X-ray assignment
BIOM9027 Medical Imaging

Ludwig Tranheden

August 2016

Contents

1 Question 1

1.1 a)

The power is given by Equation 1.

$$\text{Power [J/s]} = \frac{\text{(Effective dose [J/kg] * Weight [kg])}}{\text{Exposure time [s]}} \tag{1}$$

The result with exposure time 0.1 s and weight 75 kg is given in Table 1

X-ray type	**Gray** $[J/kg]$ 10^{-3}	**Power** $[W]$
Bone	0.001	0.00075
Abdominal	8	5.99999
Dental	0.005	0.00375
Chest	0.1	0.07500
Mammography	0.4	0.30000
Intravenous pyelogram	0.3	0.22500

Table 1: Power of different X-ray types.

1.2 b)

The visual contrast is defined by Equation 2

$$\triangle C_{visual} = log\, I_2 - log\, I_1 \tag{2}$$

The equation can be written in terms of the difference in attenuation coefficients (μ) according to Equation 3

$$\triangle C_{visual} = log\, e\, (\mu_1 - \mu_2)z \tag{3}$$

where z is the depth. So the visual contrast between different tissues is directly related to the difference in their attenuation coefficients. In figure 1 the attenuation coefficients of fat, muscle, water and bone are plotted as functions of the x-ray photon energy.

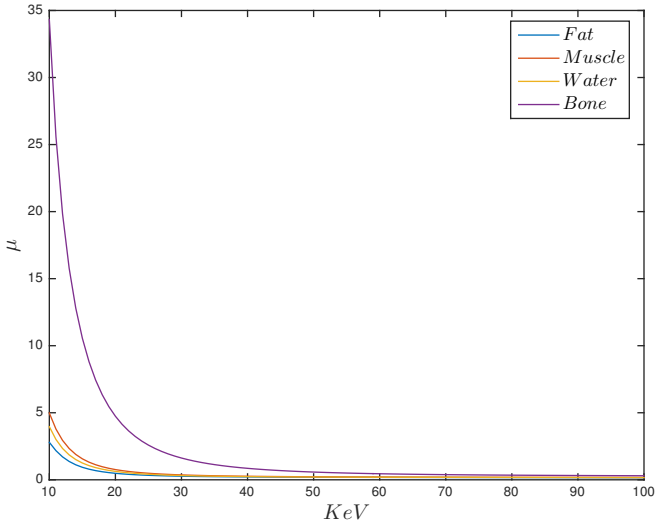

Figure 1: Attenuation coefficients as a function of x-ray photon energies for different tissues.

It's easily read from the figure that the optimal contrast between any tissues is at a photon energy of 10 keV. At higher energies the coefficients converges towards 0. The calculated visual contrast at 10 keV and unit depth is displayed in Table 2. The code used is displayed in Section 4, Listing 1

Tissues	**Visual contrast**
Fat and Water	1.6887
Muscle and Water	1.4645
Bone and Fat	45.4870
Bone and Muscle	42.3339
Bone and Water	43.7983

Table 2: Visual contrast between different tissues.

1.3 c)

The attenuation due to the photoelectric effect depends on the atomic weight (Z) and the energy of the x-ray (E). The attenuation due to Compton scatter on the other hand, because it affects free or weekly bound electrons, depends on the electron density (ED) and the energy of the x-ray. With μ being the linear attenuation coefficient (the probability of interaction per unit depth) the relationship to the above parameters is given by Equation 4 and 5.

$$\mu_{\text{photoelectric effect}} \propto \left(\frac{Z}{E}\right)^3 \tag{4}$$

$$\mu_{\text{Compton scatter}} \propto \frac{ED}{E} \tag{5}$$

With the equations in mind the photoelectric effect will be the main contribution of attenuation at low energies but decrease rapidly with increasing energy. Compton scatter attenuation will decrease slower and will be the dominating attenuation at higher energies. Since the average atomic weight of bone is higher than fat, water and muscle the relative contribution of photoelectric effect to the attenuation will be higher. The electron density is very similar between the tissues so the effect of Compton scatter will be similar. However the density in the sense of mass per unit volume will vary and contribute to the overall effect. This is reflected in Figure 1, because the dominance of photoelectric effect bone will have a high attenuation coefficient and the other tissues a almost equal lower coefficient at low energies. When the energy is increased the attenuation because of photoelectric effect fades and all tissues approach the same value but maintaining a difference due to electron density and mass per unit volume.

1.4 d)

From 1b we know that the maximum contrast is given at a x-ray energy of 10 keV. The number of photons is given by Equation 6, where the coefficient "$1.6 * 10^{-19}$" is 1eV in Joules. The total energy is the effective radiation dose multiplied with average weight of a person (75 kg).

$$\#\text{Photons} = \frac{\text{Total Energy [J]}}{\text{x-ray energy [eV] * } 1.6 * 10^{-19} \text{ [J/eV]}} \tag{6}$$

The number of photons at 10 keV are $1.87 * 10^{13}$.

2 Question 2

2.1 a)

If one consider the effective focal spot as an point source, the magnification of an object will depend on it's relative position between the x-ray source and the detector. The magnification factor (M) is given by Equation 7.

$$M = \frac{\text{(Focus to Film Distance)}}{\text{(Focus to Film Distance) - (Object to Film Distance)}} = \frac{\text{(FFD)}}{\text{(FFD - OFD)}} \tag{7}$$

In our case the magnification of the micro-calcification (side = 1 mm) is M = 1.005. Hence the magnified side of the calcification is side*M = 1.005 mm.

2.2 b)

The magnification of the focal spot (m), directly related to geometric unsharpness, can be calculated by assuming the light passes throw a pin-hole located on the object. The magnification can then be defined according to Equation 8

$$m = \frac{\text{(OFD)}}{\text{(FFD - OFD)}} = \text{M - 1} \tag{8}$$

The magnification of the focal spot for the calcification is 0.005. The focal spot at the x-ray is 1 mm, so the focal spot detected at the x-ray film is 1*0.005 = 0.005 mm.

2.3 c)

The point spread function (PSF) is a mathematical model for the response of a imaging system to a point source. In other words it's the systems impulse response. So the measured image is theoretically calculated by convoluting the PSF by the object at each point. To obtain the response to a point source the following scenario is assumed;

i Only consider the case where the point is alined with the centre of the calcification, i.e x = 0.

ii The beam originate from a theoretical point source.

iii The beams diffracts according to a normal distribution with zero mean and standard deviation one third of the magnification of the calcification.

iv All the spread out beams enters and exits the calcification along it's top respectively bottom side.

These simplifying assumptions will result in that one does not acquire the point source response, but something that should resemble it.
After a lot of trigonometric algebra you end up with the expression in 9 for the response.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2(\frac{c}{6})^2)}} e^{-\mu_B(\sqrt{(btan(arcsin(\frac{x}{R})))^2+b^2}-\sqrt{(mtan(arcsin(\frac{x}{R})))^2+m^2})} e^{-\mu_C\sqrt{(mtan(arcsin(\frac{x}{R}))^2+m^2}} \tag{9}$$

Where
$c = 1.005\ mm$, The size of the magnified calcification.
$b = 10\ mm$, The height of the breast tissue.
$L = 1000\ mm$, The FFD distance.
$R = \sqrt{((\frac{x}{2})^2 + L^2)}\ mm$, The length of the beam path.
$m = 1\ mm$, The height of the microcalcification.
$\mu_B\ mm^{-1}$, The linear attenuation coefficient for breast tissue.
$\mu_C\ mm^{-1}$, The linear attenuation coefficient for the calcification.

The substance of the equation is that if the beam originates from x = 0 as a point source the beam will diffract according to a normal distribution and attenuates to approximately zero at $x = \pm\frac{c}{2}$. The point source will not be detected by a single detector (the ideal PSF in the sense of exact measured image is the dirac delta function) but a range of detectors, hence the name "point spread function". The plot of the function in Equation 9 is given in Figure 2 with the relationship $\mu_C : 10\mu_B$.

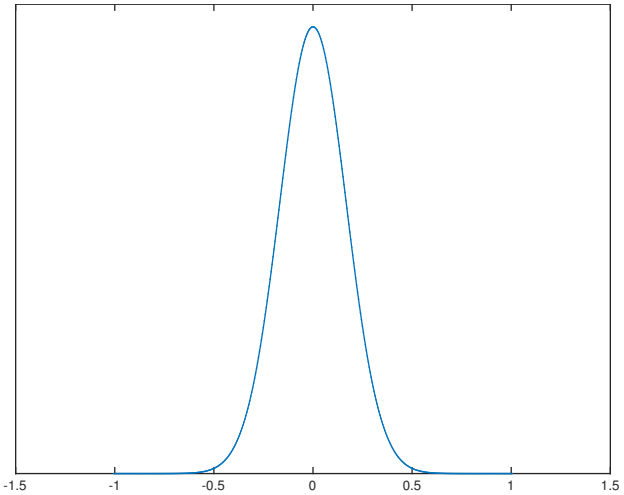

Figure 2: The response to a point source at x = 0 mm.

To calculate the image intensity as a function of x it's assumed that the beams travels orthogonally to the x-ray film and don't diffract. Using the fact that the intensity is $I_x(y) = I_0(x)e^{-\int_0^y \mu(u)du}$, where y is the depth, $\mu(y)$ is the linear attenuation coefficient (in cm^{-1}) depending on depth and $I_0(x) = \frac{1}{\sqrt{(2\pi)}}e^{-\frac{x^2}{2}}$ is the initial intensity. The image intensity as a function of x is given by Equation 10.

$$Intensity(x) = \begin{cases} I_0(x)e^{-0.9\mu_B}e^{-0.1\mu_C}, & x \in [-0.5, 0.5]\ mm \\ I_0(x)e^{-\mu_B}, & otherwise \end{cases} \tag{10}$$

If the point spread function corresponded to the dirac delta function the intensity detected at the detectors should be equal to the above expression, the expression is plotted in Figure 3 (15 keV, using attenuation values from Section 2.5).

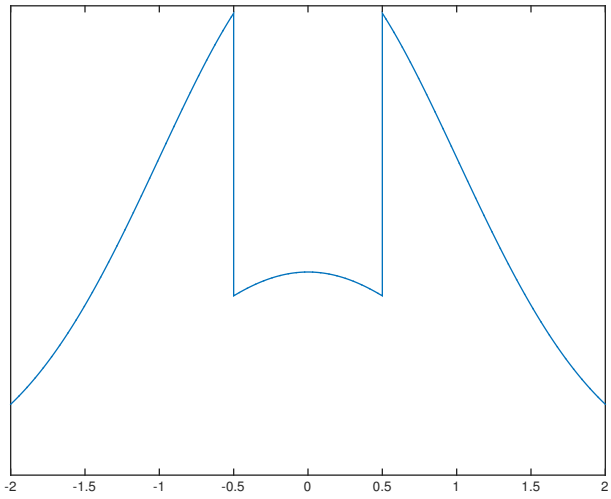

Figure 3: The theoretical intensity as a function of x.

The micro-calcification is easily identified as the submersion around x=0. In theory the image actually detected should be the above expression convoluted with the point spread function. The result of this is displayed in Figure 4.

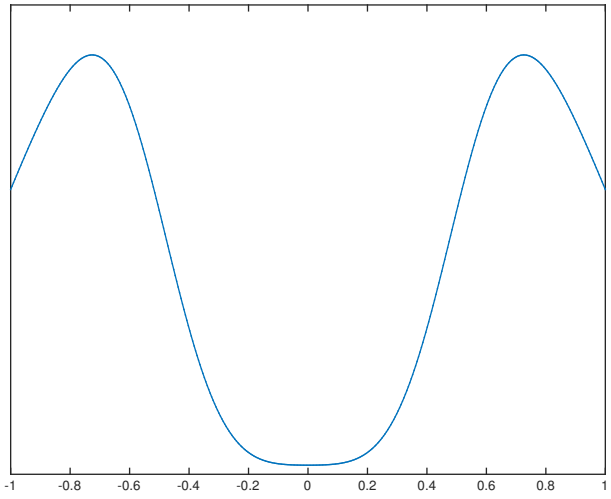

Figure 4: The convolution of the theoretical intensity and the point source function.

As seen in the figure the very fine boarder between tissues in Figure 3 is blurred, this reflect the geometric unsharpness mentioned earlier. The matlab code is displayed in Section 4 Listing 2.

2.4 d)

To detect the microcalcification on the x-ray film it has to be sampled at a appropriate rate. If not, something called aliasing will occur, the frequency components of the calcification will "overlap" and the calcification will not be detected. To ensure this do not happen the sampling frequency have to be at least two times larger then the highest frequency component of the sample-object (this makes sure the frequency-components don't overlap). The spacial frequency is defined as number of cycles per unit length (mm in our case). So the spacial frequency of the microcalcification is 1 cycle/mm. This means that we have to sample every 1/2 mm, which corresponds to a resolution of 2 lp mm^{-1}. However, the imaging system will magnify the microcalcification to a size of $\approx$ 1.005 mm, corresponding to 2.01 lp mm^{-1}. This might seem to be negligible but if the object is to small to physically sample or the resolution is a limitation, magnifying the object could be a solution.

2.5 e)

The transmittance is defined as $\frac{I(x)}{I_0(x)}$ where $I(x)$ is the "output intensity" given by Equation 10 and $I_0(x)$ is the initial intensity. Given that the x-ray photon energy is 15 keV the attenuation coefficients for breast tissue and micro calcification becomes $\mu_B = \frac{\mu_{water}+\mu_{fat}}{2} = 1.09\ cm^{-1}$ and $\mu_C = 10.56\ cm^{-1}$. The transmittance for two points, one with calcification (x=0) and one without (x = 1.5) is given by Equation 11.

$$Transmittance = \begin{cases} e^{-0.9\mu_B}e^{-0.1\mu_C} = 0.13, & x = 0\ mm \\ e^{-\mu_B} = 0.34, & x = 1.5\ mm \end{cases} \tag{11}$$

The visual contrast of the image can be estimated with these two points.

$$\triangle C_{visual} = log(\frac{I(0)}{I(1.5)}) = log(I_0(0)) - log(I_0(1.5)) + log(e)(\mu_B - 0.9\mu_B - 0.1\mu_C)$$
$$= 0.26$$

It's clear from the equation that the contrast increases as the you move the point corresponding to x=1.5 mm further away from the origin or concentrate the beam towards the centre (decrease diffraction).

3 Question 3

3.1 a)

In Figure 5 three attenuation maps of a cross-section of a thigh at different x-ray photon energies is shown. In Figure 6 the corresponding tissue map is shown.

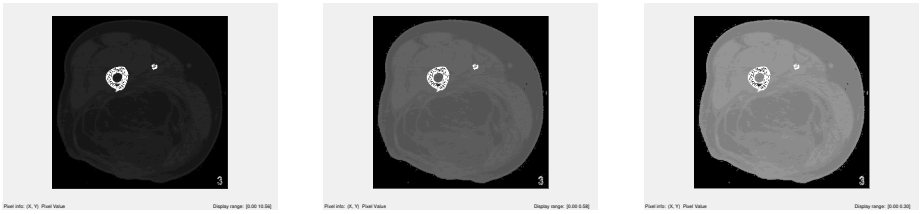

Figure 5: Scaled attenuation-maps for 15, 50 and 100 keV.

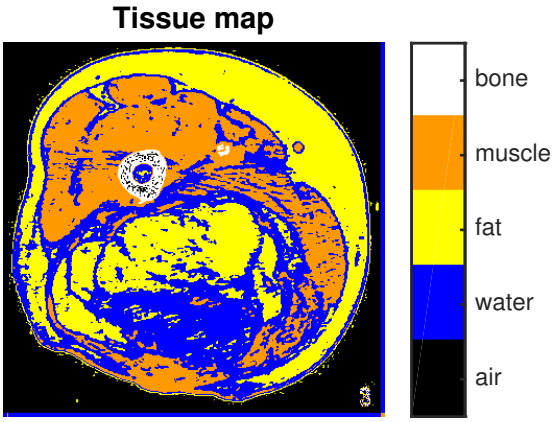

Figure 6: Tissue map

All of the pixel intensities of the pictures in Figure 5 are scaled, respectively, according to the largest and smallest pixel intensity. The visual contrast is greatest at lower photon energies. This is because the visual contrast is directly dependent on the difference between attenuation coefficients, and as discovered in question 1 these differences is greatest at low photon energies. As the energy is increased the contrast between fat and water decreases, because the attenuation becomes similar for the two tissues (compton scatter becomes the main attenuation mechanism). However the contrast between soft tissue and bone is not effected as severely (the photoelectric effect is still present for higher energies for bone). The above points is illustrated in Figure 7 where the pixel intensities of the attenuation maps are plotted as surface plots in the same order as above. Note the drastic change in the colorbar/intensity - scale for increasing photon energies. The code for generating Figure 5, 6 and 7 is displayed in section 4, Listing 3.

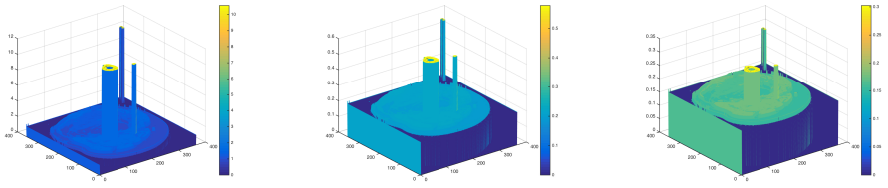

Figure 7: Surface plot of attenuation maps for 10, 50 and 100 keV.

Having a polychromatic spectrum (e.g 15 - 100 keV) of photon energies the mean photon energy increases the further you go into the material since the attenuation is higher for lower photon energies. This is called beam hardening. What this means in words are that the same tissue will have different attenuation depending on the depth of the tissue. Since the x-ray energy spectrum is polychromatic the desirable thing to do would be to filter the spectrum with a high-density material (shifting the spectrum towards higher energies), resulting in a spectrum of higher energies. Given the same spectrum as earlier one could achieve a high energy spectrum ($\approx$ 100 keV) - which still produces significant difference between different tissues without the same extent of beam hardening.

3.2 b)

Using a plane x-ray to diagnose liposarcome is probably very hard. Since you are just using one "slice" and the fact that the tissues overlap at the beam path the result will not be sufficient. Simulating a plane x-ray of the attenuation map with the help of a radon-transform at angle 0 (vertically "summing along columns") at a x-ray photon energy of 50 keV is displayed in Figure 8.

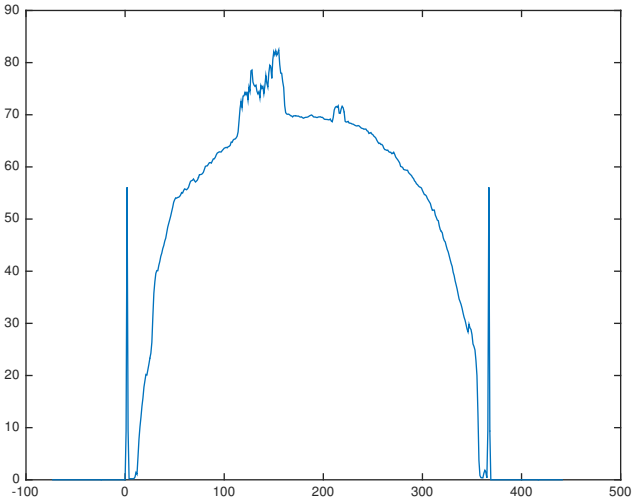

Figure 8: Plane xray.

As expected you can see the maximum attenuation in phase with the position of the bone, but it's hard the identify the tumour. The code is given in Section 4, Listing 4.

3.3 c)

Instead of a plane x-ray simulating a CT scan (100 keV) should give better result. An estimation of the number of projections set required for pi radians is given by Equation 12.

$$N_{projetions} = \frac{\pi N_{detectors}}{2} \tag{12}$$

Where the number of detectors ($N_{detectors}$) is calculated by the matlab function "radon" accordingly to the diagonal of the attenuation map. In our case $N_{detectors} = 515$ and hence $N_{projections} \approx 810$. The result of the simulation of the CT scan in comparison with the true attenuation map is shown in Figure 9.

(a) Ct scan

(b) True attenuation map

Figure 9: Attenuation maps.

As expected the image was successfully reconstructed. The code is displayed in section 4, Listing 5

4 Matlab code

Listing 1: Code for 1b.

```
data = load('Attenuation.mat');

keV = 10:1:100;
water=exp(data.watercurve(log(keV)))*1e-2;
fat=exp(data.fatcurve(log(keV)))*1e-2;
bone=exp(data.bonecurve(log(keV)))*1e-2;
muscle=exp(data.musclecurve(log(keV)))*1e-2;

figure;
plot(keV,fat)
hold on
plot(keV,muscle)
hold on
plot(keV,water)
hold on
plot(keV,bone)
hold on
ylabel('$\mu$','interpreter','latex','fontsize',16)
xlabel('$KeV$','interpreter','latex','fontsize',16)
h = legend('$Fat$','$Muscle$','$Water$','$Bone$','Location','Best');
set(h,'Interpreter','latex','fontsize',14);
saveas(gca, 'AttkeV.eps','epsc');
```

Listing 2: Code for 2c.

```
data = load('Attenuation.mat');
L = 1000;
b = 10;
a = 5;
m = 1;
c = 1.005;

keV = 15;
water=exp(data.watercurve(log(keV)))*1e-2;
fat=exp(data.fatcurve(log(keV)))*1e-2;
bone=exp(data.bonecurve(log(keV)))*1e-2;
Btissue = (fat + water)/2;
microc = bone;

T1 = exp(-1*Btissue);
T2 = exp(-0.9*Btissue)*exp(-0.1*microc);

u = -c:0.0001:c;
R = sqrt((u./2).^2 + L^2);
PSF = 1/sqrt(2*pi)*exp(-(u.^2)/((c/6).^2*2))...
```

```
    .*exp(-(Btissue/10)*(sqrt((b*tan(asin(u/R))).^2 + b^2) - sqrt((m*tan(asin(u/R))).^2 + m^2)))...
    .*exp(-(microc/10)*sqrt((m*tan(asin(u/R))).^2 + m^2));
x1 = -2:0.0001:-0.5;
REAL1 = (1/sqrt(2*pi)*exp(-(x1.^2)/(2)))*T1;
x2 = -0.5:0.0001:0.5;
REAL2 = (1/sqrt(2*pi)*exp(-(x2.^2)/(2)))*T2;
x3 = 0.5:0.0001:2;
REAL3 = (1/sqrt(2*pi)*exp(-(x3.^2)/(2)))*T1;
x = [x1 x2 x3];
REAL = [REAL1 REAL2 REAL3];
figure;
plot(u,PSF)

figure;
plot(x,REAL)
set(gca,'ytick',[])
set(gca,'yticklabel',[])
saveas(gca, 'Intensity.eps','epsc')

w = conv(PSF, REAL,'same');
figure;
plot(u,w);
set(gca,'ytick',[])
set(gca,'yticklabel',[])
saveas(gca, 'Conv.eps','epsc')
```

Listing 3: Code for 3a.

```
load('Attenuation.mat');
keV = [15 50 100];
figure;
AttenuationMap15 = getAttenuation(keV(1),TissueMap);
h12 = imtool(AttenuationMap15,[]);
saveas(h12, '15keVscaled.eps','epsc');

figure;
AttenuationMap50 = getAttenuation(keV(2),TissueMap);
h22 = imtool(AttenuationMap50,[]);
saveas(h22, '50keVscaled.eps','epsc');

figure;
AttenuationMap100 = getAttenuation(keV(3),TissueMap);
h32 = imtool(AttenuationMap100,[]);
saveas(h32, '100keVscaled.eps','epsc');

[x,y]=size(AttenuationMap15);
X=1:x;
Y=1:y;
[xx,yy]=meshgrid(Y,X);

i=im2double(AttenuationMap15);
figure;mesh(xx,yy,i);
colorbar
saveas(gca, 'surface15.eps','epsc');

i=im2double(AttenuationMap50);
```

```
figure;mesh(xx,yy,i);
colorbar
saveas(gca, 'surface50.eps','epsc');

i=im2double(AttenuationMap100);
figure;mesh(xx,yy,i);
colorbar
saveas(gca, 'surface100.eps','epsc');
```

Listing 4: Code for 3b.

```
load('Attenuation.mat');
keV = [15 50 100];
figure;
AttenuationMap50 = getAttenuation(keV(2),TissueMap,0);
[radon_,xp] = radon(AttenuationMap50,0);
centerpixel = floor((size(AttenuationMap50) + 1)/2);
xp = xp+centerpixel(2)+1;
plot(xp,radon_)
saveas(gca, 'planexray.eps','epsc');
```

Listing 5: Code for 3c.

```
load('Attenuation.mat');
figure;
AttenuationMap100 = getAttenuation(100,TissueMap,0);
N = 515;
Nperpi = ceil((515*pi)/2);
theta = 0:179/Nperpi:179;
radon_ = radon(AttenuationMap100,theta);
image = iradon(radon_,theta);
h = imtool(image,[]);
saveas(h, 'Ctscan.eps','epsc');
```