Robot localization description

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EKF-node

The `ekf_localization_node` implements the following algorithm:

1. wait for new measurements
2. **predict** next state and error covariance using a second order omnidirectional model
3. **correct** the prediction using measurements and update the error covariance
4. go to step 1.

Prediction step

When using `ekf_localization_node` in 2D-mode the next state $\tilde{s}_{t+1}$ is predicted from the current state estimate s_t using a second order omnidirectional model:

$$\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \\ \varphi \\ \dot{\varphi} \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & \delta\cos\varphi & -\delta\sin\varphi & -\frac{1}{2}\delta^2\cos\varphi & -\frac{1}{2}\delta^2\sin\varphi & 0 & 0 \\ 0 & 1 & \delta\sin\varphi & \delta\cos\varphi & \frac{1}{2}\delta^2\sin\varphi & \frac{1}{2}\delta^2\cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 & \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \\ \varphi \\ \dot{\varphi} \end{bmatrix}_{t}. \tag{1}$$

Where δ is the total time since the last measurement. See `robot_localization/src/predict.cpp` for complete function. Note that (1) is a non-linear system since the state transition matrix depends on φ.

The estimation error covariance matrix is updated according to:

$$\tilde{P}_{t+1} = JPJ^T + \delta Q \tag{2}$$

where J is the Jacobian of the motion model and Q is the process noise covariance (which must be hand tuned).

Correction step

The complete state estimate is corrected using the measurements according to:

$$s_{t+1} = \tilde{s}_{t+1} + K(m - H\tilde{s}_{t+1}) \tag{3}$$

were m is the measurement vector, H is the measurement matrix and K is the Kalman gain given by:

$$K = \tilde{P}_{t+1} H^T (H \tilde{P}_{t+1} + H^T + R)^{-1}. \tag{4}$$

The estimation error covariance matrix is updated according to:

$$P_{t+1} = (I - KH) \tilde{P}_{t+1} (I - KH)^T + KRK^T. \tag{5}$$