



PROGRAMMING METHODOLOGY (PHƯƠNG PHÁP LẬP TRÌNH)

UNIT 10: Multidimensional Arrays

Acknowledgement

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Recording of modifications

- Currently, there are no modification on these contents.

Unit 10: Multidimensional Arrays

Objective:

- Understand the concept and application of multidimensional arrays

Reference:

- Chapter 6: Numeric Arrays

Unit 10: Multidimensional Arrays (1/2)

1. One-dimensional Arrays (review)

1.1 Print Array

1.2 Find Maximum Value

1.3 Sum Elements

1.4 Sum Alternate Elements

1.5 Sum Odd Elements

1.6 Sum Last 3 Elements

1.7 Minimum Pair Difference

1.8 Accessing 1D Array Elements in Function

Unit 10: Multidimensional Arrays (2/2)

2. Multi-dimensional Arrays

2.1 Initializers

2.2 Example

2.3 Accessing 2D Array Elements in Function

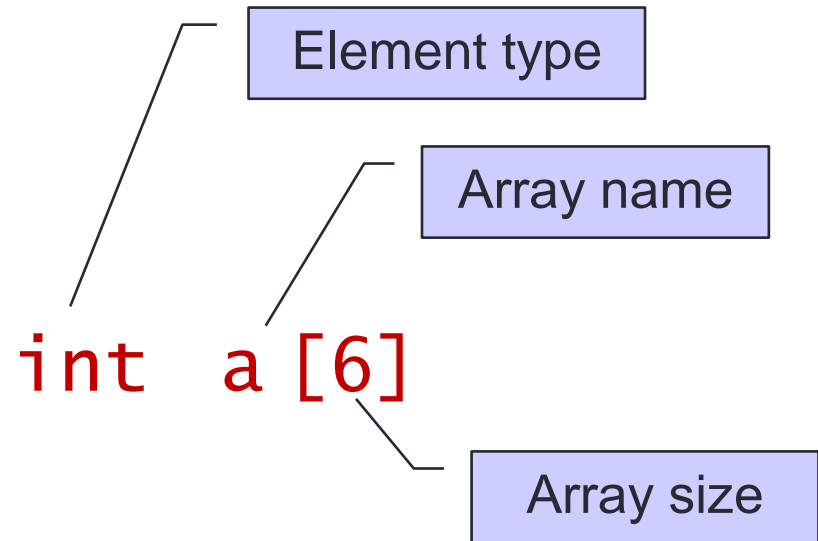
2.4 Class Enrolment

2.5 Matrix Addition

1. One-dimensional Arrays (1/2)

Array

- ▶ A collection of data, called elements, of homogeneous type



| <code>a[0]</code> | <code>a[1]</code> | <code>a[2]</code> | <code>a[3]</code> | <code>a[4]</code> | <code>a[5]</code> |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 20 | 12 | 25 | 8 | 36 | 9 |

1. One-dimensional Arrays (2/2)

- Preparing an array prior to processing:

Initialization (if values are known beforehand):

```
int main(void) {  
    int numbers[] = { 20, 12, 25, 8, 36, 9 };  
    ...  
    some_fn(numbers, 6);  
}
```

Or, read data into array:

```
int main(void) {  
    int numbers[6], i;  
    for (i = 0; i < 6; i++)  
        scanf("%d", &numbers[i]);  
    ...  
    some_fn(numbers, 6);  
}
```

1.1 Print Array

```
void printArray(int arr[], int size) {  
    int i;  
  
    for (i = 0; i < size; i++)  
        printf("%d ", arr[i]);  
    printf("\n");  
}
```

Calling:

```
int main(void) {  
    int numbers[6];  
    ...
```

```
    printArray(numbers, 6);  
    printArray(numbers, 3);  
}
```

Value must not
exceed actual
array size.

Print first 6 elements (all)

Print first 3 elements

1.2 Find Maximum Value

- `findMax(int arr[], int size)` to return the maximum value in *arr* with *size* elements
- Precond: *size* > 0

```
int findMax(int arr[], int size) {  
    int i, max;  
  
    max = arr[0];  
    for (i = 1; i < size; i++)  
        if (arr[i] > max)  
            max = arr[i];  
  
    return max;  
}
```

| i | arr | max |
|---|-----|-----|
| 6 | 20 | 36 |
| | 12 | |
| | 25 | |
| | 8 | |
| | 36 | |
| | 9 | |

1.3 Sum Elements

- `sum(int arr[], int size)` to return the sum of elements in *arr* with *size* elements
- Precond: *size* > 0

```
int sum(int arr[], int size) {  
    int i, sum = 0;  
  
    for (i = 0; i < size; i++)  
        sum += arr[i];  
  
    return sum;  
}
```

| i | arr | sum |
|---|-----|-----|
| 6 | 20 | 110 |
| | 12 | |
| | 25 | |
| | 8 | |
| | 36 | |
| | 9 | |

1.4 Sum Alternate Elements

- `sumAlt(int arr[], int size)` to return the sum of alternate elements (1st, 3rd, 5th, etc.)
- Precond: $size > 0$

```
int sumAlt(int arr[], int size) {  
    int i, sum = 0;  
  
    for (i = 0; i < size; i+=2)  
        sum += arr[i];  
  
    return sum;  
}
```

| i | arr | sum |
|---|-----|-----|
| 6 | 20 | 81 |
| | 12 | |
| | 25 | |
| | 8 | |
| | 36 | |
| | 9 | |

1.5 Sum Odd Elements

- `sumOdd(int arr[], int size)` to return the sum of elements that are odd numbers
- Precond: $size > 0$

```
int sumOdd(int arr[], int size) {  
    int i, sum = 0;  
  
    for (i = 0; i < size; i++)  
        if (arr[i]%2 == 1)  
            sum += arr[i];  
  
    return sum;  
}
```

| i | arr | sum |
|---|-----|-----|
| 6 | 20 | 34 |
| | 12 | |
| | 25 | |
| | 8 | |
| | 36 | |
| | 9 | |

1.6 Sum Last 3 Elements (1/3)

- `sumLast3(int arr[], int size)` to return the sum of the last 3 elements among *size* elements
- Precond: $size \geq 0$
- Examples:

| numbers | sumLast3(numbers, size) |
|---------------------------------------|-------------------------|
| { } | 0 |
| { 5 } | 5 |
| { 12, -3 } | 9 |
| { 20, 12, 25, 8, 36, 9 } | 53 |
| { -1, 2, -3, 4, -5, 6, -7, 8, 9, 10 } | 27 |

1.6 Sum Last 3 Elements (2/3)

Thinking...

- Last 3 elements of an array *arr*
 - *arr[size - 1]*
 - *arr[size - 2]*
 - *arr[size - 3]*
- A loop to iterate 3 times (hence, need a counter) with index starting at *size - 1* and decrementing it in each iteration



```
int i, count = 0;
for (i = size - 1; count < 3; i--) {
    . . .
    count++;
}
```

- But what if there are fewer than 3 elements in *arr*?



```
int i, count = 0;
for (i = size - 1; (i >= 0) && (count < 3); i--) {
    . . .
    count++;
}
```


1.6 Sum Last 3 Elements (3/3)

- Complete function:

```
int sumLast3(int arr[], int size) {  
    int i, count = 0, sum = 0;  
  
    for (i = size - 1; (i >= 0) && (count < 3); i--) {  
        sum += arr[i];  
        count++;  
    }  
  
    return sum;  
}
```

1.7 Minimum Pair Difference (1/3)

- Is it true that all problems on 1D arrays can be solved by single loop? Of course **not**!
- Write a function `minPairDiff(int arr[], int size)` that computes the minimum possible difference of any pair of elements in *arr*.
- For simplicity, assume *size* > 1 (i.e. there are at least 2 elements in array).

| numbers | minPairDiff(numbers, size) |
|---|----------------------------|
| { 20, 12, 25, <u>8</u> , 36, <u>9</u> } | 1 |
| { 431, 945, <u>64</u> , 841, 783, <u>107</u> , 598 } | 43 |

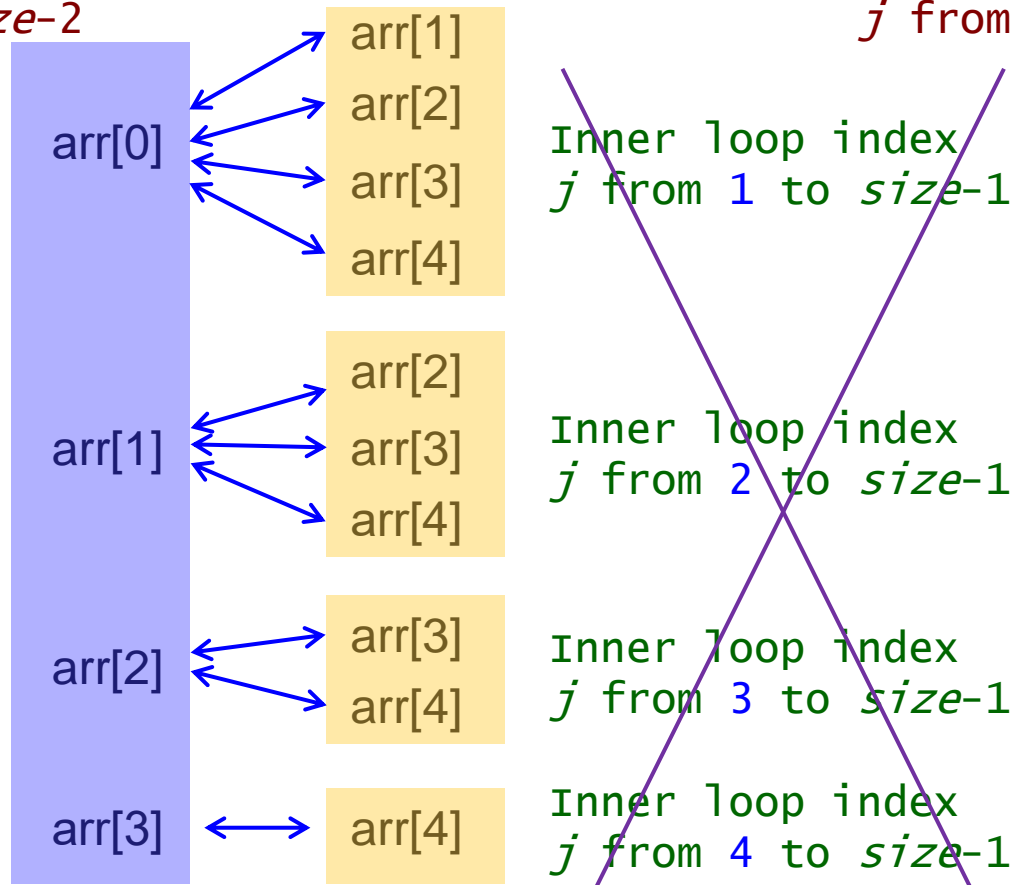
1.7 Minimum Pair Difference (2/3)

Thinking...

Eg: $size = 5$. Need to compute difference of

Outer loop index
 i from 0 to $size-2$

Inner loop index
 j from $i+1$ to $size-1$



1.7 Minimum Pair Difference (3/3)

The code...

Outer loop index
i from 0 to *size-2*

Inner loop index
j from *i+1* to *size-1*

```
int minPairDiff(int arr[], int size) {  
    int i, j, diff, minDiff;  
  
    minDiff = abs(arr[0] - arr[1]); // init min diff.  
  
    for (i = 0; i < size-1; i++)  
        for (j = i+1; j < size; j++) {  
            diff = abs(arr[i] - arr[j]);  
            if (diff < minDiff)  
                minDiff = diff;  
        }  
  
    return minDiff;  
}
```

- This kind of nested loop is found in many applications involving 1D array, for example, sorting (to be covered later).
- In fact, this problem can be solved by first sorting the array, then scan through the array once more to pick the pair of neighbours with the smallest difference.

Code Provided

- [Unit10_FindMax.c:](#)
 - Section 1.2 Find Maximum Element
- [Unit10_SumElements.c:](#)
 - Section 1.3 Sum Elements
 - Section 1.4 Sum Alternate Elements
 - Section 1.5 Sum Odd Elements
 - Section 1.6 Sum Last 3 Elements
- [Unit10_MinPairDiff.c:](#)
 - Section 1.7 Minimum Pair Difference

1.8 Accessing 1D Array Elements in Function (1/2)

A function header with array parameter,

```
int sum(int a[ ], int size)
```

Why is it not necessary to have a value in here to indicate the “real” size?

- A value is not necessary (and is ignored by compiler if provided) as accessing a particular array element requires only the following information
 - The address of the first element of the array
 - The size of each element
- Both information are known
 - For example, when the above function is called with

```
ans = sum(numbers, 6);
```

in the main(), the address of the first element, &numbers[0], is copied into the parameter a
 - The size of each element is determined since the element type (int) is given (in sunfire, an integer takes up 4 bytes)

1.8 Accessing 1D Array Elements in Function (2/2)

A function header with array parameter,

```
int sum(int a[ ], int size)
```

Why is it not necessary to have a value in here to indicate the “real” size?

- With this, the system is able to calculate the effective address of the required element, say **a[2]**, by the following formula:
Address of **a[2]** = base address + (2 × size of each element)
where base address is the address of the first element
- Hence, suppose the base address is 2400, then address of **a[2]** is 2400 + (2 × 4), or 2408.

| a[0] | a[1] | a[2] | a[3] | ... |
|-------------|-------------|-------------|-------------|-----|
| 5 | 19 | 12 | 7 | |

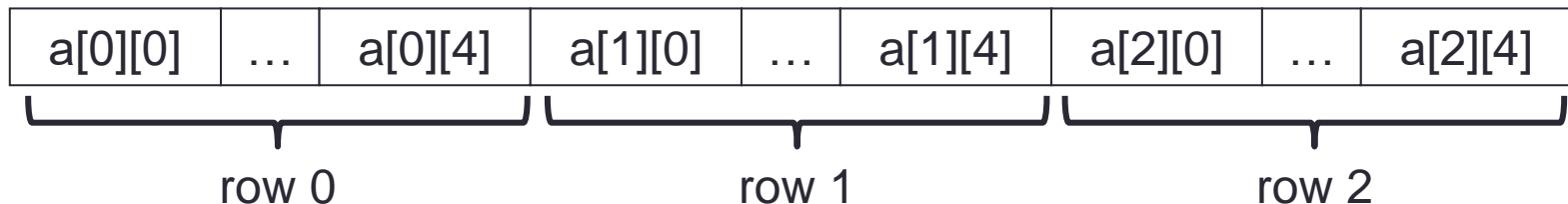
2. Multi-dimensional Arrays (1/2)

- In general, an array can have any number of dimensions
- Example of a 2-dimensional (2D) array:

```
// array with 3 rows, 5 columns  
int a[3][5];  
a[0][0] = 2;  
a[2][4] = 9;  
a[1][0] = a[2][4] + 7;
```

| | 0 | 1 | 2 | 3 | 4 |
|---|----|---|---|---|---|
| 0 | 2 | | | | |
| 1 | 16 | | | | |
| 2 | | | | | 9 |

- Arrays are stored in **row-major order**
 - That is, elements in row 0 comes before row 1, etc.



2. Multi-dimensional Arrays (2/2)

- Examples of applications:

$$\begin{pmatrix} 3 & 8 & 2 \\ -5 & 2 & 0 \\ 1 & -4 & 9 \end{pmatrix}$$

`matrix[3][3]`

| | 1 | 2 | 3 | ... | 30 | 31 |
|-----|------|------|------|-----|------|------|
| Jan | 32.1 | 31.8 | 31.9 | | 32.3 | 32.4 |
| Feb | 32.6 | 32.6 | 33.0 | | 0 | 0 |
| : | | | | ... | | |
| Dec | 31.8 | 32.3 | 30.9 | | 31.6 | 32.2 |

Daily temperatures: `temperatures[12][31]`

| Emily | | | | Zass | | | | Jerna | | | | Suisse | | | |
|-------|-----|-----|-----|------|-----|-----|-----|-------|-----|-----|-----|--------|-----|-----|-----|
| | Ex1 | Ex2 | Ex3 | | Ex1 | Ex2 | Ex3 | | Ex1 | Ex2 | Ex3 | | Ex1 | Ex2 | Ex3 |
| Lab1 | 76 | 80 | 62 | Lab1 | 59 | 68 | 60 | Lab1 | 79 | 75 | 66 | Lab1 | 52 | 50 | 45 |
| Lab2 | 60 | 72 | 48 | Lab2 | 0 | 0 | 0 | Lab2 | 90 | 83 | 77 | Lab2 | 57 | 60 | 63 |
| Lab3 | 76 | 80 | 62 | Lab3 | 67 | 71 | 75 | Lab3 | 81 | 73 | 79 | Lab3 | 52 | 59 | 66 |
| Lab4 | 60 | 72 | 48 | Lab4 | 38 | 52 | 35 | Lab4 | 58 | 64 | 52 | Lab4 | 33 | 42 | 37 |
| Lab5 | 58 | 79 | 73 | Lab5 | 78 | 86 | 82 | Lab5 | 93 | 80 | 85 | Lab5 | 68 | 68 | 72 |

Students' lab marks: `marks[4][5][3]`

2.1 Multi-dimensional Array Initializers

- Examples:

```
// nesting one-dimensional initializers
```

```
int a[3][5] = { {4, 2, 1, 0, 0},  
                {8, 3, 3, 1, 6},  
                {0, 0, 0, 0, 0} };
```

```
// the first dimension can be unspecified
```

```
int b[][5] = { {4, 2, 1, 0, 0},  
               {8, 3, 3, 1, 6},  
               {0, 0, 0, 0, 0} };
```

```
// initializer with implicit zero values
```

```
int d[3][5] = { {4, 2, 1},  
                {8, 3, 3, 1, 6} };
```

What happens to the uninitialized elements?

2.2 Multi-dimensional Array: Example

Unit10_2DArray.c

```
#include <stdio.h>
#define N 5      // number of columns in array
int sumArray(int [][][N], int); // function prototype

int main(void) {
    int foo[][N] = { {3,7,1}, {2,1}, {4,6,2} };
    printf("Sum is %d\n", sumArray(foo, 3));
    printf("Sum is %d\n", sumArray(foo, 2));
    return 0;
}

// To sum all elements in arr
int sumArray(int arr[][N], int rows) {
    int i, j, total = 0;
    for (i = 0; i < rows; i++) {
        for (j = 0; j < N; j++) {
            total += arr[i][j];
        }
    }
    return total;
}
```

Second dimension must be specified; first dimension is not required.

Sum is 26
Sum is 14

2.3 Accessing 2D Array Elements in Function

A function header with 2D array parameter,

```
function(int a[][5], ...)
```

Why second dimension must be specified, but not the first dimension?

- To access an element in a 2D array, it must know the number of columns. It needs not know the number of rows.
- For example, given the following two 2D-arrays:

A 3-column 2D array:

| | | |
|--|--|--|
| | | |
| | | |

:

A 5-column 2D array:

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |

:

- As elements are stored linearly in memory in **row-major order**, element `a[1][0]` would be the 4th element in the 3-column array, whereas it would be the 6th element in the 5-column array.
- Hence, to access `a[1][0]` correctly, we need to provide the number of columns in the array.
- For multi-dimensional arrays, all but the first dimension must be specified in the array parameter.

2.4 Class Enrolment (1/5)

- A class enrolment system can be represented by a 2D array `enrol`, where the rows represent the classes, and columns the students. For simplicity, classes and students are identified by non-negative integers.
- A '1' in `enrol[c][s]` indicates student `s` is enrolled in class `c`; a '0' means `s` is not enrolled in `c`.
- Assume at most 10 classes and 30 students.
- Example of an enrolment system with 3 classes and 8 students:

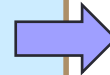
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

- **Queries:**
 - Name any class with the most number of students
 - Name all students who are enrolled in all the classes

2.4 Class Enrolment (2/5)

- Inputs:
 - Number of classes and students
 - Number of data entries
 - Each data entry consists of 2 integers *s* and *c* indicating that student *s* is enrolled in class *c*.
- Sample input:

```
Number of classes and students: 3 8
Number of data entries: 15
Enter 15 entries (student class):
3 1
0 0
0 1
1 2
2 0
2 1
2 2
3 2
7 1
6 0
5 0
4 1
4 0
6 2
6 1
```



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

2.4 Class Enrolment (3/5)

```
#define MAX_CLASSES 10
#define MAX_STUDENTS 30
int main(void) {
    int enrol[MAX_CLASSES][MAX_STUDENTS] = { {0} }, numClasses, numStudents;

    printf("Number of classes and students: ");
    scanf("%d %d", &numClasses, &numStudents);
    readInputs(enrol, numClasses, numStudents);

    return 0;
}
```

```
3 8
15
3 1
0 0
0 1
1 2
2 0
2 1
2 2
3 2
7 1
6 0
5 0
4 1
4 0
6 2
6 1
```

```
// Read data into array enrol
void readInputs(int enrol[][MAX_STUDENTS],
                int numClasses, int numStudents) {
    int entries; // number of data entries
    int i, class, student;

    printf("Number of data entries: ");
    scanf("%d", &entries);

    printf("Enter %d data entries (student class): \n", entries);
    // Read data into array enrol
    for (i = 0; i < entries; i++) {
        scanf("%d %d", &student, &class);
        enrol[class][student] = 1;
    }
}
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

2.4 Class Enrolment (4/5)

- **Query 1:** Name any class with the most number of students

```

int classWithMostStudents
    (int enrol[][MAX_STUDENTS],
     int numClasses, int numStudents) {
    int classSizes[MAX_CLASSES];
    int r, c; // row and column indices
    int maxClass, i;

    for (r = 0; r < numClasses; r++)
        classSizes[r] = 0;
    for (c = 0; c < numStudents; c++) {
        classSizes[r] += enrol[r][c];
    }

    // find the one with most students
    maxClass = 0; // assume class 0 has most students
    for (i = 1; i < numClasses; i++)
        if (classSizes[i] > classSizes[maxClass])
            maxClass = i;

    return maxClass;
}

```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Row sums |
|---|---|---|---|---|---|---|---|---|----------|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 5 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 6 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 4 |

2.4 Class Enrolment (5/5)

- **Query 2:** Name all students who are enrolled in all classes

```
// Find students who are enrolled in all classes
void busiestStudents(int enrol[][MAX_STUDENTS],
                    int numClasses, int numStudents) {
    int sum;
    int r, c;

    printf("Students who take all classes: ");
    for (c = 0; c < numStudents; c++) {
        sum = 0;
        for (r = 0; r < numClasses; r++) {
            sum += enrol[r][c];
        }
        if (sum == numClasses)
            printf("%d ", c);
    }
    printf("\n");
}
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| | 2 | 1 | 3 | 2 | 2 | 1 | 3 | 1 |

Column sums

Refer to [Unit10_ClassEnrolment.c](#) for complete program.

2.5 Matrix Addition (1/2)

- To add two matrices, both must have the same size (same number of rows and columns).
- To compute $C = A + B$, where A , B , C are matrices

$$c_{i,j} = a_{i,j} + b_{i,j}$$

- Examples:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 21 & 7 & 9 \\ 4 & 6 & 14 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 7 & 18 & 20 \\ 6 & 5 & 8 & 15 \end{pmatrix} = \begin{pmatrix} 13 & 28 & 25 & 29 \\ 10 & 11 & 22 & 20 \end{pmatrix}$$

2.5 Matrix Addition (2/2)

Unit10_MatrixOps.c

```
// To sum mtxA and mtxB to obtain mtxC
void sumMatrix(float mtxA[][MAX_COL], float mtxB[][MAX_COL],
               float mtxC[][MAX_COL], int row_size, int col_size) {
    int row, col;

    for (row=0; row<row_size; row++)
        for (col=0; col<col_size; col++)
            mtxC[row][col] = mtxA[row][col] + mtxB[row][col];
}
```

$$\begin{bmatrix} 10 & 21 & 7 & 9 \\ 4 & 6 & 14 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 7 & 18 & 20 \\ 6 & 5 & 8 & 15 \end{bmatrix} = \begin{bmatrix} 13 & 28 & 25 & 29 \\ 10 & 11 & 22 & 20 \end{bmatrix}$$

Summary

- In this unit, you have learned about
 - Declaring 2D arrays
 - Using 2D arrays in problem solving

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