On the Degrees of Freedom of the K-User Time Correlated Broadcast Channel with Delayed CSIT

Paul de Kerret, Xinping Yi and David Gesbert
Mobile Communications Department, EURECOM
Campus SophiaTech, 450 Route des Chappes, 06410 Biot, France
{dekerret,yix,gesbert}@eurecom.fr

***Abstract*—The degrees of freedom (DoF) of a K-User MISO broadcast channel (BC) is studied when the transmitter (TX) has access to a delayed channel estimate in addition to an imperfect estimate of the current channel. The current estimate could be for example obtained from prediction applied on past estimates, in the case where feedback delay is within the coherence time. Prior results in this setting are promising, yet remain limited to the two-user case. In contrast, we consider here an arbitrary number of users. We develop a new transmission scheme, called the K_α-MAT scheme, which builds upon both the principle of the MAT alignment from Maddah-Ali and Tse and zero-forcing (ZF) to achieve a larger DoF in the channel state information (CSI) configuration previously described. We also develop a new upper bound for the DoF to compare with the DoF achieved by K_α-MAT. Although not optimal, the K_α-MAT scheme performs well when the CSIT quality is not too delayed or K is small. The K_α-MAT scheme can be seen as a robust version of ZF with respect to the delay in the CSI feedback.**

I. Introduction

While in point-to-point MIMO systems, the maximal DoF, or prelog factor, can be achieved without CSI at the transmitter (CSIT), the exploitation of the multiple antennas at the TX to achieve a DoF larger than one in multi-user settings heavily relies on the availability of accurate-enough CSIT. For instance, it is well known that in the K-user MISO BC, the DoF is reduced from K to 1 in the absence of CSIT [1]–[3].

Yet, the obtaining of the CSIT represents a challenge in fast fading channels. Indeed, the channel estimate has to be fed back from the RXs which inevitably introduces some delays and some degradations. Therefore, a large literature has focused on the problem of designing efficient feedback schemes and evaluating the impact of imperfect CSIT (See [2], [4] and reference therein).

Recently, a new line of work was opened by the work from Maddah-Ali and Tse [5]. Studying a K-user MISO BC, they showed that even completely outdated CSIT, in the sense that the feedback delay exceeds the coherence period of the channel, could still be used to increase the DoF. This is accomplished through a space-time alignment of the interference referred in the literature as the MAT *alignment*.

David Gesbert acknowledges support from the FP7 European research project HARP. Paul de Kerret and Xinping Yi are partly funded by the Celtic European project SHARING.

Helpful discussions with Sheng Yang (Supelec) and Mari Kobayashi (Supelec) are gratefully acknowledged.

Furthermore, if the channel matrices are independent and identically distributed over time and across the receivers (RXs), MAT is optimal in terms of DoF [5].

This new method of exploiting completely outdated CSIT has attracted a large interest and has been extended to further network scenarios, e.g. [6], [7] among others. Going beyond completely outdated CSIT, interesting results were also obtained in the case of CSIT of alternating qualities (perfect, outdated, no CSIT) [8], [9].

A major restriction of these works is that they do not consider any correlation between the delayed CSIT and the current channel state. This assumption was first lifted in [10] where an improved DoF is shown to be achievable when the delayed CSIT is possibly correlated with the current channel state. In this case, an imperfect estimate of the current CSIT can be obtained by prediction based on the delayed CSIT.

The scheme in [10] is improved in [11], [12] to reach the maximal DoF in a two-user MISO scenario. This approach has then been extended to two-user MISO BC with *imperfect* and *evolving* delayed CSIT in [13] and to the two-user MIMO BC (and IC) in [14]. Yet, all the results in this scenario have remained restricted to the two-user case. In this work, we tackle precisely the extension to more than two users.

Specifically, our main contributions are as follows.

- As a preliminary step, we develop a new alignment scheme, called K_0-MAT, for the case of completely outdated CSIT. This scheme is optimal for $K=2$ but is outperformed by MAT for $K>2$.
- Building on K_0-MAT and ZF, we develop a new scheme, called K_α-MAT, which achieves the sum DoF $(1-\alpha)\,\mathrm{DoF}^{\mathrm{K}_0\text{-MAT}}+\alpha\,\mathrm{DoF}^{\mathrm{ZF}}$, where $\mathrm{DoF}^{\mathrm{K}_0\text{-MAT}}$ and $\mathrm{DoF}^{\mathrm{ZF}}$ are the sum DoF achieved respectively with K_0-MAT and with ZF, and α is the CSI quality exponent defined later on in (3).
- We derive an outer bound for the K-user MISO broadcast channel with delayed CSIT and imperfect current CSIT with quality exponent α. Although not provably tight, this outer bound is the first outer bound known for the K-user MISO BC with delayed CSIT correlated in time.

Note that, in parallel to this work, a similar upper bound in a CSIT setting with more general imperfect CSIT model (alternating, asymmetric,...) is derived in [15]. Yet, the achievable protocol derived in [15] makes explicit use of the alternating structure of the CSIT and cannot be applied in our context.

II. System Model

A. K-User MISO Broadcast Channel

This work considers a K-User MISO BC where the TX is equipped with M antennas and serves K single-antenna users. We assume furthermore that $M \geq K$ as this is a necessary assumption to apply the MAT scheme [5]. At any time t, the signal received at RX i can be written as

$$y_i(t) = \boldsymbol{h}_i^{\mathrm{H}}(t)\boldsymbol{x}(t) + z_i(t) \tag{1}$$

where $\boldsymbol{h}_i^{\mathrm{H}}(t) \in \mathbb{C}^{1\times M}$ is the channel to user i at time t, $\boldsymbol{x}(t) \in \mathbb{C}^{M\times 1}$ is the transmitted signal, and $z_i(t) \in \mathbb{C}^{1\times 1}$ is the normalized additive noise at RX i, independent of the channel and the transmitted signal and distributed as $\mathcal{N}_{\mathbb{C}}(0,1)$. Furthermore, the transmitted signal $\boldsymbol{x}(t)$ fulfills the average power constraint $\mathrm{E}[\|\boldsymbol{x}(t)\|^2] \leq P$.

We define further the channel matrix $\mathbf{H} \triangleq [\boldsymbol{h}_1,\ldots,\boldsymbol{h}_K]^{\mathrm{H}} \in \mathbb{C}^{K\times M}$ and introduce the notation $\mathcal{H}^t \triangleq \{\mathbf{H}(k)\}_{k=1}^{k=t}$. The channel is assumed to be drawn from a continuous ergodic distribution such that all the channel matrices and all their submatrices are full rank.

B. Delayed CSIT with Correlation in Time

The considered CSIT model builds on the delayed CSIT model introduced in [5] and generalized to account for time correlation in [10]. According to this model, the TX has access at time t to the delayed CSI. It takes the form of the CSI up to time $t-1$ which is denoted by $\mathcal{H}^{t-1}$. Furthermore, exploiting the correlation in time between the delayed CSI $\mathcal{H}^{t-1}$ and the current channel state $\mathbf{H}(t)$, the TX produces an imperfect estimate of the channel state denoted by $\hat{\mathbf{H}}(t)$. This channel estimate is then modeled such that

$$\mathbf{H}(t) = \hat{\mathbf{H}}(t) + \tilde{\mathbf{H}}(t) \tag{2}$$

where the channel estimate and the channel estimation error are independent, the channel estimation error $\tilde{\mathbf{H}}(t)$ has its elements i.i.d. $\mathcal{N}_{\mathbb{C}}(0,\sigma^2)$ while the elements of the channel estimate $\hat{\mathbf{H}}(t)$ are assumed to have a variance equal to $1-\sigma^2$. We further define $\hat{\mathcal{H}}^t \triangleq \{\hat{\mathbf{H}}(k)\}_{k=1}^{k=t}$ and $\tilde{\mathcal{H}}^t \triangleq \{\tilde{\mathbf{H}}(k)\}_{k=1}^{k=t}$. It is also assumed that the channel state $\mathbf{H}(t)$ is independent of the pair $(\hat{\mathcal{H}}^{t-1}, \tilde{\mathcal{H}}^{t-1})$ when conditioned on $\hat{\mathbf{H}}(t)$.

The variance σ^2 of the estimation error is parametrized as a function of the SNR P such that $\sigma^2 = P^{-\alpha}$ where we have defined the *CSIT quality exponent* α as

$$\alpha \triangleq \lim_{P\to\infty} \frac{-\log_2(\sigma^2)}{\log_2(P)}. \tag{3}$$

From a DoF perspective, we can restrict ourselves to $\alpha \in [0,1]$ since an estimation/quantization error scaling as P^{-1} or faster is essentially perfect while an estimation error scaling as P^0 is essentially useless in terms of DoF [2], [3].

Remark 1. This suggests that in order to keep the rate scaling in the SNR, and under a given time-correlation model, the feedback delay as a fraction of the correlation time must shrink as the SNR increases (e.g., the terminal velocity must decrease). □

Note furthermore that for any ZF precoded vector $\boldsymbol{u}$ such that $\hat{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{u} = 0$, it can easily be shown that $\mathrm{E}[|\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{u}|^2] \sim P^{-\alpha}\mathrm{E}[\|\boldsymbol{u}\|^2]$.

Following the conventional assumption from the literature of delayed CSIT (e.g., in [5]), all the RXs are assumed to receive with a certain delay both the perfect multi-user CSI and the imperfect CSI. This CSI is used only for the RX to decode its data symbols such that the only limitation for this delay lies in the delay requirement of the data transmitted. The CSI at the RX side could for example be obtained if each user broadcasts is CSI implying that the other RXs can obtain the same CSI as the TX. Another solution is to simply let the TX send its perfect delayed CSIT to all the RXs [16].

C. Degrees-of-Freedom Analysis

Albeit an incomplete measure of system performance, the DoF offers the unique advantage of allowing for analytical tractability for even complex network models and feedback scenarios such as this one. Denoting by $R(P)$ the sum rate achieved with a given precoding scheme subject to the sum power constraint P, we define the sum DoF as

$$\mathrm{DoF} \triangleq \lim_{P\to\infty} \frac{R(P)}{\log_2(P)}. \tag{4}$$

III. Main Results

We start by providing an upper bound for the DoF before describing an achievable sum DoF.

A. Upper Bound for the DoF

Theorem 1. *In the K-user MISO BC with perfect delayed CSIT and current CSIT with quality exponent α, the sum DoF is upper bounded by $\mathrm{DoF}^{\mathrm{Out}}$ given by*

$$\mathrm{DoF}^{\mathrm{Out}} = (1-\alpha)\,\mathrm{DoF}^{\mathrm{MAT}} + \alpha\,\mathrm{DoF}^{\mathrm{ZF}} \tag{5}$$

with $\mathrm{DoF}^{\mathrm{ZF}} = K$ and $\mathrm{DoF}^{\mathrm{MAT}}$ is the sum DoF achieved by the MAT scheme [5] and is equal to

$$\mathrm{DoF}^{\mathrm{MAT}} = \frac{K}{\sum_{k=1}^{K}\frac{1}{k}}. \tag{6}$$

Proof: See [17] for the proof. ■

B. Achievable DoF

The achievable DoF is obtained by developing a new scheme called the K_α-MAT scheme, which exploits both ZF and the principle behind the MAT alignment. As a building block of the K_α-MAT scheme, we have also developed a new precoding scheme for the case of completely outdated CSIT ($\alpha = 0$) which we have called the K_0-MAT scheme.

For the sake of exposition, both the K_α-MAT and the K_0-MAT schemes will be described only for the case $K = 3$ in Section IV. Indeed, the 3-user case contains the key insights but is more easily described. The general case with arbitrary number of users is described in details in [17].

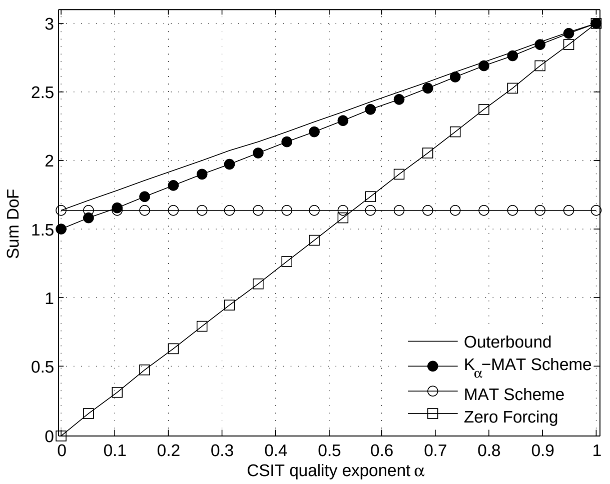

Fig. 1. Sum DoF for $K=3$ users in terms of the CSIT quality exponent α.

Theorem 2. *In the K-user MISO BC with perfect delayed CSIT and current CSIT with quality exponent α, the* K_{α}-MAT *scheme achieves* $\mathrm{DoF}^{\mathrm{K}_{\alpha}\text{-MAT}}$ *given by*

$$\mathrm{DoF}^{\mathrm{K}_{\alpha}\text{-MAT}}=(1-\alpha)\,\mathrm{DoF}^{\mathrm{K}_{0}\text{-MAT}}+\alpha\,\mathrm{DoF}^{\mathrm{ZF}} \tag{7}$$

with $\mathrm{DoF}^{\mathrm{ZF}}=K$ *and* $\mathrm{DoF}^{\mathrm{K}_{0}\text{-MAT}}$ *being the DoF achieved with the new* K_0-MAT *scheme based only on completely outdated CSIT.* $\mathrm{DoF}^{\mathrm{K}_{0}\text{-MAT}}$ *is then equal to*

$$\mathrm{DoF}^{\mathrm{K}_{0}\text{-MAT}}=\frac{2K}{K+1}+\varepsilon_{n_{\mathrm{TS}}} \tag{8}$$

where $\varepsilon_{n_{\mathrm{TS}}}\sim 1/n_{\mathrm{TS}}$ *with* n_{TS} *being the number of time-slots over which the transmission scheme is spread.*

In Fig. 1, the sum DoF is represented as a function of the CSIT quality exponent α. The K_{α}-MAT scheme outperforms ZF for every value of α and appears hence as a robust version of ZF with respect to delay in the CSIT. The DoF achieved with conventional ZF with CSIT quality exponent α is well known to be $K\alpha$ [2] such that the first term of (7) can be seen to be the DoF improvement compared to ZF.

Remark 2. $\mathrm{DoF}^{\mathrm{K}_{0}\text{-MAT}}$ converges to 2 as K increases while $\mathrm{DoF}^{\mathrm{MAT}}$ in (6) scales as $K/\log_2(K)$. Finding a precoding scheme achieving this scaling and exploiting at the same time the current CSIT available in an efficient way remains an open problem. Nevertheless, K_{α}-MAT performs well for small values of K as well as when the CSIT is not too delayed (α close to one). □

IV. The K_{α}-MAT scheme for $K=3$ Users

The structure of MAT makes it difficult to exploit the additional knowledge of an imperfect estimate of the current channel state. In the two-user case, this obstacle was circumvented in [11], [12] by using an alternative version of MAT developed by Maddah-Ali and Tse in [5]. In contrast to the original MAT, this alternative version can be nicely combined with ZF such that it was possible to reach the optimal DoF.

Yet, no such alternative version exists for more than two users. As a consequence, our first step is to develop a novel precoding scheme for the case of completely outdated CSIT ($\alpha=0$) which lends itself easily to combining with ZF whenever information on the current channel is available ($\alpha>0$). We refer to this scheme as the K_0-MAT scheme.

Similarly to [5], a DoF strictly larger than one is achieved with only delayed CSIT by exploiting the broadcast nature of the channel. Indeed, a message destined to RX j will be overheard by the other $K-1$ RXs, hence providing side information. This property is exploited to generate messages which are of interest to j RXs with $j>1$, called *order-j message*, and improve in this way the spectral efficiency.

Since no confusion is possible, we omit to mention the dependency of the channels as a function of the time t.

A. Description of the K_0-MAT *Scheme ($\alpha=0$)*

The K_0-MAT scheme consists of one initialization step, followed by a number of n "iteration" steps and is ended by a termination step. The initialization step contains 3 time slots while every other step is spread over 6 time slots such that the transmission protocol lasts $3+6n+6$ time slots.

- *Step 1–Initialization–* In this step, we aim at transmitting 4 order-1 data symbols to every RX. During the first time slot, the vector $\boldsymbol{u}_1^{\mathrm{ini}}\in\mathbb{C}^{2\times 1}$ containing 2 data symbols for RX 1 and the vector $\boldsymbol{u}_2^{\mathrm{ini}}\in\mathbb{C}^{2\times 1}$ containing 2 data symbols for RX 2 are transmitted. The received signal at RX i can then be written as
$$y_i=\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{u}_1^{\mathrm{ini}}+\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{u}_2^{\mathrm{ini}}+z_i. \tag{9}$$
Following the same philosophy as the alternative version of MAT [5], we will consider the retransmission of the interferences. Hence, $\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{u}_2$ and $\boldsymbol{h}_2^{\mathrm{H}}\boldsymbol{u}_1$ are seen as order-2 messages (i.e., messages destinated to 2 RXs) that need to be transmitted both to RX 1 and to RX 2. The successful transmission of the 4 order-1 data symbols is achieved if these 2 order-2 data symbols are transmitted. During the second (resp. the third) time slot, the same transmission scheme is used to transmit to RX 2 and RX 3 (resp. RX 3 and RX 1).
- *From step 2 to step $n+1$–iteration step–* This phase is spread over 6 time slots and is aimed at transmitting the 6 order-2 messages generated in the previous step, completed with 3 new order-1 messages for each RX.
In the first of these 6 time slots, 3 order-1 messages are transmitted to RX 1 while 2 order-2 messages are transmitted to RX 2 and RX 3. We define the vector $\boldsymbol{u}_1^{n}\in\mathbb{C}^{3\times 1}$ containing the 3 order-1 messages and the vector $\boldsymbol{u}_{23}^{n}\in\mathbb{C}^{2\times 1}$ containing the two order-2 messages. The received signal at RX i reads then as
$$y_i=\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{u}_1^{n}+\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{u}_{23}^{n}+z_i. \tag{10}$$
Considering once more the retransmission of the interference which is necessary for every RX to decode

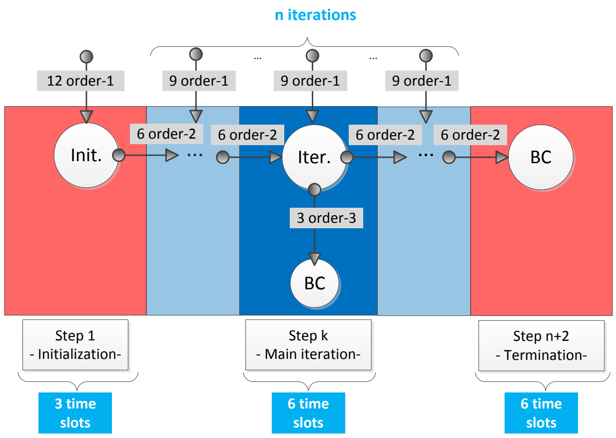

Fig. 2. Symbolic representation of the K_0-MAT scheme for $K = 3$ users.

its desired messages, $\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{u}_{23}^n$ can be seen as an order-3 message that need to be transmitted to the 3 RXs (multicast), while $\boldsymbol{h}_2^{\mathrm{H}}\boldsymbol{u}_1^n$ (resp. $\boldsymbol{h}_3^{\mathrm{H}}\boldsymbol{u}_1^n$) is an order-2 message that needs to be transmitted to RX 1 and RX 2 (resp. RX 1 and RX 3). Thus, it remains to transmit these two order-2 messages and this one order-3 message to obtain that the order-1 and the order-2 data symbols initially transmitted are successfully decoded.
During the two following time slots, the same transmission occurs after having permuted circularly the RXs.
Finally, the three order-3 data symbols which have been generated in this step are multicast, which requires 3 time slots. Note that the 6 order-2 data symbols have been transmitted while 6 new order-2 symbols have been generated and should be transmitted in the following step.

- *Step $n+2$–Termination–* At the beginning of this step, 6 order-2 data symbols have to be transmitted. This is carried out in 6 time slots by simple multicasting.

DoF Analysis: In total, $12+9n$ order-1 data symbols have been transmitted in $3+6n+6$ time slots. After simplification, a DoF in the form of (8) with $K=3$ is obtained. The DoF converges to $3/2$ as the number of iteration steps n (or the number of time slots) increases.

The K_0-MAT scheme is described schematically for $K=3$ in Fig. 2. One novelty of the K_0-MAT scheme is that symbols of *different* orders are sent at the same time.

B. *Description of the* K_α-MAT *Scheme* ($\alpha > 0$)

When the CSIT is completely outdated ($\alpha = 0$), the new K_0-MAT scheme will be used in place of MAT. In the other extreme, when $\alpha = 1$, ZF is well known to be DoF achieving [2], [3]. Thus, it remains to develop a scheme for the intermediate values of the CSIT quality exponent α. We start by describing the K_α-MAT scheme before discussing the DoF achieved.

The K_α-MAT scheme is built on the K_0-MAT scheme and follows the same structure. As a consequence, we show only how the n-th "iteration step" from K_0-MAT is adapted to exploit the available current CSIT. The modifications of the initialization and the termination are very similar and follow trivially. Furthermore, we consider only one particular circular permutation of the users while all $K=3$ circular permutations occur in the K_0-MAT scheme.

a) K_0-MAT data symbols: Conforming to the K_0-MAT scheme, the vector $\boldsymbol{u}_1^n \in \mathbb{C}^{3\times 1}$ containing 3 data symbols for RX 1 and the vector $\boldsymbol{u}_{23}^n \in \mathbb{C}^{2\times 1}$ containing 2 data symbols destinated to RX 2 and to RX 3 are transmitted during the first time slot. Yet, the data symbols are this time transmitted with the rate $(1-\alpha)\log_2(P)$ (instead of $\log_2(P)$) and they are furthermore precoded. The i-th data symbol $\{\boldsymbol{u}_1^n\}_i$ is precoded to form the vector $\boldsymbol{a}_i^{(1)} \in \mathbb{C}^{M\times 1}$ for $i=1,2,3$ while the k-th data symbol $\{\boldsymbol{u}_{23}^n\}_k$ is precoded to form the vector $\boldsymbol{a}_k^{(23)} \in \mathbb{C}^{M\times 1}$ for $k=1,2$.

The precoding is done such that $\boldsymbol{a}_1^{(1)}$ (resp. $\boldsymbol{a}_1^{(23)}$) is orthogonal to the channel estimates $\hat{\boldsymbol{h}}_2$ and $\hat{\boldsymbol{h}}_3$ (resp. $\hat{\boldsymbol{h}}_1$). The remaining precoded data symbols are chosen such that $\forall k<i, (\boldsymbol{a}_k^{(1)})^{\mathrm{H}}\boldsymbol{a}_i^{(1)} = 0$ (resp. $\forall k<i, (\boldsymbol{a}_k^{(23)})^{\mathrm{H}}\boldsymbol{a}_i^{(23)} = 0$).

The power is allocated such that

$$\begin{cases} k=1, & \mathrm{E}[\|\boldsymbol{a}_k^{(1)}\|^2] = [\frac{1}{2}(P-P^\alpha) - \frac{2}{6}P^{1-\alpha}]^+, \\ \forall k=2,3, & \mathrm{E}[\|\boldsymbol{a}_k^{(1)}\|^2] = \frac{1}{6}P^{1-\alpha} \end{cases} \tag{11}$$

and similarly

$$\begin{cases} k=1, & \mathrm{E}[\|\boldsymbol{a}_k^{(23)}\|^2] = [\frac{1}{2}(P-P^\alpha) - \frac{1}{4}P^{1-\alpha}]^+, \\ k=2, & \mathrm{E}[\|\boldsymbol{a}_k^{(23)}\|^2] = \frac{1}{4}P^{1-\alpha}. \end{cases} \tag{12}$$

We will see after *b)* that the sum power constraint is then fulfilled with equality. The reason for this particular power allocation will become clear when discussing the decoding.

b) ZF data symbols: For every RX j, one data symbol s_j is transmitted via conventional ZF at the same time as the so-called "K_0-MAT data symbols" from *a)*. The "ZF data symbol" s_j is precoded to obtain $\boldsymbol{p}_j \in \mathbb{C}^{M\times 1}$ verifying

$$\forall k \neq j, \hat{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{p}_j = 0. \tag{13}$$

The power is allocated to verify that $\forall i, \mathrm{E}[\|\boldsymbol{p}_i\|^2] = P^\alpha/3$ and each data symbol is sent with the rate $\alpha\log_2(P)$.

c) Received signal: RX 1 has then received

$$y_1 = \underbrace{\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{a}_1^{(1)}}_{\sim P} + \sum_{i=2}^{3}\underbrace{\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{a}_i^{(1)}}_{\sim P^{1-\alpha}} + \sum_{i=1}^{2}\underbrace{\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{a}_i^{(23)}}_{\sim P^{1-\alpha}} + \sum_{i=1}^{3}\underbrace{\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{p}_i}_{\sim P^\alpha} + z_1 \tag{14}$$

while we can write for RX k with $k=2,3$

$$y_k = \underbrace{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{a}_1^{(23)}}_{\sim P} + \underbrace{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{a}_2^{(23)}}_{\sim P^{1-\alpha}} + \sum_{i=1}^{3}\underbrace{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{a}_i^{(1)}}_{\sim P^{1-\alpha}} + \sum_{i=1}^{3}\underbrace{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{p}_i}_{\sim P^\alpha} + z_k. \tag{15}$$

d) Quantization and retransmission of the interference: Following a similar approach as in [11], the interferences $\sum_{i=1}^3 \boldsymbol{h}_2^{\mathrm{H}}\boldsymbol{a}_i^{(1)}$ and $\sum_{i=1}^3 \boldsymbol{h}_3^{\mathrm{H}}\boldsymbol{a}_i^{(1)}$ are quantized with $(1-\alpha)\log_2(P)$ bits, which gives a distortion scaling in P^0 (negligible in terms of DoF). As in the K_0-MAT scheme, these two messages are desired by two RXs. Thus, they form the order-2 symbols of rate $(1-\alpha)\log_2(P)$ which will be taken as input and transmitted in the next a) step of the K_α-MAT scheme.

The interference term $\sum_{i=1}^{2} \boldsymbol{h}_1^{\mathrm{H}} \boldsymbol{a}_i^{(23)}$ is also quantized with $(1-\alpha)\log_2(P)$ bits but is desired by the three RXs so that it is simply multicast. Similarly to the previous transmission, dedicated ZF data symbols are transmitted at the same time as this multicasting step, as described now.

Let us denote by $\boldsymbol{c}$ the precoded data symbol carrying the order-3 (multicast) message of rate $(1-\alpha)\log_2(P)$ with the power allocated such that $\mathrm{E}[\|\boldsymbol{c}\|^2] = P - P^\alpha$. At the same time, a ZF data symbol is transmitted to RX j with the rate $\alpha \log_2(P)$ and is precoded to obtain $\boldsymbol{q}_j \in \mathbb{C}^{M\times 1}$ with

$$\forall k \neq j, \hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{q}_j = 0. \tag{16}$$

and $\forall i, \mathrm{E}[\|\boldsymbol{q}_i\|^2] = P^\alpha/3$. The received signal at RX k is then

$$y_k = \underbrace{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{c}}_{\sim P} + \underbrace{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{q}_k}_{\sim P^\alpha} + \sum_{i=1, i\neq k}^{3} \underbrace{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{q}_i}_{\sim P^0} + z_k. \tag{17}$$

Using successive decoding, it is easily seen that each user can decode first the common data symbol of order-3 and then its dedicated ZF data symbol (See [11] for more details).

e) Decoding: We consider now that all the steps of the K_α-MAT scheme have occurred. Let us discuss first the decoding at RX 1: RX 1 has received 2 additional equations (formed by the quantized interferences $\sum_{i=1}^{3} \boldsymbol{h}_2^{\mathrm{H}} \boldsymbol{a}_i^{(1)}$ and $\sum_{i=1}^{3} \boldsymbol{h}_3^{\mathrm{H}} \boldsymbol{a}_i^{(1)}$) relative to its desired data symbols and was also able to remove the interference caused by the other "K_0-MAT data symbols" that it had received during the first transmission.

It remains then only the interference caused by the "ZF data symbols" from step b) and since the received signal $\sum_{i=1}^{3} \boldsymbol{h}_1^{\mathrm{H}} \boldsymbol{p}_i$ scales as P^α, the SNR to decode the "K_0-MAT data symbols" from step a) is also $P^{1-\alpha}$ for this equation.

RX 1 has received 3 equations having each a SNR scaling in $P^{1-\alpha}$ and can hence decode its desired precoded data symbols $\boldsymbol{a}_i^{(1)}, i=1,2,3$. Similarly, RX 2 and RX 3 have received enough equations to decode their desired data symbols.

Now that the "K_0-MAT" data symbols from step a) have been decoded, they can be subtracted from the transmitted equations. Using also the retransmitted interference, the received signal at RX k for any time slot reads then as (up to the quantization noise scaling as P^0)

$$y_k = \underbrace{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k}_{\sim P^\alpha} + \sum_{i=1, i\neq k}^{3} \underbrace{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_i}_{\sim P^0} + z_k. \tag{18}$$

The interference term in (18) is drawn in the noise due to the attenuation by $P^{-\alpha}$ from ZF precoding. As a consequence, the ZF precoded data symbol $\boldsymbol{p}_k$ is received at RX k with a SNR scaling as P^α and can be decoded.

DoF Analysis: The K_0-MAT scheme has been used to transmit data symbols of rate $(1-\alpha)\log_2(P)$ while at *every time slot* of this scheme, one data symbol has been transmitted to every user via ZF with a rate equal to $\alpha\log_2(P)$. Hence, the DoF given in (7) is achieved.

V. Conclusion

Considering a K-user MISO BC, a new transmission scheme has been developed to exploit at the same time the principles behind the MAT alignment based on delayed CSIT and ZF of the interference. Furthermore, a novel outer bound is used to evaluate the DoF achieved. The novel K_α-MAT scheme is more robust than ZF to the channel estimates being received with some delay and coincides with ZF when the CSIT received is accurate enough. Nevertheless, the K_α-MAT is not optimal and closing the gap between the achievable DoF and the upper bound represents an interesting open problem.

References

[1] S. A. Jafar and A. J. Goldsmith, "Isotropic fading vector broadcast channels: The scalar upper bound and loss in degrees of freedom," *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 848–857, Mar. 2005.

[2] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.

[3] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO achievable rates with downlink training and channel state feedback," *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2845–2866, Jun. 2010.

[4] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[5] M. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.

[6] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region of the two-user MIMO broadcast channel with delayed CSIT," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2011.

[7] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2011.

[8] N. Lee and R. W. Heath, "Not too delayed CSIT achieves the optimal degrees of freedom," in *Proc. Allerton Conference on Communication, Control, and Computing (Allerton)*, 2012.

[9] R. Tandon, S. A. Jafar, S. Shamai (Shitz), and H. V. Poor, "On the synergistic benefits of alternating CSIT for the MISO BC," *IEEE Trans. Inf. Theory*, vol. PP, no. 99, pp. 1–1, 2013.

[10] M. Kobayashi, S. Yang, D. Gesbert, and X. Yi, "On the degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2012.

[11] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, "Degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 315–328, Jan. 2013.

[12] T. Gou and S. Jafar, "Optimal use of current and outdated channel state information: Degrees of freedom of the MISO BC with mixed CSIT," *IEEE Communications Letters*, vol. 16, no. 7, pp. 1084–1087, Jul. 2012.

[13] J. Chen and P. Elia, "MISO broadcast channel with delayed and evolving CSIT," 2012, submitted to IEEE Trans. Inf. Theory. [Online]. Available: http://arxiv.org/abs/1211.1622

[14] X. Yi, S. Yang, D. Gesbert, and M. Kobayashi, "The degrees of freedom region of temporally-correlated MIMO networks with delayed CSIT," 2012, submitted to IEEE Trans. Inf. Theory. [Online]. Available: http://arxiv.org/abs/1211.3322

[15] J. Chen, S. Yang, and P. Elia, "On the fundamental feedback-vs-performance tradeoff over the MISO-BC with imperfect and delayed CSIT," to appear in IEEE International Symposium on Information Theory (ISIT) 2013. [Online]. Available: http://arxiv.org/abs/1302.0806

[16] J. Xu, J. G. Andrews, and S. A. Jafar, "MISO broadcast channels with delayed finite-rate feedback: Predict or observe?" *IEEE Trans. on Wireless Commun.*, vol. 11, no. 4, pp. 1456–1467, Apr. 2012.

[17] P. de Kerret, X. Yi, and D. Gesbert, "On the degrees of freedom of the K-User time correlated broadcast channel with delayed CSIT." [Online]. Available: http://arxiv.org/pdf/1301.2138.pdf