Mert Yildiz 1951070 / Tran Luong Bang 1956419/ Gaurav Ramse 1965564

Networking for Big Data and Laboratory / Challenge 2

1.1) Statement and discussion of system queueing model in case of SITA:

In the SITA policy, the interarrival time of the servers will be the probability of the tasks to be assigned to the respective server multiplied by the Λ rate that is the interarrival times of the dispatcher and the interarrival process of the servers will still be Poisson. As it is shown below:

$$P(S_2) = P(L > \theta) = (\frac{b}{\theta})^a \;,\; \lambda_2 = P(S_2)\Lambda,\; P(S_1) = P(L \le \theta) = 1 - P(S_2) \;,\; \lambda_1 = P(S_1)\Lambda$$

1.2) Statement of the optimization problem to find θ*:

In the SITA policy, since we are given the CCDF we have derived the CDF to get the PDF functions of the workload for both server 1 and server 2. Then we found the $E[L_1]$for the range [b,θ] and $E[L_2]$for the range (θ,∞) by taking the integral of the $x\,f_{L_i}(x)$. Lastly, after all calculations we found the mean delay of the system $E[D]$as shown below:

$$f_{L_1}(x) = \frac{ab^a}{P(S_1)x^{a+1}}\mathbb{1}_{x\le\theta} \;,\; f_{L_2}(x) = \frac{ab^a}{P(S_2)x^{a+1}}\mathbb{1}_{x>\theta} E[S_i] = \frac{E[L_i]}{\mu_i} + \frac{\lambda_i\frac{E[L_i^2]}{\mu_i^2}}{2(1-\lambda_i\frac{E[L_i]}{\mu_i})},\; E[D] = P[S_1]E[S_1] + P[S_2]E[S_2]$$

$$E[L_1] = \frac{ab^a(\theta^{1-a} - b^{1-a})}{P(S_1)(1-a)} \;,\; E[L_2] = \frac{ab^a(\theta^{1-a})}{P(S_2)(a-1)} \;,\; E[L_1^2] = \frac{ab^a(\theta^{2-a} - b^{2-a})}{P(S_1)(2-a)} \;,\; E[L_2^2] = \frac{ab^a(\theta^{2-a})}{P(S_2)(a-2)}$$

$$E[D] = (1 - (\frac{b}{\theta})^a)(\frac{E[L_1]}{\mu_1} + \frac{\lambda_1 E[L_1^2]}{2\mu_1(\mu_1-\lambda_1E[L_1])}) + ((\frac{b}{\theta})^a)(\frac{E[L_2]}{\mu_2} + \frac{\lambda_2 E[L_2^2]}{2\mu_2(\mu_2-\lambda_2E[L_2])})$$

The optimization will be done in this way: min $E[D]$ subject to; $\theta\in[b,\infty)$, $L_i > b$ with the stationary boundaries for θ as: $max\left\{b, \left(\frac{\mu_2(a-1)}{b^a a\Lambda}\right)^{\frac{1}{1-a}}\right\}<\theta< \left(\frac{1}{b^{(a-1)}} - \frac{\mu_1(a-1)}{b^a a\Lambda}\right)^{\frac{1}{1-a}}$.

1.3) Statement and discussion of system queueing model in case of RANDOM:

In the RANDOM policy, the interarrival time of the servers can also be generated as in SITA policy. As it is shown below: $\lambda_1 = p\Lambda,\;\; \lambda_2 = (1-p)\Lambda$

1.4) Statement of the optimization problem to find p*:

In the RANDOM policy, the time spend in the queue and in the server will be calculated as the sum$E[L]/\mu_i$which is the service time in the server and $\frac{\lambda_i E[X^2]}{2(1-\lambda_i E[X])}$as shown below. Lastly, the mean delay will be proportional to the probability of the tasks to be assigned to the respective server.

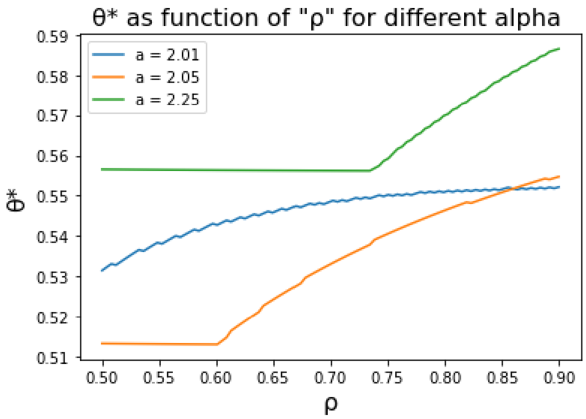
$$E[S_i] = \frac{ab}{\mu_i(a-1)} + \frac{\lambda_i\frac{ab^2}{\mu_i^2(a-2)}}{2(1-\lambda_i\frac{ab}{\mu_i(a-1)})} \;,\quad E[D] = pE[S_1] + (1-p)E[S_2]$$

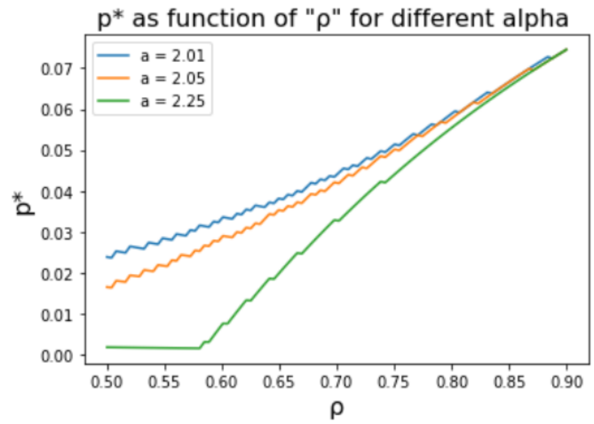
$$E[D] = p(\frac{1}{\mu_1} + \frac{\lambda_1\frac{ab^2}{\mu_1^2(a-2)}}{2(1-\lambda_1\frac{ab}{\mu_1(a-1)})}) + (1-p)(\frac{1}{\mu_2} + \frac{\lambda_2\frac{ab^2}{\mu_2^2(a-2)}}{2(1-\lambda_2\frac{ab}{\mu_2(a-1)})})$$

The optimization will be done in this way: min $E[D]$ subject to; $p\in[0,1]$, with the stationary boundaries for p as: $max\left\{0, 1 - \frac{\mu_2}{\Lambda}\right\} < p < min\left\{1, \frac{\mu_1}{\Lambda}\right\}$.

Mert Yildiz 1951070 / Tran Luong Bang 1956419/ Gaurav Ramse 1965564

2.1) Plots of θ* and p* (two separate plots) as a function of ρ for the three considered values of α;





2.2) Plots of E[D] as a function of ρ for SITA and RANDOM for the three considered values of α (make three different plots, one for each value of α).