8/30/2022

→ STATEMENTS
→ OPERATORS
→ TRUTH TABLES
→ CONDITIONALS

STATEMENTS

The first topic we're going to cover is statements, this is fundamental.

Definitions are important;

A statement is any declaritive sentence with a truth-value.

To clarify, statements are...

→ either only true or only false
Grass is green
Rome is in North America

Statements are not..

→ opinions
Pulp Fiction is the greatest movie of all time

→ meaningless
Purple clouds make time bougie

Writing out statements is meh and we're all about abstraction and generalization, so we represent statements (propositions, truth-vales) with lower-case letters, also known as variables.

$r \equiv$ Rome is in North America
$g \equiv$ Grass is green

These are fairly useless by themselves, so we manipulate them via operators.

OPERATORS

There are 3 fundamental operators in propositional logic from which all others are derived:

Negation : $\neg$: "not"

Conjunction : $\wedge$: "and"

Disjunction : $\vee$: "or"

We utilize these operators with statements to create compound - statements which are themselves statements.

The truth-values of these compound statements are dependent on the truth-values of their component statements. Negation and Conjunction are pretty intuitive but we clarify about Disjunction:

In discrete-mathematics, "or" is not exclusive

PRACTICE

$p \equiv$ Paul likes dogs

$q \equiv$ Cliff likes legos

$r \equiv$ Charlie is cute

$\neg p$: Paul does not like dogs

$q \wedge r$: Cliff likes legos and Charlie is cute

$p \vee r$: Paul likes dogs or Charlie is cute

$p \wedge \neg r$: Paul likes dogs and Charlie is not cute

TRUTHTABLES

One way to think about compound-statements is as a boolean function with a number of inputs. We can explore the domain of this function via truth tables.

Consider the compound-statement: $\neg(p \wedge q) \vee r$

The idea here is to view the output of our function for every possible input. In this class, we'll use 1 to represent true and 0 for false.

p	q	r	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \vee r$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1

Some things worth noting:

- → The number of rows in your truth-table will be 2^n where n is the number of component statements
- → when setting up your truth-table, be consistent with the example above i.e counting in binary

When filling out truth-tables you may come across a scenario where the entire column of a compound-statement is the same truth-value i.e all 1 or all 0. We have terminology for this:

Contradiction: 0 : always false

Tautology: 1 : always true

PRACTICE

p	$\neg p$	$p \wedge \neg p$	$p \vee \neg p$
0	1	0	1
1	0	0	1

$$p \wedge \neg p \equiv 0$$

$$p \vee \neg p \equiv 1$$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Observe that some compound-statements output the same truth-values. This means that they are logically equivalent. This will come up later.