CALCULUS FOR IT 501031

1 Exercises

Exercise 1: Find the specific function values:

(a) $f(x, y) = x^2 + xy^3$ at $f(0, 0), f(-1, 1), f(2, 3), f(-3, -2)$

(b) $f(x, y, z) = \dfrac{x - y}{y^2 + z^2}$ at $f(3, -1, 2), f(1, 1/2, 1/4), f(0, -1/3, 0), f(2, 2, 100)$.

Exercise 2: Plot the graph of the functions

(a) $f(x, y) = cos(x)cos(y)e^{-(\sqrt{x^2+y^2})/4}$

(b) $f(x, y) = -\dfrac{xy^2}{x^2 + y^2}$

(c) $f(x, y) = \dfrac{xy(x^2 - y^2)}{x^2 + y^2}$

(d) $f(x, y) = y^2 - y^4 - x^2$

Exercise 3: Find the first-order partial derivatives of the function $f(x, y)$, Then plot the function $f(x, y)$ and the first order partial derivative of $f(x, y)$ with regard to x, and y, respectively.

(a) $f(x, y) = 2x^2 - 3y - 4$

(b) $f(x, y) = (x^2 - 1)(y + 2)$

(c) $f(x, y) = x^2 - xy + y^2$

(d) $f(x, y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$

(e) $f(x, y) = (xy - 1)^2$

(f) $f(x, y) = (2x - 3y)^3$

(g) $f(x, y) = \sqrt{x^2 + y^2}$

(h) $f(x, y) = (x^3 + \frac{y}{2})^{\frac{2}{3}}$

(i) $f(x, y) = \dfrac{1}{x + y}$

(j) $f(x, y) = \dfrac{x}{x^2 + y^2}$

(k) $f(x, y) = \dfrac{x + y}{xy - 1}$

(l) $f(x, y) = tan^{-1}(\frac{y}{x})$

(m) $f(x, y) = e^{x+y+1}$

(n) $f(x, y) = e^{-x}sin(x + y)$

(o) $f(x, y) = ln(x + y)$

Exercise 4: Find all the second-order partial derivatives of the function $f(x, y)$. Then plot the function $f(x, y)$ and the second order partial derivative of $f(x, y)$ with regard to x, and y, respectively.

(a) $f(x, y) = x + y + xy$

(b) $f(x, y) = sin(xy)$

(c) $f(x, y) = x^2y + cosy + ysinx$

(d) $f(x, y) = xe^y + y + 1$

(e) $f(x, y) = ln(x + y)$

(f) $f(x, y) = tan^{-1}(\frac{y}{x})$

(g) $f(x, y) = x^2tan(xy)$

(h) $f(x, y) = ye^{x^2-y}$

(i) $f(x, y) = xsinx^2y$

(j) $f(x, y) = \dfrac{x - y}{x^2 + y}$

Exercise 5: Verify that $f_{xy} = f_{yx}$ or not.

(a) $f(x, y) = x\sin y + y\sin x + xy$

(b) $f(x, y) = ln(2x + 3y)$

(c) $f(x, y) = xy^2 + x^2y^3 + x^3y^4$

(d) $f(x, y) = e^x + x\ln y + y\ln x$

Exercise 6: Find the fifth-order partial derivative $\dfrac{\partial^5 f}{\partial x^2 \partial y^3}$ of the function following:

(a) $f(x, y) = y^2 x^4 e^x + 2$

(b) $f(x, y) = y^4 + y(\sin x - x^4)$

(c) $f(x, y) = x^5 + 5xy + \sin x + 7e^x$

(d) $f(x, y) = xe^{\frac{y^4}{2}}$

Exercise 7: Express $\dfrac{dw}{dt}$ as a function of t, both by using the Chain Rule and by expressing w in terms of t and differentiating directly with respect to t. Then evaluate $\dfrac{dw}{dt}$ at the given value of t.

(a) $w = x^2 + y^2, x = \cos(t), y = \sin(t), t = \pi$

(b) $w = x^2 + y^2, x = \cos(t) + \sin(t), y = \cos(t) - \sin(t), t = 0$

(c) $w = \dfrac{x}{z} + \dfrac{y}{z}, x = \cos^2(t), y = \sin^2(t), z = \dfrac{1}{t}, t = 3$

(d) $w = 2ye^x - \ln z, x = \ln(t^2 + 1), y = \tan^{-1}t, z = e^t, t = 1$

(e) $w = z - \sin(xy), x = t, y = \ln(t), z = e^{t-1}, t = 1$

Exercise 8: Use the limit definition of partial derivative to compute the partial derivatives of the functions at the specified points

(a) $f(x, y) = 1 - x + y - 3x^2y, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ at $(1, 2)$

(b) $f(x, y) = 4 + 2x - 3y - 3xy^2, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ at $(-2, 1)$

Exercise 9: Let $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$. Find the slope of line tangent to this surface at the point $(3, 2)$