11 / 28 / 2022

- → EXPECTED VALUE
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EXPECTED VALUE

Given an experiment with a set of events and the probability of them occuring, we can calculate the EXPECTED VALUE of the experiment, which is simply the average value of the events weighted by they're probabilities:

n events $a_1 \dots a_n$ with probability $p_1 \dots p_n$:

$$E = \sum_{i=1}^{n} a_i p_i$$

CONDITIONAL PROBABILITY

We denote the conditional probability that E occurs given that F has occured as $P(E|F)$.

$$P(E|F) = \frac{P(EF)}{P(F)}$$

An expression for the intersection of an arbitrary number of events:

$$P(E_1 E_2 \cdots E_n) = P(E_1)P(E_2|E_1)\cdots P(E_n|E_{n-1}\cdots E_1)$$

$n=3$ case:

$$P(E_1 \wedge E_2 \wedge E_3) = P(E_1)\,P(E_2|E_1)\,P(E_3|E_1 \wedge E_2)$$

$$= P(E_1) \cdot \frac{P(E_1 \wedge E_2)}{P(E_1)} \cdot \frac{P(E_1 \wedge E_2 \wedge E_3)}{P(E_1 \wedge E_2)}$$

INDEPENDENCE

Two events are independent if the knowledge of one event occuring does not change the probability of the other occurs:

$$P(EF) = P(E)P(F)$$

BAYES'

The idea for Baye's stems from the following statement:

$$E = (E \wedge F) \vee (E \wedge \neg F)$$

check via laws of equivalence!

In terms of probability, we may state this as ...

$$P(E) = P(E \wedge F) + P(E \wedge F^c)$$

using our definition of conditional probability...

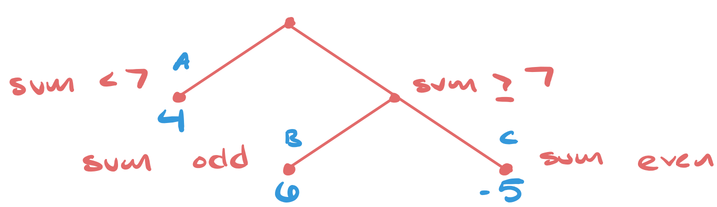
$$P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)$$

this is a valid interpretation of Bayes', although its often expressed as...

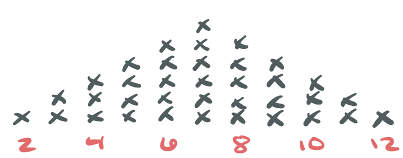
$$\boxed{P(E|F) = \frac{P(F|E) \cdot P(E)}{P(F)} = \frac{P(F|E) \cdot P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}}$$

PRACTICE

You roll 2 dice. If the sum is less than 7, you get \$4. If the sum is 7 or greater, you roll a 3rd die. If its odd, you get \$6, else you pay \$5.



SUM OF TWO DIE:



$P(A) = 15/36 \Rightarrow P(A^c) = 21/36$

The die is equally likely to be even or odd.

$P(B) = 1/2 \cdot P(A^c)$ $\quad P(C) = 1/2 \cdot P(A^c)$

Sanity check: $P(A) + P(B) + P(C)$

$= 15/36 + 21/36(1/2) + 21/36(1/2)$

$= 1$

$E = 4[15/36] + 6[21/72] - 5[21/72]$

The probability that is rains on a given day is 10%. The probability that the ground is wet on a given day is 12%. The probability the ground is wet given that it rained that day is 100%. What is the probability that it rains in a given day, given that the ground is wet?

$P(\text{rain}) = 0.1$ $\quad P(\text{wet}) = 0.12$

$P(\text{wet} | \text{rain}) = 1$ $\quad P(\text{rain} | \text{wet}) = ?$

$$P(\text{rain}|\text{wet}) = \frac{P(\text{wet}|\text{rain})}{P(\text{wet})} \cdot P(\text{rain}) = \frac{1(0.1)}{0.12} \approx 0.83$$

You run a casino with the following two phase game:

1) roll 2 fair 6-sided dice and compute the difference between them. Call this x.

2) roll x fair 6-sided dice and compute their sum. This is your payout.

Q1: How many values can x take? What are the probabilities of each?

$x =$	0	1	2	3	4	5
$P(x) =$	6/36	10/36	8/36	6/36	4/36	2/36

$P(1) = \left(\frac{2}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{4}{6}\right)\left(\frac{2}{6}\right)$

$P(2) = \left(\frac{4}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{6}\right)\left(\frac{2}{6}\right)$

$P(3) = \left(\frac{6}{6}\right)\left(\frac{1}{6}\right)$

$P(4) = \left(\frac{4}{6}\right)\left(\frac{1}{6}\right)$

Q2: What is the expected payout for each x?

$x =$	0	1	2	3	4	5
$E(x) =$	0	3.5	7	10.5	14	17.5

Q3: You want to average \$1.50 profit per game. How much should you charge?

The expected payout is...

$0\left(\frac{6}{36}\right) + 3.5\left(\frac{10}{36}\right) + 7\left(\frac{8}{36}\right) + 10.5\left(\frac{6}{36}\right) + 14\left(\frac{4}{36}\right) + 17.5\left(\frac{2}{36}\right)$

≈ 6.81

So, you should charge at least \$8.31.

You have the following grade data avalible to you:

4/5 of students who pass study

3/5 of students study

3/4 of students pass

What is the probability you pass given that you study? What is the probability you don't pass given you don't study?

$P(\text{study}) = 3/5 \qquad P(\text{pass}) = 3/4$

$P(\text{study} \mid \text{pass}) = 4/5$

$$P(\text{pass} \mid \text{study}) = \frac{P(\text{study} \mid \text{pass}) \cdot P(\text{pass})}{P(\text{study})}$$

$$= \frac{(4/5)\ (3/4)}{(3/5)} = 1$$

$$P(\text{fail} \mid \text{not study}) = 1 - P(\text{pass} \mid \text{not study})$$

$$= 1 - \frac{P(\text{not study} \mid \text{pass})\ P(\text{pass})}{P(\text{not study})}$$

$$= 1 - \frac{[1 - P(\text{study} \mid \text{pass})]\ P(\text{pass})}{[1 - P(\text{study})]}$$

$$= 1 - \frac{(1 - 4/5) \cdot (3/4)}{(1 - 3/5)} = \frac{5}{8}$$