UNIVERSIDAD AUTÓNOMA DE CHIHUAHUA

FACULTAD DE INGENIERÍA

MANUAL DE PRÁCTICAS DE MECÁNICA

LABORATORIO DE FÍSICA

LD DIDACTIC

MECÁNICA

ÍNDICE

Acoustic beats

Displaying on the oscilloscope

Objects of the experiments

- Studying acoustic beats resulting from the superposition of tuning-fork oscillations with slightly different frequencies.
- Displaying the beats on the oscilloscope.
- Determining the beat frequency f_b and the frequency f of the superposed oscillation and comparing these frequencies with the individual frequencies f_1 and f_2.

Principles

The wave character of sound becomes obvious when the superposition of two sound waves with equal amplitudes A_1 and A_2 and slightly different frequencies f_1 und f_2 is studied. At the position of the observer an oscillation comes about with the time dependence

$$y(t) = A_1 \cdot \cos(2\pi \cdot f_1 \cdot t + \varphi_1) + A_2 \cdot \cos(2\pi \cdot f_2 \cdot t + \varphi_2) \quad \text{(I)}.$$

The fact that the phases φ_1 and φ_2 of the two individual oscillations are completely arbitrary has been taken into account.

In order to calculate the beat signal, the quantities

$$A = \frac{A_1 + A_2}{2},\ \bar{A} = \frac{A_1 - A_2}{2},\ f = \frac{f_1 + f_2}{2},\ \bar{f} = \frac{f_1 - f_2}{2},$$

$$\varphi = \frac{\varphi_1 + \varphi_2}{2} \text{ und } \bar{\varphi} = \frac{\varphi_1 - \varphi_2}{2}$$

are introduced. After some transformations, the superposed signal is given by

$$y(t) = 2 \cdot A \cdot \cos(2\pi \cdot \bar{f} \cdot t + \bar{\varphi}) \cdot \cos(2\pi \cdot f \cdot t + \varphi)$$
$$- 2 \cdot \bar{A} \cdot \sin(2\pi \cdot \bar{f} \cdot t + \bar{\varphi}) \cdot \sin(2\pi \cdot f \cdot t + \varphi) \quad \text{(II)}$$

This expression becomes even simpler,

$$y(t) = 2 \cdot A \cdot \cos(2\pi \cdot \bar{f} \cdot t + \bar{\varphi}) \cdot \cos(2\pi \cdot f \cdot t + \varphi) \quad \text{(III)},$$

if the two amplitudes A_1 and A_2 agree exactly. In this case, $y(t)$ can be regarded as an oscillation with the frequency f and a time dependent amplitude:

$$y(t) = a(t) \cdot \cos(2\pi \cdot f \cdot t + \varphi) \quad \text{(IV)}$$

with

$$a(t) = 2 \cdot A \cdot \cos(2\pi \cdot \bar{f} \cdot t + \bar{\varphi}) \quad \text{(V)}.$$

The magnitude of the amplitude a varies periodically between 0 and $2A$ (see Fig. 1), the change occurring twice during one period. The number of so-called beats per second, the beat frequency f_b, is therefore

$$f_S = 2 \cdot \bar{f} = f_1 - f_2 \quad \text{(VI)}.$$

When the amplitude $a(t)$ passes zero, the sign change of the beat leads to a phase jump in the superposed oscillation.

In this experiment, the superposition of two sound waves generated with tuning-forks that are slightly out of tune is studied. The beat signal is received with a microphone and then displayed on an oscilloscope. By detuning one tuning-fork the beat frequency f_b is enhanced, or, in other words, the beat period

$$T_b = \frac{1}{f_b} \quad \text{(VII)}$$

is made shorter. The frequencies f_1, f_2, and f_b are determined by measuring the corresponding periods T_1, T_2, and T_b with the oscilloscope.

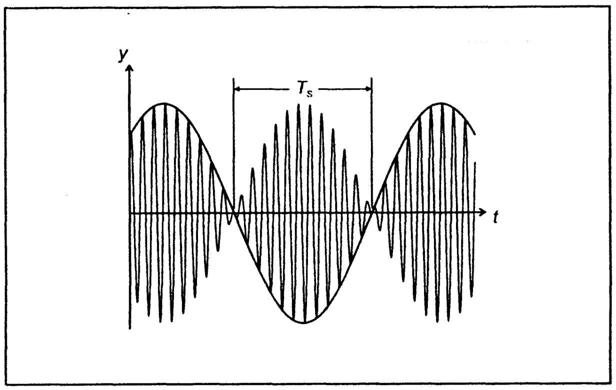


Fig. 1 Acoustic beats when the individual oscillations have the equal amplitudes.

Apparatus

1 pair of resonance tuning-forks, 440 Hz .	414 72
1 multi-purpose microphone	586 26
1 saddle base	300 11
1 two-channel oscilloscope 303	575 21
1 BNC/ 4 mm adapter, 2-pole	575 35

Setup

The experimental setup is illustrated in Fig. 2.

- Set the switch **(a)** of the multi-purpose microphone to ~ .
- Lower the frequency of one tuning-fork by means of the clamping screw **(b)**.
- Put the tuning-forks on the resonance boxes, and direct the openings of the boxes towards the microphone.
- Connect the multi-purpose microphone to the oscilloscope via the BNC/4 mm adapter:

  Zero line: middle
  Coupling: AC
  Scan: 20 mV/DIV.
  Trigger: Auto
  Time base: 20 ms/DIV.

Carrying out the experiment

- Strike the two tuning-forks with the hammer one immediately after the other, and compare the audible beats with the display on the oscilloscope.
- Determine the beat period T_b, and record it.
- Set the time base to 0.5 ms/DIV.
- In order to determine the periods T_1 und T_2, strike each tuning-fork separately while the other is removed from the resonance box.

Measuring example

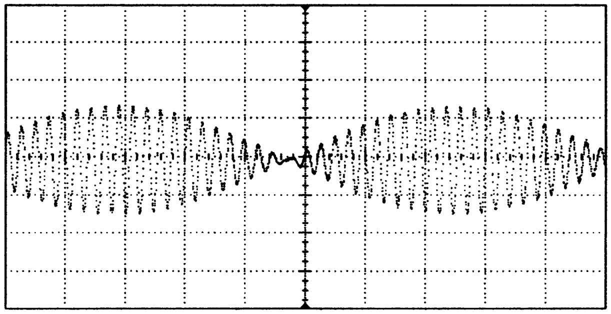
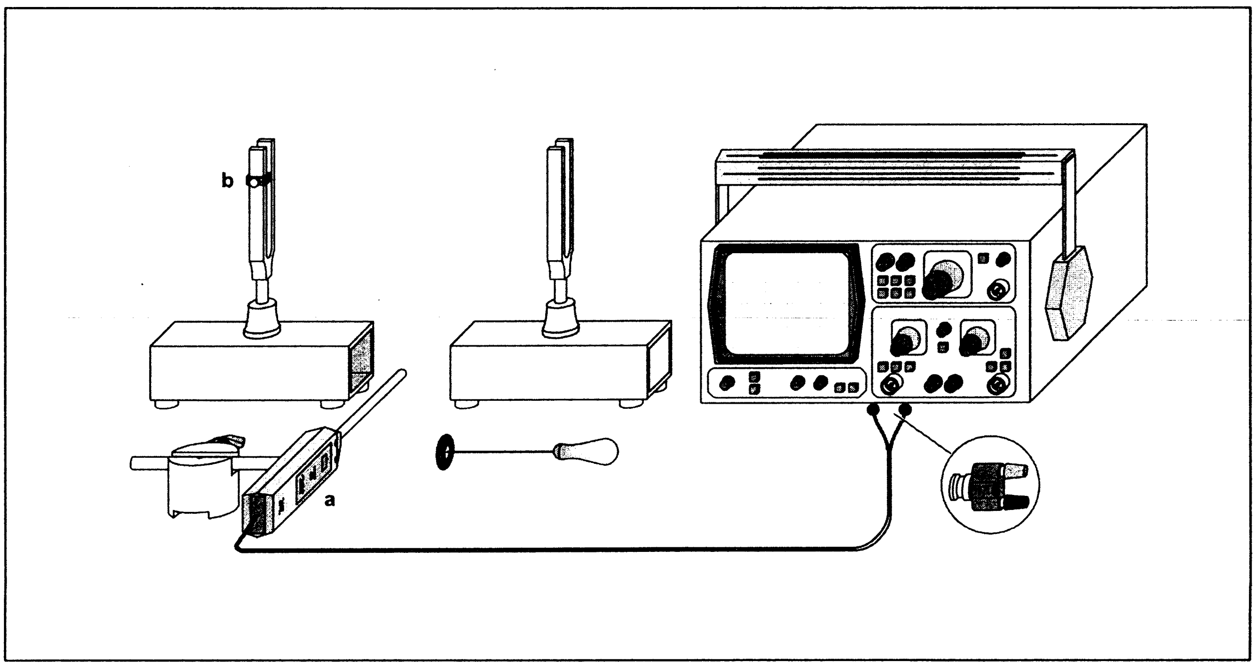


Fig. 3 Experimental setup for displaying acoustic beats on the oscilloscope

Table 1: The individual periods T_1 and T_2 and the beat period T_b:

$\frac{T_1}{\text{ms}}$	$\frac{T_2}{\text{ms}}$	$\frac{T_S}{\text{ms}}$
2.20	2.25	112

Fig. 2 Display of an acoustic beat on the oscilloscope



Evaluation

Table 2: The individual frequencies f_1 and f_2 and the beat frequency f_b

$\frac{f}{Hz}$	$\frac{f}{Hz}$	$\frac{f_S}{Hz}$	$\frac{f_1 - f_2}{Hz}$
455	444	9	11

Eq. (VI) is confirmed by comparing the measured beat frequency f_b with the difference $f_1 - f_2$.

Results

When two acoustic oscillations with a slight difference in frequency are superposed, beats that are clearly audible are generated. These beats can be displayed on an oscilloscope.

Supplementary information

More precise investigations are possible if the beats are recorded with a storage oscilloscope or with the computer-assisted data logging system CASSY.

Reflection of planar ultrasonic waves at a plane surface

Objects of the experiment

- Measuring the reflected intensity at a fixed angle of incidence as a function of the angular position of the receiver.
- Determining the angle of reflection.
- Confirming the relationship "angle of incidence = angle of reflection".

Principles

The aim of this experiment is to prove that the law of reflection "angle of incidence = angle of reflection" also applies for ultrasonic waves. The angle of reflection is defined as the angle between the perpendicular (with respect to the reflecting surface) and the maximum reflected intensity (see Fig. 1).

Two ultrasonic transducers – flexural resonators – serve as the transmitter and receiver, depending on their connection. A piezoelectric body converts electrical to mechanical energy.

When the AC voltage is applied to the piezoelectric body, the transducer configured as a transmitter supplies a sufficiently high sound amplitude at two different resonance frequencies (approx. 40 kHz and 48 kHz). Conversely, sound waves generate mechanical oscillations in the transducer when configured as a receiver. The amplitude of the resulting piezoelectric AC voltage is proportional to the sonic amplitude.

The first transducer, which can be considered as a point-type ultrasonic source, is placed in the focal point of a concave reflector so that a planar wave is formed. The signal of the second transducer, the receiver, is fed to an oscilloscope via an AC amplifier. The square of the amplitude of this signal serves as a measure of the reflected intensity.

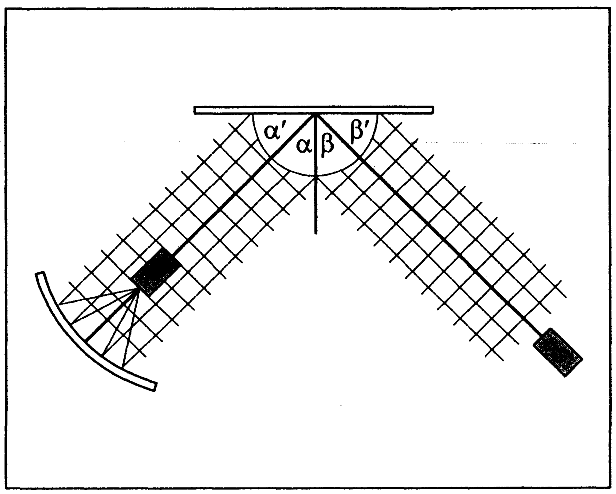


Fig. 1 Reflection of planar ultrasonic waves at a plane surface
α = angle of incidence
β = angle of reflection

Apparatus

2 Ultrasonic transducers, 40 kHz	416 000
1 Generator 40 kHz	416 012
1 AC amplifier	416 010
1 Concave mirror	389 241
1 Sensor holder for concave mirror	416 020
1 Two-channel oscilloscope 303	575 211
1 Screened cable BNC/4 mm	575 24
2 Optical benches with short lateral bracket	460 43
1 Swivel joint with angle scale	460 40
1 Reflection plate	578 66
1 Large stand base, V-shape	300 01
2 Small stand bases, V-shape	300 02
1 Stand rod, 50 cm, 10 mm dia.	301 27
1 Stand rod, 25 cm, 12 mm dia.	300 41
2 Leybold multiclamps	301 01
1 Spirit level, 30 cm long	361 03
1 Steel tape measure, 2 m	311 17

Setup

Set up the experiment as shown in Fig. 2.

First:

- Mount the two optical benches **(a)**, **(b)** on the small stand bases and connect them to the angle scale with the swivel joint **(c)**.
- Slide the stand rod (301 27) through the swivel joint **(c)** (do not fix it in place) and attach it to the large stand base.
- Carefully align the optical benches horizontally; check the alignment with the spirit level.
- Mount the reflection plate on the stand rod.
- Place the assembly consisting of the concave mirror, the sensor holder and the first ultrasonic transducer **(d)** on optical bench **(a)**.
- Attach the second ultrasonic transducer **(e)** to optical bench **(b)** and adjust it to the same height as the first ultrasonic transducer.
- Connect ultrasonic transducer **(d)** to the generator, and set the generator to continuous operation.
- Connect ultrasonic transducer **(e)** to the oscilloscope via the AC amplifier.

Adjustment:

- Set up the assembly so that $\alpha + \beta = 180°$ (see Fig. 1).
- Use the tape measure to align the reflection plate as nearly parallel as possible to the optical bench.
- Reduce the gain of the AC amplifier to minimum and observe the receiver signal on the oscilloscope.
- Adjust the frequency of the generator so that the receiver signal reaches the maximum amplitude.

If the signal is not sinusoidal, i.e. the gain is overdriven:

- Vary the frequency on the generator so that the operating frequency of the transmitter is slightly different from the resonance frequency.

Fine adjustment:

- Adjust the receiver so that it is exactly opposite the transmitter (maximum voltage amplitude of receiver signal).
- Set the transmitter arm to 45°, swivel the receiver arm and measure the voltage amplitude of the receiver signal as a function of the angle of the receiver arm.

If the voltage amplitude shows appreciable secondary maxima in addition to a maximum at approx. 45°:

- Check the adjustment of the ultrasonic transducer.

Carrying out the experiment

Part 1:

- Set a fixed angle $\alpha = 45°$ (see Fig. 1).
- Vary angle β' from 30° to 60° in steps of 1° and measure the voltage amplitude of the receiver signal.
- Write down each value $\beta = 90° - \beta'$ and the respective voltage amplitude in your experiment log.

Part 2:
Confirming the law of reflection

- Set an angle $\alpha' = 80°$.
- To determine the angle of reflection β, vary the angle β' until you find the maximum voltage reading.
- Write down the angle of incidence $\alpha = 90° - \alpha'$ and the angle of reflection $\beta = 90° - \beta'$ in your experiment log.
- Set the next value for α' (Table 2).
- Determine the angle of reflection β.

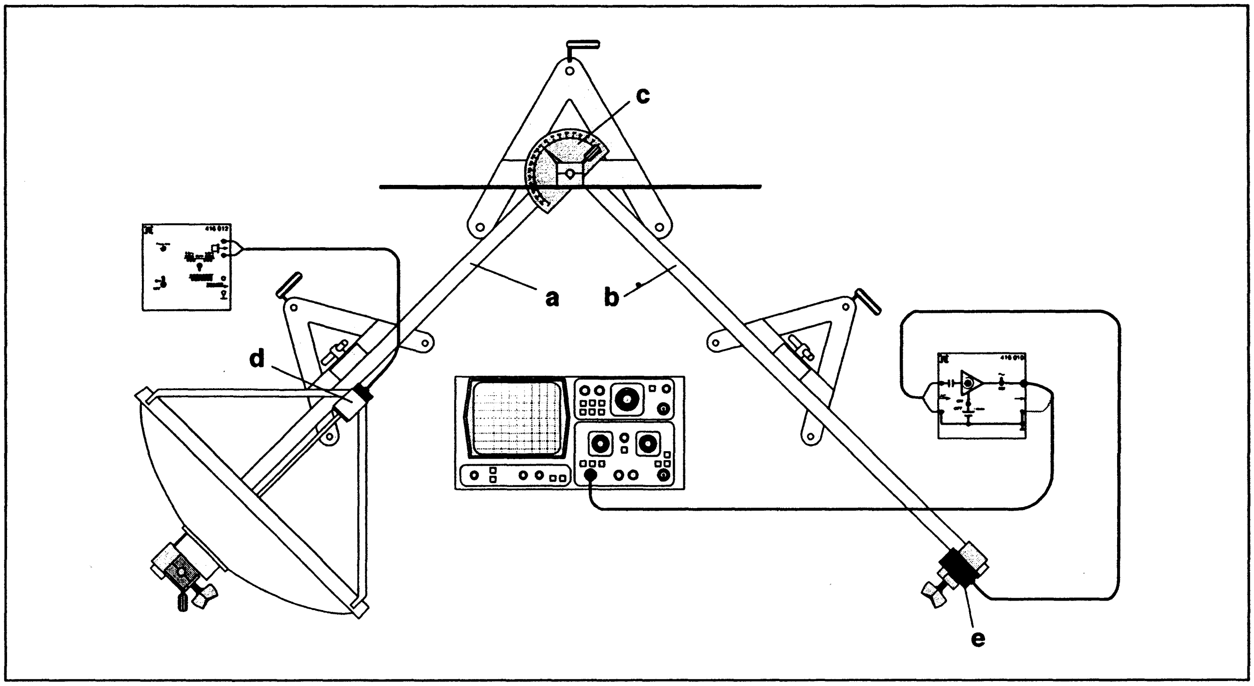


Fig. 2 Experiment setup for reflection of ultrasonic waves, top view

Measuring example

Table 1: Voltage amplitude U of the receiver signal at α = 45° as a function of β.

β	$\frac{U}{V}$
30°	0
31°	0
32°	0
33°	0.05
34°	0.12
35°	0.12
36°	0.1
37°	0.1
38°	0.3
39°	0.4
40°	0.5
41°	0.9
42°	1.25
43°	1.95
44°	2.15
45°	2.3
46°	2.1
47°	1.75
48°	1.25
49°	0.8
50°	0.45
51°	0.4
52°	0.5
53°	0.5
54°	0.5
55°	0.5
56°	0.25
57°	0
58°	0
59°	0
60°	0

Table 2: Angle of reflection β as a function of the angle of incidence α.

$\alpha = 90° - \alpha'$	$\beta = 90° - \beta'$
10°	10°
20°	20°
30°	30.5°
40°	39.5°
45°	44.5°
50°	49°
60°	58°
70°	68.5°
80°	78.5°

Evaluation and results

Fig. 3 shows the graph of the reflected intensity as a function of angle β for a fixed angle α = 45°. The intensity is greatest at the angle of reflection β = 45°. At about 3° above and below the angle of reflection, the intensity diminishes by half, i.e. the half-value width of the intensity distribution is approx. 6°.

Fig. 4 confirms the law of reflection, "angle of incidence α = angle of reflection β". Within the limits of measuring accuracy, the measured values for α and β all lie on a straight line through the origin having a slope of one.

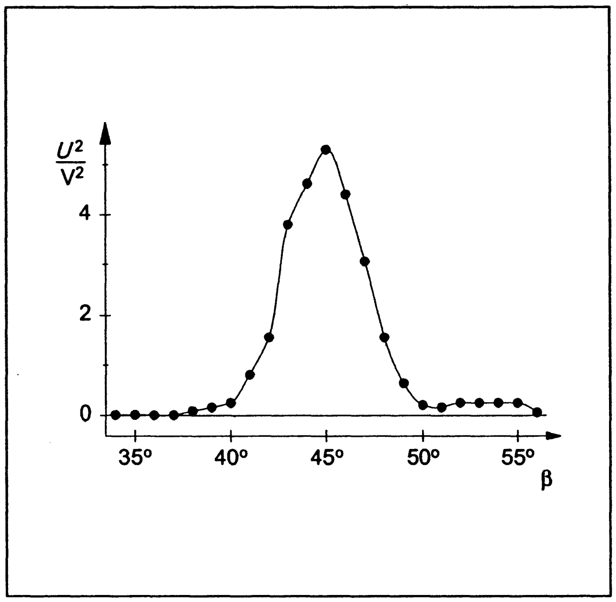


Fig. 3 Square of the voltage amplitude U at the receiver as a function of angle β

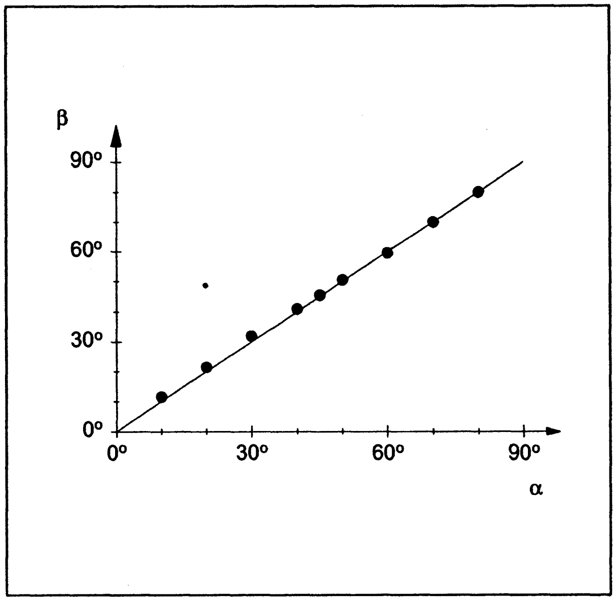


Fig. 4 Angle of reflection β as a function of the angle of incidence α

7

Investigating the Doppler effect with ultrasonic waves

Objects of the experiments

- Measuring the change of frequency perceived by an observer at rest as a function of the velocity v of the source of ultrasonic waves.
- Confirming the proportionality between the change of frequency Δf and the velocity v of the source of ultrasonic waves.
- Determining the velocity of sound c in air.

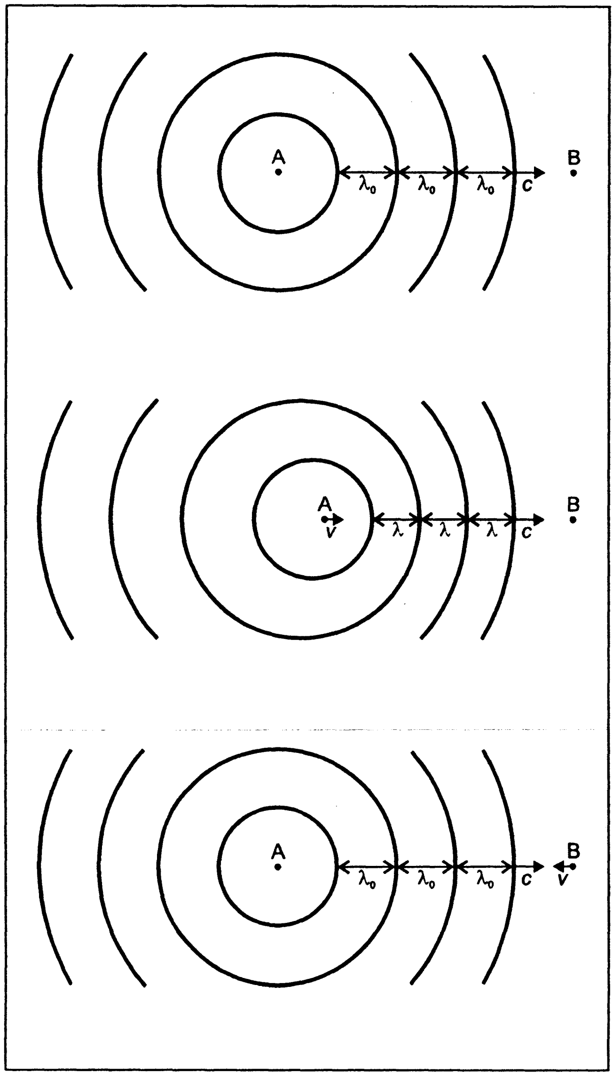


Fig. 1 The propagation of sound with the sound source and the observer at rest (above), with the sound source moving (middle), and with the observer moving (below)

Principles

The acoustic Doppler effect can often be observed in everyday life. For example, the pitch of an ambulance siren is higher while the vehicle is approaching the observer and lower while the vehicle is going away. The pitch changes abruptly at the moment when the vehicle passes the observer. Also an observer moving relatively to the sound source, which is at rest, hears a shifted frequency signal.

In order to understand this effect, consider first the case of a sound source A and an observer B both at rest with respect to the medium of propagation (see Fig. 1). The wave fronts starting from a sound source with a frequency f_0 have a distance λ_0 from each other. They approach the observer at the velocity of sound

$$c = f_0 \cdot \lambda_0 \qquad \text{(I)}$$

and reach him after the time

$$T_0 = \frac{1}{f_0} \qquad \text{(II).}$$

The situation changes when the sound source approaches the observer at the velocity v while the observer is at rest with respect to the medium of propagation. During one period of oscillation T_0, the sound source covers the distance

$$s = v \cdot T_0 \qquad \text{(III);}$$

the distance between the previous wave front and the one just generated is therefore

$$\lambda = \lambda_0 - v \cdot T_0 \qquad \text{(IV).}$$

The wave fronts propagate at the velocity c and reach the observer after the time

$$T = \frac{\lambda}{c} = T_0 \cdot \left(1 - \frac{v}{c}\right) \qquad \text{(V).}$$

Apparatus

2 ultrasonic transducers, 40 kHz	416 000
1 generator 40 kHz	416 012
1 AC-amplifier	416 010
1 trolley with electric drive	337 07
2 Mignon cells 1.5 V	200 66 264
2 precision metal rails, 1 m	460 81
1 rail connector	460 85
1 sets of pairs of feet	460 88
1 digital counter	575 48
1 two-channel oscilloscope 303	575 211
1 stopclock I, 30 s/15 min	313 07
2 saddle base	300 11
1 stand rod, 25 cm	30041
1 stand rod, 47 cm	300 42
1 Leybold multiclamp	301 01
1 clamp with ring	301 10
1 connection lead, 8 m, screened	501 031
1 set of two-way plug adapters, black	501 644
1 measuring cable BNC/4 mm	575 24
connection leads	

For the observer, the frequency emitted by the sound source is thus

$$f = \frac{1}{T} = \frac{f_0}{1 - \frac{v}{c}} \qquad \text{(VI).}$$

If, on the other hand, the observer approaches the sound source at a velocity v while the sound source is at rest, the distance between the wave fronts is λ_0. The wave fronts propagate in the medium at the velocity c, but they reach the observer with the time difference

$$T = \frac{\lambda_0}{c + v} = \frac{T_0}{1 + \frac{v}{c}} \qquad \text{(VII).}$$

For the moving observer, therefore, the frequency of the sound source at rest is

$$f = \frac{1}{T} = f_0 \cdot \left(1 + \frac{v}{c}\right) \qquad \text{(VIII).}$$

Eqs. (VI) and (VIII) give different frequencies for high velocities v. At low velocities, however, the difference is negligible. The frequency shift

$$\Delta f = f - f_0 = f_0 \cdot \frac{v}{c} \qquad \text{(IX)}$$

is then proportional to the velocity v.

In the experiment, two equal transducers serve as transmitter (sound source) and receiver (observer) depending on their connection. One transducer is attached to a trolley with electric drive, the other is fixed to a stand rod. The frequency of the observed signal is measured with a high-resolution digital counter. In order to determine the velocity

$$v = \frac{\Delta s}{\Delta t} \qquad \text{(X)}$$

of the moving transducer, the time Δt in which the trolley covers a given distance Δs is measured with a stopclock.

Setup

The experimental setup is illustrated in Figs. 2 and 3.

Basic setup:

- Connect the precision metal rails with the rail connector, and support them at both ends with the feet.
- Fix the ultrasonic transducer **(c)** with adhesive tape to the trolley with electric drive in longitudinal direction, and put the trolley onto the precision metal rail.
- Attach the clamp with ring **(d)** to the stand rod 47 cm.
- Connect the screened connection lead with the plug adapters to the pair of cables of the ultrasonic transducer, pass it through the ring **(d)**, and connect the free end to the output **(e)** of the generator 40 kHz.
- Fix the ultrasonic transducer **(f)** to the stand rod 25 cm with the Leybold multiclamp, connect it to the input **(g)** of the AC-amplifier, and align it so that both transducers are opposite each other at the same height.
- To avoid disturbing interferences with ultrasonic waves reflected at the metal rail, wrap both ultrasonic transducers with paper board or paper projecting 10 cm ahead.

Adjusting the resonance frequency:

- Set the generator 40 kHz to continuous operation and the AC-amplifier to "~".
- Switch both devices on, and wait 15 minutes until the operation is stable.
- Feed the output signal of the AC-amplifier into the oscilloscope via the measuring cable BNC/4 mm (see Fig. 2).
- Observe the output signal with the oscilloscope and improve the alignment of the two ultrasonic transducers.
- Adjust the frequency at the generator 40 kHz so that the output signal has maximum amplitude (resonance frequency).
- At the maximum distance of the trolley set the amplitude of the output signal to about 0.7 V by adjusting the amplification of the AC-amplifier.

Measuring the frequency of the ultrasonic transducer:

- Switch on the digital counter, feed the output signal of the AC-amplifier via the measuring cable BNC/4 mm into input B (see Fig. 3), and press key B.
- Press the key Frequency, and choose the unit Hz.
- Set the input threshold at input B to 0.7 V with the rotary potentiometer **(h)**.

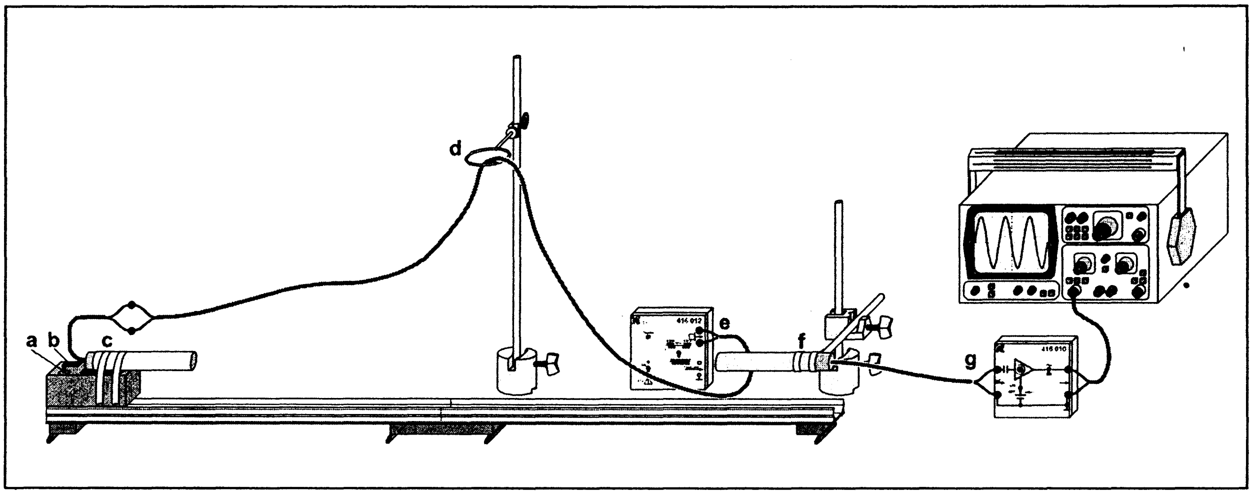


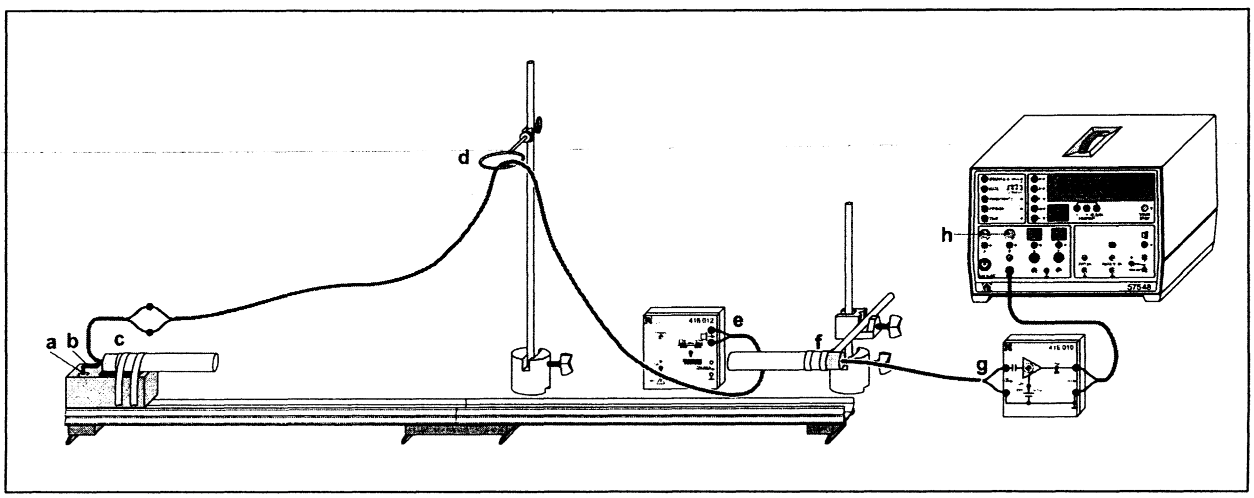
Fig. 2 Setup for adjusting the resonance frequency

Carrying out the experiment

Measuring the change of frequency with the source of ultrasonic waves moving.

- Set the velocity v of the trolley with the potentiometer **(a)**.
- Switch on the drive motor with the three-step switch **(b)**. To determine the velocity measure the time Δt in which the trolley passes a distance Δs of e.g. 1 m, and record it.
- Switch off the drive motor with the three-step switch, start the frequency measurement with the key Start Stop of the digital counter f_0, and stop it by pressing the key again.
- Press the three-step switch, measure the frequency f when the trolley moves "to the right" at a known velocity, and record the frequency.
- Switch the drive motor off with the three-step switch, and determine the rest frequency f_0 again.
- Press the three-step switch, measure the frequency f when the trolley moves with some velocity "to the left", and record it.
- Repeat the frequency measurements with the trolley moving "to the right" and "to the left".
- Set the velocity of the trolley to a smaller value. First measure the velocity v, then carry out the frequency measurements with the trolley moving "to the right" and "to the left".
- Repeat the measurements for two other velocities v.

Fig. 3 Experimental setup for investigating the Doppler effect with ultrasonic waves with the sound source moving



Measuring example

Measuring distance: Δs = 1 m

Table 1: Compilation of the measuring values of the unshifted frequency f_0 and the shifted frequency f

$\frac{\Delta t}{s}$	direction	$\frac{f_0}{Hz}$	$\frac{f}{Hz}$
5.0	right	40144	40168
	left	40143	40121
	right	40142	40166
	left	40142	40117
6.0	right	40188	40207
	left	40186	40165
	right	40186	40204
	left	40185	40166
7.7	right	40193	40207
	left	40187	40171
	right	40185	40201
	left	40183	40167
10.0	right	40147	40157
	left	40147	40135
	right	40147	40157
	left	40146	40136

Evaluation and results

Determining the change of frequency:

Table 1 contains two pairs of measuring values f and f_0 associated with one velocity v or running time Δt of the trolley respectively (the sign taken into account). After the differences $\Delta f = f - f_0$ have been calculated, their mean values are determined. The result is summarized in Table 2. The velocities v calculated from the measuring distance Δs and the running time Δt are assigned to the corresponding frequency shifts.

Tab. 2: The change of frequency $\Delta f = f - f_0$ as a function of the velocity v of the trolley.

$\frac{v}{ms^{-1}}$	$\frac{\Delta f}{Hz}$
–0.2	–23.5
–0.17	–20
–0.13	–16
–0.1	–11
0.1	10
0.13	15
0.17	18.5
0.2	24

Confirming the proportionality between the change of frequency and the velocity:

Fig. 4 is a plot of the values of Table 2. Within the accuracy of measurement they lie on a straight line through the origin. Thus the change of frequency is proportional to the velocity of the sound source.

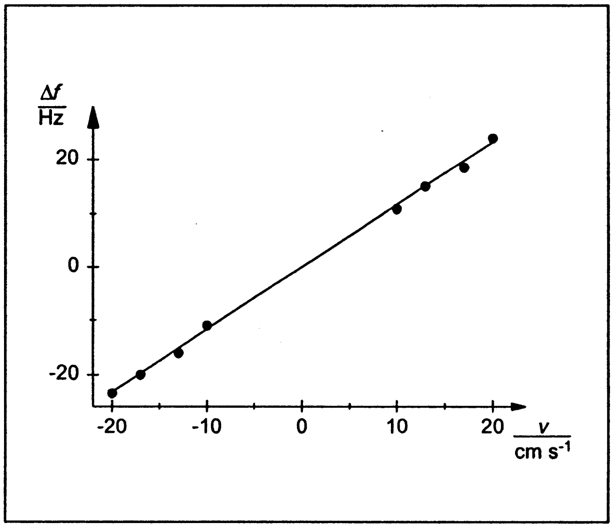


Fig. 4 The change of frequency $\Delta f = f - f_0$ as a function of the velocity v of the trolley.

Determining the velocity of sound in air:

The slope of the straight line drawn through the origin in Fig. 4 is

$$\frac{f_0}{c} = 1.162 \frac{\text{Hz}}{\text{cm s}^{-1}}$$

The mean value of the measured rest frequencies in Table 1 is f_0 = 40165 Hz. With these values the velocity of sound c in air is obtained:

$c = 346 \text{ m s}^{-1}$.

Value quoted in the literature:

$c(25\,°\text{C}) = 346.3 \text{ m s}^{-1}$.

Supplementary information

The experiment for investigating the Doppler effect can also be carried out with a moving observer. In order to do so, the ultrasonic transducer on the trolley has to be connected to the input of the AC-amplifier via the connection lead, and the fixed transducer has to be connected to the output of the generator 40 kHz.

Make certain that the ultrasonic transducers are carefully aligned with each other and that an appropriate amplification is chosen as the amplitude of the output signals must, in any case, exceed the input threshold of the digital counter.

11

Definition of pressure

It will be demonstrated that gases are in a static equilibrium when the quotients of force F and area A, that is to say the pressure $p = \frac{F}{A}$, has the same value at all points.

Unlike solid bodies, liquids and gases cannot maintain transverse forces. Consequently, the equilibrium is not simply characterized by the disappearance of the sum of all forces, but rather by a new value; this is "pressure", defined as the force per unit area.

In the experiments with gases, it is assumed here that they are not subject to the forces of gravity. In this case, the same pressure is to be found at all points in the gas when it is in equilibrium. This fact will be verified in the experiment.

Three air-filled syringes are connected by means of hoses (see Fig. 1) and loaded with various loads. The cross-sectional areas of the syringes are in a ratio of 1:2:4.

The load will be determined for the individual syringes at which the syringes are in mutual equilibrium.

Apparatus:

1 Set of 2 syringes	361 30	Or
1 Syringe 50 ml	361 27	375 58 *
1 Set of 10 weights, 100 g	590 24	
1 Large stand base	300 01	
2 Stand rods, 47 cm	300 42	
1 Universal stand clamp	302 65	*
1 Leybold multiclamps	301 01	*

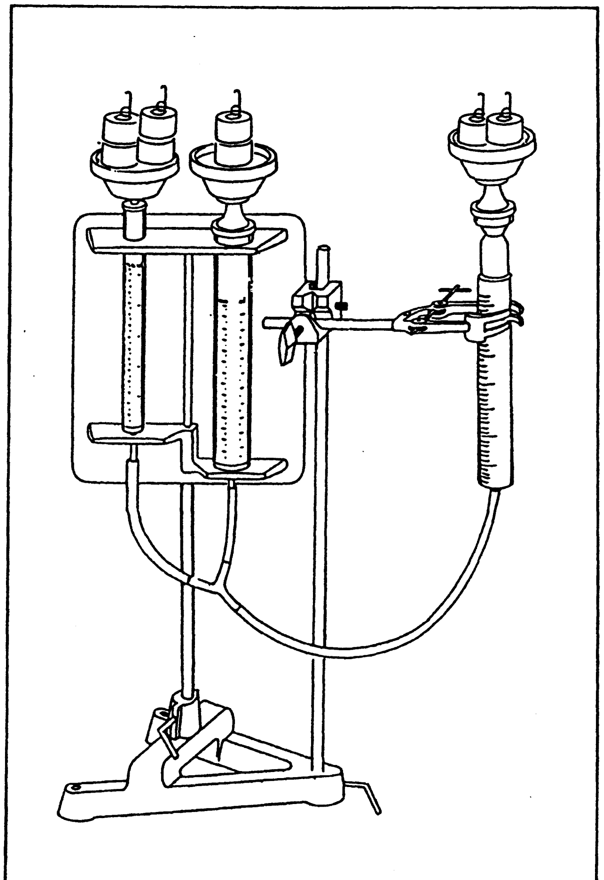


Fig. 1: Experiment setup for the definition of pressure.

Setting up:

Note:

The experiment will only be successful if all syringes are completely clean and free of grease. If necessary, clean the cylinders and syringes with a solvent (acetone, benzine) before the experiment. Never touch the syringe wall with your fingers.

In the experiment setup shown here, the cross-sectional areas of the syringes have a ratio of 1:4:2 from left to right.

Set up the syringes in accordance with Fig. 1, but do not yet place any load upon them. Clamp the limiting thread of the right syringe below the stand clamp. Pull out the center syringe half-way before pushing the hose into the right syringe.

Carrying out the experiment:

Place various combinations of weights on the syringe dishes and check in each case whether the system is in equilibrium. Arrange the weights symmetrically on the dishes so that the syringes do not tilt or stick.

Criterion for equilibrium: the syringes remain stationary if they are released in any position.

Evaluation and results:

An equilibrium is only created if the syringes are loaded with 100 g, 400 g and 200 g from left to right (Fig. 1).

The syringes are clearly in a mutual equilibrium when the load forces are proportional to the areas.

If we call the minimum syringe area A, it follows that
the syringe area = 4 A for the center syringe
and that the syringe area = 2 A for the right syringe.

Thus,

$$p = \frac{100\ g}{A} = \frac{400\ g}{4\ A} = \frac{200\ g}{2\ A}.$$

The quotient $\frac{F}{A}$ is always the same for a gas in equilibrium, i.e. in gases, the same pressure p is to be found everywhere when they are in equilibrium.

Dependence of hydrostatic pressure on depth and direction

It is shown that hydrostatic pressure decreases proportionally with immersion depth and is independent of direction, that is to say a scalar quantity.

Measuring principle
An air-filled metal liquid pressure gauge with a rubber membrane is immersed in a vessel filled with water. The air in the liquid pressure gauge is compressed by the hydrostatic pressure. The pressure is measured with a U-tube manometer and related to the immersion depth.

The liquid pressure gauge can be rotated about two axes so that the side with the membrane can be moved to face any desired direction. In this way, the independence of pressure on direction can be checked.

Equipment list

1 Liquid pressure gauge	361 56
1 Steel tape measure	311 77
1 Colouring soluble	309 42

Setup
Fill the glass container 2/3 full with water. Rotate the liquid pressure gauge until the membrane is at the top, and shift the metal tube *c* until the membrane is immediately below the water surface. Secure with the aid of clamp *e*.

Remove manometer tube *a* (pull it off the metal tube *c* together with the rubber socket *b*) and, ensuring there are no bubbles, half fill it with coloured water.

Wait for temperature equalisation between the air in the liquid pressure gauge and the water (approx. 5 min). Then fit the manometer tube on the metal tube again. When fitting it, incline the manometer tube slightly to the left so that, after it is fitted, both meniscuses are at the same level when the manometer is positioned vertically. Shift scale *d* until both meniscuses are at the zero mark.

Procedure
Lower the liquid pressure gauge in steps and, at each step, measure and make a note of the immersion depth *h* (see Fig. 1) and the height difference Δh of the manometer. In the last measurement, additionally rotate the liquid pressure gauge (insert the hook in the eye and move the liquid pressure gauge) without altering the immersion depth.

Note:
It is simpler, when lowering the liquid pressure gauge, to observe the manometer and to make the measurements when the manometer readings are round numbers, since the depth of immersion can be determined simply with the steel tape measure.

Measurement example

Table 1
Manometer readings Δh at various immersion depths h

h/cm	Δh/cm
1.3	1
2.5	2
3.9	3
5.1	4
6.4	5
7.8	6
8.1	7
10.4	8
11.7	9
13.1	10

Result

If the measured values of Table 1 are plotted, a straight line is obtained (Fig. 2). Immersion depth and hydrostatic pressure are clearly proportional to each other.

As the manometer is filled with water, h and Δh should be identical. The restoring force of the rubber membrane does, however, cause Δh always to be too low. The experiment is therefore not suitable for quantitative determination of hydrostatic pressure.

When the liquid pressure gauge is rotated, the indicated pressure remains constant. Hydrostatic pressure is thus a scalar quantity.

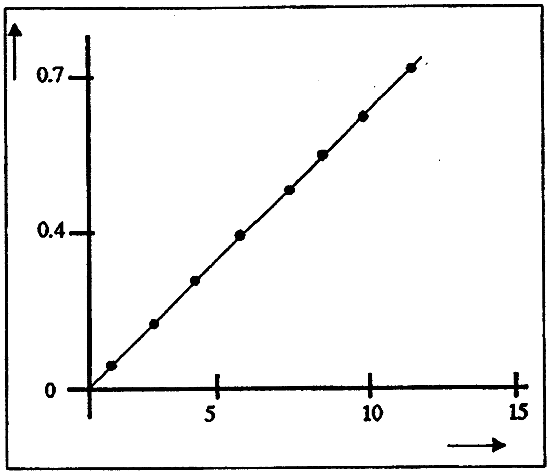
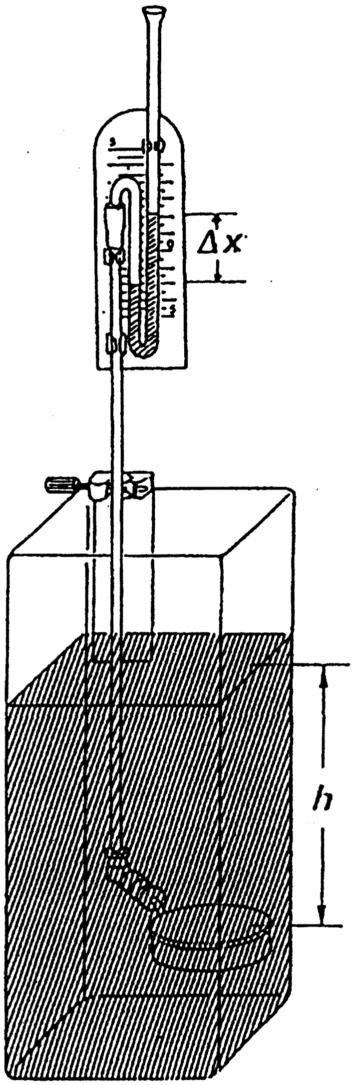
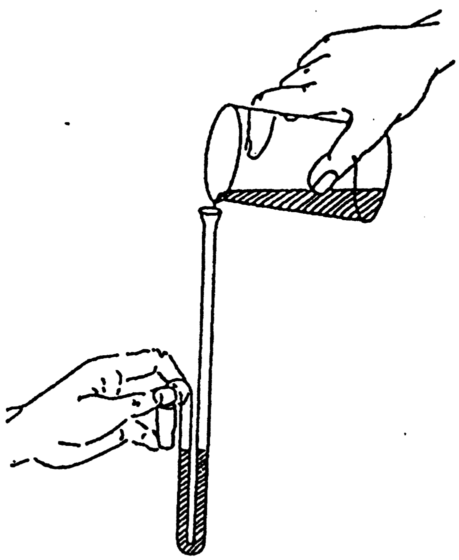


Fig. 2 Measurement example: Manometer reading Δh as a function of immersion depth h.

Fig. 1
Setup





Assembling a falling-ball viscosimeter to determine the viscosity of viscous fluids

Objects of the experiments

- Assembling a falling-ball viscosimeter.
- Determining the viscosity of glycerine.

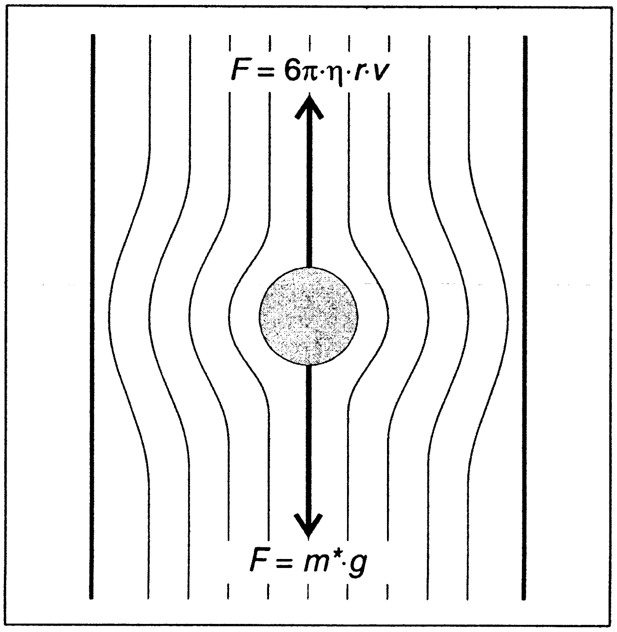
Principles

A body moving in a fluid is acted on by a frictional force in the opposite direction of its velocity. The magnitude of this force depends on the geometry of the body, its velocity, and the internal friction of the fluid. A measure for the internal friction is given by the dynamic viscosity η. For a sphere of radius r moving at velocity v in an infinitely extended fluid of dynamic viscosity η, *G. G. Stokes* derived the frictional force

$$F_1 = 6\pi \cdot \eta \cdot v \cdot r \qquad \text{(I)}.$$

If the sphere falls down vertically in the fluid, it will move at a constant velocity v after a certain time, and there will be an equilibrium between all forces acting on the sphere: the frictional force F_1, which acts upward, the buoyancy force

$$F_2 = \frac{4\pi}{3} \cdot r^3 \cdot \rho_1 \cdot g \qquad \text{(II)},$$



which acts upward too, and the downward acting gravitational force

$$F_3 = \frac{4\pi}{3} \cdot r^3 \cdot \rho_2 \cdot g \qquad \text{(III)}$$

ρ_1: density of the fluid
ρ_2: density of the sphere
g: acceleration of free fall

These forces fulfil the relation

$$F_1 + F_2 = F_3 \qquad \text{(IV)}$$

The viscosity can, therefore, be determined by measuring the rate of fall v.

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g}{v} \qquad \text{(V)}$$

where v is to be determined from the distance s and the time t of fall. The viscosity then is

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g \cdot t}{s} \qquad \text{(VI)}$$

In practice, Eq. (I) has to be corrected since the assumption of an infinitely extended fluid is unrealistic and the velocity distribution of the fluid particles with respect to the surface of the sphere is influenced by the finite dimensions of the fluid. For the movement of the sphere along the axis of a fluid cylinder of radius R and infinite length, for example, the frictional force is

$$F_1 = 6\pi \cdot \eta \cdot r \cdot v \cdot \left(1 + 2.4 \cdot \frac{r}{R}\right) \qquad \text{(VII)}$$

Eq. (V) thus is changed into

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g \cdot t}{s} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}} \qquad \text{(VIII)}$$

If the finite length L of the fluid cylinder is taken into account, there are further corrections of the order $\frac{r}{L}$.

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Apparatus

1 steel ball, 16 mm dia.	200 67 288
1 guinea-and-feather apparatus	379 001
6 glycerine, 99 %, 250 ml	672 121
1 counter P	575 45
1 holding magnet with clamp	336 21
1 low-voltage power supply, 3, 6 ,9 ,12 V	522 16
1 morse key	504 52
1 stand base, V-shape	300 01
1 stand rod, 100 cm	300 44
1 stand rod, 25 cm	300 41
1 Leybold multiclamp	301 01
1 clamp with jaw clamp	301 11
1 steel tape measure, 2 m	311 77
1 pair of magnets, cylindrical	510 48
connection leads	
additionally recommended:	
1 precision vernier callipers	311 54
1 measuring cylinder, 100 ml, plastic	590 08
1 electronic balance LS 200, 200 g: 0,1 g	667 793

Setup

The experimental setup is illustrated in Fig. 1:

- Assemble the stand material.
- Hold the guinea-and-feather apparatus inclined, and fill the glycerine in slowly, if possible without bubbles, almost up to the top.

Note:

Air bubbles in the fluid influence the viscosity and the density. If there are small air bubbles in the fluid after the filling, wait a few hours before carrying out the experiment.

- Fix the guinea-and-feather apparatus in the clamp with jaw clamp **(c)** so that it is propped up on the experiment table.
- Turn the knurled screw **(a)** of the holding magnet down until stopping, so that the iron core **(b)** sticks out of the coil former.
- Connect the holding magnet to the DC output of the low-voltage power supply with the morse key in the line coming from the negative pole so that the connection is closed when the morse key is in the rest position.
- Supply an output voltage of 12 V, and hang the steel ball up on the iron core **(b)**.
- Turn the knurled screw **(a)** upward by about five turns.
- Position the holding magnet with the steel ball above the fluid column in a way that the steel ball is on concenter with the cylinder axis and completely dipped in.
- Make a mark at the guinea-and-feather apparatus some centimetres above its bottom and measure the distance of fall s between the lower edge of the ball and the mark.

Connection of the counter P:

- Connect the ground socket of the counter P to the feeder socket **(d)** of the morse key, the start input to socket **(e)**, and the stop input to socket **(f)**.
- Choose the measuring range ms.

Carrying out the experiment

- Set the counter P to zero by pressing the key "0".
- Trigger off the morse key, and observe the falling ball.
- As soon as the ball has reached the mark **(c)**, release the morse key.
- Read the time of fall t from the counter P and record it.

If the ball does not fall at all or if it falls with a delay:

- Check the connections.
- Turn the iron core a bit upward.
- Choose a lower voltage for the holding magnet.

If the ball falls without the morse key's being triggered:

- Turn the iron core a bit downward.

Repeating the measurement:

- Turn the voltage for the holding magnet to 12 V and turn the knurled screw **(a)** to stop.
- Get grip of the steel ball from outside on the bottom of the vessel with the pair of magnets sticking together (red mark outward), and move the ball slowly upward along the wall of the vessel until it reaches the holding magnet. Using a bent piece of wire, for example, push the ball exactly below the iron core (see Fig. 2).
- Turn the knurled screw upward again, set the counter P to zero, and repeat the measurement of the time of fall.

If the devices recommended in addition are available (see above):

- Determine the inner diameter D of the guinea-and-feather apparatus, and the diameter d and the mass m_2 of the steel ball.
- Put the measuring cylinder on the electronic balance, and counterbalance.
- Fill 100 ml of glycerine from the storage bottle into the measuring cylinder, and determine its mass.

Measuring example

Table 1: times of fall t

n	$\frac{t}{ms}$
1	2084
2	2110
3	2104
4	2036
5	2116

distance of fall: s = 66.6 cm

diameter of the ball: d = 16.0 mm

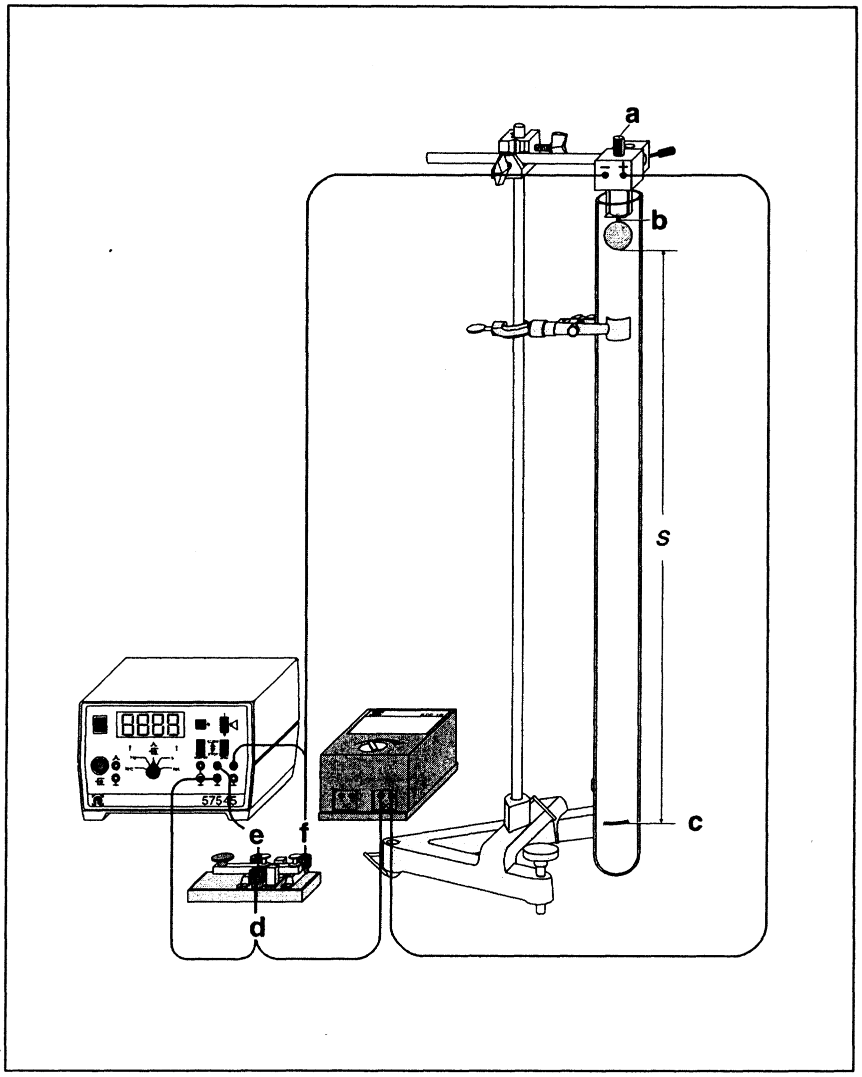
diameter of the guinea-and-feather apparatus: D = 44 mm

mass of the ball: m_2 = 16.7 g

mass of 100 ml of glycerine: m_1 = 125.4 g

*) If these quantities are not measured, use the following values for the further evaluation:

r = 8 mm, R = 22 mm, ρ_1 = 1260 kg m^{-3}, ρ_2 = 7790 kg m^{-3}



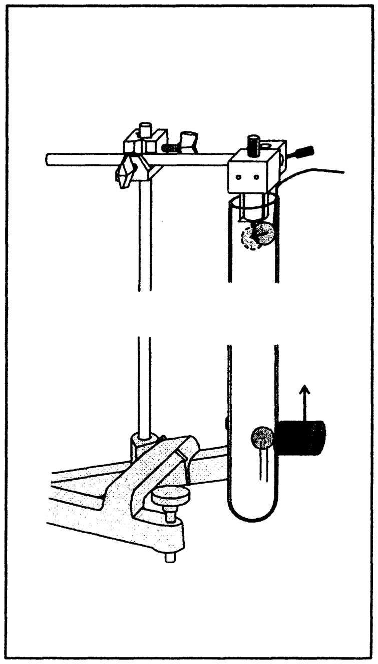


Fig. 2 Returning the steel ball ▲

◀
Fig. 1 Experimental setup for the determination of the viscosity of glycerine

Evaluation and results

Time of fall:

Mean value of the measuring results of Table 1: t = 2.090 s

Density of the ball:

From the measuring results you find

$V_2 = 2.14\ \mathrm{cm}^3$ and $\rho_2 = \frac{m_2}{V_2} = 7787\ \mathrm{kg\ m^{-3}}$

Density of glycerine:

$\rho_1 = 1254\ \mathrm{kg\ m^{-3}}$

According to Eq. (VIII), the viscosity thus is:

$\eta = 1.53\ \mathrm{kg\ m^{-1}\ s^{-1}}$

The value quoted in the literature is (ϑ = 20 °C):

$\eta = 1.480\ \mathrm{kg\ m^{-1}\ s^{-1}}$

When you compare your result with the value taken from the literature, keep in mind that the viscosity of glycerine strongly depends on the temperature.

Supplementary information

After application of Eqs. (II), (III) and (VII), the equation of motion of the falling ball

$$m \cdot \frac{dv}{dt} = F_3 - F_2 - F_1$$

can be converted into the following differential equation:

$$\frac{dv}{dt} = \frac{\rho_2 - \rho_1}{\rho_2} \cdot g - \frac{v}{\tau}$$

with the time constant $\tau = \frac{2}{9} \cdot \frac{r^2 \cdot \rho_2}{\eta} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}}$

With the initial condition $v(t = 0) = 0$, its solution is:

$$v = \frac{2}{9} \cdot \frac{r^2 \cdot (\rho_2 - \rho_1) \cdot g}{\eta} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}} \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$

With the parameters of our measurement you find τ = 39 ms; the total time of fall is 2.1 s. The assumption of a constant rate of fall thus turns out to be a justified approximation.

Flow rate measurement using the Venturi tube

The quantity of air flowing through a Venturi tube will be measured per unit of time by measuring the pressure difference between two points of the tube with known cross-sections.

Bernoulli's law states the relationship between static pressure p and flow velocity v, whereby the following applies to a friction-free, horizontally flowing stream through a stationary flow tube between two points labelled 0 and 1:

$$p_0 + \frac{\rho}{2} v_0^2 = p_1 + \frac{\rho}{2} v_1^2 \qquad (1)$$

ρ is the density of the flow medium.

In the experiment described here, air flows through a Venturi tube whose diameter varies between 100 mm (at both ends) and 50 mm (at the center). The cross-sectional areas are therefore in a ratio of 1 to 4. We shall measure the static pressure p_0 at the tube entrance and the static pressure p_1 at the center of the tube. Due to the incompressibility of air, which we can assume without reservation for the flow velocities occurring here, the following equation applies to the flow velocities v_0 and v_1 and the cross-sectional areas A_0 and A_1 at the two measurement points:

$$v_0 A_0 = v_1 A_1 \text{ (continuity equation)}, \qquad (2)$$

since the products of $v \cdot A$ represent the volume flowing through the tube cross-section per unit of time.

From (1), it follows that:

$$p_0 - p_1 = \frac{\rho}{2}(v_1^2 - v_0^2).$$

If we substitute the velocity v_0 in this equation by

$$v_0 = v_1 \cdot \frac{A_1}{A_0},$$

we can resolve it for the velocity v_1 at the center of the tube:

$$v_1 = \sqrt{\frac{2(p_0 - p_1)}{\rho \cdot \left(1 - \frac{A_1^2}{A_0^2}\right)}} \qquad (3)$$

v_1 can be calculated by means of pressure difference measurement for known cross-sectional areas.

The flow rate to be determined is the product of v_1 and A_1.

Apparatus:

1 Blower	373 04
1 Venturi tube with 7 measurement points	373 09
1 Precision manometer	373 10
1 Pressure sensor	373 13
2 Leybold multiclamps	301 01
1 Stand rod, 47 cm	300 42
1 Small stand base	300 02
1 Saddle base	300 11
1 Stand rod, 25 cm	300 41

Additionally recommended:

1 Multi-manoscope	373 11

Setting up:

Equip the blower with the 100 mm nozzle and the Venturi tube, and set up horizontally on the base (Fig. 1). Support the Venturi nozzle using the saddle base, 25 cm stand rod and clamp (Fig. 2). Do not overtighten screw (d). Mount the precision manometer on stand material and align exactly horizontally using the built-in spirit level.

Connect the overpressure side (a) of the precision manometer to the first measurement outlet by means of a hose and the vacuum side (b) to the center measurement outlet on the Venturi tube.

Carrying out the experiment:

Set the blower to its minimum speed (left limit position of the potentiometer (c) on the blower control unit), and only then switch on. Read off the pressure difference on the manometer and note this.

Important:

The manometer fluid is sucked in and atomized by the Venturi tube if the flow velocity is too high.

Measurement example:

$\Delta p = p_0 - p_1 = 50$ Pa

$\frac{A_1}{A_0} = 0{,}25$

Evaluation and results:

If we insert 1.26 kg · m^{-3} for ρ_{air} in (3), we obtain:

$$v_1 = \sqrt{\frac{2 \cdot 50}{1{,}26\ (1 - 0{,}25^2)}} \cdot m \cdot s^{-1} = 9{,}2\ m\ s^{-1}.$$

Where $A_1 = (0.025)^2 \cdot \pi\ m^2 = 1.9 \cdot 10^{-3}\ m^2$, it

follows that the flow rate is given by

$v_1 \cdot A_1 = 0{,}017\ m^3\ s^{-1} = 17$ l/s.

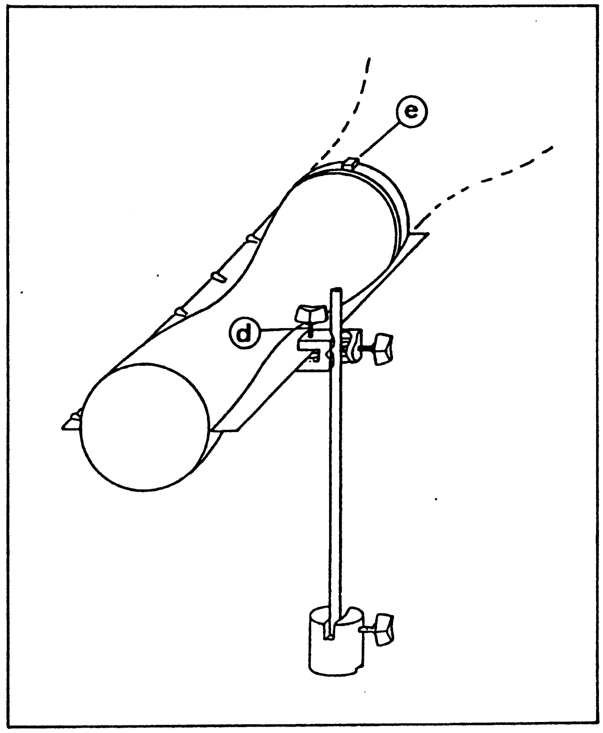


Fig. 2: Mounting the Venturi nozzle on the blower

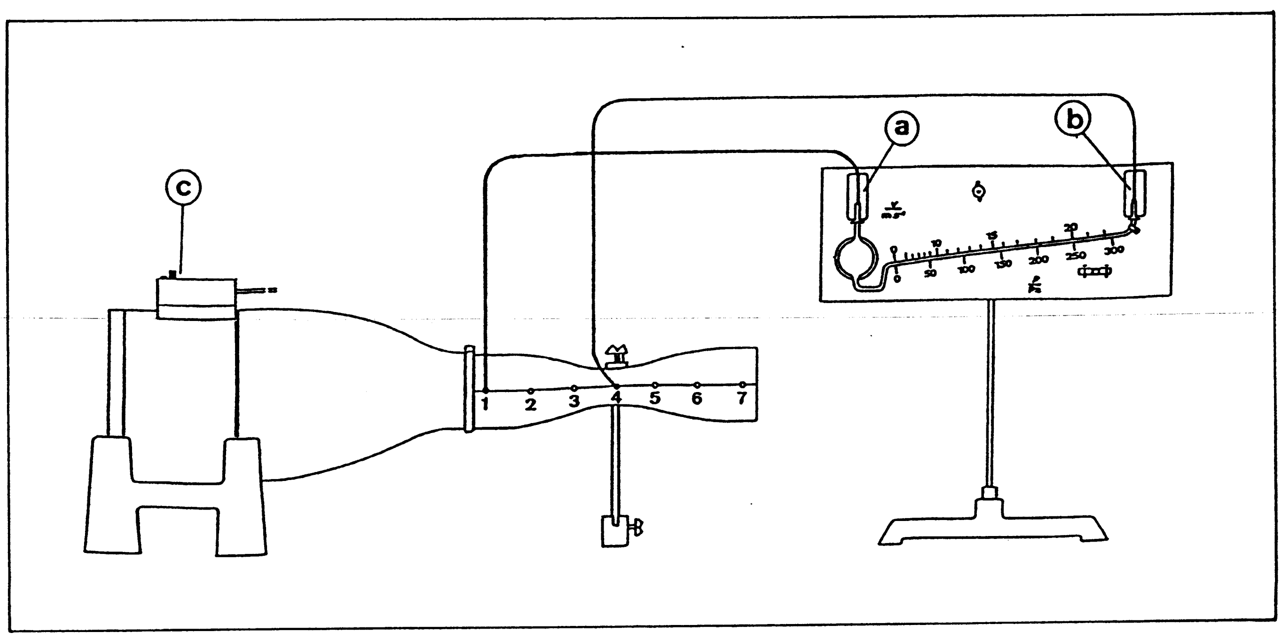


Fig. 1: Experiment setup

Notes:

- The Venturi tube is actually used in engineering to determine the flow rates of liquids or gases.

- The pressure pattern in the Venturi tube can be demonstrated using the multi-manoscope. This can be done in the following way:

Assembling the multi-manoscope.

Press the hoses into the multi-manoscope grooves starting from the bottom so that the assembled device is like that shown in Fig. 3.

Important: Press the bottom 5 cm of the hoses into the grooves particularly firmly.

Connect the Venturi tube measurement points with the multi-manoscope; press the connection nipples firmly into the measurement outlets (Fig. 3), and then mount the Venturi tube on the 100 mm nozzle as shown in Fig. 2 so that the bulge passes over the square nut (e); again use the clamp to support the Venturi tube in horizontal position, but do not overtighten screw (d).

Pour colored water, to which an extremely small quantity of washing-up liquid has been added (max. 1 drop to 500 ml), into the manoscope trough until the liquid is approx. 1 cm high in the hoses.

Switch on the blower with the speed potentiometer (c) (Fig. 1) set to the minimum setting and slowly increase the wind speed until the liquid column in the manometer hose (1) has fallen by approx. 0.5 cm (overpressure display at the blower end of the Venturi tube).

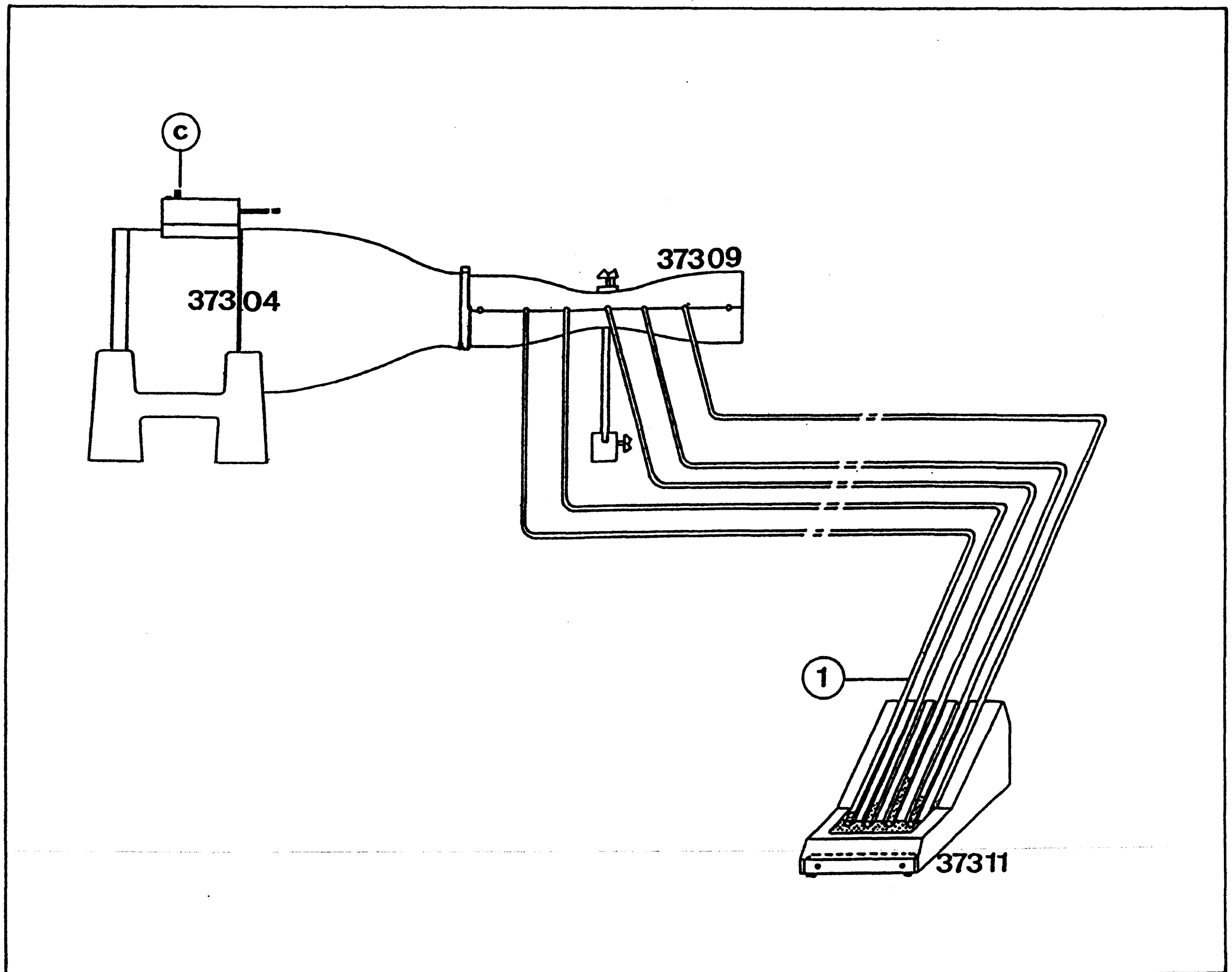


Fig. 3: Connecting the multi-manoscope

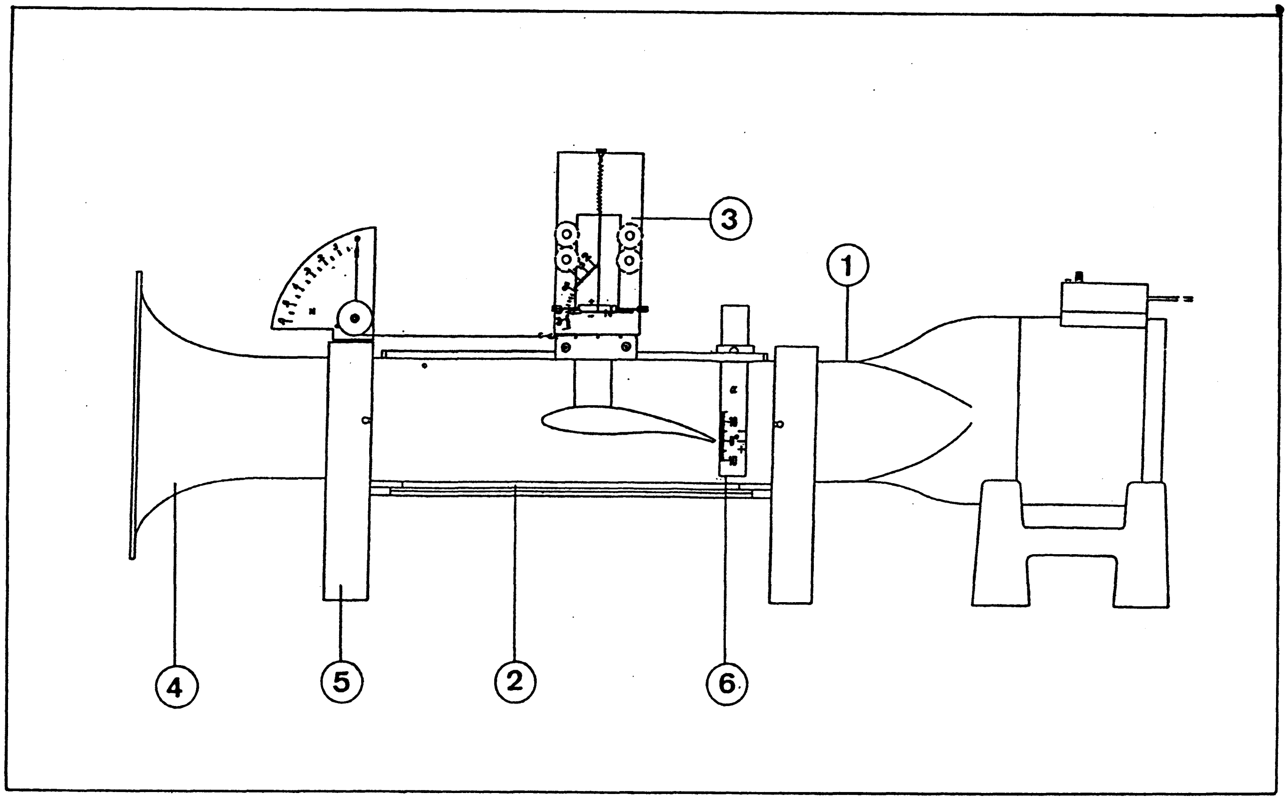


Fig. 2: Experiment setup to measure the resisting force F_W and the lifting force F_a of an

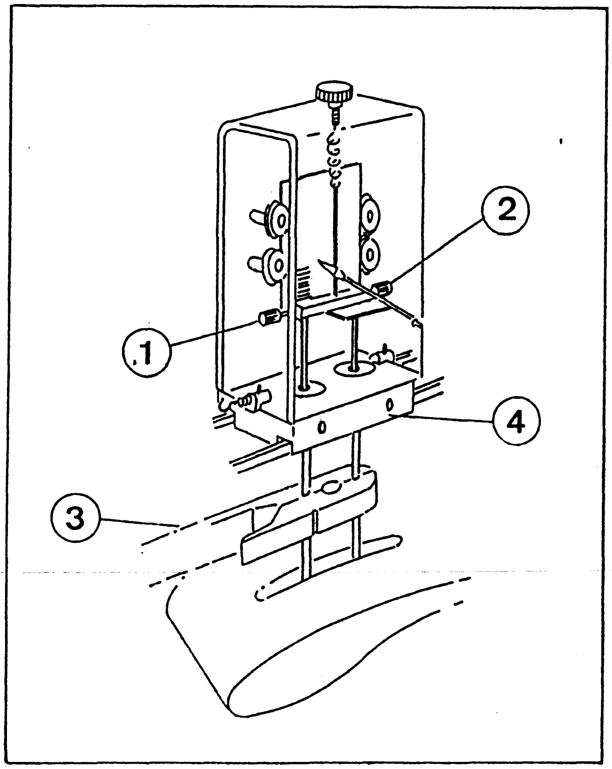


Fig. 3: Mounting the wing on the lift balance
(1) Front knurled screw
(2) Rear knurled screw
(3) Holding clamp
(4) Measurement trolley

<u>Carrying out the experiment:</u>

Set an angle of attack of 15 degrees by unscrewing the front knurled screw on the coupling block and tilting the aerofil wing so that the rear edge touches the 15-degree mark on the angle of attack scale. Then tighten the knurled screw again and remove the angle of attack scale from the wind tunnel.

Switch on the blower and adjust so that a resisting force of 0.5 N is indicated by the dynamometer. Place both paper strips on the slot in the plexiglas cover and push under the measurement trolley so that they just do not touch the holding rods of the aerofoil wing. This prevents air being sucked in through the slots in the plexiglas cover (leave a distance of approx. 5 mm).

Continuously check the position of the paper strips during measurement and adjust if necessary.

Starting at 15 degrees, measure the resisting force and lifting force. Lower the angle of attack step-by-step and carry out measurements in each case. Always adjust the angle of attack when the air velocity is zero (switch off the blower at the mains switch; do not change the position of the potentiometer for adjusting the air velocity; remove the angle of attack scale from the wind tunnel during force measurement). Wait for at least 30 seconds after switching on the blower before each force measurement to enable the blower to reach its maximum speed.

Recording the polar curve of an aerofoil wing in a wind tunnel

The air resisting and lifting force of an aerofoil wing will be measured in a wind tunnel as a function of angle of attack.

The force F exerted by an airstream on an aerofoil wing (air force) can be divided into two components. The component parallel to the flow direction is called the resisting force F_w and the component perpendicular to this, the rising force F_a (cf. Fig. 1). both components depend on the angle of attack α, that is to say the angle between the flow direction and the tangent to the wing on the side (cf. Fig. 1).
In the experiment, F_w and F_a are measured as a function of α at constant flow velocity. If we plot F_a as a function of F_w (curve parameter α), we obtain the so-called "polar curve" of the aerofoil wing, from which we can find out important data on the flight qualities of the profile used.

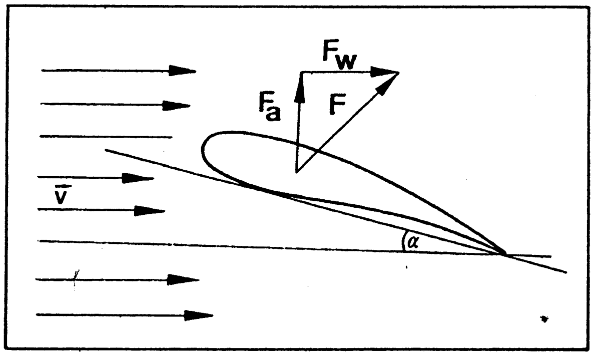


Fig. 1: Forces acting on an aerofoil wing in an airstream

Apparatus:

1 Wind tunnel	373 12
1 Blower	373 04
1 Measurement accessories 1	373 07
1 Measurement accessories 2	373 08
1 Sector dynamometer	373 14

Setting up:

Set up the wind tunnel with blower as shown in Fig. 2. Push the blower into the wind tunnel diffuser (1) so that the air is sucked in through the wind tunnel during the experiment. Ensure that there is a space of approx. 1 m in front of the intake nozzle and behind the blower so that the air can be sucked in by the channel without turbulences. Carry out the experiment without the smoothing filter. Secure the flat base (2) below the plexiglas cover using the four screws.

Mount the lift balance (3) on the measurement trolley from measurement accessories set 1. Attach the hook to the trolley and place on the rail on the plexiglas cover of the wind tunnel. Set up the measurement section horizontally by adjusting the levelling screws on the base of the intake nozzle (4). The measurement trolley should now not start moving independently.
Mount the sector dynamometer on the input gate (5) and set the pointer to zero. Attach the adjustable eye of the dynamometer line to the hook on the measurement trolley and move the trolley along the line so that it is approximately in the center of the measurement section.
Insert the aerofoil wing in the wind tunnel through the intake nozzle.
Push holding rods through the slot in the plexiglas cover. Using the holding clamps from measurement accessory set 2, clamp the aerofoil wing so close to the surface that the holding rods pass through the clamping holes of the holding clamps (Fig. 3). Rise the measurement trolley and lift balance a little and push the aerofoil wing holding rods through the appropriate openings in the measurement trolley from below. Place the trolley on the rails again without releasing the wing. Insert the holding rods in the coupling block of the plexiglas slide in the lift balance, and carefully tighten the knurled screws of this block so that that aerofoil wing is suspended from the lift balance. Remove the holding clamps.

Adjustment:
Hold the aerofoil wing with one hand through the inlet nozzle of the wind tunnel, unscrew the knurled screws of the coupling block with the other hand and adjust the wing so that the tips of the holding rods are over the zero mark of the angle of attack scale on the plexiglas slide. Tighten the knurled screws again.

Suspend the second angle of attack scale (demonstration scale) in the wind tunnel (6) (cf. Fig. 2), and push up to the rear edge of the aerofoil wing. After undoing the knurled screw, move the scale strip of the angle of attack scale in a vertical direction so that the rear edge of the wing points exactly at the zero marking.

Note:
When adjusting the angle of attack during a series of measurements, always unscrew only the front knurled screw of the coupling block in the lift balance. The settings are lost if the rear screw is undone.
Cut out two paper strips (30 mm wide, at least 30 cm long) before starting measurements.

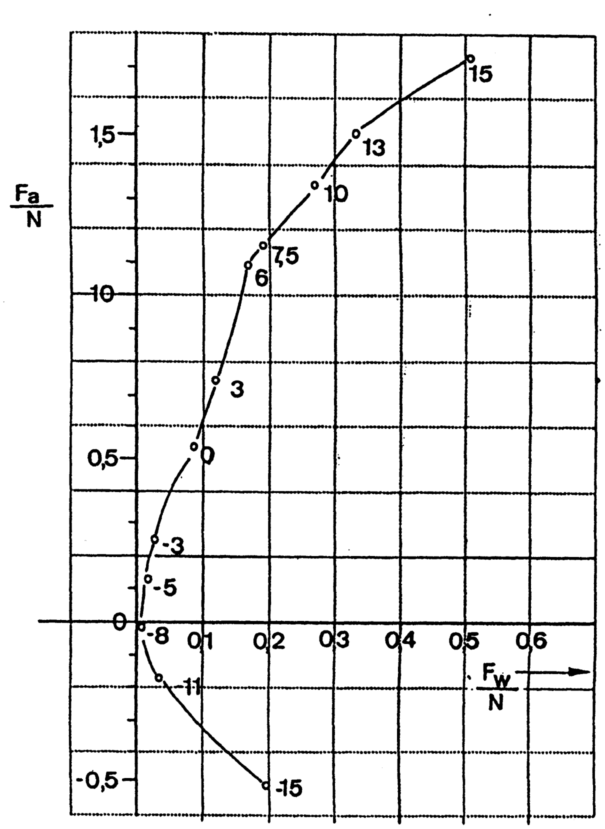


Fig. 4: F_a as a function of F_w; curve parameter: angle of attack α

Strictly speaking, lift coefficient c_a and drag coefficient c_w must be plotted instead of F_a and F_w in a polar diagram. Due to the fact that

$$F_a = \frac{\rho}{2} c_a v^2 A$$

and

$$F_w = \frac{\rho}{2} c_w v^2 A$$

A = Area of the aerofoil wing: ρ = Density of air

this is equivalent to renaming the axis and does not change the shape of the polar curve.

Evaluation and results:

Fig. 4 shows the polar curve of the aerofoil wing profile used here (F_a as a function of F_w).

The aerofoil wing profile used clearly produces a lifting force, even at negative angles of attack.

$F_w > 0$ and $\frac{F_a}{F_w} > 0$

Note:

Errors occur at angles greater than 10°.
The rise of the polar curve and the lifting force in this angle range are due to the influence of the wind tunnel, since its clear width is noticeably blocked by the profile of the aerofoil wing at these angles of attack with the result that the measured values for resisting and lifting forces are too large.

The rise in a real polar diagram becomes smaller in the angle range $\alpha > 10°$, and the polar diagram approaches a maximum (mostly where α is equal to 15° - 16°).

Analysis of polar diagrams enables us to determine the shape an aerofoil wing profile should have in order to produce a minimum quotient $\frac{F_w}{F_a}$ for a given angle of attack α.