MRI assignment
BIOM9027 Medical Imaging

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Contents

Constants: Proton gyromagnetic ratio $\gamma = 42.58 MHz/T$

1 Question 1

1.1 a)

MRI uses the magnetic properties of the body to generate detailed images of it. The main chemical compound used is hydrogen, the nucleus consists of a single proton. When the patient is placed in a strong static magnetic field the protons split up into two different energy-states (and hence spin-states) (because of the spin quantum number of hydrogen), i.e the protons align against or along the magnetic field. The magnetic moment will precess about the magnetic field with the Larmor frequency (Larmor precession). Looking at a volume-element (voxel) consisting of many protons and summing up the individual magnetic moments (dipole vectors) yields a macroscopic magnetization (resultant) aligned with the field. As long as this voxel contains enough protons it behaves as a classical dipole (Bulk magnetization).

Since the energy-gap is dependent on the Larmor frequency by introducing transverse magnetization components on-resonance one can deflect the bulk magnetization towards the transverse plane. This simplifies substantially using a rotating frame. The magnetization free precession in the transverse plane is detected by receive coils. The free induction decay signal is projected onto the transverse magnetization component-planes.

After flipping the magnetization into the transverse plane the magnetization will eventually return to the direction of the static field, Spin–lattice relaxation. It's characterised by the spin–lattice relaxation time T1. The transverse magnitization will decay towards zero by spin–spin relaxation. It is characterised by the spin–spin relaxation time T2.

1.2 b)

The signal generated doesn't contain any spatial information. By applying a linear gradient field different spatial locations have different precession frequencies, local magnetization Larmor frequencies. Using this fact you can by the resonance condition choose to excite a specific slice using a excitation field that matches the local Larmor frequency in the object. Through a shift of variables (k-space) in the free precession induction signal there is a Fourier relationship to the local magnetisation. To recover the local magnetisation one has to move around and collect values in k-space. This is done by using the gradient fields and the time in which they are turned on (frequency and phase encoding). By just measuring the free induction decay during a gradient switching the main signal is acquired is the beginning of the switching. Since the gradient switching is not ideal and instantaneous the most important part of the signal is acquired on the gradient ramp which is non-linear and liable to suffer from distortions. To skip the ramp part instead weakens the signal. To shift the most important part of the signal one can use gradient echo. By applying another gradient with opposite polarity and twice the time-length the important part is instead recorded in the middle of the latter gradient.

K-space contains complex valued values, that is a magnitude and phase. Each data point in the space corresponds to a plane-wave with a frequency and direction specified by it's coordinate. The low spatial frequencies (near the centre of k-space) contain the contrast information of the image and high spatial frequencies contains image resolution information. Because of the symmetry of k-space a image can be reconstructed only using half of k-space, allowing time saving techniques as half Fourier or half-echo.

1.3 c)

The main components of an MRI scanner is the main magnet, RF-receiver coils, RF-transmitter coils and Gradient coils.

Main magnet

The purpose of the main magnet is to introduce the main magnetic field. The main magnetic field generates the bulk magnetisation which then can be manipulated by the RF-transmitter and Gradients. Since small variations in the field will result in variations in precession frequencies the field homogeneity must be very high. You make sure of this by using optimised discrete coil geometries and shimming.

Gradient coils

Gradient coils are used to spatially encode the positions of protons. High gradient strength allows, e.g thinner slices, smaller FOV, higher resolution and shorter Echo times. At the same time, especially when dealing with large gradient amplitudes, you want to keep the slew rate low. As in the case with the main magnet you want to avoid non-linearity. The z-gradient is based on Helmholtz geometry with two coils with opposite current coils. The transverse (x- and y-) gradients have a Golay coil configuration. Complex shapes as "Finger-print" coils leads to increased linearity and better gradient strength and slew rate.

RF-transmitter coils

The transmitter coil delivers a RF-field causing the protons to alter their magnetisation alignment relative to the main field. The transmission coil must deliver a uniform homogeneous transmission field, hence a large volume coil in a cylindrical geometry in appropriate.

RF-receiver coils

A short time after a sequence of RF pulses is transmitted, the resonating tissue will respond by returning an RF signal. This signal are picked up by the receiver-coils. The most important criteria for a receive coil is Signal-to-Noise-Ratio, hence the coil should have a small reception, small receiver bandwidth and a placement that yields a beneficial local field. Receive coils are usually built for predefined specific regions. There are two major kinds of receiver coils with different advantages and disadvantages, Volume receive coils and Surface receive coils.

2 Question 2

2.1 a)

Using two independent spin echo sequences S_1 and S_2 with $TR_2 = 2TR_1$ but otherwise identical parameters the relaxation constant T_1 can be quantified.

$$\frac{S_2}{S_1} = \frac{S_0(1 - e^{-\frac{2TR_1}{T_1}})e^{-\frac{TE}{T_2}}}{S_0(1 - e^{-\frac{TR_1}{T_1}})e^{-\frac{TE}{T_2}}} = \frac{(1 - e^{-\frac{2TR_1}{T_1}})}{(1 - e^{-\frac{TR_1}{T_1}})} = 1 + e^{-\frac{TR_1}{T_1}} \implies$$

$$-\frac{TR_1}{T_1} = ln(\frac{S_2}{S_1} - 1) \implies$$

$$T_1 = \frac{TR_1}{ln(\frac{1}{\frac{S_2}{S_1} - 1})}$$

2.2 b)

To acquire the maximum contrast with respect to the repetition time the absolute difference between the signals are maximised.

$$\frac{d(S_2 - S_1)}{dTR_1} = S_0 e^{-\frac{TE}{T_2}}(-\frac{1}{T_1}e^{-\frac{TR_1}{T_1}} + \frac{2}{T_1}e^{-\frac{2TR_1}{T_1}}) = 0 \implies$$

$$e^{-\frac{TR_1}{T_1}} = 2e^{-\frac{2TR_1}{T_1}} \implies$$

$$TR_{1,opt} = T_1 ln(2)$$

So with $T_1 = 1000ms$ the optimal repetition time to maximise contrast is $TR = 0.6931s$.

3 Question 3

3.1 a)

When a frequency encode gradient $\vec{G}$ is applied the precession frequencies will be dependent on position, as stated in Equation 1.

$$v_{local}(\vec{r}) = \gamma(B_0 + \vec{G}\vec{r}) \tag{1}$$

Where $B_0 = 3$T is the main magnetic field. When a gradient in x,y and z direction is applied the precession frequencies change according to Equation 1. The three cases when a gradient of $1\frac{mT}{m}$ is applied are displayed below in Figure 1.

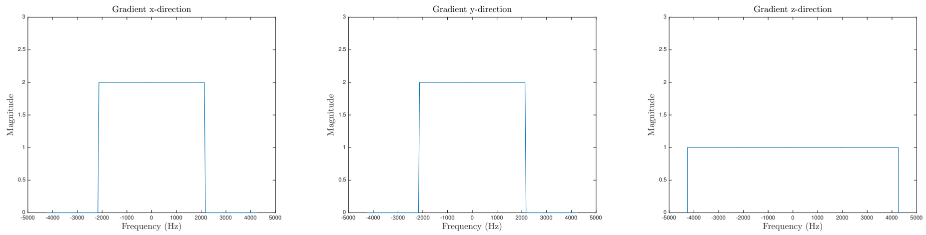

Figure 1: Frequency profiles for gradient in x,y and z-direction relative to the Larmor frequency at the origin $v_{origin} = \gamma B_0 = 127.7Mhz$.

The cutoff of the box functions represent the "end" of the object in the spatial domain. The object boundary frequencies are given by Table 1.

X Gradient	Y Gradient	Z Gradient
-2129 Hz	-2129 Hz	-4258 Hz
2129 Hz	2129 Hz	4258 Hz

Table 1: Object boundary frequencies relative to v_{origin}.

3.2 b)

When gradients are applied in both the x and y direction simultaneously, $|G_x| = |G_y| = \frac{1}{\sqrt{2}}\frac{mT}{m}$, the frequency profile changes. Equation 1 turns into:

$$v_{local}(x, y) = \gamma(B_0 + \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}})$$

And the new frequency profile will look like a triangle centred at the zero frequency (v_{origin}). It is displayed relative to v_{origin} below in Figure 2.

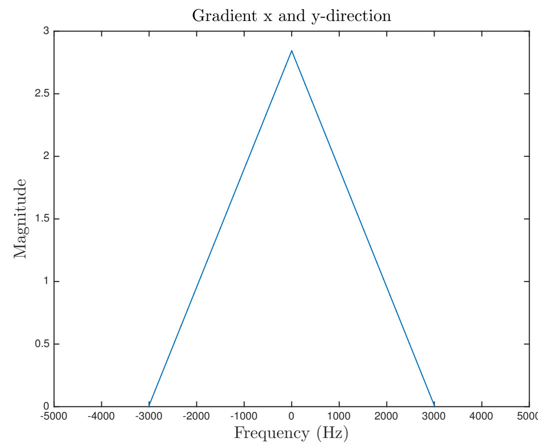

Figure 2: Frequency profile for gradient in x and y-direction relative to v_{origin}.

The new maximum and minimum frequencies is $\pm 3011 Hz$

3.3 c)

When a gradient along x-direction is applied the corresponding coil induction signal is given (in a rotating frame) by Equation 2.

$$S(k_x) = \int_{-\infty}^{\infty} m(x) e^{i2\pi k_x x} dx \tag{2}$$

$$k_x = \gamma \frac{1}{2\pi} G_x t$$

If the magnetisation m(x) is assumed to be homogeneous over the whole sample it can be properly represented as a constant over the interval where the object is defined, i.e a (sort of) rect-function. Also by realising that the induction signal simply is the Fourier transform of the magnetisation and that the Fourier transform of a rect-function is a sinc the following expression is acquired for the signal.

$$S(t) = Asinc(\gamma G_x \triangle xt)$$

The signal is plotted in Figure 3 as if it was acquired during a echo sequence.

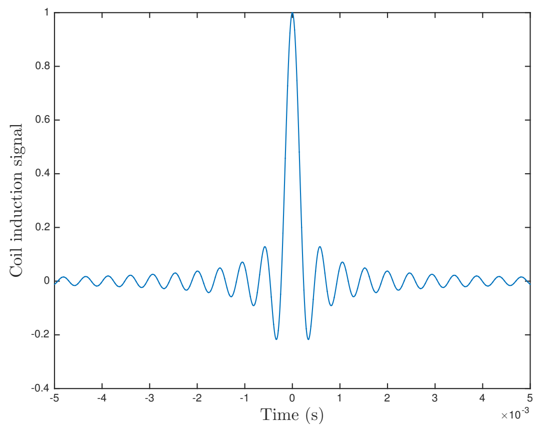

Figure 3: Coil induction signal, the time is relative to the center of the gradient switching time.

3.4 d)

The frequency profiles shown in Figure 1 and 2 is projections of the object. If linear combinations of the gradient field is applied several projections can be acquired, corresponding to taking projections at different angles around the object. By taking the Fourier transform of the received coil signal one gets the wanted projections, note that the Fourier transform of the signal in c) is the frequency profile when an gradient in x-direction is applied. This method of image acquisition corresponds moving radially through k-space. The method described is essentially the same used in CT where you also rotate around the object to acquire projections at different angles. Hence the image can as in CT be reconstructed from the projections through filtered back-projection.

3.5 e)

A tiny iron particle on the middle of one of the quadratic surfaces increases the main magnetic field. In other words the first term of Equation 1 becomes spatially dependent. When an gradient in x or y-direction is applied a portion of the zero frequency components will be increased. When an gradient in z-direction is applied a portion of the highest frequency component will be increased.

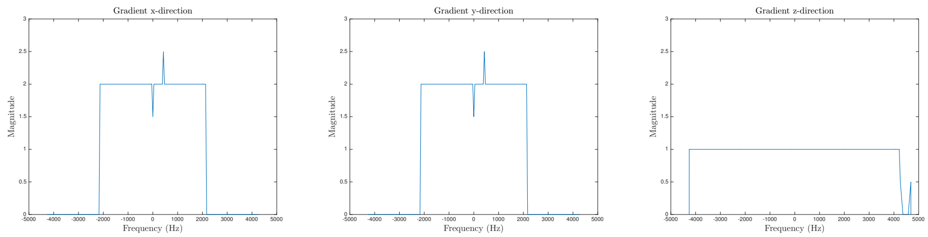

Figure 4: Frequency profiles for gradient in x,y and z-direction with an iron particle, relative to v_{origin}.

The new maximum and minimum frequencies is displayed in Table 2.

X Gradient	Y Gradient	Z Gradient
-2129 Hz	-2129 Hz	-4258 Hz
2129 Hz	2129 Hz	4683 Hz

Table 2: Object boundary frequencies relative to v_{origin}.

3.6 f)

The cuboid consists of two equally large portions of canola oil and water. Due to the canola oil protons there will have an average Larmor frequency shift of -3.3ppm. For the x-gradient case half the frequency profile will have an -3.3ppm shift, i.e the frequency profile will be slightly compressed and shifted to the lower frequencies. In the y and z-gradient case negative frequencies relative to the origin frequency will have a frequency shift of -3.3ppm. The profiles are displayed in Figure 5.

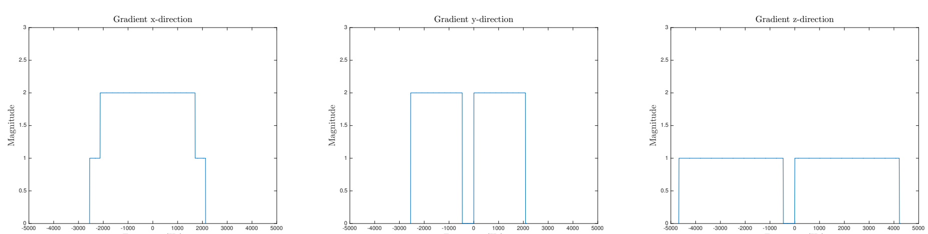

Figure 5: Frequency profiles for gradient in x,y and z-direction with canola oil and water, relative to v_{origin}.

The new maximum and minimum frequencies is displayed in Table 3.

X Gradient	Y Gradient	Z Gradient
-2551 Hz	-2551 Hz	-4679 Hz
2129 Hz	2129 Hz	4258 Hz

Table 3: Object boundary frequencies relative to v_{origin}.

3.7 g)

Fatty tissues

An MRI scanner uses frequency to indicate spatial locations. Hence the difference in frequency, illustrated in Figure 5, due to that fat protons precess slower than water protons maps to an faulty spatial positional difference. As a consequence artifacts as white or dark bands on the sides of the object may arise, the idea is illustrated in Figure 6

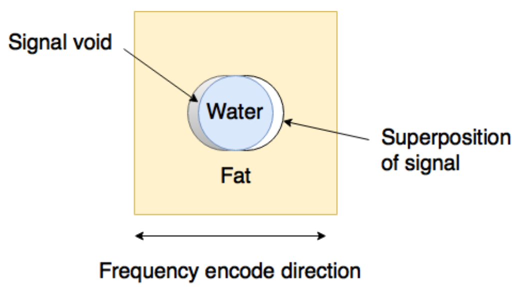

Figure 6: Illustration of how artifacts arise from chemical shift.

To avoid chemical shift one can lower the static magnetic field strength, the shift of frequency becomes lower. Another thing to do is to increase the amplitude of the applied gradient since the relative difference becomes lower. The result of increasing the gradient amplitude from $1\frac{mT}{m}$ to $10\frac{mT}{m}$ is displayed below and shows how the shift is damped in comparison with Figure 5.

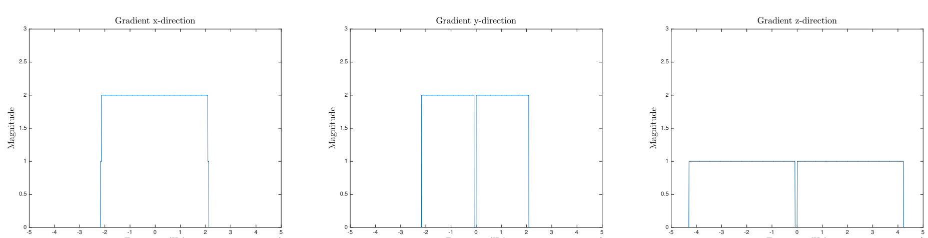

Figure 7: Frequency profiles for increased gradient amplitude in x,y and z-direction with canola oil and water, relative to v_{origin}. Note the increased bandwidth.

Metal screws

When a ferromagnetic metal is present in the object to be imaged the magnetic field applied will suffer inhomogeneities around the metal. This leads to an precession frequency offset in the area close to the metal as seen in Figure 4. This will create a local signal void (the submersion) which is accompanied by an area of high signal intensity (the peak). This translates to bright areas and dark areas in close proximity to the ferromagnetic metal in the image. The artefacts can be minimised by, as in the case with fatty tissues, lowering the static magnetic field or increasing the gradient amplitude.

4 Question 4

4.1 a)

To select a slice of $\delta L = 3mm$ the bandwidth of the RF-pulse $\triangle v = 4kHz$ must match the difference of the local frequencys induced by the gradient field according to Equation 3.

$$\triangle\, v = \gamma G_{Slice} \delta L \tag{3}$$

Solving for the gradient amplitude yields:

$$G_{Slice} = \frac{\triangle v}{\gamma \delta L} = 31.3 \frac{mT}{m}$$

To regain coherent spin phases after the slice selection a re-phasing gradient is applied and Equation 4 has to be satisfied.

$$G_{Slice} t_{Dephase} = G_{RP} t_{Rephase} \tag{4}$$

The time $t_{Dephase}$ is defined as half the RF-pulse duration $t_{rf} = 1.5ms$. Given that the re-phasing gradient amplitude $G_{RP} = 45 \frac{mT}{m}$ is applied one can solve for the rephasing time:

$$t_{Rephase} = \frac{G_{Slice} t_{Dephase}}{G_{RP}} = 0.5219ms \tag{5}$$

4.2 b)

The field of view (FOV_f) and resolution in frequency direction (r_f) can be caculated through Equation 6.

$$r_f = \frac{1}{\triangle K_f^{max}} = \frac{1}{\gamma G_{freq} T_{ADC}} = 0.7828mm/pixel \tag{6}$$

The field of view is then given by mulitplying the resolution by the number of pixels (m=256), $FOV_f = r_f m = 200.4mm$.

4.3 c)

Having a field of view in the phase encoding direction of $FOV_p = 200mm$ implies a resolution in the phase encoding direction of $\triangle p = \frac{200}{m} = 0,7813mm/pixel$. The maximum size or spread of the gradient ($\triangle G_p^{max}$) and the gradient increments ($\triangle G_p$) can then be calculated by Equation 7 - 10.

$$\triangle K_p^{max} = \frac{1}{\triangle p} \tag{7}$$

$$\triangle K_p = \frac{\triangle K_p^{max}}{m} \tag{8}$$

$$\triangle G_p = \frac{\triangle K_p}{\gamma t_p} \tag{9}$$

$$\triangle G_p^{max} = \frac{\triangle K_p^{max}}{\gamma t_p} \tag{10}$$

The equations yields $\triangle G_p = 0.12 \frac{mT}{m}$ and $\triangle G_p^{max} = 30 \frac{mT}{m}$.

4.4 d)

The phase encoding gradient cannot cant be switched instantaneously but is ramped. Then the effective gradient applied is given by the integral over the time in which it is applied, i.e if the amplitude of the gradient at the platue part is G_p the effective gradient is given by $G_{p,app} = \frac{3}{2} G_p t_p$, where t_p is the time it stays in the plateau phase.

Hence the gradient increment is now given by the relation $\frac{3}{2} \triangle G_p \gamma t_p = \triangle K_p \Longrightarrow \triangle G_p = \frac{2}{3} \frac{\triangle K_p}{\gamma t_p} = \frac{2}{3} 0.12 \frac{mT}{m} = 0.078 \frac{mT}{m}$.

4.5 e)

The minimum echo time achievable is given by using the maximum gradient strength were possible to minimise the times that makes up the echo time. The time during the slice pre-phasing is given by 4.1 a), i.e $t_{slicerephase} = 0.5329ms$. Using the maximum gradient amplitude for G_p^{max} and G_{freq} the phase encoding can be done in $t_p = \frac{\triangle K_p^{max}}{\gamma G_p^{max}} = 0.6680ms$ and the signal acquisition in $T_{ADC} = \frac{1}{r_f \gamma G_{freq}} = 0.6667ms$. Hence the echo time TE is given by Equation 11.

$$TE = \frac{t_{rf}}{2} + t_{slicerephase} + max(t_p, \frac{T_{ADC}}{2}) + \frac{T_{ADC}}{2} = 2.2730ms. \tag{11}$$

With the equations presented in this section the echo time could be further reduced by decreasing the bandwidth of the RF-pulse (hence decreasing the size of the slice) or by reducing the spatial resolution. Also a stronger gradient will reduce TE.