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A Comparison of Superposition Coding Schemes

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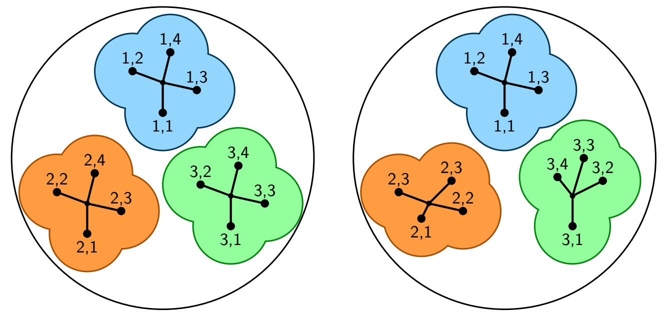
***Abstract*—There are two variants of superposition coding schemes. Cover's original superposition coding scheme has code clouds of identical shape, while Bergmans's superposition coding scheme has code clouds of independently generated shapes. These two schemes yield identical achievable rate regions in several scenarios, such as the capacity region for degraded broadcast channels. This paper shows that under optimal decoding, these two superposition coding schemes can result in different rate regions. In particular, it is shown that for the two-receiver broadcast channel, Cover's scheme achieves a larger rate region than Bergmans's scheme in general.**



(a) Homogeneous coding **(b)** Heterogeneous coding

Figure 1. Superposition codebooks for which (a) the structure within each cloud is identical and (b) the structure is nonidentical between clouds. Codewords (dots) are annotated by "m_1, m_2", where m_1 is the coarse layer message and m_2 is the fine layer message.

I. Introduction

Superposition coding is one of the fundamental building blocks of coding schemes in network information theory. This idea was first introduced by Cover in 1970 at the IEEE International Symposium on Information Theory, Noordwijk, the Netherlands, in a talk titled "Simultaneous Communication," and appeared in his 1972 paper [6]. Subsequently, Bergmans [2] adapted Cover's superposition coding scheme to the general degraded broadcast channel (this scheme is actually applicable to any nondegraded broadcast channel), which establishes the capacity region along with the converse proof by Gallager [12]. Since then, superposition coding has been applied in numerous problems, including multiple access channels [13], interference channels [3], [5], [14], relay channels [8], channels with feedback [9], [16], and wiretap channels [4], [10].

The objective of superposition coding is to communicate two message simultaneously by encoding them into a single signal in two layers. A "better" receiver of the signal can then recover the messages on both layers while a "worse" receiver can recover the message on the coarse layer of the signal and ignore the one on the fine layer.

There are two variants of the superposition coding idea in the literature, which differ in how the codebooks are generated. The first variant is described in Cover's original 1972 paper [6] and later in [17] and [7]. Both messages are first encoded independently via separate random codebooks of auxiliary sequences. To send a message pair, the auxiliary sequences associated with each message are then mapped through a symbol-by-symbol superposition function (such as addition) to generate the actual codeword. One can visualize the image of one of the codebooks centered around a fixed codeword from the other as a "cloud" (see the illustration in Figure 1(a)). Since all clouds are images of the same random codebook (around different cloud centers), we refer to this variant as *homogeneous superposition coding*. Note that in this variant, the messages are treated equally and the corresponding auxiliary sequences play the same role. Thus, there is no natural distinction between "coarse" and "fine" layers and there are two ways to group the resulting superposition codebook into clouds.

The second variant was introduced in Bergmans's 1973 paper [2]. Here, the coarse message is encoded into a random codebook of auxiliary sequences. For each auxiliary sequence, a random satellite codebook is generated conditionally independently to represent the fine message. This naturally results in clouds of codewords around each auxiliary sequence. Since all clouds are generated independently, we refer to this variant as *heterogeneous superposition coding*. This is illustrated in Figure 1(b).

A natural question is whether these two variants are fundamentally different, and if so, which of the two is preferable. It is known that both variants achieve the capacity region of the degraded broadcast channel [2]. For the two-user-pair interference channel, the two variants again achieve identical rate regions, namely, the Han–Kobayashi inner bound (see [14] for homogeneous superposition coding and [5] for heterogeneous superposition coding). Since heterogeneous superposition coding usually yields a simpler characterization of the achievable rate region with fewer auxiliary random variables, one may be tempted to prefer this variant.

However, we show in this paper that homogeneous superposition coding always achieves a rate region at least as large as that of heterogeneous superposition coding for two-user broadcast channels, provided that the optimal (maximum likelihood) decoding rule is used. Furthermore, this dominance can be strict in general. Intuitively speaking, homogeneous superposition coding results in more structured interference from the undesired layer and this structure can be exploited under optimal decoding.

Throughout the paper, we closely follow the notation in [11]. In particular, for $X \sim p(x)$ and $\epsilon \in (0,1)$, we define the set of ϵ-typical n-sequences x^n (or the typical set in short) [15] as $\mathcal{T}_\epsilon^{(n)}(X) = \{x^n : |\#\{i : x_i = x\}/n - p(x)| \le \epsilon p(x) \text{ for all } x \in \mathcal{X}\}$.

II. Rate Regions for the Two-Receiver BC

Consider the two-receiver discrete memoryless broadcast channel depicted in Figure 2. The sender wishes to communicate message M_1 to receiver 1 and message M_2 to receiver 2. We define a $(2^{nR_1}, 2^{nR_2}, n)$ code by an encoder $x^n(m_1, m_2)$ and two decoders $\hat{m}_1(y_1^n)$ and $\hat{m}_2(y_2^n)$. We assume that the message pair (M_1, M_2) is uniformly distributed over $[1:2^{nR_1}] \times [1:2^{nR_2}]$, i.e., the messages are independent of each other. The average probability of error is defined as $P_e^{(n)} = \mathsf{P}\{(M_1, M_2) \neq (\hat{M}_1, \hat{M}_2)\}$. A rate pair (R_1, R_2) is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $\lim_{n\to\infty} P_e^{(n)} = 0$.

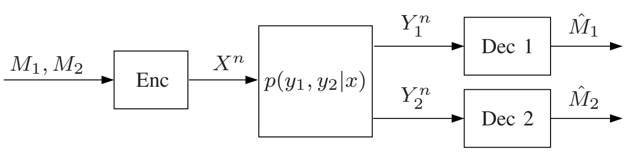


Figure 2. Two-receiver broadcast channel.

We now describe the two variants of superposition coding and compare their achievable rate regions for this channel under optimal decoding.

A. Homogeneous Superposition Coding (UV Scheme)

Codebook generation. Fix a pmf $p(u)\,p(v)$ and a function $x(u,v)$. Randomly and independently generate 2^{nR_1} sequences $u^n(m_1)$, $m_1 \in [1:2^{nR_1}]$, from $\prod_{i=1}^n p_U(u_i)$, and 2^{nR_2} sequences $v^n(m_2)$, $m_2 \in [1:2^{nR_2}]$ from $\prod_{i=1}^n p_V(v_i)$.

Encoding. To send the message pair (m_1, m_2), transmit x_1^n, where $x_i = x(u_i(m_1), v_i(m_2))$.

Decoding. Both receivers use simultaneous nonunique decoding, which achieves the same rate region as maximum likelihood decoding [1] for this codebook ensemble. In particular, upon receiving y_1^n, receiver 1 declares that $\hat{m}_1$ was sent if it is the unique message such that

$$\big(u^n(\hat{m}_1), v^n(m_2), y_1^n\big) \in \mathcal{T}_\epsilon^{(n)}$$

for some m_2. If there is no unique $\hat{m}_1$, it declares an error. Similarly, upon receiving y_2^n, receiver 2 declares that $\hat{m}_2$ was sent if it is the unique message such that

$$\big(u^n(m_1), v^n(\hat{m}_2), y_2^n\big) \in \mathcal{T}_\epsilon^{(n)}$$

for some m_1. If there is no unique $\hat{m}_2$, it declares an error. Standard typicality arguments show that receiver 1 will succeed if

$$R_1 < I(U;Y_1) \qquad \text{or} \qquad \begin{aligned} R_1 + R_2 &< I(X;Y_1), \\ R_1 &< I(X;Y_1\,|\,V), \end{aligned} \tag{1}$$

or, equivalently, if

$$\begin{aligned} R_1 &< I(X;Y_1\,|\,V), \\ R_1 + \min\{R_2, I(X;Y_1\,|\,U)\} &< I(X;Y_1). \end{aligned}$$

Similarly, receiver 2 will succeed if

$$R_2 < I(V;Y_2) \qquad \text{or} \qquad \begin{aligned} R_1 + R_2 &< I(X;Y_2), \\ R_2 &< I(X;Y_2\,|\,U), \end{aligned} \tag{2}$$

or, equivalently, if

$$\begin{aligned} R_2 &< I(X;Y_2\,|\,U), \\ R_2 + \min\{R_1, I(X;Y_2\,|\,V)\} &< I(X;Y_2). \end{aligned}$$

The regions for both receivers are depicted in Table 1. Letting $\mathscr{R}_{UV}(p)$ denote the set of rate pairs (R_1, R_2) satisfying (1) and (2) under the given pmf $p(u)\,p(v)$ and function $x(u,v)$, it follows that the rate region

$$\mathscr{R}_{UV} = \mathrm{co}\Big(\bigcup_{p \in \mathcal{P}_{UV}} \mathscr{R}_{UV}(p)\Big)$$

is achievable. Here, $\mathrm{co}(\cdot)$ denotes convex hull, and $\mathcal{P}_{UV}$ is the set of distributions of the form $p = p(u)\,p(v)\,p(x|u,v)$ where $p(x|u,v)$ represents the function $x(u,v)$.

B. Heterogeneous Superposition Coding (UX Scheme)

Codebook generation. Fix a pmf $p(u,x)$. Randomly and independently generate 2^{nR_1} sequences $u^n(m_1)$, $m_1 \in [1:2^{nR_1}]$ from $\prod_{i=1}^n p_U(u_i)$. For each message $m_1 \in [1:2^{nR_1}]$, randomly and conditionally independently generate 2^{nR_2} sequences $x^n(m_1, m_2)$, $m_2 \in [1:2^{nR_2}]$ from $\prod_{i=1}^n p_{X|U}(x_i|u_i(m_1))$.

Encoding. To send (m_1, m_2), transmit $x^n(m_1, m_2)$.

Decoding. Both receivers use simultaneous nonunique decoding, which is rate-optimal as shown below. In particular, upon receiving y_1^n, receiver 1 declares $\hat{m}_1$ was sent if it is the unique message such that

$$\big(u^n(\hat{m}_1), x^n(\hat{m}_1, m_2), y_1^n\big) \in \mathcal{T}_\epsilon^{(n)}$$

for some m_2. If there is no unique $\hat{m}_1$, it declares an error. Similarly, upon receiving y_2^n, receiver 2 declares $\hat{m}_2$ was sent if it is the unique message such that

$$\big(u^n(m_1), x^n(m_1, \hat{m}_2), y_2^n\big) \in \mathcal{T}_\epsilon^{(n)}$$

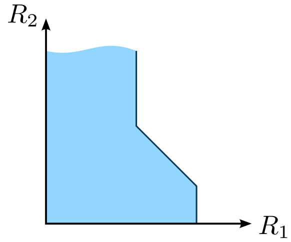
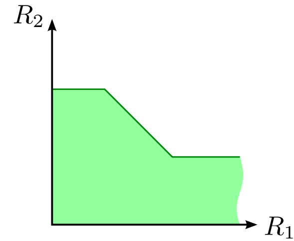
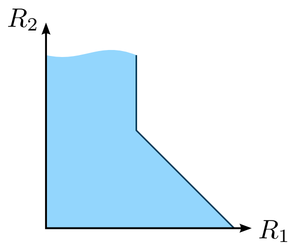
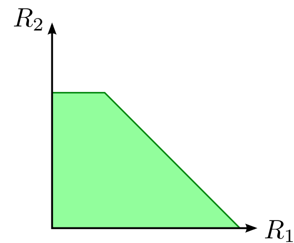
	Receiver 1	Receiver 2
$\mathscr{R}_{UV}(p)$ $p = p(u)\,p(v)\,p(x\|u,v)$	$R_1 < I(X;Y_1\,\|\,V)$, $R_1 + \min\{R_2, I(X;Y_1\,\|\,U)\} < I(X;Y_1)$ 	$R_2 < I(X;Y_2\,\|\,U)$, $R_2 + \min\{R_1, I(X;Y_2\,\|\,V)\} < I(X;Y_2)$ 
$\mathscr{R}_{UX}(p)$ $p = p(u,x)$	$R_1 + \min\{R_2, I(X;Y_1\,\|\,U)\} < I(X;Y_1)$ 	$R_2 < I(X;Y_2\,\|\,U)$, $R_1 + R_2 < I(X;Y_2)$ 

Table 1. Rate regions for homogeneous and heterogeneous superposition coding.

for some m_1. If there is no unique $\hat{m}_2$, it declares an error. Standard arguments show that receiver 1 will succeed if

$$R_1 < I(U;Y_1) \qquad \text{or} \qquad R_1 + R_2 < I(X;Y_1), \tag{3}$$

or, equivalently, if

$$R_1 + \min\{R_2, I(X;Y_1\,|\,U)\} < I(X;Y_1).$$

Similarly, receiver 2 will succeed if

$$\begin{aligned} R_2 &< I(X;Y_2\,|\,U), \\ R_1 + R_2 &< I(X;Y_2). \end{aligned} \tag{4}$$

A similar argument to the one in [1] readily shows that the region in (3) cannot be improved by using maximum likelihood decoding. It is also shown in the Appendix that applying maximum likelihood decoding does not improve the region in (4).

The regions for both receivers are depicted in Table 1. Let $\mathscr{R}_{UX}(p)$ denote the set of all rate pairs (R_1, R_2) satisfying both (3) and (4). Clearly, the rate region

$$\mathscr{R}_{UX} = \mathrm{co}\Bigl(\bigcup_{p \in \mathcal{P}_{UX}} \mathscr{R}_{UX}(p)\Bigr)$$

is achievable. Here, $\mathcal{P}_{UX}$ is the set of distributions of the form $p = p(u,x)$.

If the roles of m_1 and m_2 in codebook generation are reversed, one can also achieve the region $\mathscr{R}_{VX} = \mathrm{co}(\cup_p \mathscr{R}_{VX}(p))$ obtained by swapping Y_1 with Y_2 and R_1 with R_2 in the definition of $\mathscr{R}_{UX}(p)$.

It is worth reiterating that the two schemes above differ only in the dependence/independence between clouds around different u^n sequences, and not in the underlying distributions from which the clouds are generated. Indeed, it is well known that the classes of distributions $\mathcal{P}_{UX}$ and $\mathcal{P}_{UV}$ are equivalent in the sense that for every $p(u,x) \in \mathcal{P}_{UX}$, there exists a $q(u)\,q(v)\,q(x|u,v) \in \mathcal{P}_{UV}$ such that $\sum_v q(u)\,q(v)\,q(x|u,v) = p(u,x)$ (see, for example, [11, p. 626]).

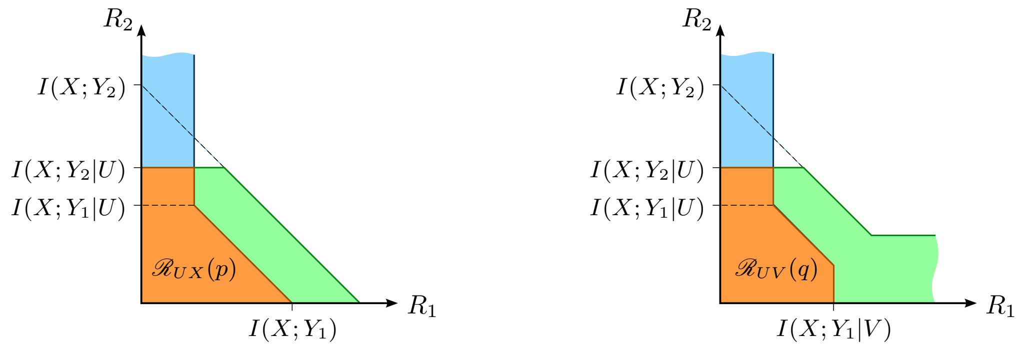
III. Main Result

Theorem 1. The rate region achieved by homogeneous superposition coding includes the rate region achieved by heterogeneous superposition coding, i.e.,

$$\mathrm{co}\bigl(\mathscr{R}_{UX} \cup \mathscr{R}_{VX}\bigr) \subseteq \mathscr{R}_{UV}.$$

Moreover, there are channels for which the inclusion is strict.

Proof: Due to the convexity of $\mathscr{R}_{UV}$ and the symmetry between the definitions of $\mathscr{R}_{UX}$ and $\mathscr{R}_{VX}$, it suffices to show that $\mathscr{R}_{UX}(p) \subseteq \mathscr{R}_{UV}$ for all $p \in \mathcal{P}_{UX}$. Fix a $p \in \mathcal{P}_{UX}$. Let $q' \in \mathcal{P}_{UV}$ be such that $U = X$, $V = \emptyset$, and $q'(x) = p(x)$. Let $q'' \in \mathcal{P}_{UV}$ be such that $V = X, U = \emptyset$, and $q''(x) = p(x)$. An inspection of (1)–(4) and Table 1 reveals that $\mathscr{R}_{UV}(q')$ is



(a) Rate region in (6).

(b) Rate region in (7).

Figure 3. Rate regions for the proof of Theorem 1.

the set of rate pairs satisfying

$$R_2 = 0,$$
$$R_1 < I(X;Y_1),$$

and $\mathscr{R}_{UV}(q'')$ is the set of rate pairs satisfying

$$R_1 = 0,$$
$$R_2 < I(X;Y_2).$$

It then follows that $\mathrm{co}\big(\mathscr{R}_{UV}(q') \cup \mathscr{R}_{UV}(q'')\big)$ includes the rate region

$$R_1 + R_2 < \min\big\{I(X;Y_1), I(X;Y_2)\big\}. \tag{5}$$

We will consider three cases and prove the claim for each.

- If $I(X;Y_1) \geq I(X;Y_2)$ then $\mathscr{R}_{UX}(p)$ reduces to the rate region
$$R_2 < I(X;Y_2 \,|\, U),$$
$$R_1 + R_2 < I(X;Y_2),$$
which is included in the rate region in (5), and therefore in $\mathscr{R}_{UV}$.
- If $I(X;Y_1) < I(X;Y_2)$ and $I(X;Y_1 \,|\, U) \geq I(X;Y_2 \,|\, U)$, then $\mathscr{R}_{UX}(p)$ reduces to the rate region
$$R_2 < I(X;Y_2 \,|\, U),$$
$$R_1 + R_2 < I(X;Y_1),$$
which is also included in the rate region in (5), and therefore in $\mathscr{R}_{UV}$.
- If $I(X;Y_1) < I(X;Y_2)$ and $I(X;Y_1 \,|\, U) < I(X;Y_2 \,|\, U)$, then $\mathscr{R}_{UX}(p)$ reduces to the rate region (see Figure 3(a))
$$\begin{aligned} R_2 &< I(X;Y_2 \,|\, U), \\ R_1 + \min\{R_2, I(X;Y_1 \,|\, U)\} &< I(X;Y_1). \end{aligned} \tag{6}$$
Find a $q \in \mathcal{P}_{UV}$ with $q(u,x) = p(u,x)$, and note that $\mathscr{R}_{UV}(q)$ is described by the bounds
$$\begin{aligned} R_2 &< I(X;Y_2 \,|\, U), \\ R_1 &< I(X;Y_1 \,|\, V), \\ R_1 + \min\{R_2, I(X;Y_1 \,|\, U)\} &< I(X;Y_1). \end{aligned} \tag{7}$$
Comparing (6) with (7) (Figure 3(b)), one sees that $\mathscr{R}_{UX}(p) \subseteq \mathrm{co}\big(\mathscr{R}_{UV}(q) \cup \mathscr{R}_{UV}(q')\big)$. This proves the first claim of the theorem.

To show that the inclusion can be strict, consider the vector broadcast channel with binary inputs (X_1, X_2) and outputs $(Y_1, Y_2) = (X_1, X_2)$. For all $p \in \mathcal{P}_{UX}$, we have from (4) that $R_1 + R_2 < I(X_1 X_2; Y_2) < 1$, and thus $\mathscr{R}_{UX}$ is included in the rate region $R_1 + R_2 < 1$, and by symmetry, so is $\mathscr{R}_{VX}$. Note, however, that the rate pair $(1,1)$ is achievable by homogeneous superposition coding, setting $U = X_1$ and $V = X_2$. This proves the second claim. ∎

IV. Discussion

In addition to the basic superposition coding schemes presented in Section II, one can consider coded time sharing [11], which could potentially enlarge the achievable rate regions. In the present setting, however, it can be easily checked that coded time sharing does not enlarge $\mathscr{R}_{UX}$. Thus, the conclusion of Theorem 1 continues to hold and homogeneous superposition coding with coded time sharing outperforms heterogeneous superposition coding with coded time sharing.

Appendix
Optimality of the Rate Region in (4)

We show that the heterogeneous superposition coding ensemble cannot achieve a rate region larger than the one in (4) under any decoding rule. To that end, denote the random codebook by

$$\mathcal{C} = (U^n(1), U^n(2), \ldots, X^n(1,1), X^n(1,2), \ldots).$$

By Fano's inequality,

$$\begin{aligned} &H(M_2 \,|\, Y_2^n, \mathcal{C}) \\ &\quad = \sum_c \mathsf{P}\{\mathcal{C} = c\} H(M_2 \,|\, Y_2^n, \mathcal{C} = c) \\ &\quad \leq \sum_c \mathsf{P}\{\mathcal{C} = c\}[1 + nR_2 \, \mathsf{P}\{\hat{M}_2 \neq M_2 \mid \mathcal{C} = c\}] \\ &\quad \leq n\epsilon_n, \end{aligned} \tag{8}$$

where $\epsilon_n \to 0$ as $n \to \infty$. Thus,

$$\begin{aligned} nR_2 &= H(M_2 \mid \mathcal{C}, M_1) \\ &\leq I(M_2; Y_2^n \mid \mathcal{C}, M_1) + n\epsilon_n \\ &= H(Y_2^n \mid \mathcal{C}, M_1) - H(Y_2^n \mid \mathcal{C}, M_1, M_2) + n\epsilon_n \\ &\overset{\text{(a)}}{\leq} nH(Y_2 \mid U) - nH(Y_2 \mid X) + n\epsilon_n \\ &= nI(X; Y_2 \mid U) + n\epsilon_n, \end{aligned}$$

where (a) follows by the definition of the codebook ensemble and the memoryless property.

To see the second inequality, first consider the case

$$R_1 < I(X; Y_2). \tag{9}$$

By (8), we have

$$\begin{aligned} H(M_1, M_2 \mid Y_2^n, \mathcal{C}) &= H(M_2 \mid Y_2^n, \mathcal{C}) + H(M_1 \mid Y_2^n, \mathcal{C}, M_2) \\ &\leq n\epsilon_n + H(M_1 \mid Y_2^n, \mathcal{C}, M_2). \end{aligned}$$

To bound the last term, note that given $M_2 = m_2$, the codebook reduces to

$$\mathcal{C}' = (X^n(1, m_2), X^n(2, m_2), X^n(3, m_2), \dots).$$

These codewords are pairwise independent since they do not share common U^n sequences, and thus $\mathcal{C}'$ is a nonlayered random codebook of rate R_1. Since (9) holds, receiver 2 can reliably recover M_1 from $(Y_2^n, \mathcal{C}, M_2)$ by using, for example, a typicality decoder. Thus we have

$$H(M_1 \mid Y_2^n, \mathcal{C}, M_2) \leq n\epsilon_n.$$

The sum-rate can then be bounded as

$$\begin{aligned} n(R_1 + R_2) &= H(M_1, M_2) \\ &\leq I(M_1, M_2; Y_2^n \mid \mathcal{C}) + 2n\epsilon_n \\ &\leq nI(X; Y_2) + 2n\epsilon_n. \end{aligned} \tag{10}$$

To conclude the argument, suppose that there exists a decoding rule that achieves a rate point (R_1, R_2) with $R_1 \geq I(X; Y_2)$. Then, this decoding rule must also achieve $(R_1', R_2') = (I(X; Y_2) - R_2/2, R_2)$ with the heterogeneous superposition coding ensemble, since (R_1, R_2) dominates (R_1', R_2'). Note that $R_1' < I(X; Y_2)$. It thus follows from the discussion above that $R_1' + R_2' \leq I(X; Y_2)$, which yields a contradiction.

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