

DEPARTMENT OF MATHEMATICS

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Special Offerings

2024-2025 Special Course Offerings

(Subject to change, check the time schedule for most current information.)

Undergraduate Special Topics

- Summer 2024
 - Math 480: Computing and AI for Mathematics
- Autumn 2024
 - Math 180: Mathematics and Democracy
 - Math 380: Math That Lies
- Winter 2025
 - Math 180: The math class you wished you had in school
 - Math 480: Mathematical Introduction to Data Analysis and Image Processing
- Spring 2025
 - Math 180: Mathematics in Games
 - Math 480 A: The Burnside Problem
 - Math 480 B: Introduction to Mathematical Formalization

Summer 2024, Math 480: Computing and AI for Mathematics

This is a course in which we'll learn computational and machine learning based approaches to solving problems in mathematics. Passing familiarity with Python will be helpful, but we will review the fundamentals of programming in the first two weeks.

The first half of the course will cover basic complexity analysis, algorithmic concepts such as depth-first search / breadth-first search and max flow-min cut, linear programming, and Gröbner bases. Additionally, we will cover modern tooling and integration of AI code generation to speed up the programming process.

The second half of the course will cover the basics of machine learning using PyTorch. We will cover multilayer perceptrons, convolutional neural networks, and transformers. Our examples will be mathematical in nature and we will examine interpretability techniques to extract what our models have "learned". We will additionally have some guest lecturers present on how programming and machine learning are impacting pure mathematics.

Prerequisite: Any one of CSE 123, CSE 142, or CSE 163 (or equivalent). MATH 208 and 300 (or equivalent). MATH 402 recommended but not required.

Textbook (optional but recommended): "Computational Mathematics: An Introduction to Numerical Analysis and Scientific Computing with Python" by Dimitrios Mitsotakis

Autumn 2024, Math 180: Mathematics and Democracy

We'll take a mathematical look at problems that arise in a representative democracy. Some questions we may consider:

- How can we determine the winner in a ranked-choice election? What are the advantages and disadvantages of different systems?
- In a voting system with unequal weights like the Electoral College, how can we measure the relative power of different groups?
- In the House of Representatives, each state is assigned representatives according to their population. How exactly does that work?
- What algorithms can we use to divide limited resources, or settle disputes between parties with different goals and priorities?
- Can mathematics help us create fair congressional districts or detect gerrymandering? Along the way, we'll learn about different areas of math that can help us answer these questions.

Prerequisite: None

Autumn 2024, Math 380: Math That Lies, Communicating Why Some Quantitative Arguments Are Misleading or Bogus (a Calderwood Seminar in public writing)

The Calderwood Seminars started at Wellesley College in 2013 and later spread to a few dozen colleges and universities nationwide. The Calderwood seminars at UW started in 2019. The seminar offers a unique opportunity to work collaboratively with fellow students in the writing and peer-editing process. The course

requires commitment, curiosity, and a critical mindset. It is open to juniors and seniors, and qualifies for W-credit and DIV-credit.

Although it's important to educate the public about the ways that our field contributes to technological advance, medical progress, etc., mathematically trained people also have a special role to play in critiquing the fallacious and misleading ways that statistics, equations, and mathematical models are often used. Students in the seminar will learn:

- how bogus statistical arguments were used to bolster white supremacist pseudoscience (from a book by paleontologist Stephen Jay Gould);
- how proxy data fed into algorithms led to a revival of redlining, a practice that had supposedly been outlawed by the civil rights legislation of the 1960s (from a book by mathematician Cathy O'Neil);
- how misuse of probabilistic arguments has repeatedly led to the conviction of innocent people (from a book by Leila Schneps and Coralie Colmez, a mother-and-daughter team of mathematicians);
- how mathematical modeling early in the Covid-19 pandemic that was based on false assumptions gave support to Donald Trump's claims that Covid-19 would be no worse than the flu.

In a given week half of the class will write commentary on the reading, in the form of a book review, a blog posting, a newspaper article, or a letter to the editor; the other half will be student-editors who meet with the writers to suggest corrections and improvements in their drafts. That will be followed by workshopping all the 2nd drafts during class time.

LEARNING GOALS

- To increase your skill and confidence as writers.
- To learn how to collaborate effectively as editors and workshop participants.

- To learn how to process, analyze, and criticize mathematical arguments related to socially important controversies.
- To learn how to communicate in clear, crisp, lively, and error-free prose about the challenges and pitfalls in interpreting quantitative information.

Prerequisite: None

Please note: This is an intensive 5-credit course. Every week you need to (1) do all the assigned reading (whether you're a writer or editor), and do it carefully; (2) proofread your written work and revise it carefully in response to feedback, if you're a writer; and (3) give the writer extensive helpful corrections and suggestions, if you're an editor. During the first few weeks of the quarter I will check (1) by giving quizzes on the assigned reading and monitor (3) by viewing Zoom recordings of the editor-writer meetings and by noting what shape the written work is in when we workshop it in class. In the event that you do not do a conscientious job as editor in your first Zoom meeting with a writer, I will ask you to continue making Zoom recordings of your editing sessions so I can check for rapid improvement in that crucial aspect of the seminar. Of course, (2) will also affect the grade directly, because the final version of the written work will be a major component of the grade.

Winter 2025, Math 180: The math class you wished you had in school

What is the course about? This course is a friendly introduction to Mathematics as a science. We all know that memorizing trig identities isn't really anyone's idea of a good time -- so how come there are people who actively work on mathematics? The goal of this course is to show that mathematics is primarily a collection of ideas -- and often very simple and beautiful ideas that are much more fun than some complicated formulae. Frequently, such ideas can also be turned into very beautiful pictures. Moreover, just as there are things about the natural Universe that we do not understand (Is Time Travel really impossible?), there are very strange but very simple things about the good old integers that remain elusive -- and these are fun to see! Lastly, we'll discuss a little about language, truth and logic – what do they really mean?

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Who is it aimed at? This course is aimed at first-year students, students who haven't taken too much math (but everyone is welcome) and anyone who'd like to find out more about it. Just as not everyone who takes 'History 101' ends up a historian, not everyone taking this class is going to be a math major -but it's fun to know a little bit about history (especially the fun parts) and the same is true for math (especially the fun parts).

Prerequisites: No prerequisites are required but it might still help to be familiar with High School Algebra, and know things like the expansion of (a+b)^2. No calculus is required.

Winter 2025, Math 480: Mathematical Introduction to Data Analysis and Image **Processing**

This class will discuss theoretical aspects of data analysis and image processing techniques. We will cover principal component analysis and its applications, graph theory, Fourier-based techniques such as compressing and denoising, and (if time permits) an introduction to neural networks. There will be a few Python projects, but no preliminary knowledge of programming is necessary. Lecture notes: https://wwwusers.cse.umn.edu/~jwcalder/5467Notes.pdf [2]

Prerequisites: A proof-based real analysis class such as Math 327 or 335; and an advanced linear algebra class such as Math 318 or 340.

MATH 480 REGISTRATION REQUEST FORM ☑

Spring 2025, Math 180: Mathematics in Games

In this course we will explore the mathematics behind games. We will look into winning strategies, probability of winning and patterns the games reveal. This course will connect the best of two worlds: having fun playing and analyzing the situation through the logic lense of a mathematician. Anyone who likes to get a glimpse into proofs and the methodical ways of abstraction is welcome to join. Many ideas will be taken from the book 'Math Games with Bad Drawings' by Ben Orlin.

Prerequisite: None

Spring 2025, Math 480 A: The Burnside Problem

The Burnside Problem(s) tantalized the algebra community over the 20th century. The general version, posed in 1902, asks whether a finitely generated group in which every element has a finite order must be finite. It was resolved by Golod and Shafarevich more than 60 years afterwards, discovering fascinating methods in combinatorial ring theory which found surprising applications in algebraic number theory, lattices and more. Later on, dramatic developments in group theory enabled Adian-Novikov and later Olshanskii (using geometric methods) to solve the bounded version of the problem: must a finitely generated group with a bound on the orders (that is, finite exponents) of its elements be finite? Finally, in the early 90s, Efim Zelmanov solved the Restricted Burnside Problem, showing that there is a largest finite group with any given number of generators and a given exponent, using ingenious connections between p-groups, Lie algebras, Jordan algebras and polynomial identities. We will discuss these three versions of the Burnside Problem and overview some of the core ideas behind their solutions. We will focus on the novel concepts that these solutions required and revealed, and on their impacts on different mathematical disciplines.

Textbook: * A. I. Kostrikin, "Around Burnside", Ergeb. Math. Grenzgeb. (3), 20 [Results in Mathematics and Related Areas (3)] Springer-Verlag, Berlin, 1990. * A. Yu. Ol'shanskii, "Geometry of defining relations in groups", Kluwer Academic Publishers Group, Dordrecht, 1991.

Prerequisites: Math 402 - familiarity with groups and rings

Spring 2025, Math 480 B: Introduction to Mathematical Formalization

This course will explore how mathematical theorems can be formalized in computer proof systems such as the Lean Theorem Prover. We will focus on topics in undergraduate mathematics such as set theory, logic, functions, induction, number theory, and other topics typically encountered in Math 300: Introduction to Mathematical Reasoning. For each topic, we will review the mathematical foundations, and then attempt to formalize main theorems and exercises. As the course progresses, we will learn more and more sophisticated high-level tactics that will assist in the formalization. By the end of the quarter, we will have group projects, where each group chooses a mathematical statement to formalize, depending on the interests of the students.

Prerequisites: Math 300, programming experience

Graduate Special Topics 2024-2025

NOTE: courses are restricted to current Mathematics graduate students or those who have completed a minimum of three 500-level core math courses. If you have questions about this enrollment requirement visit our page about <u>registering for 500-level courses</u>.

Core Classes

- <u>504/505/506 (A/W/Sp): McGovern Modern Algebra</u>
- 524/525 (W/Sp): Real Analysis
- 534/535 (A/W): Rohde Complex Analysis

Autumn 2024

- 581 A: Wilson Classical & Multilinear Harmonic Analysis
- 581 B: Novik Discrete Geometry Potporrui
- 581 C: Billey Schubert Calculus of Flag Varieties and Grassmannian Varieties
- 581 D: Kovacs Moduli Theory of Higher Dimensional varieties
- <u>581 E: Viray Algebraic Points on Curves</u>

Winter 2025

- 582 A: Sanmarco Symmetric Tensor Categories
- <u>582 B: Steinerberger Graduate-level Linear Algebra</u>
- 582 C: Liu, R Dynamical Algebraic Combinatorics
- 582 D: Vinzant Tropical Geometry
- 582 E: Paternain Microlocal methods in dynamical systems

Spring 2025

- 583 A: Zhang Invariant Theory of Noncommutative Algebras
- <u>583 B: Steinerberger Graduate-level Linear Algebra</u>
- 583 C: Athreya Translation Surfaces and their Moduli Spaces
- 583 D: Thomas Symmetry Reduction in Optimization
- 583 E: Uhlmann Mathematics of Medical Imaging
- 583 F: Zhu Minimal Surfaces

504/505/506 McGovern - Modern Algebra

See <u>Syllabus.DVI (washington.edu)</u> for the full list of topics.

524/525: Real Analysis

524: We will cover essentially all of Chapters 1 through 3 of "Real Analysis: Modern Techniques and Their Applications" by Folland (2nd Edition)

525: We will cover Chapter 5, and parts of Chapters 6 - 7 of Folland. If time permits I may do a quick review of point set topology along the way, depending on whether students have seen it.

534/535: Rohde - Complex Analysis

The course covers the following topics:

- The complex exponential and Polar form
- Complex logarithms and roots
- Complex differentiation and the Cauchy-Riemann equations
- The Riemann sphere
- Linear fractional transformations
- Contours and line integrals
- Cauchy's theorem and Cauchy's formula
- Power series expansions of analytic functions
- Zeroes of analytic functions
- Laurent expansions
- The structure of isolated singularities
- The residue theorem
- Integrals over the real line and trigonometric integrals
- The argument principal
- Theorems of Rouche and Hurwitz
- Local inverse theorem
- Simple connectivity

- Maximum modulus theorem and Schwarz's lemmma
- Conformal automorphisms of D and H.

581 A: Wilson - Classical & Multilinear Harmonic Analysis

This course will focus on the study of classical and multilinear harmonic analysis. In particular, the main goal of the class will be to build the theory of wave packet analysis developed by Carleson and used to solve problems related to convergence of Fourier series and elliptic boundary value problems. We will follow the texts of Muscalu and Schlag ("Classical and Multilinear Har- monic Analysis: Volume 1 and Volume 2") as well as the text of Thiele ("Wave packet analysis"). If time permits, we will end with more recent methods of decoupling due to Bourgain and Demeter.

Prerequisites: Math 524 and Math 525

581 B: Novik - Discrete Geometry Potporrui

Discrete Geometry deals with finite sets of points, lines, planes, circles, and many other seemingly simple geometric objects. It has deep connections to combinatorics, optimization, number theory, and computer science. It is also very rich in simple-to-state yet long-unsolved problems. This course will be a sampler of a few of the topics in this vast field. We will start with classical theorems due to Radon, Helly, and Caratheodory as well as the Steinitz theorem and Dehn's solution to Hilbert's third problem. These 100year-old results very quickly lead to much more modern developments such as (1) fractional, colorful, and topological versions of some of these theorems, (2) Kahn--Kalai's counterexample to Borsuk's problem, (3) Paco Santos' disproof of 53-year-old Hirsch's conjecture, (4) Florian Frick's counterexamples to the topological Tverberg's conjecture, (5) high-dimensional paradoxes, (6) face enumeration on polytopes. We will cover some of these topics (and perhaps a few others). We will not follow any particular textbook. Instead, we'll use quite a few sources (including several recent papers). Some of the textbooks that might be handy are:

- Imre Barany, "Combinatorial convexity", 2021
- Jiri Matousek, "Lectures on Discrete Geometry", 2002
- Igor Pak, "Lectures on Discrete and Polyhedral Geometry", available at https://www.math.ucla.edu/~pak/book.htm

Prerequisites: good understanding of core 400-level math classes.

581 C: Billey - Schubert Calculus of Flag Varieties and Grassmannian Varieties

The study of Schubert calculus of Grassmannians and flag manifolds and the associated structures in algebraic geometry, algebraic topology, combinatorics and representation theory inspired some of the great advances in mathematics in the twentieth century. Inherently, the subject rests firmly on the study of matrices, determinants, intersections of linear spaces (meaning vector spaces and their translates), symmetry, and computation. We will introduce the flag variety from a modern perspective and present some of the tools used in intersection theory and the cohomology rings mostly from an algebraic and combinatorial perspective.

Textbooks:

- 1. "Introduction to the Cohomology of the Flag Variety" by Sara Billey, Yibo Gao, and Brendan Pawlowski. To appear 2024. This article is intended to be the first chapter of a book entitled "Handbook of Combinatorial Algebraic Geometry: Subvarieties of the Flag Variety" that is a compendium of topics in this area. The book is being edited by Erik Insko, Martha Precup, and Ed Richmond. The other chapters will each be written by different authors. Selected experts from other chapters will also be covered.
- 2. "Young Tableaux; With Applications To Representation Theory And Geometry" by William Fulton. Cambridge University Press. 1997.

Prerequisites: Math 504/505/506 and 561 or permission from the instructor are required.

581 D: Kovacs - Moduli Theory of Higher Dimensional varieties

Topics to be included:

- Overview and some history of moduli theory, statements of the main results
- Comparison to M_g. Why is this a lot harder? (difficulties, obstacles, long time roadblocks)
- Stable and related singularities and their deformations
- Moduli theory without a boundary
- Moduli of pairs I: flat divisorial part, "large" coefficients
- Moduli of pairs II: K-flatness, marked pairs.

581 E: Viray - Algebraic Points on Curves

This course will give an overview of the arithmetic of curves over global fields, with a focus on results on points of higher degree and on the arithmetic of modular curves. Students should have background in Galois theory and algebraic geometry.

582 A: Sanmarco - Symmetric Tensor Categories

582 B: Steinerberger - Graduate-level Linear Algebra

Graduate Level Introduction to Linear Algebra, no prerequisites.

582 C: Liu, R - Dynamical Algebraic Combinatorics

Dynamical algebraic combinatorics is the study of group actions on objects commonly studied in algebraic combinatorics such as words, partitions, or Young tableaux. In this course we will study various types of actions, most commonly symmetric group or cyclic group actions, often generated by simple local operations, and study the resulting behavior and emergent phenomena using techniques from combinatorics and representation theory. Particular emphasis will be placed on concrete examples.

Topics will include:

- group actions on sets and posets, Polya counting, and cycle index polynomials
- the q=-1 and cyclic sieving phenomena
- homomesy and resonance phenomena
- Bender-Knuth involutions and crystal operators, promotion, and evacuation
- toggles and rowmotion on order ideals and plane partitions
- piecewise-linear and birational analogues.
- Additional topics will be covered depending on time and interest.

Prerequisites: Some familiarity with algebra (504/505/506) and combinatorics (561/562), or permission of the instructor

582 D: Vinzant - Tropical Geometry

Description: Tropical geometry is the study of certain combinatorial shadows of solutions to systems of polynomial equations. It is based on tropical algebra, where the sum of two numbers is their minimum and the product is their sum. This turns polynomials into piecewise-linear functions, and their zero sets into polyhedral complexes. These combinatorial structures retain a surprising amount of information about their classical counterparts. This course will introduce and survey some topics in tropical geometry, including Puiseux series and valuations, Grobner complexes, tropical varieties, hyperplane arrangements and matroids, Bernstein's theorem, Viro's patchworking, and combinatorial Hodge theory.

Some familiarity with ideals and varieties will be assumed. Grades will be based on occasional homework assignments and a final project.

582 E: Paternain - Microlocal methods in dynamical systems

Description: In recent years there has been notable progress in the application of microlocal techniques to hyperbolic dynamics, showing for example that the Ruelle zeta function admits a meromorphic continuation to the entire complex plane (a conjecture of Smale from the 60s) or giving an alternative proof to the ergodicity of the geodesic flow in negative curvature. The objective of the course would be to introduce students to this circle of ideas.

No background is necessary in either microlocal analysis or dynamics; it will be provided during the course. Familiarity with manifolds, Fourier transforms and ODEs is important.

583 A: Zhang - Invariant Theory of Noncommutative Algebras

We will cover some of recent developments in noncommutative invariant theory. Here is a list of potential topics:

- 1. Local cohomology and Artin-Schelter (AS) regular algebras.
- 2. Group actions, Hopf algebra actions, and Homological determinant.

- 3. Shephard-Todd-Chevalley Theorem for AS regular algebras.
- 4. Gorenstein property and Watanabe's Theorem.
- 5. Auslander theorem, pertinency and McKay quiver.

Prerequisites: First year graduate algebra sequence

583 B: Steinerberger - Graduate-level Linear Algebra

Graduate Level Introduction to Linear Algebra, no prerequisites.

583 C: Athreya - Translation Surfaces and their Moduli Spaces

This will be a course translation surfaces and their moduli spaces, with a focus on associated dynamical systems, counting problems, and group actions. In recent years, translation surfaces and their moduli spaces have been the objects of extensive study and interest, with connections to widely-varied fields including (but not limited to), geometry and topology; Teichmuller theory; low-dimensional dynamical systems; homogeneous dynamics and Diophantine approximation; and algebraic and complex geometry.

Recommended (but not required) prerequisites would be complex analysis and measure theory, and background in manifolds may be helpful.

583 D: Thomas - Symmetry Reduction in Optimization

Techniques from representation theory can be used to reduce the size of optimization problems that are amenable to group actions. This is sometimes the only way to make these optimization problems tractable and there have been many successful applications of this idea in recent years. This course will develop the needed techniques via an array of examples from graph optimization, sum of squares optimization, geometric (sphere) packing, graph homomorphism inequalities etc.

Prerequisites: Some background in group theory and optimization will be helpful. No representation theory background is needed.

583 E: Uhlmann - Mathematics of Medical Imaging

583 F: Zhu: Minimal Surfaces

Course description: The study of the minimal surface spanning a wire is a central problem in differential geometry, dating back to Lagrange in the 1760s but still bringing forth deep mathematical challenges and innovations today. We'll cover the basic theory of minimal surfaces, towards characterisations of stability. If time permits, we'll discuss some recent breakthroughs related to the positive mass theorem and the classification of stable minimal surfaces.

Prerequisites: Manifolds sequence, some familiarity with Geometric Structures/PDE

Share

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