π ; $2c_1 = 0$ and $c_1 = 2c_2$.

Then the sequence a has the following property:

$$R_a(\tau) = 0, \quad \tau \in (0, 2^{m-3}] \cup [3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}] \cup [7 \cdot 2^{m-3}, 2^m).$$

In other words, in one period $[0, 2^m)$, it has three zero autocorrelation zones of respective length 2^{m-3}, $2^{m-2} + 1$, 2^{m-3}, given by $(0, 2^{m-3}]$, $[3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}]$ and $[7 \cdot 2^{m-3}, 2^m)$.

B. Zero Autocorrelation Zone of 4^q-QAM Golay Sequences

In this subsection, we will consider the ZACZ of 4^q-QAM Golay sequences defined by (3), which are based on the quaternary Golay sequences with large ZACZ property. Due to the space limitation, the proofs for all theorems in this paper are omitted.

Theorem 7: Let a be the 4^q-QAM Golay sequence defined by (3) with $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(m)}$ or $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(1)}$ for any $d_{e,0}, d_{e,1} \in \mathbf{Z}_4$, where $\{a_{i,0}\}_{i=0}^{2^m-1}$ satisfies one of the conditions listed in (A), then the sequence a has the following ZACZ:

$$R_a(\tau) = 0, \quad \tau \in (0, 2^{m-2}] \cup [3 \cdot 2^{m-2}, 2^m).$$

In other words, in one period $[0, 2^m)$, it has two ZACZ of length 2^{m-2}, given in the range of $(0, 2^{m-2}]$ and $[3 \cdot 2^{m-2}, 2^m)$.

Theorem 8: Let a be the 4^q-QAM Golay sequence defined by (3) with $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(m)}$ or $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(1)}$ for any $d_{e,0}, d_{e,1} \in \mathbf{Z}_4$, where $\{a_{i,0}\}_{i=0}^{2^m-1}$ satisfies one of the conditions listed in (B), then the sequence a has the following ZACZ:

$$R_a(\tau) = 0, \quad \tau \in [2^{m-2}, 3 \cdot 2^{m-2}].$$

In other words, in one period $[0, 2^m)$, it has one ZACZ of length $2^{m-1} + 1$, given in the range of $[2^{m-2}, 3 \cdot 2^{m-2}]$.

Theorem 9: Let a be the 4^q-QAM Golay sequence defined by (3) with $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(m)}$ or $s_{i,e} = d_{e,0} + d_{e,1} i_{\pi(1)}$ for any $d_{e,0}, d_{e,1} \in \mathbf{Z}_4$, where $\{a_{i,0}\}_{i=0}^{2^m-1}$ satisfies one of the conditions listed in (C), then the sequence a has the following ZACZ:

$$R_a(\tau) = 0, \quad \tau \in (0, 2^{m-3}] \cup [3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}] \cup [7 \cdot 2^{m-3}, 2^m).$$

In other words, in one period $[0, 2^m)$, it has three ZACZ of respective length 2^{m-3}, $2^{m-2} + 1$, 2^{m-3}, given in the range of $(0, 2^{m-3}]$, $[3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}]$ and $[7 \cdot 2^{m-3}, 2^m)$.

We have presented the ZACZ for certain QAM Golay sequences that satisfies cases 1 and 2 in Fact 3. For QAM Golay sequences of case 3, we have not generalized a method in which it contains a large ZACZ. However, by computer search, we have found that, if $q = 2$, $m = \{4, 5\}$, $\pi(1) = 1$, $\pi(2) = 2$ and $2c_1 = 0$, and $2d_0^{(1)} + d_1^{(1)} + d_2^{(1)} = 0$, then the 16-QAM Golay sequence a defined by (3) has $R_a(\tau) = 0$ for $\tau \in (0, 2^{m-2}] \cup [3 \cdot 2^{m-2}, 2^m)$, which corresponds to the same ZACZ as Theorem 8. This is illustrated in the following example.

Example 10: Let $q = 2$ and $m = 5$. Let $\pi = (1)$, $c_1 = 0$ and $s_i^{(1)} = 1 + i_{\pi(2)} + i_{\pi(3)}$. Then this 16-QAM Golay defined by equality (3), has $R_A(\tau) = 0$ for $\tau \in (0, 8] \cup [24, 32)$, or has two zero autocorrelation zones of length 8.

IV. Crosscorrelation of Golay Pairs

In this section, we will show there exists Golay pairs with large zero crosscorrelation zone.

Theorem 11: Let $a = \{a_i\}_{i=0}^{2^m-1}$ be the sequence satisfying the condition (A) given in Theorem 4. Define $b = \{b_i\}_{i=0}^{2^m-1}$, where $b_i = a_i + \frac{H}{2} i_{\pi(m)}$ if a is defined by π, or $b_i = a_i + \frac{H}{2} i_{\pi(1)}$ if a is defined by the reciprocal of π. Then the crosscorrelation between sequences a and b has the following property:

$$R_{a,b}(\tau) = 0, \quad \tau \in (0, 2^{m-2}].$$

Remark 12: In [5], the authors proposed three constructions of binary ZCZ sequence sets, one of which is a Golay sequence defined by identity permutation. Other than Golay sequences defined by identity permutation, Theorem 11 has included cases where Golay sequences are defined by non-identity permutations.

Remark 13: For any given ZCZ sequence set consisting of M sequences of period N, the upper bound of zone correlation zone Z_{cz} given by (1) is reduced as $Z_{cz} \leq \frac{N}{M}$. It is conjectured that the bound on binary ZCZ sequence set would be greatly reduced to $Z_{cz} \leq \frac{N}{2M}$ (see [19] for example). If $N = 2^m$ and $M = 2$, then $Z_{cz} \leq 2^{m-2}$. Therefore, Theorem 11 proposed some optimal binary Golay sequences.

Theorem 14: Let $a = \{a_i\}_{i=0}^{2^m-1}$ be the sequence satisfying the condition (B) given in Theorem 4. Define $b = \{b_i\}_{i=0}^{2^m-1}$, where $b_i = a_i + \frac{H}{2} i_{\pi(m)}$ if a is defined by π, or $b_i = a_i + \frac{H}{2} i_{\pi(1)}$ if a is defined by the reciprocal of π. Then the crosscorrelation between sequences a and b has the following property:

$$R_{a,b}(\tau) = 0, \quad \tau \in [2^{m-2}, 3 \cdot 2^{m-2}].$$

Theorem 15: Let $a = \{a_i\}_{i=0}^{2^m-1}$ be the sequence satisfying the condition (C) given in Theorem 4. Define $b = \{b_i\}_{i=0}^{2^m-1}$, where $b_i = a_i + \frac{H}{2} i_{\pi(m)}$ if a is defined by π, or $b_i = a_i + \frac{H}{2} i_{\pi(1)}$ if a is defined by the reciprocal of π. Then the crosscorrelation between sequences a and b has the following property:

$$R_{a,b}(\tau) = 0, \tau \in (0, 2^{m-3}] \cup [3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}] \cup [7 \cdot 2^{m-3}, 2^m).$$

We will use empirical results to demonstrate these three categories of ZCZ. A total of 3 Golay pairs $a^{(k)}$ and $b^{(k)}$, $1 \leq k \leq 3$, and their autocorrelation and crosscorrelation are given in Tables I-III.

TABLE I
BINARY SEQUENCES $a^{(1)}$ AND $b^{(1)}$ OF LENGTH 32 WITH ZCZ PROPERTY FROM THEOREM 11

Conditions	$\pi = (1), H = 2$ $(c_0, c_1, c_2, c_3, c_4, c_5) = (0, 0, 1, 1, 0, 0)$
Sequence $a^{(1)}$	$(0,0,0,1,1,1,0,1,1,1,1,0,1,1,0,$ $1,0,0,0,1,1,1,0,1,0,0,0,1,0,0,1,0)$
$\{R_{a^{(1)}}(\tau)\}_0^{31}$	$(32,0,0,0,0,0,0,0,0,-4,0,-4,0,-12,0,4,$ $0,4,0,-12,0,-4,0,-4,0,0,0,0,0,0,0,0)$
Sequence $b^{(1)}$	$(0,1,0,0,1,0,0,0,1,0,1,1,1,0,0,0,$ $0,1,0,0,1,0,0,0,0,1,0,0,0,1,1,1)$
$\{R_{b^{(1)}}(\tau)\}_0^{31}$	$(32,0,0,0,0,0,0,0,0,4,0,4,0,12,0,-4,$ $0,-4,0,12,0,4,0,4,0,0,0,0,0,0,0,0)$
$\{R_{a^{(1)},b^{(1)}}(\tau)\}_0^{31}$	$(0,0,0,0,0,0,0,0,0,0,-4,8,-12,0,4,-8,$ $-4,0,4,-8,-4,0,12,8,4,0,0,0,0,0,0,0,0)$

V. Conclusion

In this paper, we have presented three constructions of 4^q-QAM Golay sequences that contain a large ZACZ, where q is an integer greater than or equal to 2. We have also presented three constructions of Golay sequences over $\mathbf{Z}_H$ that contain a large zero crosscorrelation zone, where H is a positive even integer.

The three zero autocorrelation zones are characterized as follows:

(i) $R_a(\tau) = 0$ for $\tau \in (0, 2^{m-2}] \cup [3 \cdot 2^{m-2}, 2^m)$.
(ii) $R_a(\tau) = 0$ for $\tau \in [2^{m-2}, 3 \cdot 2^{m-2}]$.
(iii) $R_a(\tau) = 0$ for $\tau \in (0, 2^{m-3}] \cup [3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}] \cup [7 \cdot 2^{m-3}, 2^m)$.

TABLE II
BINARY SEQUENCES $a^{(2)}$ AND $b^{(2)}$ OF LENGTH 32 WITH ZACZ FROM THEOREM 14

Conditions	$\pi = (12), H = 2,$ $(c_0, c_1, c_2, c_3, c_4, c_5) = (0, 0, 0, 0, 0, 1)$
Sequence $a^{(2)}$	$(0,1,0,0,0,1,1,1,0,1,0,0,0,1,1,1,$ $0,1,0,0,1,0,0,0,1,0,1,1,0,1,1,1)$
$\{R_{a^{(2)}}(\tau)\}_0^{31}$	$(32,-4,0,-4,0,-12,0,4,0,0,0,0,0,0,0,0,$ $0,0,0,0,0,0,0,0,0,4,0,-12,0,-4,0,-4)$
Sequence $b^{(2)}$	$(0,0,0,1,0,0,1,0,0,0,0,1,0,0,1,0,$ $0,0,0,1,1,1,0,1,1,1,1,0,0,0,1,0)$
$\{R_{b^{(2)}}(\tau)\}_0^{31}$	$(32,4,0,4,0,12,0,-4,0,0,0,0,0,0,0,$ $0,0,0,0,0,0,0,0,0,0,0,-4,0,12,0,4,0,4)$
$\{R_{a^{(2)},b^{(2)}}(\tau)\}_0^{31}$	$(0,-4,-8,-12,0,4,8,-4,0,0,0,0,0,0,0,$ $0,0,0,0,0,0,0,0,0,0,4,8,-4,0,12,-8,4)$

TABLE III
BINARY SEQUENCES $a^{(3)}$ AND $b^{(3)}$ OF LENGTH 32 WITH ZCZ FROM THEOREM 15

Conditions	$\pi = (23), H = 2,$ $(c_0, c_1, c_2, c_3, c_4, c_5) = (0, 0, 0, 0, 0, 1)$
Sequence $a^{(3)}$	$(0,1,0,0,0,1,0,0,0,1,1,1,1,$ $0,0,0,0,1,0,0,1,0,1,1,0,1,1,1,0,1,1,1)$
$\{R_{a^{(3)}}(\tau)\}_0^{31}$	$(32,0,0,0,0,-4,0,-4,0,-12,0,4,0,0,0,0,$ $0,0,0,0,0,4,0,-12,0,-4,0,-4,0,0,0,0)$
Sequences $b^{(3)}$	$(0,0,0,1,0,0,0,1,0,0,1,0,1,1,0,$ $1,0,0,0,1,1,1,1,0,0,0,1,0,0,0,1,0)$
$\{R_{b^{(3)}}(\tau)\}_0^{31}$	$(32,0,0,0,0,4,0,4,0,12,0,-4,0,0,0,0,0,$ $0,0,0,0,-4,0,12,0,4,0,4,0,0,0,0)$
$\{R_{a^{(3)},b^{(3)}}(\tau)\}_0^{31}$	$(0,0,0,0,0,-4,-8,-12,0,4,8,-4,0,0,0,$ $0,0,0,0,0,0,4,8,-4,0,12,-8,4,0,0,0,0)$

Furthermore, the three zero crosscorrelation zones are characterized as follows:

(i) $R_{a,b}(\tau) = 0$ for $\tau \in (0, 2^{m-2}]$.
(ii) $R_{a,b}(\tau) = 0$ for $\tau \in [2^{m-2}, 3 \cdot 2^{m-2}]$.
(iii) $R_{a,b}(\tau) = 0$ for $\tau \in (0, 2^{m-3}] \cup [3 \cdot 2^{m-3}, 5 \cdot 3^{m-3}] \cup [7 \cdot 2^{m-3}, 2^m)$.

Along with the results from [10], it can be observed that Golay sequences can contain large zero autocorrelation zone and large zero crosscorrelation zone simultaneously. As a result, the binary Golay pairs with large zero autocorrelation and crosscorrelation zone of length 2^{m-2} have the best known parameters.

Acknowledgment

The work is supported by NSERC Discovery Grant.

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