

Attenuation of Shock Waves Exiting From an Open Tube

Mitchell J. Robertson*

UNSW Canberra

In many engineering applications, such as exhaust systems for internal combustion engines, the attenuation of moving shock waves is desired to decrease noise and to increase performance. In this paper, the effect of angular and circular diffusers on shock decay is determined through a parametric study of the characteristic parameters that define the diffuser. The reduction in shock strength, measured through peak overpressure, has been determined numerically using a second-order Godunov-type scheme applied to the Euler equations. The results demonstrate that a diffuser's effectiveness is limited to the region immediately after the exit. At distances further downstream attenuation is limited by the sonic condition and the effect of a diffuser is negligible. The outcomes of the parametric study demonstrate that an angular diffuser is more effective than a circular diffuser of equivalent area ratio. Further, the shock strength can be minimised by maximising divergence angle/radius and transition length as allowed by physical constraints.

I. Introduction & Background

THE drive for increased performance from the internal combustion engine has lead to the prevalence of very high cylinder pressures and direct exhaust systems. The resulting pressure ratios lead to the production of compression waves, which coalesce into a moving shock wave of up to Mach 1.1 under high load conditions.¹ The shock wave and the resultant pressure variations are known to produce an extremely loud exhaust noise in a similar manner to a sonic boom.¹ While typically not a concern for the racing industry, such exhaust sounds are generally adverse to the desired image for production vehicles, requiring significant effort and expense to design an appropriate muffler. This requirement is made even more critical by the introduction of modern noise abatement standards.

In the reactive-type muffler used with most consumer vehicles, the pressure wave is attenuated through an expansion chamber containing a number of area discontinuities to promote reflection and destructive interference. However, with every direction change in the muffler, the exhaust back pressure increases, which reduces the overall power output.² While the arrangement of these discontinuities has been extensively studied,^{3,4} the entrance to the muffler remains (in general) a simple 90° expansion. The primary hypothesis of this project is that tempering this area change with a diffuser will increase the shock attenuation and reduce the requirements for discontinuities in the muffler, reducing overall back pressure.

Among the first to study shock wave attenuation was Chisnell through a theoretical approach. In doing so, a closed-form relation was developed between the channel area and the shock strength.⁵ Qualitatively, an increase in channel area will result in a decrease in shock strength. In his work, Chisnell defined shock strength as the ratio of pressure behind the shock wave to that in front ($Z = P/P_0$).⁵ This definition has

*School of Engineering and Information Technology, ZEIT4500-4501

become the typical measure for the strength of a shock wave and will also be used in this study. Chisnell's approach neglects downstream wave interactions and assumes quasi-steady, planar flow in parallel sided ducting. However, experimental work, such as Deckker & Gururaja (1969⁶), has demonstrated that the theory can estimate shock strength during diffraction reasonably accurately.

The topic of diffraction was subsequently investigated theoretically by Whitham^{7,8} and experimentally by Skews.^{9,10} Through shock tube studies, Skews was able to refine the approximate theoretical approximations and describe the shape of the shock wave as it diffracts around convex corners. For both rounded and angular corners, the shock turns to follow the wall, forming a curved shock front. Numerical data inferred from the flow visualisation showed that the Mach number at the wall was less than that of the incident shock, with the difference directly proportional to the divergence angle/radius and incident Mach number.⁹ While the difference in diffraction patterns described are important to the diffuser geometry to be designed, Skews' work does not characterise the axial decay of the shock through such geometry changes.

Nettleton expanded upon Skews' work to investigate the effect of divergence angle and area ratio on shock attenuation through an area change.¹¹ It was observed that the expansion fans that drive the diffraction propagate towards the axis, weakening the shock wave as it passes. Through shock tube studies and direct pressure measurements, shock strength downstream of the area change was determined. The results indicate that the majority of the decay occurs within 3 hydraulic diameters of the area change and that larger divergence angles produce higher degrees of attenuation.¹¹ Additionally, a rounded corner was shown to marginally increase the rate of attenuation compared to a sharp 90° corner. However, these experiments were limited to a small range of divergence angles ($\theta < 15^\circ$) and high Mach numbers ($Ma > 1.8$).¹¹

The qualitative findings of these works indicate that an increase in shock wave attenuation can be incurred by introducing a diffuser to the muffler inlet. However, the literature is primarily concerned with planar flows, not the axisymmetric flow found in most practical applications. Furthermore, the study of low Mach number shock waves is particularly limited when considering diffuser geometry and will be expanded in the present work.

II. Method

The diffuser geometry selected for this study was largely influenced by the work of Nettleton, with an elaboration on his work with rounded corners. As the baseline for comparison, a simple 90° corner was selected. Following this, the geometry was altered to include a circular or angular diffuser. As seen Fig. (1), the circular diffuser contains one characteristic dimension, the radius (R), whereas the angular diffuser has two; the transition length (L_T) and the divergence angle (θ). In order for the results to be scalable, the nozzle parameters have been non-dimensionalised in terms of the inlet pipe diameter (D). As the present work is only concerned with passage through the area change, the expansion chamber was modelled as 4.5 x 6 hydraulic diameters. This was found to be large enough that the shock did not reflect off the sides and influence the flow field.

Consideration of the theory of Chisnell gives weight to the idea that the only factor affecting attenuation is the area ratio through the diffuser, not the shape of the diffuser itself. To examine this, a simple comparison study was conducted between an angular and circular diffuser with an inlet:outlet area ratio of 9. These simulations were also compared to shock strengths produced through the Chisnell theory to determine its applicability to the present study. The results of this comparison, discussed in detail in Section 3, indicate that the Chisnell theory is inadequate for the present work and analysis through experimentation and computational fluid dynamics (CFD) was required.

The **Masterix** software package¹² has been used for analysis. The second-order Godunov-type scheme used by this program has been shown to be a suitable compromise of accuracy and efficiency compared

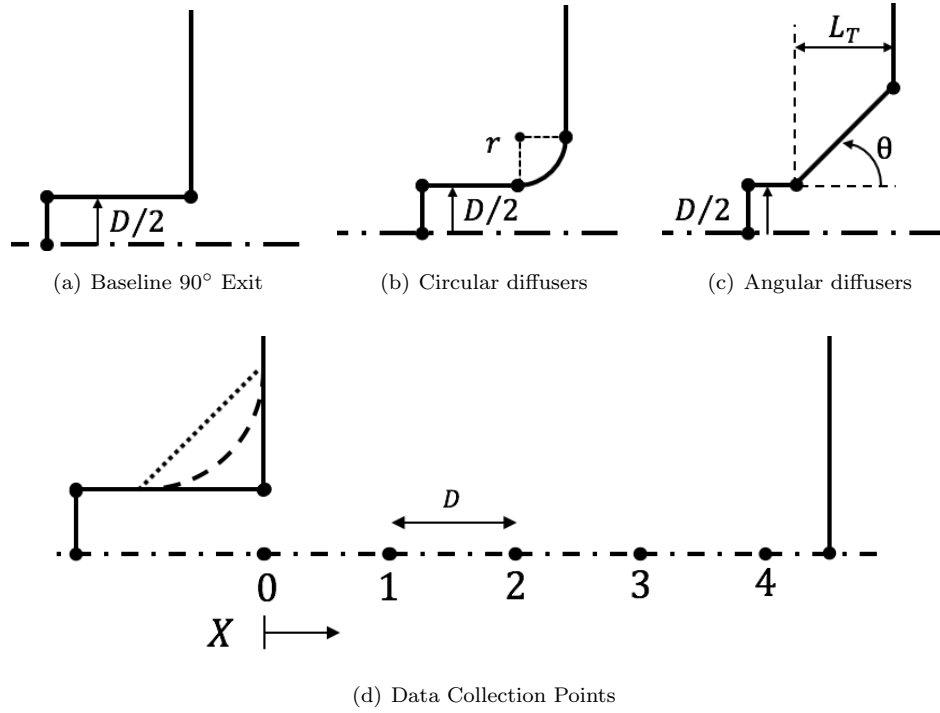


Figure 1. Nozzle geometry for the variety of diffusers to be examined in terms of characteristic dimensions.

to other available techniques.¹³ Furthermore, *Masterix*¹² operates with the inviscid Eulerian gas model described by Saito et al. (2001).¹⁴ As a result, flow features such as boundary layers are not modelled by the simulation. Regardless, previous studies into shock wave attenuation have shown this assumption to be valid. One such example is the work of Yu and Grönig (1996),¹⁵ where numerical results were compared to experimental data and close agreement was found. Finally, the software uses an unstructured triangular mesh to better capture the vortices and other complexities in the flow field. Automatic (adaptive) grid refinement is achieved through a velocity gradient criterion between cells to determine whether additional mesh points are required. This allows for a more coarse mesh to be used overall, with high levels of refinement only at shock front and other points of interest. This leads to an order of magnitude improvement in memory efficiency over a uniform, fine mesh.¹⁴

The attenuation of the shock was analysed by comparing the shock strength (Z) as the flow progresses downstream. As shown in Fig. (1d), the measurement points were chosen to lie on the axis of symmetry, where the shock strength will be highest. To characterise the axial decay of the shock wave, total of 17 points evenly distributed from the diffuser exit to $4D$ downstream were initially used for the baseline case. As this resulted in a significant increase in computation time, the measurement points were altered to 5 locations evenly spaced one diameter apart.

As the circular geometry only depends on one factor, a simple experimental design of 9 simulations where diffuser radius is varied from $0.2D$ to $1.0D$ has been used. This range has been selected as it effectively characterised the range of diffuser sizes that could practically be applied and results in a reasonable amount of computation time.

For the angular geometry, the presence of two characteristic dimensions complicates the experimental design. To adequately capture the effects of both of these factors and maintain a reasonable computation time, a faced central composite experimental design has been selected.¹⁶ This uses 9 combinations of 3 levels for each factor. The levels were chosen to lie across the parametric range at $L_T = 0.2D, 0.6D, 1.0D$ and

$\theta = 15^\circ, 45^\circ, 75^\circ$. This selection allows a study of effects of both parameters individually and the interaction between them.¹⁶

Verification of the numerical method was conducted before commencing the simulations. A candidate central to the parametric range for each geometry class was selected and a series of simulations was conducted. With each test, the initial mesh was refined and the adaptive refinement parameters were adjusted to allow a larger degree of cell splitting. The shock strength data at the five locations outlined earlier were used to compare the results. Grid independence was confirmed when three successive refinements showed less than 1% variance across all five data points. For the angular diffusers and the baseline case this was achieved with a 0.25×0.5 initial mesh with refinement parameter `MaxLevel` = 8. The presence of curved geometry in the circular diffusers meant that a 0.125×0.25 initial mesh with refinement parameter `MaxLevel` = 6 was required for grid independent results.

The numerical simulation methodology was validated through comparison to shock tube experiments. The candidate geometry were scaled and designed to attach to the exit of a cylindrical shock tube with inner diameter 20mm. Manufacturing was completed by fused filament fabrication with acrylonitrile butadiene styrene (ABS) and surface refinement with acetone vapour. The shock tube used, detailed in Fig. (2), consists of an 325mm driver section, and a 1165mm driven section separated by a aluminium foil diaphragm. Through calibration runs it was determined that the diaphragm burst under a driver section gauge pressure of 120kPa, which produced a shock wave of approximately Mach 1.2. Although this is above the desired shock speed of Mach 1.1, it proved to be much more reliable than manually rupturing the diaphragm at lower pressures in attempt to reach lower Mach numbers.

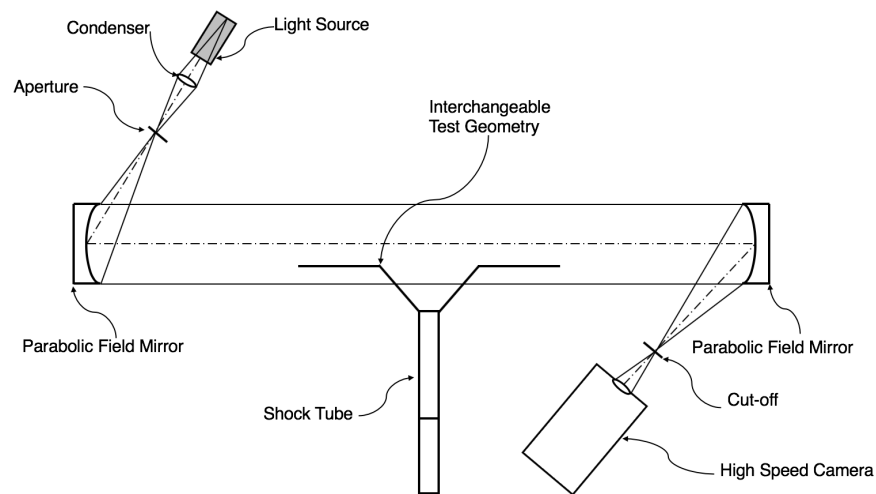


Figure 2. Shock tube and flow visualisation apparatus.

Flow visualisation through the schlieren technique was used to both characterise the shock wave diffraction and infer numerical data. A monochrome, omnidirectional method was used, implemented by a simple Z-configuration.¹⁷ This consisted of a circular aperture and cut-off with image data collected by a Shimadzu Hypervision high speed camera. The shock Mach number at the tube exit was determined by comparing pressure peaks from a pair of Kistler Type 603B pressure transducers displayed on a Yokogawa DLM2024 Oscilloscope.

As the present work is not concerned with the reflections in the expansion chamber, the shock wave is discharged into the atmosphere. Collecting accurate pressure data from the axis is difficult, requiring specialist apparatus to position transducers in the flow field. However, such systems disturb the flow downstream, limiting data collection to a single point, which is not suitable for the present applications. Another method

involves inferring the shock strength from the flow visualisation. This is achieved by determining the shock velocity from the motion of the shock and the camera frame rate. This is then converted to shock Mach number and then to shock strength through knowledge of the ambient conditions and the shock wave jump equations. This process is detailed by the relations in Eq. (1).

$$v_s = lf \rightarrow M_s = \frac{v_s}{\sqrt{\gamma RT_a}} \rightarrow Z = \frac{2\gamma M_s^2 - (\gamma - 1)}{\gamma + 1}$$

Where:

$l \equiv$ distance travelled between two frames

$f \equiv$ the camera frame rate

$T_a \equiv$ ambient temperature

(1)

III. Results & Discussion

A. Validation of the Numerical Results

The schlieren imagery from the validation cases are illustrated in Fig. (3). In all cases, the shape of the primary shock front is represented quite well by the numerical simulations. However, the significant detail of the complexities of the flow field behind the shock wave has been lost. This is due to the low Mach number of the primary shock causing these features to be quite weak. As a result, they fail to produce a large enough velocity gradient to trip the adaptive refinement criterion.

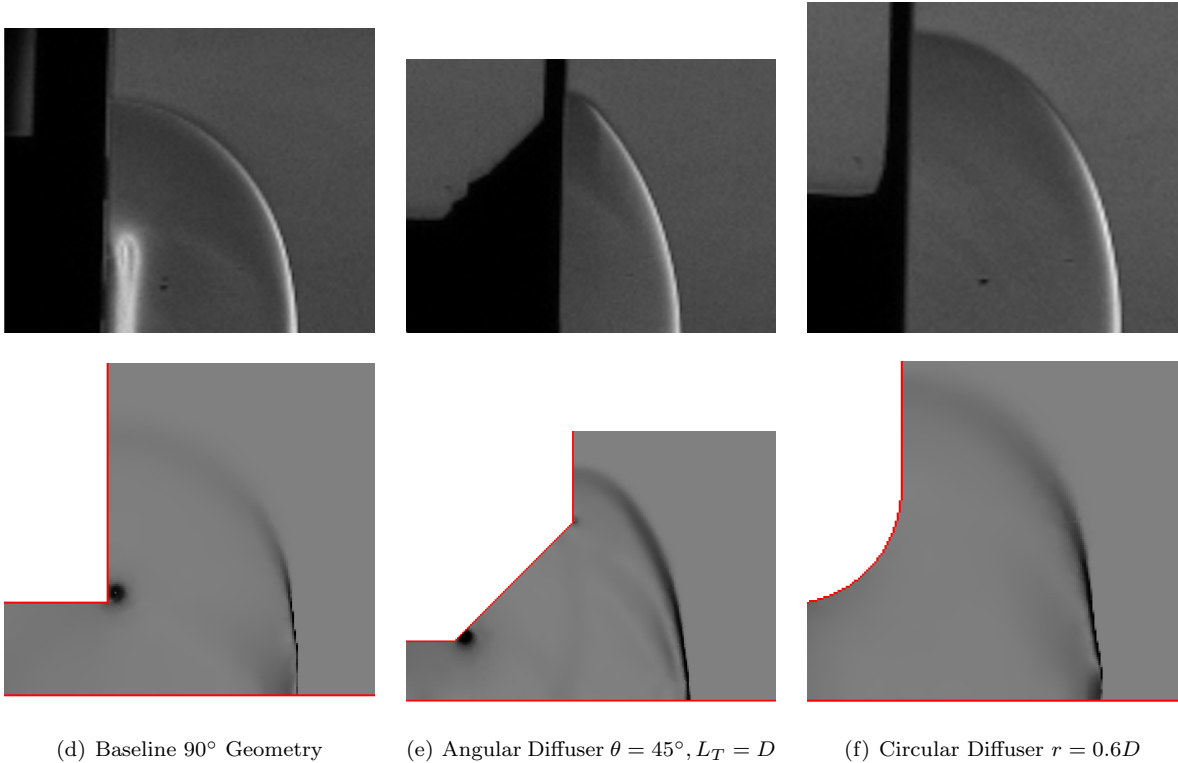


Figure 3. Experimental (Top) and numerical (Bottom) schlieren imagery of the baseline (Left), angular (Middle) and circular (Right) geometry studied. In all cases the shock wave is at $Ma=1.2$ when it reaches the diffuser.

A comparison between the shock strength data determined from the CFD and shock tube experiments is

illustrated in Fig. (4). A number of remarks must be made about the experimental data before conclusions can be drawn. Firstly, the large degree of uncertainty in the shock strength measurements. As the data is determined from image processing, one can only estimate the shock location to within one pixel. With the relatively low resolution camera used, this leads to an approximate 10% relative uncertainty. When propagated through to the shock strength, this becomes quite significant. This leads to the second issue; the shock seemingly decays below the sonic limit at 3-4 diameters downstream. This is obviously not a physical result, the shock strength must be at least 1. This is taken to be an result of the resolution limitations of the camera.

Despite these issues, the shock strength data are in agreement with the numerical results (within the range of uncertainty) and the general trends are represented. While further experimental work is desired to confirm the accuracy of the numerical results, the validity of the simulations is confirmed for the purposes of a broad parametric study of diffuser geometry.

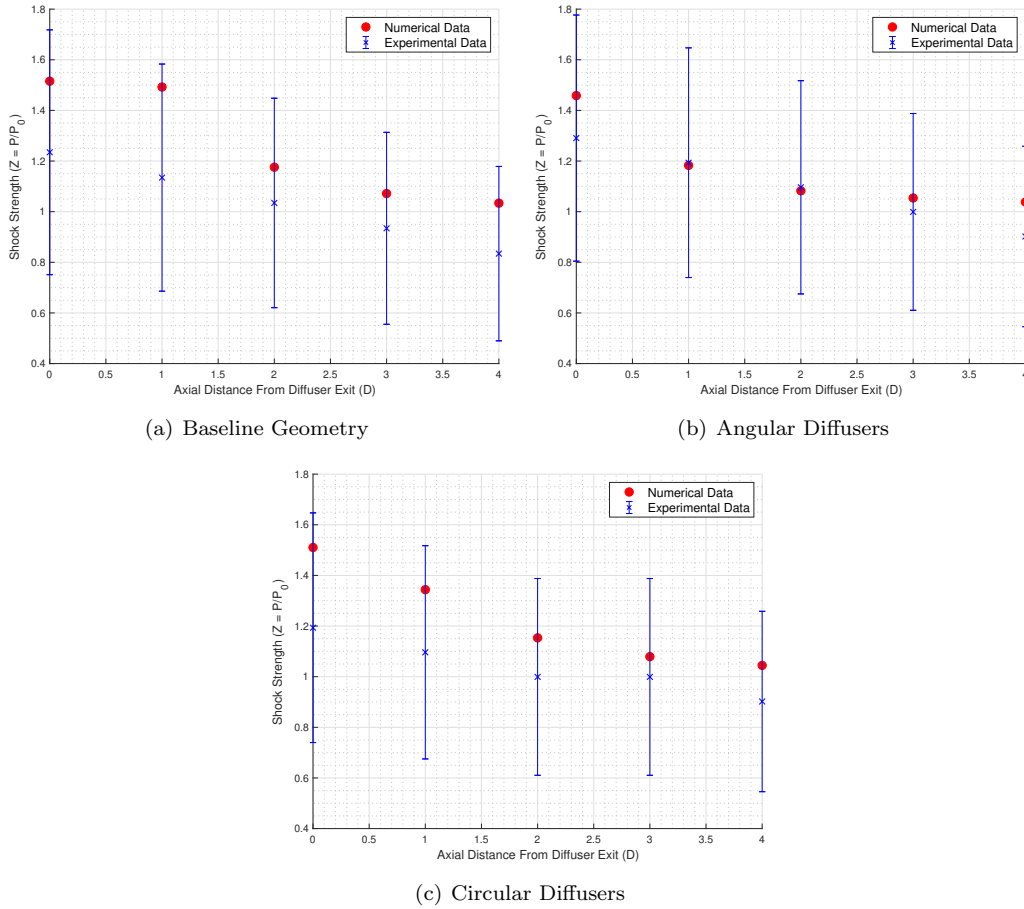


Figure 4. Comparison between shock strength data collected experimentally and numerically. In each case the shock wave is at $Ma=1.2$ when it reaches the diffuser.

B. Theoretical Approaches

A comparison of the decay through an angular and circular diffuser of area ratio 9 is show in Fig. (4). It is observed that while Chisnell's equation can accurately predict shock strength in the inlet tube, the shock strength through the diffuser and expansion chamber is considerably higher than predicted. This discrepancy is seen to decrease at large distances downstream, however this is outside the area of interest for this study. Comparing now the difference in attenuation between the two diffusers we see that the difference

in attenuation through the diffuser ($0 < X < 1$) is negligible. However, a significant difference is present in the expansion chamber ($1 < X < 5$). According to the Chisnell theory the two results should be identical, clearly the theory does not account for diffraction effects through the diffuser. As such, it is not suitable for analysis in the present work.

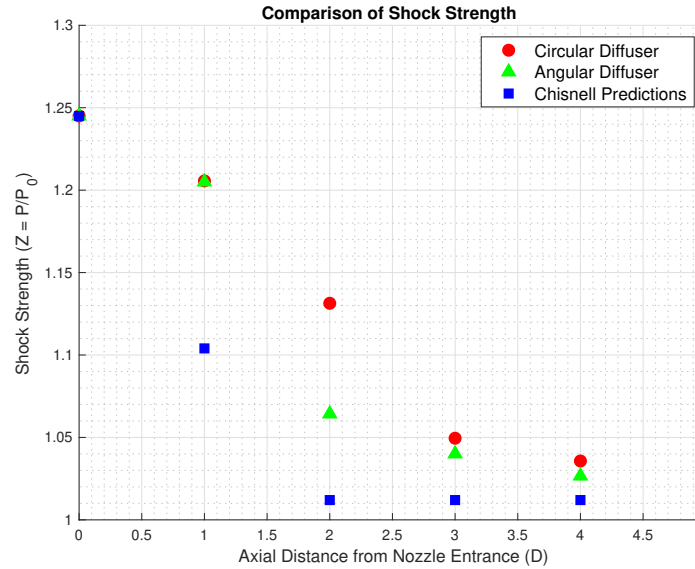


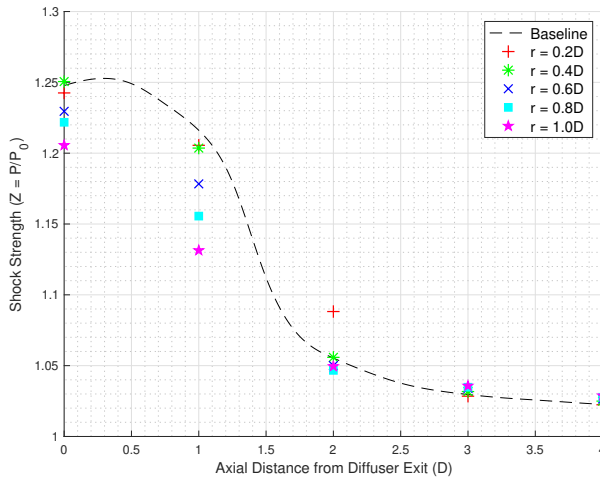
Figure 5. Comparison of diffusers of equal area ratio with the theory of Chisnell. Theoretical approaches are seen to overestimate the amount of decay throughout the flow field. Additionally, a large difference in attenuation is seen between the geometry types, which indicates limited validity of the theory to the present application.

From the schlieren imagery depicted in Fig. (3), a clear difference in shock diffraction is evident. The baseline exit (Left), exhibits a comparatively large vortex in the wake of the wall shock turning the corner. Further, the shock front curves so that it is almost circular. In contrast, the angular diffuser in (Middle), produces a smaller vortex, which travels much more slowly. Additionally, the shock front is seen to stretch in the vertical direction indicating additional expansion. The circular diffuser (Right), displays a weak vortex that does not propagate downstream until the shock front has travelled approximately 3 diameters from the diffuser exit. The shock front is also seen to stretch in the vertical direction, although it is to a lesser extent than the angular diffuser. It is this difference in vertical expansion that is hypothesised to cause the difference in axial attenuation.

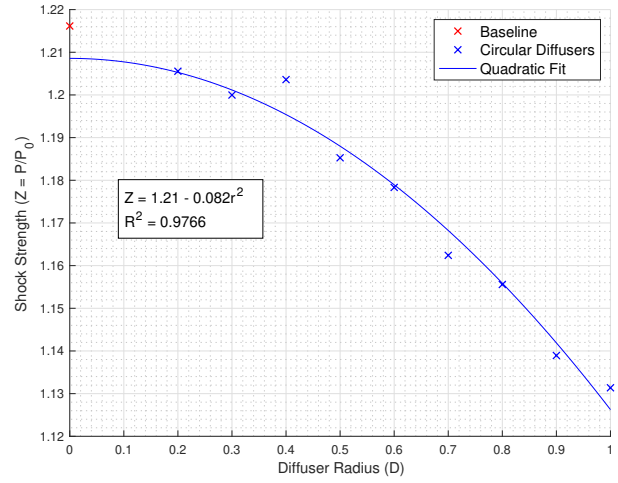
C. Parametric Study

The attenuation of the shock wave is characterised by the decrease in shock strength with axial propagation. Thus an effective diffuser will display a lower shock strength (Z) than the baseline at each downstream location. The shock strengths for the circular diffusers are compared to the baseline case in Fig. (6a).

After a small reduction in shock strength through the diffuser ($X = 0$), Fig. (6a) demonstrates significant attenuation compared to the baseline case at $X = 1$. The difference in shock strength appears to be proportional to diffuser radius, with the largest radius test demonstrating an approximate 7% improvement over the baseline. Following this the shock strengths reconverge and are approximately equivalent by 3 – 4 diameters downstream. The large reduction in the rate of decay is a result of the shock waves approaching the sonic limit where their strength is unity. This limit to the decay means that the shock wave from the baseline case is able to ‘catch-up’ in terms of attenuation. This result implies that a diffuser is only relevant at small distances from the area change.



(a) Shock strength at increasing axial distances from the nozzle exit for the range of circular diffuser radii.



(b) Effect of varying diffuser radius on the peak shock strength measured on the axis of symmetry at a distance of 1 hydraulic diameter from the diffuser exit.

Figure 6. Effect of a circular diffuser on the attenuation of a planar shock. The incident shock strength was $Z = 1.245$ ($Ma = 1.1$) in each case.

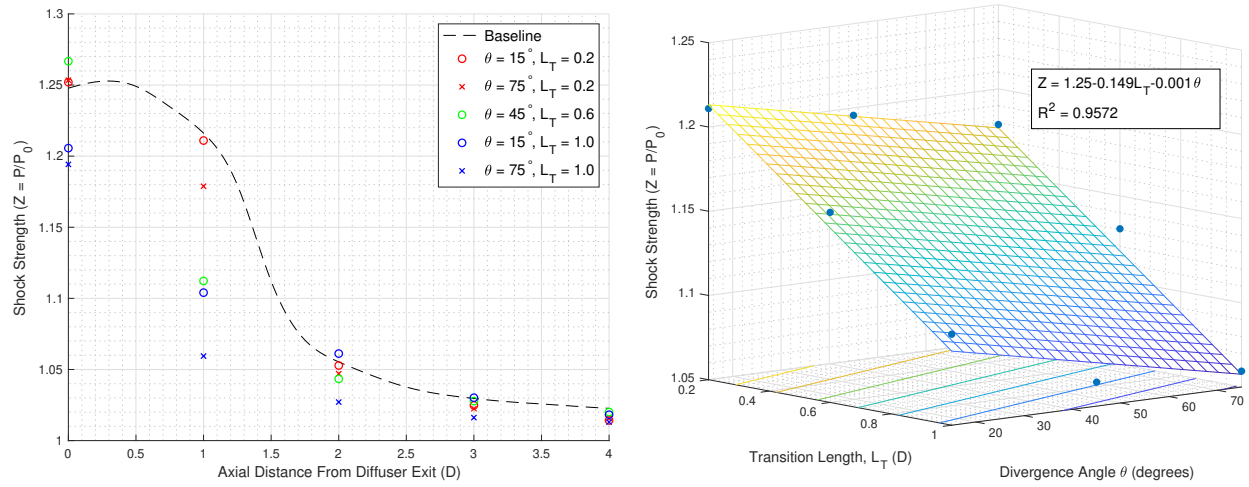
However, modern mufflers are generally designed to feature baffling geometry within this range, meaning that the results from further downstream will not eventuate in such applications. The relationship between diffuser radius and shock strength at $X = 1$ from the diffuser exit is demonstrated in Fig. (6b). A quadratic decay model was found to be statistically significant and represent the numerical data quite well. The range of radii test indicates that increased attenuation can be achieved by increasing the radius. This suggests that the largest diffuser allowed by physical constraints should be used in muffler design.

The data from the angular diffuser simulations displayed in Fig. (7a) are qualitatively similar to those from the circular diffusers. The tight grouping of data at $X = 0$ demonstrates the relatively small degree of attenuation through the diffuser. The shock then decays rapidly as it progresses downstream. Again, the greatest increase in decay is seen at $X = 1$ with the variance proportional to both divergence angle and transition length. The results for the largest diffuser demonstrated an approximate 13% improvement over the baseline. Moving further downstream, the difference in shock strength becomes less pronounced, with only a marginal improvement over the baseline case from $X = 3$ onwards. Through similar reasoning to the circular diffusers, it is anticipated that this is a result of the shock approaching the sonic limit.

As the attenuation immediately after the diffuser exit is most relevant to the design of exhaust systems, the shock strength at $X = 1$ was used for further analysis. A response surface¹⁶ indicating the variance of shock strength with both transition length and divergence angle is given in Fig. (7b). The surface fit to the data is represented in terms of constant and linear terms as all higher order terms (θ^2 , L_T^2 , θL_T) were found to be statistically insignificant.

This result has a number of implications, firstly the lack of a coupled term (θL_T) indicates that there is no significant interaction between the factors. That is, that the attenuation is simply a result of the sum of the influence of the two factors. Furthermore, the significant difference between the coefficients indicates a higher sensitivity to transition length than divergence angle. For the range tested, an increase in the transition length improves the attenuation of the shock by up to 9%, whereas divergence angle only varies the strength by approximately 2%.

The results from $X = 1$ indicate that increased attenuation can be achieved by increasing both factors. Practically, this means setting the divergence angle such that the inlet pipe is linked to the expansion chamber



(a) Shock strength at increasing axial distances from the nozzle exit for a selection of angular diffusers. The remaining diffuser sizes have been omitted for clarity

(b) Response surface demonstrating the influence of transition length and divergence angle on shock strength measured one diameter downstream from the diffuser exit.

Figure 7. Effect of an angular diffuser on the attenuation of a planar shock. The incident shock strength was $Z = 1.245$ ($Ma = 1.1$) in each case.

without a vertical wall. For a transition length of $L_T = 1D$ and an expansion chamber as shown, this angle is approximately 80° . No such convergence is seen for variations in transition length, suggesting that the effect of increasing L_T is limited only by the practical considerations that prevent further extension of the diffuser.

IV. Conclusions

In the design of the exhausts systems of automobile engines, it is desirable to enhance the attenuation of shock waves to limit noise and improve performance. The current methods involving complicated flow paths and many reflections in a muffler have been extensively studied and are quite effective. However, the fact remains that each direction change increases exhaust back pressure and reduces engine performance. This report investigated a new method of achieving shock wave attenuation through the use of a small diffuser before the muffler entrance to temper the sharp 90° corner present in most systems. After the analytical approach of Chisnell was shown to be insufficient, a parametric study of circular and angular diffuser geometry was conducted. Simulations were conducted to determine the effect of diffuser geometry on the shock strength in a muffler.

Diffraction in the baseline 90° corner produces a large vortex downstream of the corner. Using an angular or circular diffuser reduces the size of the vortex and its propagation speed. Accordingly, the change in diffraction pattern leads to a change in downstream shock strength. While only a small change in strength was noted at the diffuser exit, significant attenuation compared to the baseline was observed a small distance downstream. At one diameter from the diffuser exit a circular diffuser was able to increase the attenuation by up to 7% over the baseline. Similarly, an angular diffuser was able to produce a 13% improvement to shock strength attenuation. No coupling was found between the length of the diffuser and its divergence angle. Additionally, the shock wave attenuation was found to be insensitive to changes divergence angle compared to transition length. However, it was found in both cases that the increased attenuation over the baseline became less significant downstream, particularly at four diameters where the difference in shock strength was insignificant.

The results of this study allow a number of conclusions to be drawn.

1. The attenuation of a shock wave exiting a tube can be increased by the installation of a small diffuser at the exit.
2. An angular diffuser, is better at attenuating the shock wave than the circular diffuser with corresponding area ratio.
3. The effectiveness of an angular diffuser is more sensitive to changes in transition length than divergence angle. Thus, the attenuation through an angular diffuser can be enhanced by designing the transition length to be as large as possible within practical constraints.
4. Regardless of the diffuser design employed, the low Mach number shock waves present in many applications quickly approach the sonic condition following an area change. This limits the effective region of any diffuser to areas close to the diffuser exit.

V. Recommendations

Further experimental studies are required to complete the validation of the numerical simulations to the flow parameters studied in this report. It is recommended that a camera with a higher resolution be used and the field of view restricted to the first four diameters after the diffuser exit. While a wider view was useful for characterising the diffraction pattern and the shock propagation, it limits the pixel density in the region of interest and increases the uncertainty in the shock strength data.

The study of circular diffusers can be expanded by including elliptical curves. This would allow a two factor study of such geometry, with the semi-major and semi-minor axes analogous to the transition length and divergence angle. Time constraints prevent the author from completing this work, but the geometry script is available on request.

While this study has thoroughly investigated the effect of diffuser geometry on the downstream shock strength, the influence of these changes on noise production requires a significant amount of additional analysis. An acoustic study of engines featuring mufflers with these geometries would be able to characterise their effect on reducing overall engine noise.

References

- ¹Sekine, N., Matsumura, S., Aoki, K., and Takayama, K., “Generation and Propagation of Shock Waves in the Exhaust Pipe of a 4 Cycle Automobile Engine,” *AIP Conference Proceedings*, Vol. 208, AIP, 1990, pp. 671–677.
- ²Potente, D., “General Design Principles for an Automotive Muffler,” *Proceedings of ACOUSTICS*, 2005, pp. 9–11.
- ³Lee, J. W., “Optimal topology of reactive muffler achieving target transmission loss values: Design and experiment,” *Applied acoustics*, Vol. 88, 2015, pp. 104–113.
- ⁴Yedeg, E. L., Wadbro, E., and Berggren, M., “Interior layout topology optimization of a reactive muffler,” *Structural and Multidisciplinary Optimization*, Vol. 53, No. 4, Apr 2016, pp. 645–656.
- ⁵Chisnell, R. F., “The Motion of a Shock Wave in a Channel, with Applications to Cylindrical and Spherical Shock Waves,” *Journal of Fluid Mechanics*, Vol. 2, No. 3, 1957, pp. 286–298.
- ⁶Deckker, B. and Gururaja, J., “Paper 4: An Investigation of Shock Wave Behaviour in Ducts with a Gradual or Sudden Enlargement in Cross-Sectional Area,” *Proceedings of the Institution of Mechanical Engineers, Conference Proceedings*, Vol. 184, SAGE Publications Sage UK: London, England, 1969, pp. 17–27.
- ⁷Whitham, G., “A New Approach to Problems of Shock Dynamics Part I Two-Dimensional Problems,” *Journal of Fluid Mechanics*, Vol. 2, No. 2, 1957, pp. 145–171.
- ⁸Whitham, G., “A New Approach to Problems of Shock Dynamics Part 2. Three-Dimensional Problems,” *Journal of Fluid Mechanics*, Vol. 5, No. 3, 1959, pp. 369–386.
- ⁹Skews, B. W., “The Shape of a Diffracting Shock Wave,” *Journal of Fluid Mechanics*, Vol. 29, No. 2, 1967, pp. 297–304.
- ¹⁰Skews, B., “Shock Diffraction on Rounded Corners,” *Proceedings of the Third Australasian Conference on Hydraulics and Fluid Mechanics, Sydney*, 1968, pp. 20–22.
- ¹¹Nettleton, M., “Shock Attenuation in a ‘Gradual’ Area Expansion,” *Journal of Fluid Mechanics*, Vol. 60, No. 2, 1973, pp. 209–223.
- ¹²Masterix, ver. 3.40, RBT Consultants, Toronto, Ontario, 2005–2018.
- ¹³Fursenko, A., Sharov, D., Timofeev, E., and Voinovich, P., “High-Resolution Schemes and Unstructured Grids in Transient Shocked Flow Simulation,” *Thirteenth International Conference on Numerical Methods in Fluid Dynamics*, Springer, 1993, pp. 250–254.
- ¹⁴Saito, T., Voinovich, P., Timofeev, E., and Takayama, K., “Development and Application of High-Resolution Adaptive Numerical Techniques in Shock Wave Research Center,” *Godunov Methods*, Springer, 2001, pp. 763–784.
- ¹⁵Yu, Q. and Grönig, H., “Shock Waves from an Open-Ended Shock Tube with Different Shapes,” *Shock waves*, Vol. 6, No. 5, 1996, pp. 249–258.
- ¹⁶Lorenzen, T. and Anderson, V., *Design of experiments: a no-name approach*, CRC Press, 1993.
- ¹⁷Settle, G. S., *Schlieren and Shadowgraph Techniques: Visualizing Phenomena in Transparent Media*, Berlin: Springer, 2001.