Data Structures and Algorithms

Census Problem

Acknowledgement

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- We greatly appreciate support from Dr. Steven Halim for kindly sharing these materials.

Policies for students

- These contents are only used for students PERSONALLY.
- Students are NOT allowed to modify or deliver these contents to anywhere or anyone for any purpose.

Recording of modifications

- Currently, there are no modification on these contents.

Outline

Motivation: Census Problem

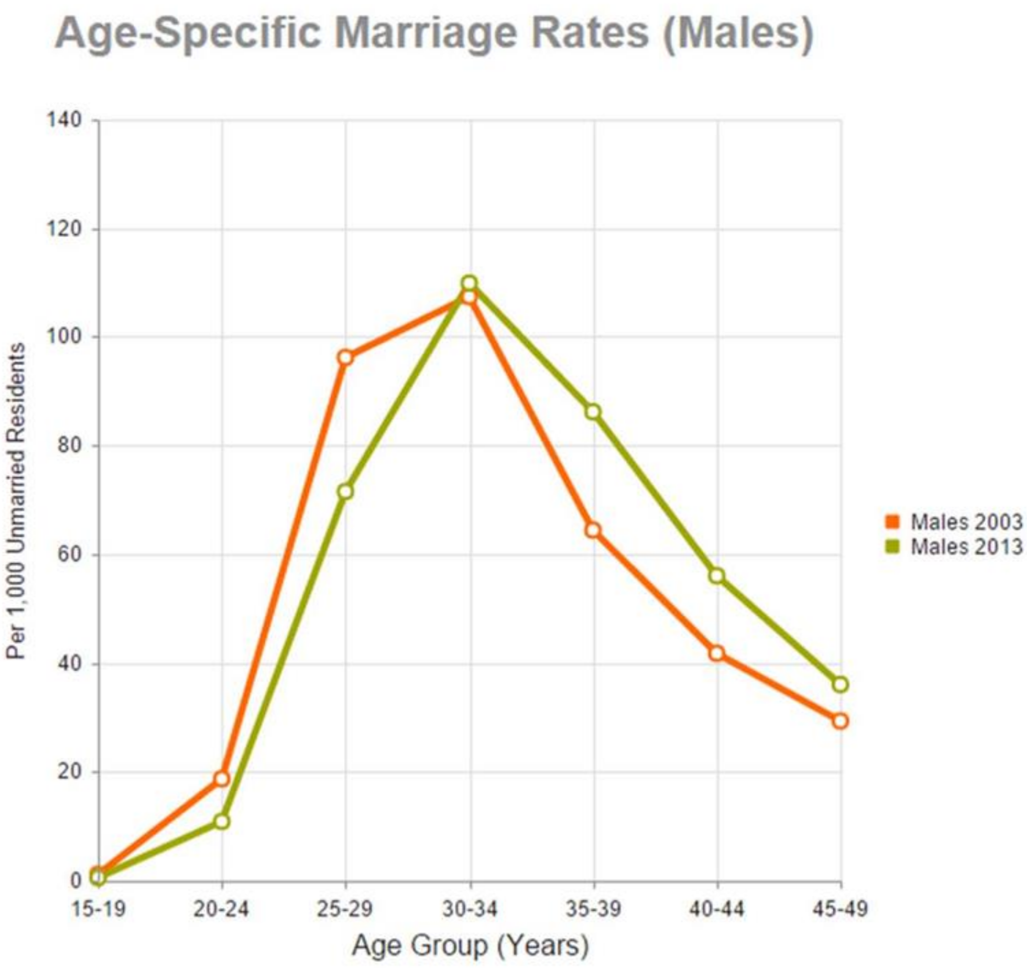
- Abstract Data Type (ADT) Table
- Solving Census Problem with CS1020 Knowledge
- The “performance issue”

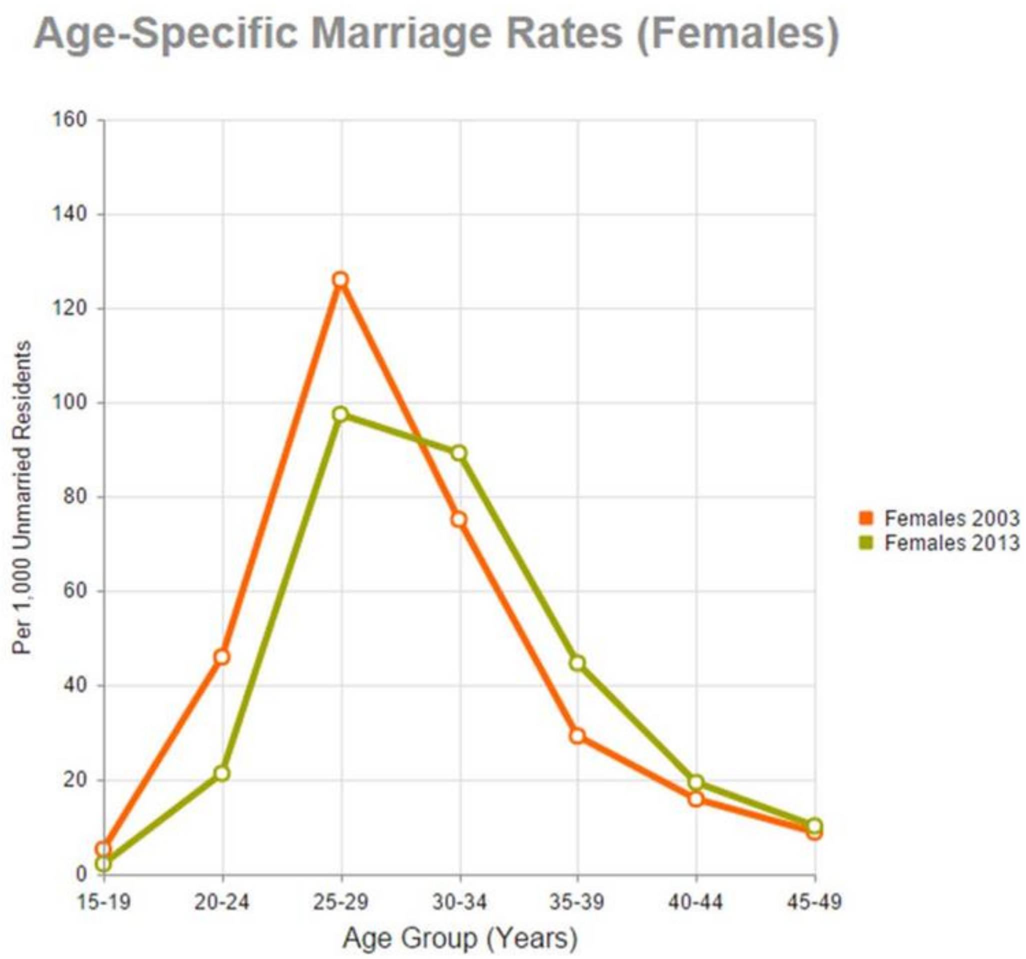
Binary Search Tree (BST)

- Heavy usage of VisuAlgo Binary Search Tree Visualization
- Simple analysis of BST operations
- Java Implementation

PS2 Preview

Census is Important!





Source:
http://www.singstat.gov.sg

Sun Tzu's Art of War

Chapter 1 "The Calculations"

知彼知己百战不殆

zhī bǐ zhī jǐ bǎi zhàn bù dài

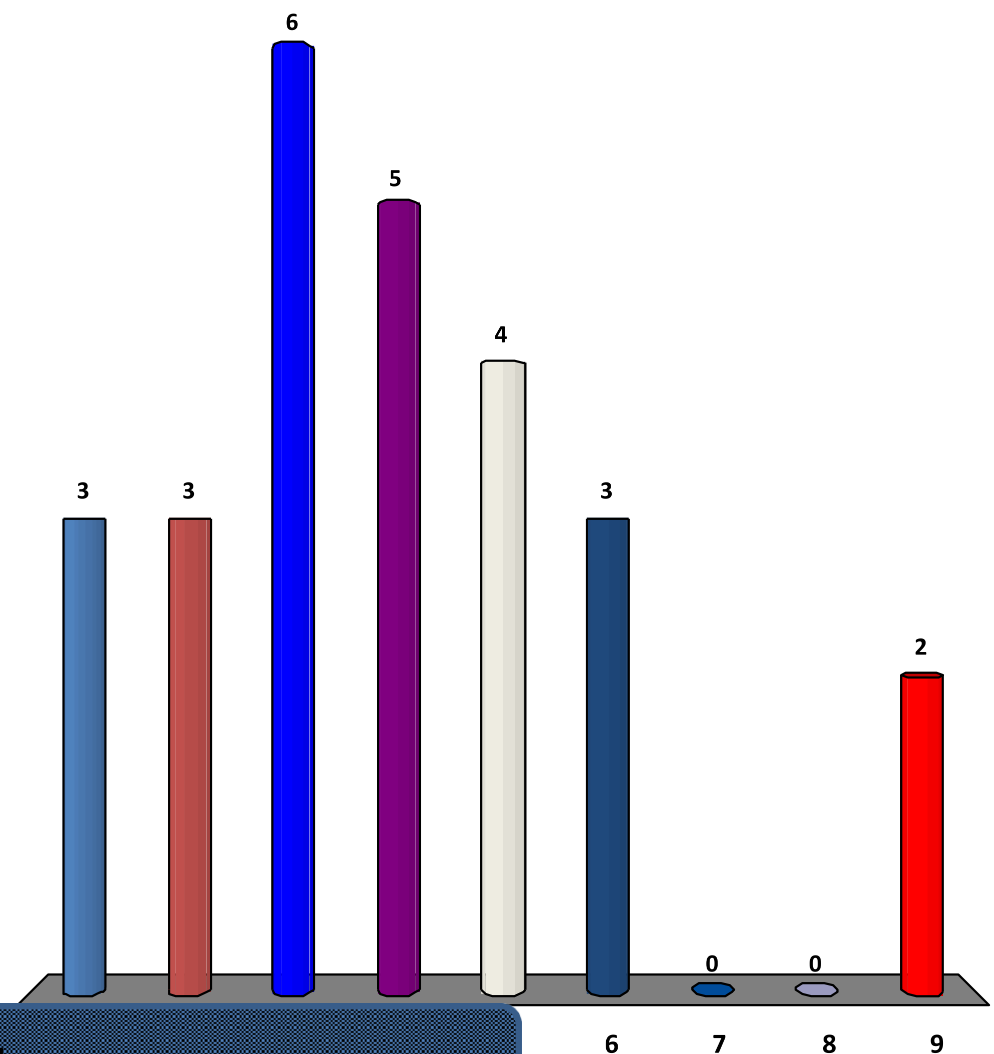
(If you know your enemies and know yourself, you will not be imperiled in a hundred battles)

Your Age (2013 data)

'[' (or ']') means that endpoint is included (closed)

1. [24 … ∞)
2. [23 … 24)
3. [22 … 23)
4. [21 … 22)
5. [20 … 21)
6. [19 … 20)

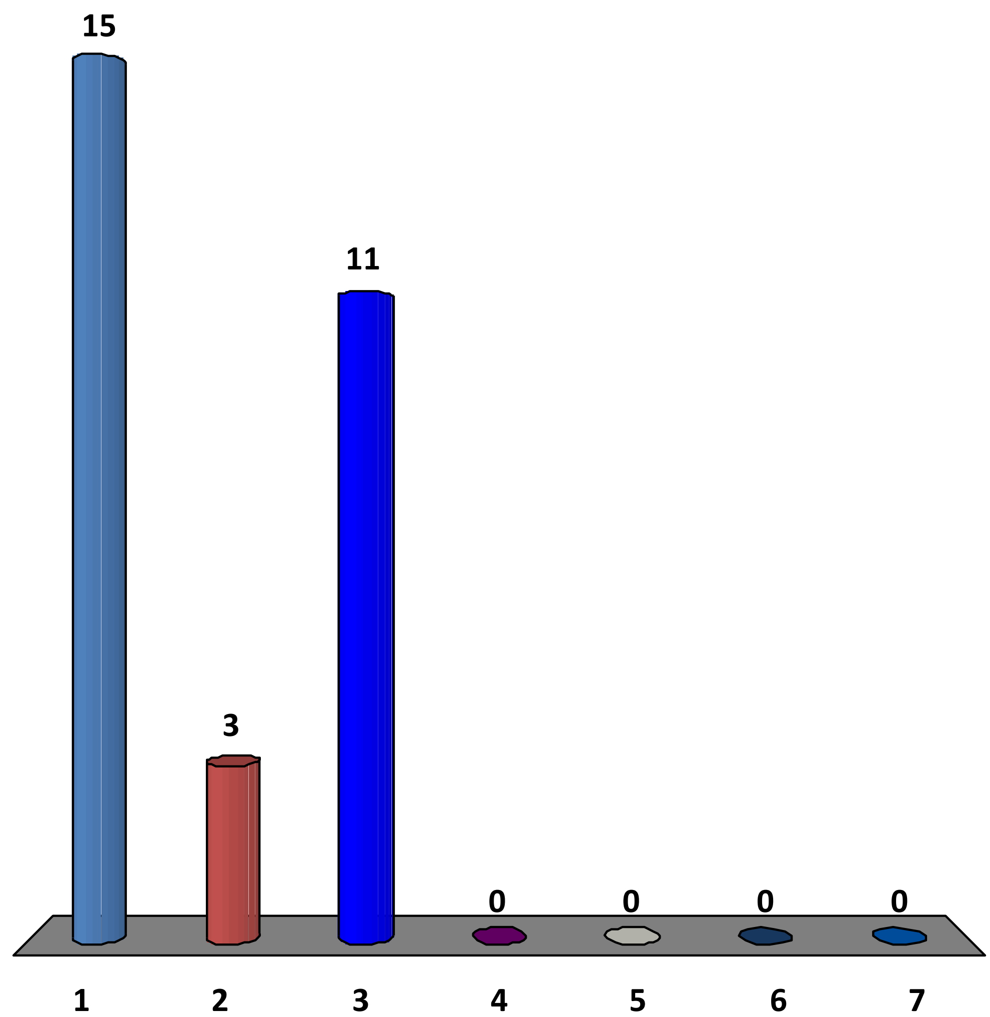
'(' (or ')') means that endpoint is **not** included (open)



Mean = 3.96

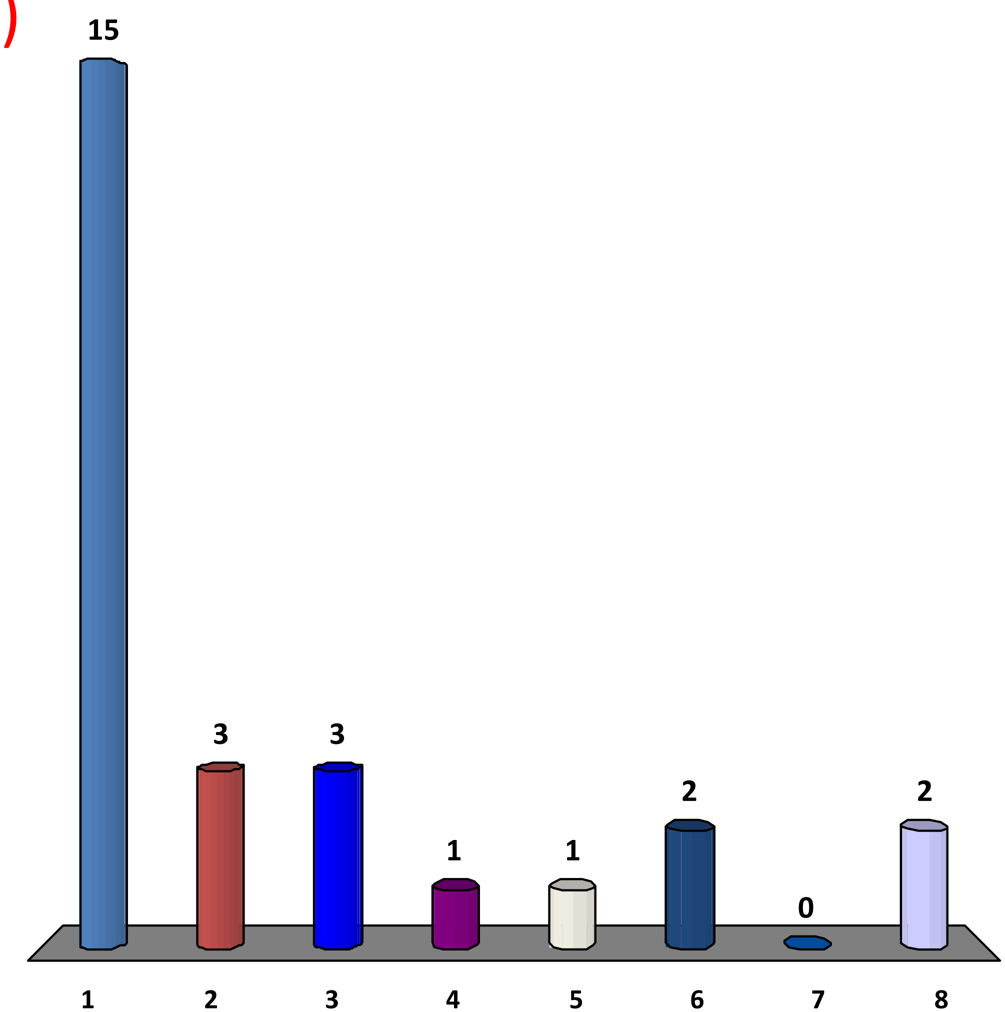
Your Major (2013 data)

1. Computer Science (CS)
2. ~~Communications and Media (C&M)~~
3. Computer Engineering (CEG/CEC)
4. Comp. Biology (CB)
5. Information System (IS)
6. Science Maths (SCI)
7. None of the above :O



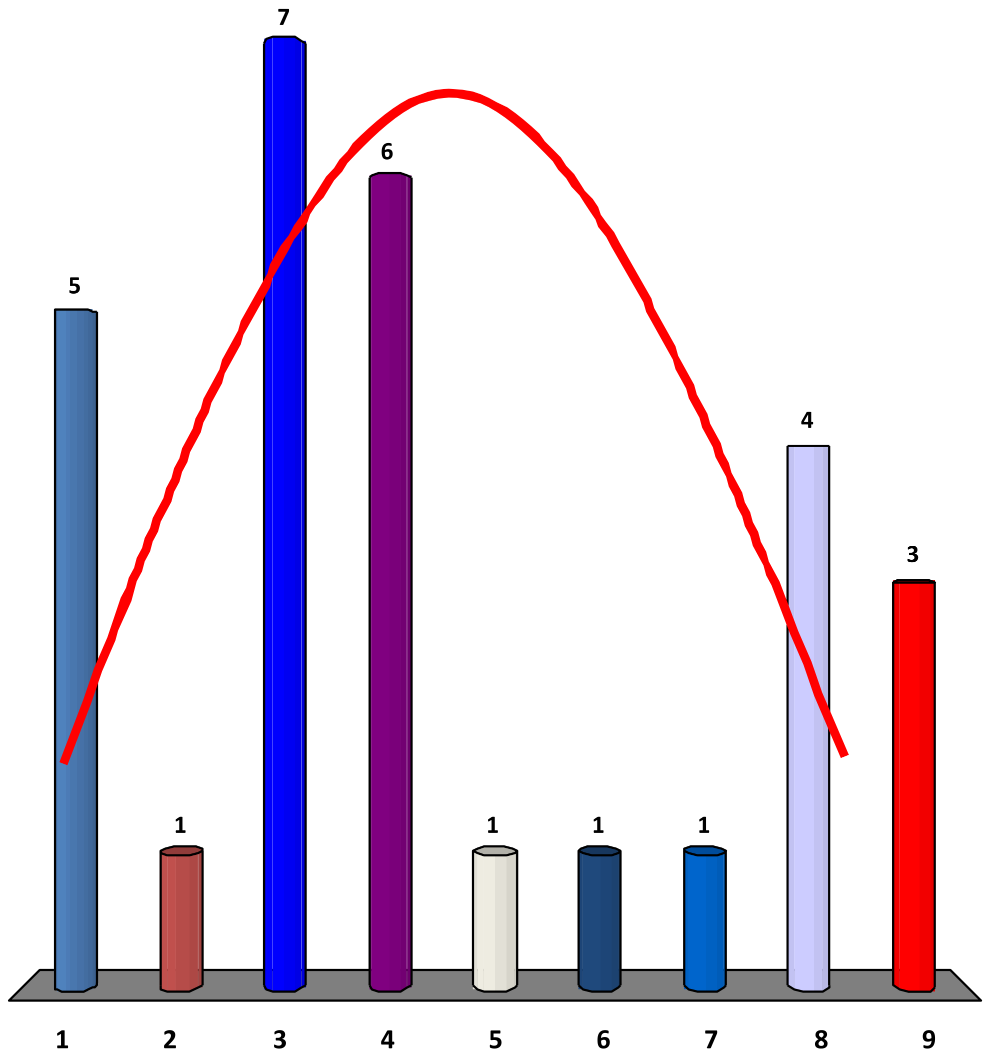
Your Nationality (2013 data)

1. Singaporean (should be ≥70% according to MOE rules)
2. Chinese
3. Indian
4. Indonesian
5. Vietnamese
6. Malaysian
7. European
8. None of the above



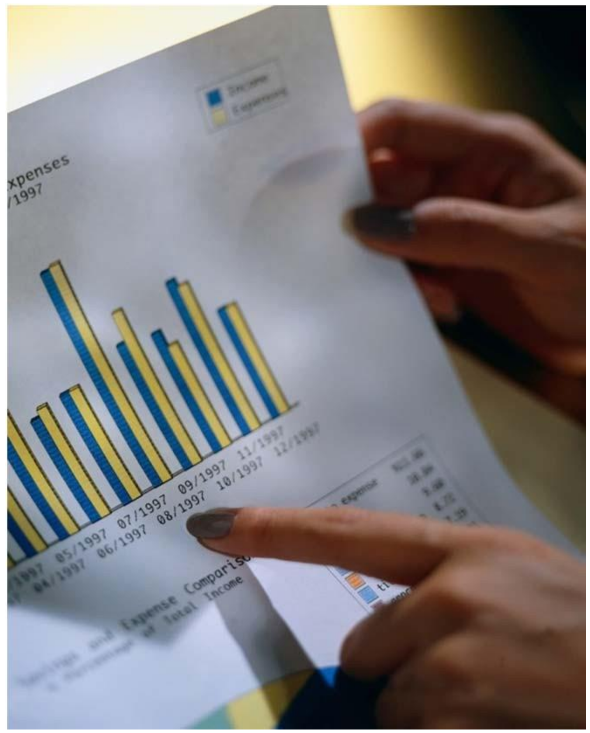
Your CAP (2013 data)

1. [4.5 … 5.0]
2. [4.25 … 4.5)
3. [4.0 … 4.25)
4. [3.75 … 4.0)
5. [3.5 … 3.75)
6. [3.25 … 3.5)
7. [3.0 … 3.25)
8. [0.0 … 3.00)
9. I do not want to tell



What Happen After Census?

Data
Mining



Statistica
l
Analysis

Abstract Data Type (ADT) Table

Let’s deal with one aspect of our census: **Age**

To simplify this lecture, we assume that students’ age ranges from [0 … 100), all integers, and distinct

Required operations:

1. Search whether there is a student with a certain age?
2. Insert a new student (that is, insert his/her age)
3. Determine the youngest and oldest student
4. List down the ages of students in sorted order
5. Find a student slightly older than a certain age!
6. Delete existing student (that is, remove his/her age)
7. Determine the median age of students
8. How many students are younger than a certain age?

CS1020: Unsorted Array

Index	0	1	2	3	4	5	6	7	
A	5	7	71	50	23	4	6	15	

No	Operation	Time Complexity
1	Search(age)	O(n)
2	Insert(age)	O(1)
3	FindOldest()	O(n)
4	ListSortedAges()	O(n log n)
5	NextOlder(age)	O(n)
6	Remove(age)	O(n)
7	GetMedian()	O(n log n)/O(n)
8	NumYounger(age)	O(n log n)

CS1020: Sorted Array

Index	0	1	2	3	4	5	6	7	
A	4	5	6	7	15	23	50	71	

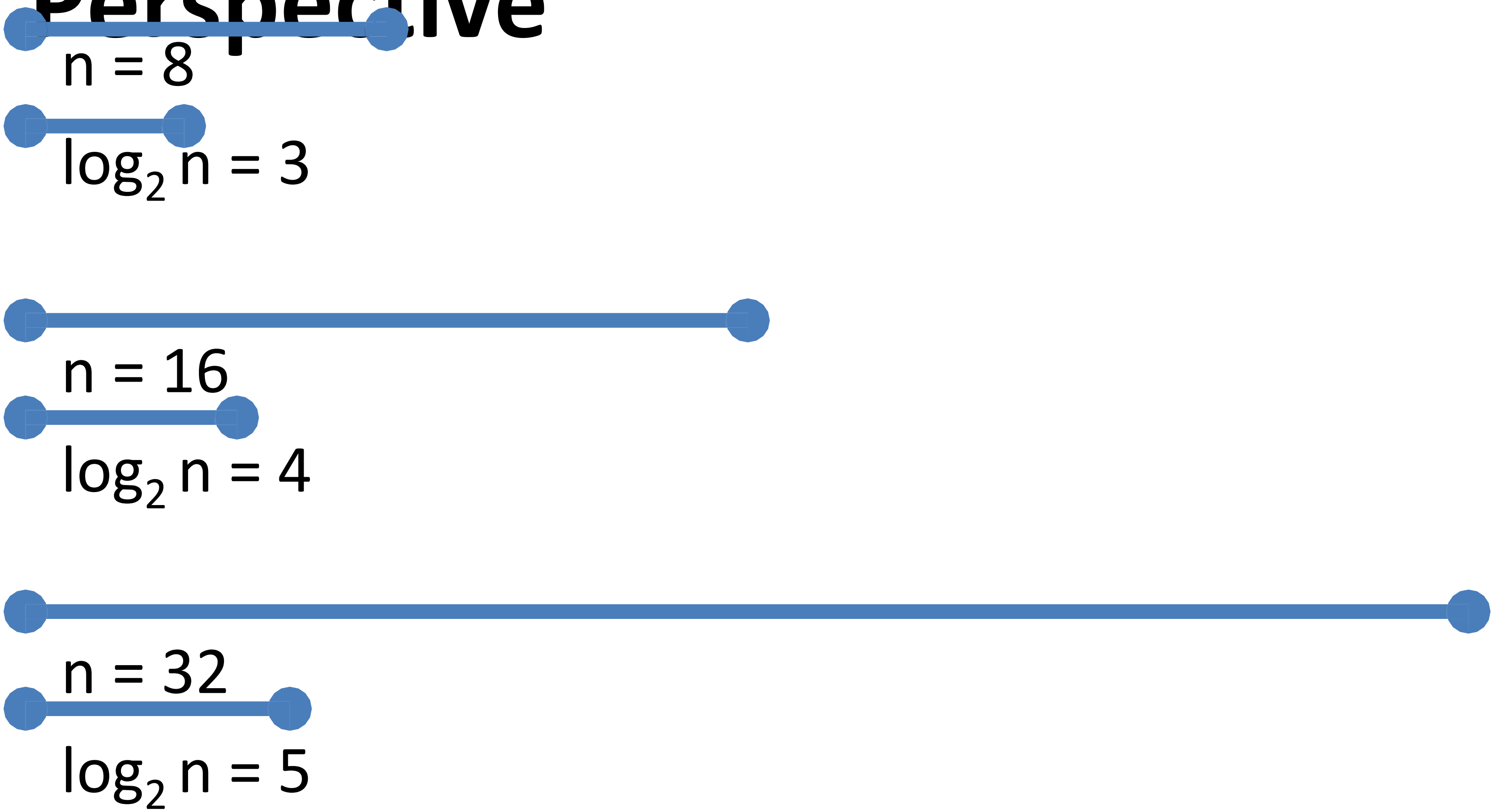
No	Operation	Time Complexity
1	Search(age)	O(log n)
2	Insert(age)	O(n)
3	FindOldest()	O(1)
4	ListSortedAges()	O(n)
5	NextOlder(age)	O(log n)
6	Remove(age)	O(n)
7	GetMedian()	O(1)
8	NumYounger(age)	O(log n)

With Just CS1020 Knowledge

No	Operation	Unsorted Array	Sorted Array
1	Search(age)	O(n)	O(log n)
2	Insert(age)	O(1)	O(n)
3	FindOldest()	O(n)	O(1)
4	ListSortedAge	(n log n)	O(n)
5	NextOlder(ag	O(n)	O(log n)
6	Remove(age)	O(n)	O(n)
7	GetMedian()	O(n log n) / O(n)	O(1)
8	NumYounger(age)	O(n log n)	O(log n)

Dynamic data s struc ture operations

O(n) versus O(log n): A Perspective



Try larger n, e.g. n = 1000000…

A Versatile, Non-Linear Data Structure

BINARY SEARCH TREE (BST)

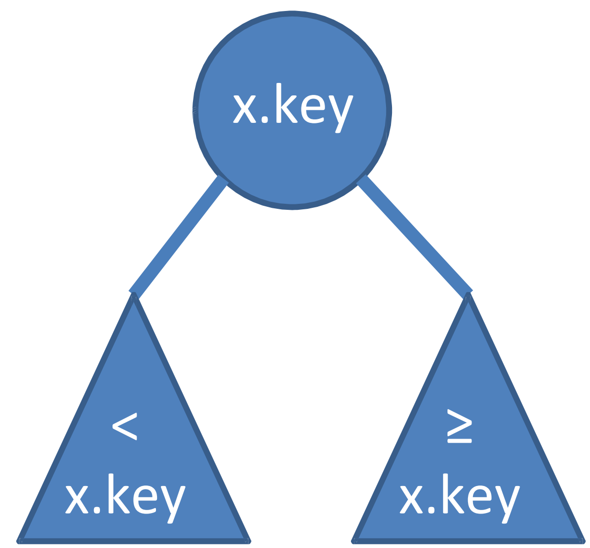
Binary Search Tree (BST) Vertex

For every vertex x, we define:

- x.left = the left child of x
- x.right = the right child of x
- x.parent = the parent of x
- x.key (or x.value, x.data) = the value stored at x

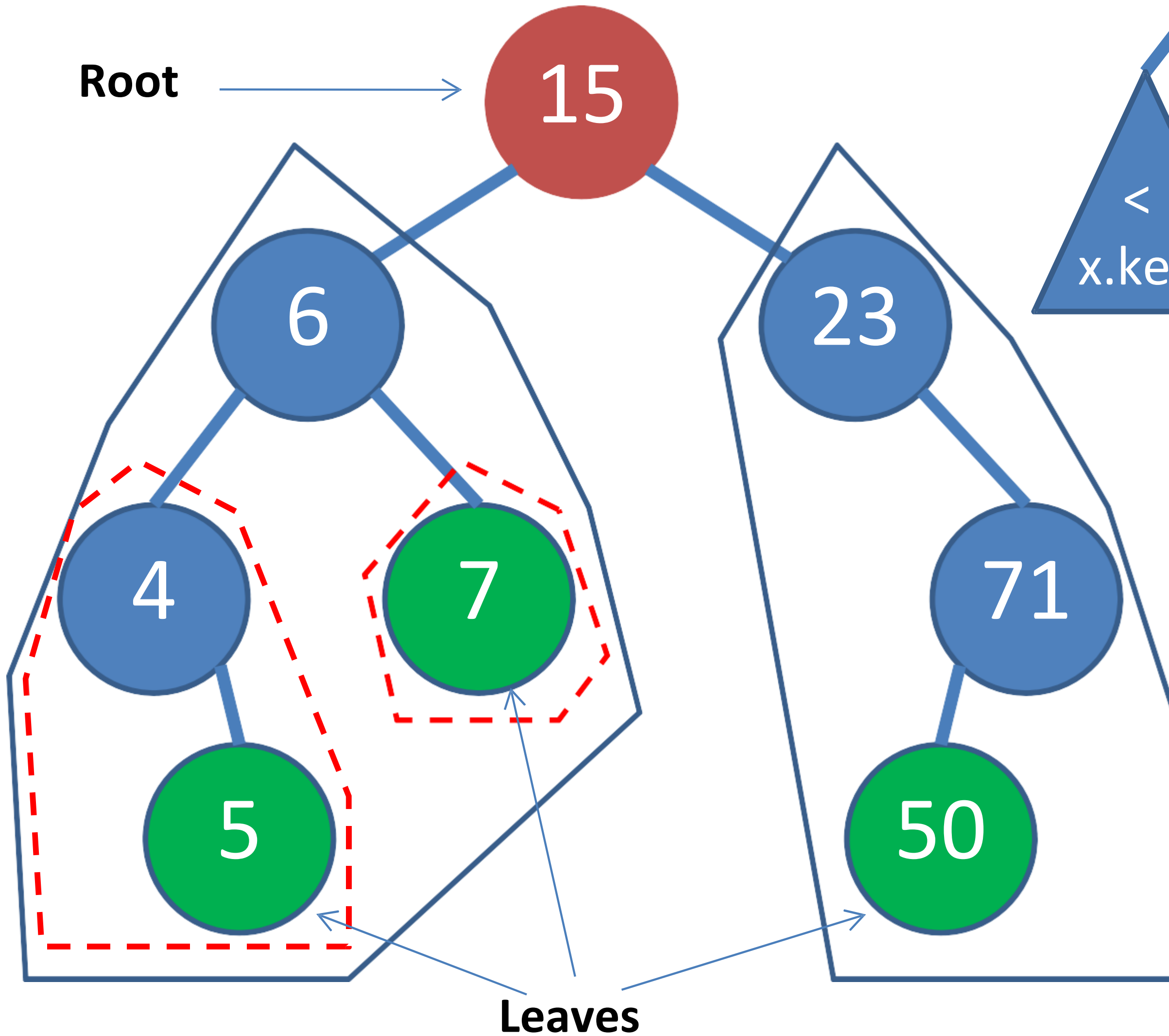
BST Property:

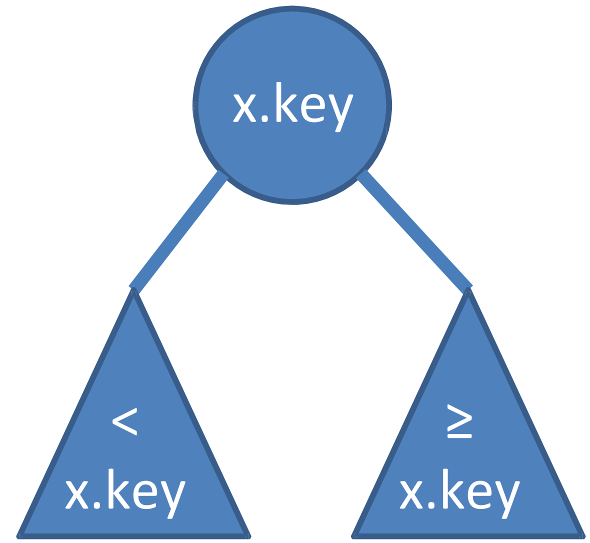
- x.left.key < x.key ≤ x.right.key
- For simplicity, we assume that the keys are unique
  so that we can change ≥ to >



BST: An Example, Keys = Ages

Recursive definition



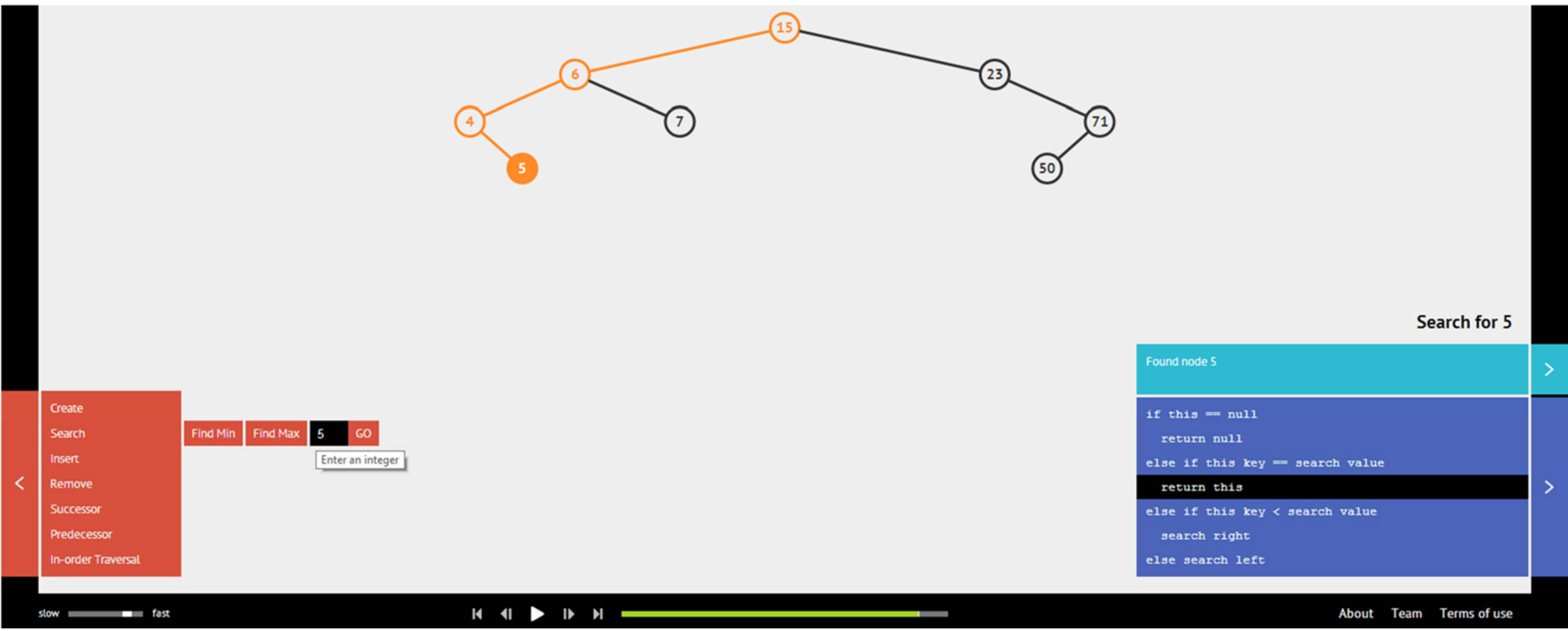


All other vertices other than 15, 5, 7, and 50 are **Internal vertices**

BST: Search/Min/Max Operations

Ask VisuAlgo to perform various search operations on the sample BST, including find min and find max

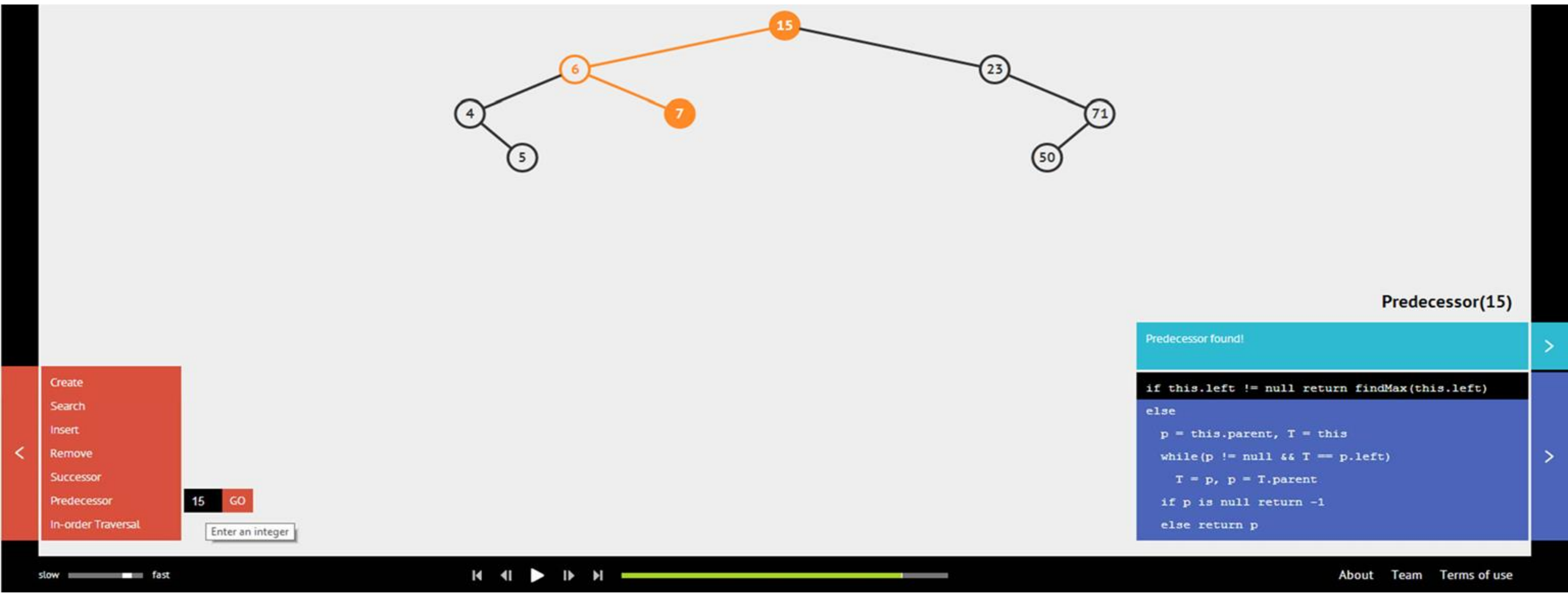
In the screen shot below, we show **search(5)**



BST: Succ/Predec-essor Operations

Ask VisuAlgo to perform Succ/Pred operations on the sample BST

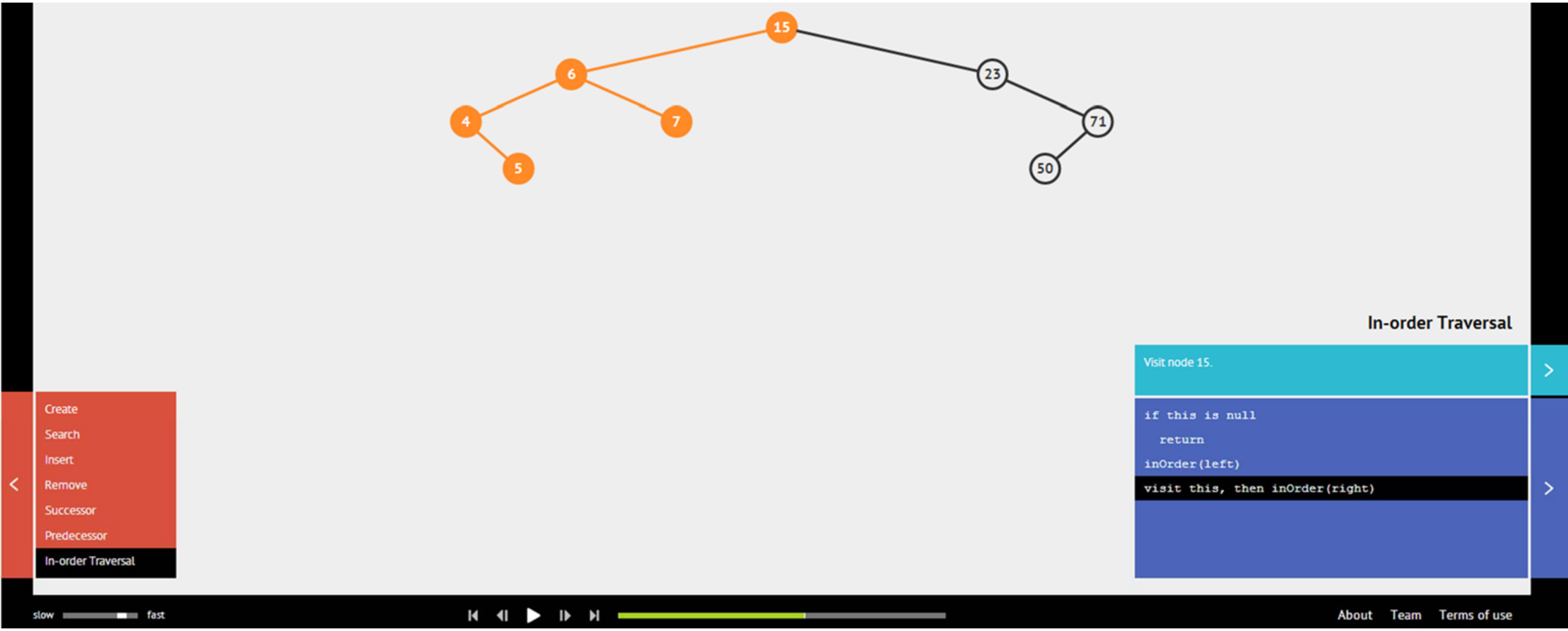
In the screen shot below, we show **pred(15)**



BST: Inorder Traversal Operation

Ask VisuAlgo to perform inorder traversal operation on the sample BST

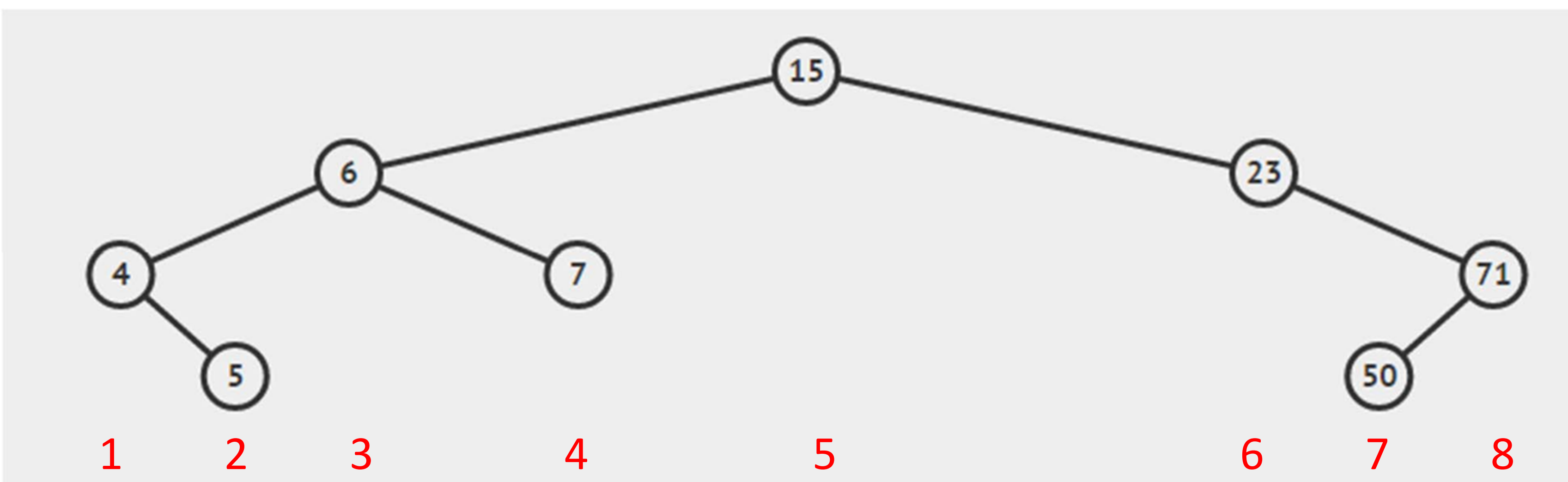
In the screen shot below, we *partial* inorder traversal



BST: Select/Rank Operations

These 2 operations will be added to VisuAlgo BST visualization *soon*; for now, here are the concepts:

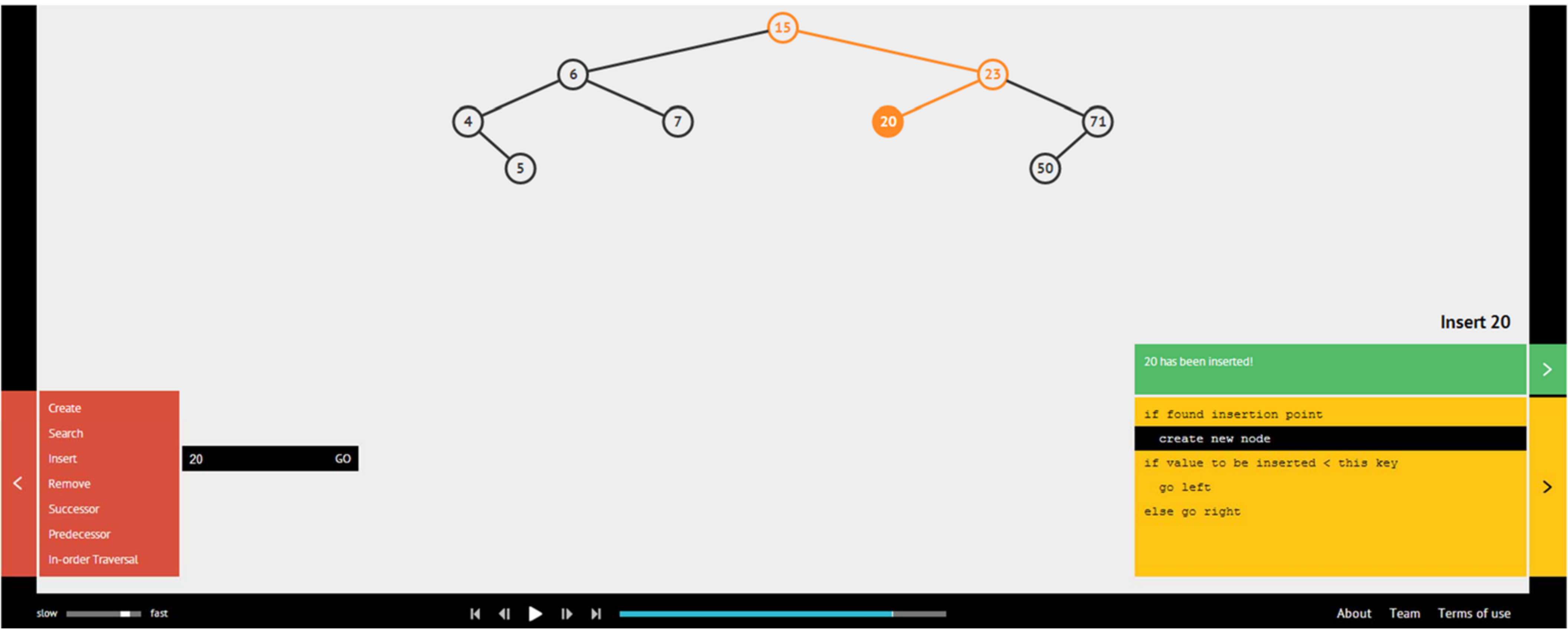
- Select(k) – Return the value v of k-th smallest* element
  - Examples: Select(1) = 4, Select(3) = 6, Select(8) = 71, etc (1-based index)
- Rank(v) – Return the ranking* k of element v
  - Examples: Rank(4) = 1, Rank(6) = 3, Rank(71) = 8, etc
- Details will be discussed in the next lecture



BST: Insert Operation

Ask VisuAlgo to perform various insert operations on the sample BST

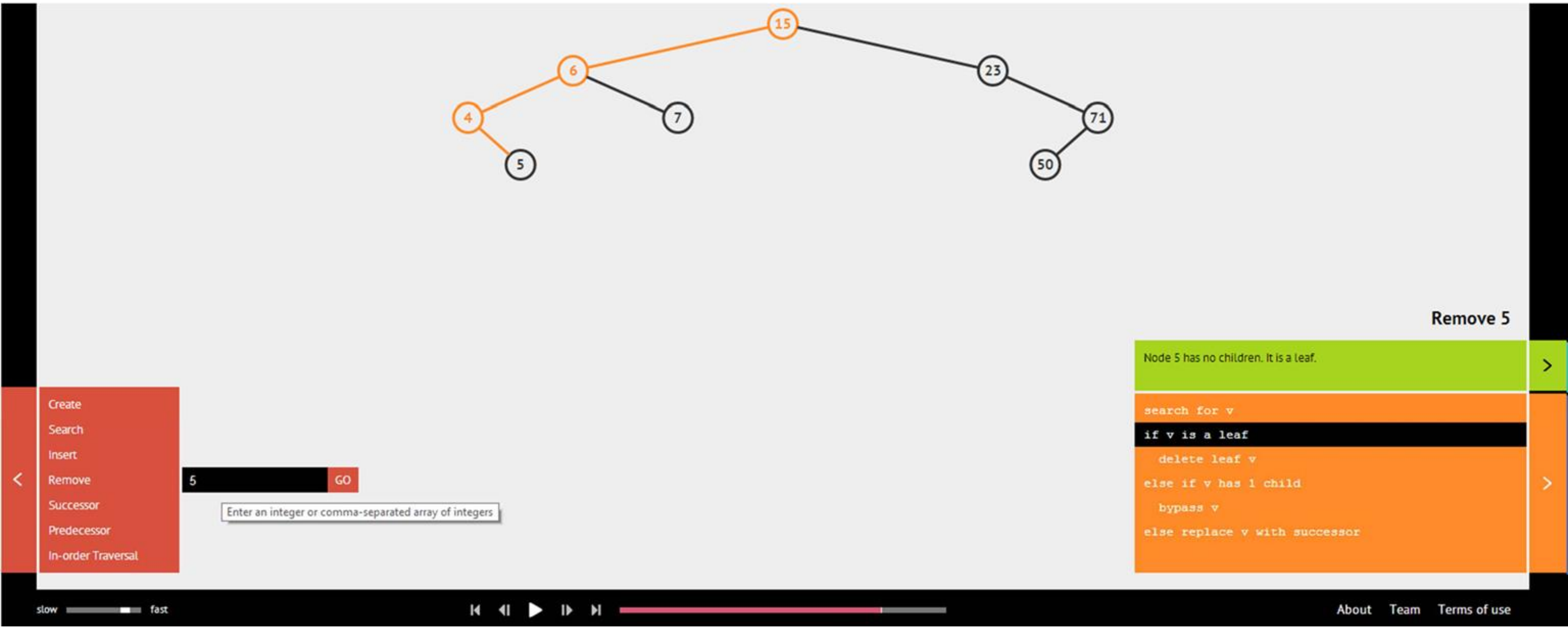
In the screen shot below, we show **insert(20)**



BST: Delete/Remove Operation (1)

Ask VisuAlgo to perform various delete operations on the sample BST (3 cases, this is **delete leaf**)

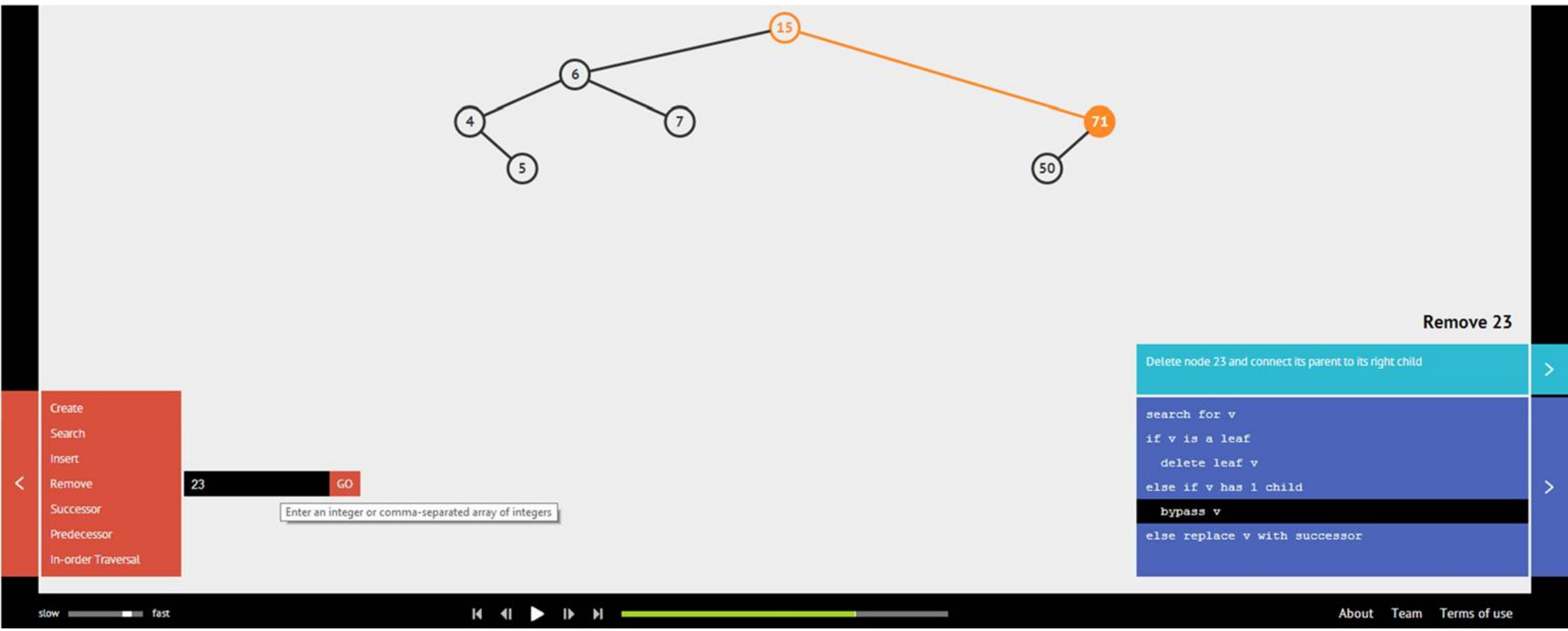
In the screen shot below, we show **remove(5)** before deletion



BST: Delete/Remove Operation (2)

Ask VisuAlgo to perform various delete operations on the sample BST (this is **delete vertex with one child**)

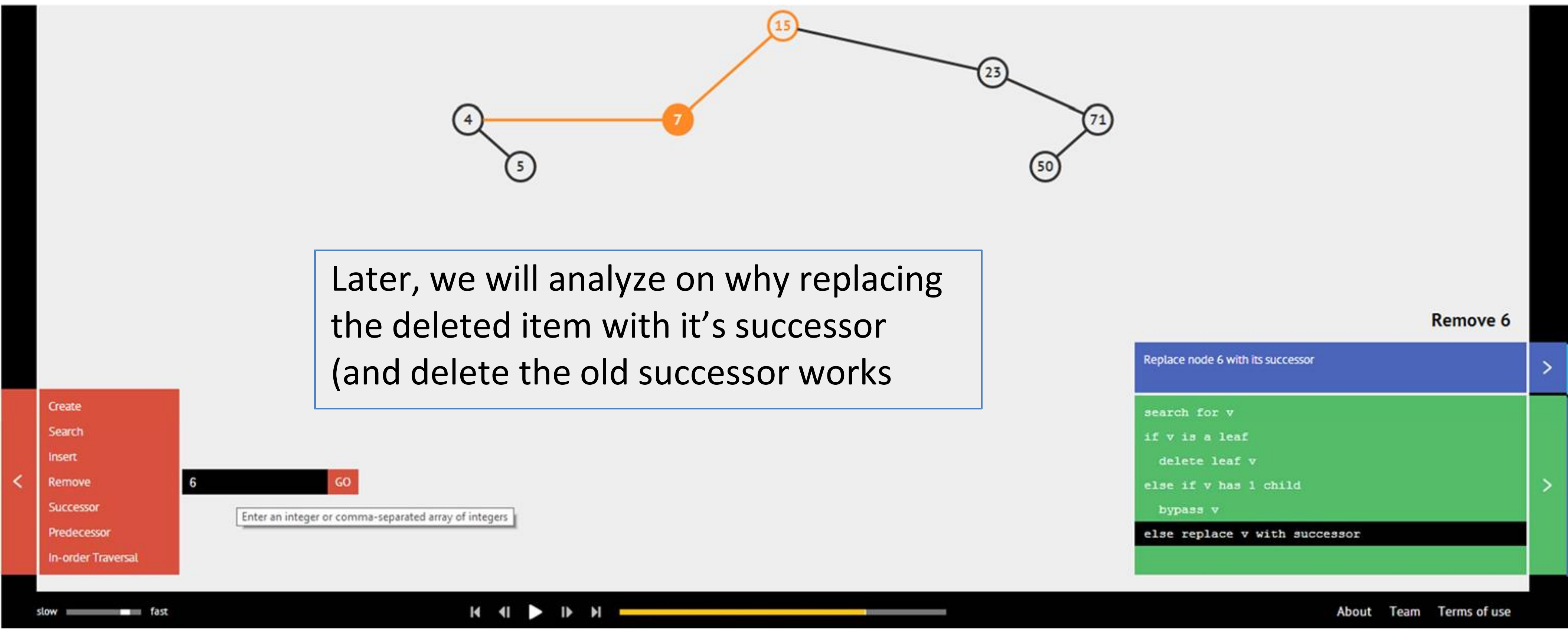
In the screen shot below, we show **remove(23)** before relayout



BST: Delete/Remove Operation (3)

Ask VisuAlgo to perform various delete operations on the sample BST (delete **vertex with two children**)

In the screen shot below, we show **remove(6)** before relayout



Later, we will analyze on why replacing the deleted item with it's successor (and delete the old successor works

ANALYSIS OF BST OPERATIONS

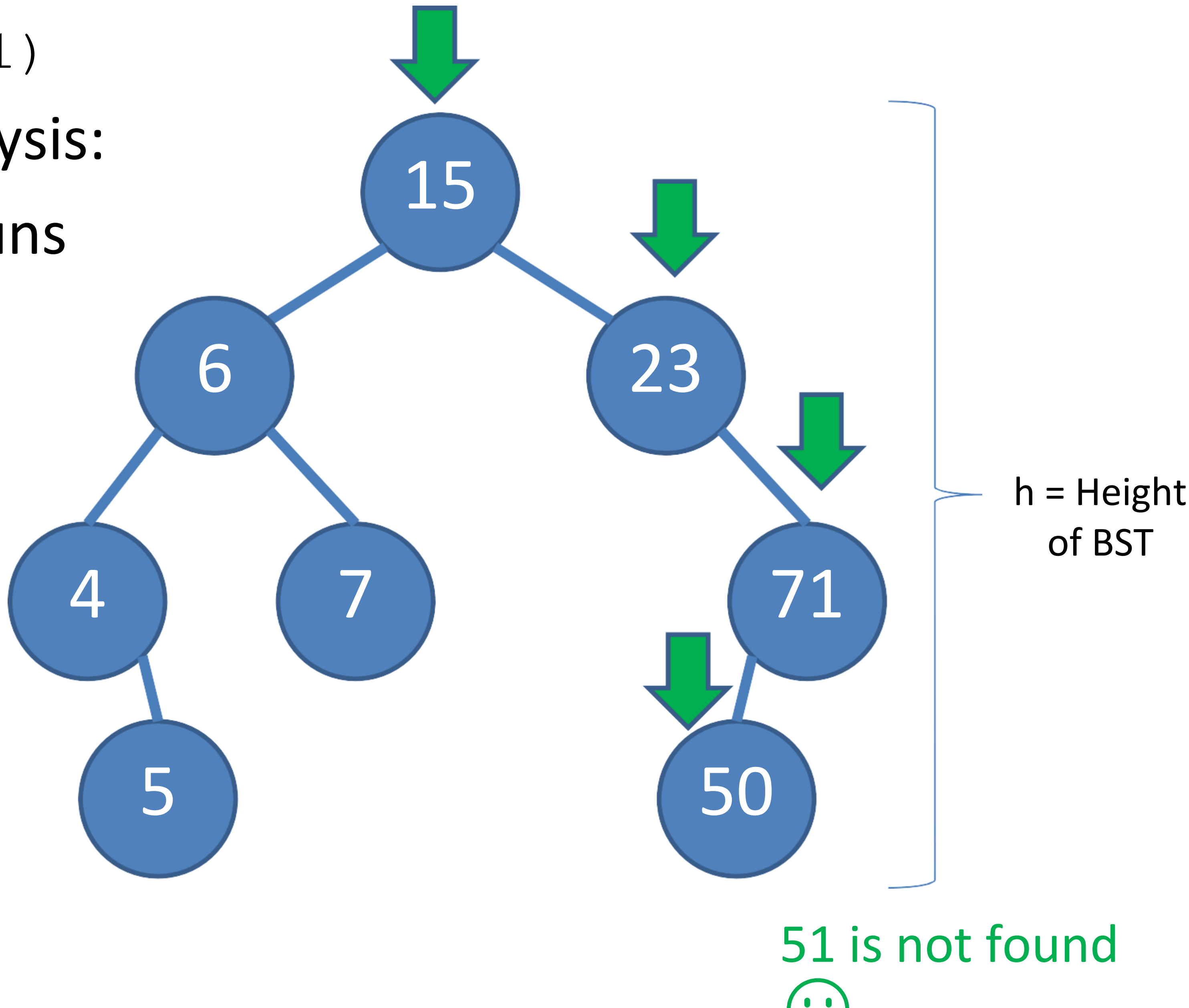
BST: Search Analysis

`search(51)`

Quick analysis:

`search` runs

in **O(h)**

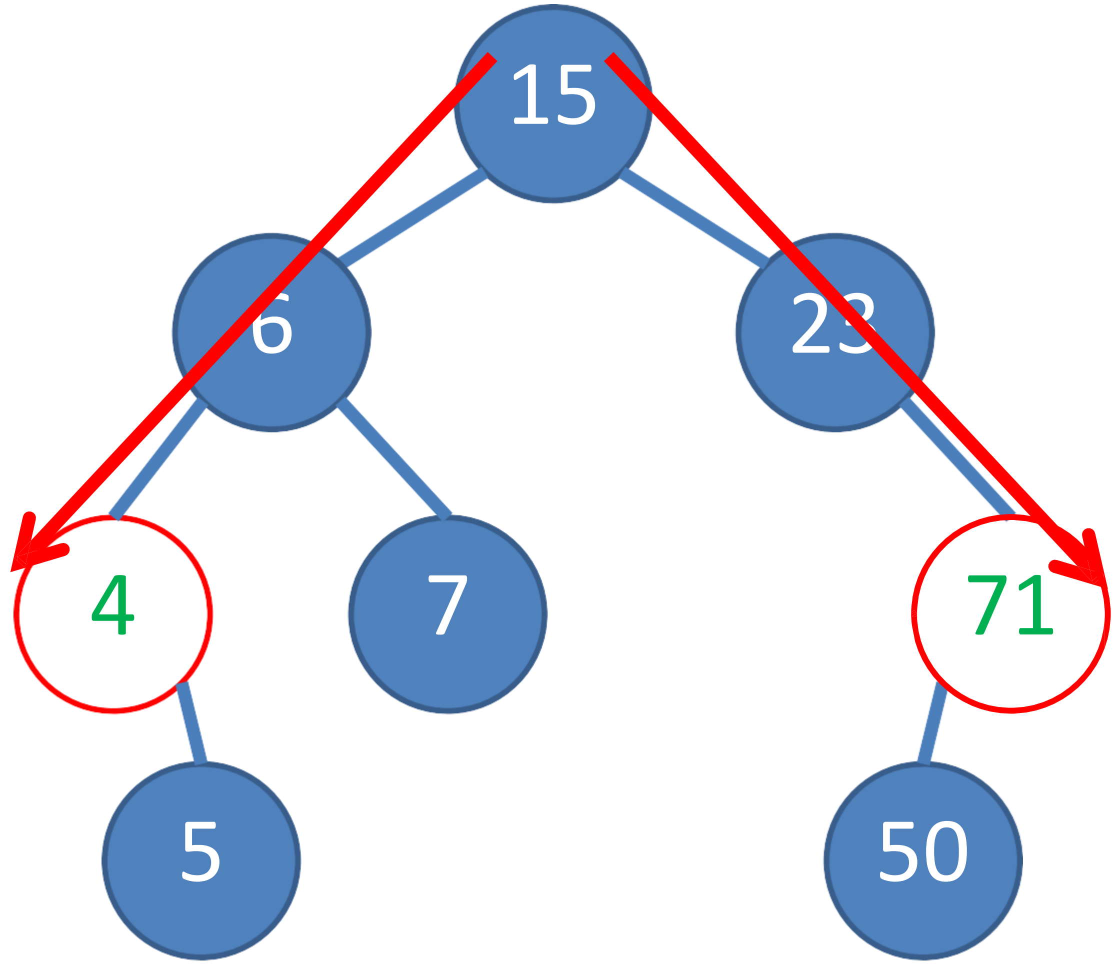


BST: Find Min/Max Analysis

Quick analysis:

`findMin/findMax`

also runs in **O(h)**



BST: Successor/Predecessor Analysis

Assumption, we already done an O(h) search(71) before

`successor(71)`

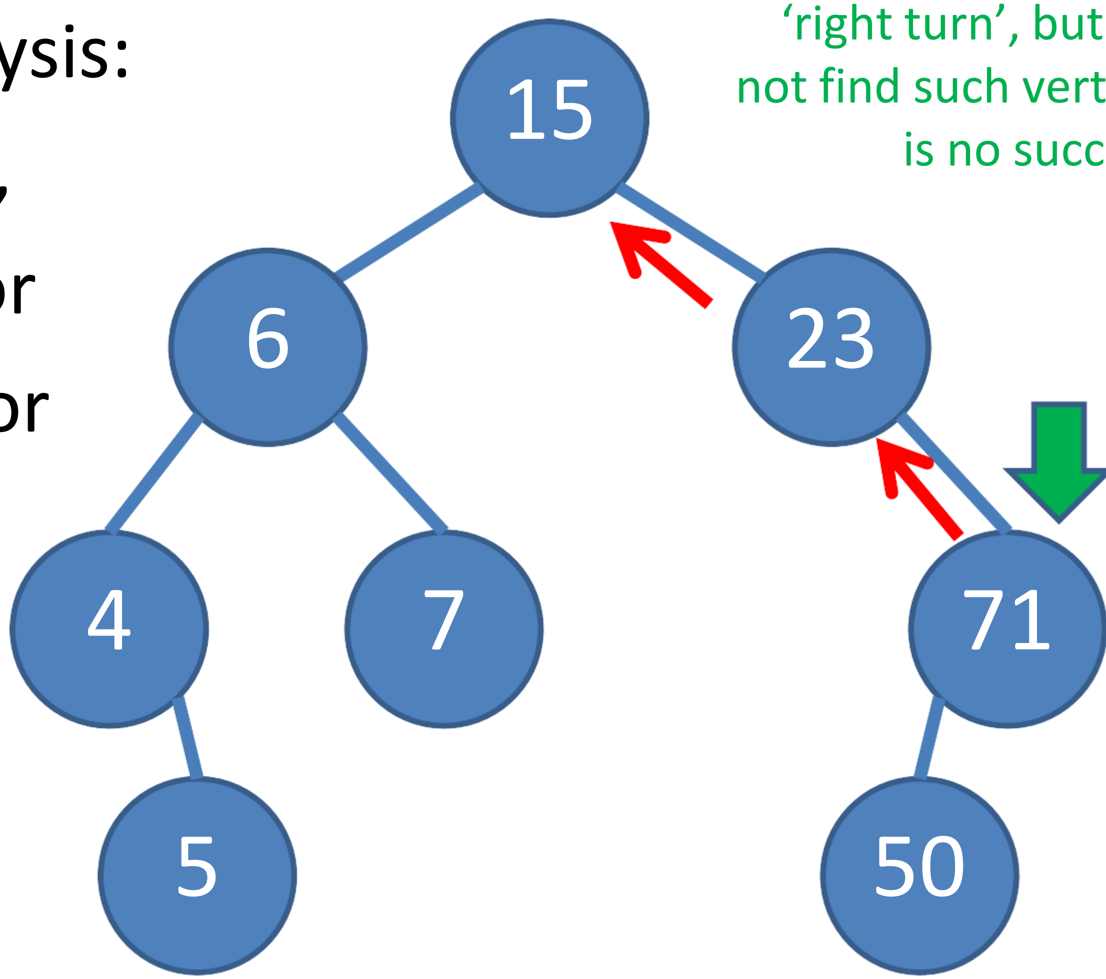
Quick analysis:

O(h) again,

similarly for

predecessor

Keep going up until we make a 'right turn', but here we do not find such vertex, so there is no successor for 71



No right child

BST: Inorder Traversal Analysis

Using a *new* analysis technique

Ask this question:

- How many times a vertex is *touched* during inorder traversal from the start until the end?

Answer:

- Three times: from parent and from left + right children (even if one or both of them is/are empty/NULL)
- $O(3n) = O(n)$

BST: Select/Rank Analysis

We have not explored the operations in detail yet

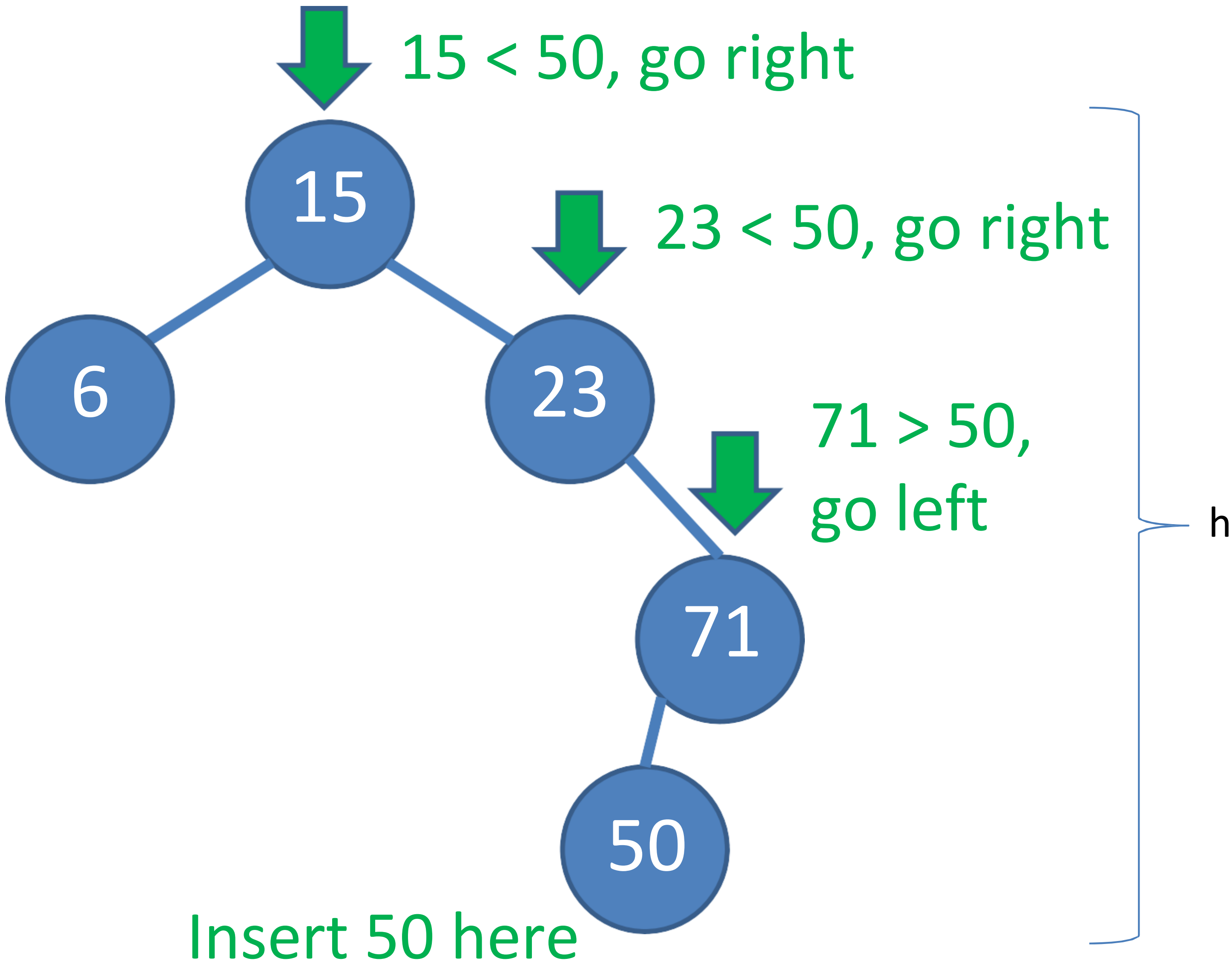
This will be discussed in more details in the next lecture

BST: Insertion Analysis

`insert(50)`

Quick analysis:

`insert` also runs in **O(h)**



Why successor of x can be used for deletion of a BST vertex x with 2 children?

Claim: Successor of **x** has at most 1 child!

- Easier to delete and will not violate BST property

Proof:

- Vertex **x** has two children
- Therefore, vertex **x** must have **a right child**
- Successor of **x** must then be the minimum of the right subtree
- A minimum element of a BST has no left child!!
- *So, successor of **x** has at most 1 child!* ☺

BST: Deletion Analysis

Delete a BST vertex **v**, find **v** in O(**h**), then three cases:

- Vertex **v** has no children:
  - Just remove the corresponding BST vertex **v** → O(1)
- Vertex **v** has 1 child (either left or right):
  - Connect **v**.left (or **v**.right) to **v**.parent and vice versa → O(1)
  - Then remove v → O(1)
- Vertex **v** has 2 children:
  - Find **x** = successor(**v**) → O(**h**)
  - Replace **v**.key with **x**.key → O(1)
  - Then delete **x** in **v**.right (otherwise we have duplicate) → O(**h**)

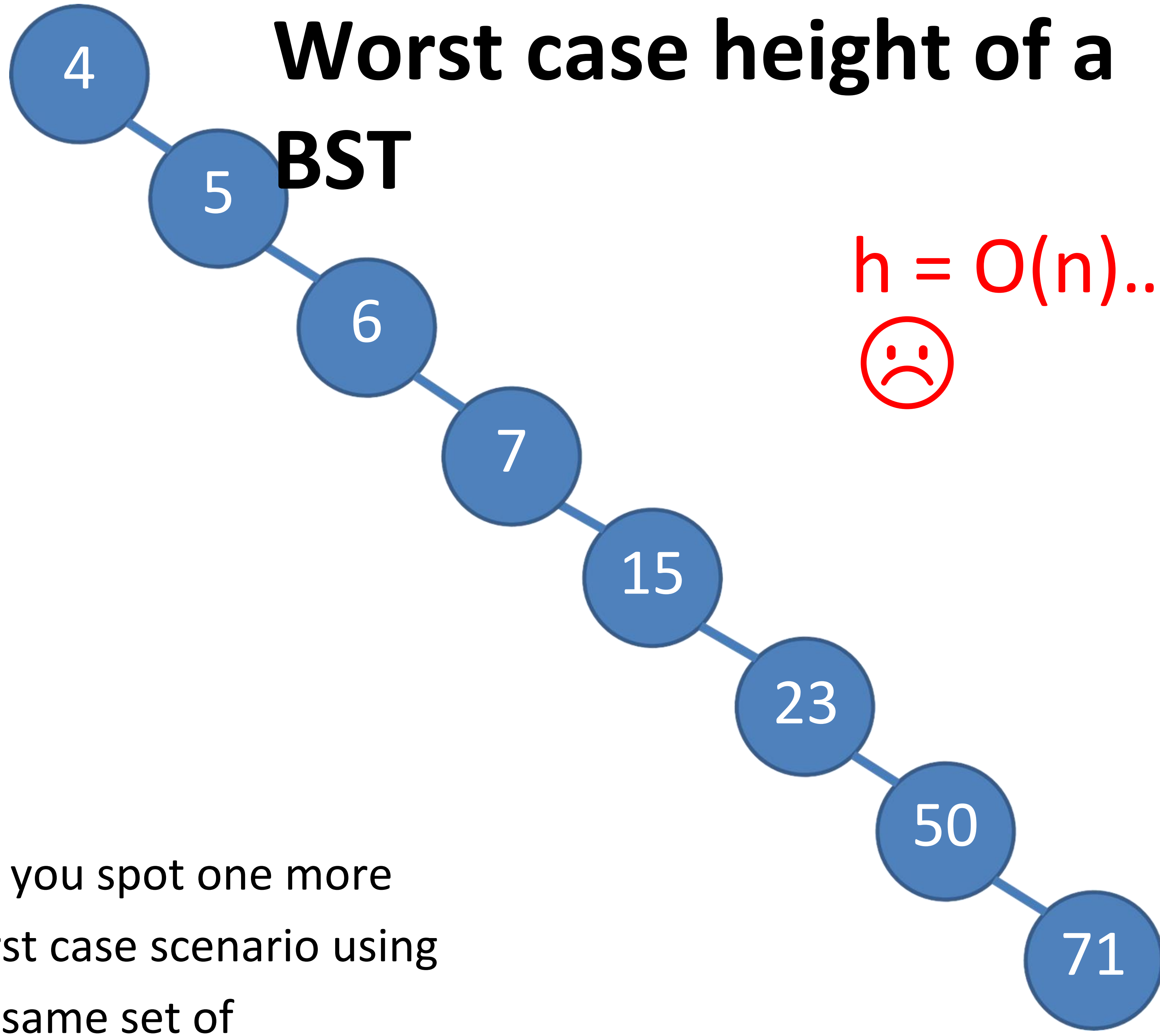
Running time: O(**h**)

Now, after we learn BST…

No	Operation	Unsorted Array	Sorted Array	BST
1	Search(age)	O(n)	O(log n)	**O(h)**
2	Insert(age)	O(1)	O(n)	**O(h)**
3	FindOldest()	O(n)	O(1)	**O(h)**
4	ListSortedAges()	O(n log n)	O(n)	O(n)
5	NextOlder(age)	O(n)	O(log n)	**O(h)**
6	Remove(age)	O(n)	O(n)	**O(h)**
7	GetMedian()	O(n log n)	O(1)	**O(h)**
8	Rank(age)	O(n log n)	O(log n)	**?**

It is all now depends on **‘h’**… next lecture

Worst case height of a BST



h = O(n)… ☹

Can you spot one more worst case scenario using the same set of numbers?

Java Implementation

See BSTDemo.java (you can use this for PS2)

Concepts covered:

1. Java Object Oriented Programming (OOP) implementation of BST data structure
2. Java Error Handling: Throw & Catch Exception

The Baby Names Problem (PS2)

Given a list of male and female baby names suggestions *(from your parents, in-laws, friends, yourself, Internet,* ***etc****), your task is to* answer some queries (see the next slide)

This problem is always encountered by every parents with new baby

(Including the search for baby Joshua name, born on 16 July 2014)

PS2 Queries

(Note: Unlike this lecture with integer keys, the keys in PS1 are strings)

Easy: How many names start with a certain letter?

Medium: How many names start with a certain prefix?

Definition: A prefix of a string $T = T_0T_1 \ldots T_{n-1}$ *with length* n *is string* $P = T_0T_1 \ldots T_m$ *where* $m < n$.

Hard: Can you do it without Java API library code?

CS2010R: How many names have a certain substring?

Definition: A substring of a string $T = T_0T_1 \ldots T_{n-1}$ with length n *is string* $S = T_iT_{i+1} \ldots T_{j-1}T_j$ *where* $0 \leq i \leq j < n$.

You need efficient DS(es) to answer those queries

End of Lecture Quiz 😚

After Lecture 03, I will set a random test mode @ VisuAlgo to see if you understand BST

Go to:

[http://visualgo.net/**test.html**](http://visualgo.net/test.html)

Use your CS2010 account to try the 5 BST questions (medium difficulty, 5 minutes)

Meanwhile, train first 😚

[http://visualgo.net/**training.html**](http://visualgo.net/training.html)