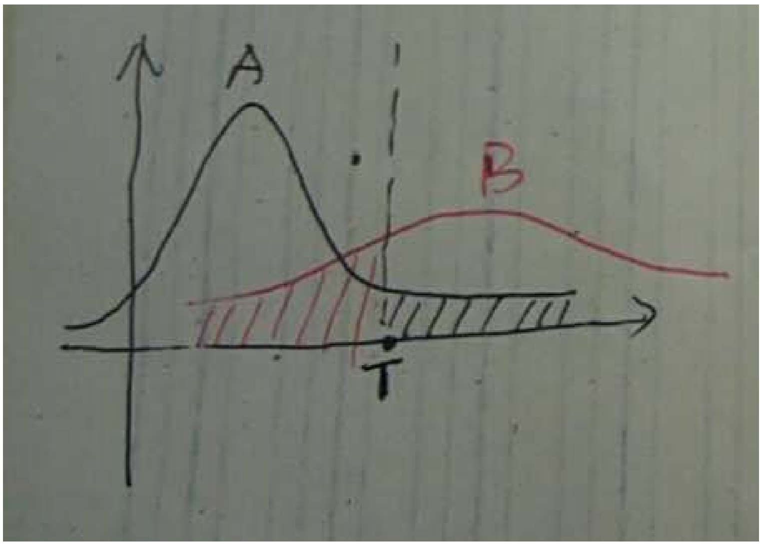
Given:

$$A \sim N(\mu_A, \sigma_A), B \sim N(\mu_B, \sigma_B), 0 < \mu_A < \mu_B, 0 < \alpha < 1$$

Evaluate:

$$\underset{\mu_A < T < \mu_B}{\arg\min} \quad \alpha \cdot \int_{-\infty}^{T} P_B(x)dx + (1-\alpha) \cdot \int_{T}^{+\infty} P_A(x)dx$$



Let:

$$\begin{aligned} g(T) &= \alpha \cdot \int_{-\infty}^{T} P_B(x)dx + (1-\alpha) \cdot \int_{T}^{+\infty} P_A(x)dx \ , \ \mu_A < T < \mu_B \\ &= \alpha \cdot \int_{-\infty}^{T} P_B(x)dx + (1-\alpha) \cdot \Big(1 - \int_{-\infty}^{T} P_A(x)dx\Big) \ , \ \mu_A < T < \mu_B \end{aligned} \tag{1}$$

Then:

$$\begin{aligned} g'(T) &= \alpha P_B(T) - (1-\alpha) P_A(T) \\ &= \frac{\alpha}{\sigma_B\sqrt{2\pi}} e^{-(\frac{T-\mu_B}{\sigma_B\sqrt{2}})^2} - \frac{1-\alpha}{\sigma_A\sqrt{2\pi}} e^{-(\frac{\mu_A-T}{\sigma_A\sqrt{2}})^2} \end{aligned} \tag{2}$$

$$\begin{aligned} g''(T) &= \frac{\alpha}{\sigma_B\sqrt{2\pi}} e^{-(\frac{T-\mu_B}{\sigma_B\sqrt{2}})^2} \cdot (-2) \cdot \frac{T-\mu_B}{\sigma_B\sqrt{2}} \cdot \frac{1}{\sigma_B\sqrt{2}} - \frac{1-\alpha}{\sigma_A\sqrt{2\pi}} e^{-(\frac{\mu_A-T}{\sigma_A\sqrt{2}})^2} \cdot (-2) \cdot \frac{\mu_A-T}{\sigma_A\sqrt{2}} \cdot \frac{-1}{\sigma_A\sqrt{2}} \\ &= \frac{\alpha(\mu_B-T)}{\sigma_B^3\sqrt{2\pi}} e^{-(\frac{T-\mu_B}{\sigma_B\sqrt{2}})^2} + \frac{(1-\alpha)(T-\mu_A)}{\sigma_A^3\sqrt{2\pi}} e^{-(\frac{\mu_A-T}{\sigma_A\sqrt{2}})^2} \end{aligned} \tag{3}$$

According to (3), $g''(T) > 0$ for $\forall T \in (\mu_A, \mu_B)$
According to (2),

$$\left.\begin{aligned} g'(\mu_A) < 0 \\ g'(\mu_B) > 0 \end{aligned}\right\} \Rightarrow \frac{\sigma_B \cdot e^{-(\frac{\mu_A-\mu_B}{\sigma_A\sqrt{2}})^2}}{\sigma_A + \sigma_B \cdot e^{-(\frac{\mu_A-\mu_B}{\sigma_A\sqrt{2}})^2}} < \alpha < \frac{\sigma_B}{\sigma_A \cdot e^{-(\frac{\mu_A-\mu_B}{\sigma_B\sqrt{2}})^2} + \sigma_B} \tag{4}$$

Let: $g'(T) = 0$
Then:

$$\begin{aligned} \frac{\alpha}{\sigma_B\sqrt{2\pi}} e^{-(\frac{T-\mu_B}{\sigma_B\sqrt{2}})^2} &= \frac{1-\alpha}{\sigma_A\sqrt{2\pi}} e^{-(\frac{\mu_A-T}{\sigma_A\sqrt{2}})^2} \\ ln\Big(\frac{\alpha}{\sigma_B}\Big) - \Big(\frac{T-\mu_B}{\sigma_B\sqrt{2}}\Big)^2 &= ln\Big(\frac{1-\alpha}{\sigma_A}\Big) - \Big(\frac{\mu_A-T}{\sigma_A\sqrt{2}}\Big)^2 \end{aligned} \tag{5}$$