CALCULUS FOR IT - 501031

1 Exercises

Exercise 1: Write a computer program to find the derivative of functions

(a) $f(x) = 4 - x^2$
(b) $f(x) = (x-1)^2 + 1$
(c) $g(t) = \frac{1}{t^2}$
(d) $k(z) = \frac{1-z}{2z}$
(e) $k(z) = \sqrt{(3z)}$
(f) $k(z) = \sqrt{(2z+1)}$

Exercise 2: Find the equation of the line tangent of the following functions, then draw the graph.

(a) $f(x) = x^2 + 1$, (2, 5)
(b) $f(x) = x - 2x^2$, (1, -1)
(c) $f(x) = \frac{x}{x-2}$, (3, 3)
(d) $g(x) = \frac{8}{x^2}$, (2, 2)
(e) $g(x) = \sqrt{x}$, (4, 2)
(f) $h(t) = t^3 + 3t$, (1, 4)
(g) $f(x) = \frac{8}{\sqrt{x-2}}$, (6, 4).
(h) $g(z) = 1 + \sqrt{4-z}$, (3, 2).

Exercise 3: Find the slope of the curve at the point indicated, and then find the equation of the corresponding tangent.

(a) $f(x) = 5x - 3x^2$, $x = 1$
(b) $f(x) = \frac{1}{x-1}$, $x = 3$
(c) $f(x) = x^3 - 2x + 7$, $x = -2$
(d) $f(x) = \frac{x-1}{x+1}$, $x = 0$

Exercise 4: Find the derivative of function $y = -\frac{2x^2}{3} + x$ at $x = 0$ by the definition

Hint:

- $f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$
- Calculate function value at a point.

Exercise 5: Using the definition formula of derivatives f to find the values of the derivatives as specified.

(a) $f(x) = 4 - x^2$, $f'(-3), f'(0), f'(1)$
(b) $F(x) = (x-1)^2 + 1$, $F'(-1), F'(0), F'(2)$
(c) $g(t) = \frac{1}{t^2}$, $g'(-1), g'(2), g'(\sqrt{3})$
(d) $k(z) = \frac{1-z}{2z}$, $k'(-1), k'(1), k'(\sqrt{2})$

Exercise 6: Use the formula below

$$f'(x) = lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

to find the derivative of the functions below:

(a) $f(x) = \dfrac{1}{x+2}$

(b) $f(x) = x^2 - 3x + 4$

(c) $f(x) = \dfrac{x}{x-1}$

(d) $f(x) = 1 + \sqrt{x}$

Exercise 7: Write a computer program to perform the following steps

- Step 1: Plot $y = f(x)$ over the interval $(x_0 - 1/2 \leq x \leq (x_0 + 3))$
- Step 2: Holding x_0 fixed, the difference quotient
$$q(h) = \frac{f(x_0 + h) - f(x_0)}{h}$$
at x_0 becomes a function of the step size h.
- Step 3: Find the limit of q as $h \to 0$.
- Step 4: Define the tangent lines $y = f(x_0) + q(x - x_0)$ for $h = 3, 2$ and 1. Graph them together with f and the tangent line over the interval in step 1.

Evaluate program by the functions

(a) $f(x) = x^3 + 2x$, $x_0 = 0$

(b) $f(x) = x + \dfrac{5}{x}$, $x_0 = 1$

(c) $f(x) = x + sin(2x)$, $x_0 = \pi/2$

(d) $f(x) = cosx + 4sin(2x)$, $x_0 = \pi$

Exercise 8: Given $f(x) = x^3 - 3x + 1$ (C). Find the tangent line of (C) in the cases:

(a) At a point $x_0 = 3$

(b) The tangent line is parallel to $y = 9x + 2$

(c) The tangent line at $A = (\dfrac{2}{3}, -1)$

Exercise 9: Find $f'(x)$ and use it to find equations of the tangent lines to curve $f(x) = 4x^2 - x^3$ at points $(2, 8)$ and $(3, 9)$. Illustrate your result by graphing the curve and the tangent lines on the same graph.

Exercise 10: Determine the differentiable function or not

(a) $f_1(x) = (x-1)^{\frac{1}{3}}$ at $x = 1$

(b) $f_2(x) = \begin{cases} -(x+2), \text{ if } x \leq -2 \\ x + 2, \text{ if } x > -2 \end{cases}$

(c) $f_3(x) = \begin{cases} x^2, \text{ if } x \geq 0 \\ 0, \text{ if } x < 0 \end{cases}$

Exercise 11: Determine whether $f'(0)$ exists ($f(x)$ is differentiable at $x = 0$)

(a) $f(x) = \begin{cases} xsin\dfrac{1}{x}, \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$

(b) $f(x) = \begin{cases} x^2 sin\dfrac{1}{x}, \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$

Exercise 12: Suppose that it costs

$$c(x) = x^3 - 6x^2 + 15x$$

dollars to produce x radiators when 8 to 30 radiatord are produced. Your shop currently produces 10 radiators a day. Write a program to compute how much extra will it cost to produce one more radiator a day.

Exercise 13: Suppose that the revenue from selling x washing machines is

$$r(x) = 20,000(1 - \frac{1}{x}) \text{ dollars}$$

Write a program to find the marginal revenue when 100 machines are produced.

Exercise 14: When a bactericide was added to a nutrient broth in which bacteria were growing, the bacterium population continued to grow for a while, but then stopped growing and began to decline. The size of the population at time t (hours) was

$$b = 10^6 + 10^4 t - 10^3 t^2$$

Find the growth rates at

(a) $t = 0$ hours

(b) $t = 5$ hours

(c) $t = 10$ hours

Exercise 15: A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s = 24t - 0.8t^2$ m in t sec. Write a program to

(a) Find the rock's velocity and acceleration at time t.

(b) How long does it take the rock to reach its highest point?

(c) How high does the rock go?

Exercise 16: Write a program to implement Newton algorithm, find the approximation of the root function. Perform Newton-Raphson method by

(a) $f(x) = 2x^3 + 3x - 1$ with starting interval $p_0 = 2$ and a tolerance $\epsilon = 10^{-8}$. Then, put the results in a table and plot the graph.

(b) $f(x) = x^3 - 4$, perform 3 iterations with starting point $p_0 = 2$. Then, put the results in a table and plot the graph.

Algorithm 1 Newton Method

Let $f : R \longrightarrow R$ be a differentiable function. The following algorithm computes an approximate solution x^* to the equation $f(x) = 0$

Choose an initial guess x_0

for $k = 1, 2, 3, ...$ **do**
  if $f(x_k)$ is sufficiently small **then**
    $x^* = x_k$
    return x^*
  end
  $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$
  if $\mid x_{k+1} - x_k \mid$ is sufficiently small **then**
    $x^* = x_{k+1}$
    return x^*
  end
end