Multiuser Space-Time Block Code System Design

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I. PROBLEM

Design multiuser Space-Time Block Code (STBC) system with six transmitter's users and one receiver. Each transmitter's user, which uses QPSK modulation, has four antennas and can be transmitted in four timeslots. The receiver has six antennas and using four timeslots to receive signals.

II. SOLUTION

A. Motivation

First of all, thank you very much for inspiring me with this problem, as it is well designed and really interesting. It is a non-trivial problem as it includes many aspects, which need to be considered, to improve the performance, i.e., bit error rate (BER).

B. Transmitter Design

Simulation code for this design is file: transmitter.m

As six transmitter's users share four timeslots for transmitting signals, to reduce inter-user interference (and hence, improve the system performance), all users are separated into two groups. Each group has three users. In the first two timeslots, all users in the first group transmit their STBC signals, and each user uses only its first two transmit antennas. Then, in the last two timeslots, all users in the second group take their turn, i.e., they transmit their STBC signals, and each user also uses only its first two transmit antennas.

C. Receiver Design

Simulation code for this design is file: receiver.m

At the receiver, maximum likelihood (ML) detection algorithm is used to detect signals. If only user 1 uses first two antennas to transmit STBC signal in the first two timeslots, then the signals received at the first antenna of the receiver for the first two timeslots can be expressed as

$$
\underbrace{\begin{bmatrix} y_{1,1} \\ y_{2,1}^* \end{bmatrix}}_{\mathbf{y}_1} = \underbrace{\begin{bmatrix} h_{1,1}^{(1)} & h_{2,1}^{(1)} \\ \left(h_{2,1}^{(1)}\right)^* & -\left(h_{1,1}^{(1)}\right)^* \end{bmatrix}}_{\mathbf{A}_1} \cdot \underbrace{\begin{bmatrix} x_{1,1}^{(1)} \\ x_{1,2}^{(1)} \end{bmatrix}}_{\mathbf{x}^{(1)}} + \underbrace{\begin{bmatrix} n_{1,1} \\ n_{2,1}^* \end{bmatrix}}_{\mathbf{n}_1}, \tag{1}
$$

where $\mathbf{y}_1 \triangleq \begin{bmatrix} y_{1,1} & y_{2,1}^* \end{bmatrix}^T$; $h_{i,j}^{(k)}$ is channel coefficient between the i-th antenna of transmitter's user k and the j-th receiver's antenna; $\mathbf{x}^{(1)} \triangleq \begin{bmatrix} x_{1,1}^{(1)} & x_{1,2}^{(1)} \end{bmatrix}^T$ is the vector of transmitted symbols at user 1; and $\mathbf{n}_1$ is AWGN noise's vector.

When users in the first group transmit their STBC signals, and the receiver uses all its six receive antennas, the signals received at the receiver can be represented

$$
\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \\ \mathbf{y}_5 \\ \mathbf{y}_6 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 & \mathbf{C}_1 \\ \mathbf{A}_2 & \mathbf{B}_2 & \mathbf{C}_2 \\ \mathbf{A}_3 & \mathbf{B}_3 & \mathbf{C}_3 \\ \mathbf{A}_4 & \mathbf{B}_4 & \mathbf{C}_4 \\ \mathbf{A}_5 & \mathbf{B}_5 & \mathbf{C}_5 \\ \mathbf{A}_6 & \mathbf{B}_6 & \mathbf{C}_6 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \\ \mathbf{x}^{(3)} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \\ \mathbf{n}_4 \\ \mathbf{n}_5 \\ \mathbf{n}_6 \end{bmatrix}, \tag{2}
$$

where $\mathbf{y}_i \triangleq \begin{bmatrix} y_{1,i} & y_{2,i}^* \end{bmatrix}^T$ for $i = 1, 2, ..., 6$, is the received vector at antenna i in the first and second timeslots; $\mathbf{x}^{(j)} \triangleq \begin{bmatrix} x_{1,1}^{(j)} & x_{1,2}^{(j)} \end{bmatrix}^T$ for $j = 1, 2, 3$, is the vector of transmitted symbols of user j; $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ for $i = 1, 2, ..., 6$, are channel matrices between the transmitter's user 1, 2, 3 and the i-th receiver's antenna, respectively. Note that all channel matrices $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ are formed by using a similar format as $\mathbf{A}_1$ given in (1), i.e.,

$$
\mathbf{A}_i \triangleq \begin{bmatrix} h_{1,i}^{(1)} & h_{2,i}^{(1)} \\ \left(h_{2,i}^{(1)}\right)^* & -\left(h_{1,i}^{(1)}\right)^* \end{bmatrix}, \tag{3}
$$

$$
\mathbf{B}_i \triangleq \begin{bmatrix} h_{1,i}^{(2)} & h_{2,i}^{(2)} \\ \left(h_{2,i}^{(2)}\right)^* & -\left(h_{1,i}^{(2)}\right)^* \end{bmatrix}, \tag{4}
$$

and

$$
\mathbf{C}_i \triangleq \begin{bmatrix} h_{1,i}^{(3)} & h_{2,i}^{(3)} \\ \left(h_{2,i}^{(3)}\right)^* & -\left(h_{1,i}^{(3)}\right)^* \end{bmatrix}. \tag{5}
$$

Then, ML detection algorithm for simultaneously detecting signals of first three users in the first group is

$$\underset{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}}{\operatorname{argmin}} \left\| \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \\ \mathbf{y}_5 \\ \mathbf{y}_6 \end{bmatrix} - \begin{bmatrix} \mathbf{A}_1 \ \mathbf{B}_1 \ \mathbf{C}_1 \\ \mathbf{A}_2 \ \mathbf{B}_2 \ \mathbf{C}_2 \\ \mathbf{A}_3 \ \mathbf{B}_3 \ \mathbf{C}_3 \\ \mathbf{A}_4 \ \mathbf{B}_4 \ \mathbf{C}_4 \\ \mathbf{A}_5 \ \mathbf{B}_5 \ \mathbf{C}_5 \\ \mathbf{A}_6 \ \mathbf{B}_6 \ \mathbf{C}_6 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \\ \mathbf{x}^{(3)} \end{bmatrix} \right\|^2 . \tag{6}$$

Similarity, ML detection algorithm for simultaneously detecting signals of the last three users in the second group is

$$\underset{\mathbf{x}^{(4)}, \mathbf{x}^{(5)}, \mathbf{x}^{(6)}}{\operatorname{argmin}} \left\| \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \mathbf{r}_4 \\ \mathbf{r}_5 \\ \mathbf{r}_6 \end{bmatrix} - \begin{bmatrix} \mathbf{D}_1 \ \mathbf{E}_1 \ \mathbf{F}_1 \\ \mathbf{D}_2 \ \mathbf{E}_2 \ \mathbf{F}_2 \\ \mathbf{D}_3 \ \mathbf{E}_3 \ \mathbf{F}_3 \\ \mathbf{D}_4 \ \mathbf{E}_4 \ \mathbf{F}_4 \\ \mathbf{D}_5 \ \mathbf{E}_5 \ \mathbf{F}_5 \\ \mathbf{D}_6 \ \mathbf{E}_6 \ \mathbf{F}_6 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{(4)} \\ \mathbf{x}^{(5)} \\ \mathbf{x}^{(6)} \end{bmatrix} \right\|^2 , \tag{7}$$

where $\mathbf{r}_i \triangleq \begin{bmatrix} y_{3,i} \ y_{4,i}^* \end{bmatrix}^T$ for $i = 1, 2, ..., 6$, is the received vector at antenna i in the 3-rd and 4-th timeslots; $\mathbf{x}^{(j)} = \begin{bmatrix} x_{1,1}^{(j)} \ x_{1,2}^{(j)} \end{bmatrix}^T$ for $j = 4, 5, 6$, is the vector of transmitted symbols of user j; $\mathbf{D}_i$, $\mathbf{E}_i$, $\mathbf{F}_i$ for $i = 1, 2, ..., 6$, are channel matrices between the transmitter's user 4, 5, 6 and the i-th receiver's antennas, respectively, given by

$$\mathbf{D}_i \triangleq \begin{bmatrix} h_{1,i}^{(4)} & h_{2,i}^{(4)} \\ \left(h_{2,i}^{(4)} \right)^* & -\left(h_{1,i}^{(4)} \right)^* \end{bmatrix}, \tag{8}$$

$$\mathbf{E}_i \triangleq \begin{bmatrix} h_{1,i}^{(5)} & h_{2,i}^{(5)} \\ \left(h_{2,i}^{(5)} \right)^* & -\left(h_{1,i}^{(5)} \right)^* \end{bmatrix}, \tag{9}$$

and

$$\mathbf{F}_i \triangleq \begin{bmatrix} h_{1,i}^{(6)} & h_{2,i}^{(6)} \\ \left(h_{2,i}^{(6)} \right)^* & -\left(h_{1,i}^{(6)} \right)^* \end{bmatrix}. \tag{10}$$

D. Numerical Results and Discussion

1) Numerical results: Simulation codes include 3 files: transmitter.m for transmitter design, receiver.m for receiver design, and pgen.m for generating a combination of a group of numbers. As shown in Figure 1, my model solution achieves BERs of 4.34×10^{-4} for all six users at $1/N_0 = 1$ dB.
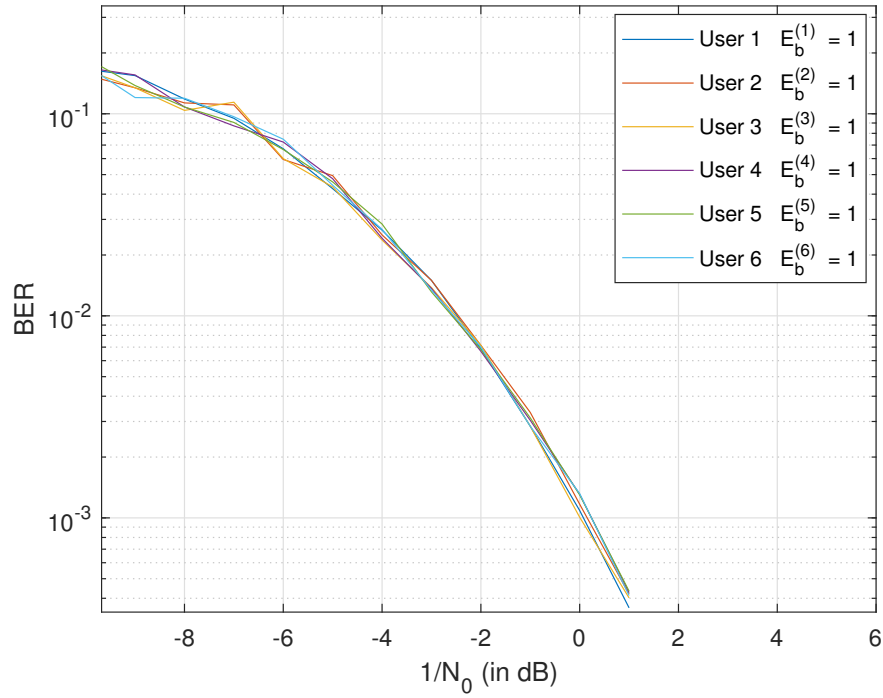
Fig. 1. Performance of the ML detection algorithm.

2) Discussion:

- By choosing suitable design for both the transmitter and receiver, the model can achieve high performance with an acceptable complexity.

- Linear Minimum Mean Square Error (MMSE) detection algorithm can be used for trading off between performance and complexity. However, this technique cannot be applied to solve this problem as signal-to-noise ratio (SNR), which also needs to be known at the receiver, is not provided in this problem, (i.e., provide at the receiver input).

- As the receiver uses ML detection algorithm, the complexity of this algorithm exponentially grows with the number of transmitter's antennas and the order of modulation. The solution may not suitable with systems using large number of antennas and high order modulation.

- The system performance may improve if all antennas of the transmitter are used (so that the array gain, multiplexing gain, and diversity gain can be optimally used).