Contingency-Based Zonal Reserve Modeling and Pricing in a Co-Optimized Energy and Reserve Market

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Abstract—Ancillary service markets are a very important part of the Standard Market Design and are under development or implementation on a centralized basis in major U.S. electricity markets. This paper presents the market clearing framework for the co-optimized real-time energy-reserve market that has been developed and implemented in the ISO New England (ISO NE). In this co-optimized market clearing model, reserve products are procured on a zonal basis to satisfy the first and the second contingency protection criteria required by the ISO NE's real-time operations for both the entire area and typical import-constrained areas. The proposed zonal reserve model is derived from simulating each contingency event in the predefined reserve zones. Differing from standard industrial practices, the proposed zonal reserve model recognizes reserve deliverability for nested reserve zones and allocates the zonal total transfer capability between energy and reserves in an economic way. Numerical examples for a two-zone system are presented to demonstrate the validity of this modeling technique.

Index Terms—Ancillary service market, electricity market, locational marginal pricing, operating reserve, optimization, power systems.

NOMENCLATURE

10	Superscript or subscript that indicates a capacity that can be achieved in 10 min.
30	Superscript or subscript that indicates a capacity that can be achieved in 30 min.
b	Bid cost function for a price sensitive demand.
c	Offer cost function for a generator.
C_R	Set of reserve categories, $\{sp, nsp, op\}$.
D	Fixed demand.
$flow$	Function that represents the active power flow over a transmission line in both pre- and post-contingent state.
i	Index for all generators.
$Iflow$	Function that represents the active power flow over an interface.
j	Index for all price sensitive demands.
k	Index for all transmission constraints.
l	Index for all local reserve zones.

L	Control variable that indicates the consumption level of the price sensitive demand.
lr	Superscript that indicates the local reserve.
m	Index for all contingencies in a reserve zone.
max	Superscript that indicates the maximum value.
min	Superscript that indicates the minimum value.
n	Index for all buses in the system.
N	Set of the generators; its subscripts can be reserve zones.
P	Control variable that indicates the generation output.
PB	Amount of power imbalance.
pl	Parent reserve zone of zone l. A parent reserve zone of l is the one in which zone l is completely located.
P_{loss}	Linear function that represents the total system active power losses.
Q	Amount associated with local or system-wide contingencies or reserve requirements.
r	Index for all resources i and j.
R	Control variable that indicates reserve quantity or designation.
RC	Ramping capability for a generator or load; its superscript indicates the ramping time-interval.
RCPF	Reserve constraint penalty factor.
RMCP	Reserve market clearing price.
s	Index for all categories of system-wide reserve constraints, including the 10-min spinning reserve (10s), the total 10-min reserve (10t), and the total 30-min operating reserve (30t).
S	Control variable that indicates the reserve deficit.
sr	Superscript that indicates the system-wide reserve.
t	Index for all reserve categories including 10-min spinning (sp), 10-min non-spinning (nsp), and 30-min operating (op) reserves.
$\delta^x_{t,i\,\text{or}\,j}$	Binary parameter that is 1 when the type t reserve from the generator i or the load j belongs to a set x (i.e., local reserve zone l, system-wide reserve constraints $s = \{10s, 10t, 30t\}$).

I. INTRODUCTION

ANCILLARY service markets have been a very important part of the Standard Market Design (SMD) recommended by the Federal Energy Regulatory Commission (FERC). In the

FERC's proposal, an independent transmission provider would "establish schedules for transmission service, and sales and purchases of energy, regulation, and both operating reserves (spinning and supplemental), to ensure the most efficient use of the transmission grid" [1].

Ancillary services are essential to the reliable operation of a power system. Market-based ancillary service procurement can be the least costly way of ensuring system reliability. In general, ancillary services include [1]:

- scheduling, system control, and dispatch services;
- reactive supply and voltage control;
- regulation and frequency response;
- energy imbalance;
- operating reserve—spinning;
- operating reserve—supplemental.

However, only regulation and reserve services are procured through a market mechanism in the majority of electricity markets in the United States. The rest of them are procured on a regulated basis. In this paper, we will focus on the operating reserve market only.

Currently, reserves as ancillary service products have been purchased centrally by the independent system operator (ISO) to ensure the system reliability. ISO NE had used a sequential market clearing approach to clear reserve bids in the real-time before its SMD implementation [2] and has established forward reserve markets to encourage long-term investments in the fast-start units. PJM established tiered reserve markets without energy and reserve co-optimization [3]. In NY ISO [4], operating reserves are procured on a zonal basis without explicitly considering zonal transfer capabilities. California ISO implemented ancillary services procurement using sequential markets combined with a rational buyer algorithm [5] without considering the coupling effect between energy and reserve products. Similar to CAISO, ERCOT purchases operating reserves without energy-reserve co-optimization [6]. Ontario IESO implemented the operating reserves market on a system-wide basis with a joint energy and reserve clearing approach [7].

Different market structures determine different market clearing processes; however, the simultaneous energy and reserve clearing approach provides the most efficient way of allocating resources. Energy-reserve co-optimization has been discussed extensively in the literature: [8] and [9] modeled responsive reserves by simulating every contingency event; [10] and [11] proposed a zonal reserve model based on predefined single zonal reserve requirement without considering the impact of different types of local contingency on other reserve zones; while [12]–[17] considered the system-wide reserve requirements only.

Recognizing the convenience for trading, we propose a zonal reserve model while considering reserve deliverability among reserve zones. In this model, a reserve zone is established for each import-constrained area based on historical studies. A zonal reserve requirement is then determined based on the simulation of the second contingency event inside the local reserve zone. To satisfy local reserve requirements, resources, both inside and outside of the reserve zone, are utilized. The zonal total transfer capability (TTC) is honored when outside reserves are delivered to the reserve zone during the local reserve event. In this way, the zonal TTC is allocated to both energy and reserve in a least cost fashion. Different from existing practices, this model is much simpler and recognizes the reserve deliverability for nested zones.

The rest of this paper is organized as follows: Section II describes features of the co-optimized real-time market in the ISO NE; Section III provides the detailed derivation of the proposed zonal reserve model; Section IV presents the co-optimization formulation; Section V discusses the energy and reserve pricing properties; Section VI presents numerical examples; and the conclusion is drawn in Section VII.

II. ISO NE's REAL-TIME MARKET

On October 1, 2006, ISO NE successfully implemented its real-time reserve market, completing the ancillary service market development for SMD. The SMD ancillary service markets at the ISO NE include three components: forward reserve market, regulation market, and real-time reserve market. The forward reserve market is designed to attract long-term investment for quick start units. The regulation market is to arrange real-time frequency control on a market basis. The real-time reserve market is designed to ensure the system reliability. In this section, we will only concentrate on the discussion of the real-time reserve market.

The design of the real-time reserve market is based on the needs for the contingency recovery required by power system operations. The North America Reliability Committee (NERC) standard requires that the system be able to recover in 10 and 30 min upon the loss of the first and the second generating resources. Therefore, the reserve products in the real-time market include the 10-min spinning reserve (sp), the 10-min non-spinning reserve (nsp), and the 30-min operating reserve (op).

To cover the system-wide generation contingency without considering the transmission network, system-wide reserve requirements are established. There are three system-wide reserve requirements: the 10-min spinning reserve requirement, the total 10-min reserve requirement, and the total 30-min operating reserve requirement.

In order to protect the local area from the loss of the second contingency element in the area, ISO NE establishes zonal reserve requirements. Currently, there is only a 30-min operating reserve requirement for each identified reserve zone, assuming that the local area is already protected from the first contingency by operating the system under the n-1 security criterion.

The real-time reserve market is cleared jointly with the energy market in a co-optimized fashion. Each generating resource or dispatchable demand eligible to participate in the reserve market is designated to provide reserves of the category that it is qualified for and is paid the reserve market clearing price at the reserve zone where it is located. In the meantime, it also participates in the energy market and receives energy payments. Although each qualified resource participates in energy and reserve markets, it can submit only an energy offer or bid. The reserve availability bid is always zero, assuming that there is no incremental cost for a resource to provide reserves if it is ready to be dispatched and committed to serve energy.
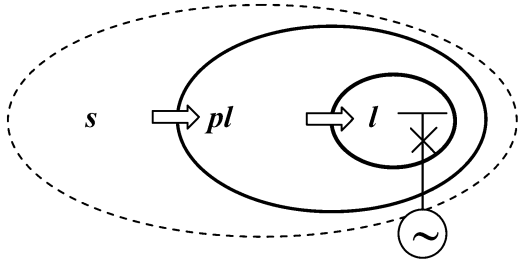
Fig. 1. Nested-reserve-zone configuration.

In order to deal with reserve scarcity conditions, reserve constraint penalty factors (RCPF) are introduced to the real-time energy-reserve co-optimization. One RCPF is set for each system reserve constraint and each local reserve event. In the event of reserve scarcity, the RCPFs will set the reserve market clearing prices and affect energy prices.

III. ZONAL RESERVE REQUIREMENTS AND MODELING

During real-time operation at the ISO NE, operators have the responsibility to maintain system reliability. In addition to system-wide protection against the loss of generating capacity, an import-constrained area needs to be able to recover from the loss of any second contingency element in the area within 30 min. It is such an operating criterion that determines the need for the modeling of locational reserves.

To satisfy real-time operation needs, we propose a contingency-based zonal reserve model by simulating each individual second contingency event in a local area. Compared to other locational reserve models, the proposed model considers the available transfer capability of the tie line interface that carries reserves from the resources located outside of the area. Like any other model, this zonal reserve model makes certain assumptions, which are listed below.

1) The model is established on a zonal basis. The reserve zone is in general an import-constrained area identified by offline studies. The energy normally flows into the area.
2) The reserve zone must be separated by a closed interface, and the interface import limit can be established in offline studies.
3) Each reserve zone must be located either completely outside or completely inside another reserve zone.
4) Reserves inside a reserve zone can be deployed to respond to the local contingency event without congestion. In other words, the intra-zonal congestion will not prevent reserve deployment in the reserve zone.
5) Reserves in a reserve zone can be fully delivered to its parent zone or the system during any contingency event without violating the export limit of its interface.
6) The transmission losses for each area of the system do not change before and after the contingency event.

Basically, the model considers contingency events on a zonal basis, subject to the interface import constraint. To derive the zonal reserve model, let us investigate how we respond to the contingency event in a nested-reserve-zone example as shown in Fig. 1. Reserve zone l is located completely inside its parent reserve zone pl. s represents the overall system.

In general, the system has 30 min to recover from any second contingency event in a local area. Any second contingency event m in reserve zone l could result in the following:

a) the loss of interface transfer capability. The interface limit associated with reserve zone l is reduced from the n-1 limit to the n-2 limit;
b) the loss of local generation;
c) the deployment of reserves from all resources;
d) corrective local actions such as switching of local generation to the outside area, load swapping, load shedding, and emergency actions.

To come up with a simplified local reserve model, we will formulate the power flow equations for the given system before and after the mth contingency event based on above assumptions.

A. Pre-Contingent State

Before the contingency event, power is balanced at both the system and reserve zone levels as follows:

$$\sum_i P_i = \sum_n D_n \tag{1.1}$$

$$\sum_{i \in G_{pl}} P_i + \text{Iflow}_{pl} = \sum_{n \in N_{pl}} D_n \tag{1.2}$$

$$\sum_{i \in G_l} P_i + \text{Iflow}_l = \sum_{n \in N_l} D_n \tag{1.3}$$

where G_l and G_{pl} are sets of generators that are located in zones l and pl, respectively; N_l and N_{pl} are sets of buses that are located in zones l and pl, respectively.

Equation (1.1) describes the power balance for the system; (1.2) is the power balance for the parent reserve zone pl, the power that flows into zone pl is subject to the n-1 interface limit of zone pl; (1.3) presents the power flow equation for zone l, where the power that flows into zone l is subject to the n-1 interface limit of zone l.

B. Post-Contingent State

In the post-contingent state, the power flow equations are shown in the following:

$$\sum_i \left(P_i + \sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} = \sum_{n \in N_s} D_n + PB_{l,s}^m \tag{2.1}$$

$$\sum_{i \in G_{pl}} \left(P_i + \sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} + \text{Iflow}_{pl}^m = \sum_{n \in N_{pl}} D_n + PB_{l,pl}^m \tag{2.2}$$

$$\sum_{i \in G_l} \left(P_i + \sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} + \text{Iflow}_l^m = \sum_{n \in N_l} D_n + PB_{l,l}^m. \tag{2.3}$$

Equation (2.1) is the system-wide power balance in the post-contingent state under contingency m. In the post-contingent

state, we assume each generator is generating at its pre-contin-gent output level plus its reserve responses to the contingency $m (\sum_{t \in C_R} R_{t,i}^m)$. The power imbalance for the system s due to the local reserve event is reflected in the variable $PB_{l,s}^m$, which includes all effects of the generation loss and corrective actions. In addition, $S_l^{lr,m}$ represents the energy deficit due to the local event, assuming that the total reserve deployed cannot cover the contingency. Similarly, (2.2) and (2.3) are the power balance for zones pl and l, and $PB_{l,pl}^m$ and $PB_{l,l}^m$ are the power imbalance for zones pl and l, respectively, due to the mth contingency event. Since all post-contingent interface flows must be under the n-2 interface limit, we have

$$\text{Iflow}_{pl}^m \leq \text{Iflow}_{pl}^{m,n-2} \tag{3.1}$$

$$\text{Iflow}_l^m \leq \text{Iflow}_l^{m,n-2} \tag{3.2}$$

where $\text{Iflow}_l^{m,n-2}$ and $\text{Iflow}_{pl}^{m,n-2}$ are the n-2 interface limits under the mth contingency event for zones l and pl, respectively.

From (1–3), we have the following for each contingency m:

$$\sum_i \left(\sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} = PB_{l,s}^m \tag{4.1}$$

$$\sum_{i \in G_{pl}} \left(\sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} + \left(\text{Iflow}_{pl}^{n-1} - \text{Iflow}_{pl} \right)$$
$$\geq \text{Iflow}_{pl}^{n-1} - \text{Iflow}_{pl}^{m,n-2} + PB_{l,pl}^m \tag{4.2}$$

$$\sum_{i \in G_l} \left(\sum_{t \in C_R} R_{t,i}^m \right) + S_l^{lr,m} + \left(\text{Iflow}_l^{n-1} - \text{Iflow}_l \right)$$
$$\geq \text{Iflow}_l^{n-1} - \text{Iflow}_l^{m,n-2} + PB_{l,l}^m \tag{4.3}$$

where Iflow_l^{n-1} and Iflow_{pl}^{n-1} are n-1 interface limits for zones l and pl, respectively.

In order to cover any contingency, we should clear the max-imum of all reserve responses to each individual contingency m. In other words

$$R_{t,i} = \underset{m}{\text{Max}} \left(R_{t,i}^m \right)$$

$$S_l^{lr} = \underset{m}{\text{Max}} \left(S_l^{lr,m} \right).$$

After combining all the contingencies, we should have

$$\sum_i \left(\sum_{t \in C_R} R_{t,i} \right) + S_l^{lr} \geq Q_{l,s} \tag{5.1}$$

$$\sum_{i \in G_{pl}} \left(\sum_{t \in C_R} R_{t,i} \right) + S_l^{lr} + \left(\text{Iflow}_{pl}^{n-1} - \text{Iflow}_{pl} \right)$$
$$\geq Q_{l,pl} \tag{5.2}$$

$$\sum_{i \in G_l} \left(\sum_{t \in C_R} R_{t,i} \right) + S_l^{lr} + \left(\text{Iflow}_l^{n-1} - \text{Iflow}_l \right)$$
$$\geq Q_{l,l} \tag{5.3}$$

where

$$Q_{l,s} = \underset{m}{\text{Max}} \left(PB_{l,s}^m \right) \tag{5.4}$$

$$Q_{l,pl} = \underset{m}{\text{Max}} \left(\text{Iflow}_{pl}^{n-1} - \text{Iflow}_{pl}^{m,n-2} + PB_{l,pl}^m \right) \tag{5.5}$$

$$Q_{l,l} = \underset{m}{\text{Max}} \left(\text{Iflow}_l^{n-1} - \text{Iflow}_l^{m,n-2} + PB_{l,l}^m \right). \tag{5.6}$$

As seen from the derivation of the zonal reserve model, the interface transmission constraint is considered in each post-con-tingent state. Therefore, (5) ensures the reserve deliverability in the post-contingent state. It should be noted that each individual transmission line can be monitored to ensure 100% of reserve deliverability. However, such a method needs online simulation of every contingency event, which is a very complex process. The proposed zonal reserve model is a compromise between full reserve deliverability and complexity.

IV. REAL-TIME CO-OPTIMIZATION FORMULATION

The real-time dispatch problem of the ISO NE market is for-mulated as a linear programming problem with the objective to maximize the social welfare, subject to real-time operating constraints and the physical characteristics of resources. Under the co-optimized energy and reserve market, system-wide and locational reserve constraints are enforced by the market oper-ator to procure enough reserves to cover the first and the second contingency events. In short, the energy-reserve co-optimization problem is formulated as

$$\underset{P,R,L,S}{\text{Min}} \sum_i [c_i(P_i)] - \sum_j [b_j(L_j)]$$
$$+ \sum_l \left(\text{RCPF}_l^{lr} \cdot S_l^{\text{lr}} \right) + \sum_{s=10s,10t,30t} \left(\text{RCPF}_s^{sr} \cdot S_s^{sr} \right)$$

subject to the following constraints:
1) System-Wide Energy Balance Constraint

$$(\lambda): \sum_i P_i = \sum_j L_j + \sum_n D_n + P_{\text{loss}} \tag{6.1}$$

where the power supply must meet the power demand and the system losses. The system losses are derived from the base case power flow study and are represented by a linear function of bus net injections [18].
2) Transmission Constraints

$$(\mu_k \leq 0): \text{flow}_k \leq \text{flow}_k^{\max}, \forall k \tag{6.2}$$

where the transmission flow must be less than its limit. The transmission flow is a linear function of bus net injections in the pre-contingent state. The ISO NE's real-time opera-tion requires the n-1 security protection, and therefore, the transmission flow presented here can be either pre-contin-gent or post-contingent line or interface flows.

3) System-Wide Reserve Constraints

$$(\alpha_s \geq 0): \sum_{r=i,j} \sum_{t \in C_R} \left(\delta_{t,r}^s \cdot R_{t,i} \right)$$
$$+ S_s^{\mathrm{sr}} \geq Q_s, s \in \{10s, 10t, 30t\} \quad (6.3)$$

where the sum of different categories of reserves over the entire system, including the reserve deficit, must meet the reserve requirement. In order to recognize the value of different categories of reserves (spinning, non-spinning, and operating) and prevent price reversal, ISO NE currently defines three system-wide reserve requirements: the 10-min spinning $(10s)$, the 10-min total $(10t)$, and the 30-min total $(30t)$. In this way, the 10-min spinning reserve can be utilized to satisfy the $10s$, the $10t$, and the $30t$ reserve requirements; the 10-min non-spinning reserve can be used to satisfy the $10t$ as well as the $30t$ reserve requirements; however, the 30-min operating reserve can only be used to satisfy the $30t$ reserve requirement.

4) Locational Reserve Constraints
For any local reserve zone l, we have

$$(\alpha_{l,l} \geq 0): \sum_{r=i,j} \sum_{t \in C_R} \left(\delta_{t,r}^l \cdot R_{t,r} \right)$$
$$+ (\mathrm{Iflow}_l^{\max} - \mathrm{Iflow}_l) + S_l^{lr} \geq Q_{l,l} \text{ for any } l. \quad (6.4)$$

For any of the parent zone *pl of reserve zone l*, we have

$$(\alpha_{l,pl} \geq 0): \sum_{r=i,j} \sum_{t \in C_R} \left(\delta_{t,r}^{\mathrm{pl}} \cdot R_{t,r} \right)$$
$$+ (\mathrm{Iflow}_{pl}^{\max} - \mathrm{Iflow}_{pl}) + S_l^{lr} \geq Q_{l,pl} \text{ for any } pl. \quad (6.5)$$

In the system-wide, we have

$$(\alpha_{l,30t} \geq 0): \sum_{r=i,j} \sum_{t \in C_R} \left(\delta_{t,r}^{30t} \cdot R_{t,i} \right)$$
$$+ S_l^{lr} \geq Q_{l,s} \text{ for any } l. \quad (6.6)$$

$Q_{l,l}, Q_{l,pl}$, and $Q_{l,s}$ are the reserve requirements or power imbalance caused by zone l's second contingencies in zone l, parent zone pl, and the system, respectively. These values can be calculated using (5.4)–(5.6) in the offline study. Iflow is the active power flow over an interface of a reserve zone and is a linear function of bus injections in the pre-contingent state. Equations(6.4)–(6.6) ensure the second contingency coverage of reserve zone l while considering the reserve deliverability. This model is derived in Section III. Equations (6.4)–(6.6) are the same as (5.3), (5.2), and (5.1), respectively, when we add dispatchable demand. Note that for any zone l, we have $\sum_i \left(\sum_{t \in C_R} \left(\delta_{t,i}^l \cdot R_{t,i} \right) \right) = \sum_{i \in G_l} \left(\sum_{t \in C_R} R_{t,i} \right)$, since $\delta_{t,i}^l$ indicates whether a resource is located inside the reserve zone l. We also have $\sum_i \left(\sum_{t \in C_R} \left(\delta_{t,i}^{30t} \cdot R_{t,i} \right) \right) = \sum_{i \in G_l} \left(\sum_{t \in C_R} R_{t,i} \right)$, since all resources are able to contribute to the system-wide 30-min reserve constraint.

5) Resource Capacity Constraints

$$P_i + \sum_{t \in C_R} (R_{t,i}) \leq P_i^{\max} \text{ for any online generator } i \quad (6.7)$$

$$\sum_{t \in C_R} (R_{t,j}) \leq L_j - L_j^{\min}, \forall j. \quad (6.8)$$

Equation (6.7) shows that the total amount of reserve and energy cleared in the market cannot exceed the capacity of the generating resource. Also, (6.8) ensures that the total consumption cleared must be at least the total amount of cleared reserves and its minimum consumption level for a dispatchable demand.

6) 10-min Reserve Capability Constraints

$$R_{\mathrm{sp},r} + R_{\mathrm{nsp},r} \leq \mathrm{RC}_r^{10}, \forall r \in \{i,j\} \quad (6.9)$$

where the total of 10-min spinning and non-spinning reserves cleared in the market for an individual resource cannot exceed its online or offline 10-min response during the contingency event.

7) 30-min Reserve Capability Constraints

$$R_{\mathrm{sp},r} + R_{\mathrm{nsp},r} + R_{\mathrm{op},r} \leq \mathrm{RC}_r^{30}, \forall r \in \{i,j\} \quad (6.10)$$

where the total amount of 10-min spinning, non-spinning, and 30-min operating reserves cleared in the market for an individual resource cannot exceed its total capability that can be achieved in 30 min.

8) Other Resource Level Constraints
Constraints for each resource's physical characteristics are also considered in the real-time market. Such constraints include the energy ramp limit (the ramp rate is considered to be constant in this formulation), the regulation capability, the minimum generation, as well as upper and lower bounds for each decision variable.

In the above formulation, λ, μ, and α are the shadow prices for constraints (6.1)–(6.6), respectively. The decision variables are the generation output P_i, the consumption level L_j, the reserve designation $R_{t,i \text{ or } j}$, and the reserve deficit S. Equations (6.1)–(6.10) present the overall formulation of the co-optimized real-time market, which is cleared every 5 min. The problem is a linear programming problem, which is solved by a commercial solver. The overall market clearing software was implemented by AREVA T&D.

V. ENERGY-RESERVE PRICING AND PROPERTIES

In this section, we will first define energy and reserve prices derived from the optimization problem presented in Section IV, then discuss the reserve shortage pricing, and finally present a way of understanding market clearing results.

A. Energy and Reserves Prices Definitions

Based on the marginal pricing concept [19], the energy price or LMP for each bus n and the reserve market clearing price

(*RMCP*) of each reserve product t for each resource r are defined as (7.1) and (7.2) at the bottom of the page.

The energy price shown in (7.1) is defined for each network location n. In general, it is decomposed into three components [18]: energy or reference, marginal loss, and congestion as shown in the three pairs of braces in (7.1), respectively. Although the RMCP shown in (7.2) is defined at the resource level, RMCPs for resources located in the same reserve zone are identical. In addition, RMCPs have the following properties.

1) The reserve prices are determined by the shadow prices of reserve related constraints (6.3)–(6.6). When there are sufficient reserve supply (no reserve shortage or energy re-dispatch), reserve prices will be zero.

2) The RMCP for a higher quality reserve product is no less than that of a lower quality reserve product. In other words, for any resource r, we have

$$\mathrm{RMCP}_{\mathrm{sp},r} \geq \mathrm{RMCP}_{\mathrm{nsp},r} \geq \mathrm{RMCP}_{\mathrm{op},r}. \qquad (8.1)$$

3) The RMCP for a reserve zone is no less than that of the same type of reserve for its parent reserve zone. This means that for any reserve product t, if resource r1 is in reserve zone l and r2 is in its parent reserve zone pl, we have

$$\mathrm{RMCP}_{t,r1} \geq \mathrm{RMCP}_{t,r2}. \qquad (8.2)$$

Finally, (7.1) and (7.2) show that the energy and reserve prices are coupled directly through the shadow prices of the local reserve constraints ($\alpha_{l,l}$ and $\alpha_{l,pl}$). This indicates that the congestion component of the energy price at a local reserve zone can be directly related to the reserve price for a local reserve zone. This is because the energy import into the reserve zone is coupled with reserves in the local area, as shown in (6.4) and (6.5). Changing the energy import into a local reserve zone does impact the reserve contribution of a local area, affecting both energy and reserve prices in a local area.

B. Reserve Scarcity Pricing

The ISO NE's market rule requires pricing energy and reserve products under the reserve scarcity event. Such a pricing rule is implemented through reserve constraint penalty factors. The values of those factors are set administratively by the ISO and approved by the market participants.

The RCPFs are associated with both system-wide constraints and local reserve events. For the system-wide reserve constraints, there is one RCPF value corresponding to each reserve constraint (10s, 10t, and 30t); and each reserve constraint has a slack variable S that indicates the reserve shortage amount

and has the cost of the corresponding RCPF value. Based on the analysis of the optimality condition of the co-optimization problem, we can see that the system-wide RCPF is a cap for the shadow price of the corresponding reserve constraint. That is, for each system-wide reserve constraint s, the following holds:

$$0 \leq \alpha_s \leq \mathrm{RCPF}_s^{\mathrm{sr}}. \qquad (9)$$

Differing from the system-wide reserve constraint slack variable, the local reserve slack variable is assigned to the zone. The cost of the slack variable is the RCPF value. As discussed in the previous section, this type of modeling assumes that the local reserve event should be completely resolved using local actions at the cost of local RCPF value. Therefore, the slack variables for constraints (6.4)–(6.6) of each local reserve zone are the same. As a result, the zonal RCPF value becomes a cap of the total of zonal reserve constraint shadow prices as follows:

$$\alpha_{l,l} + \alpha_{l,30t} + \sum_{pl} \alpha_{l,pl} \leq \mathrm{RCPF}_l^{lr}. \qquad (10)$$

Equations (9) and (10) show that the zonal RCPF is the cap for the reserve constraint shadow prices, and it indirectly sets the ceiling price for the zonal reserve products.

One important point in the application of the slack variable and the RCPF is that the value of RCPF does affect the prices (both energy and reserves) and market clearing quantities (both energy and reserves). This is because the RCPF is a threshold of the re-dispatch cost for reserves. Different RCPF values could result in different system re-dispatches.

C. Understanding Energy and Reserve Prices

The real-time co-optimization recognizes the coupling effects between energy and reserve products, which are also presented indirectly in the energy and reserve prices defined in (7.1) and (7.2). To understand the prices and cleared quantities for energy and reserves, we can study them at the individual resource level.

The co-optimization problem presented in Section IV is a linear programming problem. Therefore, the marginal prices derived from its dual problem are market equilibrium prices under the perfectly competitive market condition. This means that the energy and reserve prices maximize each resource's as-bid profit/utility. Mathematically, the market clearing quantity pair $(P_i^*, R_{t,i}^*)$ of an online generator i is also the optimal solution of the following problem:

$$\mathrm{LP\_O}: \max_{P_i, R_{t,i}} [\mathrm{LMP}_i \cdot P_i - c_i(P_i)] + \sum_{t \in C_R} (\mathrm{RMCP}_{t,i} \cdot R_{t,i})$$

$$\mathrm{LMP}_n = \{\lambda\} + \left\{\lambda \cdot \frac{\partial P_{\mathrm{loss}}}{\partial D_n}\right\} + \left\{ \begin{array}{c} \sum_k \left[-\mu_k \cdot \frac{\partial \mathrm{flow}_k}{\partial D_n}\right] + \\ \sum_l \left[\left(\alpha_{l,l} \cdot \frac{\partial \mathrm{Iflow}_l}{\partial D_n}\right) + \sum_{pl}\left(\alpha_{l,pl} \cdot \frac{\partial \mathrm{Iflow}_{pl}}{\partial D_n}\right)\right] \end{array} \right\} \qquad (7.1)$$

$$\mathrm{RMCP}_{t,r} = \sum_{s=10s,10t,30t} \left[\alpha_s \cdot \delta_{t,r}^s\right] + \sum_l \left[\alpha_{l,l} \cdot \delta_{t,r}^{l,l} + \alpha_{l,30t} \cdot \delta_{t,r}^{30t} + \sum_{pl}\left(\alpha_{l,pl} \cdot \delta_{t,r}^{pl}\right)\right] \qquad (7.2)$$

subject to (6.7), (6.9), (6.10), and other resource level constraints.

Equation (6.7) represents the coupling between energy and reserve products. To unbundle energy from reserves, we can evaluate the as-bid profit maximization strategy under different generation levels.

1) When $P_i^* \in [P_i^{\max} - \text{RC}_i^{10}, P_i^{\max}]$, the cleared reserve quantities will be: $(R_{\text{sp},i}^*, R_{\text{nsp},i}^*, R_{\text{op},i}^*) = (P_i^{\max} - P_i^*, 0, 0)$.

This is due to the price cascading relationship presented in (8.1). Generator i will trade off energy with its *sp* reserve. Therefore, the LP_O becomes

$$\text{LP\_1}: \underset{P_i}{\text{Max}}\{LMP_i \cdot P_i - [c_i(P_i) + \text{RMCP}_{\text{sp},i} \cdot P_i]\} \\ + \text{RMCP}_{\text{sp},i} \cdot P_i^{\max}$$

subject to resource level constraints, except (6.7), (6.9), and (6.10).

This is equivalent to increasing the incremental energy offer curve by the amount of $\text{RMCP}_{\text{sp},i}$ and solving for energy only.

2) When $P_i^* \in [P_i^{\max} - \text{RC}_i^{30}, P_i^{\max} - \text{RC}_i^{10}]$, the cleared reserve quantities $(R_{\text{sp},i}^*, R_{\text{nsp},i}^*, R_{\text{op},i}^*) = (\text{RC}_i^{10}, 0, P_i^{\max} - \text{RC}_i^{10} - P_i^*)$.

This is due to the price cascading relationship presented in (8.1). Generator i will trade off energy with its *op* reserve. Therefore, the LP_O becomes

$$\text{LP\_2}: \underset{P_i}{\text{Max}}\{\text{LMP}_i \cdot P_i - [c_i(P_i) + \text{RMCP}_{\text{op},i} \cdot P_i]\} \\ + (\text{RMCP}_{\text{sp},i} - \text{RMCP}_{\text{op},i}) \cdot \text{RC}_i^{10} + \text{RMCP}_{\text{op},i} \cdot P_i^{\max}$$

subject to resource level constraints, excluding (6.7), (6.9), and (6.10).

This is the same as increasing the incremental energy offer curve by the amount of $\text{RMCP}_{\text{op},i}$ and searching for the energy solution only.

3) When $P_i^* \in [0, P_i^{\max} - \text{RC}_i^{30}]$, the cleared reserve quantities will be: $(R_{\text{sp},i}^*, R_{\text{nsp},i}^*, R_{\text{op},i}^*) = (\text{RC}_i^{10}, 0, \text{RC}_i^{30} - \text{RC}_i^{10})$.

As indicated by (8.1), generator i will not trade off energy with any reserve due to the fact that all reserves are cleared to their maximum. Therefore, the LP_O becomes

$$\text{LP\_3}: \underset{P_i}{\text{Max}}[\text{LMP}_i \cdot P_i - c_i(P_i)] \\ + \text{RMCP}_{\text{sp},i} \cdot \text{RC}_i^{10} + \text{RMCP}_{\text{op},i} \cdot (\text{RC}_i^{30} - \text{RC}_i^{10})$$

subject to resource level constraints, excluding (6.7), (6.9), and (6.10).

This is the same as maximizing the energy-only profit with given reserve profit.

To summarize, solving LP_O problem can be translated into maximizing energy-only profit with offer curve modified by RMCPs accordingly, which can be called the all-in offer curve. Given LMP and RMCPs, the optimal generation level should be
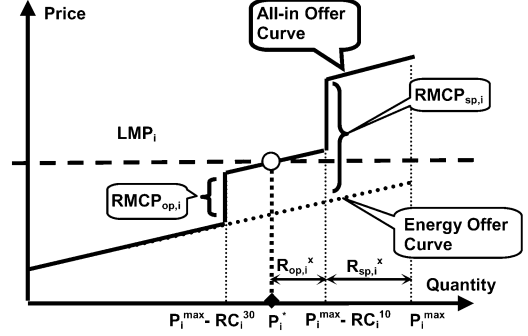

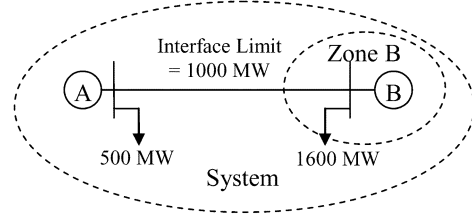Fig. 2. Price and quantity relationship for an online generator.

Fig. 3. Two-zone system diagram.

where the LMP equals to the all-in offer price if the resource is not constrained by any other conditions. Fig. 2 shows the concept of finding the best solution for an online generator facing both energy and reserve markets.

Like the generators, demand bids also have the same price-quantity relationship if we consider demands as negative generators.

The proposed presentation of the equivalent energy offer curve to take into account the trade-off between energy and reserve is very helpful in the analysis of the clearing results or bid selection for market participants or the ISO.

VI. NUMERICAL EXAMPLE

To illustrate the zonal reserve modeling and pricing concept, we provide a two-zone example system. The system diagram is shown in Fig. 3.

In this system, there is only one local reserve zone B, which contains generator B. Generator A is located outside of the reserve zone B. The bus loads are constant for all cases below. The interface between buses A and B has limited transfer capability of 1000 MW. Since the focus of this example is to illustrate the reserve modeling concept, only one category of reserve product is procured to meet both system-wide and local reserve requirements, and the system losses are ignored. In addition, zone B's and the system's reserve requirements are constant in the following examples. Traditionally, in electricity markets, the power quantities calculated by economic dispatch are being called "energy" due to consecutive use in settlements that deal with integrated values. Therefore, for convenience, the following examples use $/MWh for the price, and MW for the "energy" cleared for the dispatch interval.

Case 1: In this case, we create a local reserve shortage scenario such that the energy and reserve prices in the local reserve zone are affected by the value of the RCPF. The generators' bid

TABLE I
GENERATORS' BIDS INFORMATION FOR CASE 1

Generator	Min (MW)	Max (MW)	Offer Price ($/MWh)	Maximum Reserve (MW)
A	0	1800	20	1000
B	0	900	25	800

TABLE II
RESERVE INFORMATION FOR CASE 1

Reserve Zone	Reserve Requirement (MWh)	RCPF ($/MWh)
System	550	50
Zone B	500	50

TABLE III
MARKET CLEARING RESULTS FOR CASE 1

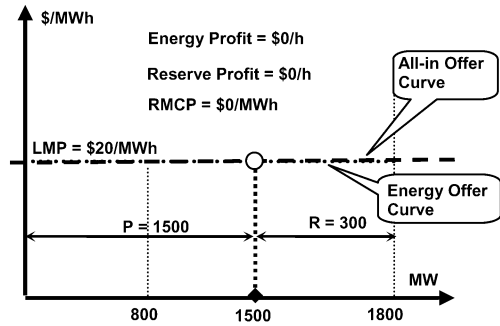| Generator	Cleared Energy (MW)	Cleared Reserve (MW)	Energy Price ($/MWh)	Reserve Price ($/MWh)
A	1500	300	20	0
B	600	300	75	50

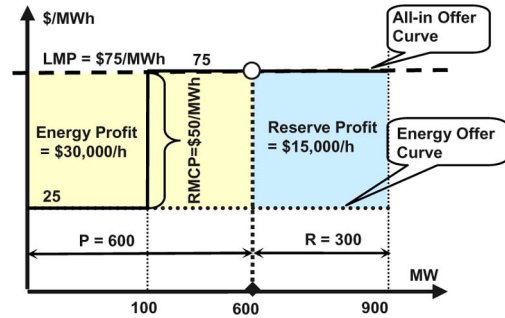Fig. 4. Generator A's profit maximization in Case 1.

Fig. 5. Generator B's profit maximization in Case 1.

TABLE IV
GENERATORS' BIDS INFORMATION FOR CASE 2

Generator	Min (MW)	Max (MW)	Offer Price ($/MWh)	Maximum Reserve (MW)
A	0	1800	20	250
B	0	1200	25	200

information is presented in Table I, and the reserve requirements as well as the associated RCPF values are listed in Table II.

As shown in Tables I and II, the reserve requirement in zone B is 500 MW. The total capacity that is required to serve both load and reserves in zone B is the sum of 1600 MW of load and 500 MW reserve, or 2100 MW in total. However, the total capacity available to zone B is 1000 MW of import and 900 MW of capacity from generator B, or 1900 MW in total. This shows that the system is in the reserve shortage condition, and the RCPF will affect the market clearing prices.

The market clearing results are summarized in Table III. Generator A is relatively cheaper and is dispatched to 1500 MW, and designated for 300 MW of reserve due to the transmission limitation between A and B; generator B is designated 600 MW of energy and 300 MW of reserve. The energy price at A is 20/MWh, which is set by generator A. The total reserve cleared in the system is 600 MW, which is higher than the system reserve requirement 550 MW. Therefore, the reserve price at A is $0/MWh. The reserve price at B is set by the value of RCPF for reserve zone B at $50/MWh, since the system does not have enough reserves (200 MW shortages) that can be delivered to zone B upon the contingency event in zone B. The energy price at B that is $50/MWh above the offer price of generator B is a direct result of the reserve shortage condition in zone B. This is due to the fact that to serve the next MW of energy at B, generator B has to produce 1 MW of energy at a cost of $20/h and reduce 1 MW of reserve contribution at a cost $50/h.

Looking at the market clearing results from each individual resource's point of view, we can use the proposed all-in offer curve. Figs. 4 and 5 illustrate the market clearing results from the all-in offer curves of generators A and B. Since the reserve clearing price for A is $0/MWh, its all-in offer curve is the same

as its incremental energy offer curve. Facing the $20/MWh energy price, generator A could produce at any output level between 0 and 1800 MW. This is because no matter where it generates, the total as-bid profit from energy and reserve is always zero. For generator B, the reserve price is $50/MWh, and its all-in offer curve will be different from its energy offer curve as shown in Fig. 5. According to the $75/MWh energy price, it could generate anywhere between 100 and 900 MW, since its maximum reserve capability is 800 MW. The total as-bid profit from both energy and reserve is $45 000/h, which is the maximum it can achieve.

The analysis also shows how helpful the use of the "all-in" curve in analyzing and explaining the results of co-optimization.

Case 2: In this case, we create a reserve shortage condition for both system and local reserves and study the relationship between RCPF values and reserve market clearing prices. To create such scenario, we reduced the reserve capacity (column "Maximum Reserve" in Table IV) of generators A and B and increased generator B's total capacity to 1200 MW. The generators' bid information is presented in Table IV, and the reserve requirements as well as RCPF values are listed in Table V.

The total capacity from generators A and B is $1800 + 1200$ or 3000 MW, which is higher than the total of system load 2100

TABLE V
RESERVE INFORMATION FOR CASE 2

Reserve Zone	Reserve Requirement (MW)	RCPF ($/MWh)
System	550	100
Zone B	500	50

TABLE VI
MARKET CLEARING RESULTS FOR CASE 2

Generator	Cleared Energy (MW)	Cleared Reserve (MW)	Energy Price ($/MWh)	Reserve Price ($/MWh)
A	1250	250	20	145
B	850	200	25	150

Fig. 6. Generator A's profit maximization in Case 2.

Fig. 7. Generator B's profit maximization in Case 2.

MW and system reserve requirement 550 MW. This indicates that there may not be trade-offs between energy and reserve products. Since the maximum reserve available is $250 + 200$ or 450 MW, which is less than any of the reserve requirements, the system is in the reserve shortage condition.

The market clearing results are summarized in Table VI. Generator A is cleared 1250 MW of energy and 250 MW of reserve; while generator B is cleared 850 MW of energy and 200 MW of reserve. The total energy import to zone B is 750 MW. None of the two units is cleared at their full capacity. No trade-off is present in the energy and reserve designations. The energy price at A is $20/MWh, which is set by generator A; the energy price at B is $25/MWh that is set by generator B, since the cheaper energy from A cannot flow through the interface that leaves 250 MW of capacity for the reserve import to zone B. The reserve price at A is $145/MWh because the increase of 1 MW of reserve supply at A reduces reserve violations for both system ($100/h) and local area ($50/h) through re-dispatching generators A and B at a cost of $5/h to create one more MW of tie-line room to guarantee the deliverability of the additional reserve supply at A. The reserve price at B is $150/MWh, which is the sum of the system and local reserve RCPF values.

In addition, this case shows that the co-optimization optimally allocates the interface capacity 1000 MW between energy import 750 MW and reserve import 250 MW, which is exactly equal to the total reserve available at A. Such interface capacity allocation ensures that reserves cleared at A can be delivered to zone B upon the reserve event in zone B. Furthermore, it also shows that the reserve price can be higher than the energy price due to the decoupling of energy and reserve under limited ramping capacity condition.

As mentioned before, the market clearing results should maximize all generators' as-bid profit. This can be shown in the all-in offer curve developed in Section V-C. Fig. 6 shows the market clearing results for generator A. Since the reserve clearing price for A is $145/MWh, the all-in offer curve is different from its incremental energy offer curve. Facing the $20/MWh energy price, generator A can produce at any output level between 0 and 1550 MW. This is because no matter where
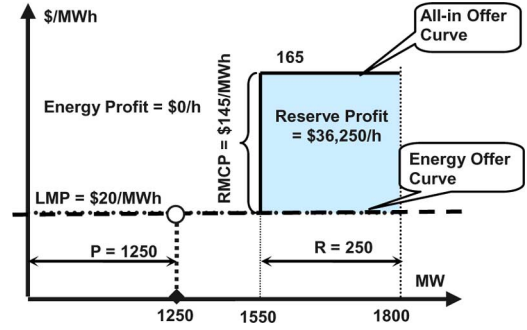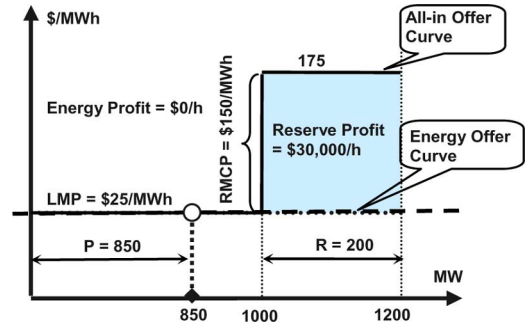
it generates, the as-bid profit from energy is always zero and its reserve profit is already maximized at $36 250/h. For generator B, the reserve price is $150/MWh, and its all-in offer curve is shown in Fig. 7. According to the $25/MWh energy price, it can generate anywhere between 0 and 1000 MW, since the energy profit is zero. The total as-bid profit from both energy and reserve is $30 000/h, the maximum it can achieve. Both generators A and B maximize their as-bid profit and contribute the maximum amount of reserves that they have.

VII. CONCLUSION

This paper presents a new zonal reserve model implemented in the ISO NE's real-time market. In this model, energy and reserve products are cleared in a co-optimized fashion, recognizing the coupling between energy and reserves. The nested zonal reserve model is derived based on the need of the second contingency protection for import-constrained areas, while considering the deliverability of reserves during contingency events. In addition, this model optimally allocates energy and reserves for the reserve zone interface capacity, resulting in more efficient resource allocation for both energy and reserve products. Admittedly, the zonal reserve model cannot fully address the issue of reserve deliverability, since it only considers the reserve zone interface limitation during reserve events. However, such a simplification is a trade-off between modeling accuracy and the bidding convenience. Furthermore, the proposed all-in offer curve presentation simplifies the understanding and analysis of the market clearing results for both market participants and the ISO, and it improves market transparency.

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