

1 Linear Weight

$$T = \frac{\sigma_B \mu_A + \sigma_A \mu_B}{\sigma_B + \sigma_A}$$

		True Negative					True Positive			
1st Order	Centralized	41.09	32.95	40.70	38.25	100.00	100.00	100.00	100.00	
	Equalized	37.69	38.89	41.26	39.28	62.96	64.00	53.85	60.27	
2nd Order	Centralized	55.68	46.59	47.96	50.08	100.00	100.00	100.00	100.00	
	Equalized	46.95	44.36	50.96	47.42	100.00	100.00	100.00	100.00	

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	34	172	89	0	0.2833	37	111	146	1	0.3978
	33	166	96	0	0.2845	39	141	115	0	0.3562
	23	178	94	0	0.2054	27	122	146	0	0.3068
	31	148	116	0	0.2952	34	116	145	0	0.3696
	26	173	96	0	0.2311	34	146	115	0	0.3178
Equalized	18	168	99	10	0.1682	25	134	136	0	0.2717
	17	170	99	9	0.1596	31	136	128	0	0.3131
	14	161	108	12	0.1393	30	140	125	0	0.3000
	25	157	98	15	0.2252	28	115	152	0	0.3275
	18	145	117	15	0.1837	23	136	136	0	0.2527

2 Square-Root Weight

$$T = \frac{\sqrt{\sigma_B} \mu_A + \sqrt{\sigma_A} \mu_B}{\sqrt{\sigma_B} + \sqrt{\sigma_A}}$$

		True Negative					True Positive			
1st Order	Centralized	97.22	98.15	97.45	97.61	100.00	100.00	100.00	100.00	
	Equalized	68.50	70.59	68.87	69.32	19.51	30.43	39.47	29.80	
2nd Order	Centralized	71.22	73.86	78.68	74.59	95.83	96.77	91.89	94.83	
	Equalized	77.57	81.27	80.59	79.81	100.00	100.00	100.00	100.00	

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	30	6	259	0	0.9091	20	48	227	0	0.4545
	35	8	252	0	0.8974	24	25	246	0	0.6575
	27	7	261	0	0.8852	30	48	217	0	0.5556
	36	6	253	0	0.9231	33	48	214	0	0.5789
	23	7	256	0	0.8679	32	48	215	0	0.5714
Equalized	7	73	194	21	0.1296	33	23	239	0	0.7416
	11	68	190	26	0.1897	27	45	223	0	0.5455
	10	90	171	24	0.1493	27	66	202	0	0.4500
	9	81	181	24	0.1463	31	30	234	0	0.6739
	5	83	193	14	0.0935	24	46	225	0	0.5106

3 Logarithmic Weight

$$T = \frac{(\ln \sigma_B) \mu_A + (\ln \sigma_A) \mu_B}{\ln \sigma_B + \ln \sigma_A}$$

		True Negative					True Positive			
1st Order	Centralized	99.23	99.25	99.63	99.37	55.88	53.57	70.37	59.94	
	Equalized	95.80	95.04	95.11	95.32	9.09	3.03	3.45	5.19	
2nd Order	Centralized	91.37	94.74	90.49	92.20	97.50	68.97	93.75	86.74	
	Equalized	97.70	99.26	99.26	98.74	85.29	95.83	96.00	92.37	

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	13	2	264	16	0.5909	23	24	246	2	0.6389
	10	2	270	13	0.5714	24	16	250	5	0.6957
	29	2	242	22	0.7073	23	24	245	3	0.6301
	14	2	262	17	0.5957	23	32	234	1	0.6292
	14	1	263	17	0.6087	21	23	248	3	0.6176
Equalized	1	13	260	21	0.0556	35	9	250	1	0.8750
	1	13	260	21	0.0556	22	5	267	1	0.8800
	2	12	254	17	0.0930	27	2	261	5	0.8852
	5	13	246	31	0.1852	33	8	253	1	0.8800
	3	13	253	26	0.1333	26	6	261	2	0.8667

4 Square-Log Weight

$$T = \frac{(\sqrt{|\ln \sigma_B|})\mu_A + (\sqrt{|\ln \sigma_A|})\mu_B}{\sqrt{|\ln \sigma_B|} + \sqrt{|\ln \sigma_A|}}$$

		True Negative				True Positive			
1st Order	Centralized	99.24	99.26	99.25	99.25	53.13	52.17	56.67	53.99
	Equalized	93.89	94.81	95.45	94.72	3.03	8.00	6.45	5.83
2nd Order	Centralized	91.92	89.92	88.01	89.95	65.71	92.59	85.71	81.34
	Equalized	95.88	99.23	97.76	97.62	100.00	88.89	100.00	96.30

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	21	1	260	13	0.7500	29	26	240	0	0.6905
	15	1	267	12	0.6977	28	41	221	5	0.5490
	9	2	263	21	0.4390	29	14	249	3	0.7733
	11	2	264	18	0.5238	28	28	236	3	0.6437
	8	2	274	11	0.5517	31	18	243	3	0.7470
Equalized	1	16	249	29	0.0426	21	11	263	0	0.7925
	5	10	249	31	0.1961	24	15	256	0	0.7619
	1	18	250	26	0.0435	35	8	252	0	0.8974
	2	13	243	37	0.0741	25	5	263	2	0.8772
	3	12	250	30	0.1250	32	4	253	6	0.8649

5 Log-Square Weight

$$T = \frac{(\ln \sqrt{\sigma_B})\mu_A + (\ln \sqrt{\sigma_A})\mu_B}{\ln \sqrt{\sigma_B} + \ln \sqrt{\sigma_A}}$$

		True Negative				True Positive			
1st Order	Centralized	99.25	99.25	99.62	99.37	51.85	40.74	43.33	45.31
	Equalized	95.45	93.51	94.93	94.63	6.45	6.06	5.26	5.92
2nd Order	Centralized	91.70	93.21	89.27	91.39	83.33	100.00	94.12	92.48
	Equalized	99.62	99.63	97.37	98.87	93.10	80.77	96.55	90.14

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	11	2	263	19	0.5116	25	20	248	2	0.6944
	7	1	262	25	0.3500	25	18	250	2	0.7143
	11	2	272	10	0.6471	29	16	248	2	0.7632
	11	2	264	18	0.5238	23	20	244	8	0.6216
	15	2	248	30	0.4839	27	20	242	6	0.6750
Equalized	2	12	253	28	0.0909	25	4	264	2	0.8929
	0	17	249	29	0.0000	28	11	255	1	0.8235
	1	13	250	31	0.0435	23	4	263	5	0.8364
	2	14	254	25	0.0930	21	5	267	2	0.8571
	1	13	261	20	0.0571	25	1	266	3	0.9259

6 Linear Additional Weight

$$T = \frac{\sigma_B(1 - \alpha)\mu_A + \sigma_A\alpha\mu_B}{\sigma_B(1 - \alpha) + \sigma_A\alpha}$$

		True Negative				True Positive			
1st Order	Centralized	20.44	23.11	22.05	21.87	100.00	100.00	96.88	98.96
	Equalized	22.26	22.93	23.79	22.99	76.19	75.86	80.77	77.61
2nd Order	Centralized	17.10	17.60	16.54	17.08	100.00	100.00	100.00	100.00
	Equalized	17.18	17.23	17.36	17.26	100.00	100.00	100.00	100.00

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	34	202	59	0	0.2519	29	222	44	0	0.2071
	30	202	63	0	0.2290	33	217	45	0	0.2332
	24	216	55	0	0.1818	33	216	46	0	0.2340
	31	201	63	0	0.2357	33	216	46	0	0.2340
	36	202	57	0	0.2628	40	207	48	0	0.2787
Equalized	23	208	55	9	0.1749	25	228	42	0	0.1799
	20	206	59	10	0.1563	25	225	45	0	0.1818
	22	203	65	5	0.1746	17	234	44	0	0.1269
	15	217	55	8	0.1176	34	220	41	0	0.2361
	22	210	53	10	0.1667	23	228	44	0	0.1679

7 Square-Root Additional Weight

$$T = \frac{(\sigma_B\sqrt{1 - \alpha})\mu_A + (\sigma_A\sqrt{\alpha})\mu_B}{\sigma_B\sqrt{1 - \alpha} + \sigma_A\sqrt{\alpha}}$$

		True Negative				True Positive			
1st Order	Centralized	25.77	24.34	29.30	26.47	100.00	100.00	100.00	100.00
	Equalized	27.31	26.64	27.76	27.24	83.33	77.78	78.13	79.75
2nd Order	Centralized	20.91	18.96	19.10	19.66	100.00	100.00	100.00	100.00
	Equalized	17.74	17.16	17.91	17.60	100.00	100.00	100.00	100.00

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	33	193	63	0	0.2548	28	219	48	0	0.2036
	24	199	72	0	0.1943	36	207	52	0	0.2581
	38	183	74	0	0.2934	36	206	53	0	0.2590
	25	201	69	0	0.1992	35	205	55	0	0.2545
	28	199	68	0	0.2196	26	216	53	0	0.1940
Equalized	26	190	76	3	0.2122	22	224	49	0	0.1642
	17	201	70	7	0.1405	25	222	48	0	0.1838
	22	191	77	5	0.1833	26	220	49	0	0.1912
	16	200	73	6	0.1345	26	224	45	0	0.1884
	17	202	71	5	0.1411	29	219	47	0	0.2094

8 Logarithmic Additional Weight

$$T = \frac{[\sigma_B\ln(1 - \alpha)]\mu_A + [\sigma_A\ln\alpha]\mu_B}{\sigma_B\ln(1 - \alpha) + \sigma_A\ln\alpha}$$

		True Negative				True Positive			
1st Order	Centralized	99.62	99.62	99.61	99.62	23.53	25.00	19.44	22.66
	Equalized	96.85	96.18	97.73	96.92	4.88	3.03	12.90	6.94
2nd Order	Centralized	97.72	96.63	98.87	97.74	25.00	42.86	48.28	38.71
	Equalized	100.00	99.62	100.00	100.00	47.83	37.93	46.15	43.97

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	6	2	267	18	0.3750	7	5	264	19	0.3684
	10	2	270	13	0.5714	7	4	263	21	0.3590
	8	1	262	24	0.3902	12	5	265	13	0.5714
	11	1	259	24	0.4681	7	4	263	21	0.3590
	12	1	262	20	0.5333	12	2	265	16	0.5714
Equalized	0	11	252	32	0.0000	12	0	262	21	0.5333
	0	11	253	31	0.0000	10	0	270	15	0.5714
	1	12	258	24	0.0526	18	0	254	23	0.6101
	2	9	261	23	0.1111	10	0	267	18	0.5263
	2	9	257	27	0.1000	13	0	260	22	0.5417

9 Square-Log Additional Weight

$$T = \frac{\mu_A \sigma_B \sqrt{-\ln(1-\alpha)} + \mu_B \sigma_A \sqrt{-\ln \alpha}}{\sigma_B \sqrt{-\ln(1-\alpha)} + \sigma_A \sqrt{-\ln \alpha}}$$

		True Negative				True Positive			
1st Order	Centralized	98.47	98.48	98.13	98.36	82.35	100.00	100.00	94.12
	Equalized	88.93	89.02	89.63	89.19	9.09	22.58	16.00	15.89
2nd Order	Centralized	96.28	95.09	95.44	95.60	73.08	66.67	84.38	74.71
	Equalized	99.26	98.49	96.67	98.14	91.30	100.00	96.00	95.77

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	24	2	266	3	0.9057	19	12	252	12	0.6129
	28	3	264	0	0.9492	29	14	248	4	0.7632
	27	4	264	0	0.9310	23	14	253	5	0.7077
	26	3	265	1	0.9286	17	13	256	9	0.6071
	31	4	257	3	0.8986	18	14	257	6	0.6429
Equalized	8	25	242	20	0.2623	24	10	261	0	0.8276
	4	27	243	21	0.1429	30	6	258	1	0.8955
	3	32	239	21	0.1017	34	9	252	0	0.8831
	4	29	229	33	0.1143	20	1	262	12	0.7547
	1	33	235	26	0.0328	34	2	257	2	0.9444

10 Log-Square Additional Weight

$$T = \frac{\mu_A \sigma_B \ln \sqrt{1-\alpha} + \mu_B \sigma_A \ln \sqrt{\alpha}}{\sigma_B \ln \sqrt{1-\alpha} + \sigma_A \ln \sqrt{\alpha}}$$

		True Negative				True Positive			
1st Order	Centralized	99.24	99.63	99.63	99.50	18.18	21.43	18.52	19.38
	Equalized	95.91	96.01	95.20	95.70	0.00	0.00	0.00	0.00
2nd Order	Centralized	98.48	98.43	99.63	98.85	25.00	27.50	37.04	29.85
	Equalized	100.00	99.27	100.00	99.76	32.14	54.55	31.03	39.24

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	11	1	262	21	0.5000	8	5	259	23	0.3636
	5	1	266	23	0.2941	9	3	271	12	0.5455
	6	2	264	23	0.3243	10	2	267	16	0.5263
	6	1	270	18	0.3871	16	3	256	20	0.5818
	7	0	267	21	0.4000	9	6	261	19	0.4186
Equalized	0	11	262	22	0.0000	17	0	254	24	0.5862
	1	9	257	28	0.0513	6	0	269	20	0.3750
	1	10	258	26	0.0526	13	0	264	18	0.5909
	0	12	255	28	0.0000	15	0	255	25	0.5455
	1	10	238	46	0.0345	13	0	264	18	0.5909

11 Square-Log Combined Weight

$$T = \frac{\mu_A \sqrt{|\ln[\sigma_B(1-\alpha)]|} + \mu_B \sqrt{|\ln(\sigma_A\alpha)|}}{\sqrt{|\ln[\sigma_B(1-\alpha)]|} + \sqrt{|\ln(\sigma_A\alpha)|}}$$

		True Negative				True Positive			
1st Order	Centralized	99.26	99.22	99.62	99.37	50.00	59.46	39.39	49.62
	Equalized	94.70	94.70	95.20	94.87	12.90	0.00	4.17	5.69
2nd Order	Centralized	94.98	95.17	89.59	93.25	87.50	88.46	92.31	89.42
	Equalized	97.00	99.23	95.96	97.40	100.00	76.47	95.65	90.71

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	25	2	256	12	0.7813	22	17	250	6	0.6567
	15	2	260	18	0.6000	23	23	247	2	0.6479
	12	1	262	20	0.5333	24	15	250	6	0.6957
	13	2	267	13	0.6341	28	27	235	5	0.6364
	12	0	272	11	0.6857	21	29	242	3	0.5676
Equalized	2	12	256	25	0.0976	20	3	270	2	0.8889
	2	12	238	43	0.0678	43	7	244	1	0.9149
	3	11	244	37	0.1111	16	1	276	2	0.9143
	0	17	245	33	0.0000	29	7	259	0	0.8923
	3	11	259	22	0.1538	27	5	261	2	0.8852

12 Log-Square Combined Weight

$$T = \frac{\mu_A \ln \sqrt{\sigma_B(1-\alpha)} + \mu_B \ln \sqrt{\sigma_A\alpha}}{\ln \sqrt{\sigma_B(1-\alpha)} + \ln \sqrt{\sigma_A\alpha}}$$

		True Negative				True Positive			
1st Order	Centralized	99.26	99.62	99.64	99.51	36.00	45.71	38.89	40.20
	Equalized	95.57	95.17	95.51	95.42	4.17	0.00	3.57	2.58
2nd Order	Centralized	94.31	93.94	94.30	94.18	87.10	63.52	78.13	76.25
	Equalized	99.62	98.89	99.62	99.38	78.79	70.83	75.86	75.16

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	12	1	263	19	0.5455	24	14	247	10	0.6667
	6	1	274	14	0.4444	19	20	254	2	0.6333
	19	0	248	28	0.5758	16	17	256	6	0.5818
	19	1	259	16	0.6909	21	17	250	7	0.6364
	8	0	269	18	0.4706	15	14	261	5	0.6122
Equalized	3	11	258	23	0.1500	23	2	264	6	0.8519
	0	13	247	35	0.0000	24	0	261	10	0.8276
	0	12	257	26	0.0000	19	0	272	4	0.9048
	2	12	255	26	0.0952	15	1	266	13	0.6818
	2	11	255	27	0.0952	26	3	264	2	0.9123

13 Linear Balanced

$$T = \arg \min_{\mu_A \leq T \leq \mu_B} \alpha \cdot \int_{-\infty}^T P_B(x) dx + (1-\alpha) \cdot \int_T^{+\infty} P_A(x) dx$$

		True Negative				True Positive			
1st Order	Centralized	95.47	88.81	85.50	89.93	100.00	96.30	100.00	98.77
	Equalized	73.99	80.52	79.85	78.12	40.91	30.14	11.11	27.39
2nd Order	Centralized	77.70	78.88	76.87	77.82	96.15	84.09	96.30	92.18
	Equalized	72.35	78.83	77.17	76.12	100.00	100.00	100.00	100.00

	1st Order					2nd Order				
	TP	FP	TN	FN	F1 Score	TP	FP	TN	FN	F1 Score
Centralized	24	17	254	0	0.7385	25	57	210	3	0.4545
	30	11	254	0	0.8451	30	58	206	1	0.5042
	26	33	236	0	0.6118	28	59	203	5	0.4667
	27	28	240	0	0.6585	27	52	216	0	0.5094
	32	14	249	0	0.8205	32	60	203	0	0.5161
Equalized	6	63	199	27	0.1176	32	51	212	0	0.5565
	13	53	214	15	0.2766	35	48	212	0	0.5932
	5	66	194	30	0.0943	22	36	237	0	0.5500
	10	50	212	23	0.2151	23	51	221	0	0.4742
	3	62	208	22	0.0667	38	55	202	0	0.5802

Total

	1st Order		2nd Order		
No.	Centralized	Equalized	Centralized	Equalized	Sum
1	0.2410	0.1752	0.3496	0.2930	1.0588
2	0.8965	0.1417	0.5636	0.5843	2.1861
3	0.6148	0.1045	0.6423	0.8774	2.2390
4	0.5924	0.0963	0.6807	0.8388	2.2082
5	0.5033	0.0569	0.6937	0.8672	2.1211
6	0.2322	0.1580	0.2001	0.1785	0.7688
7	0.2323	0.1623	0.2338	0.1874	0.8158
8	0.4676	0.0527	0.4458	0.5566	1.5227
9	0.9226	0.1508	0.6668	0.8611	2.6013
10	0.3811	0.0277	0.4872	0.5195	1.4155
11	0.6469	0.0861	0.6409	0.8991	2.2730
12	0.5454	0.0681	0.6261	0.8357	2.0753
13	0.7349	0.1541	0.4902	0.5508	1.9300

Order:
9 > 11 > 3 > 4 > 2 > 5 > 12 > 13 > 8 > 10 > 1 > 7 > 6

Test with Gaussian Data

	F1	False Rate(%)
1	0.5810	17.12
2	0.5718	18.06
3	0.3860	31.50
4	0.5515	19.31
5	0.3811	30.31
6	0.3515	36.16
7	0.4510	26.54
8	0.5523	13.32
9	0.5791	14.71
10	0.5578	13.23
11	0.5600	17.98
12	0.2808	27.01
13	0.6072	12.69