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# ECE 2202
# Circuit Analysis II

# Lecture Set #1
# Thévenin's and Norton's Theorems including Dependent Sources

Version 20

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# *Thévenin’s and Norton’s Equivalents and Dependent Sources*

# *Overview*
# *Thévenin's and Norton's Equivalents and Dependent Sources*

In this part, we will cover the following topics:

- Dependent Sources and Equivalent Resistance
- The Test-Source Method
- Example of finding an equivalent resistance with a dependent source present

# *Textbook Coverage*

This material is introduced in different ways in different textbooks. Approximately this same material is covered in the Nilsson and Riedel textbook in the following sections:

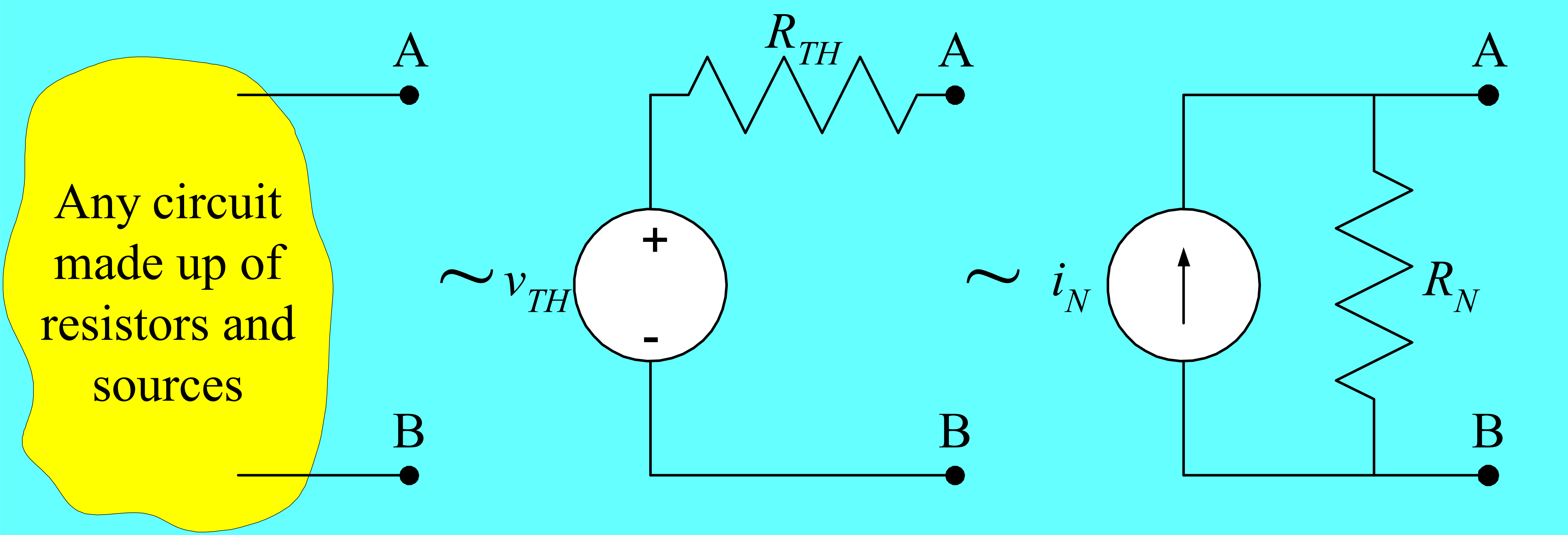
- Electric Circuits 10th Ed., by Nilsson and Riedel: Section 4.11

# Thévenin's and Norton's Theorems Reviewed

Thévenin's Theorem and Norton's Theorem can be stated as follows:

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance, or to a current source in parallel with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, the current source is equal to the short-circuit current for that circuit, and the resistance is equal to the equivalent resistance of that circuit.



# Extra note

We have shown that for the Thevenin equivalent and for the Norton equivalent, the open-circuit voltage is equal to the short-circuit current times the equivalent resistance. This is fundamental and important.

$$\boxed{v_{OC} = i_{SC} R_{EQ}.}$$

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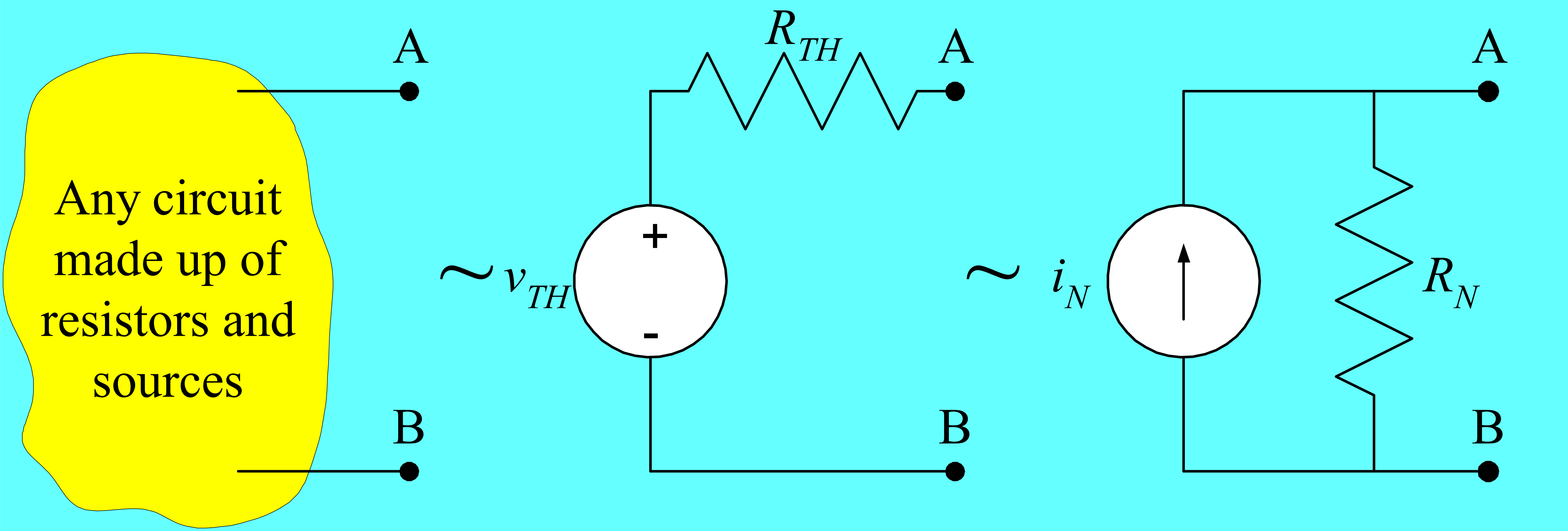
# Equivalent Resistance Reviewed

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

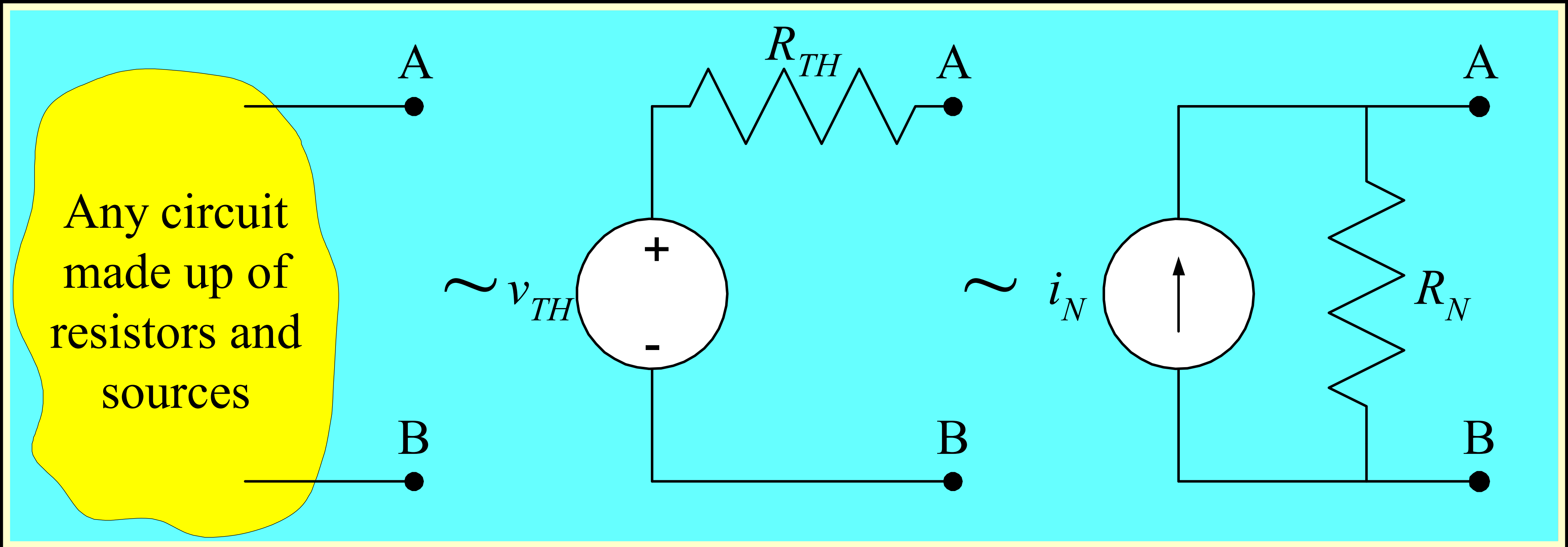
When a dependent source is present, trying to find the equivalent resistance results in a situation we have not dealt with yet. What do we mean by the equivalent resistance of a dependent source?

The answer must be stated carefully. If the ratio of voltage to current for something is a constant, then that something can be said to have an equivalent resistance, since it is behaving as a resistance.



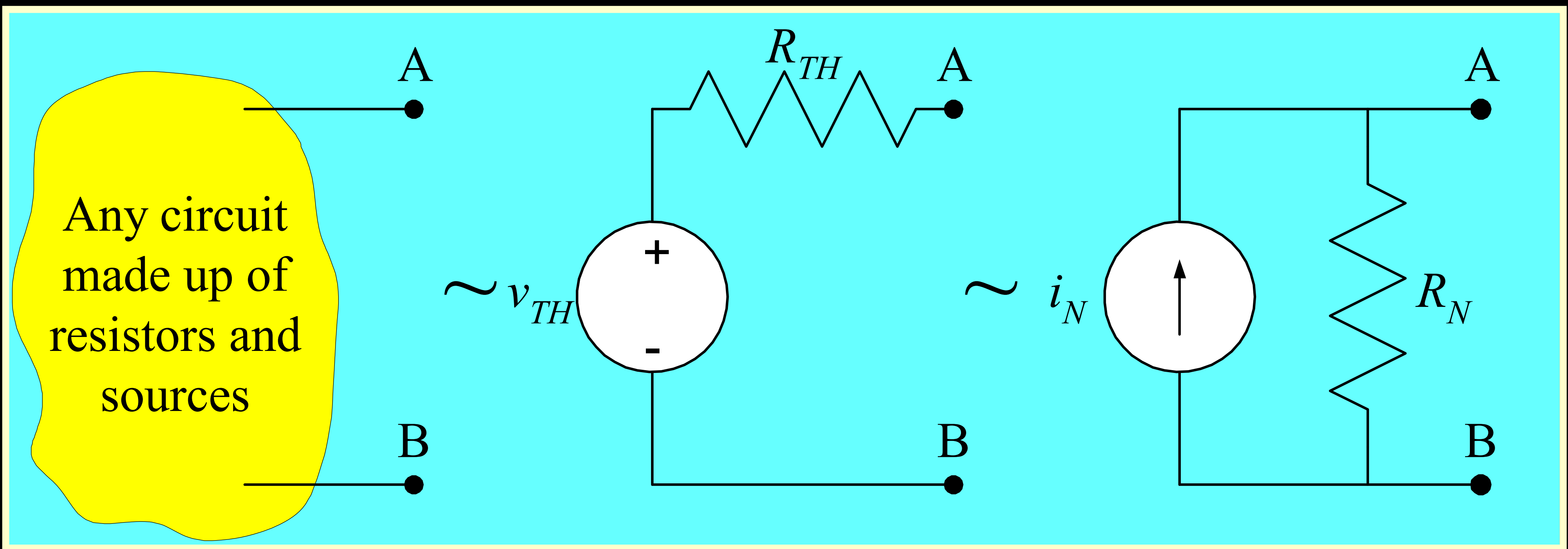
# *Equivalent Resistance of a Source*

So, what we mean by the equivalent resistance of a dependent source is that in this case the ratio of voltage to current is a constant. Then the source can be said to have an equivalent resistance, since it is behaving as a resistance. The equivalent resistance of a dependent source depends on what voltage or current it depends on, and where that voltage or current is in the circuit. It is not easy to predict the answer.



# No Equivalent Resistance for an Independent Source

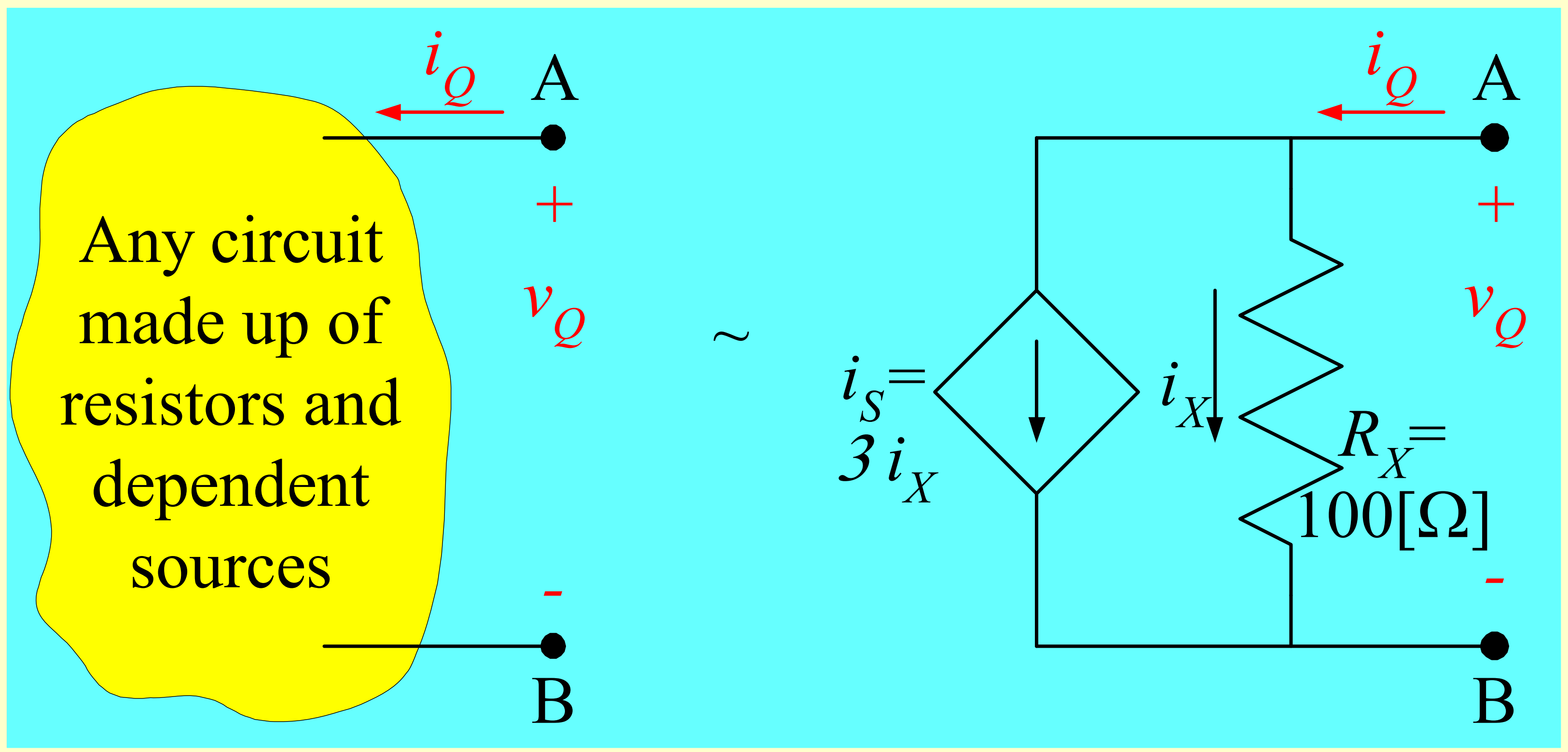
The equivalent resistance of a dependent source, in this case, is the ratio of voltage to current, which is a constant. Then the source can be said to have an equivalent resistance, since it is behaving as a resistance. This will only be meaningful for a dependent source. It is not meaningful to talk about the equivalent resistance of an independent source. The ratio of voltage to current will not be constant for an independent source.



# Simple Example with a Dependent Source

We will try to explain this by starting with a simple example. We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B.

This will mean that the ratio of the voltage across the circuit, labeled $v_Q$, to the ratio of the current through the circuit, labeled $i_Q$, must be a constant. Let's find that constant by finding the ratio.



# Simple Example with a Dependent Source – Step 1

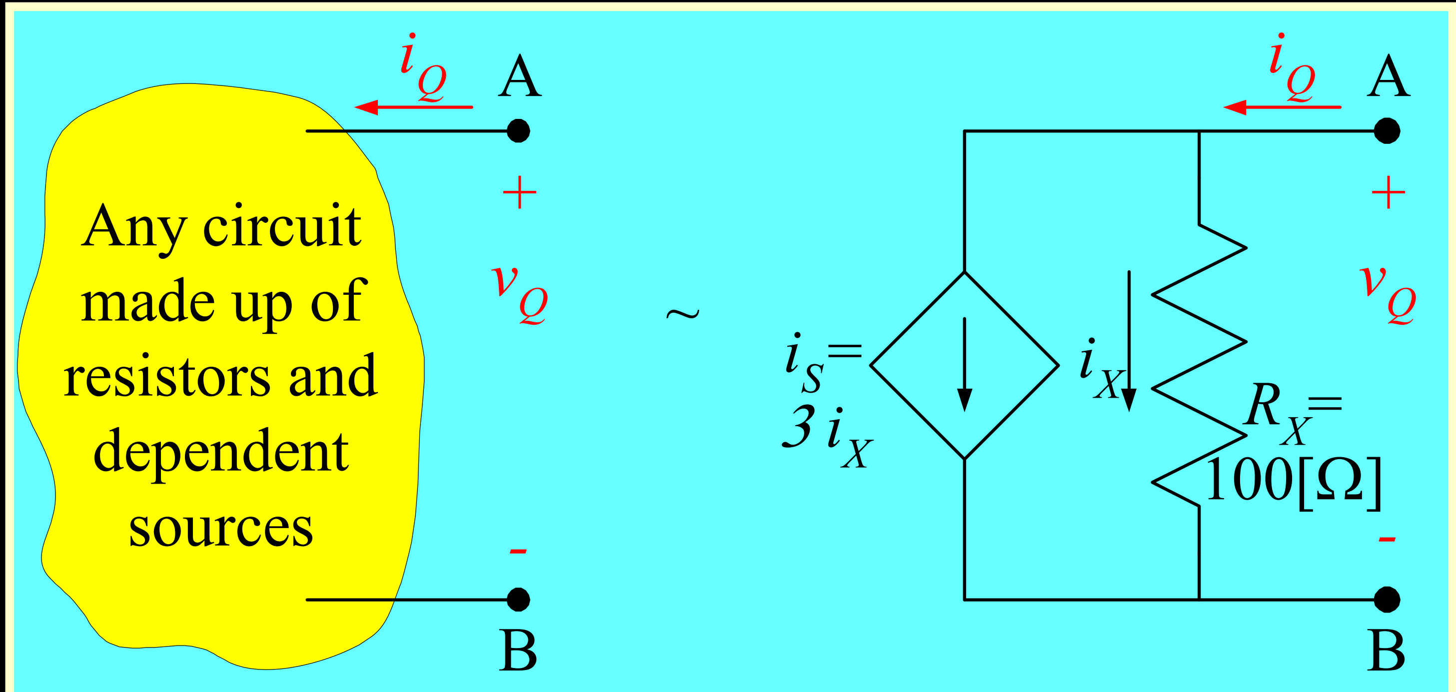
We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B.

Let's find the ratio of the voltage across the circuit, labeled $v_Q$, to the ratio of the current through the circuit, labeled $i_Q$. This must be a constant. Let's look first at the circuit equivalent on the right. We note that from Ohm's Law applied to $R_X$, we can say

$$v_Q = i_X R_X .$$

Next, we apply KCL at the A node to write that

$$i_Q = i_X + 3i_X .$$



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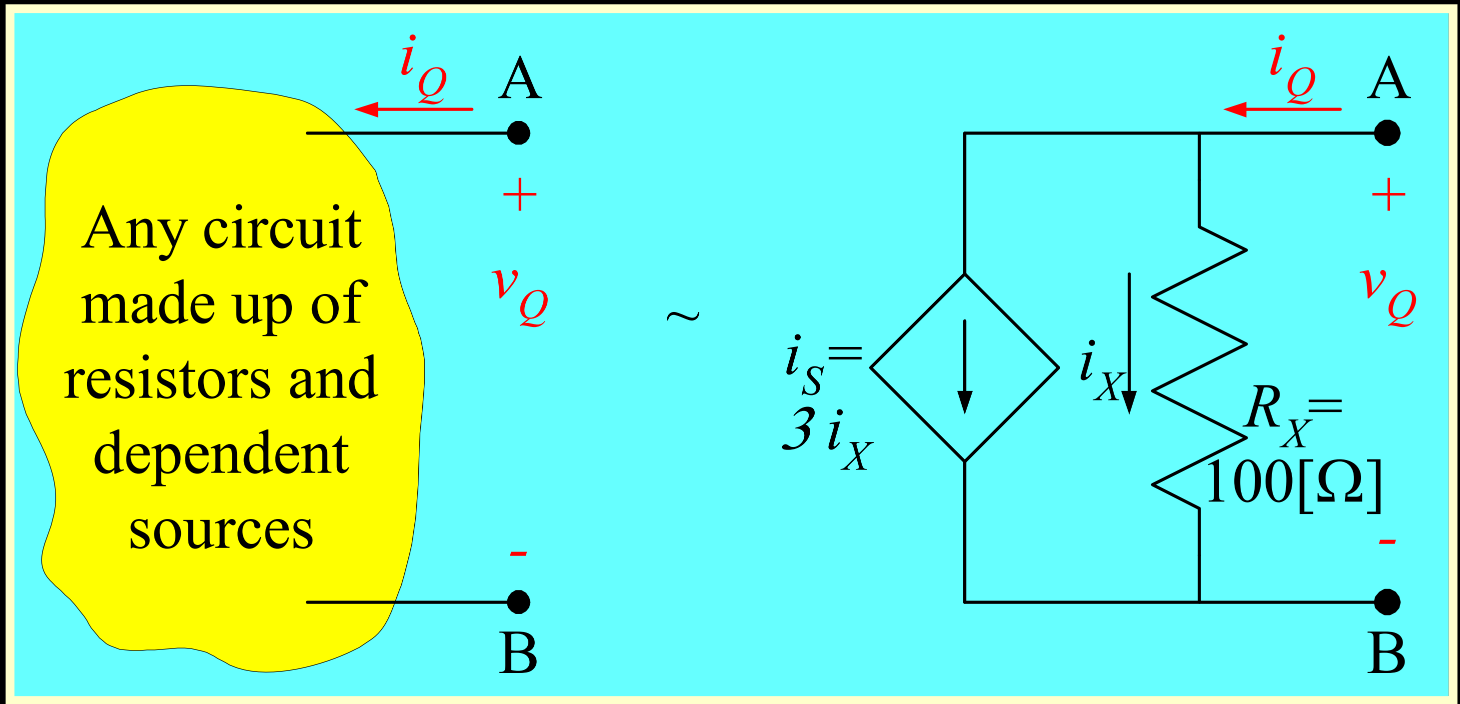
# Simple Example with a Dependent Source – Step 2

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. On the last slide we found $v_Q$, and we found $i_Q$. We take the ratio of them, and plug in the expressions that we found for each. When we do this, we get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X + 3 i_X} = \frac{i_X R_X}{4 i_X} = \frac{R_X}{4} = \frac{100[\Omega]}{4} = 25[\Omega].$$

Note that ratio is a constant. The ratio has units of resistance, which is what we expect when we take a ratio of a voltage to a current.



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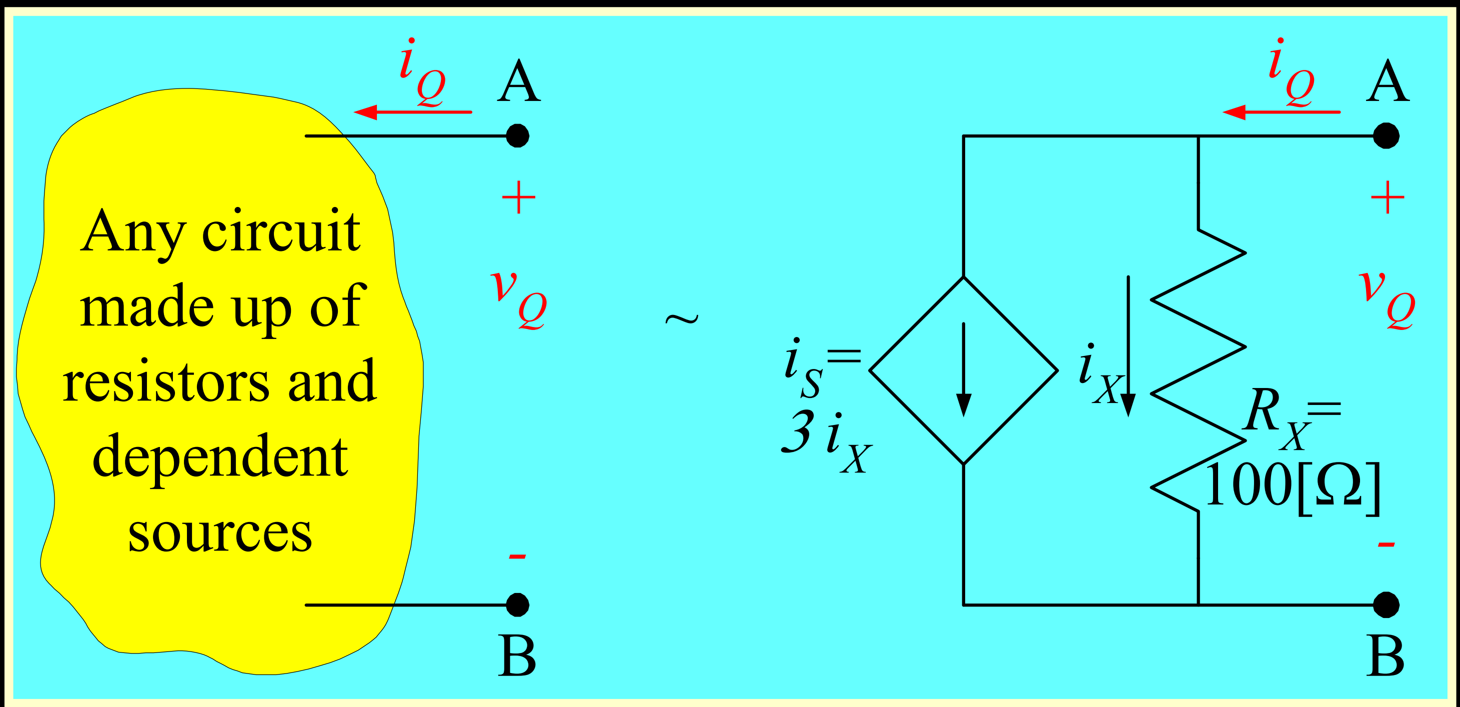
# Simple Example with a Dependent Source – Step 2 (Note)

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. Let's find the ratio of the voltage across the circuit, labeled $v_Q$, to the ratio of the current through the circuit, labeled $i_Q$. We take the ratio of them, and get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X + 3i_X} = \frac{i_X R_X}{4i_X} = \frac{R_X}{4} = \frac{100[\Omega]}{4} = 25[\Omega].$$

The dependent source is in parallel with the resistor $R_X$. Since the parallel combination is 25[Ω], the dependent source must be behaving as if it were a 33.33[Ω] resistor. However, this value depends on $R_X$; in fact, it is $R_X/3$.



# $2^{nd}$ Simple Example with a Dependent Source – Step 1

We wish to find the equivalent resistance of a second circuit, given below, as seen at terminals A and B.

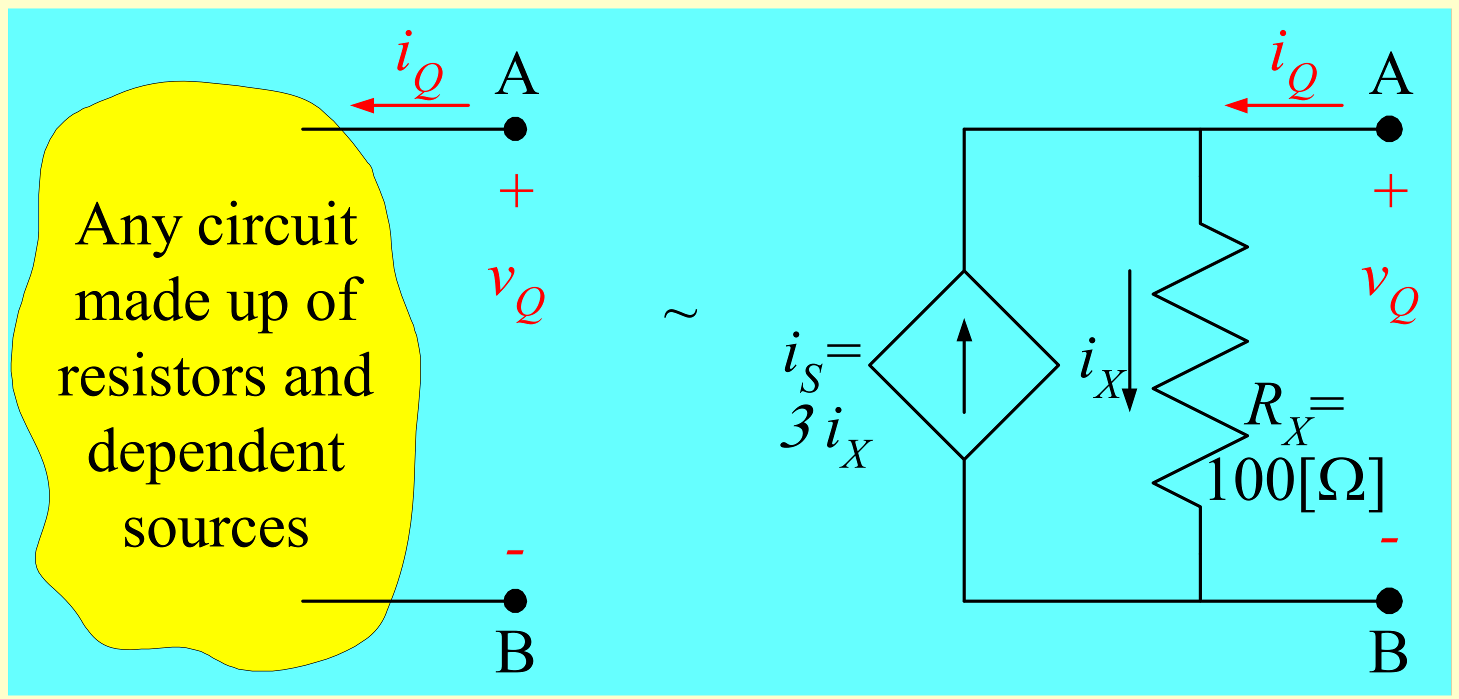
Let's find the ratio of the voltage across the circuit, labeled $v_Q$, to the ratio of the current through the circuit, labeled $i_Q$. This must be a constant. We note that from Ohm's Law applied to $R_X$, we can say that

$$v_Q = i_X R_X.$$

Next, we apply KCL at the A node to write that

$$i_Q = i_X - 3i_X.$$

Note the change in polarity for the source, from the previous example.



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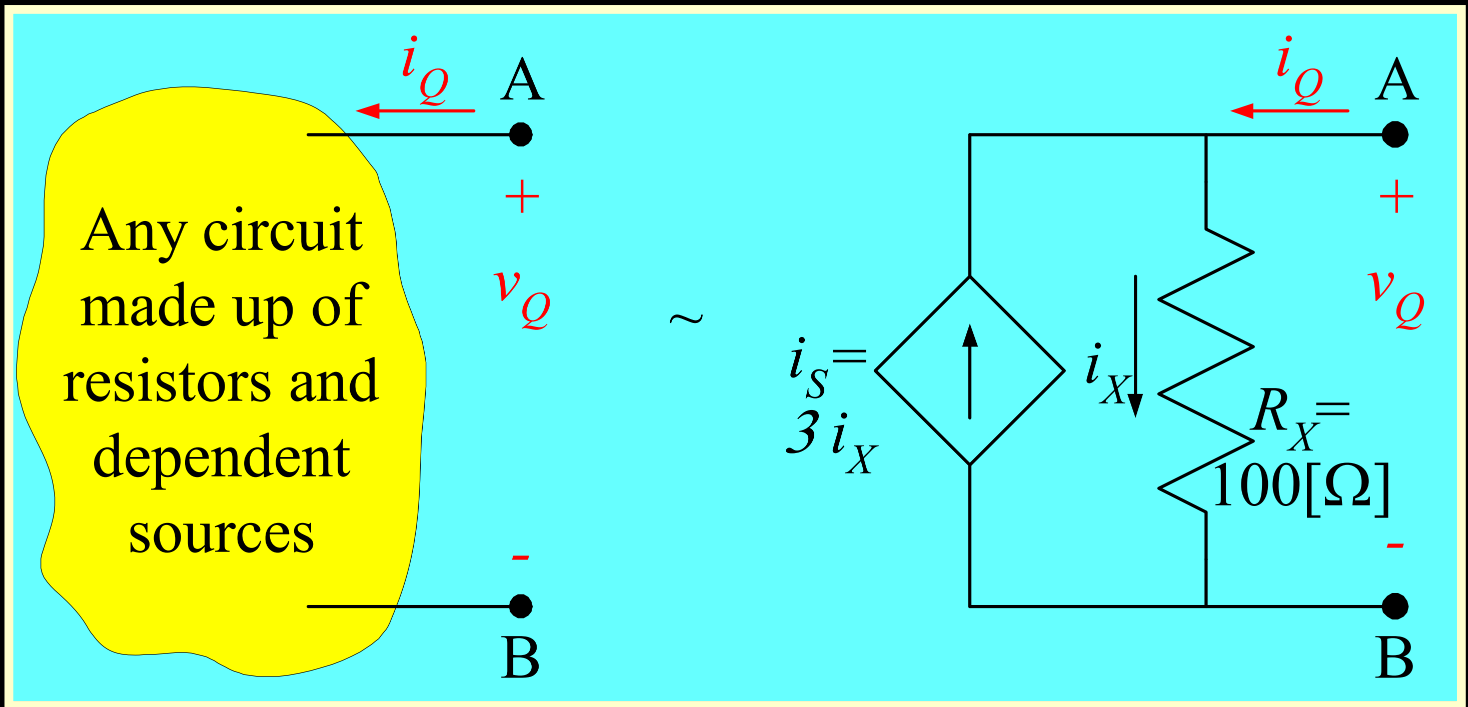
# 2nd Simple Example with a Dependent Source – Step 2

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. On the last slide we found $v_Q$, and we found $i_Q$. We take the ratio of them, and plug in the expressions that we found for each. When we do this, we get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X - 3i_X} = \frac{i_X R_X}{-2i_X} = \frac{R_X}{-2} = \frac{100[\Omega]}{-2} = -50[\Omega].$$

Note that ratio has changed when we simply changed the polarity of the dependent source. The magnitude is not the only thing that changed; the equivalent resistance is now **negative**!

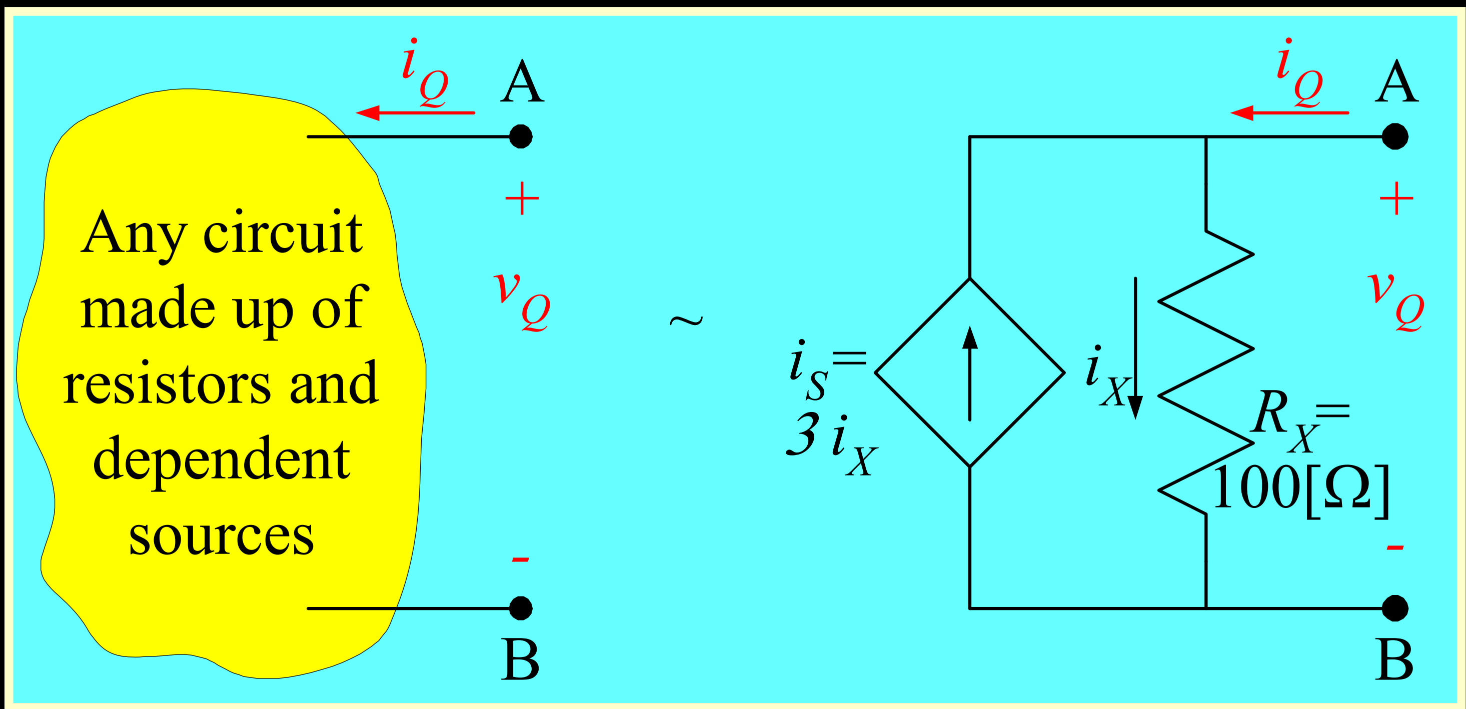


# Simple Example with a Dependent Source – Step 2 (Note)

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. Let's find the ratio of the voltage across the circuit, labeled $v_Q$, to the ratio of the current through the circuit, labeled $i_Q$. We take the ratio of them, and get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X - 3i_X} = \frac{i_X R_X}{-2i_X} = \frac{R_X}{-2} = \frac{100[\Omega]}{-2} = -50[\Omega].$$

The dependent source is in parallel with the resistor $R_X$. Since the parallel combination is -50[Ω], the dependent source must be behaving as if it were a -33.33[Ω] resistor. This value depends on $R_X$; in fact, it is $-R_X/3$.



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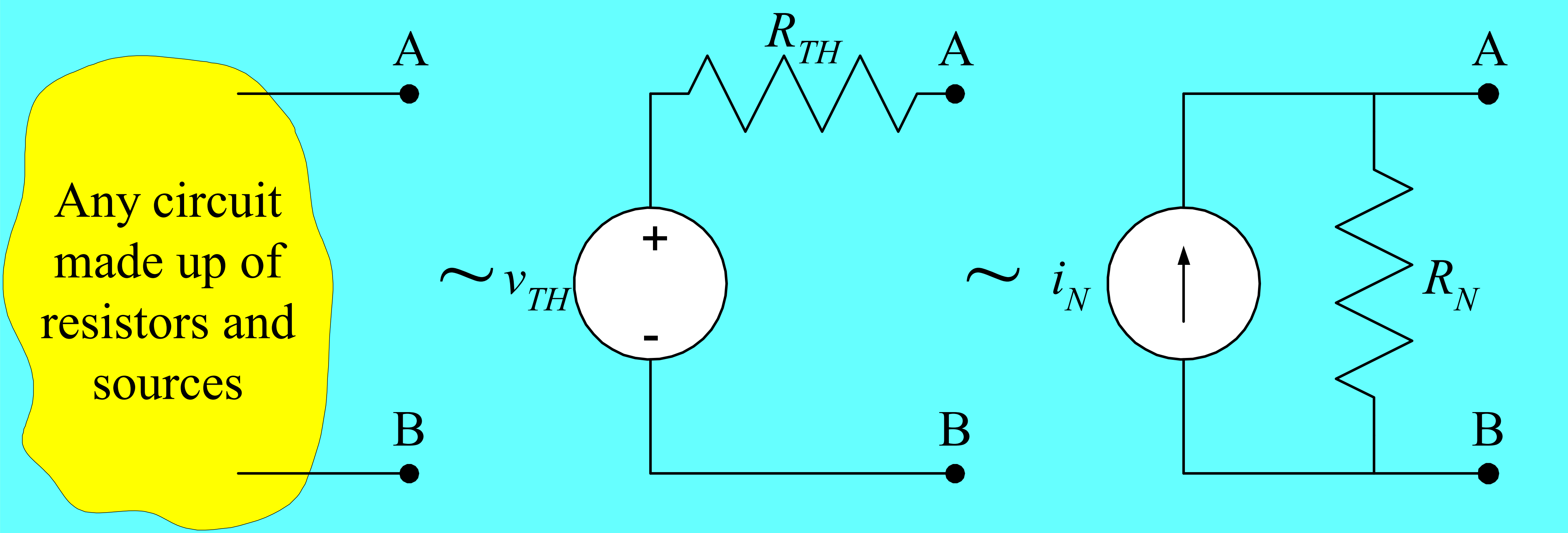
# Note 1

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

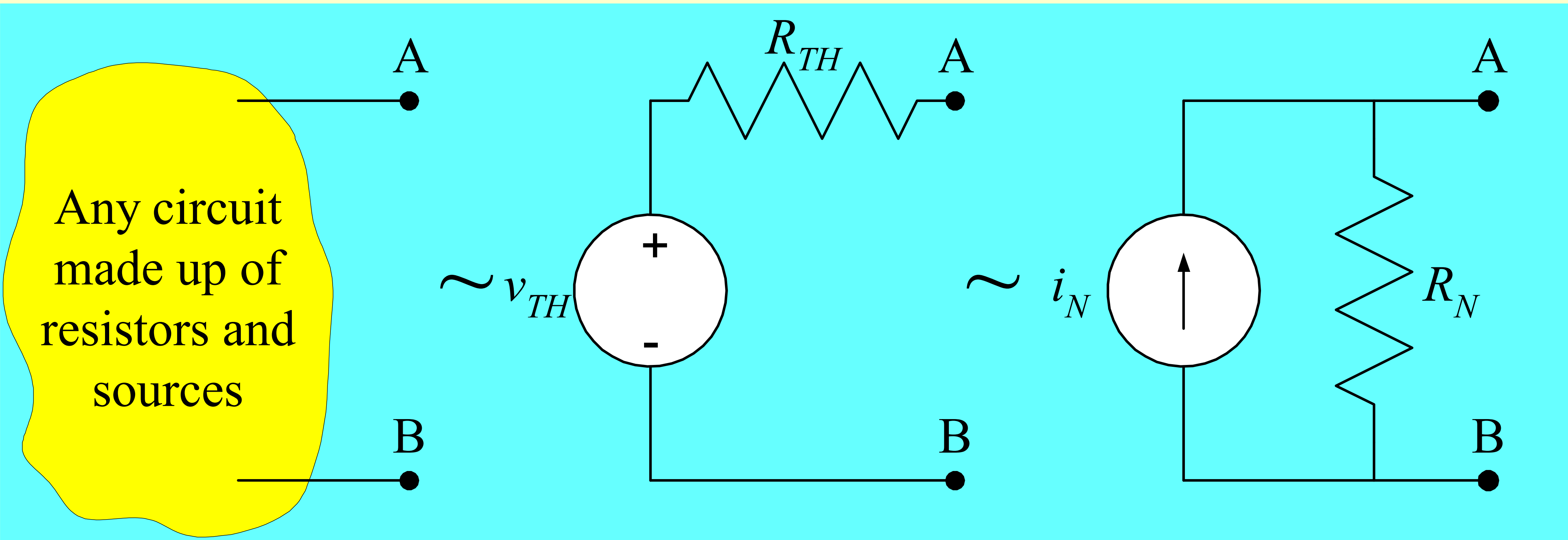
We can see that the equivalent resistance can be negative. This is one reason why we have been so careful about polarities all along. We need to get the polarities right to be able to get our signs right.



# Note 2

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

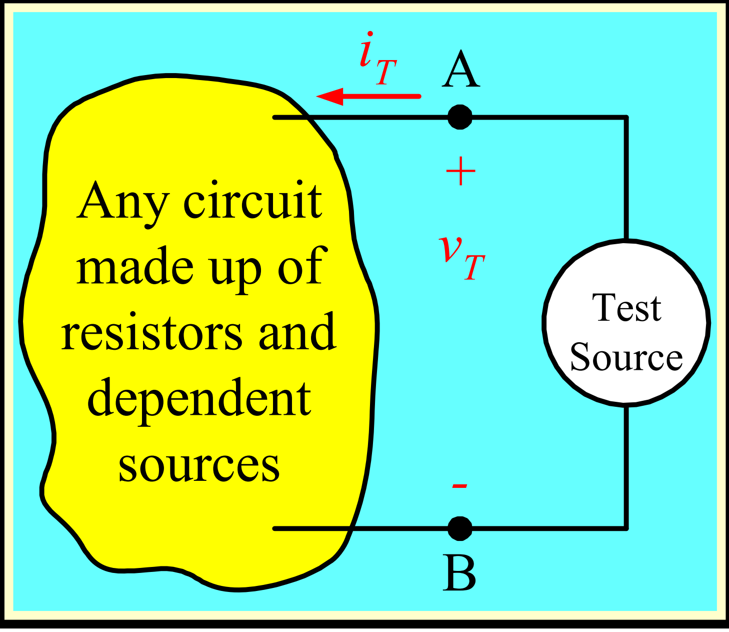
In the simple examples that we just did, we were effectively applying a source to the terminals of the circuit. This results in a circuit like others that we have solved before, and we can find the ratio of voltage to current. This is usually easier to think about for most students. It is as if we were applying a source just to test the circuit; we call this method the Test-Source Method.



# Test-Source Method – Defined

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to zero.
2) Find the equivalent resistance.
   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

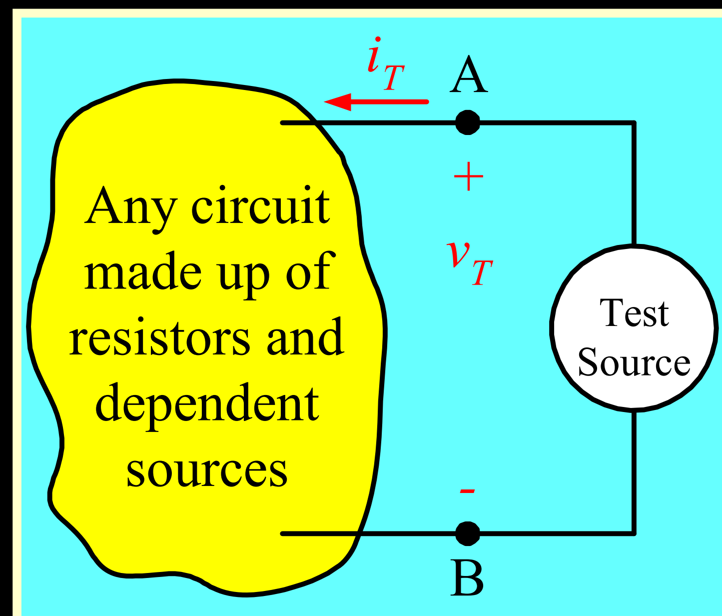


# Test-Source Method – Note 1

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to zero.
2) Find the equivalent resistance.
   a) If there are no dependent sour
   the equivalent resistance rules
   include series combinations, pa
   equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

Don't forget this step. It is **always** applied when finding equivalent resistance.



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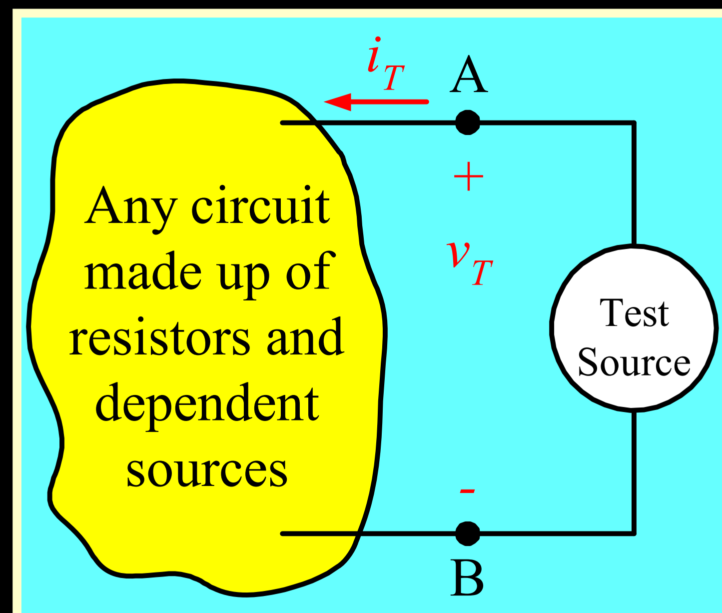
# Test-Source Method – Note 2

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z

Note that step 2 has two options (a or b). Pick one. You don't need to do both.

2) Find the equivalent resistance.
   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.



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# Test-Source Method – Note 3

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z

2) Find the equivalent resistance.

   a) If there are no dependent sources, find the equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.

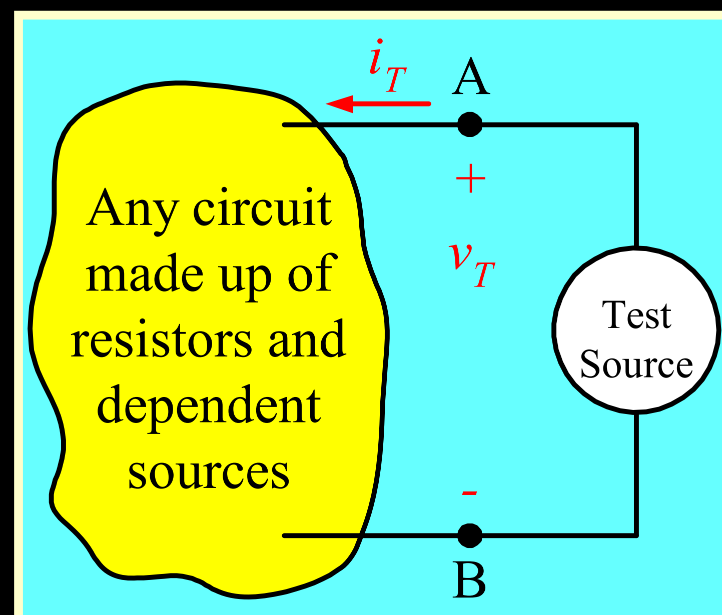
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.

      1) If you apply a voltage source, find the current through that voltage source.

      2) If you apply a current source, find the voltage across that current source.

      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

You could actually pick option b) every time, but option a) is easier. Use it if you can.



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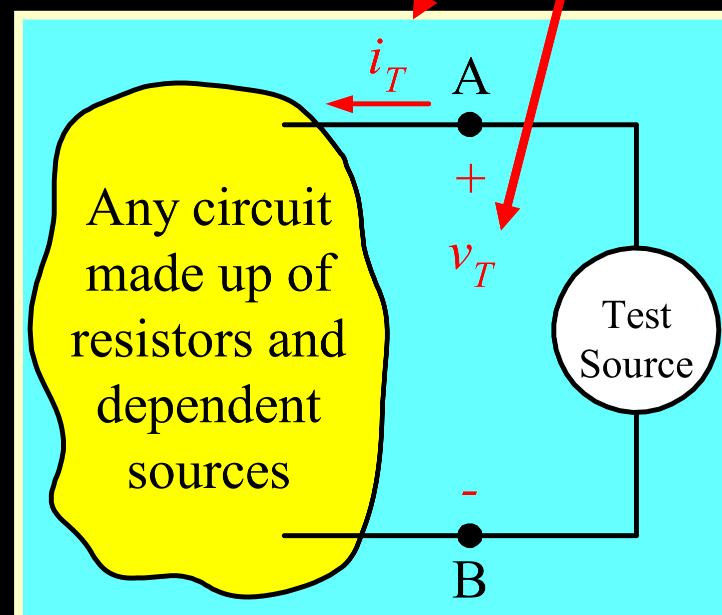
# Test-Source Method – Note 4

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

When you apply these voltages and currents, apply them in the active sign relationship for the source. This gives the "proper" sign.

1) Set all independent sources equal to z
2) Find the equivalent resistance.
   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.



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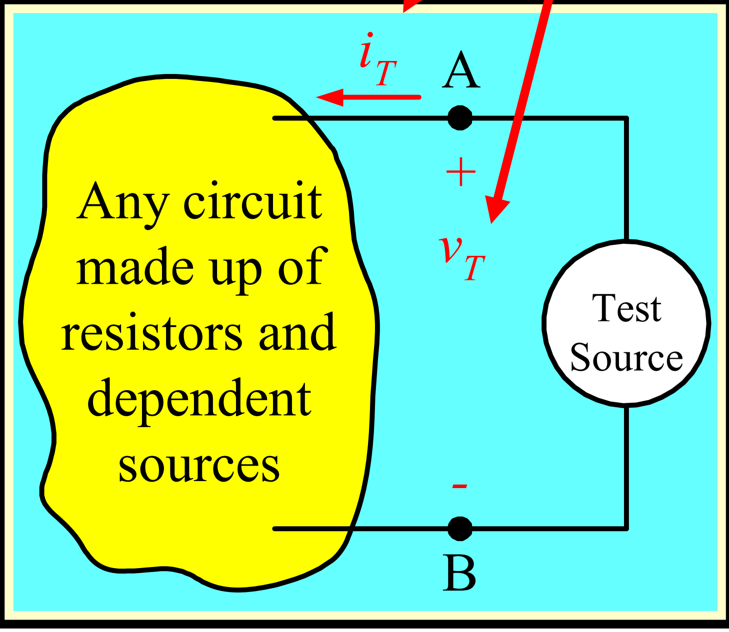
# Test-Source Method – Note 5

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z
2) Find the equivalent resistance.
   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

The active sign relationship for the test source gives the passive sign relationship for the circuit, which is the resistance, by Ohm's Law.



Go back to Overview slide.

# Notes

1. The Test-Source Method usually requires some practice before it becomes natural for students. It is important to work several problems to get this practice in.
2. There is a tendency to assume that one could just ignore the Test-Source Method, and just find the open-circuit voltage and short-circuit current whenever a dependent source is present. However, sometimes this does not work. In particular, when the open-circuit voltage and short-circuit current are zero, we must use the Test-Source Method. Learn how to use it.

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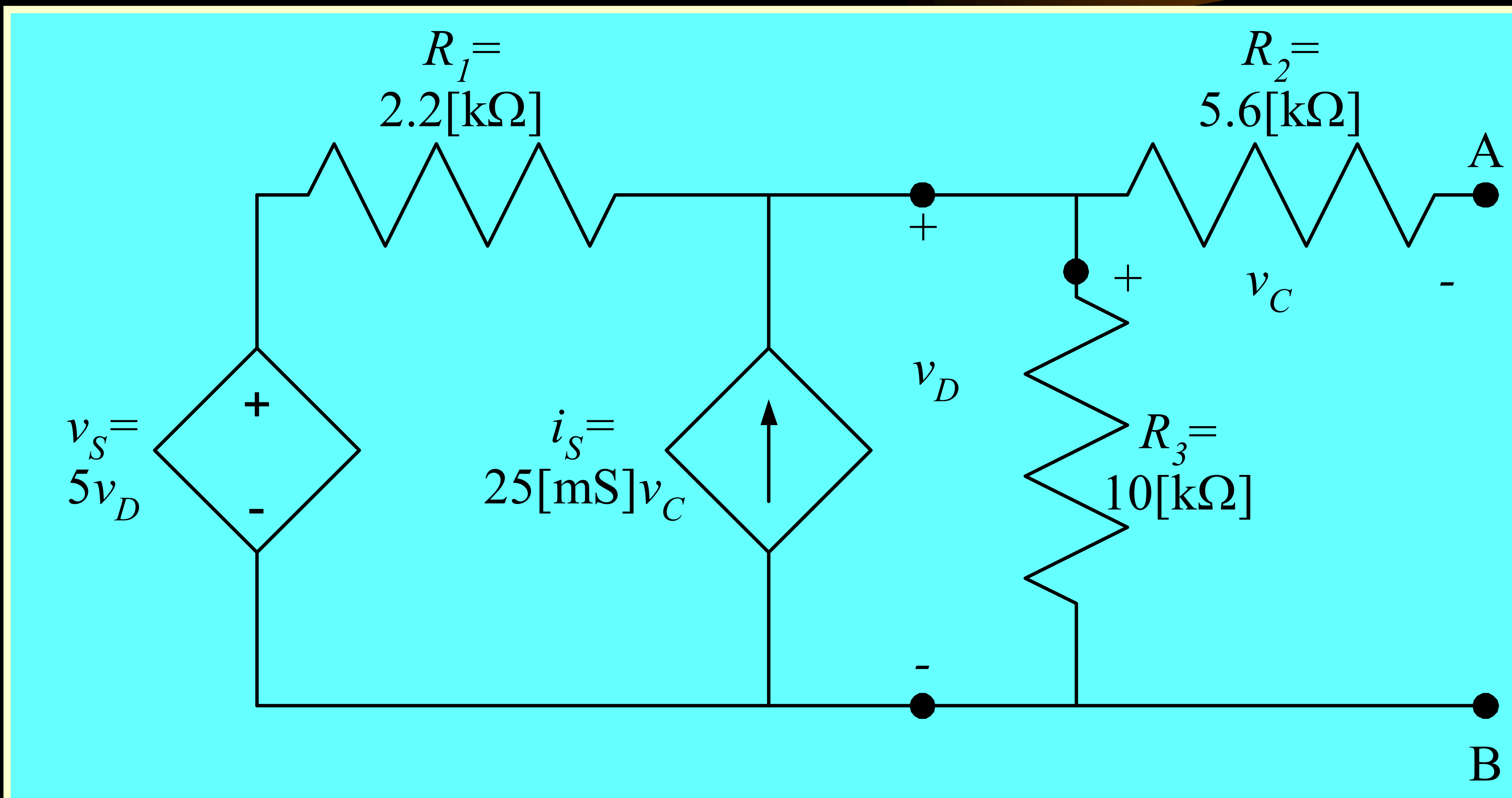
# Example Problem

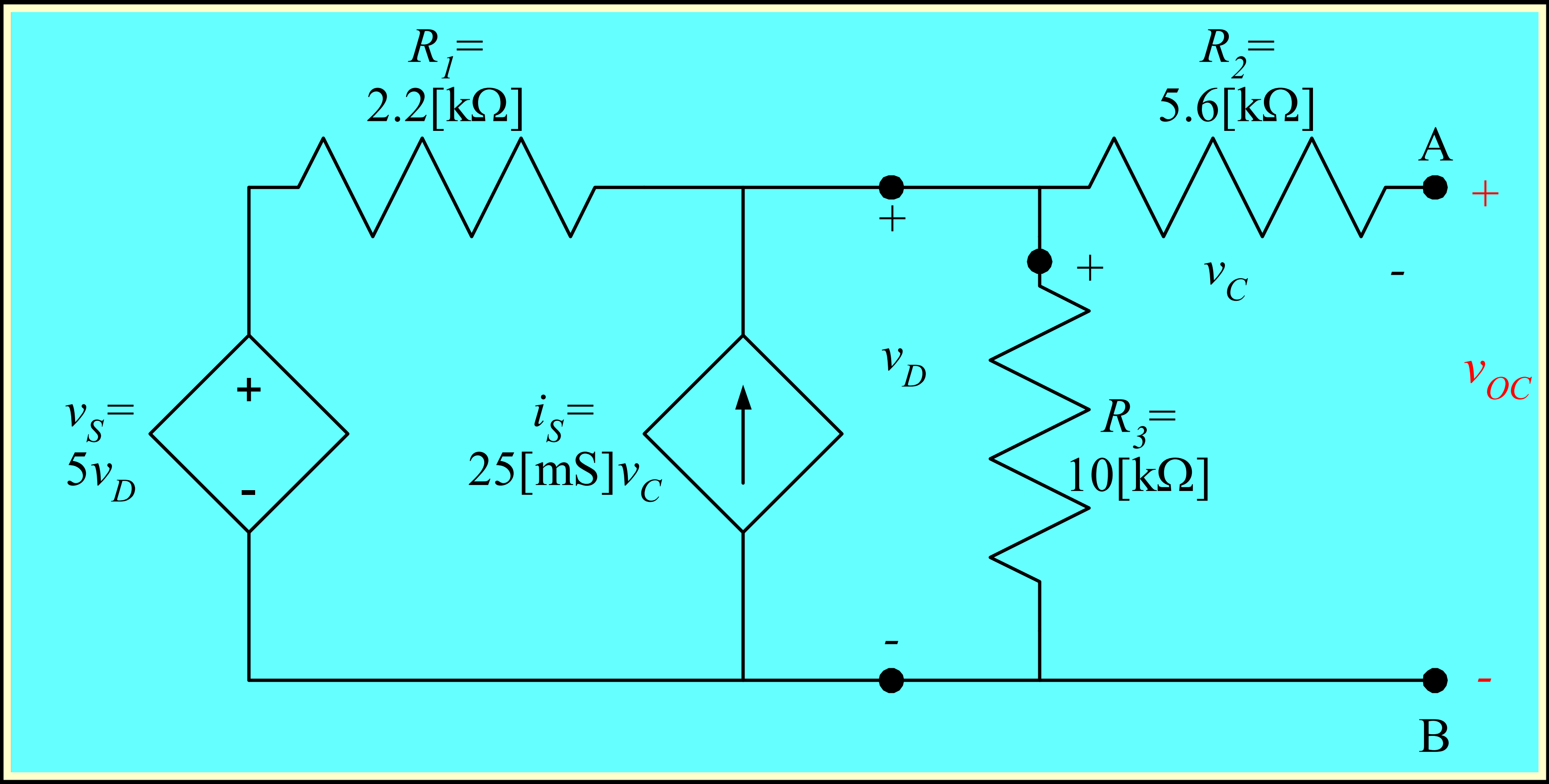
We wish to find the Thévenin equivalent of the circuit below, as seen from terminals A and B.



# Example Problem – Step 1

We wish to find the Thévenin equivalent of the circuit below, as seen from terminals A and B.

We will start by find the open-circuit voltage at the terminals, as defined below.



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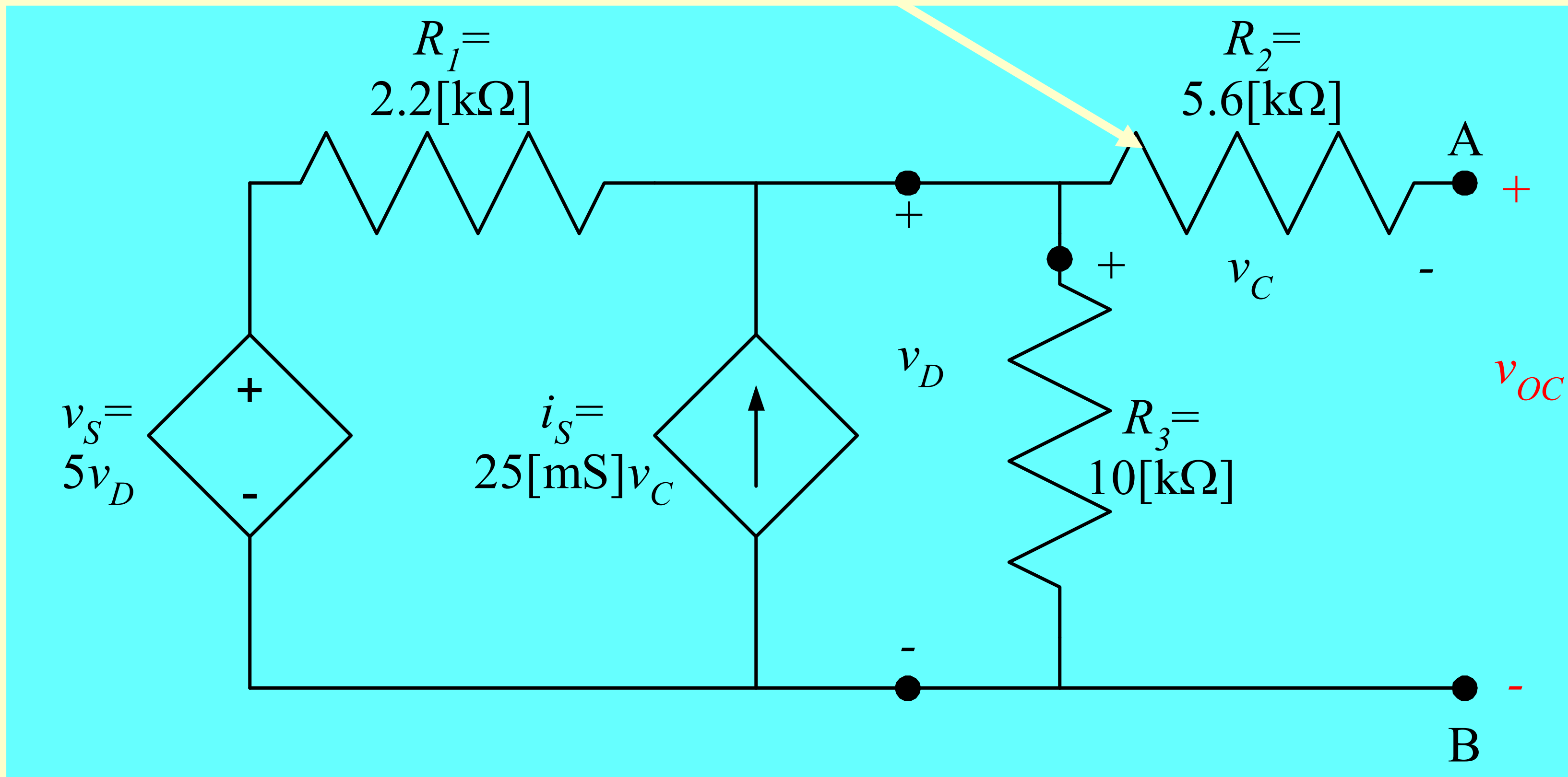
# Example Problem – Step 2

To find $v_{OC}$, we will first find $v_D$, by writing KCL at the top center node. We have

$$\frac{v_D - 5v_D}{R_1} + 0 + \frac{v_D}{R_3} - i_S = 0.$$

Note that we recognize that the current through $R_2$ must be zero since $R_2$ is in series with an open circuit.



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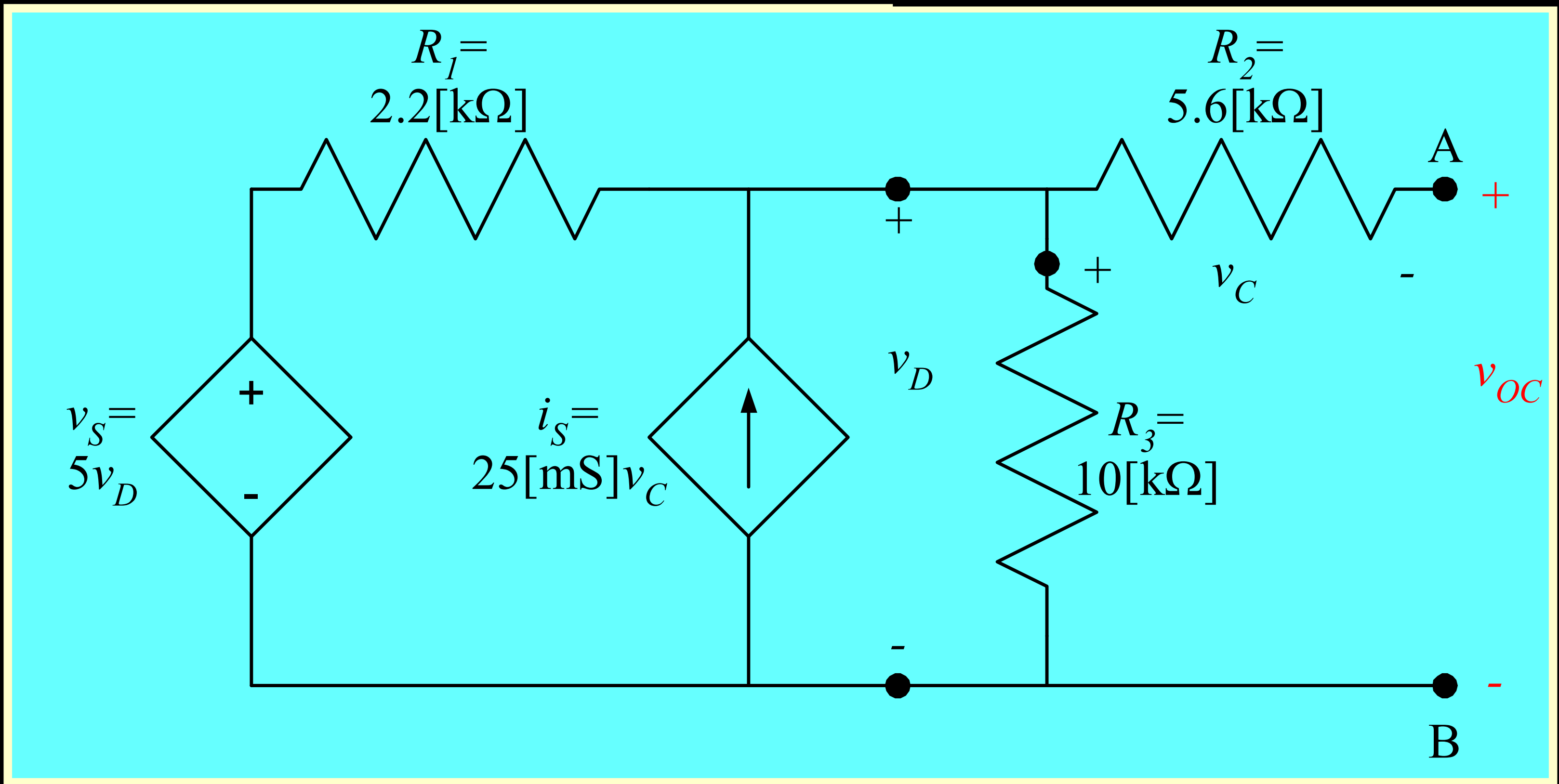
# Example Problem – Step 3

We can substitute in the value for $i_S$, 25[mS]$v_C$. We note that since the current through $R_2$ is zero, the voltage across it is zero, so $v_C$ is zero. So, we write

$$\frac{v_D - 5v_D}{R_1} + 0 + \frac{v_D}{R_3} - 25[\mathrm{mS}]v_C = 0, \text{ or}$$

$$\frac{v_D - 5v_D}{R_1} + \frac{v_D}{R_3} = 0.$$

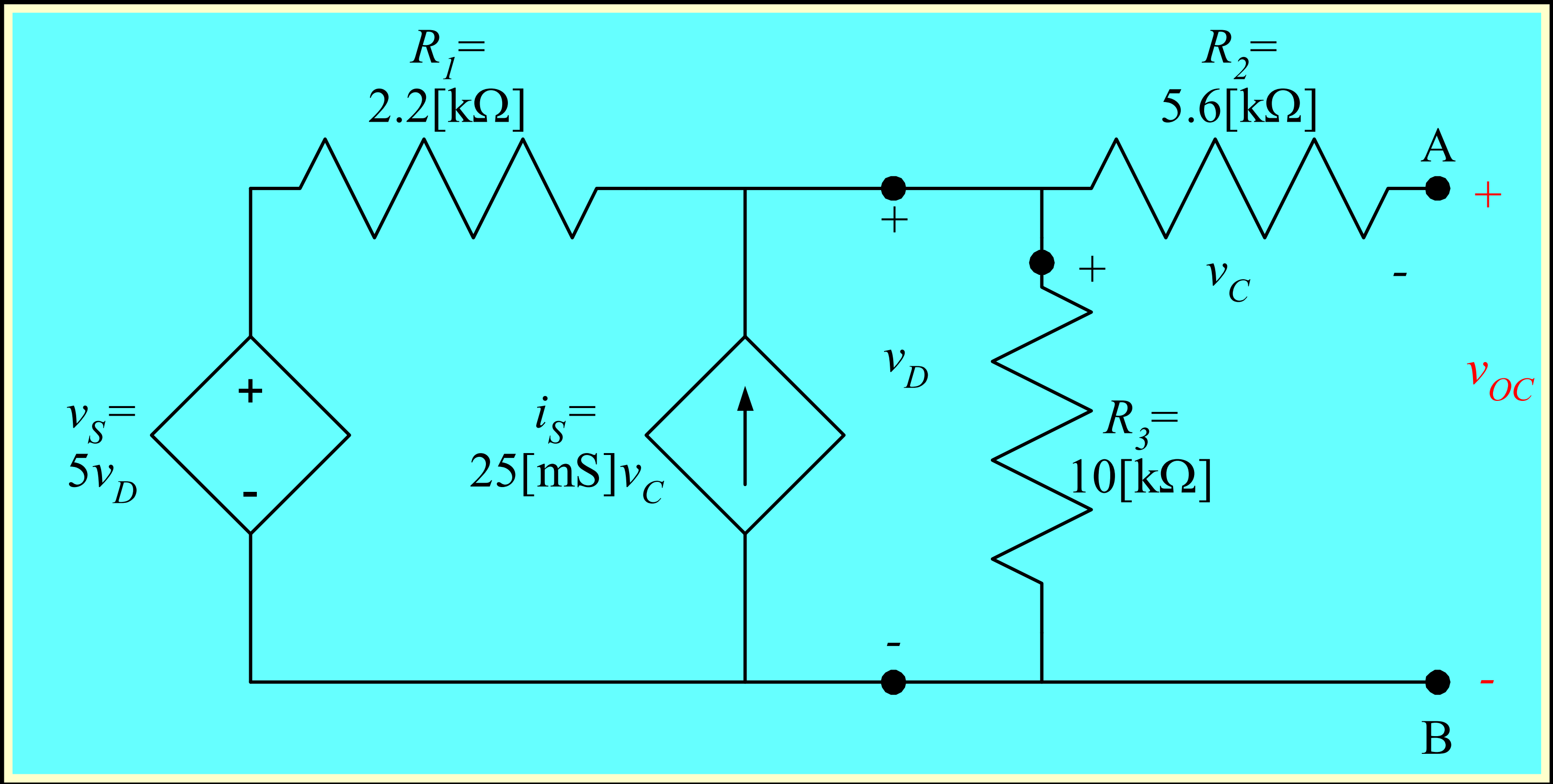


# Example Problem – Step 4

Next, we substitute in values and solve for $v_D$. We write

$$\frac{-4v_D}{2.2[\text{k}\Omega]} + \frac{v_D}{10[\text{k}\Omega]} = 0.$$ With some math, we find

$$v_D = 0.$$

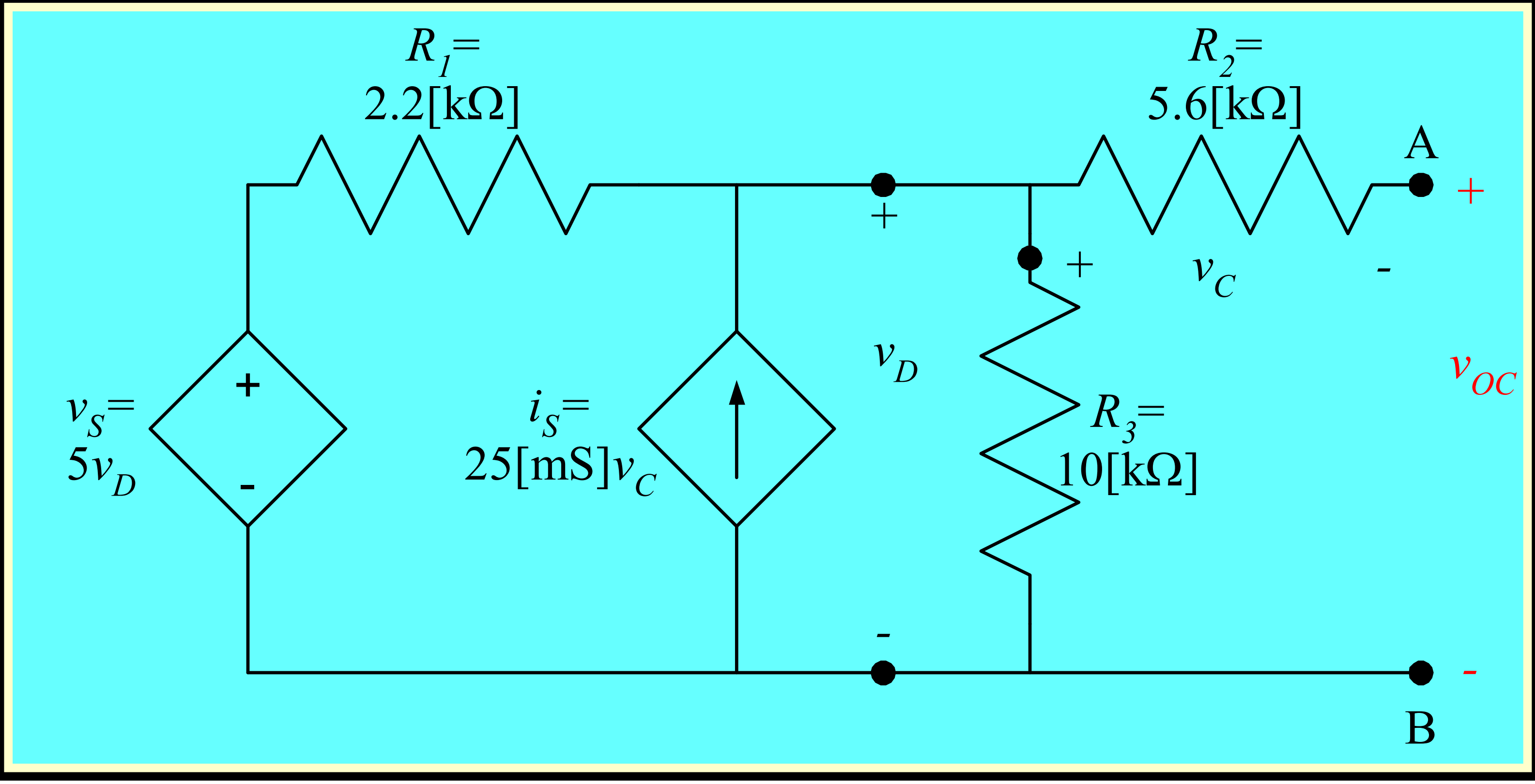


# Example Problem – Step 5

Now, we can take KVL around the loop, and we write

$$-v_D + v_C + v_{OC} = 0, \text{ and so}$$
$$v_{OC} = 0.$$

The Thévenin voltage is equal to this open-circuit voltage, so the Thévenin voltage must be zero. The short-circuit current will also be zero. To get the resistance, we need to use the Test-Source Method.



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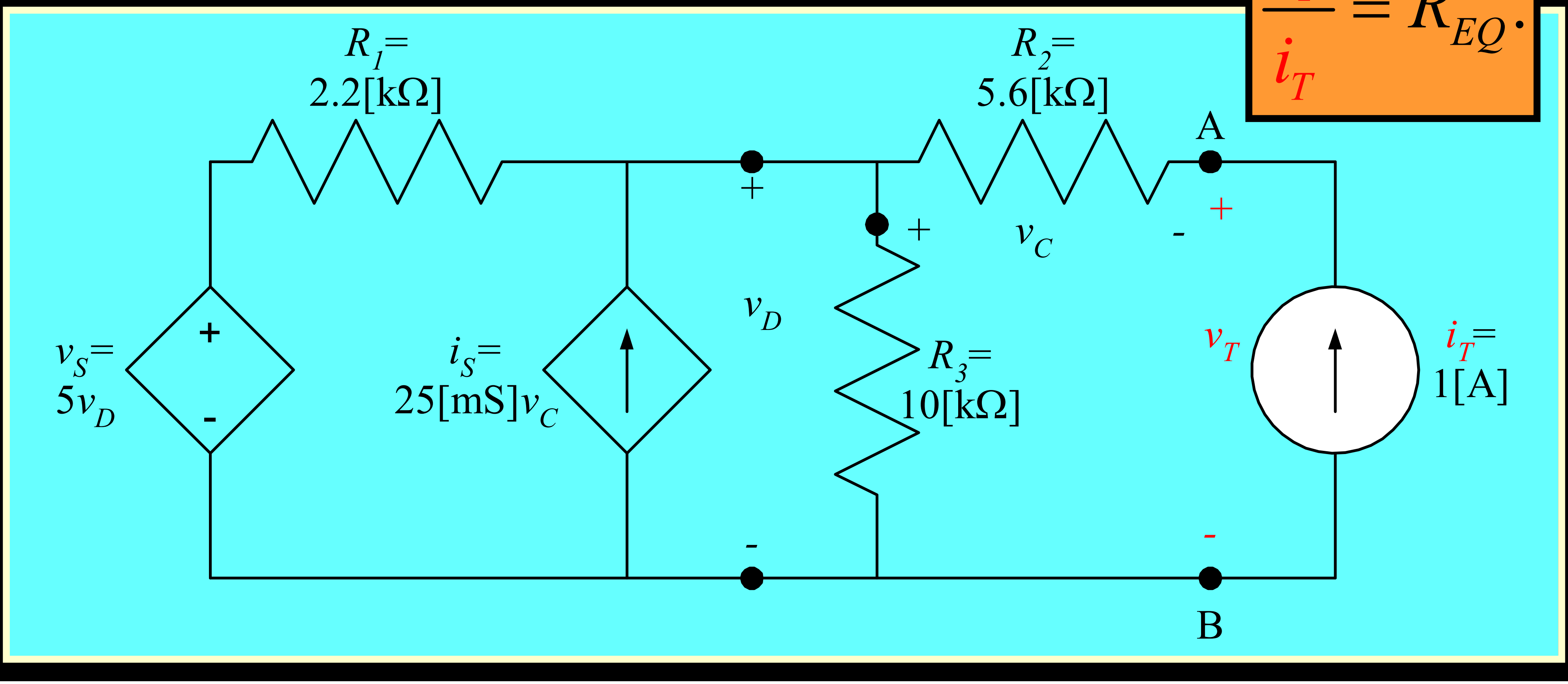
# Example Problem – Step 6

We have applied a test current source to the two terminals. We have also labeled a voltage across this current source, $v_T$. This voltage has been defined in the active sign relationship for the current source. As noted earlier, this will give us the passive sign relationship for $v_T$ and $i_T$ for the circuit that we are finding the equivalent resistance of. Thus, we will have

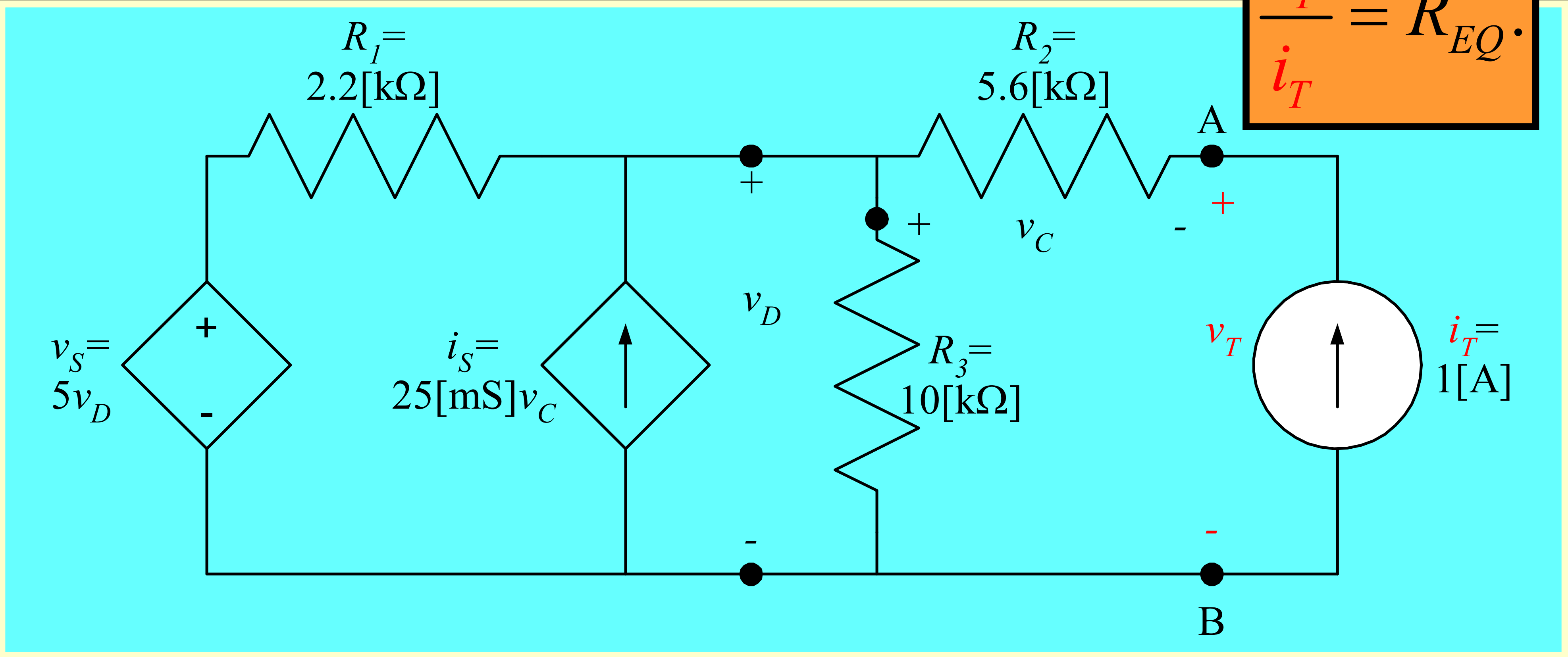
$$\frac{v_T}{i_T} = R_{EQ}.$$



# Example Problem – Step 7

We have applied a ***test*** current ***source*** to the two terminals. We don't need to do this, but doing so makes it clear that we are now just solving another circuit, like the many that we have solved before. We have even given the source a value, in this case, 1[A]. This is just a convenience. Many people choose to leave this as an arbitrary source. We choose to use a value, an easy value like 1[A], to allow us to find an actual value for $v_T$.

$$\frac{v_T}{i_T} = R_{EQ}.$$



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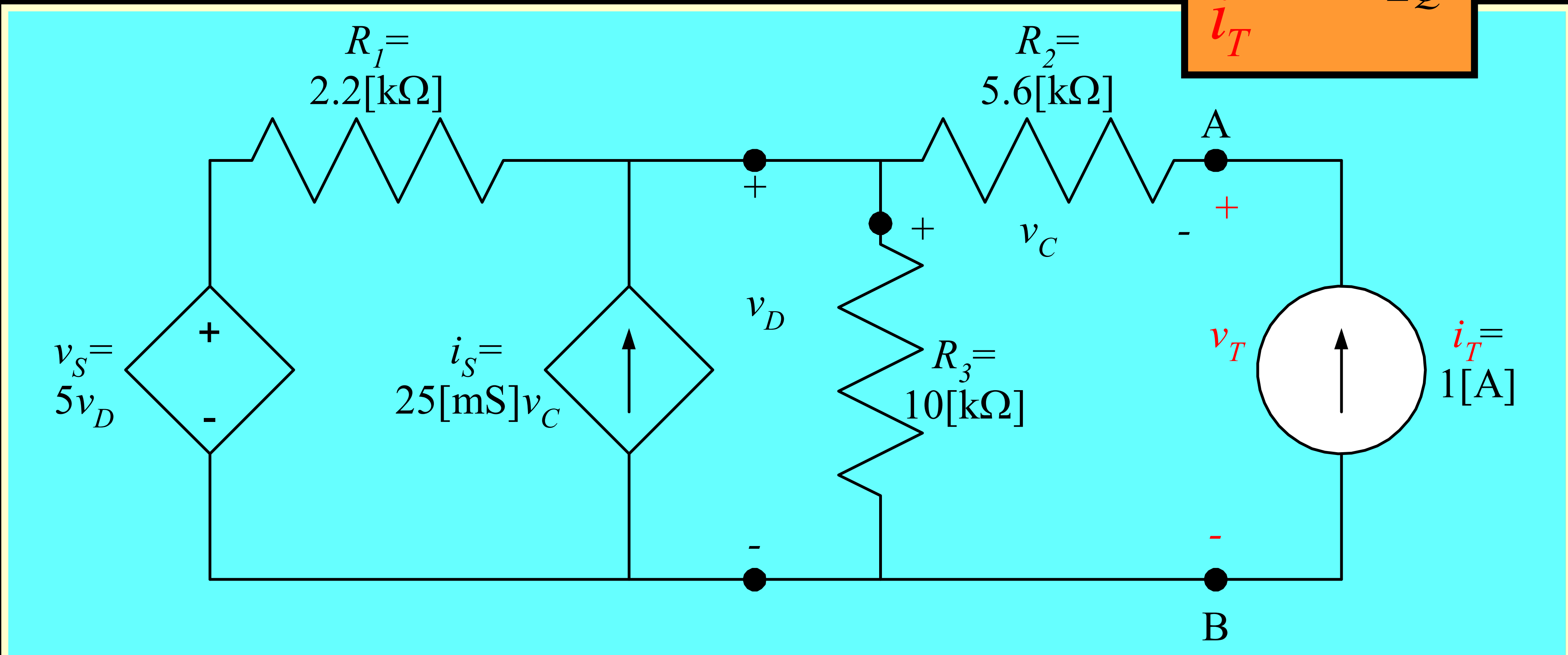
# Example Problem – Step 8

We have applied a test ***current*** source to the two terminals. A test ***voltage*** source would have been just as good. We chose a current source because we thought it might make the solution a little easier, since we can find $v_C$ so easily now. But it really does not matter. Don't worry about which one to choose. Let's solve.

$$\frac{v_T}{i_T} = R_{EQ}.$$



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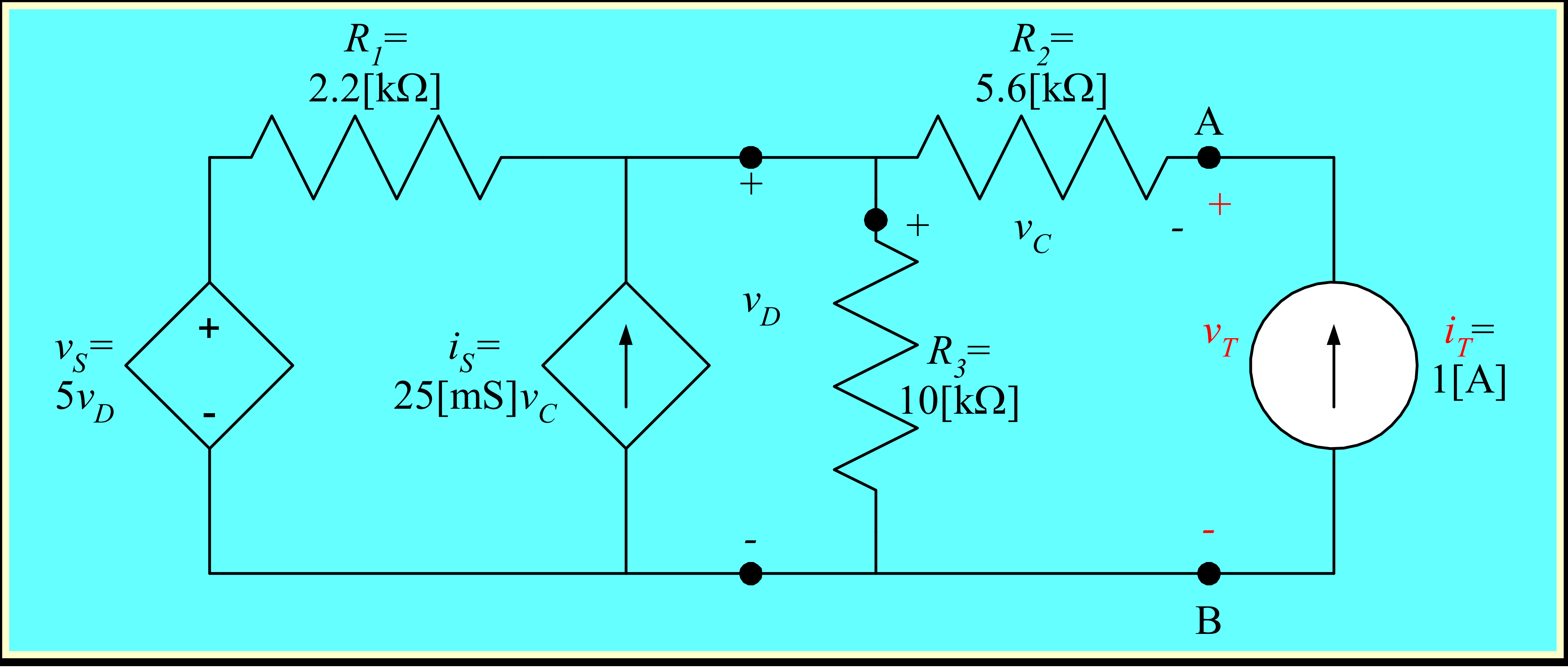
# Example Problem – Step 9

Let's solve for $v_T$. We note that we can write an expression for $v_C$ using Ohm's Law, and get

$$v_C = -1[\mathrm{A}]R_2 = -5600[\mathrm{V}].$$

This voltage may seem very large. Don't let this bother you. We do not actually have this voltage; it is just for calculating the resistance.

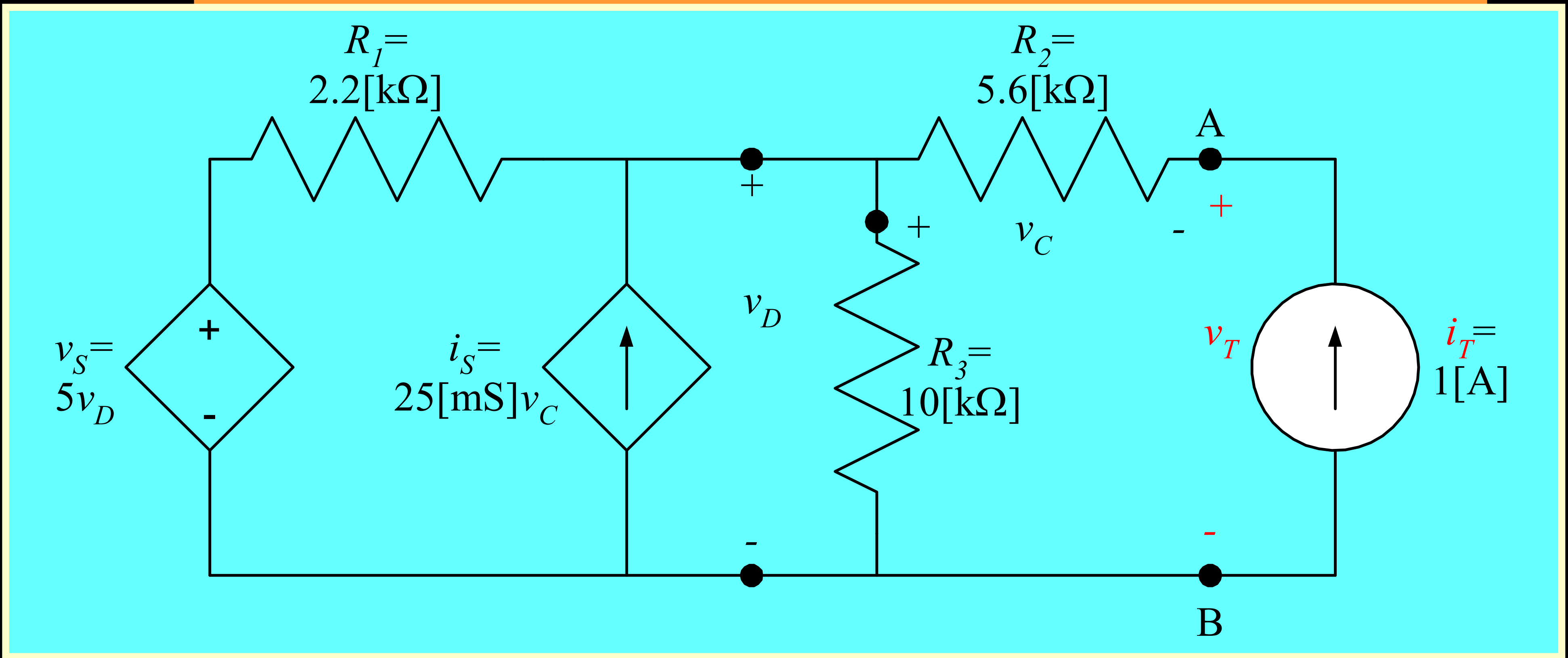


# Example Problem – Step 10

Next, let's write KCL for the top center node. We get

$$\frac{v_D - 5v_D}{R_1} - i_T + \frac{v_D}{R_3} - i_S = 0, \text{ or by substituting,}$$

$$\frac{-4v_D}{2.2[\mathrm{k}\Omega]} - 1[\mathrm{A}] + \frac{v_D}{10[\mathrm{k}\Omega]} - 25[\mathrm{mS}]\left(-5600[\mathrm{V}]\right) = 0.$$



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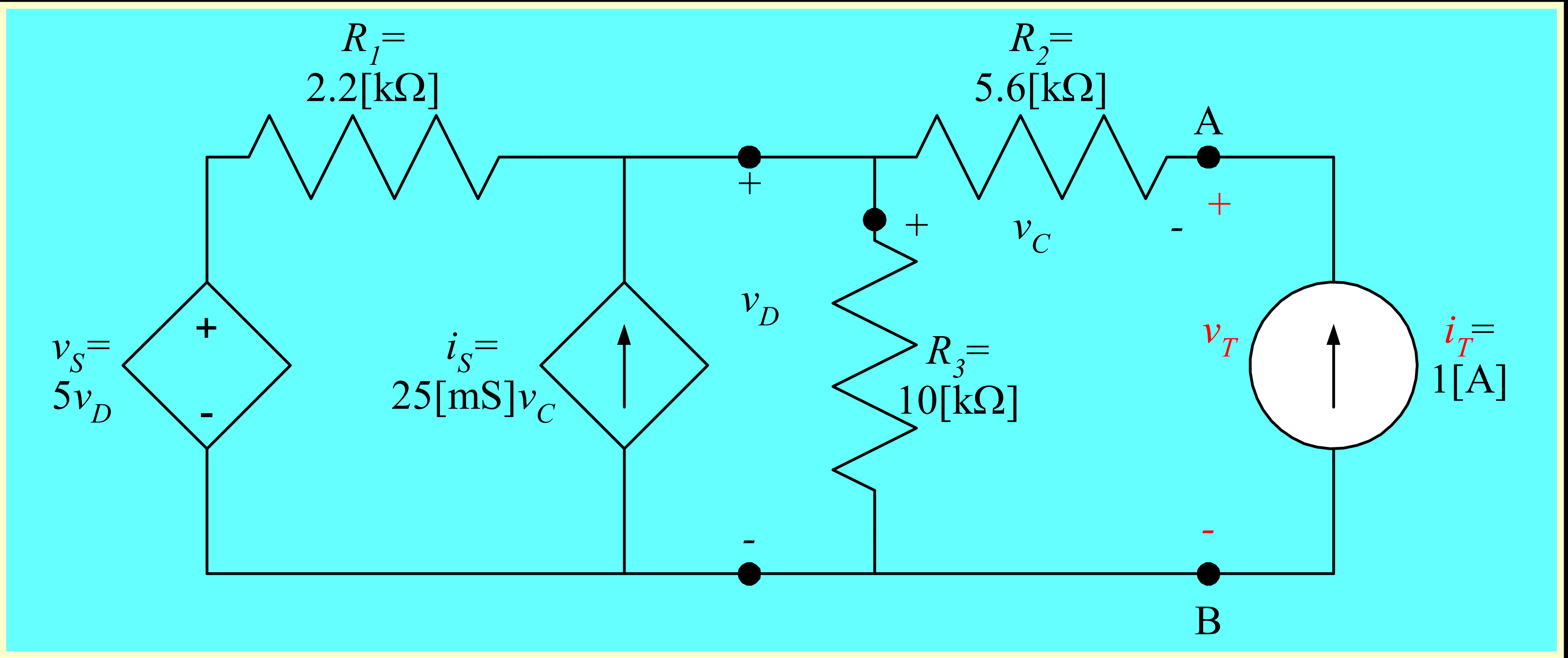
# Example Problem – Step 11

Solving for $v_D$ yields

$$\frac{-4v_D}{2.2[\text{k}\Omega]} + \frac{v_D}{10[\text{k}\Omega]} = -139[\text{A}], \text{ or}$$

$$\left(-1.72[\text{mS}]\right) v_D = -139[\text{A}], \text{ or}$$

$$v_D = 80,900[\text{V}].$$



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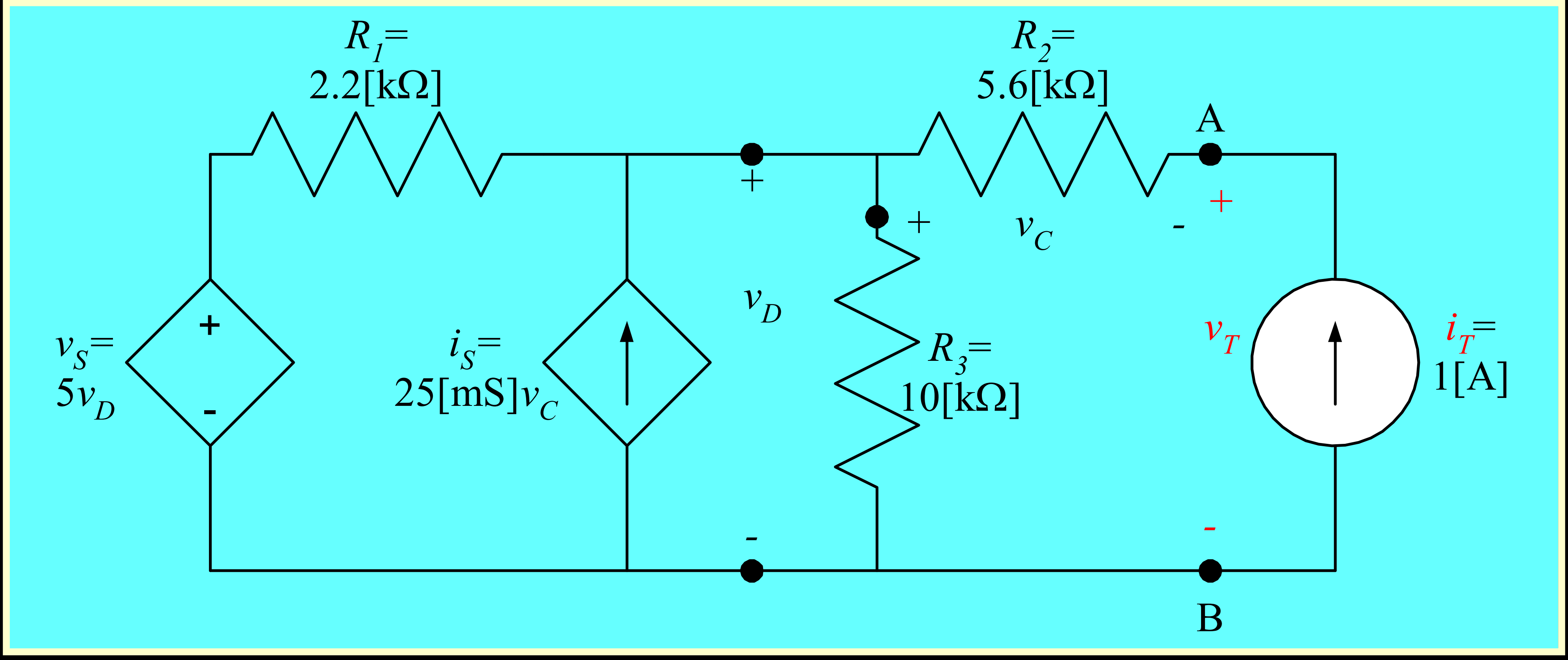
# Example Problem – Step 12

Taking KVL, we get

$$-v_D + v_C + v_T = 0, \text{ or}$$
$$v_T = v_D - v_C = 80{,}900[\text{V}] - \left(-5600[\text{V}]\right) = 86{,}500[\text{V}].$$



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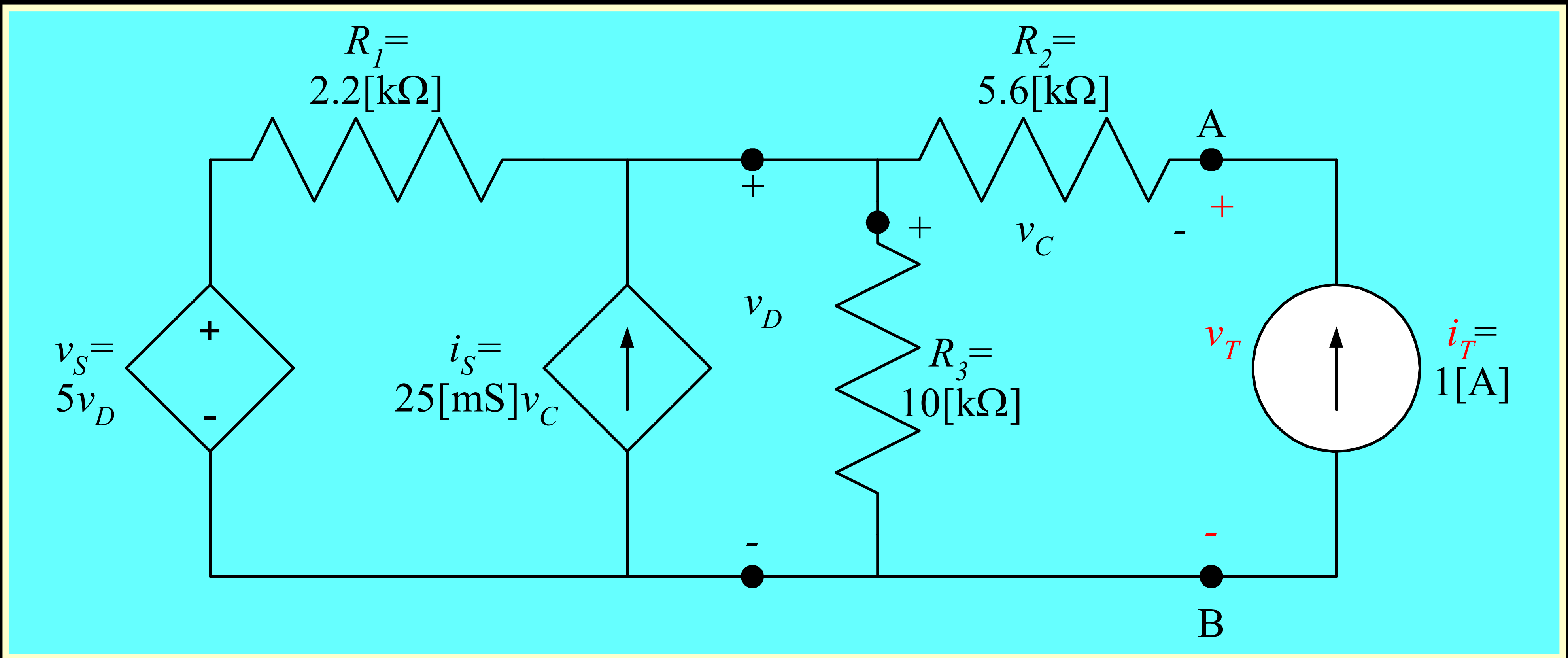
# Example Problem – Step 13

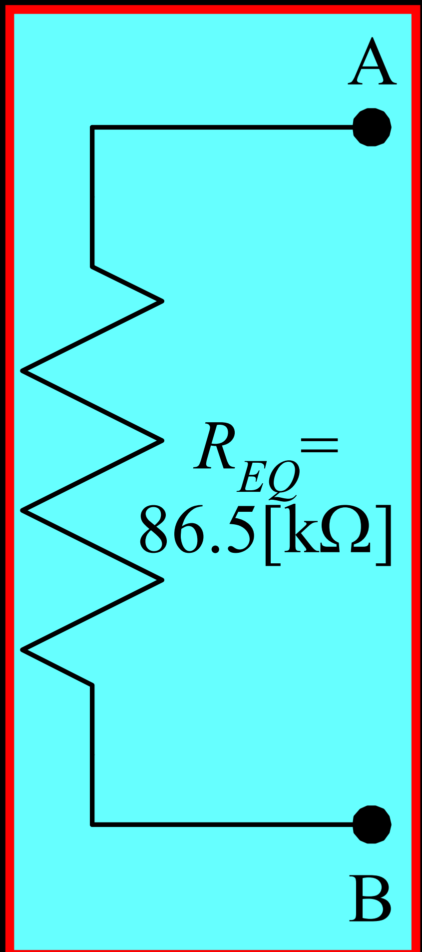
So, we can find the equivalent resistance by finding

$$R_{EQ} = \frac{v_T}{i_T} = \frac{86{,}500[\mathrm{V}]}{1[\mathrm{A}]} = 86.5[\mathrm{k\Omega}].$$



# Example Problem – Step 14

So, the Thévenin equivalent is given in the circuit below. Note that the Thévenin voltage is zero, and so we don't even show the voltage source at all. The Thévenin resistance is shown, and in this case, it is the Thévenin equivalent.



# *Is the Test-Source Method Really* **That** *Important?*

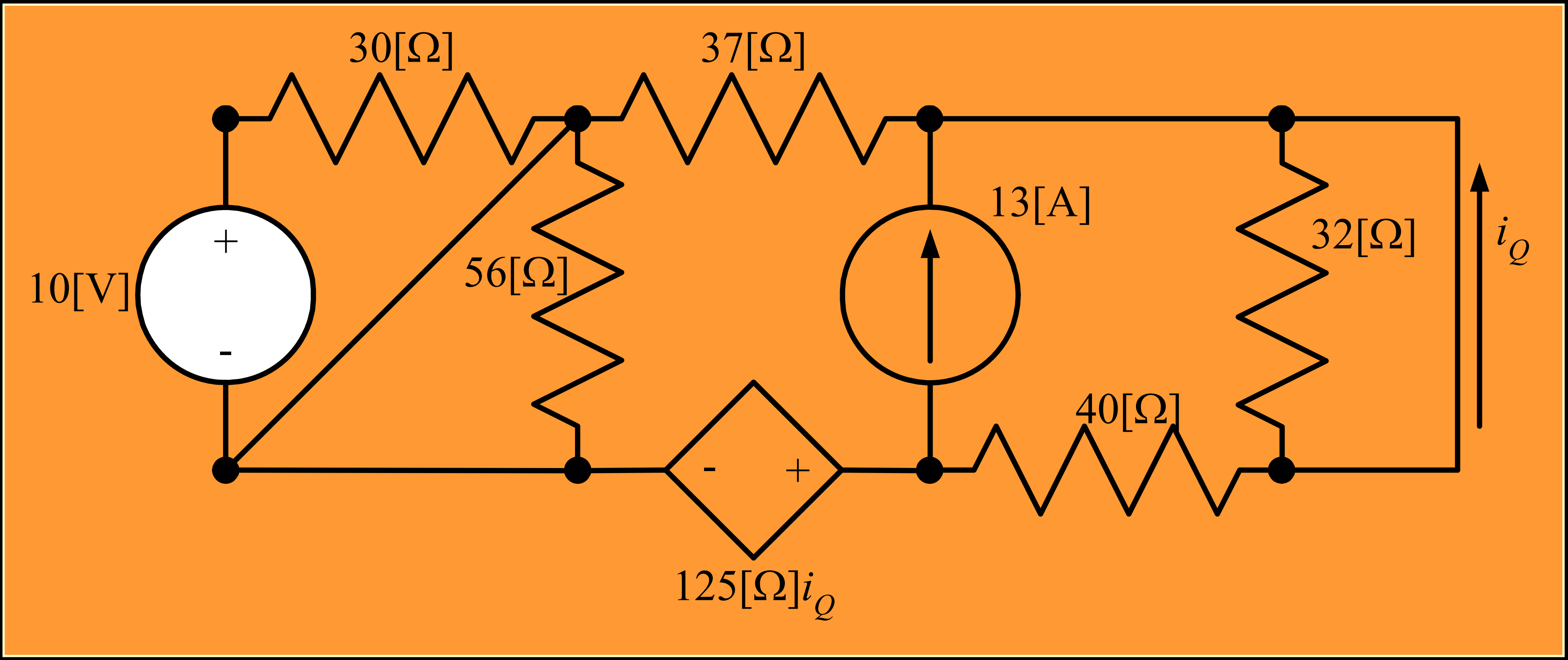
- This is a good question. Basically, the answer is yes. There are many cases where we have dependent sources present, and wish to use a Thévenin equivalent. While in some cases we can get the Thévenin resistance from the open-circuit voltage and the short-circuit current, there are others where we cannot. The Test-Source Method is also quicker in some cases.
- Some students go to great lengths to avoid learning this method. This seems like a waste of energy. Just learn it and use it.


Go back to Overview slide.

# Example Problem #1

For the circuit given below, find the Norton equivalent as seen by the current source.

Find the power delivered by the current source in this circuit.



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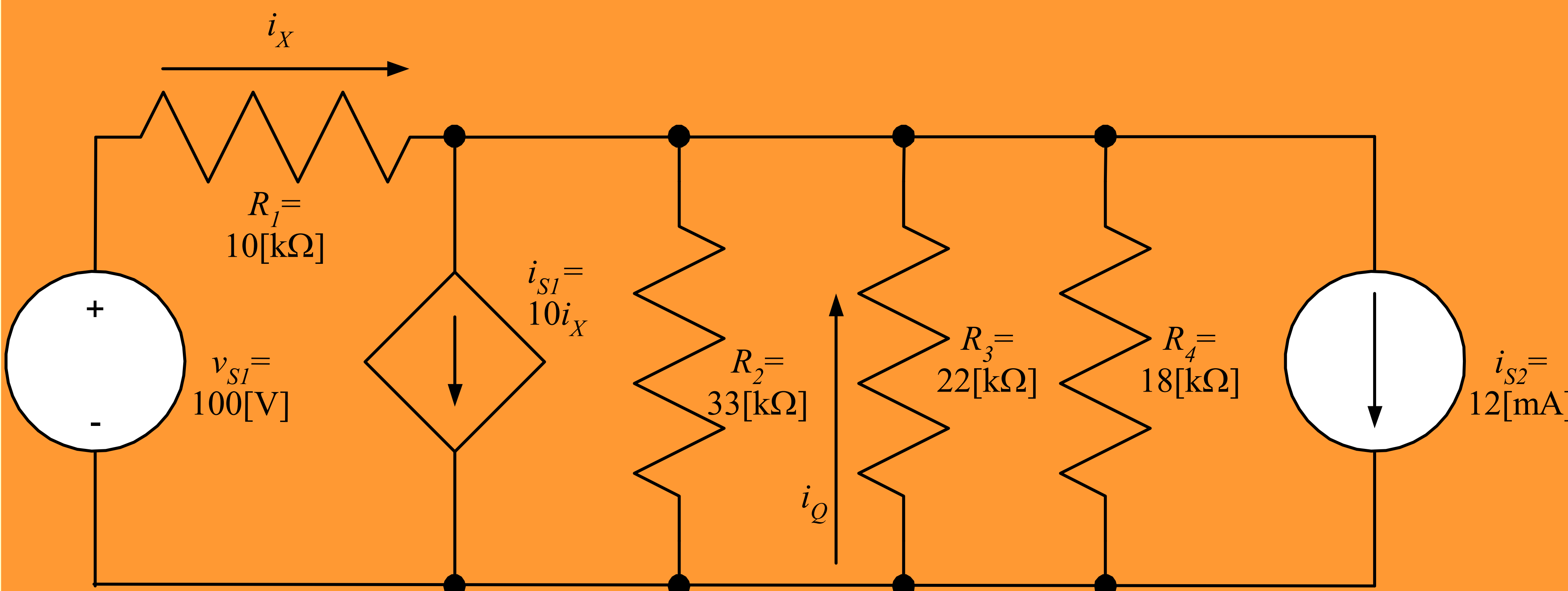
# Sample Problem #2

a) For the circuit shown below, find the Norton equivalent as seen by the 22[kΩ] resistor.
b) Use this equivalent circuit to solve for $i_Q$.



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# Sample Problem #2

a) For the circuit shown below, find the Norton equivalent as seen by the 22[kΩ] resistor.

b) Use this equivalent circuit to solve for $i_Q$.

Soln: a) $i_N$ = -102[mA], $R_N$ = -1.228[kΩ]

b) -5.988[mA]