LESSON 1 NUMBER SYSTEM

LEARNING IN THIS CHAPTER

- Decimal Number System
- Binary Number System
- Octal Number System
- Hexadecimal Number System
- Computer Arithmetic

In the earlier times, when there were no means of counting, people used to count with the help of fingers, stones, pebbles, sticks, etc. These methods were not adequate and had many limitations. To overcome these limitations, many number systems were introduced with the passage of time, like:

- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

A number system is a set of values used to represent different quantities, such as number of students in a class, number of viewers watching a particular show, etc. We use the decimal number system in our day to day life, whereas a computer represents all kinds of data and information (text, numbers, graphics, etc.) in the binary number system.

The total number of digits used in a number system is called its **Base** or **Radix**. The base is written after the number as a subscript, for example, $(15)_{10}$.

DECIMAL NUMBER SYSTEM

The need for counting paved the way to introduce the Decimal number system, in which 0,1,2,3...9 are used to form any number. Most of our arithmetic operations are performed with decimal numbers.

Decimal number system consists of ten digits, i.e., 0 to 9 with the base 10. Each number can be used individually or they can be grouped to form a numeric value, e.g., 82, -256, 52.87, etc. The value of each digit in a number depends upon the following:

- The face value of the digit
- The base of the number system
- The position of the digit in the number

Each position represents a specific power of the base (10). The right most digit of a number is called **Least Significant Digit**, whereas the left most digit is called **Most Significant Digit**. For example, the number 547 can be represented in the following way:

Example

5 4 7

7 →	**$7 * 10^0$ units**	=	**7**
4 →	**$4 * 10^1$ tens**	=	**40**
5 →	**$5 * 10^2$ hundreds**	=	**500**
			547

OBSERVATION

The positional value of each digit increases ten folds as we move from right to left. In the above mentioned example, 5, 4, and 7 are the face values and their place values are hundreds, tens, and units, respectively. The place value of a digit depends on its position in the number.

Now we will discuss the various types of number systems that are used in a computer.

BINARY NUMBER SYSTEM

The Binary number system consists of only two digits, i.e., zero and one (0 and 1). Since this system uses two digits, it has the base 2. All digital computers use this number system and convert the input data from the decimal format into its binary equivalent.

WHY BINARY?

A computer cannot understand human language, rather it understands only the binary code. Therefore, the data that is entered into a computer is converted into its binary equivalent. It further converts the binary result into its decimal equivalent to generate an output.

CONVERSION OF DECIMAL INTO BINARY NUMBER SYSTEM

The equivalence between binary and decimal numbers can be understood with the given examples. To convert a decimal number into a binary number, follow the given steps:

Step 1: Divide the given decimal number with the base 2.

Step 2: Write down the remainder and divide the quotient again by 2.

Step 3: Repeat the step 2 till the quotient is zero.

Step 4: Write the remainders obtained in each step in the reverse order to form the binary equivalent of the given decimal number, i.e., placing the Least Significant Digit at the top and the Most Significant Digit at the bottom.

Consider the following examples:

Example

Example 1:

2	25		
2	12	1	→ Least Significant Digit
2	6	0	
2	3	0	
2	1	1	
	0	1	→ Most Significant Digit

Thus $(25)_{10} = (11001)_2$

The base of number is given as subscript

Example 2:

2	321		
2	160	1	→ Least Significant Digit
2	80	0	
2	40	0	
2	20	0	
2	10	0	
2	5	0	
2	2	1	
2	1	0	
	0	1	→ Most Significant Digit

Thus $(321)_{10} = (101000001)_2$

Fact File

Aryabhat was India's greatest mathematician and astronomer. He introduced the concept of 0 (zero) without which modern computer technology would have been non-existent.

Fact File

Gottfried Leibniz, a German mathematician is credited with the invention of the modern Binary number system.

Let Us Recall

Which number system do we use in our day to day life?

Observe, that the remainders obtained in each step are written in the reverse order.

BINARY TO DECIMAL NUMBER

To convert a binary number into its equivalent decimal number, follow these steps :

- Multiply each binary number with its positional value, which is in terms of power of 2, starting from the extreme right digit.
- Increase the power one by one, keeping the base fixed as 2.
- Calculate the sum of all the products to get the decimal number.

Example

Example 1:

$(1\ 0\ 1\ 0)_2$

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

Thus $(1010)_2 = (10)_{10}$

Example 2:

$(1\ 0\ 0\ 1)_2$

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

Thus $(1001)_2 = (9)_{10}$

Example 3:

$(110001001)_2 = (1 \times 2)^8 + (1 \times 2)^7 + (0 \times 2)^6 + (0 \times 2)^5 + (0 \times 2)^4 + (1 \times 2)^3 + (0 \times 2)^2 + (0 \times 2)^1 + (1 \times 2)^0$

$= 256+128+0+0+0+8+0+0+1$

$= 393$

Thus $(110001001)_2 = (393)_{10}$

OCTAL NUMBER SYSTEM

The Octal number system (Oct) consists of 8 digits, from 0 to 7, with the base 8. The concept of Octal number system came from the Native Americans as they used to count numbers by using the space between their fingers rather than using their fingers. The procedure of 'Octal to Decimal' conversion is similar to 'Binary to Decimal' conversion. The only difference is the change of base.

To convert an octal number to decimal number, start multiplying the digits of the number from the right-hand side with increasing power of 8 starting from 0 and finally calculating the sum of all the products.

Example

Example 1:

$(345)_8$

$=(3 \times 8^2) + (4 \times 8^1) + (5 \times 8^0)$

$=192 + 32 + 5 = 229$

Thus $(345)_8 = (229)_{10}$

Example 2:

$(317)_8$

$=(3 \times 8^2) + (1 \times 8^1) + (7 \times 8^0)$

$=192 + 8 + 7 = 207$

Thus $(317)_8 = (207)_{10}$

HEXADECIMAL NUMBER SYSTEM

This number system consists of 16 digits, numbers 0-9 and the letters A-F, where A-F represent decimal numbers from 10 to 15. That means, A is equivalent to 10, B is equivalent to 11, C refers to 12, and so on. The base of this number system is 16. This number system is also known as **Hex**. The procedure of converting hexadecimal to decimal is similar to the methods shown in the previous pages, with the only difference being the change of base.

To convert a hexadecimal number to decimal, multiply the numbers starting from the right-hand side with increasing power of 16, starting from 0 and then calculating the sum of all the products.

Example

Example 1:

$(3B)_{16}$

$=(3 \times 16^1)+(11 \times 16^0)$

$=48 + 11 = 59$

$\boxed{\text{Thus } (3B)_{16} = (59)_{10}}$

Example 2:

$(4D2)_{16}$

$=(4 \times 16^2) + (13 \times 16^1) + (2 \times 16^0)$

$=1024 + 208 + 2 = 1234$

$\boxed{\text{Thus } (4D2)_{16} = (1234)_{10}}$

Know the Fact

4 bits = 1 nibble
2 nibbles = 8 bits
8 bits = 1 byte

Let's Discuss

Why do we use Binary Number system in computers?

COMPUTER ARITHMETIC

As a computer understands only the binary code, so the data entered into the computer by the user is converted into binary code for processing. This processing may involve various kinds of arithmetic operations, such as addition, subtraction, multiplication, division, etc., on the binary numbers.

Let's Know More

Most computer operations use the byte, or a multiple of the byte (16, 24, 32, 64 bits, etc.). Hexadecimal makes it easier to work with long binary numbers as a byte of binary data can be represented using just two hexadecimal digits.

BINARY ADDITION

The technique used to add binary numbers inside the computer is very easy and simple. This is performed in the same way as you perform addition with decimal numbers.

The following table illustrates the addition of two binary digits:

a	b	a + b = c
0	0	0 + 0 = 0
0	1	0 + 1 = 1
1	0	1 + 0 = 1
1	1	1 + 1 = 10

TIP

While adding 1 + 1, the output will be 10, where 0 is written under the same column and carry over 1 is shifted to the next place, as it happens in the decimal number addition.

Quick Quiz

How will you find whether a number is represented in Decimal/Binary/Octal or Hexadecimal system?

Example

Example 1:

Compute $(1000)_2 + (1111)_2$

```
  1000
 +1111
 -----
 10111
```

Example 2:

Compute $(11111)_2 + (1011)_2$

```
  1111    Carry over
  11111
+ 01011
-------
 101010
```

Let Us Recall

Which number system has '8' as its base?

BINARY SUBTRACTION

The rules given in the table must be followed to perform binary subtraction:

a	b	a - b = c
0	0	0 - 0 = 0
1	0	1 - 0 = 1
1	1	1 - 1 = 0
0	1	0 - 1 = 1

NOTE

The number is borrowed when 1 is subtracted from 0 (10 - 1 = 1).

Example

Example 1:

Compute $(1111)_2$ - $(1010)_2$

```
   1 1 1 1
 - 1 0 1 0
 ---------
   0 1 0 1
 ---------
```

Example 2:

Compute $(1100)_2$ - $(11)_2$

```
          Borrowed 1   Again Borrowed 1
          Balance   Balance   Number is now
             0    10   1          10
   1         1         0          0 -1
                                     1
 - 0         0         1          1
 ----------------------------------
   1         0         0          1
 ----------------------------------
```

BINARY MULTIPLICATION

The rules for performing multiplication using binary numbers is same as that of the decimal numbers. The given table illustrates the multiplication of two binary digits:

a	b	a * b = c
0	0	0 * 0 = 0
0	1	0 * 1 = 0
1	0	1 * 0 = 0
1	1	1 * 1 = 1

Example

Example 1:

Compute $(101)_2 \times (11)_2$

```
       101
     ×  11
     -----
       101
     +101×
     -----
Sum = 1111
     -----
```

Example 2:

Compute $(1111)_2 \times (101)_2$

```
     1111
    × 101
    -----
     1111
    0000×
 +1111××
 -------
 1001011
 -------
```

BINARY DIVISION

The method to perform division of two binary numbers is same as that of decimal numbers. See the example given below:

Example

Example 1: Compute $(110)_2 \div (10)_2$

```
      11   <------ Quotient
   10|110  <------ Dividend
   |  10
Divisor --
     010
      10
     ---
      00   <------ Remainder
     ---
```

Example 2: Compute $(10000111)_2 \div (1001)_2$

```
          01111     <------ Quotient
     1001|10000111  <------ Dividend
     |    1001
Divisor  --------
          001111
            1001
         --------
           01101
            1001
         --------
           01001
            1001
         --------
            0000    <------ Remainder
         --------
```

Recap

- The commonly used number system is Decimal number system with the base 10.
- The right most digit of a number is called Least Significant Digit, whereas the left most digit is called Most Significant Digit.
- The Decimal number system consists of 10 digits, from 0 to 9, and has the base 10.
- The Binary number system consists of two digits, i.e., 0 and 1, and has the base 2.
- The Octal number system consists of 8 digits from 0 to7 with the base 8.
- The Hexadecimal number system consists of 16 digits [ten digits (0 to 9) and six letters (A to F)], and has the base 16.

Brain DEVELOPER

A. Fill in the blanks:

1. The base of Binary number system is ....
2. The base of .... number system is 10.
3. Octal Number system consists of .... digits.
4. In Binary addition, 1+1 equals to ....
5. .... number system is understood by the computer system.
6. .... number system uses 16 digits, from 0 to 9 and the letters A to F.
7. In Binary subtraction, 1-1 equals to ....

HINTS • 0 • Binary • Decimal • Hexadecimal • 2 • 8 • 10

B. State True or False:

1. You cannot perform arithmetical operations on binary numbers. ☐
2. The decimal number system consists of 10 digits, i.e., 0 to 9. ☐
3. The method to perform division of two binary numbers is not the same as that of decimal numbers. ☐
4. 1 multiplied by 0 equals to 0. ☐
5. Charles Babbage introduced the concept of 0 (Zero). ☐
6. The numbers used in Octal number system are 1 to 7. ☐

C. Application Based Questions:

1. Ratika's computer teacher has asked her to convert an Octal number to Decimal number. Suggest her the method which she should apply in converting the Octal number.

....

2. The teacher has given an assignment to Saurabh on Binary subtraction. Saurabh is confused about how to subtract 1 from 0. Help him in solving the problem.

....

D. Multiple Choice Questions:

1. .... introduced the concept of 0 (Zero).

a. Ada Lovelace b. Aryabhat c. Charles Babbage

2. A .... converts the decimal format into its binary equivalent.

a. Digital Computer　　b. Cell Phone　　c. Abacus

3. A computer understands only the .... code.

a. English　　b. French　　c. Binary

4. In Binary multiplication, 1x1 equals to ....

a. 0　　b. 1　　c. 2

5. To convert a Decimal number into a Binary number, divide the number by ....

a. 2　　b. 8　　c. 10

E. Answer the following:

1. What is a number system? Explain its commonly used types.

....

....

....

2. What are the rules to convert a Decimal number into a Binary number?

....

....

....

3. Write the rules to multiply two Binary numbers.

....

....

....

4. Briefly explain the Octal number system.

....

....

....

5. What do you understand by the Hexadecimal number system?

....

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