A SMALL CONTRIBUTION TO THE THEORY OF TRANSCENDENTAL FUNCTIONS

Consider the integral

$$p = \int \frac{qdx}{x-a},$$

where q is a function of x which does not depend on a. Differentiating p with respect to a we find

$$\frac{dp}{da} = \int \frac{qdx}{(x-a)^2}.$$

If q is chosen so that $\int \frac{qdx}{(x-a)^2}$ can be reduced to the integral $\int \frac{qdx}{x-a}$, then one can find a linear differential equation relating p and a, which can be used to express p as a function of a. In this way, one can find relationships between many different integrals, some taken with respect to x, and others with respect to a. Since this gives rise to several interesting theorems, I will attempt to develop it in a very general case in which the aforementioned reduction of the integral $\int \frac{qdx}{(x-a)^2}$ is possible. Namely, this can always be done if $q = \phi x.e^{fx}$, where fx is a rational algebraic function of x, and ϕx is given by an equation of the form

$$\phi x = k(x+\alpha)^{\beta}(x+\alpha')^{\beta'} \cdots (x+\alpha^{(n)})^{\beta^{(n)}},$$

where $\alpha, \alpha', \alpha'', \dots$ are constants, and $\beta, \beta', \beta'', \dots$ are arbitrary rational numbers. In this case one has

$$p = \int \frac{e^{fx}\phi x.dx}{x-a},$$

$$\frac{dp}{da} = \int \frac{e^{fx}\phi x.dx}{(x-a)^2}.$$