Lab 1 Preparatory Questions
Applied Estimation EL2320

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1 Linear Kalman Filter

1. z_t is an observer, i.e. the measured data observed. x_t is the state vector of which we can predict/find given z_t and u_t. u_t is the control data, which we know the noise and the uncertainty. u_t is not related to the dataset itself.

2. Yes, the belief is stated as $Bel(X) = P(X|z_0 = a_1, ..., z_n = a_n)$ (all a_n are constants), by a mean μ_t and the covariance Σ_t. If the measured data is in some way bad/corrupt then the mean μ_t and Σ_t will change and can change in a way that it increases the uncertainty of the belief.

3. The input of the Kalman filter is given as the belief at the time $t-1$ along with μ_{t-1} and Σ_{t-1}. If we look at the update formula $\mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t)$, if the Kalman gain is high then the measured "next" data z_t will have a higher impact on the mean. But if the Kalman gain is low then the previous value will have a lower impact.

4. If we look at the formula $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$, then the result of using too large covariance matrix Q_t will give a small Kalman gain due to the inverse (the denominator becomes larger). This will also result that the new belief will have a small impact and the new belief will approximately be the same as the previous.

5. If we look at the formula $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$, then there will be an incresed effect in the measued data if there exist a greater difference between z_t and the mean u_t and C_t, or if the Kalman gain is relatively high.

6. Because of reason that the next state transition is stochastic, there will exists an uncertainty that grows. By looking at the formula:
   $\bar{\Sigma}_t = Q_t + A_t \Sigma_{t-1} A_t^T$ we can see that the covariance will often grow at the next step prediction with $A_t \Sigma_{t-1} A_t^T$.

7. It is due to the Kalman filters construction, it is an assumption you make that the filter is already optimal. We know that the Gaussian priori distribution will be the distribution that is exact. If you conclude any other distribution it will be less exact with more variance. We also know that the filter works under Markov assumption which gives the optimal filter.

8. The MLE follows as $\lambda_{MLE} = argmax_\lambda p(z_i|\lambda)$ where we assume that both the measured data and distribution is Gaussian. The MAP follows as $\lambda_{MAP} = argmax_\lambda p(z_i|\lambda)P(\lambda)$ and we know the distribution $p(\lambda)$. Therefor in this case the Kalman filter is both MLE and MAP because it fulfills the criterias for both.

2 Extended Kalman Filter

9. They are basically the same, but the extended Kalman filter can handle non-linear functions while the regular can not.

10. No, this is because of the non-linear transform, that the Gaussians become distorted. Also it could be that the original noise perhaps is not Gaussian from the beginning.

11. Yes, there are a some methods that can be used, one major is to regulate the covariance Q_t to reduce the change of diverging. For example if it becomes to small it does not take new measurements and it can diverge.

3 Localization

12. Location is unknown, we have the posterior belief as:
$\bar{bel}(x_t) = \int p(x_t|u_t, x_{t-1,\underline{m}})bel(x_{t-1})dx$
$bel(x_t) = \eta p(r_t|x_t, m)\bar{bel}(x_t)$,
where we get the location as $x_t = [x, y, \theta]$

13. In this case θ (the bearing) is known. We create a new vector, for example $\bar{W}$ that now contains both r_t and θ.

14. If we assume that the angle θ is uncertain, and the robot is moving, then there exist a risk where the "spread"-angle can be bigger while it moves. The placement is then stochastic and there is a wider range where the robots location might be, that concludes that it is more uncertain where it is located.

15. If there exists an uncertainty in the measured bearing, then it becomes a risk that the robot picks another feature predicted from the EKF.

4 Part II

Q1. When looking at the given matrices, we can see that the dimensions for ϵ_k is 2x1 and for δ_k it is 1, then it will be valid with the dimension of z_k and x_{k+1}.
We know that the mean value is already zero with a white Gaussian. It is distributed accordingly to $N(0, \sigma^2)$, therefore the parameter we need is the variance.

Q2.
x | State space variable/vector
$xhat$ | Predicted belief/state
P | Predicted covariance matrix
G | Identity matrix
D | Constant
Q | Process noise
R | Measurement noise
$wStdP$ | Noise on simulated position
$wStdV$ | Noise on simulated velocity
$vStd$ | Simulated measurement noise on position
u | Control data, where we know the noise and uncertainty
PP | Saves the P matrix as a 4x1 matrix PP

Q3.

First we write the Kalman gain in terms of the new variables:
$K_t = \bar{P}_t C^T (C\bar{P}_t C^T + DQ_t D^T)^{-1}$
And also we can deduce an equation for the covariance matrix P as:
$P = (I - K_t C_t)(R_t + A_t P_{t_1} A_t^T)$ (*from slide 25 lecture about Kalman filter prediction and observation*)
and the mean as $\bar{x} = \bar{x} + K_t(y - C_t\bar{x})$

With the help of these we can see that if we want to increase the mean, we want to increase the Kalman gain. So the larger the Kalman gain the faster the system gets. If we look at figure 1 with the initial values of the process noise (Q_t) and measurement noise (R_t), we observe the systems position and speed with standard settings. Now if we increase Q_t we expect, according to $K_t = \bar{P}_t C^T (C\bar{P}_t C^T + DQ_t D^T)^{-1}$, that the Kalman gain will get smaller because of the covariance matrix decreases, and therefore the system slower. This can be visualized in figure 2 where Q_t is 100 times larger. However, if we increase R_t also we expect the covariance matrix to increase which concludes that the Kalman gain increases. Therefore we expect a faster changing system. Which is clearly visible in figure 3. If we decrease both the measurement noise and process noise with the same factor, say with 1/100, then according to the equations and simulation the system will be a lot faster.

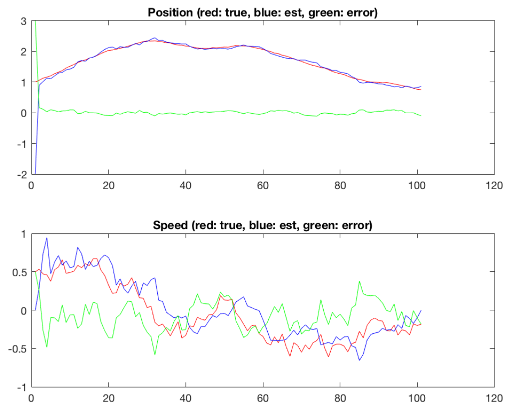

Figure 1: Q_t and R_t with its original values

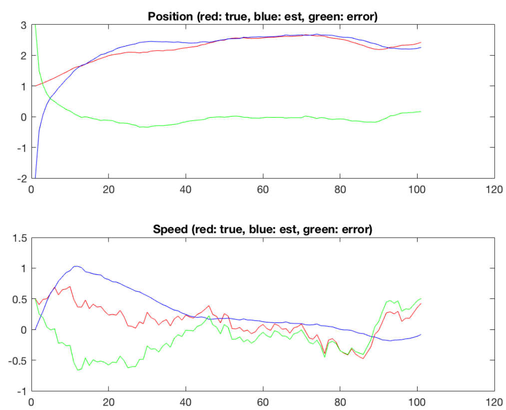

Figure 2: Q_t 100 times larger

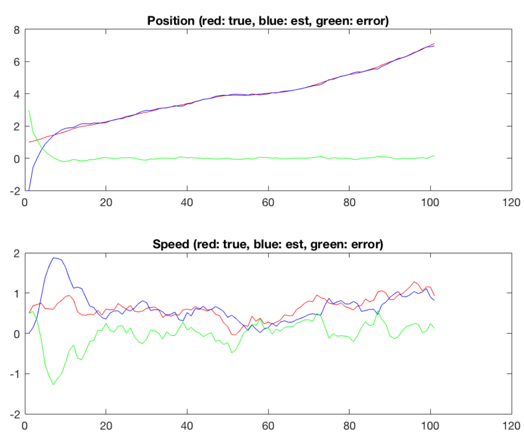

Figure 3: Q_t and R_t 100 times larger

Q4.