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→ RULES OF INFRENCE

RULES OF INFRENCE

We have used laws to prove equivalence. Now, we're interested in combining certain statements (premises) to derive new information (conclusions).

The importance of this in a proofs class should be evident. This will be achived via rules of infrence:

Modus Ponens	Modus Tollens	Generalization
$p \Rightarrow q$ p $\therefore\ q$	$p \Rightarrow q$ $\sim q$ $\therefore\ \sim p$	p $\therefore\ p \vee q$
Specialization	**Conjunction**	**Elimination**
$p \wedge q$ $\therefore\ p$	p q $\therefore\ p \wedge q$	$p \vee q$ $\sim p$ $\therefore\ q$
Transitivity	**Cases**	**Contradiction**
$p \Rightarrow q$ $q \Rightarrow r$ $\therefore\ p \Rightarrow r$	$p \vee q$ $p \Rightarrow r$ $q \Rightarrow r$ $\therefore\ r$	$\sim p \Rightarrow 0$ $\therefore\ p$
Dilema		
$(p \Rightarrow q) \wedge (r \Rightarrow s)$ $(p \vee r)$ $\therefore\ (q \vee s)$		

An example:

P
Q
$(P \vee Q) \Rightarrow R$
$(P \wedge Q) \Rightarrow S$

$\therefore R \wedge S$

1	P	
2	Q	
3	$(P \vee Q) \Rightarrow R$	
4	$(P \wedge Q) \Rightarrow S$	
5	$P \vee Q$	Generalization (1)
6	$P \wedge Q$	Conjunction (1, 2)
7	R	Modus Pones (3, 5)
8	S	Modus Pones (4, 6)
$\therefore$	$R \wedge S$	Conjunction (7, 8)

PRACTICE

$$(S \vee R) \wedge (R \Rightarrow P) \wedge (\neg S) \Rightarrow P$$

1	$S \vee R$	
2	$R \Rightarrow P$	
3	$\neg S$	
4	R	Elimination (1, 3)
$\therefore$	P	Modus Ponens (2, 4)

$$(P \vee Q) \wedge (\neg P \Rightarrow \neg R) \wedge (R) \wedge (\neg Q \Rightarrow \neg(P \vee Q)) \Rightarrow P \wedge Q$$

1	$P \vee Q$	
2	$\neg P \Rightarrow \neg R$	
3	R	
4	$\neg Q \Rightarrow \neg(P \vee Q)$	
5	Q	Modus Tollens (1, 4)
6	P	Modus Tollens (2, 3)
$\therefore$	$P \wedge Q$	Conjunction (5, 6)

$$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge ((P \Rightarrow R) \Rightarrow S) \Rightarrow S$$

1	$P \Rightarrow Q$	
2	$Q \Rightarrow R$	
3	$(P \Rightarrow R) \Rightarrow S$	
4	$P \Rightarrow R$	Transivity (1, 2)
$\therefore$	S	Modus Pones (3, 4)

$$(P \wedge Q) \wedge (P \Rightarrow R) \wedge (S \Rightarrow \neg P) \Rightarrow \neg S \wedge R$$

1	$P \wedge Q$	
2	$P \Rightarrow R$	
3	$S \Rightarrow \neg P$	
4	P	Specialization (1)
5	R	Modus Pones (2, 4)
6	$\neg S$	Modus Tollens (3, 4)
$\therefore$	$\neg S \wedge R$	Conjunction (5, 6)