EE4070 Numerical Analysis

Final Exam, June 22, 2021

ID: ____________________ Name: ____________________

All numerical answers should have 6 significant digits.

1. (24%) Please find the smallest positive real number, x, that satisfies the following equations.

   1.1. (4%) $\exp(x) + \exp(\exp(x)) + \exp(\exp(\exp(x))) = 20.$

   Answer: $x =$

   1.2. (4%) $\log(x) + \log(\log(x)) + \log(\log(\log(x))) = 3.$

   Answer: $x =$

   1.3. (4%) $\log(x) + \log(x) \cdot \log(x) + \log(x) \cdot \log(x) \cdot \log(x) = 5.$

   Answer: $x =$

   1.4. (4%) $\dfrac{1}{\log(x)} + \dfrac{1}{\log(x)} \cdot \dfrac{1}{\log(x)} + \dfrac{1}{\log(x)} \cdot \dfrac{1}{\log(x)} \cdot \dfrac{1}{\log(x)} = 0.5.$

   Answer: $x =$

   1.5. (4%) $\sin(x) + \sin(\sin(x)) + \sin(\sin(\sin(x))) = 0.9.$

   Answer: $x =$

   1.6. (4%) $\cos(x) + \cos(\cos(x)) + \cos(\cos(\cos(x))) = 2.0.$

   Answer: $x =$

2. (40%) Please find a set of solution for the nonlinear systems shown below. It is suggested to use the zero vector, $\mathbf{x}_0 = \mathbf{0}$, as the <u>initial guess</u> for the <u>Newton's iterations</u>.

2.1. (8%)

$$y = x^2 + 3x - 3.81$$
$$x = 3y^2 + y - 1.07$$

Answer: $x =$
$y =$

2.2. (8%)

$$y = x^3 + x + 1.375$$
$$x = y^3 + 2y - 11.5$$

Answer: $x =$
$y =$

2.3. (8%)

$$y = \sqrt{x+1} - 0.60384$$
$$x = \sqrt{y+2} - 0.943168$$

Answer: $x =$
$y =$

2.4. (8%)

$$y = \frac{1}{x+1} + \frac{5}{3}$$
$$x = \frac{2}{y+1} + \frac{4}{3}$$

Answer: $x =$
$y =$

2.5. (8%)

$$y = \sin(x) + 0.208793$$
$$x = \cos(y) + 0.646404$$

Answer: $x =$
$y =$

3. (36%) The following 2nd order ordinary differential equation needs to be solve.

$$\frac{d^2x}{dt^2} + 0.1 \cdot \frac{dx}{dt} + x = 0.$$

with the initial conditions: $x(0) = 1$ and $\frac{dx(0)}{dt} = 0$. One can define $y_1 = x$ and $y_2 = \frac{dx}{dt}$, then the 2nd order ODE becomes a dynamic system as

$$\frac{dy_1}{dt} = y_2,$$
$$\frac{dy_2}{dt} = -0.1 \cdot y_2 - y_1,$$

with initial conditions: $y_1(0) = 1$ and $y_2(0) = 0$.

3.1. (12%) Please use forward Euler method with time step $h = 0.1$ to solve the ODE and find $x(t)$ and $\frac{dx(t)}{dt}$ at $t = 1, 2, 3$.

$t = 0$	$x(t) = 1$	$dx(t)/dt = 0$
$t = 1$	$x(t) =$	$dx(t)/dt =$
$t = 2$	$x(t) =$	$dx(t)/dt =$
$t = 3$	$x(t) =$	$dx(t)/dt =$

3.2. (12%) Please use backward Euler method with time step $h = 0.1$ to solve the ODE and find $x(t)$ and $\frac{dx(t)}{dt}$ at $t = 1, 2, 3$.

$t = 0$	$x(t) = 1$	$dx(t)/dt = 0$
$t = 1$	$x(t) =$	$dx(t)/dt =$
$t = 2$	$x(t) =$	$dx(t)/dt =$
$t = 3$	$x(t) =$	$dx(t)/dt =$

3.3. (12%) Please use trapezoidal method with time step $h = 0.1$ to solve the ODE and find $x(t)$ and $\frac{dx(t)}{dt}$ at $t = 1, 2, 3$.

$t = 0$	$x(t) = 1$	$dx(t)/dt = 0$
$t = 1$	$x(t) =$	$dx(t)/dt =$
$t = 2$	$x(t) =$	$dx(t)/dt =$
$t = 3$	$x(t) =$	$dx(t)/dt =$