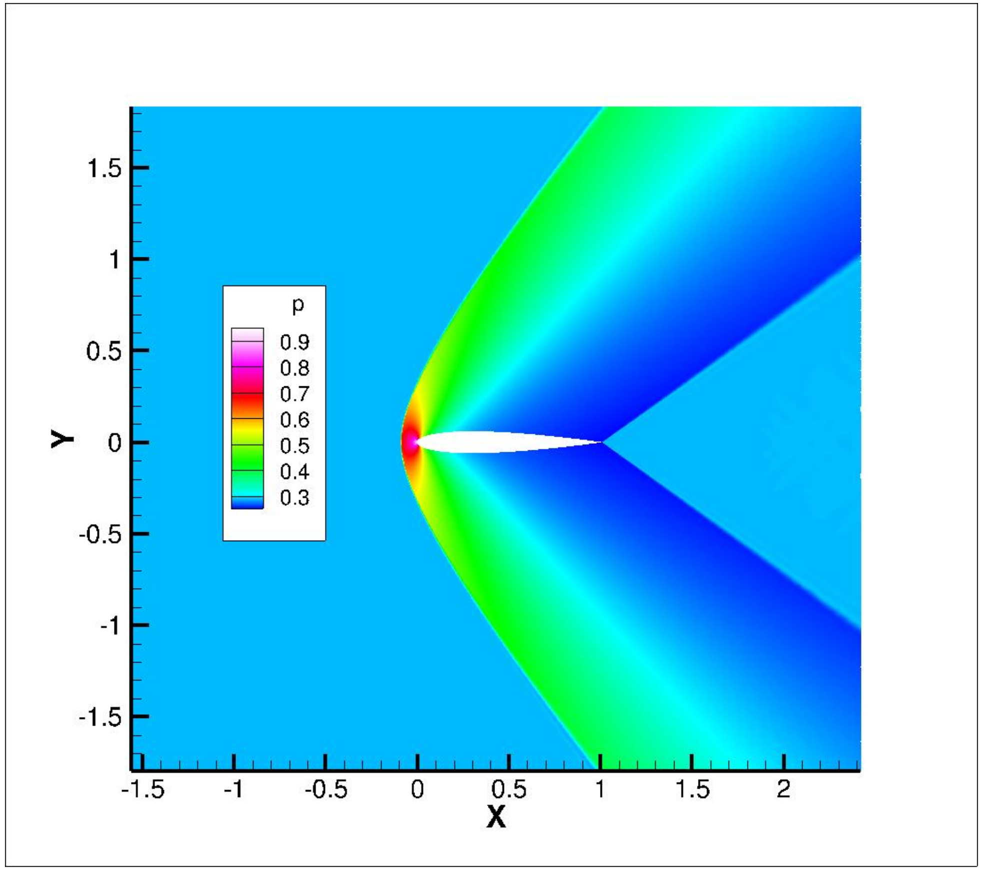
POLYTECHNIQUE MONTRÉAL

# Homework 3
# Development of a 2D Euler CFD solver

MEC6602E – Transonic Aerodynamics



# 2D Euler equations

- **Local form**

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \cancel{\frac{\partial (\rho w)}{\partial z}} = 0$$

Continuity equation

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho u \vec{V}) + \nabla p = 0$$

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho v \vec{V}) + \nabla p = 0$$

Momentum equations

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho E \vec{V}) + \nabla \cdot (p \vec{V}) = 0$$

Energy equations

# 2D Euler equations

- **Integral form :** used in the numerical solver

$$\int_{\Omega} \frac{\partial \vec{W}}{\partial t} + \oint_{S} \vec{F}_c \mathrm{d}S = 0$$

$$\vec{W} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \cancel{\rho w} \\ \rho E \end{pmatrix} \quad \text{3D}$$

$$\vec{F}_c = \begin{pmatrix} \rho V_c \\ \rho u V_c + n_x p \\ \rho v V_c + n_y p \\ \cancel{\rho w V_c + n_z p} \\ \rho E V_c + p V_c \end{pmatrix} \quad \text{3D}$$

Contravariant velocity: $V_c = \vec{V} \cdot \vec{n}$

# Code overview

- **Language** : Python

  - ***Cons :***

  Lack of low-level optimizations : No direct memory management or hardware-level optimizations.
  Higher memory usage : Inefficient for handling large-scale simulations compared to low-level languages.
  Interpreted language : Slower execution compared to compiled languages like C or Fortran.

  - ***Pros :***

  High level language : User-friendly.
  Object-oriented programming language : not the case of C language
  Possibility of relying on pre-compiled libraries (e.g. Numpy) to accelerate numerical computations

- **Structure**: Oriented object

  - ***Classes : Mesh, ConservedVariables, FluxDiscretizationScheme, TimeIntegrationMethod, Monitor***

# Code overview

- **Type of CFD solver** : structured, cell-centered
- **Numerical methods**
  - Flux discretization : **Central scheme with Artificial Dissipation** (order 1,2)
  - Time Integration : **Explicit Euler** (order 1), **RK2** (order 2)
  - Global & Local Time stepping
- **Inputs : Mesh (plot3D format), AOA, Mach number, Numerical Parameters**
- **Outputs : Flow solution** (.dat file)**, CL, CD, $CM_{c/4}$** coefficients**, residuals evolution, Cp curve, computational time**

# Validation case : NACA0012 airfoil in freestream flow

- **Symmetrical airfoil** : CL = 0, $CM_{c/4}$ = 0 @ 0 deg. AOA
- Comparison with results from Vassbert et Jameson [1] for different flow conditions (see Tab.1)
- Family of grids used : **O-grids, from (8x8 to 2049x2049)**
- BCs : wall, farfield, connect

| $(M, \alpha)$ | Nonlifting | Lifting |
|---|---|---|
| Subcritical | $(0.5M, 0\text{ deg})$ | $(0.5M, 1.25\text{ deg})$ |
| Transonic | $(0.8M, 0\text{ deg})$ | $(0.8M, 1.25\text{ deg})$ |

*Tab.1 : Comparison cases with [1]*

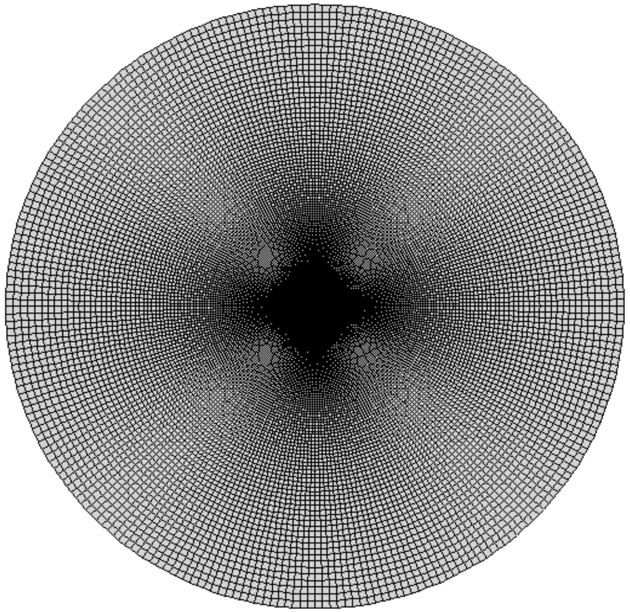


*Fig: Overview of the whole mesh (256x256)*

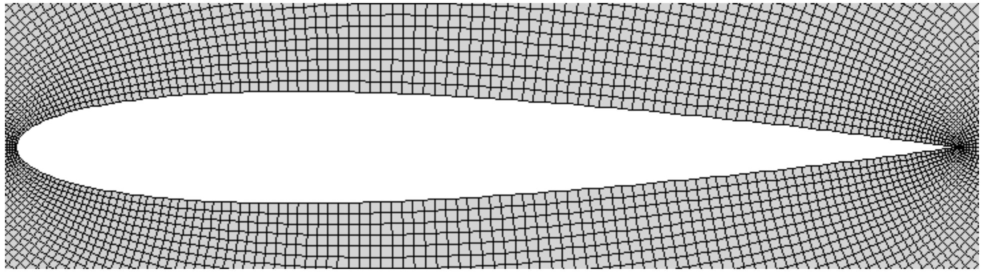
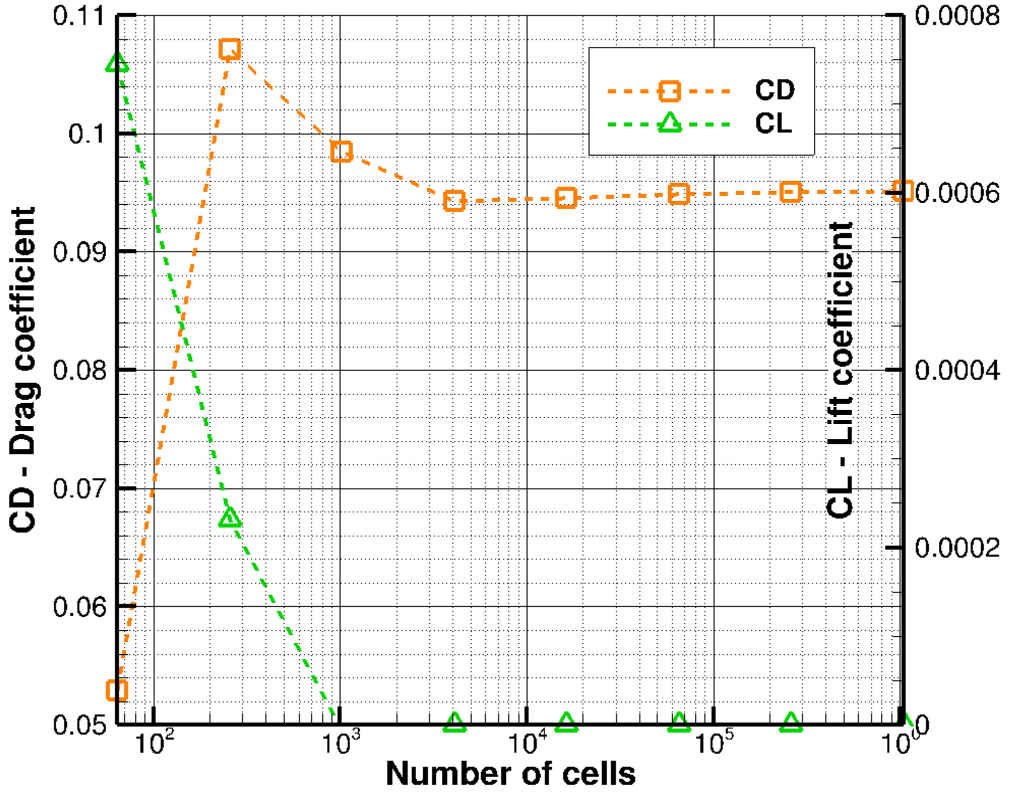


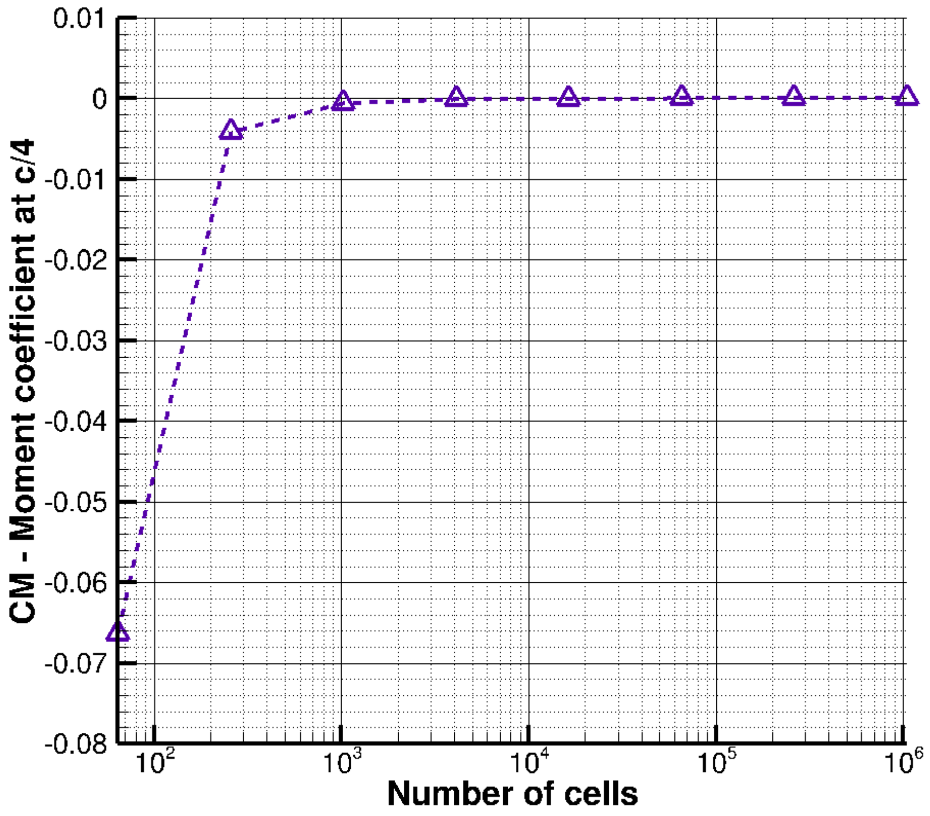
*Fig: Closeup view of the mesh near the airfoil (256x256)*

# Grid Convergence

- Performed for supersonic flow : **M=1.5, AOA=0°**
- Numerical parameters (CFL, dissipation constants) are kept constant across the cases to compare convergence and computational time
- Time Integration : RK2 (CFL = 1)
- Flux discretization : Central scheme with AD ($k_2$ = ½ , $k_4$= 1/64)
- Residual decrease (density) : $10^{-10}$ to consider the case as converged

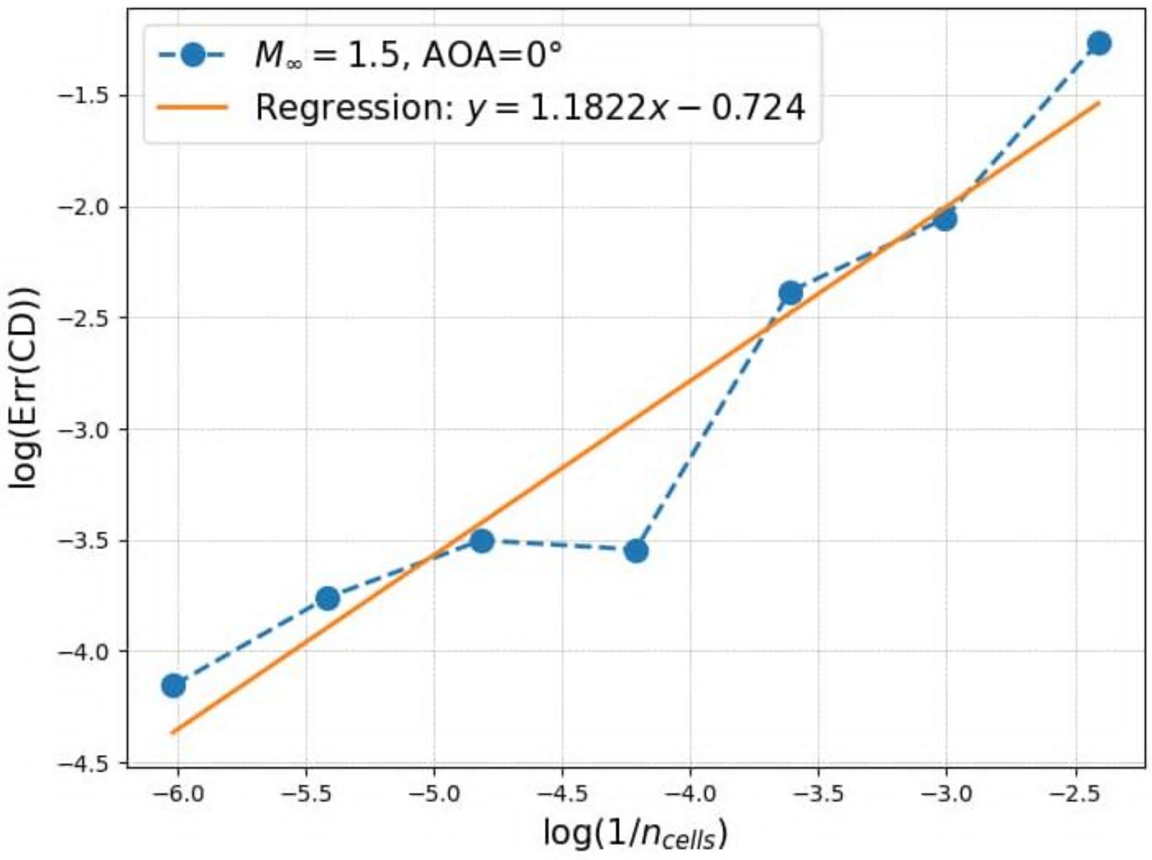
# Grid Convergence




- **Convergence in aerodynamic coefficients reached for the 128x128 mesh**
- **Converged values : CL = 0, $CM_{c/4}$ = 0, CD = 0.095134 (due to shock)**

# Order of Convergence



# Grid Convergence

- **Exponential rise in computational time** observed at grid sizes of 1024 and 2048 cells.

| Mesh Size | Convergence | Runtime [hh:mm:ss] |
|---|---|---|
| 1024 | $10^{-10}$ | 8:58:23 |
| 2048 | $10^{-6}$ | 51:52:42 |

- Optimized **parallel processing** or **advanced algorithms** are necessary for scalability in industrial applications

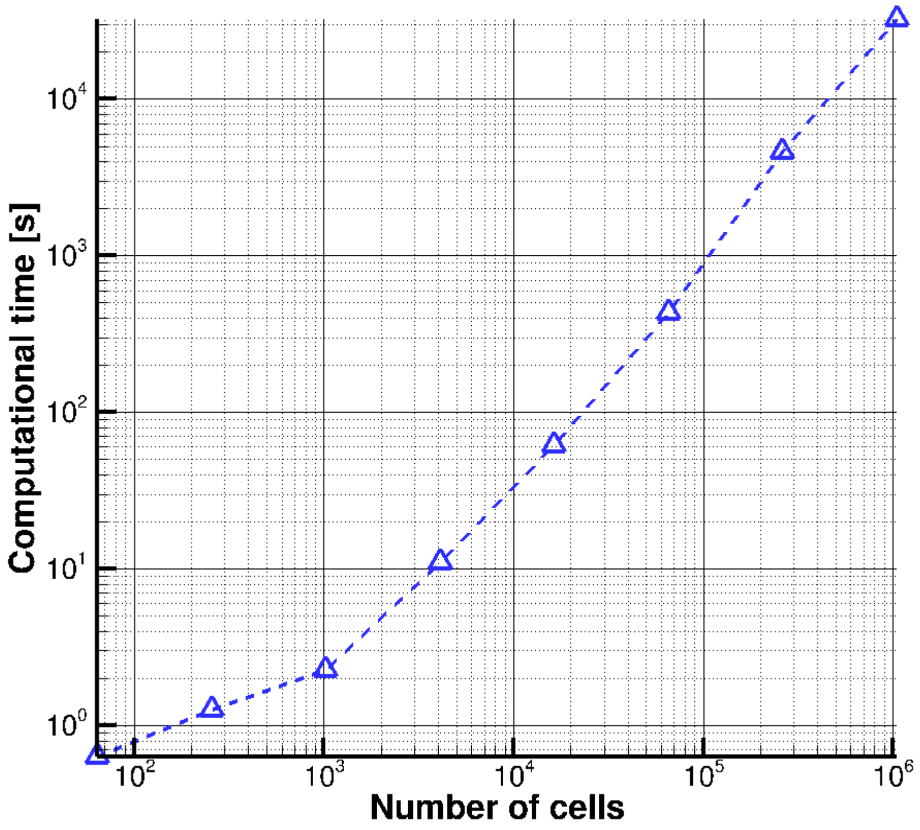
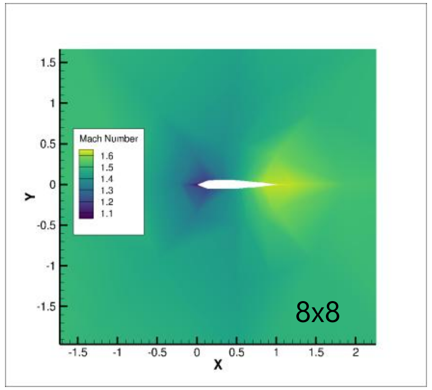
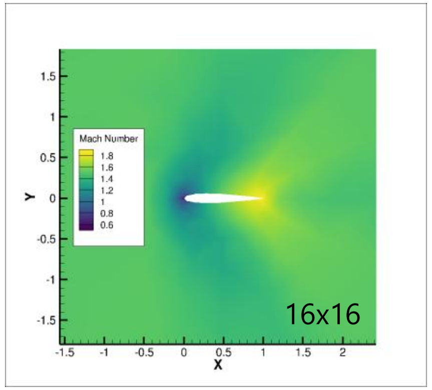


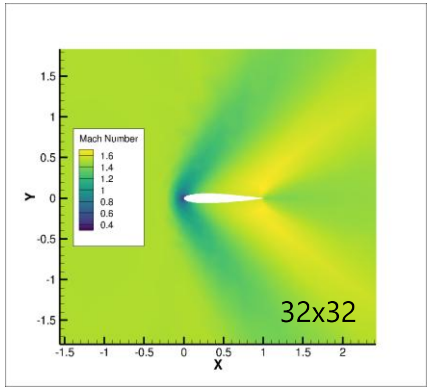

**Figure:** Computational Time vs. Grid Size: Impact of Increasing Resolution [1].

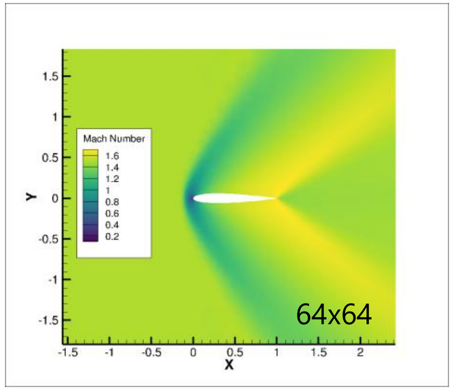
[1] Results for grid convergence have not been presented for the largest mesh. That mesh was only computed till a residual of $10^{-6}$.

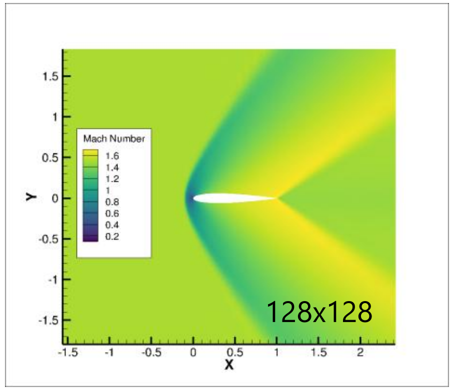
# Grid Convergence

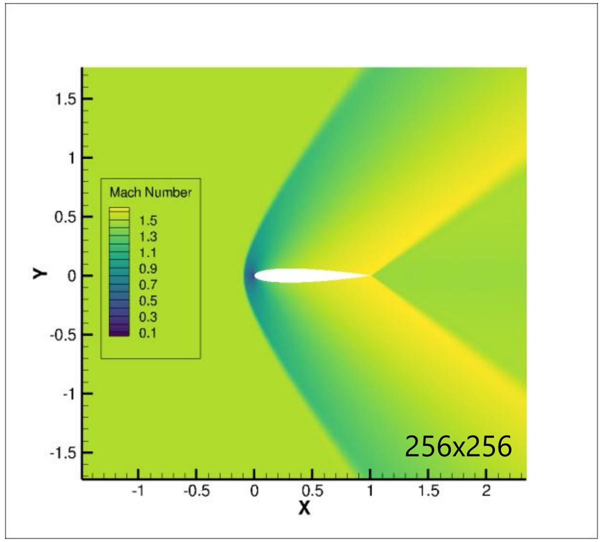


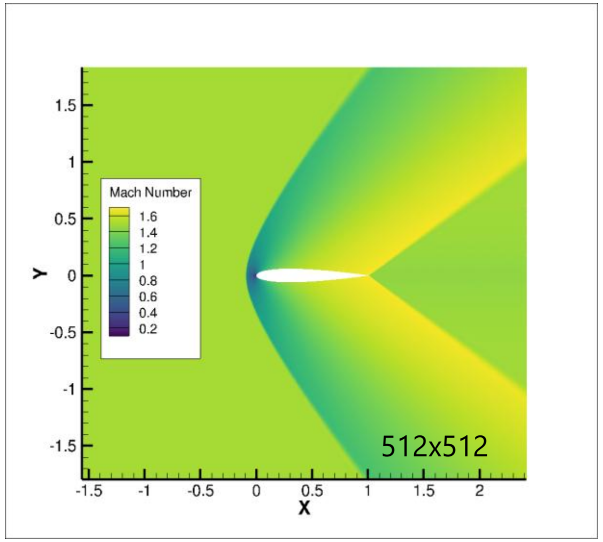



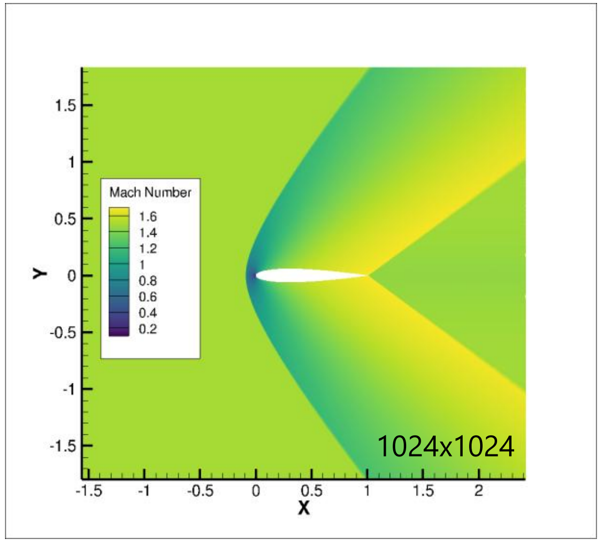



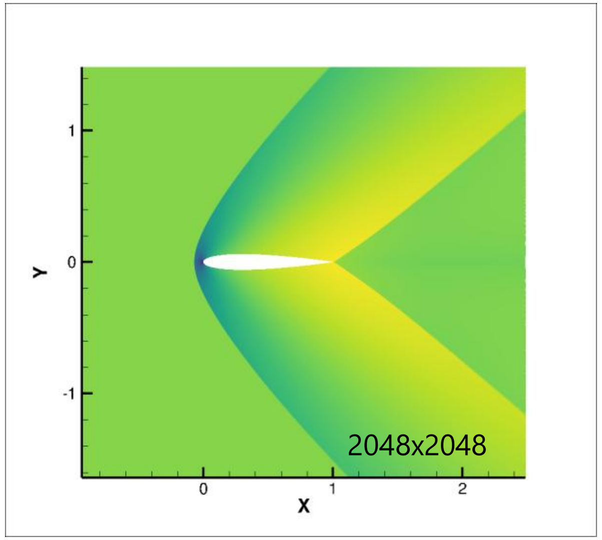



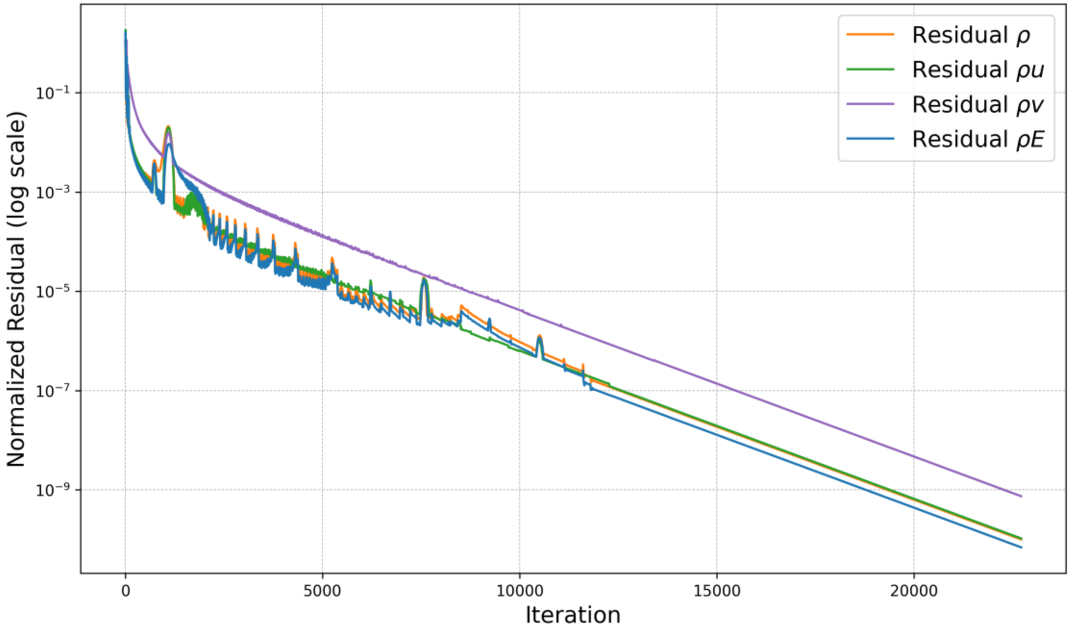



# Analysis of Results for the 1024x1024 Grid

- Bow shock at LE and shock at TE
- Symmetrical solution since the problem is symmetric



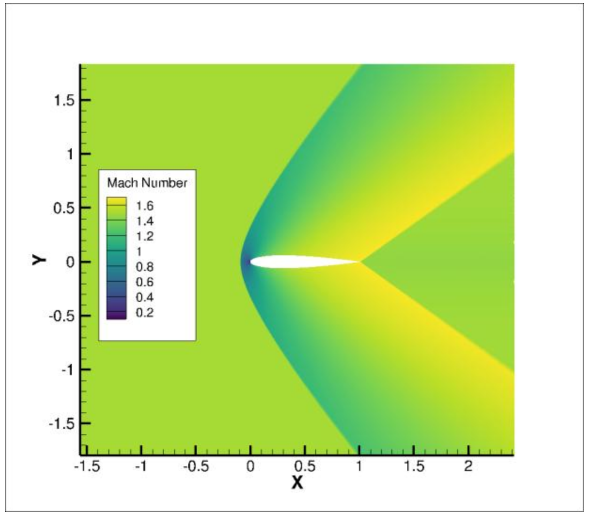




**Figure:** Mach Number.

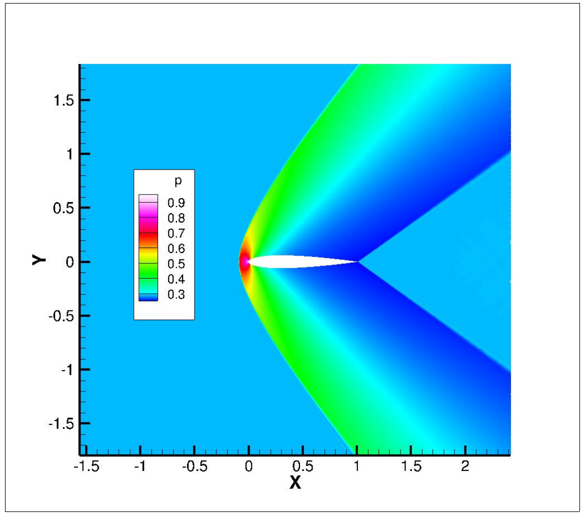


**Figure:** Density.

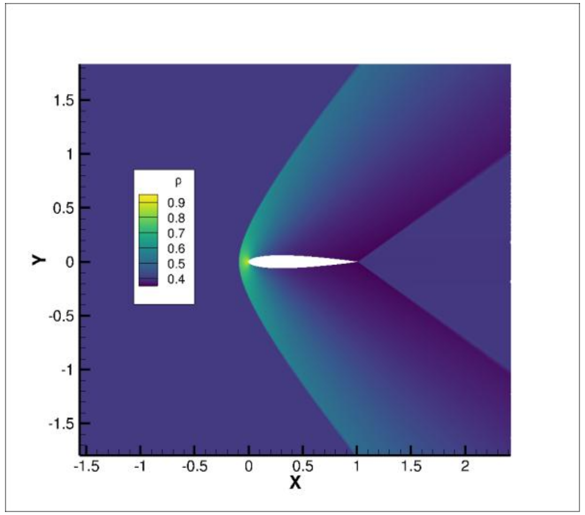


**Figure:** Pressure.

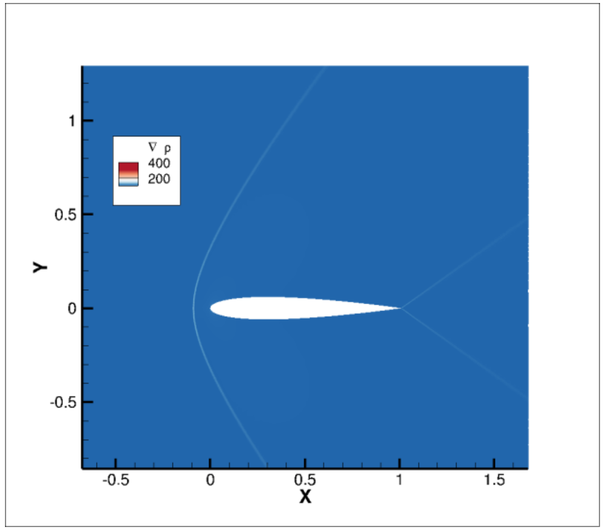
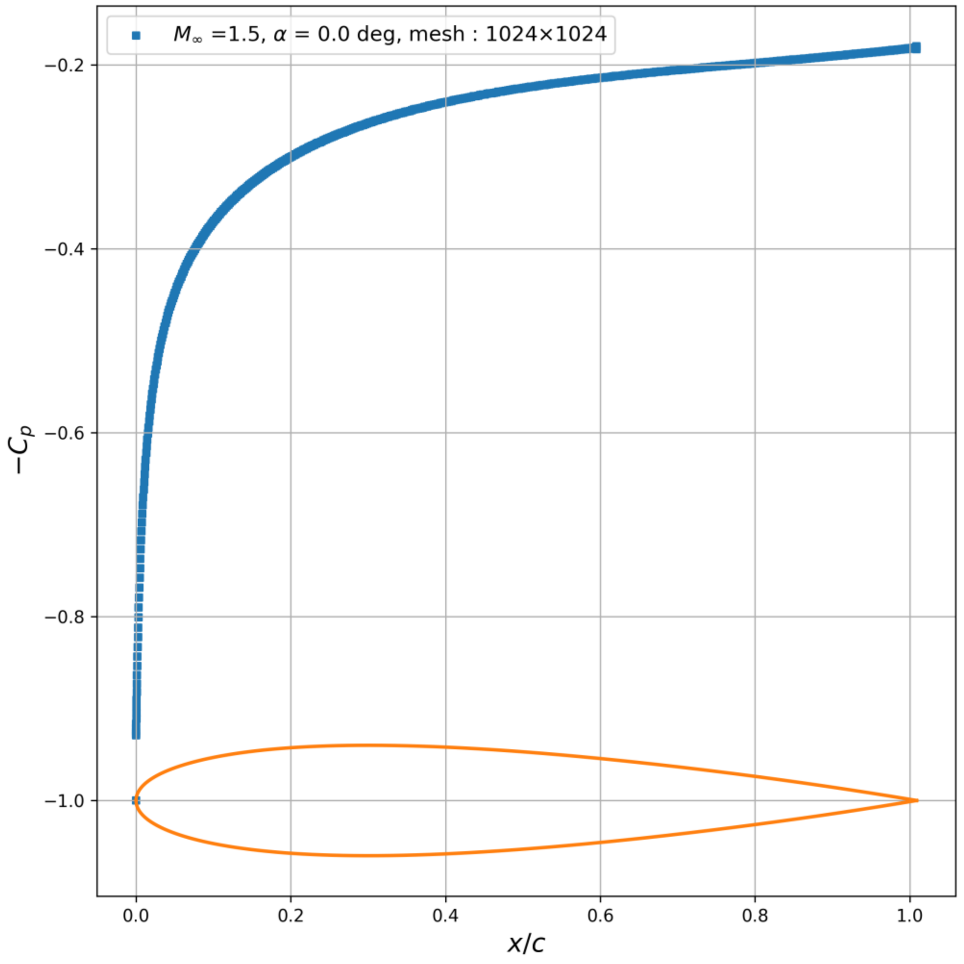


**Figure:** Density gradient.

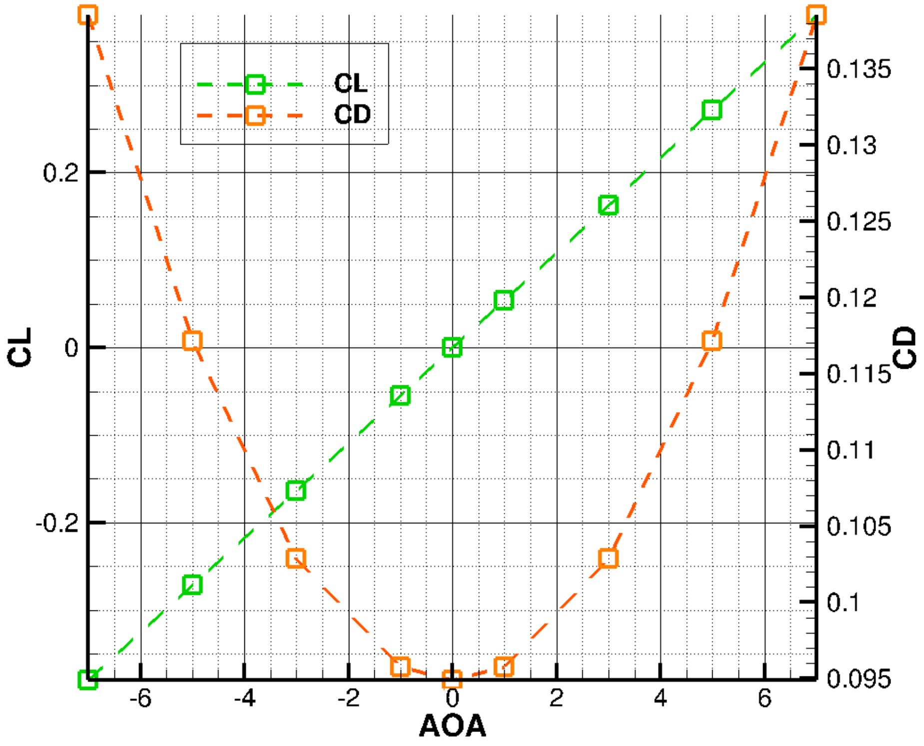
# Analysis of Results for the 1024x1024 Grid

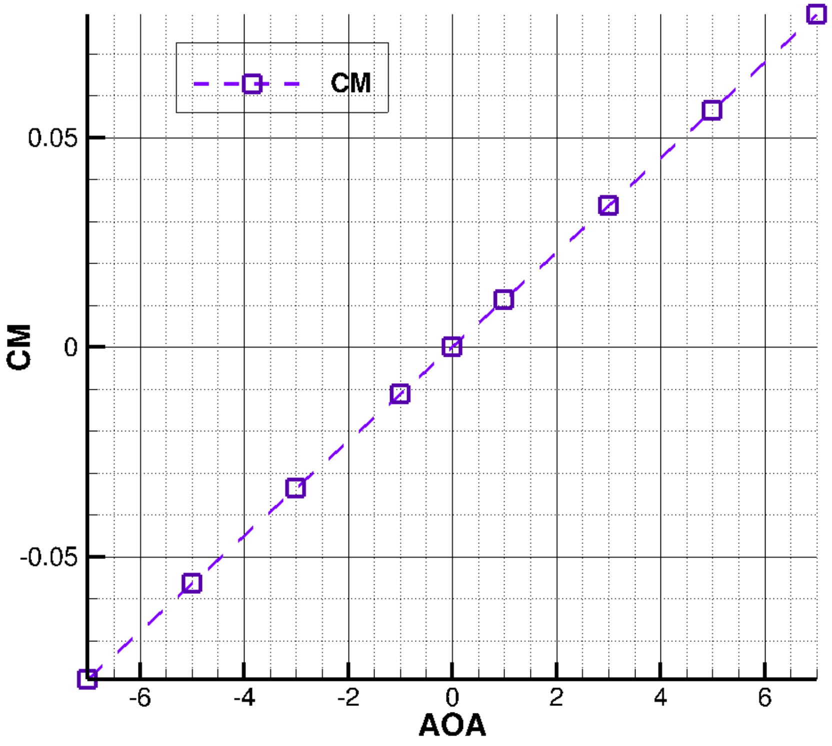
- Pressure distribution on the airfoil physically coherent with the case studied
- Pressure distribution is identic for lower and upper surfaces : CL = 0



# CL/CD/CM vs AOA curves

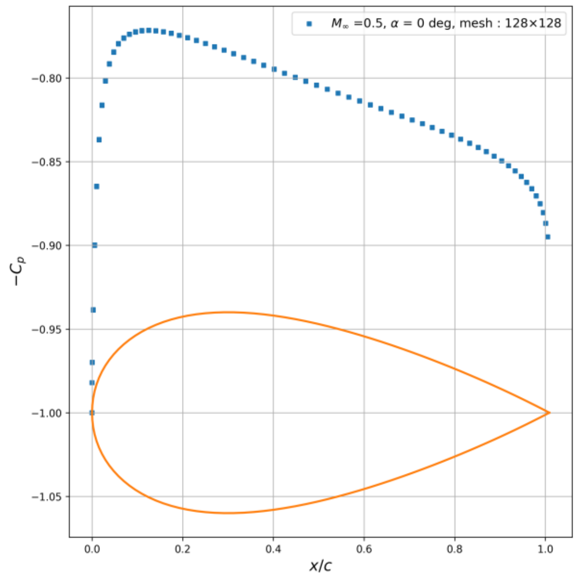
- Three flow regimes studied : subsonic (M=0.5), transonic (M=0.8), supersonic (M=1.5)
- Results for supersonic case (M=1.5) using 256x256 grid

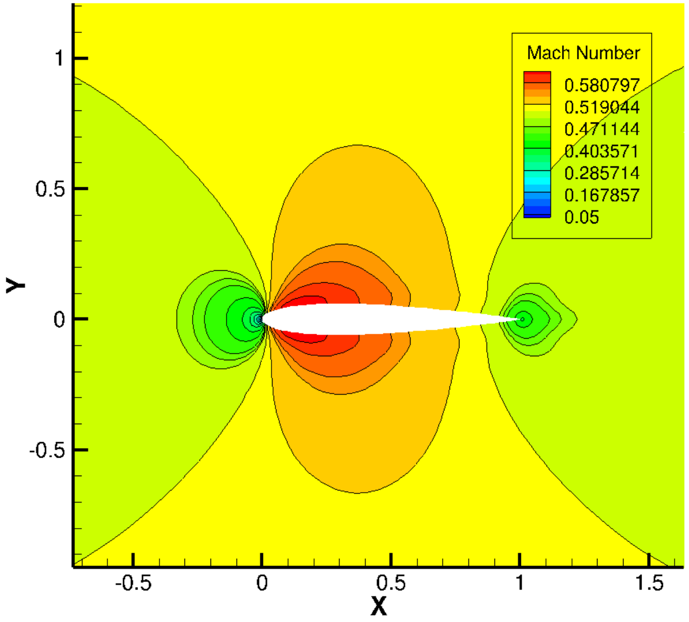


# Comparison with Jameson & Vassberg results [1]

- Results for subsonic case (M=0.5) using 128x128 grid – AOA = 0°



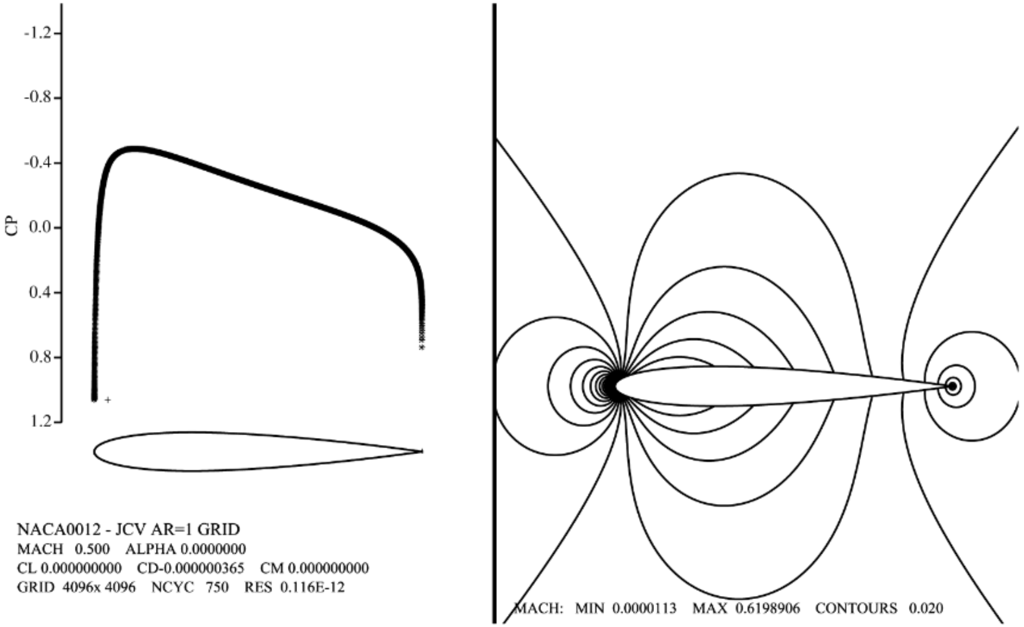
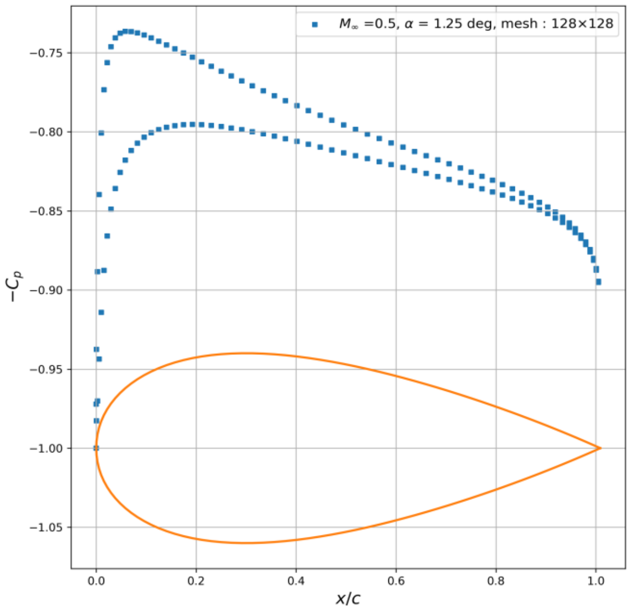



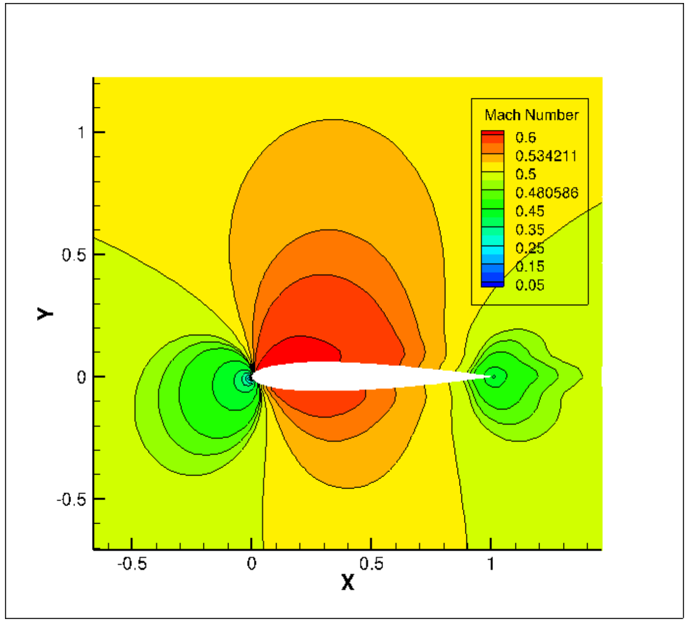

**Fig. 13** **FLO82 solution $M = 0.50$, $\alpha = 0.0$ deg.**

- Results agree well with the reference publication

# Comparison with Jameson & Vassberg results [1]

- **Results for subsonic case (M=0.5) using 128x128 grid – AOA = 1.25°**



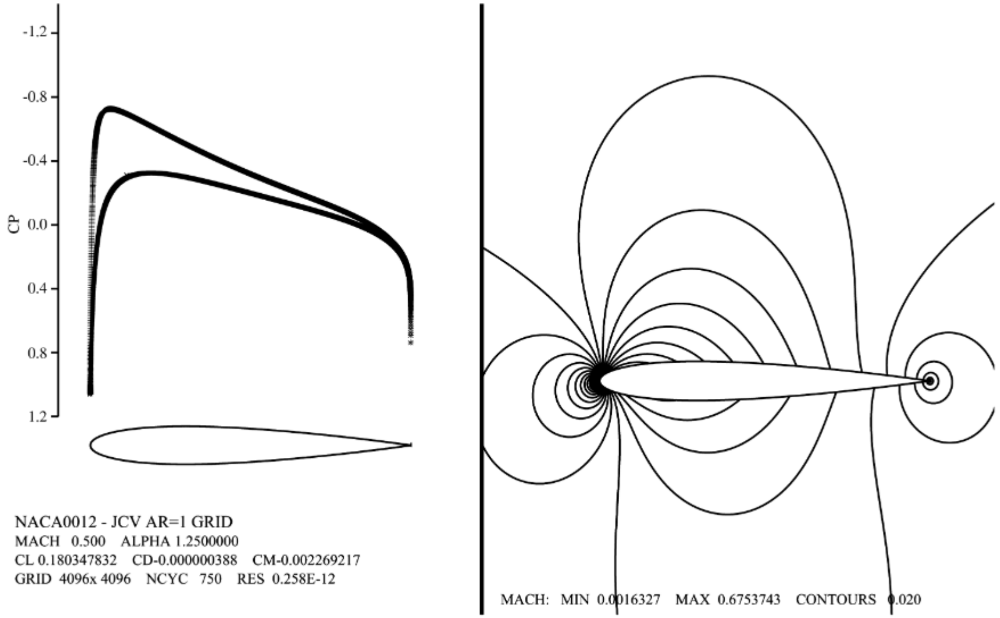



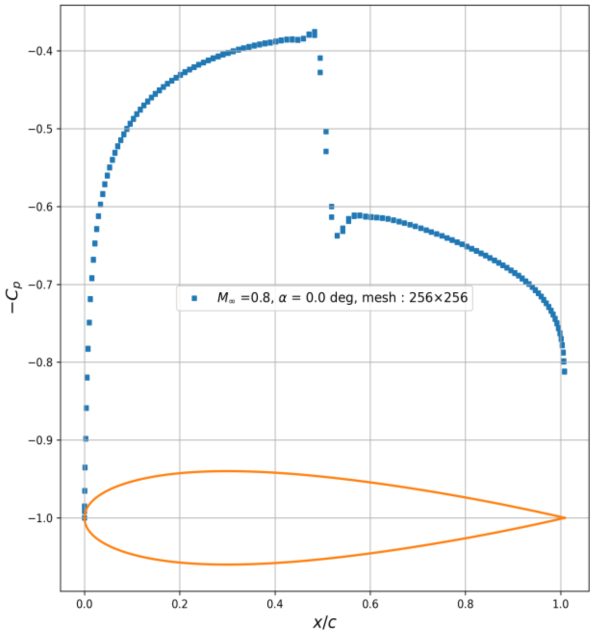

**Fig. 14** **FLO82 solution $M = 0.50$, $\alpha = 1.25$ deg.**

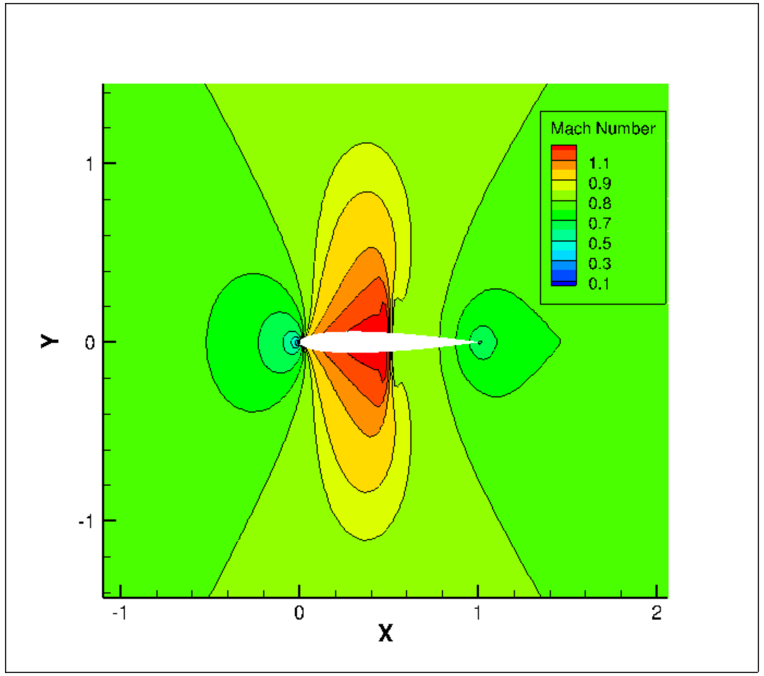
- **CL =** 0.177, **CD** = 0.000358, $\mathbf{CM_{c/4}}$ **=** 0.00211
- **CL =** 0.180, **CD** = 0.000298, $\mathbf{CM_{c/4}}$ **=** 0.00229

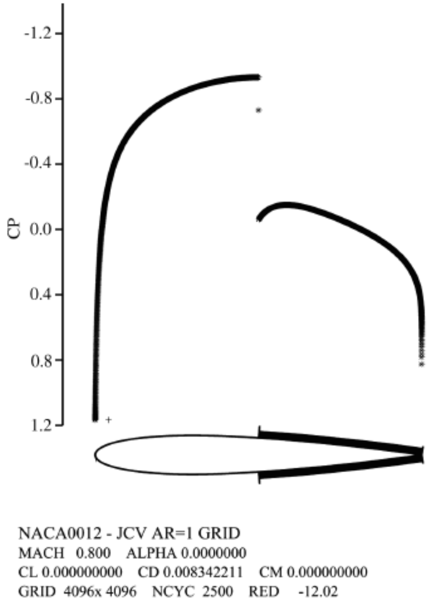
- **Results agree well with the reference publication**

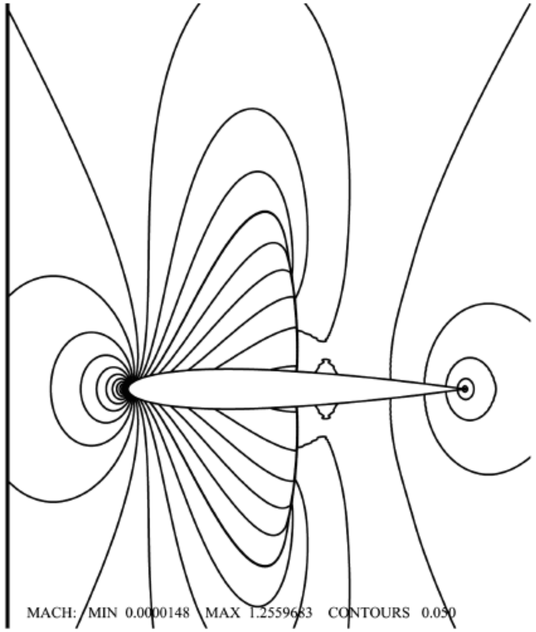
# Comparison with Jameson & Vassberg results [1]

- Results for transonic case (M=0.8) using 256x256 grid – AOA = 0°



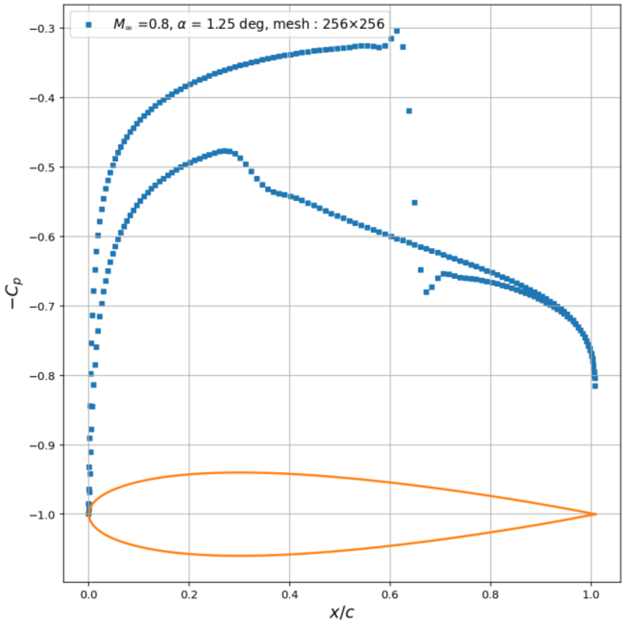

- **CD** = 0.008131

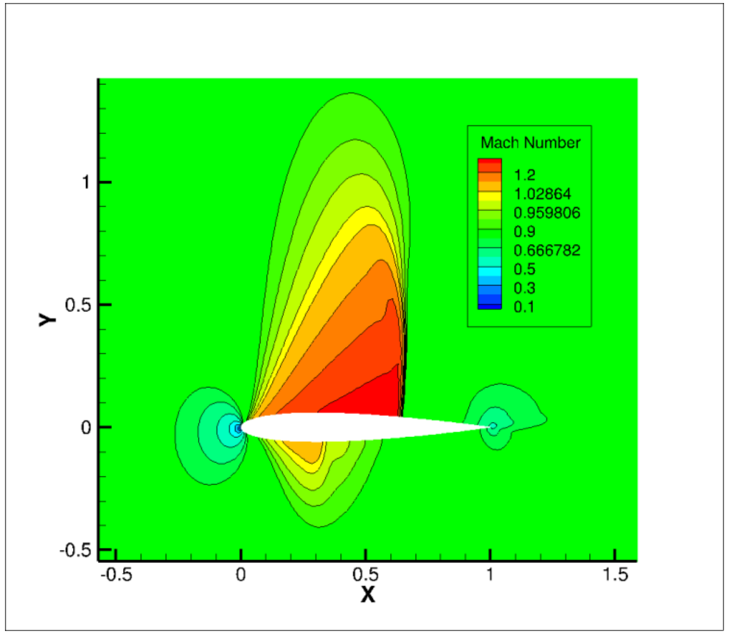
- **CD** = 0.008312

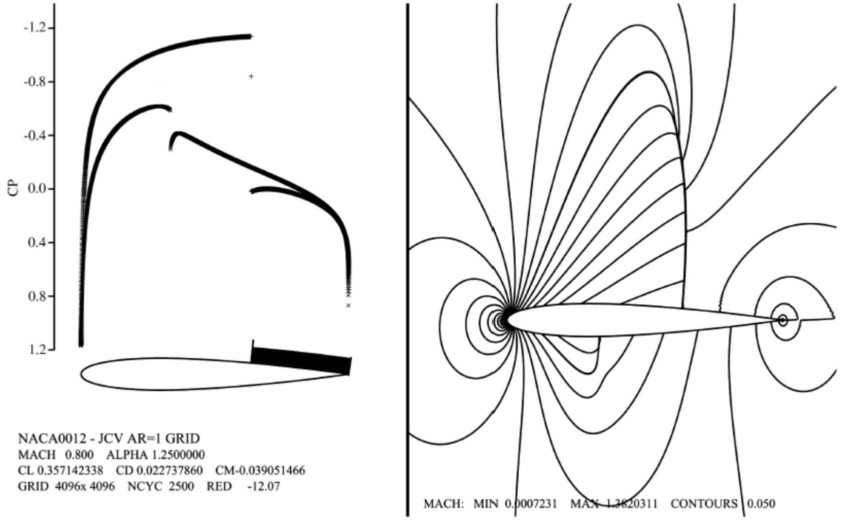
- Results agree well with the reference publication

# Comparison with Jameson & Vassberg results [1]

- **Results for transonic case (M=0.8) using 256x256 grid – AOA = 1.25°**




- **CL =** 0.364, **CD** = 0.0226, **CM$_{c/4}$ =** 0.0415
- **CL =** 0.357, **CD** = 0.0227, **CM$_{c/4}$ =** 0.0391

- **Results agree well with the reference publication**

# Improvements

- Development of implicit time integration schemes to enable faster convergence through the use of higher CFL numbers.
- Development of acceleration techniques, such as multigrid methods and preconditioning, to enhance computational efficiency.
- Optimization of the code, including rewriting it in a low-level language, implementing parallelization strategies, and exploring GPU acceleration to improve performance.
- Verify the implementation of subsonic far-field boundary conditions.
- Incorporate additional validation cases and design the code with greater abstraction to ensure it is not specialized solely for the NACA0012 problem.

# References

[1] Vassberg, J. & Jameson, Antony. (2010). In Pursuit of Grid Convergence, Part I: Two-Dimensional Euler Solutions. Journal of Aircraft - J AIRCRAFT. 47. 1152-1166. 10.2514/1.46737.

[2] Blazek, J. (2005). Computational Fluid Dynamics: Principles and Applications. 10.1016/B978-0-08-044506-9.X5000-0.