Quantum Computing: A Gentle Introduction

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2.2

c. Let

$$|v\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

and

$$|v'\rangle = \frac{1}{\sqrt{2}}\left(-|0\rangle + i\,|1\rangle\right) .$$

Measuring with respect to $|v\rangle$,

$$|\langle v|v\rangle|^2 = 1$$

while

$$\begin{aligned}
\langle v|v'\rangle &= \left[\frac{1}{\sqrt{2}}\left(\langle 0| + \langle 1|\right)\right]\left[\frac{1}{\sqrt{2}}\left(-|0\rangle + i\,|1\rangle\right)\right] \\
&= \frac{1}{2}\left(\langle 0| + \langle 1|\right)\left(-|0\rangle + i\,|1\rangle\right) \\
&= \frac{1}{2}\left(-\langle 0|0\rangle + i\,\langle 0|1\rangle - \langle 1|0\rangle + \langle 1|1\rangle\right) \\
&= \frac{1}{2}\left(-1 + 1\right) \\
&= \frac{1}{2}(0) \\
&= 0
\end{aligned}$$

and therefore,

$$|\langle v|v'\rangle|^2 = 0 \neq |\langle v|v\rangle|^2 .$$

h. Note that

$$\begin{aligned}
|v\rangle &= \frac{1}{\sqrt{2}}\left(|i\rangle - |{-i}\rangle\right) \\
&= \frac{1}{2}\left(|0\rangle + i\,|1\rangle - |0\rangle + i\,|1\rangle\right) \\
&= i\,|1\rangle .
\end{aligned}$$

Since $\exists c = e^{i\phi},\ \phi \in \mathbb{R}$ such that $|v\rangle = c\,|1\rangle$, namely $\phi = \frac{\pi}{2}$, the two states are congruent.