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→ PRACTICE

A doctor is in a sick ward where 90% of the patients have the flu and the other 10% have measles. If you have measles there is a 95% chance you have a rash, and if you have the flu there is an 8% chance you have a rash. The doctor finds a patient with a rash. What is the probability they have measles?

$$P(M \mid R) = \frac{P(R \mid M)\, P(M)}{P(R \mid M)\, P(M) + P(R \mid F)\, P(F)}$$

$$= \frac{0.95 \cdot 0.10}{0.95 \cdot 0.10 + 0.08 \cdot 0.90} \approx 0.57$$

Amongst women with breast cancer, 99% have benign tumors while 1% have cancerous ones. There exists a mammogram test that correctly classifies 90% of benign tumors and 80% of cancerous tumors. Given that a mammogram test returns positive for a cancerous tumor, what is the probability the patient actually has a cancerous tumor.

$$P(C \mid +) = \frac{P(+ \mid C)\, P(C)}{P(+ \mid C)\, P(C) + P(+ \mid B)\, P(B)}$$

$$= \frac{0.80 \cdot 0.01}{0.80 \cdot 0.01 + 0.10 \cdot 0.99} \approx 0.075$$

You have 3 cards which have one solid color either side; one black-black, one red-red, one red-black. One card is randomly drawn. If the top is red, what is the probability that the other side is black?

$$P(RB \mid R) = \frac{P(RB \cap R)}{P(R)}$$

$$= \frac{P(R \mid RB)\, P(RB)}{P(R \mid RR)\, P(RR) + P(R \mid RB)\, P(RB) + P(R \mid BB)\, P(BB)}$$

$$= (1/2 \cdot 1/3) \,/\, (1 \cdot 1/3 + 1/2 \cdot 1/3 + 0 \cdot 1/3) = 1/3$$

Its estimated that 50% of emails are spam. A particular spam-blocking software claims to block 99% of spam while having a false-positive rate of 5%. i.e a non-spam email detected as spam. If the software detects an email as spam, what is the probability that it is in fact a non-spam email?

× ← software identifies as spam
✓ ← software identifies as non-spam

$$P(N \mid \times) = \frac{P(\times \mid N)\, P(N)}{P(\times \mid S)\, P(S) + P(\times \mid N)\, P(N)}$$

$$= \frac{0.05 \cdot 0.50}{0.99 \cdot 0.5 + 0.05 \cdot 0.50} \approx 0.048$$

Assume that it takes at least 9 votes from a 12 person jury to convict a defendant. Assume that the probability a juror votes a guilty person innocent is 20%, and the probability that a juror votes an innocent person guilty is 10%. Assume also that every juror acts independently and that 65% of defendants are guilty.

A. What is the probability that the jury renders a correct decision

B. What percentage of defendants are convicted

A. A juror votes liable (L) or not liable (N)

A defendant is innocent (I) or guilty (G)

$P(N \mid G) = 0.2$ $\quad P(L \mid I) = 0.1$ $\quad P(G) = 0.65$

$P(L \mid G) = 0.8$ $\quad P(N \mid I) = 0.9$ $\quad P(I) = 0.35$

$$P(\text{CORRECT\_WHEN\_INNOCENT}) = \sum_{i=4}^{12} \binom{12}{i} P(N \mid I)^{i}\, P(L \mid I)^{12-i} = 0.9999\ldots$$

$$P(\text{CORRECT\_WHEN\_GUILTY}) = \sum_{i=9}^{12} \binom{12}{i} P(L \mid G)^{i}\, P(N \mid G)^{12-i} = 0.79457$$

$$P(\text{CORRECT}) = P(\text{CORRECT\_WHEN\_INNOCENT}) \cdot P(I) + P(\text{CORRECT\_WHEN\_GUILTY}) \cdot P(G)$$

$$P(\text{CORRECT}) = 0.99999\,(0.35) + 0.79457\,(0.65)$$

$$P(\text{CORRECT}) = 0.8664$$

B.

$$P(\text{CONVICTED}) = P(G) \cdot \sum_{i=9}^{12} \binom{12}{i} P(L|G)^i P(N|G)^{12-i} + P(I) \cdot \sum_{i=9}^{12} \binom{12}{i} P(L|I)^i P(N|I)^{12-i}$$

$$P(\text{CONVICTED}) = 0.65\,(0.794569) + 0.35\,(1.658 \cdot 10^{-7})$$

$$P(\text{CONVICTED}) = 0.51647$$