

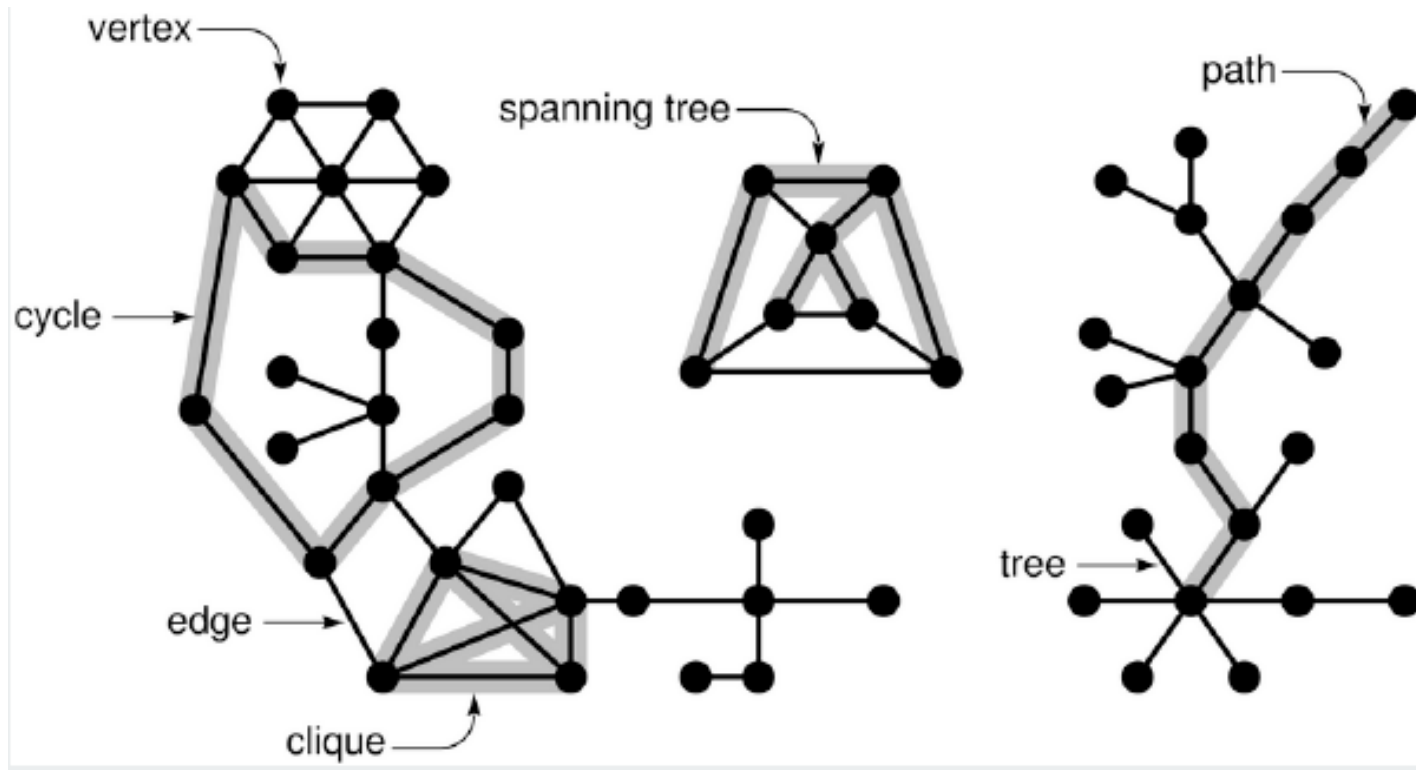
Undirected graphs

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Undirected graphs

- A graph $G=(V, E)$ where V is a set of vertices connected pairwise by edges E .
- Why study graph algorithms?
 - Interesting and broadly useful abstraction.
 - Challenging branch of computer science and discrete math.
 - Hundreds of graph algorithms known.
 - Thousands of practical applications.
 - Communication, circuits, transportation, scheduling, software systems, internet, games, social network, neural networks, ...

Graph terminology

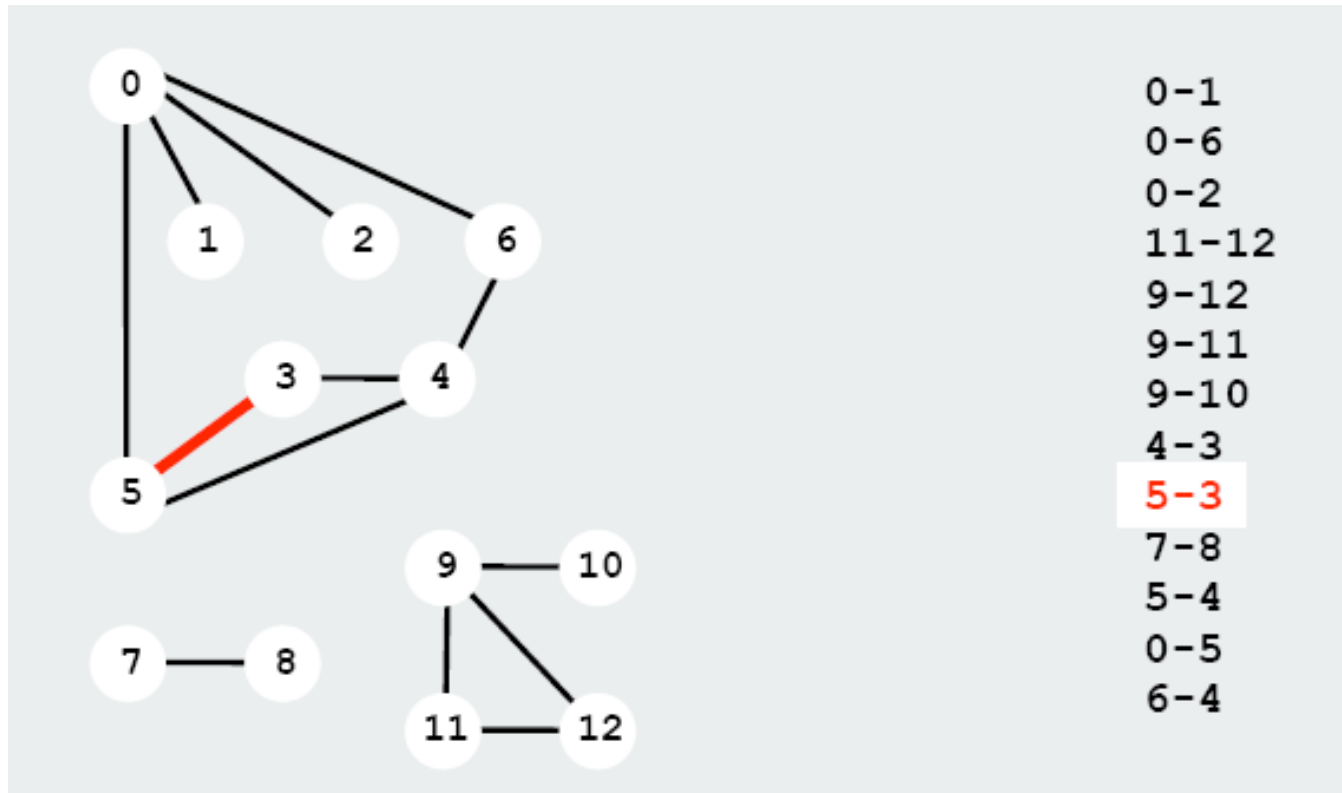


Some graph-processing problems

- Path: Is there a path between s to t ?
- Shortest path: What is the shortest path between s and t ?
- Cycle: Is there a cycle in the graph?
- Euler tour: Is there a cycle that uses each edge exactly once?
- Hamilton tour: Is there a cycle that uses each vertex exactly once?
- Connectivity: Is there a way to connect all of the vertices?
- MST: What is the best way to connect all of the vertices?
- Biconnectivity: Is there a vertex whose removal disconnects the graph?

Graph representation (1)

- Maintain a list of the edges

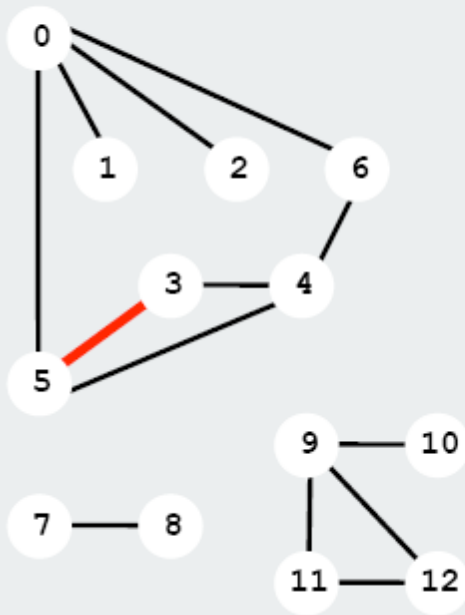


- Not suitable for searching

Graph representation (2)

- Maintain an adjacency matrix.

For each edge $v-w$ in graph: $\text{adj}[v][w] = \text{adj}[w][v] = \text{true}$.



two entries for each edge

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	0	0	1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	1	0	0	0	0	0	0
5	1	0	0	1	1	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	1
12	0	0	0	0	0	0	0	0	0	1	0	1	0

- Suitable for random accesses to the edges

A graph data structure

- Use a dynamic array to represent a graph as the following

```
typedef struct {  
    int * matrix;  
    int sizemax;  
} Graph;
```

- Define the following API

```
Graph createGraph(int sizemax);  
void addEdge(Graph graph, int v1, int v2);  
int adjacent(Graph graph, int v1, int v2);  
int getAdjacentVertices(Graph graph, int vertex, int* output); //  
    return the number of adjacent vertices.  
void dropGraph(Graph graph);
```

How to use the API?

```
int i, n, output[100];
Graph g = createGraph(100);
addEdge(g, 0, 1);
addEdge(g, 0, 2);
addEdge(g, 1, 2);
addEdge(g, 1, 3);
n = getAdjacentVertices (g, 1, output);
if (n==0) printf("No adjacent vertices of node 1\n");
else {
    printf("Adjacent vertices of node 1:");
    for (i=0; i<n; i++) printf("%5d", output[i]);
}
```


Quiz 1

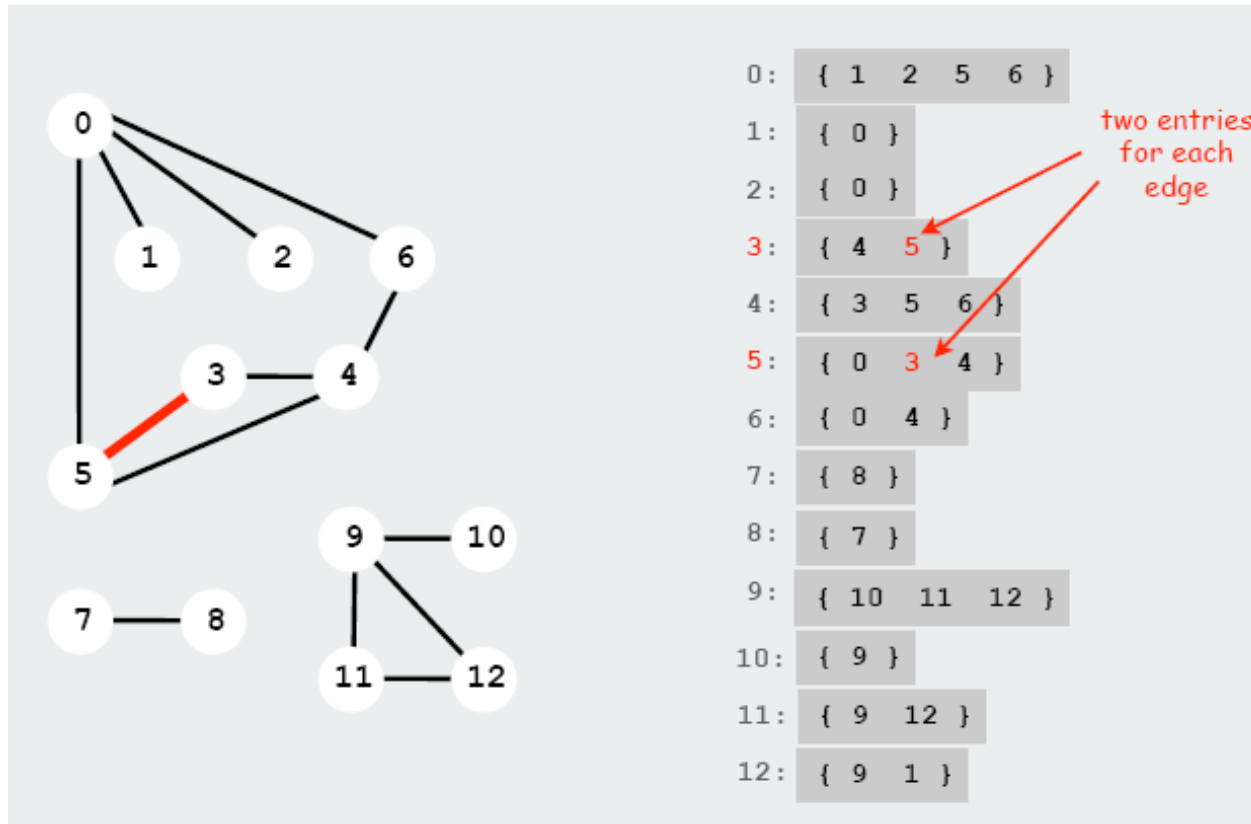
- Write the implementation for the API defined in the previous slide
- Use the example to test your API

Solution

- graph_matrix.c

Graph representation (3)

- Maintain an adjacency list.



Comparison

- Adjacency List is usually preferred, because it provides a compact way to represent **sparse** graphs – those for which $|E|$ is much less than $|V|^2$
- Adjacency Matrix may be preferred when the graph is **dense**, or when we need to be able to tell quickly if there is an edge connecting two given vertices

Implementation

- The red black tree can be used to store such a graph where each node in the tree is a vertex and its value is a list of adjacent vertices.
- Such a list of adjacent vertices can be stored in a red black tree as well.

Quiz 2

- Rewrite the API defined for graphs using the libfdr library as the following

```
#include "jrb.h"
```

```
typedef JRB Graph;
```

```
Graph createGraph();
```

```
void addEdge(Graph graph, int v1, int v2);
```

```
int adjacent(Graph graph, int v1, int v2);
```

```
int getAdjacentVertices (Graph graph, int v, int*  
    output);
```

```
void dropGraph(Graph graph);
```

Instructions (1)

- To create a graph
Simply call `make_jrb()`
- To add a new edge $(v1, v2)$ to graph `g`
`tree = make_jrb();`
`jrb_insert_int(g, v1, new_jval_v(tree));`
`jrb_insert_int(tree, v2, new_jval_i(1));`
- If the node `v1` is already allocated in the graph
`node = jrb_find_int(g, v1);`
`tree = (JRB) jval_v(node->val);`
`jrb_insert_int(tree, v2, new_jval_i(1));`

Instructions (2)

- To get adjacent vertices of v in graph g

```
node = jrb_find_int(g, v);
```

```
tree = (JRB) jval_v(node->val);
```

```
total = 0;
```

```
jrb_traverse(node, tree)
```

```
    output[total++] = jval_i(node->key);
```

- To delete/free a graph

```
jrb_traverse(node, graph)
```

```
    jrb_free_tree( jval_v(node->val) );
```


Solution

- graph_jrb.c

Homework

- In order to describe the metro lines of a city, we can store the data in a file as the following.

[STATIONS]

S1=Name of station 1

S2=Name of station 2

...

[LINES]

M1=S1 S2 S4 S3 S7

M2=S3 S5 S6 S8 S9

...

- Make a program to read such a file and establish the network of metro stations in the memory using the defined API.
- Write a function to find all the stations adjacent to a station given by its name.