Correction of aberration

December 5, 2014

Abstract

Description of procedure of aberration correction

0.1 Ray tracing

0.1.1 Lens system description

In our experiment we use lenses which made of BK7 glass. Parameters of lens are presented on figure

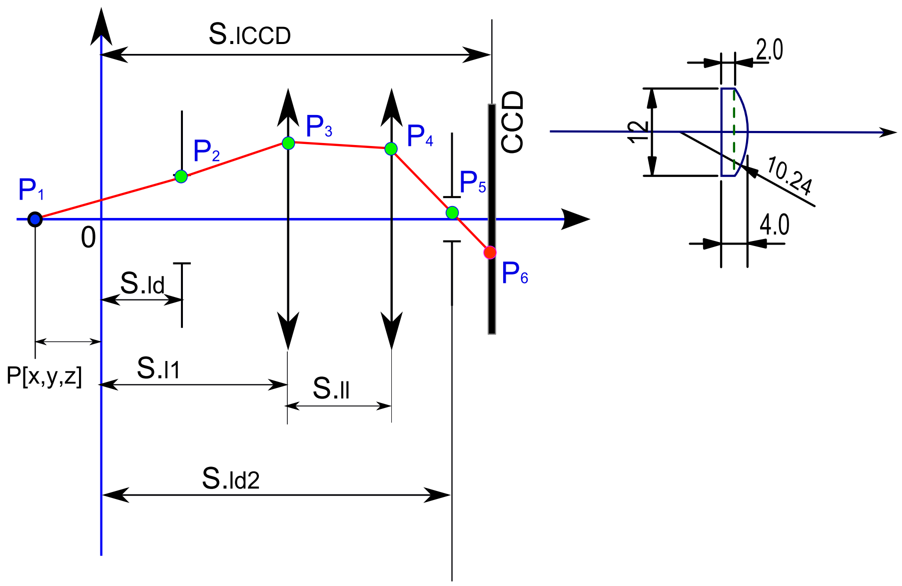

Figure 1: Parameters of lens and lens system.

This function creates initial structure for objective and CCD parameters.

All parameters are in [mm]

The origin of the system coordinate is the center of trap.

Parameters of lens:

Larger diameter lens simplifies the calculation.

Real diameter S.D = 12;

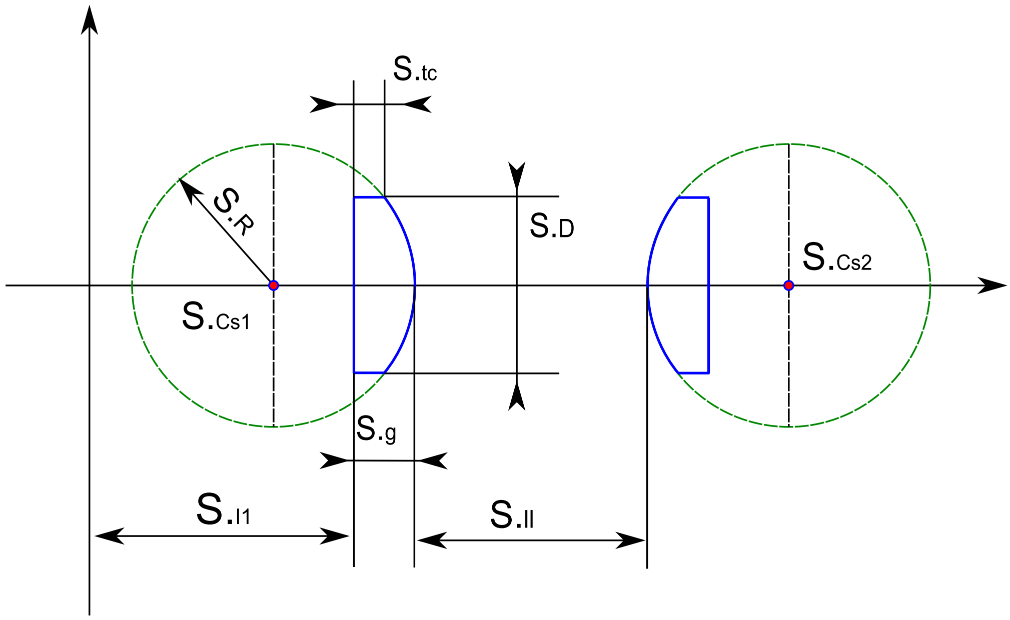

Figure 2: Parameters of lens and lens system.

S.D = 13; lens diametr

S.efD = 13; 11.8; - effective diameter of lens

S.R = 10.3; radius of lens curvature

S.tc = 1.8; thickness of the lenses wall

S.g = 4; thickness of the whole lens along optical axes

Distances:

S.ld = 15; Distance between center of the trap and first diaphragm

S.l1 = 18; Distance between center of the trap and first lens

S.ll = 37.3; Distance between lenses

S.ld2 = 76.6; Distance to second diaphragm

First Diaphragm

S.dW = 9; width of diaphragm

S.dH = 4; height of diaphragm

Second diaphragm

S.RDph = 1; Radius of aperture

S.W2 = 2; thickness of the diaphragm wall

CCD parameters

S.lCCD = 87.4; Distance to CCD detector

S.CCDPH = 480; width of CCD [Pix]

S.CCDPW = 640; height of CCD [Pix]

S.PixSize = 9.9e-3; Pixel size

S.CCDH = S.CCDPH * S.PixSize; height of CCD

S.CCDW = S.CCDPW * S.PixSize; width of CCD

Droplet position

S.Pk = [1,0,0]; Position of droplet relative to the origin of coordinates system

Wavelength of incident ray

S.lambda = 532;

0.1.2 Algorithm description

In order to correct the aberration we need to find some function,which links the point before the lens system and the point after lens system. It is very difficult to find analytic function for lens system. So we try to conducts ray through the lens system by means of ray tracing. The figure 3 represents our lens system. In

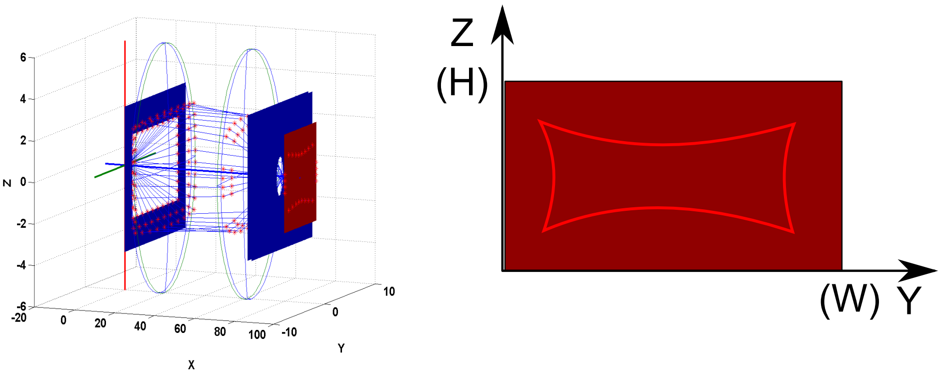

Figure 3: Lens System (Objective)

order to calculate the ray trajectory we have to set of two space point. The first represents the droplet position, the second point is placed into the plane of first diaphragm. Then we can conducts the line through this points. To do this we build a vector parallel to the straight line. It's convenient to take the normalized vector:

$$\vec{V} = \frac{P_2 - P_1}{||P_2 - P_1||} \tag{1}$$

where $P_1(x,y,z)$ is the droplets coordinates; $P_2(x,y,z)$ is the coordinates of some point onto first diaphragm. Then we are looking for the point of intersection of the line and lenses. In our case the first surface of lens is flat and parallel to coordinate plane Z,Y so For this we need write the line equation in parametric form:

$$\begin{cases} x = x_0 + \alpha \cdot t; \\ y = y_0 + \beta \cdot t; \\ z = z_0 + \gamma \cdot t; \end{cases} \quad (2)$$

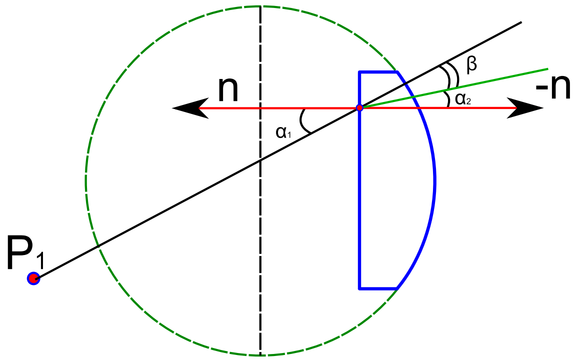

Figure 4: Angles

Calculation angle between ray and normal vector.

$$\left(-\vec{n} \cdot \vec{V}\right) = |-\vec{n}| \cdot \left|\vec{V}\right| \cdot \cos\alpha \quad (3)$$

$$\alpha = \cos^{-1}\left(\frac{\left(-\vec{n} \cdot \vec{V}\right)}{|-\vec{n}| \cdot \left|\vec{V}\right|}\right) \quad (4)$$

We need to create vector for next ray and this ray has to be in the same plane with incident vector.

$$\beta = \alpha_1 - \alpha_2; \quad (5)$$

Creating vector perpendicular to incident plane

$$\vec{S} = \vec{n} \times \vec{V} \tag{6}$$

now we looking for vector perpendicular to S and vector which has angle β with vector V. We know that

$$\begin{aligned} \left(\vec{V}_1 \cdot \vec{V}_2\right) &= \left|\vec{V}_1\right| \cdot \left|\vec{V}_2\right| \cdot \cos\beta \\ \left(\vec{n} \cdot \vec{V}_2\right) &= \left|\vec{V}_1\right| \cdot \left|\vec{V}_2\right| \cdot \cos\alpha_1 \\ \left(\vec{S} \cdot \vec{V}_2\right) &= 0 \end{aligned} \tag{7}$$

We deals with normalised vectors so:

$$\begin{aligned} \left(\vec{V}_1 \cdot \vec{V}_2\right) &= \cos\beta \\ \left(\vec{n} \cdot \vec{V}_2\right) &= \cos\alpha_2 \\ \left(\vec{S} \cdot \vec{V}_2\right) &= 0 \end{aligned} \tag{8}$$

from another hand:

$$\begin{cases} V_{x1} \cdot V_{x2} + V_{y1} \cdot V_{y2} + V_{z1} \cdot V_{z2} & = & \cos\beta \\ n_x \cdot V_{x2} + n_y \cdot V_{y2} + n_z \cdot V_{z2} & = & \cos\alpha_2 \\ S_x \cdot V_{x2} + S_y \cdot V_{y2} + S_z \cdot V_{z2} & = & 0; \end{cases} \tag{9}$$

we need to solve this system of equation and found V_2:

$$A = \begin{bmatrix} V_{x1} & V_{y1} & V_{z1} \\ n_x & n_y & n_z \\ S_x & S_y & S_z \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} \cos\beta \\ \cos\alpha_2 \\ 0 \end{bmatrix} \tag{11}$$

In matlab in order to solve this equation we need to do this:

$$\begin{aligned} V_2 &= A^{-1} \cdot B \\ V_2 &= A \setminus B; \end{aligned} \tag{12}$$

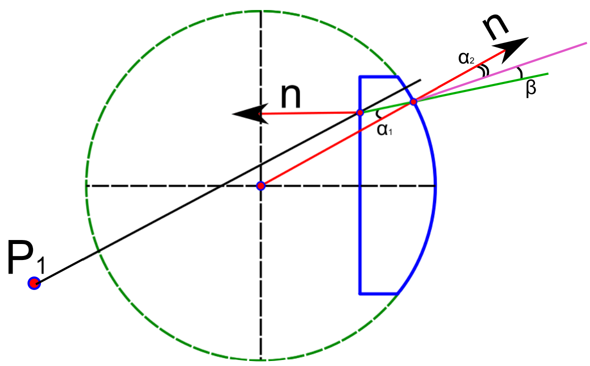

Figure 5: Intersection ray with surface of lens

In order to find point of intersection ray with surface of lens we need to solve system of equation

$$\begin{cases} x &= x_0 + V_x \cdot t; \\ y &= y_0 + V_y \cdot t; \\ z &= z_0 + V_z \cdot t; \\ R^2 &= x^2 + y^2 + z^2; \end{cases} \tag{13}$$

where $\vec{V}$ is directional vector of the line;

$$\begin{aligned} (x_0 + V_x \cdot t)^2 + (y_0 + V_y \cdot t)^2 + (z_0 + V_z \cdot t)^2 &= R^2 \\ t^2 \cdot (V_x^2 + V_y^2 + V_z^2) + 2 \cdot t \cdot (x_0 \cdot V_x + y_0 \cdot V_y + z_0 \cdot V_z) + x_0^2 + y_0^2 + z_0^2 - R^2 &= 0; \end{aligned} \tag{14}$$

so we need to solve square equation $At^2 + Bt + C = 0$ where:

$$\begin{aligned} A &= V_x^2 + V_y^2 + V_z^2; \\ B &= 2 \cdot (x_0 \cdot V_x + y_0 \cdot V_y + z_0 \cdot V_z); \\ C &= x_0^2 + y_0^2 + z_0^2; \\ D &= B^2 - 4 \cdot A \cdot C; \\ t_{1,2} &= \tfrac{-B \pm \sqrt{D}}{2A}; \\ \vec{r_c} &= \vec{r} + \vec{V} \cdot t \end{aligned} \tag{15}$$

0.2 Pixel size determination

The measurement by microscope.

The slit width has 9 marks.

The etalon wire with has diameter of 240 μm and 49 marks.

So the one mark has 4.8980μm and slit width has 44.0816 μm

528.7 0.2W 501.7 0.3W 496.5 0.5W 476.5 0.5W 472.7 0.12W 465.8 0.1W 457.9 0.25W 454.5 0.05W

Single slit diffraction:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{Pix \cdot P_{Size}}{L}\right) \\ X(\theta) &= \frac{d \cdot \pi \cdot \sin\theta}{\lambda} \\ I(\theta) &= I_0 \cdot \frac{sinX}{X} \end{aligned} \tag{16}$$

0.2.1 Another way to determine the pixel size

0.3 Comparing of the theoretical and experimental image

Size of experimental image is given in pixels. But RayTracing function calculates distances in millimeters. So, if we want to compare two images we need transform millimeters into pixels.

$$P_X = \left(\frac{W_{CCD}}{2} + P\right) \cdot \frac{1}{P_{Size}}; \tag{17}$$

where P is the coordinates of points on the border of the trap's aperture, W_{CCD} is width of CCD element in $[mm]$, P_{Size} size of the one pixel $[\frac{mm}{Pix}]$. Now we can draw experimental image with theoretical border. Next step, we have to change parameters of the lens system in order to overlap theoretical and experimental images. To do it you need click one of the figures with experimental image and next press one of these keyboard buttons:

1 - change of the mask's position;()

2 - change of the CCD position; (change distance between droplet and CCD element. CCD element is moved)

3 - change of the droplet position;(change distance between droplet and CCD element. Droplet is moved)

4 - change of the size of effective aperture;

5 - change of the line position;

c - set contrast;

m - set masks;

q - exit.

And drag mouse pointer across the image.

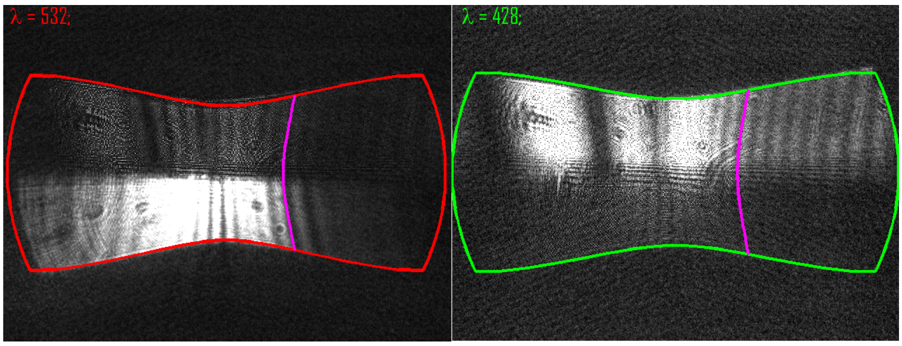

Figure 6: first and second channels with theoretical calculated aberration

0.4 Angle calculation

First step - find points of interest($[r,c] = find(HG.H.BWR)$), where r,c are row and column vectors (pixel's numbers) of length equal 640*480 non-zero elements. Recalculation pixels to meters and shifting image at the origin

$$\begin{aligned} Y &= \left(c - \frac{PW_{CCD}}{2} - Sh_X\right) \cdot PixSize \\ Z &= \left(r - \frac{PH_{CCD}}{2} - Sh_Y\right) \cdot PixSize \end{aligned} \tag{18}$$

where PW and PH are the width and height of element CCD in Pixels H.Min = zeros(length(Y),2); H.Mout = zeros(length(Z),2); ThetaPhiR = zeros(length(Z),3);

H.Min(i,1) = Y0; matrix of input coordinates

H.Min(i,2) = Z0; matrix of input coordinates

H.Mout(i,1) = R(7,2); matrix of output coordinates

H.Mout(i,2) = R(7,3); matrix of output coordinates

$$\begin{aligned} ThetaPhiR(i,1) &= \arctan(\tfrac{Y0}{S1.ld - R(1,1)}); \\ ThetaPhiR(i,2) &= atan(Z0/(S1.ld - R(1,1))); \\ ThetaPhiR(i,3) &= norm([(S1.ld - R(1,1)), Y0, Z0]); \end{aligned} \tag{19}$$

HG.H.ThetaPhiR1 - matrix - containing angle theta, phi, and distance from droplet to first aperture HG.H.ThetaPhiR2 - the same but for second channel. The structure is saved in the main object (aviscat figure) of aviscatt

0.5 Kalman Filtering

0.6 Near-axial approximation

We can reduce the aperture of lens to size when aberration effect could be negligible. Then we can calculate distance between ”droplet” position and CCD matrix. Then We can calculate the angle distributions because we know pixel size and distance to CCD/

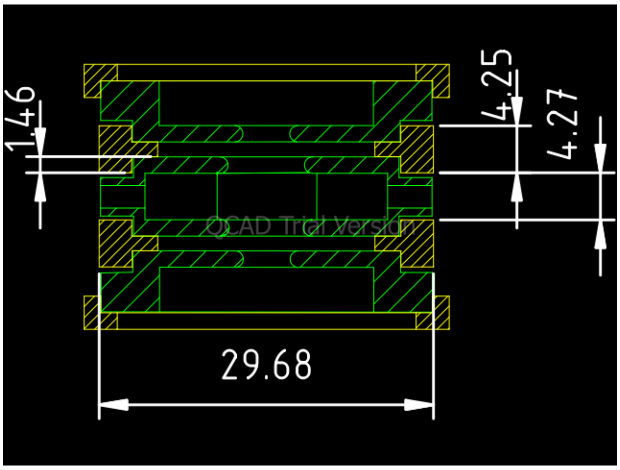

Figure 7: Aperture Parameters

0.7 Objective properties: troubleshooting

In our experiment we spotted a problem. Our experimental data didn’t fit to theory. There could be a several origin of our problem.

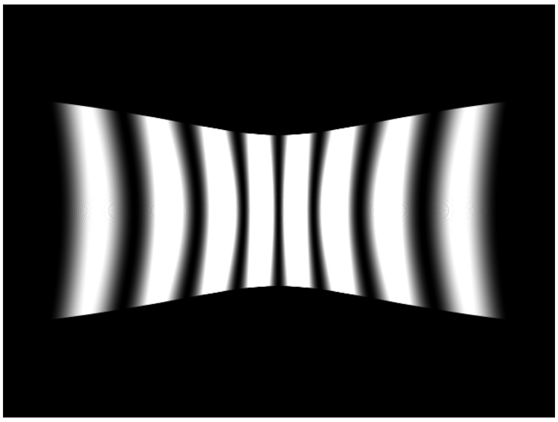

Figure 8: Intensity distribution across CCD element.

0.7.1 Mie calculation

In Mie theory, intensity is calculated for constant radius. But in our experiment we have flat surface instead of spherical one. So we need take into account the changes of radius.

0.7.2 Photometric problem

According to[1]

When we calculate intensity distribution across CCD element we should keep in mind that after lens system we'll have distortion in intensity field. First step which we need to do, it is checking self-consistency of our methods

Point light approximation. We put point light into origin of our system and calculates intensity produced by that source. We've chose rectangle aperture and we've calculated 2500X2500 points.

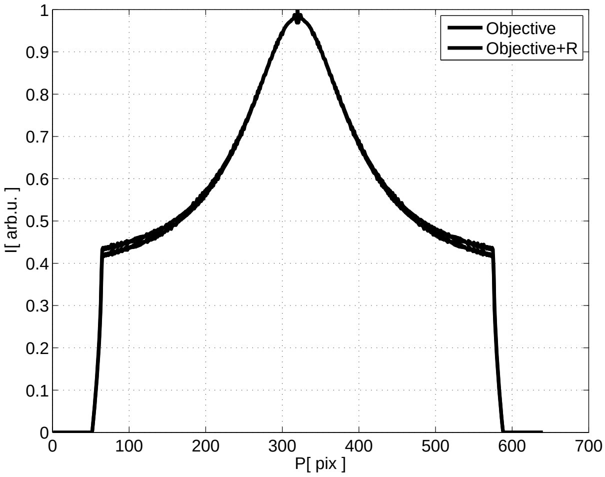

Figure 9: Intensity distribution across CCD element.

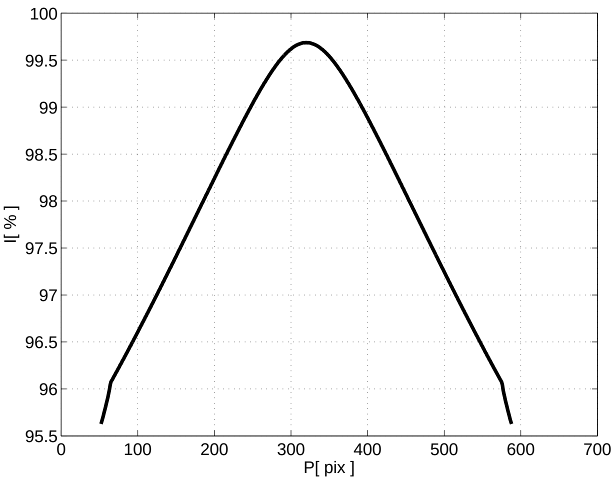

Figure 10: Intensity distribution across CCD element.

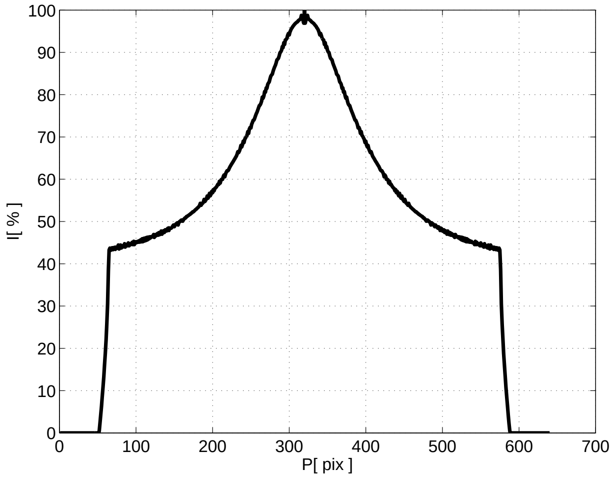

Figure 11: Intensity distribution across CCD element.

Bibliography

[1] Born, M.; Wolf, E. *Principles of Optics*; Pergamon: Oxford, 1970.