11/16/2022

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SAMPLE / EVENT SPACES

The set of all possible outcomes of an experiment is known as the SAMPLE SPACE and is denoted with a S.

Any subset E of the sample space is known as an EVENT. A UNION of two events $\cup$ is a set of all outcomes in either event, while the INTERSECTION $-\cap-$ is the set of outcomes in both events.

An event that does not contain any outcomes is known as the NULL EVENT and is denoted $\emptyset$. If the intersection of two events is the null event, they are said to be mutually exclusive.

The COMPLIMANT of event E, denoted E^c, are all the outcomes in the sample space not in E.
$* \; S^c = \emptyset$

If all the outcomes of event E are contained within event F, then E is a SUBSET of F; $E \subset F$, and F is a SUPERSET of E.

As you can see, a lot of set theory shows up in probability.

PROBABILITY DEFINITION

We denote the PROBABILITY OF EVENT E as $P(E)$ and define it to be...

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

... where n is the number of repititions of the experiment and $n(E)$ is the number of times event E occurs.

PROBABILITY PROPERTIES

For any event E:

$$0 \leq P(E) \leq 1$$

And for any sample space ...

$$P(S) = 1$$

For any set disjoint events $E_1, \dots, E_n$:

$$P\left(\bigcup_{1}^{n} E_i\right) = \sum_{1}^{n} P(E_i)$$

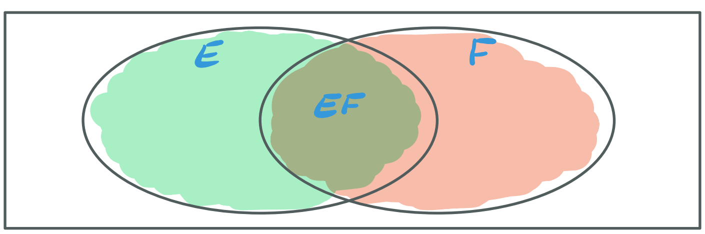
The probability an event does not occur is one, minus the probability it does:

$$P(E^c) = 1 - P(E)$$

The INCLUSION-EXCLUSION PRINCIPLE we introduced in countability holds analogously for probability:

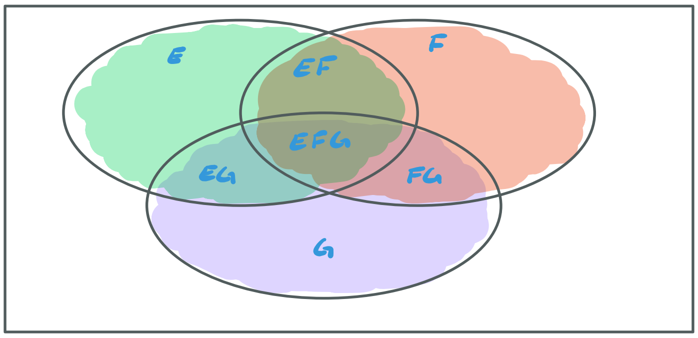
$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{r=1}^{n}\left((-1)^{r+1} \sum_{i_1<\cdots<i_r} P(E_{i_1}\cdots E_{i_r})\right)$$

This looks pretty complicated but recall it represents a simple pattern:



$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\begin{aligned} P(E \cup F \cup G) = {} & P(E) + P(F) + P(G) \\ & - P(E \cap F) - P(E \cap G) - P(F \cap G) \\ & + P(E \cap F \cap G) \end{aligned}$$



COMPUTATIONAL METHODS

For many experiments, all outcomes are equally likely to occur i.e for a finite sample space $S = \{1, 2, \dots N\}$;

$$P(\{1\}) = P(\{2\}) = \dots = P(\{N\})$$

$$P(\{i\}) = \frac{1}{N} \quad i = 1, 2, \dots N$$

In such cases, we can define the probability of an event occuring as the ratio of outcomes in the event to the number of outcomes in the sample space.

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in } S}$$

This method essentially allows you to turn a probability question into two combinatoric questions. Just be careful that all outcomes are equally likely.

PRACTICE

A box contains 12 red balls, 16 blue, and 18 green. You draw 7. What is the probability that...

3 red, 2 blue, 2 green are drawn?

$$\binom{12}{3}\binom{16}{2}\binom{18}{2} \Big/ \binom{46}{7}$$

$\frac{16}{3}$

all 7 are the same color?

$$\binom{12}{7} + \binom{16}{7} + \binom{18}{7} \Big/ \binom{46}{7}$$

at least 2 are red?

$$\sum_{i=2}^{7} \binom{12}{i}\binom{34}{7-i} \Big/ \binom{46}{7}$$

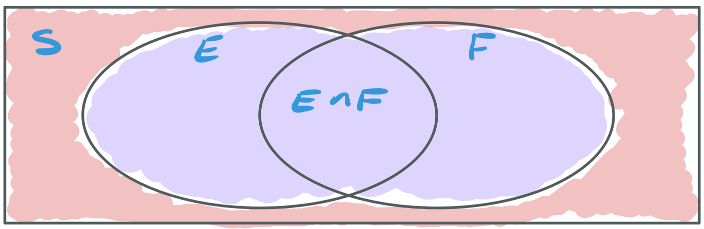
exactly 3 are red or exactly 3 are blue?

$$\binom{12}{3}\binom{34}{4} + \binom{16}{3}\binom{30}{4} - \binom{12}{3}\binom{16}{3}\binom{18}{1} \Big/ \binom{46}{7}$$

Prove Bonferroni's Inequality:

$$P(E \cap F) \geq P(E) + P(F) - 1$$

Consider the following disjoint sets that cover the sample space S:



$$P(S) = P(E \cup F) + P([E \cup F]^c) = P(E \cup F) + P(E^c \cap F^c)$$

$$1 = P(E) + P(F) - P(E \cap F) + P(E^c \cap F^c)$$

$$P(E \cap F) - P(E^c \cap F^c) = P(E) + P(F) - 1$$

$$P(E \cap F) \geq P(E) + P(F) - 1$$

You flip a coin 10 times. What is the probability of getting at least one head?

$$P(\text{at least one head}) = 1 - P(\text{no heads})$$

$$= 1 - (1/2)^{10}$$

You scramble the word "NUMBER". What is the probability that the new word starts with "M" or has "ER" in it?

$$\frac{5! + 5! - 4!}{6!}$$

A lake contains 120 fish, of which 40 are captured tagged. Some time later, 30 more fish are caught. What is the probability that 10 of those 30 are tagged?

$$\frac{\binom{40}{10}\binom{80}{20}}{\binom{120}{30}}$$