10/03/2022

→ ADVANCED SETS
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ADVANCED SETS

At this point, we've introduced methodologies for defining sets as well as many operations on sets. We now discuss numerous structures that we can spawn from a given set, these will be useful later.

CARDINALITY: This is actually a property of sets; for a given set S we refer to the number of elements in S as its cardinality and denote it $|S|$.

$S :\equiv \{ \text{purple}, \text{blue}, \text{red}, \text{green} \}$, then $|S| = 4$

A reasonable question here is how we utilize the concept of cardinality with infinite sets. This actually leads to the study of COUNTABILITY which we will cover later.

POWERSET: For a given set S, we refer to the set of subsets of S as its powerset and denote it $P(S)$.

$$P(S) :\equiv \{ S' \mid S' \subseteq S \}$$

PARTITION: For a given set S, we call a collection of disjoint subsets $\{S_1, \dots, S_n\}$ a partition of S if they collectively contain all the elements of S.

$$S_1 \cup \dots \cup S_n \equiv S \quad \wedge \quad \forall i,j \in [1,n] \cap \mathbb{N},\ i \neq j \Rightarrow S_i \cap S_j \equiv \emptyset$$

$$\Updownarrow$$

$$\{S_1, \dots, S_n\} \text{ is a partition of } S$$

CARTESIAN PRODUCT: Given two sets S, T we call the set of ordered pairs (s,t) such that $s \in S$ and $t \in T$ the cartesian product of S and T and denote it $S \times T$.

$$S \times T \equiv \{ (s,t) \mid s \in S \wedge t \in T \}$$

We can take cartesian products of more than just two sets, for example for sets R, S, T

$$R \times S \times T \equiv \{ (r,s,t) \mid r \in R \wedge s \in S \wedge t \in T \}$$

Its quite common to take the cartesian product of a given set S with itself. We sometimes denote this as follows:

$$S^n \equiv \{ (s_1, \dots, s_n) \mid s_1, \dots, s_n \in S \}$$

PRACTICE

$\{3,4\} \in P(\{3,4\})$	TRUE
$\mathbb{Z} \times \emptyset = \emptyset$	TRUE
$\{\mathbb{Q}, \mathbb{R} - \mathbb{Q}\}$ is a partition of $\mathbb{R}$	TRUE
$\{\emptyset, \mathbb{Z}\}$ is a partition of $\mathbb{Z}$	FALSE
$\forall S \in \{S \mid S \subseteq U\}, \emptyset \in P(A)$	TRUE

$P(\{1,2\}) \equiv$	$\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$
$\lvert A \times B \rvert =$	$\lvert A \rvert \cdot \lvert B \rvert$
$\lvert P(S) \rvert =$	$2^{\lvert S \rvert}$

COMMON MISTAKES

We saw a lot of incorrect syntax around sets and quantifiers so we've compiled some incorrect examples below:

$$\forall(x \in S \land \neg(x \in T)),$$

$$\forall x \in \{S\}, x \notin \{T\},$$

$$(\exists x \in S, x \forall T) \; [K(x)]$$

b) $(\exists x \in S \;\land\; x \notin T)[K(x)]$

$$(\forall x(S \notin T))[$$

$$(\forall x \in S)(\exists x \notin T)[k($$