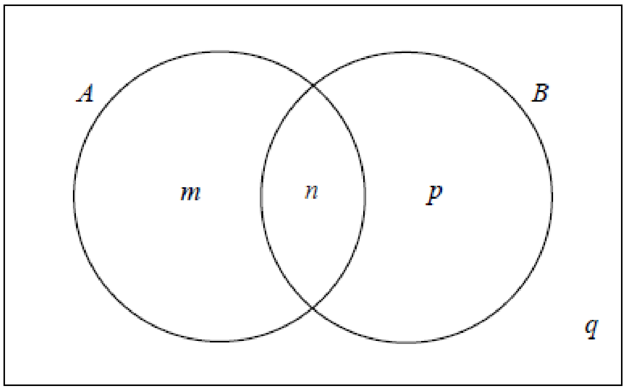
SL Paper 1

The Venn diagram below shows events A and B where $\mathrm{P}(A) = 0.3$, $\mathrm{P}(A \cup B) = 0.6$ and $\mathrm{P}(A \cap B) = 0.1$. The values m , n , p and q are probabilities.



a(i) and (ii). (i) Write down the value of n . [4]

(ii) Find the value of m , of p , and of q .

b. Find $\mathrm{P}(B')$. [2]

Markscheme

a(i) and (ii). (i) $n = 0.1$ ***A1 N1***

(ii) $m = 0.2$, $p = 0.3$, $q = 0.4$ ***A1A1A1 N3***

[4 marks]

b. appropriate approach

e.g. $\mathrm{P}(B') = 1 - P(B)$, $m + q$, $1 - (n + p)$ ***(M1)***

$\mathrm{P}(B') = 0.6$ ***A1 N2***

[2 marks]

Examiners report

a(i) and (ii). Most candidates were able to find the correct values for the Venn diagram. Unfortunately, however, there were many candidates who did not understand what each region of the diagram represents. For example, a very common error was thinking that $\mathrm{P}(B) = p$, rather than the correct $\mathrm{P}(B) = p + n$.

b. Candidates seemed to understand the idea of the complement in part (b), but some were not able to find the correct answer because of confusion over the separation of the different regions in the diagram.

A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble and then selects a second marble.

a. Write down the probability that the first marble Anna selects is red. [1]

b. Find the probability that Anna selects two red marbles. [2]

c. Find the probability that one marble is red and one marble is blue. [3]

Markscheme

a. **Note**: In this question, method marks may be awarded for selecting without replacement, as noted in the examples.

$\mathrm{P}(R) = \frac{6}{8}\left(= \frac{3}{4}\right)$ ***A1 N1***

[1 mark]

b. attempt to find $\mathrm{P}(\text{Red}) \times \mathrm{P}(\text{Red})$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(R)$, $\frac{3}{4} \times \frac{3}{4}$, $\frac{6}{8} \times \frac{5}{7}$

$\mathrm{P}(2R) = \frac{36}{64}\left(= \frac{9}{16}\right)$ ***A1 N2***

[2 marks]

c. **METHOD 1**

attempt to find $\mathrm{P}(\text{Red}) \times \mathrm{P}(\text{Blue})$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(B)$, $\frac{6}{8} \times \frac{2}{8}$, $\frac{6}{8} \times \frac{2}{7}$

recognizing two ways to get one red, one blue ***(M1)***

e.g. $\mathrm{P}(RB) + \mathrm{P}(BR)$, $2\left(\frac{12}{64}\right)$, $\frac{6}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{6}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64}\left(= \frac{3}{8}\right)$ ***A1 N2***

[3 marks]

METHOD 2

recognizing that $\mathrm{P}(1R, 1B)$ is $1 - \mathrm{P}(2B) - \mathrm{P}(2R)$ ***(M1)***

attempt to find $\mathrm{P}(2R)$ and $\mathrm{P}(2B)$ ***(M1)***

e.g. $\mathrm{P}(2R) = \frac{3}{4} \times \frac{3}{4}$, $\frac{6}{8} \times \frac{5}{7}$; $\mathrm{P}(2B) = \frac{1}{4} \times \frac{1}{4}$, $\frac{2}{8} \times \frac{1}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64}\left(= \frac{3}{8}\right)$ ***A1 N2***

[3 marks]

Examiners report

a. Candidates did very well on parts (a) and (b) of this probability question.

b. Candidates did very well on parts (a) and (b) of this probability question, and knew to multiply the probabilities of independent events in part (b). However, in part (c), very few candidates considered that there are two ways to draw one red and one blue marble, and therefore did not earn full marks on this question. There were also some candidates who tried to add, rather than multiply, the probabilities in parts (b) and (c).

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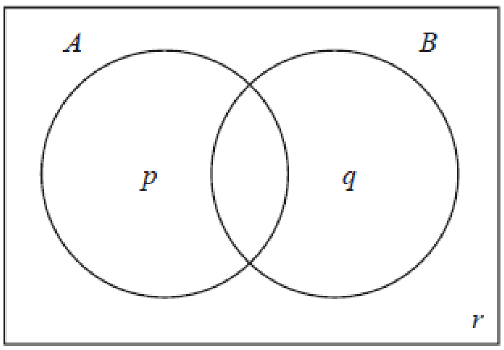
Consider the events *A* and *B*, where $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A \cap B) = 0.3$.

The Venn diagram below shows the events *A* and *B*, and the probabilities *p*, *q* and *r*.



a(i), (ii) and (iii). Write down the value of [3]

(i) p ;

(ii) q ;

(iii) r.

b. Find the value of $\mathrm{P}(A|B')$. [2]

c. Hence, or otherwise, show that the events *A* and *B* are **not** independent. [1]

Markscheme

a(i), (ii) and (iii). (i) $p = 0.2$ ***A1 N1***

(ii) $q = 0.4$ ***A1 N1***

(iii) $r = 0.1$ ***A1 N1***

[3 marks]

b. $\mathrm{P}(A|B') = \frac{2}{3}$ ***A2 N2***

Note: Award ***A1*** for an unfinished answer such as $\frac{0.2}{0.3}$.

[2 marks]

c. valid reason ***R1***

e.g. $\frac{2}{3} \neq 0.5$, $0.35 \neq 0.3$

thus, A and B are not independent ***AG*** ***N0***

[1 mark]

Examiners report

a(i), (ii) and (iii). As the definitions of p and q were not clear to candidates, both responses of $p = 0.2$, $q = 0.4$ and $p = 0.5$, $q = 0.7$ were accepted for full marks. However, finding r eluded many.

b. Few candidates answered the conditional probability correctly. Many attempted to use the formula in the booklet without considering the complement, and there was little evidence of the Venn diagram being utilized as a helpful aid.

c. To show the events are not independent, many correctly reasoned that $0.3 \neq 0.35$. A handful recognized that $\mathrm{P}(A|B') \neq \mathrm{P}(A)$ is an alternative approach that uses the answer in part (b). Some candidates do not know the difference between **independent** and **mutually exclusive**.

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

a. Find the values of k such that $f(x) = 0$ has two equal roots. [4]

b. Each value of k is equally likely for $-5 \leq k \leq 5$. Find the probability that $f(x) = 0$ has no roots. [4]

Markscheme

a. **METHOD 1**

evidence of discriminant ***(M1)***

e.g. $b^2 - 4ac$, discriminant = 0

correct substitution into discriminant ***A1***

e.g. $k^2 - 4 \times \frac{1}{2} \times 8$, $k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

recognizing that equal roots means perfect square ***(R1)***

e.g. attempt to complete the square, $\frac{1}{2}(x^2 + 2kx + 16)$

correct working

e.g. $\frac{1}{2}(x + k)^2$, $\frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

b. evidence of appropriate approach ***(M1)***

e.g. $b^2 - 4ac < 0$

correct working for k ***A1***

e.g. $-4 < k < 4$, $k^2 < 16$, list all correct values of k

$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

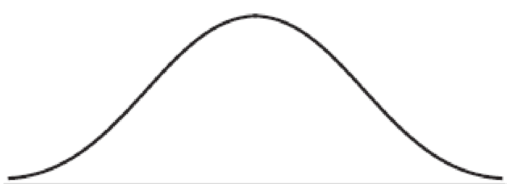
Examiners report

a. A good number of candidates were successful in using the discriminant to find the correct values of k in part (a), however, there were many who tried to use the quadratic formula without recognizing the significance of the discriminant.

b. Part (b) was very poorly done by nearly all candidates. Common errors included finding the wrong values for k, and not realizing that there were 11 possible values for k.

Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

a. On the diagram below, shade the region representing $\mathrm{P}(X > 105)$. [2]

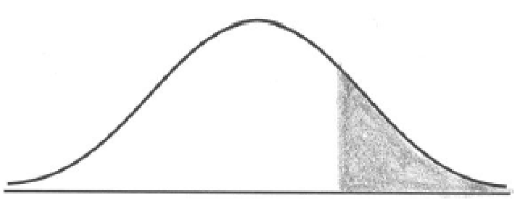


b. Given that $\mathrm{P}(X < d) = \mathrm{P}(X > 105)$, find the value of d . [2]

c. Given that $\mathrm{P}(X > 105) = 0.16$ (correct to two significant figures), find $\mathrm{P}(d < X < 105)$. [2]

Markscheme

a.



A1A1 ***N2***

Note: Award ***A1*** for vertical line to right of mean, ***A1*** for shading to right of **their** vertical line.

b. evidence of recognizing symmetry ***(M1)***

e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part, $105 - 100 = 100 - d$

$d = 95$ ***A1*** ***N2***

[2 marks]

c. evidence of using complement ***(M1)***

e.g. $1-0.32$, $1-p$

$\mathrm{P}(d<X<105)=0.68$ ***A1*** ***N2***

[2 marks]

Examiners report

a. Most candidates did very well on part (a), shading the area under the normal curve.

b. Not all candidates realised that the problem could be solved by only using the symmetry of the normal distribution curve and the information given. Some of them saw the need to use tables and others just left it blank.

Candidates were only moderately successful on parts (b) and (c), which required understanding of the symmetry of the curve. Many candidates resorted to formulae or tables instead of reasoning through the question.

c. Not all candidates realised that the problem could be solved by only using the symmetry of the normal distribution curve and the information given. Some of them saw the need to use tables and others just left it blank.

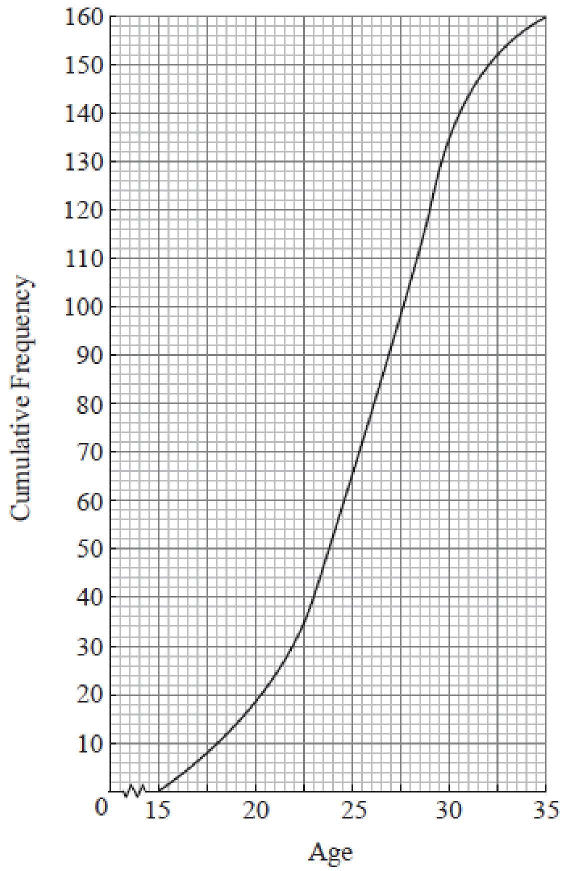
Candidates were only moderately successful on parts (b) and (c), which required understanding of the symmetry of the curve. Many candidates resorted to formulae or tables instead of reasoning through the question.

The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

a. Find p . [2]

b(i) and (ii). The cumulative frequency diagram is given below. [5]



Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range.

Markscheme

a. evidence of valid approach ***(M1)***

e.g. $92 + 52$, line on graph at $x = 31$

$p = 144$ ***A1*** ***N2***

[2 marks]

b(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. line on graph, 0.8×160 , using complement

$= 29.5$ ***A1*** ***N2***

(ii) $Q_1 = 23$; $Q_3 = 29$ ***(A1)(A1)***

$\text{IQR} = 6$ (accept any notation that suggests an interval) ***A1*** ***N3***

[5 marks]

Examiners report

a. Part (a) was generally answered correctly, with most candidates showing a good grasp of cumulative frequency from a table.

b(i) And (ii) A surprising number of candidates had difficulty reading values off the cumulative frequency curve. A common incorrect answer for (b)(i) was 29, indicating carelessness with the given scale. Too many candidates gave 40 and 120 for the quartile values.

The letters of the word PROBABILITY are written on 11 cards as shown below.

P	R	O	B	A	B	I	L	I	T	Y

Two cards are drawn at random without replacement.

Let A be the event the first card drawn is the letter A.

Let B be the event the second card drawn is the letter B.

a. Find $\mathrm{P}(A)$. [1]

b. Find $\mathrm{P}(B|A)$. [2]

c. Find $\mathrm{P}(A \cap B)$. [3]

Markscheme

a. $\mathrm{P}(A) = \frac{1}{11}$ ***A1 N1***

[1 mark]

b. $\mathrm{P}(B|A) = \frac{2}{10}$ ***A2 N2***

[2 marks]

c. recognising that $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B|A)$ ***(M1)***

correct values ***(A1)***

e.g. $\mathrm{P}(A \cap B) = \frac{1}{11} \times \frac{2}{10}$

$\mathrm{P}(A \cap B) = \frac{2}{110}$ ***A1 N3***

[3 marks]

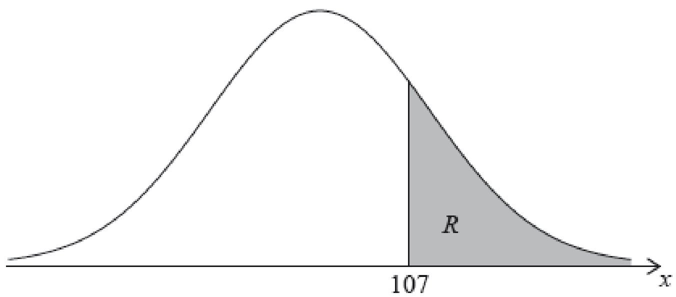
Examiners report

a. Most candidates answered part (a) correctly.

b. Few candidates used the concept of "B given A" to simply "write down" the answer of $\frac{2}{10}$. Instead, most reached for the formula in the booklet, with which few were successful.

c. Few also made the connection that part (c) could be answered using both previous answers. Many found $\mathrm{P}(A \cap B)$ correctly even when answering part (b) incorrectly, although some candidates did not decrease the denominator for the second event.

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

a. Write down $\mathrm{P}(X > 107)$. [1]

b. Find $\mathrm{P}(100 < X < 107)$. [3]

c. Find $\mathrm{P}(93 < X < 107)$. [2]

Markscheme

a. $\mathrm{P}(X > 107) = 0.24 \quad \left(= \frac{6}{25},\ 24\%\right)$ ***A1 N1***

[1 mark]

b. valid approach ***(M1)***

eg $\mathrm{P}(X > 100) = 0.5,\ \mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24,\ 0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \quad \left(= \frac{13}{50},\ 26\%\right)$ ***A1 N2***

[3 marks]

c. valid approach ***(M1)***

eg $2 \times 0.26,\ 1 - 2(0.24),\ \mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52 \quad \left(= \frac{13}{25},\ 52\%\right)$ ***A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

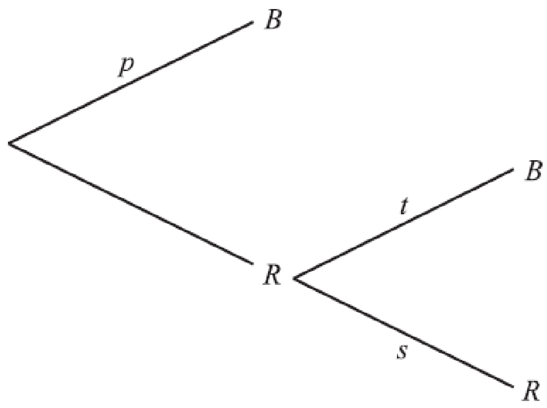
A **four-sided** die has three blue faces and one red face. The die is rolled.

Let B be the event a blue face lands down, and R be the event a red face lands down.

a. Write down [2]

(i) $\mathrm{P}(B)$;
(ii) $\mathrm{P}(R)$.

b. If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities. [2]



Find the value of p, of s and of t.

c. Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained. [3]

(i) Show that $\mathrm{P}(X=3)=\frac{3}{16}$.
(ii) Find $\mathrm{P}(X=2)$.

d. (i) Construct a probability distribution table for X. [5]

(ii) Calculate the expected value of X.

e. If the total score is 3, Guiseppi wins \$10. If the total score is 2, Guiseppi gets nothing. [4]

Guiseppi plays the game twice. Find the probability that he wins exactly \$10.

Markscheme

a. (i) $\mathrm{P}(B)=\frac{3}{4}$ ***A1 N1***

(ii) $\mathrm{P}(R)=\frac{1}{4}$ ***A1 N1***

[2 marks]

b. $p=\frac{3}{4}$ ***A1 N1***

$s=\frac{1}{4}, t=\frac{3}{4}$ ***A1 N1***

[2 marks]

c. (i) $\mathrm{P}(X=3)$

$=\mathrm{P}\text{ (getting 1 and 2)}=\frac{1}{4}\times\frac{3}{4}$ ***A1***

$=\frac{3}{16}$ ***AG N0***

(ii) $\mathrm{P}(X=2)=\frac{1}{4}\times\frac{1}{4}+\frac{3}{4}\left(\text{or } 1-\frac{3}{16}\right)$ ***(A1)***

$= \frac{13}{16}$ ***A1 N2***

[3 marks]

d. (i)

X	2	3
$\mathrm{P}(X=x)$	$\frac{13}{16}$	$\frac{3}{16}$

A2 N2

(ii) evidence of using $\mathrm{E}(X) = \sum x\mathrm{P}(X=x)$ ***(M1)***

$\mathrm{E}(X) = 2\left(\frac{13}{16}\right) + 3\left(\frac{3}{16}\right)$ ***(A1)***

$= \frac{35}{16}\ \left(= 2\frac{3}{16}\right)$ ***A1 N2***

[5 marks]

e. win \$10 $\Rightarrow$ scores 3 one time, 2 other time ***(M1)***

$\mathrm{P}(3) \times \mathrm{P}(2) = \frac{13}{16} \times \frac{3}{16}$ (seen anywhere) ***A1***

evidence of recognising there are different ways of winning \$10 ***(M1)***

e.g. $\mathrm{P}(3) \times \mathrm{P}(2) + \mathrm{P}(2) \times \mathrm{P}(3)$, $2\left(\frac{13}{16} \times \frac{3}{16}\right)$, $\frac{36}{256} + \frac{3}{256} + \frac{36}{256} + \frac{3}{256}$

$\mathrm{P}(\text{win } \$10) = \frac{78}{256}\ \left(= \frac{39}{128}\right)$ ***A1 N3***

[4 marks]

Examiners report

a. This was the most difficult of the extended response questions for the candidates. Finding *s* and *t* correctly in part (b) was difficult, with many confused between writing appropriate probabilities on a single branch compared to at the final end of a multiple branch. Many candidates had no idea what to write for a probability distribution and those who did often had probabilities that did not sum to 1. Candidates who wrote a probability distribution often could correctly compute the expected value. The final part was the most challenging, but some good answers were seen. The most common error was not recognizing that there were two different ways of winning.

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Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

a. Find the probability that [5]

(i) none of the marbles are green;

(ii) exactly one marble is green.

b. Find the expected number of green marbles drawn from the jar. [3]

c. (i) Write down the probability that the marble is drawn from jar B. [2]

(ii) Given that the marble was drawn from jar B, write down the probability that it is red.

d. Given that the marble is red, find the probability that it was drawn from jar A. [6]

Markscheme

a. (i) attempt to find $\mathrm{P(red)} \times \mathrm{P(red)}$ ***(M1)***

eg $\frac{3}{8} \times \frac{2}{7}$, $\frac{3}{8} \times \frac{3}{8}$, $\frac{3}{8} \times \frac{2}{8}$

$\mathrm{P(none\ green)} = \frac{6}{56}\ \left(= \frac{3}{28}\right)$ ***A1 N2***

(ii) attempt to find $\mathrm{P(red)} \times \mathrm{P(green)}$ ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7}$, $\frac{3}{8} \times \frac{5}{8}$, $\frac{15}{56}$

recognizing two ways to get one red, one green ***(M1)***

eg $2\mathrm{P}(R) \times \mathrm{P}(G)$, $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$, $\frac{3}{8} \times \frac{5}{8} \times 2$

$\mathrm{P(exactly\ one\ green)} = \frac{30}{56}\ \left(= \frac{15}{28}\right)$ ***A1 N2***

[5 marks]

b. $\mathrm{P(both\ green)} = \frac{20}{56}$ (seen anywhere) ***(A1)***

correct substitution into formula for $\mathrm{E}(X)$ ***A1***

eg $0 \times \frac{6}{56} + 1 \times \frac{30}{56} + 2 \times \frac{20}{56}$, $\frac{30}{64} + \frac{50}{64}$

expected number of green marbles is $\frac{70}{56}\ \left(= \frac{5}{4}\right)$ ***A1 N2***

[3 marks]

c. (i) $\mathrm{P(jar\ B)} = \frac{4}{6}\ \left(= \frac{2}{3}\right)$ ***A1 N1***

(ii) $\mathrm{P(red|\ jar\ B)} = \frac{6}{8}\ \left(= \frac{3}{4}\right)$ ***A1 N1***

[2 marks]

d. recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|R)$, $\frac{\mathrm{P(jar\ A\ and\ red)}}{\mathrm{P(red)}}$, tree diagram

attempt to multiply along either branch (may be seen on diagram) ***(M1)***

eg $\mathrm{P(jar\ A\ and\ red)} = \frac{1}{3} \times \frac{3}{8}\ \left(= \frac{1}{8}\right)$

attempt to multiply along **other** branch ***(M1)***

eg $\mathrm{P(jar\ B\ and\ red)} = \frac{2}{3} \times \frac{6}{8}\ \left(= \frac{1}{2}\right)$

adding the probabilities of two mutually exclusive paths ***(A1)***

eg $\mathrm{P(red)} = \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}$

correct substitution

eg $\mathrm{P(jar\ A|red)} = \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}}$, $\frac{\frac{1}{8}}{\frac{5}{8}}$ ***A1***

$\mathrm{P(jar\ A|red)} = \frac{1}{5}$ ***A1 N3***

[6 marks]

Examiners report

a. Many candidates correctly found the probability of selecting no green marbles in two draws, although some candidates treated the second draw as if replacing the first. When finding the probability for exactly one green marble, candidates often failed to recognize two pathways for selecting one of each color.

b. Few candidates understood the concept of expected value in this context, often leaving this blank or treating as if a binomial experiment. Successful candidates often made a distribution table before making the final calculation.

c. Most candidates answered part (c) correctly. However, many overcomplicated (c)(ii) by using the conditional probability formula. Those with a clear understanding of the concept easily followed the “write down” instruction.

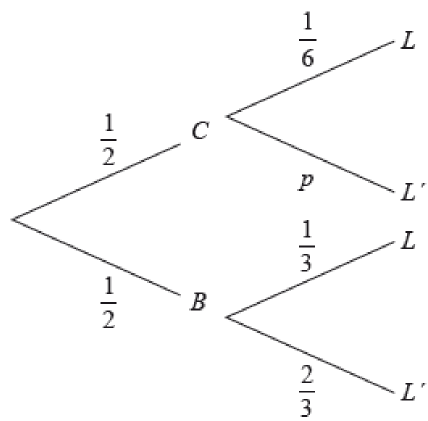
d. Only a handful of candidates correctly applied conditional probability to find $P(A|R)$ in part (d). While some wrote down the formula, or drew a tree diagram, few correctly calculated $P(red) = \frac{5}{8}$. A common error was to combine the marbles in the two jars to give $P(red) = \frac{9}{16}$.

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



a. Find the value of p. [2]

b. Find the probability that Adam will travel by car and be late for school. [2]

c. Find the probability that Adam will be late for school. [4]

d. Given that Adam is late for school, find the probability that he travelled by car. [3]

e. Adam will go to school three times next week. [4]

Find the probability that Adam will be late exactly once.

Markscheme

a. correct working ***(A1)***

eg $1 - \frac{1}{6}$

$p = \frac{5}{6}$ ***A1 N2***

[2 marks]

b. multiplying along correct branches ***(A1)***

eg $\frac{1}{2} \times \frac{1}{6}$

$\mathrm{P}(C \cap L) = \frac{1}{12}$ ***A1 N2***

[2 marks]

c. multiplying along the other branch ***(M1)*** *eg* $\frac{1}{2} \times \frac{1}{3}$

adding probabilities of their 2 mutually exclusive paths ***(M1)***

eg $\frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{3}$

correct working ***(A1)***

eg $\frac{1}{12} + \frac{1}{6}$

$\mathrm{P}(L) = \frac{3}{12} \quad \left(= \frac{1}{4}\right)$ ***A1 N3***

[4 marks]

d. recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(C|L)$

correct substitution of **their** values into formula ***(A1)***

eg $\frac{\frac{1}{12}}{\frac{3}{12}}$

$\mathrm{P}(C|L) = \frac{1}{3}$ ***A1 N2***

[3 marks]

e. valid approach ***(M1)***

eg $X \sim B\left(3, \frac{1}{4}\right), \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2, \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, three ways it could happen

correct substitution ***(A1)***

eg $\begin{pmatrix} 3 \\ 1 \end{pmatrix}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2, \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

correct working ***(A1)***

eg $3\left(\frac{1}{4}\right)\left(\frac{9}{16}\right), \frac{9}{64} + \frac{9}{64} + \frac{9}{64}$

$\frac{27}{64}$ ***A1 N2***

[4 marks]

Total [15 marks]

Examiners report

a. Parts (a) and (b) of this question were answered correctly by nearly all candidates, and the majority earned full marks on part (c), as well. Unfortunately, there were a number of candidates who made arithmetic errors when multiplying or adding fractions.

b. Parts (a) and (b) of this question were answered correctly by nearly all candidates, and the majority earned full marks on part (c), as well. Unfortunately, there were a number of candidates who made arithmetic errors when multiplying or adding fractions. Candidates were not as successful in parts (d) and (e) of this question. Although many knew that conditional probability was necessary in part (d), many did not know to use their values from parts (b) and (c), and started from scratch with brand new, and often incorrect, calculations for the numerator and denominator. A majority of candidates did not recognize that binomial probability was needed in part (e), not realizing that there were three ways for Adam to be "late exactly once". A very common incorrect solution to part (e) was $\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64}$.

c. Parts (a) and (b) of this question were answered correctly by nearly all candidates, and the majority earned full marks on part (c), as well. Unfortunately, there were a number of candidates who made arithmetic errors when multiplying or adding fractions. Candidates were not as successful in parts (d) and (e) of this question. Although many knew that conditional probability was necessary in part (d), many did not know to use their values from parts (b) and (c), and started from scratch with brand new, and often incorrect, calculations for the numerator and denominator. A majority of candidates did not recognize that binomial probability was needed in part (e), not realizing that there were three ways for Adam to be "late exactly once". A very common incorrect solution to part (e) was $\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64}$.

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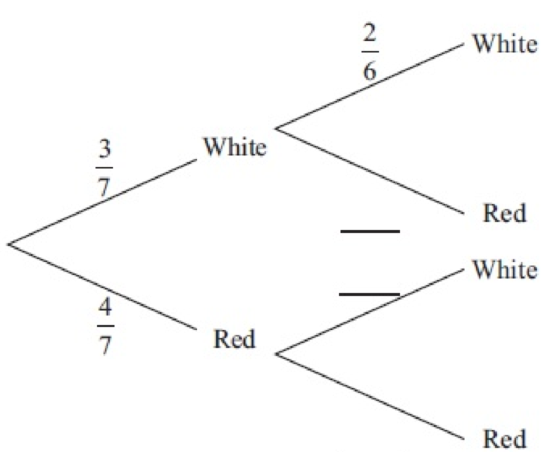
a(i) Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement. [5]

(i) Copy and complete the following tree diagram.



(ii) Find the probability that two white balls are chosen.

b. Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement. [5]

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

Find the probability that the two balls are white.

c. Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement. [4]

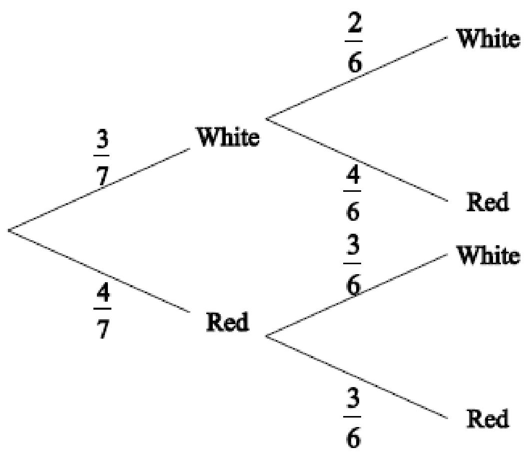
Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

Given that both balls are white, find the probability that they were chosen from bag A.

Markscheme

a(i) and (ii).



$\frac{4}{6}, \frac{3}{6} \text{and} \frac{3}{6} \left(\frac{2}{3}, \frac{1}{2} \text{and} \frac{1}{2}\right)$ ***A1A1A1 N3***

(ii) multiplying along the correct branches (may be seen on diagram) ***(A1)***

e.g. $\frac{3}{7} \times \frac{2}{6}$

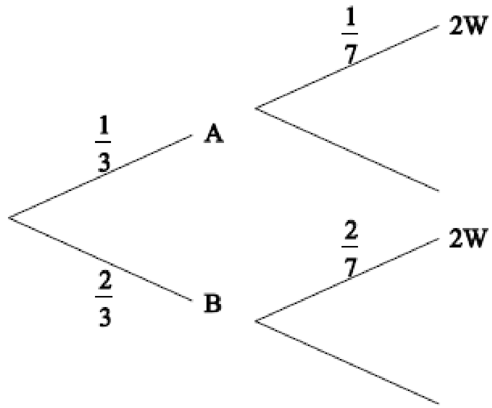
$\frac{6}{42} \left(= \frac{1}{7}\right)$ ***A1 N2***

[5 marks]

b. $\mathrm{P(bagA)} = \frac{2}{6} \left(= \frac{1}{3}\right)$, $\mathrm{P(bagB)} = \frac{4}{6} \left(= \frac{2}{3}\right)$ (seen anywhere) ***(A1)(A1)***

appropriate approach ***(M1)***

e.g. $\mathrm{P}(WW \cap A) + \mathrm{P}(WW \cap B)$



correct calculation ***A1***

e.g. $\frac{1}{3} \times \frac{1}{7} + \frac{2}{3} \times \frac{2}{7}$, $\frac{2}{42} + \frac{8}{42}$

$\mathrm{P}(2W) = \frac{60}{252} \left(= \frac{5}{21}\right)$ ***A1 N3***

[5 marks]

c. recognizing conditional probability ***(M1)***

e.g. $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A|WW) = \frac{\mathrm{P}(WW \cap A)}{\mathrm{P}(WW)}$

correct numerator ***(A1)***

e.g. $\mathrm{P}(A \cap WW) = \frac{6}{42} \times \frac{2}{6}, \frac{1}{21}$

correct denominator ***(A1)***

e.g. $\frac{6}{252}, \frac{5}{21}$

probability $\frac{84}{420} \left(= \frac{1}{5}\right)$ ***A1 N3***

[4 marks]

Examiners report

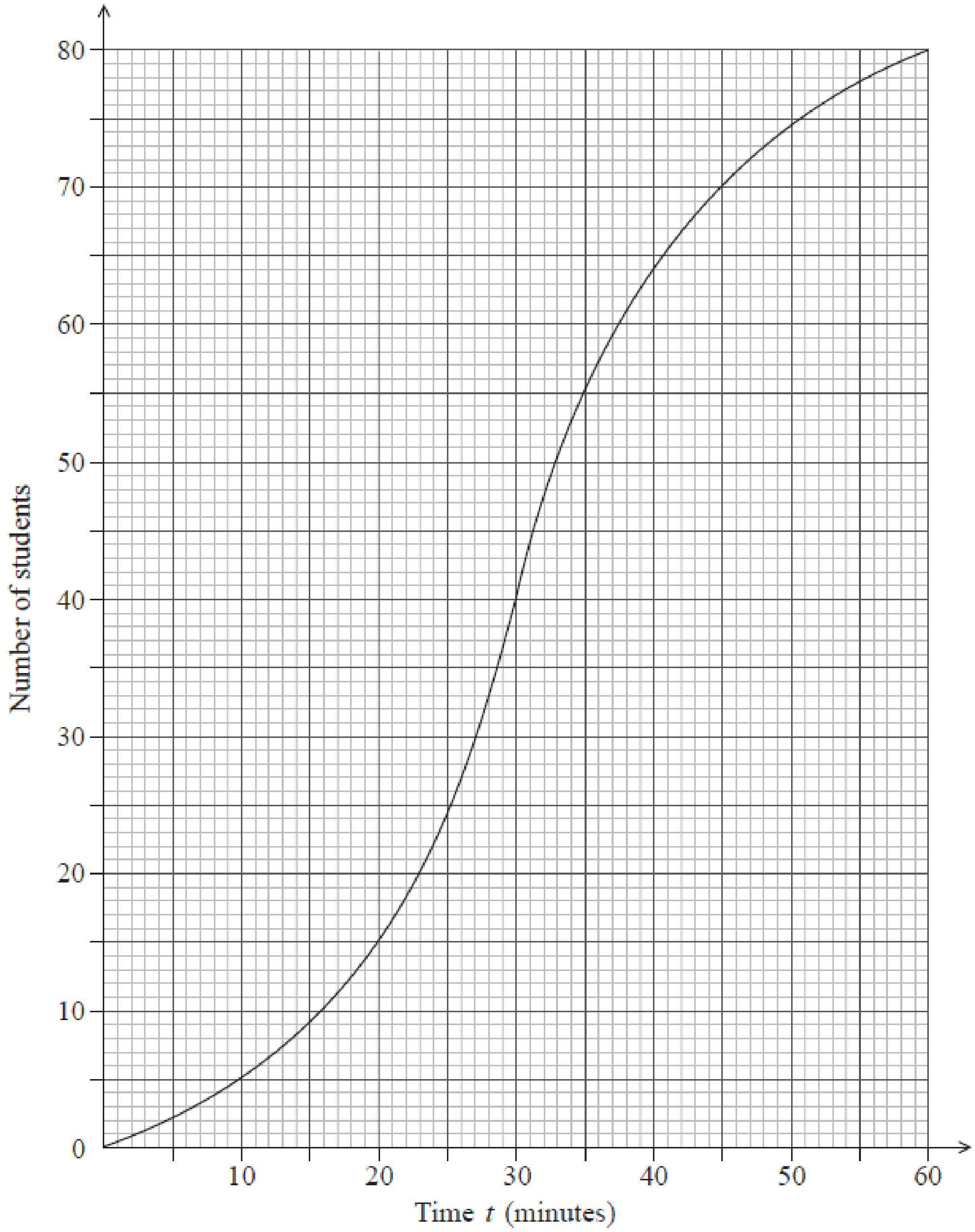
a(i) and (ii). Part (a) of this question was answered correctly by the large majority of candidates. There were some who did not follow the instruction to copy and complete the tree diagram on their separate paper, and simply filled in the blanks on the exam paper.

b. In part (b), many candidates struggled with finding the compound probability, and did not use the provided information in the appropriate manner. Quite a few candidates seemed to be confused about when they should add the probabilities or when they should multiply.

c. In part (c), many recognized that the question dealt with conditional probability, and many tried to use the formula from the information booklet, but failed to realize that they had already found the required values for the numerator and denominator in their working for part (b).

Throughout this question, it was discouraging to see the large number of candidates making arithmetic errors. There were a surprising number of candidates who multiplied fractions incorrectly, or found an incorrect value for simple multiplication such as $2 \times 4 = 6$ or $6 \times 7 = 43$.

The following is a cumulative frequency diagram for the time *t*, in minutes, taken by 80 students to complete a task.



a. Find the number of students who completed the task in less than 45 minutes. [2]

b. Find the number of students who took between 35 and 45 minutes to complete the task. [3]

c. Given that 50 students take less than k minutes to complete the task, find the value of k. [2]

Markscheme

a. attempt to find number who took less than 45 minutes ***(M1)***

eg line on graph (vertical at approx 45, or horizontal at approx 70)
70 students (accept 69) ***A1 N2***
[2 marks]

b. 55 students completed task in less than 35 minutes ***(A1)***

subtracting **their** values ***(M1)***
eg 70 – 55
15 students ***A1 N2***

[3 marks]

c. correct approach *(A1)*

eg line from y-axis on 50

$k = 33$ ***A1*** ***N2***

[2 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

Let A and B be independent events, where $\mathrm{P}(A) = 0.6$ and $\mathrm{P}(B) = x$.

a. Write down an expression for $\mathrm{P}(A \cap B)$. [1]

b(i) and (ii). Given that $\mathrm{P}(A \cup B) = 0.8$, [4]

(i) find x ;

(ii) find $\mathrm{P}(A \cap B)$.

c. **Hence**, explain why A and B are **not** mutually exclusive. [1]

Markscheme

a. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) (= 0.6x)$ ***A1*** ***N1***

[1 mark]

b(i) and (ii). (i) evidence of using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ***(M1)***

correct substitution ***A1***

e.g. $0.8 = 0.6 + x - 0.6x$, $0.2 = 0.4x$

$x = 0.5$ ***A1*** ***N2***

(ii) $\mathrm{P}(A \cap B) = 0.3$ ***A1*** ***N1***

[4 marks]

c. valid reason, with reference to $\mathrm{P}(A \cap B)$ ***R1*** ***N1***

e.g. $\mathrm{P}(A \cap B) \neq 0$

[1 mark]

Examiners report

a. This question was well done by most candidates.

b(i) and (ii). This question was well done by most candidates. When errors were made, candidates confused the terms "independent" and "mutually exclusive" and did not subtract the intersection when finding $\mathrm{P}(A \cup B)$.

c. Candidates should also be aware of the command term "hence" used in part (c) where they were expected to provide a reason that involved $\mathrm{P}(A \cap B)$ from their work in part (b). It seemed that many turned to the formula in the booklet instead of considering the conceptual meaning of the term.

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	p

a. Find p. [3]

b. Find $\mathrm{E}(X)$. [3]

Markscheme

a. summing probabilities to 1 ***(M1)***

eg, $\sum = 1,\ 3 + 4 + 2 + x = 10$

correct working ***(A1)***

$\frac{3}{10} + \frac{4}{10} + \frac{2}{10} + p = 1,\ p = 1 - \frac{9}{10}$

$p = \frac{1}{10}$ ***A1 N3***

[3 marks]

b. correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $0\left(\frac{3}{10}\right) + \ldots + 3(p)$

correct working ***(A1)***

eg $\frac{4}{10} + \frac{4}{10} + \frac{3}{10}$

$\mathrm{E}(X) = \frac{11}{10}\quad(1.1)$ ***A1 N2***

[3 marks]

Total [6 marks]

Examiners report

a. Most candidates were able to find p, however expectation emerged as surprisingly more difficult. Quite often $\mathrm{E}(X)/4$ was found or candidates wrote the formula with no further work.

b. Most candidates were able to find p, however expectation emerged as surprisingly more difficult. Quite often $\mathrm{E}(X)/4$ was found or candidates wrote the formula with no further work.

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	Tennis	Hockey
Female	5	3	3
Male	4	2	3

a(i) and (ii). One student is selected at random. [4]

(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football.

b. Two students are selected at random. Calculate the probability that neither student plays football. [3]

Markscheme

a(i) and (ii). (i) correct calculation *(A1)*

e.g. $\frac{9}{20}+\frac{5}{20}-\frac{2}{20}$, $\frac{4+2+3+3}{20}$

$\mathrm{P(male\ or\ tennis)}=\frac{12}{20}$ ***A1 N2***

(ii) correct calculation *(A1)*

e.g. $\frac{6}{20}\div\frac{11}{20}$, $\frac{3+3}{11}$

$\mathrm{P(not\ football|female)}=\frac{6}{11}$ ***A1 N2***

[4 marks]

b. $\mathrm{P(first\ not\ football)}=\frac{11}{20}$, $\mathrm{P(second\ not\ football)}=\frac{10}{19}$ ***A1***

$\mathrm{P(neither\ football)}=\frac{11}{20}\times\frac{10}{19}$ ***A1***

$\mathrm{P(neither\ football)}=\frac{110}{380}$ ***A1 N1***

[3 marks]

Examiners report

a(i) and (ii). Many candidates had difficulty with this question, usually as a result of seeking to solve the problem by formula instead of looking carefully at the table frequencies.

b. A very common error in part (b) was to assume identical probabilities for each selection instead of dependent probabilities where there is no replacement.

Let $\boldsymbol{A}=\begin{pmatrix}0 & 3\\ -2 & 4\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}-4 & 0\\ 5 & 1\end{pmatrix}$.

a. Find $\boldsymbol{AB}$. [3]

b. Given that $\boldsymbol{X} - 2\boldsymbol{A} = \boldsymbol{B}$, find $\boldsymbol{X}$. [3]

Markscheme

a. evidence of multiplying ***(M1)***

e.g. one correct element, $(0 \times -4) + (3 \times 5)$

$\boldsymbol{AB} = \begin{pmatrix} 15 & 3 \\ 28 & 4 \end{pmatrix}$ ***A2 N3***

Note: Award ***A1*** for three correct elements.

[3 marks]

b. finding $2\boldsymbol{A} = \begin{pmatrix} 0 & 6 \\ -4 & 8 \end{pmatrix}$ ***(A1)***

adding $2\boldsymbol{A}$ to both sides (may be seen first) ***(M1)***

e.g. $\boldsymbol{X} = \boldsymbol{B} + 2\boldsymbol{A}$

$\boldsymbol{X} = \begin{pmatrix} -4 & 6 \\ 1 & 9 \end{pmatrix}$ ***A1 N2***

[3 marks]

Examiners report

a. The large majority of candidates answered this question successfully. There were only a small number of candidates who seemed to have never worked with matrices before. Occasionally a candidate would incorrectly approach part (b) by finding an inverse of matrix $\boldsymbol{A}$.

b. The large majority of candidates answered this question successfully. There were only a small number of candidates who seemed to have never worked with matrices before. Occasionally a candidate would incorrectly approach part (b) by finding an inverse of matrix $\boldsymbol{A}$.

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X = x) = \frac{x^2}{14},\, x \in \{1, 2, k\}\,, \text{where} k > 0$$

.

a. Write down $\mathrm{P}(X = 2)$. [1]

b. Show that $k = 3$. [4]

c. Find $\mathrm{E}(X)$. [2]

Markscheme

a. $\mathrm{P}(X = 2) = \frac{4}{14} \left(= \frac{2}{7}\right)$ ***A1 N1***

[1 mark]

b. $\mathrm{P}(X=1)=\frac{1}{14}$ ***(A1)***

$\mathrm{P}(X=k)=\frac{k^2}{14}$ ***(A1)***

setting the sum of probabilities $=1$ ***M1***

e.g. $\frac{1}{14}+\frac{4}{14}+\frac{k^2}{14}=1$, $5+k^2=14$

$k^2=9$ (accept $\frac{k^2}{14}=\frac{9}{14}$) ***A1***

$k=3$ ***AG*** ***N0***

[4 marks]

c. correct substitution into $\mathrm{E}(X)=\sum x\mathrm{P}(X=x)$ ***A1***

e.g. $1\left(\frac{1}{14}\right)+2\left(\frac{4}{14}\right)+3\left(\frac{9}{14}\right)$

$\mathrm{E}(X)=\frac{36}{14}\left(=\frac{18}{7}\right)$ ***A1*** ***N1***

[2 marks]

Examiners report

a. Although many candidates were successful in working with the probability function, students had difficulty following the "show that" instruction of this question. Many substituted $k=3$ and worked backwards to show that the sum of probabilities is 1. Some would argue that $k=4$ does not work, but were unable to give a complete justification for $k=3$. A good number of students seemed unprepared to find an expected value. Many candidates wrote a formula and did not know what to do with it, while others divided $\mathrm{E}(X)$ by 3 or by 6, which confuses the concept of a mean in a probability distribution with the more common understanding.

b. Although many candidates were successful in working with the probability function, students had difficulty following the "show that" instruction of this question. Many substituted $k=3$ and worked backwards to show that the sum of probabilities is 1. Some would argue that $k=4$ does not work, but were unable to give a complete justification for $k=3$.

c. A good number of students seemed unprepared to find an expected value. Many candidates wrote a formula and did not know what to do with it, while others divided $\mathrm{E}(X)$ by 3 or by 6, which confuses the concept of a mean in a probability distribution with the more common understanding.

The following table shows the probability distribution of a discrete random variable X .

x	0	2	5	9
$\mathrm{P}(X=x)$	0.3	k	$2k$	0.1

a. Find the value of k . [3]

b. Find $\mathrm{E}(X)$. [3]

Markscheme

a. evidence of summing to 1 ***(M1)***

e.g. $\sum p = 1,\, 0.3 + k + 2k + 0.1 = 1$

correct working ***(A1)***

e.g. $0.4 + 3k,\, 3k = 0.6$

$k = 0.2$ ***A1*** ***N2***

[3 marks]

b. correct substitution into formula $\mathrm{E}(X)$ ***(A1)***

e.g. $0(0.3) + 2(k) + 5(2k) + 9(0.1),\, 12k + 0.9$

correct working

e.g. $0(0.3) + 2(0.2) + 5(0.4) + 9(0.1),\, 0.4 + 2.0 + 0.9$ ***(A1)***

$\mathrm{E}(X)$= 3.3 ***A1*** ***N2***

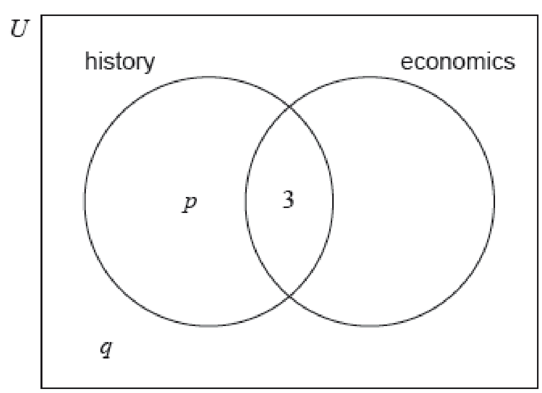
[3 marks]

Examiners report

a. Overall, this question was very well done. A few candidates left this question blank, or used methods which would indicate they were unfamiliar with discrete random variables. In part (b), there were a good number of candidates who set up their work correctly, but then had trouble adding or multiplying decimals without a calculator. A common type of error for these candidates was $5(0.4) = 0.2$.

b. Overall, this question was very well done. A few candidates left this question blank, or used methods which would indicate they were unfamiliar with discrete random variables. In part (b), there were a good number of candidates who set up their work correctly, but then had trouble adding or multiplying decimals without a calculator. A common type of error for these candidates was $5(0.4) = 0.2$.

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



a.i. Find the value of p; [2]

a.ii.Find the value of q. [2]

b. A girl is selected at random. Find the probability that she takes economics but not history. [2]

Markscheme

a.i. valid approach ***(M1)***

eg $p+3=13,\ 13-3$

$p=10$ ***A1 N2***

[2 marks]

a.ii.valid approach ***(M1)***

eg $p+3+5+q=20,\ 10-10-8$

$q=2$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $20-p-q-3,\ 1-\frac{15}{20},\ n(E\cap H')=5$

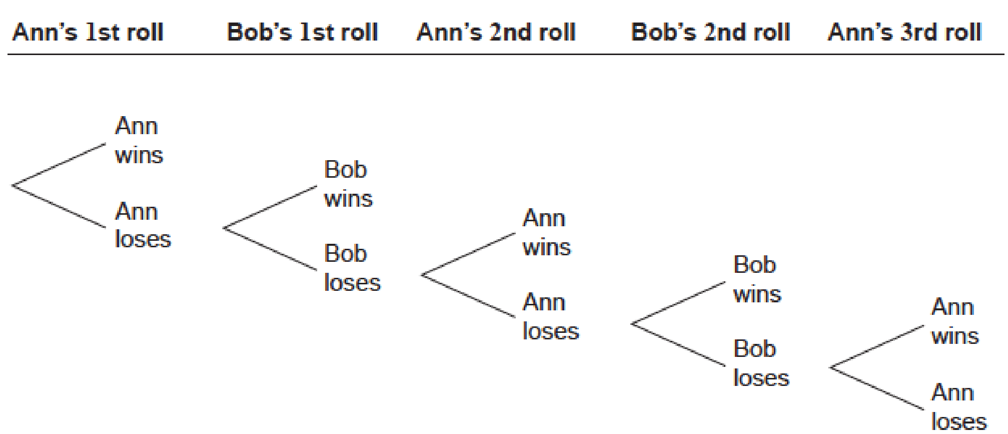
$\frac{5}{20}\ \left(\frac{1}{4}\right)$ ***A1 N2***

[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



a. Find the probability that Ann wins on her first roll. [2]

b. (i) The probability that Ann wins on her third roll is $\frac{5}{8} \times \frac{4}{8} \times p \times q \times \frac{3}{8}$. [6]

Write down the value of p and of q.

(ii) The probability that Ann wins on her tenth roll is $\frac{3}{8} r^k$ where $r \in \mathbb{Q},\ k \in \mathbb{Z}$.

Find the value of r and of k.

c. Find the probability that Ann wins the game. [7]

Markscheme

a. recognizing Ann rolls green ***(M1)***

eg $\mathrm{P(G)}$

$\frac{3}{8}$ ***A1 N2***

[2 marks]

b. (i) $p = \frac{4}{8},\ q = \frac{5}{8}$ or $q = \frac{4}{8},\ p = \frac{5}{8}$ ***A1A1 N2***

(ii) recognizes Ann and Bob lose 9 times ***(M1)***

eg\(\;\;\;\(\overbrace {{A_L}{B_\(\overbrace {{A_L}{B_ \ldots \(\overbrace {{A_L}{B_{\text{ 9 times, }}\underbrace {\left({\frac{5}{8} \times \frac{4}{8}} \right) \times \ldots \times \left({\frac{5}{8} \times \frac{4}{8}} \right)}_{{\text{9 times}}}\)

$k = 9$ (seen anywhere) ***A1 N2***

correct working ***(A1)***

eg $\left(\frac{5}{8} \times \frac{4}{8}\right)^9 \times \frac{3}{8},\ \left(\frac{5}{8} \times \frac{4}{8}\right) \times \ldots \times \left(\frac{5}{8} \times \frac{4}{8}\right) \times \frac{3}{8}$

$r = \frac{20}{64}\ \left(= \frac{5}{16}\right)$ ***A1 N2***

[6 marks]

c. recognize the probability is an infinite sum ***(M1)***

eg Ann wins on her 1^{st} roll or 2^{nd} roll or 3^{rd} roll…, S_∞

recognizing GP ***(M1)***

$u_1 = \frac{3}{8}$ (seen anywhere) ***A1***

$r = \frac{20}{64}$ (seen anywhere) ***A1***

correct substitution into infinite sum of GP ***A1***

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}},\ \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right),\ \frac{1}{1-\frac{5}{16}}$

correct working ***(A1)***

eg $\frac{\frac{3}{8}}{\frac{11}{16}},\ \frac{3}{8} \times \frac{16}{11}$

$\mathrm{P\ (Ann\ wins)} = \frac{48}{88}\ \left(= \frac{6}{11}\right)$ ***A1 N1***

[7 marks]

Total [15 marks]

Examiners report

a. Some teachers' comments suggested that the word 'loses' in the diagram was misleading. But candidate scripts did not indicate any adverse effect.

a) Very well answered.

b) i) Probabilities p and q were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

b. Some teachers' comments suggested that the word 'loses' in the diagram was misleading, But candidate scripts did not indicate any adverse effect.

a) Very well answered.

b) i) Probabilities p and q were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

c. Some teachers' comments suggested that the word 'loses' in the diagram was misleading, But candidate scripts did not indicate any adverse effect.

a) Very well answered.

b) i) Probabilities p and q were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

A data set has a mean of 20 and a standard deviation of 6.

a(i) and (ii). Each value in the data set has 10 added to it. Write down the value of [2]

(i) the new mean;

(ii) the new standard deviation.

b(i) and (ii). Each value in the original data set is multiplied by 10. [3]

(i) Write down the value of the new mean.

(ii) Find the value of the new variance.

Markscheme

a(i) and (ii). (i) new mean is $20 + 10 = 30$ ***A1 N1***

(ii) new sd is 6 ***A1 N1***

[2 marks]

b(i) and (ii). (i) new mean is $20 \times 10 = 200$ ***A1 N1***

(ii) **METHOD 1**

variance is 36 ***A1***

new variance is $36 \times 100 = 3600$ ***A1 N2***

METHOD 2

new sd is 60 ***A1***

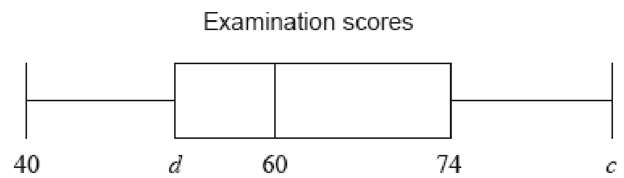
new variance is $60^2 = 3600$ ***A1 N2***

[3 marks]

Examiners report

a(i) and (ii). [N/A]
b(i) and (ii). [N/A]

The following box-and-whisker plot represents the examination scores of a group of students.



a. Write down the median score. [1]

The range of the scores is 47 marks, and the interquartile range is 22 marks.

b. Find the value of [4]

(i) c;

(ii) d.

Markscheme

a. 60 ***A1*** ***N1***

b. (i) valid approach ***(M1)***

eg $\max - \min = \text{range}, c = 40 + 47$

$c = 87$ ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg $Q3 - Q1 = IQR,\ 74 - 22$

$d = 52$ ***A1*** ***N2***

Examiners report

a. [N/A]
b. [N/A]

A data set has *n* items. The sum of the items is 800 and the mean is 20.

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

a. Find *n*. [2]

b.i. Write down the value of the new mean. [1]

b.ii. Find the value of the new variance. [3]

Markscheme

a. correct approach ***(A1)***

eg $\frac{800}{n} = 20$

40 ***A1 N2***

[2 marks]

b.i. 200 ***A1 N1***

[1 mark]

b.ii. **METHOD 1**

recognizing variance = σ^2 ***(M1)***

eg $3^2 = 9$

correct working to find new variance ***(A1)***

eg $\sigma^2 \times 10^2$, 9×100

900 ***A1 N3***

METHOD 2

new standard deviation is 30 ***(A1)***

recognizing variance = σ^2 ***(M1)***

eg $3^2 = 9$, 30^2

900 ***A1 N3***

[3 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]

There are 10 items in a data set. The sum of the items is 60.

The variance of this data set is 3. Each value in the set is multiplied by 4.

a. Find the mean. [2]

b. (i) Write down the value of the new mean. [3]

(ii) Find the value of the new variance.

Markscheme

a. correct approach *(A1)*

eg $\frac{60}{10}$

$\text{mean} = 6$ ***A1 N2***

b. (i) new mean $= 24$ ***A1 N1***

(ii) valid approach *(M1)*

eg $\text{variance} \times (4)^2,\ 3 \times 16$, new standard deviation $= 4\sqrt{3}$

new variance $= 48$ ***A1 N2***

[3 marks]

Examiners report

a. While most candidates were able to answer part (a) of this question correctly, they were not as successful in part (b). It seems that the item on the Maths SL syllabus dealing with the "effect of constant changes to the original data" was skipped over in many schools.

b. While most candidates were able to answer part (a) of this question correctly, they were not as successful in part (b). It seems that the item on the Maths SL syllabus dealing with the "effect of constant changes to the original data" was skipped over in many schools.

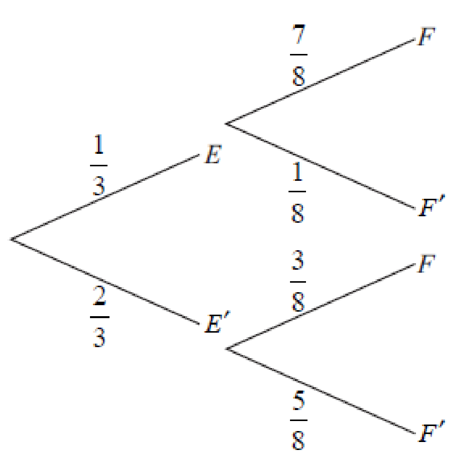
José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event "he misses his bus" and F the event "he is late for school".

The information above is shown on the following tree diagram.



a(i) and (ii). Find [4]

(i) $\mathrm{P}(E \cap F)$;

(ii) $\mathrm{P}(F)$.

b(i) and (ii). Find the probability that [5]

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

c. The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday. [3]

Copy and complete the probability distribution table.

X (cost in euros)	0	3	6
P(X)	$\frac{1}{9}$		

d. The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday. [2]

Find the expected cost for José for both days.

Markscheme

a(i) and (ii). (i) $\frac{7}{24}$ ***A1 N1***

(ii) evidence of **multiplying** along the branches ***(M1)***

e.g. $\frac{2}{3} \times \frac{5}{8}$, $\frac{1}{3} \times \frac{7}{8}$

adding probabilities of two mutually exclusive paths ***(M1)***

e.g. $\left(\frac{1}{3} \times \frac{7}{8}\right) + \left(\frac{2}{3} \times \frac{3}{8}\right)$, $\left(\frac{1}{3} \times \frac{1}{8}\right) + \left(\frac{2}{3} \times \frac{5}{8}\right)$

$\mathrm{P}(F) = \frac{13}{24}$ ***A1 N2***

[4 marks]

b(i) and (ii). (i) $\frac{1}{3} \times \frac{1}{8}$ ***(A1)***

$\frac{1}{24}$ ***A1***

(ii) recognizing this is $\mathrm{P}(E|F)$ ***(M1)***

e.g. $\frac{7}{24} \div \frac{13}{24}$

$\frac{168}{312} \left(= \frac{7}{13}\right)$ ***A2 N3***

[5 marks]

c.

X (cost in euros)	0	3	6
P(X)	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

A2A1 N3

[3 marks]

d. correct substitution into $\mathrm{E}(X)$ formula ***(M1)***

e.g. $0 \times \frac{1}{9} + 3 \times \frac{4}{9} + 6 \times \frac{4}{9}$, $\frac{12}{9} + \frac{24}{9}$

$\mathrm{E}(X) = 4$ (euros) ***A1 N2***

[2 marks]

Examiners report

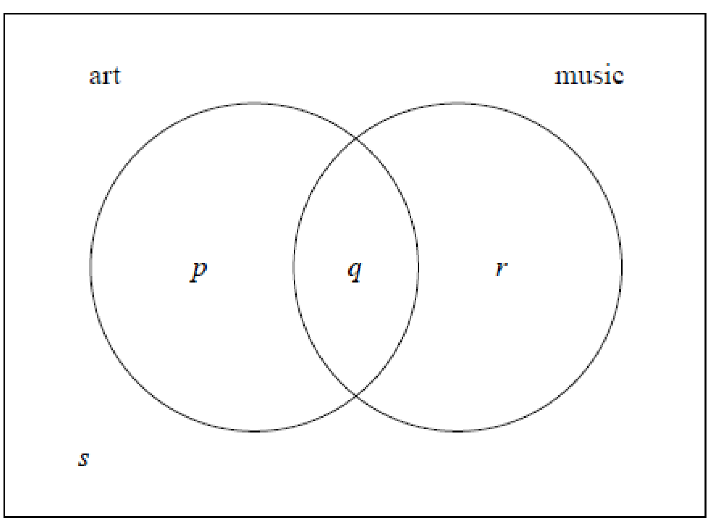
a(i) and (ii). Candidates generally handled some or all of parts (a) and (b) well. Errors included adding probabilities along branches and trying to use the union formula from the information booklet.

b(i) and (ii). Candidates generally handled some or all of parts (a) and (b) well. Errors included adding probabilities along branches and trying to use the union formula from the information booklet. On part (b)(ii), many candidates knew that they were supposed to use some type of conditional probability but did not know how to find $\mathrm{P}(E|F)$. Many candidates made errors working with fractions. Some candidates who missed part (a)(ii) were able to earn follow-through credit on part (b)(ii).

c. Many candidates had difficulty completing the probability distribution table. While the common error of finding the probability for $x = 3$ as $\frac{2}{9}$ was understandable as the candidate did not appreciate that there were two ways of paying three euros, it was disappointing that these candidates often correctly found $\mathrm{P}(X = 4)$ as $\frac{4}{9}$ and did not note that the probabilities failed to sum to one. These candidates could not earn full follow-through marks on their expected value calculation in part (d). Some candidates did use the probabilities summing to one with incorrect probabilities in part (c); these candidates often earned full follow-through marks in part (d), as a majority of candidates knew the method for finding expected value.

d. Many candidates had difficulty completing the probability distribution table. While the common error of finding the probability for $x = 3$ as $\frac{2}{9}$ was understandable as the candidate did not appreciate that there were two ways of paying three Euros, it was disappointing that these candidates often correctly found $\mathrm{P}(X) = 4$ as $\frac{4}{9}$ and did not note that the probabilities failed to sum to one. These candidates could not earn full follow-through marks on their expected value calculation in part (d). Some candidates did use the probabilities summing to one with incorrect probabilities in part (c); these candidates often earned full follow-through marks in part (d), as a majority of candidates knew the method for finding expected value.

In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p , q , r and s represent numbers of students.



a(i), (ii) and (iii). (i) Write down the value of s . [5]

(ii) Find the value of q .

(iii) Write down the value of p and of r .

b(i) and (ii). (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art. [4]

(ii) **Hence**, show that taking music and taking art are **not** independent events.

c. Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art. [4]

Markscheme

a(i), (ii) and (iii). (i) $s = 1$ ***A1 N1***

(ii) evidence of appropriate approach ***(M1)***

e.g. $21 - 16$, $12 + 8 - q = 15$

$q = 5$ ***A1 N2***

(iii) $p = 7$, $r = 3$ ***A1A1 N2***

[5 marks]

b(i) and (ii). (i) $\mathrm{P}(\mathrm{art}|\mathrm{music}) = \frac{5}{8}$ ***A2 N2***

(ii) **METHOD 1**

$\mathrm{P}(\mathrm{art}) = \frac{12}{16}\left(=\frac{3}{4}\right)$ ***A1***

evidence of correct reasoning ***R1***

e.g. $\frac{3}{4} \neq \frac{5}{8}$

the events are not independent ***AG N0***

METHOD 2

$\mathrm{P}(\mathrm{art}) \times \mathrm{P}(\mathrm{music}) = \frac{96}{256}\left(=\frac{3}{8}\right)$ ***A1***

evidence of correct reasoning ***R1***

e.g. $\frac{12}{16} \times \frac{8}{16} \neq \frac{5}{16}$

the events are not independent ***AG N0***

[4 marks]

c. $\mathrm{P}(\text{first takes only music}) = \frac{3}{16}$ (seen anywhere) ***A1***

$\mathrm{P}(\text{second takes only art}) = \frac{7}{15}$ (seen anywhere) ***A1***

evidence of valid approach ***(M1)***

e.g. $\frac{3}{16} \times \frac{7}{15}$

$\mathrm{P}(\text{music and art}) = \frac{21}{240}\left(=\frac{7}{80}\right)$ ***A1 N2***

[4 marks]

Examiners report

a(i), (ii) and (iii). A majority of candidates found the values in the Venn diagram easily. Common errors include giving $s = 16$, and also neglecting s in finding $q = 4$ (e.g. $12 + 8 - 16$) . Some interpreted the values as probabilities, despite the question explicitly stating that p, q, r and s represent numbers of students. Occasionally the values for p and r were misinterpreted as being inclusive of q. Follow-through marks were often earned in subsequent parts for such cases.

b(i) and (ii). For (b), rather than think of the situations conceptually, most candidates reached for the formula for conditional probability, with mixed results. Few candidates considered that independence means $\mathrm{P}(A|M) = \mathrm{P}(A)$. Most applied $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, with many giving incomplete or incorrect calculations. Some candidates compared the wrong things and showed, for example, that $\frac{5}{8} \neq \frac{3}{8}$, which incorrectly compares $\mathrm{P}(A|M)$ with $\mathrm{P}(A \cap M)$. Others stated that because there is an intersection, the events are independent, which is an insufficient explanation.

c. Part (c) was commonly answered as if there is replacement, with many candidates calculating $\frac{3}{16} \times \frac{7}{16}$. However, implicit in the phrasing "one after the other" is that there is no replacement.

A running club organizes a race to select girls to represent the club in a competition.

The times taken by the group of girls to complete the race are shown in the table below.

Time t minutes	$10 \le t < 12$	$12 \le t < 14$	$14 \le t < 20$	$20 \le t < 26$	$26 \le t < 28$	$28 \le t < 30$
Frequency	50	20	p	40	20	20
Cumulative Frequency	50	70	120	q	180	200

a. Find the value of p and of q . [4]

b. A girl is chosen at random. [3]

(i) Find the probability that the time she takes is less than 14 minutes.

(ii) Find the probability that the time she takes is at least 26 minutes.

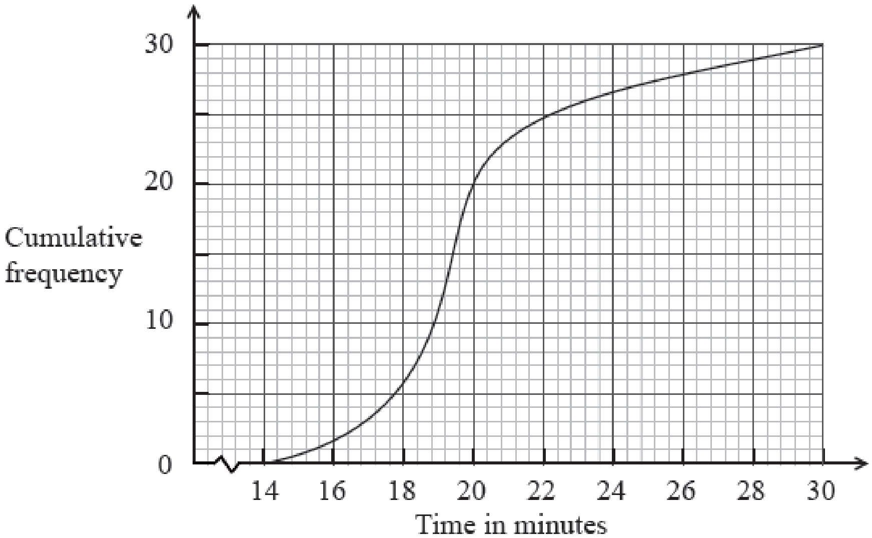
c. A girl is selected for the competition if she takes less than x minutes to complete the race. [4]

Given that 40% of the girls are not selected,

(i) find the number of girls who are not selected;

(ii) find x .

d. Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below. [4]



(i) Write down the number of girls who were allowed another chance.

(ii) Find the percentage of the whole group who were selected.

Markscheme

a. attempt to find p ***(M1)***

eg $120 - 70$, $50 + 20 + x = 120$

$p = 50$ ***A1 N2***

attempt to find q ***(M1)***

eg $180 - 20$, $200 - 20 - 20$

$q = 160$ ***A1 N2***

[4 marks]

b. (i) $\frac{70}{200}\left(=\frac{7}{20}\right)$ ***A1 N1***

(ii) valid approach ***(M1)***

eg $20 + 20$, $200 - 160$

$\frac{40}{200}\left(=\frac{1}{5}\right)$ ***A1 N2***

[3 marks]

c. (i) attempt to find number of girls ***(M1)***

eg 0.4, $\frac{40}{100} \times 200$

80 are not selected ***A1 N2***

(ii) 120 are selected ***(A1)***

$x = 20$ ***A1 N2***

[4 marks]

d. (i) 30 given second chance ***A1 N1***

(ii) 20 took less than 20 minutes *(A1)*

attempt to find **their** selected total (may be seen in % calculation) *(M1)*

eg $120 + 20\ (= 140)$, 120+ **their** answer from (i)

70 (%) ***A1*** ***N3***

[4 marks]

Examiners report

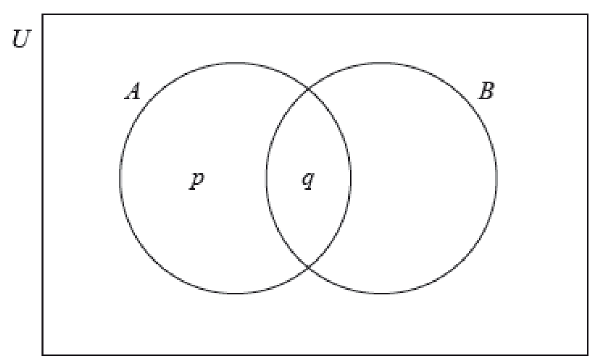
a. Overall, candidates were very successful in parts (a), (b) and (c) of this question. Most of the errors in these parts had to do with candidates not understanding terms such as "at least" or "less than".

b. Overall, candidates were very successful in parts (a), (b) and (c) of this question. Most of the errors in these parts had to do with candidates not understanding terms such as "at least" or "less than".

c. Overall, candidates were very successful in parts (a), (b) and (c) of this question. Most of the errors in these parts had to do with candidates not understanding terms such as "at least" or "less than".

d. Part (d) was quite challenging for candidates, who may not have read the question carefully and studied the values in the diagram. Many seemed confused by the idea that not all the girls who were given a second chance were selected. In part (d)(ii), many did not find the percentage of the whole group, but rather the percentage of the girls who were given a second chance.

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.4,\ \mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



a. (i) Write down the value of q. [3]

(ii) Find the value of p.

b. Find $\mathrm{P}(B)$. [3]

Markscheme

a. (i) $q = 0.1$ ***A1*** ***N1***

(ii) appropriate approach ***(M1)***

eg $\mathrm{P}(A) - q,\ 0.4 - 0.1$

$p = 0.3$ ***A1*** ***N2***

[3 marks]

b. valid approach ***(M1)***

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values ***(A1)***

eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1,\ 0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ ***A1*** ***N2***

[3 marks]

Examiners report

a. This question was well done by most candidates. In part (b), the intersection $\mathrm{P}(A \cap B)$ was sometimes overlooked, incorrectly using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ instead.

b. This question was well done by most candidates. In part (b), the intersection $\mathrm{P}(A \cap B)$ was sometimes overlooked, incorrectly using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ instead.

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

a. Find $\mathrm{P}(B)$. [2]

b. Find $\mathrm{P}(A \cup B)$. [4]

Markscheme

a. valid interpretation (may be seen on a Venn diagram) ***(M1)***

eg $\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B),\ 0.2 + 0.6$

$\mathrm{P}(B) = 0.8$ ***A1*** ***N2***

[2 marks]

b. valid attempt to find $\mathrm{P}(A)$ ***(M1)***

eg $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B),\ 0.8 \times A = 0.2$

correct working for $\mathrm{P}(A)$ ***(A1)***

eg $0.25,\ \frac{0.2}{0.8}$

correct working for $\mathrm{P}(A \cup B)$ ***(A1)***

eg $0.25 + 0.8 - 0.2,\ 0.6 + 0.2 + 0.05$

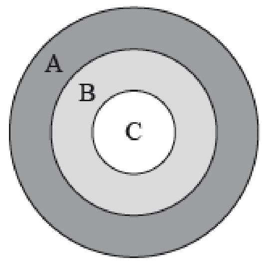
$\mathrm{P}(A \cup B) = 0.85$ ***A1*** ***N3***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

The following diagram shows a board which is divided into three regions A, B and C.



A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

a. Find the probability that the dart does **not** hit the board. [3]

b. The contestant scores points as shown in the following table. [4]

Region	A	B	C	Does not hit the board
Points	0	q	10	-3

Given that the game is fair, find the value of q.

Markscheme

a. evidence of summing probabilities to 1 ***(M1)***

eg $\frac{5}{20} + \frac{4}{20} + \frac{1}{20} + p = 1, \quad \sum = 1$

correct working ***(A1)***

eg $p = 1 - \frac{10}{20}$

$p = \frac{10}{20} \quad \left(= \frac{1}{2}\right)$ ***A1*** ***N2***

[3 marks]

b. correct substitution into $\mathrm{E}(X)$ ***(A1)***

eg $\frac{4}{20}(q) + \frac{1}{20}(10) + \frac{10}{20}(-3)$

valid reasoning for fair game (seen anywhere, including equation) ***(M1)***

eg $\mathrm{E}(X) = 0$, points lost=points gained correct working ***(A1)***

eg $4q+10-30=0, \quad \frac{4}{20}q+\frac{10}{20}=\frac{30}{20}$

$q=5$ ***A1*** ***N2***

[4 marks]

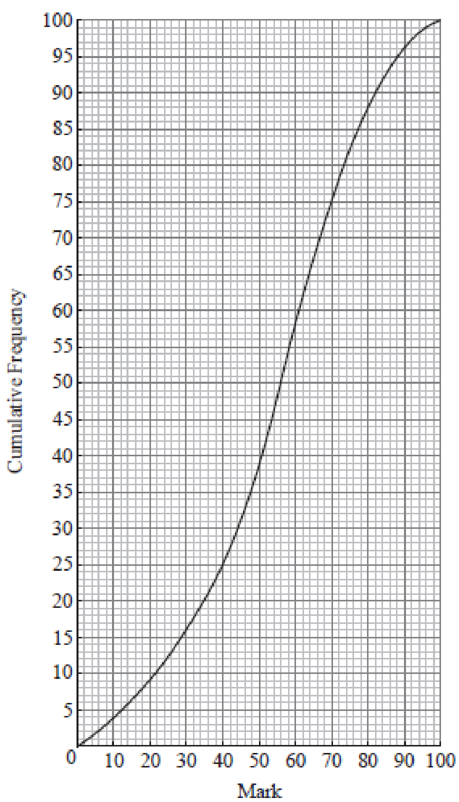
Total [7 marks]

Examiners report

a. The large majority of candidates answered part (a) of the question correctly by summing the probabilities to 1.

b. Part (b), however was not as well done. Many candidates seemed to be unfamiliar with the idea of a "fair game", despite this topic being listed in the syllabus. The most common error in part (b) was setting $E(X)=1$ rather than $E(X)=0$.

The cumulative frequency curve below represents the marks obtained by 100 students.



a. Find the median mark. [2]

b. Find the interquartile range. [3]

Markscheme

a. evidence of median position ***(M1)***

e.g. 50, line on sketch

median is 56 ***A1*** ***N2***

[2 marks]

b. lower quartile $= 40$, upper quartile $= 70$ ***(A1)(A1)***

interquartile range $= 30$ ***A1*** ***N3***

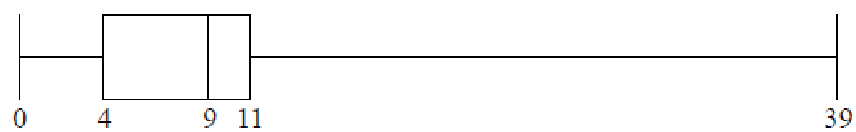
[3 marks]

Examiners report

a. Overall, this question was done well by candidates. In part (a), a surprising number of candidates found the median position (the cumulative frequency) on the y-axis, but did not find the median mark on the x-axis.

b. Overall, this question was done well by candidates. In part (a), a surprising number of candidates found the median position (the cumulative frequency) on the y-axis, but did not find the median mark on the x-axis. Similar misunderstanding was shown by some candidates in part (b), when attempting to find the interquartile range.

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.



a. Find the value of the interquartile range. [2]

b. One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k. [4]

Markscheme

a. recognizing Q_1 or Q_3 (seen anywhere) ***(M1)***

eg 4,11 , indicated on diagram

IQR = 7 ***A1 N2***

[2 marks]

b. recognizing the need to find 1.5 IQR ***(M1)***

eg 1.5 × IQR, 1.5 × 7

valid approach to find k ***(M1)***

eg 10.5 + 11, 1.5 × IQR + Q_3

21.5 ***(A1)***

$k = 22$ ***A1 N3***

Note: If no working shown, award ***N2*** for an answer of 21.5.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

The random variable X has the following probability distribution.

x	1	2	3
$\mathrm{P}(X=x)$	s	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find q .

Markscheme

correct substitution into $\mathrm{E}(X) = \sum px$ (seen anywhere) ***A1***

e.g. $1s + 2 \times 0.3 + 3q = 1.7$, $s + 3q = 1.1$

recognizing $\sum p = 1$ (seen anywhere) ***(M1)***

correct substitution into $\sum p = 1$ ***A1***

e.g. $s + 0.3 + q = 1$

attempt to solve simultaneous equations ***(M1)***

correct working ***(A1)***

e.g. $0.3 + 2q = 0.7$, $2s = 1$

$q = 0.2$ ***A1*** ***N4***

[6 marks]

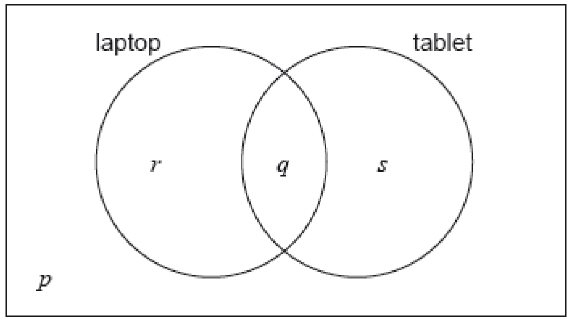
Examiners report

Candidates generally earned either full marks or only one mark on this question. The most common error was where candidates only wrote the equation for $\mathrm{E}(X) = 1.7$, and tried to rearrange that equation to solve for q. The candidates who also knew that the sum of the probabilities must be equal to 1 were very successful in solving the resulting system of equations.

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.

The following Venn diagram shows the events “own a laptop” and “own a tablet”.

The values p, q, r and s represent numbers of students.



A student is selected at random from the class.

Two students are randomly selected from the class. Let L be the event a “student owns a laptop”.

a. (i) Write down the value of p. [5]

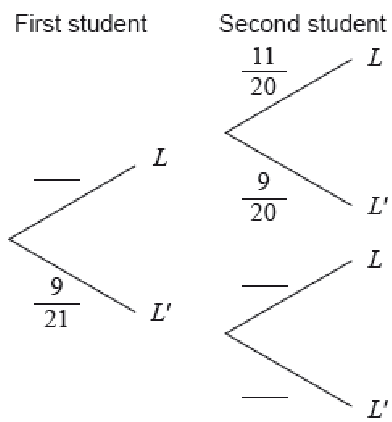
(ii) Find the value of q.

(iii) Write down the value of r and of s.

b. (i) Write down the probability that this student owns a laptop. [4]

(ii) Find the probability that this student owns a laptop or a tablet but not both.

c. (i) **Copy** and complete the following tree diagram. (Do **not** write on this page.) [4]



(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop.

Markscheme

a. (i) $p = 3$ ***A1 N1***

(ii) valid approach ***(M1)***

eg $(12 + 10 + 3) - 21,\ 22 - 18$

$q = 4$ ***A1 N2***

(iii) $r = 8,\ s = 6$ ***A1A1 N2***

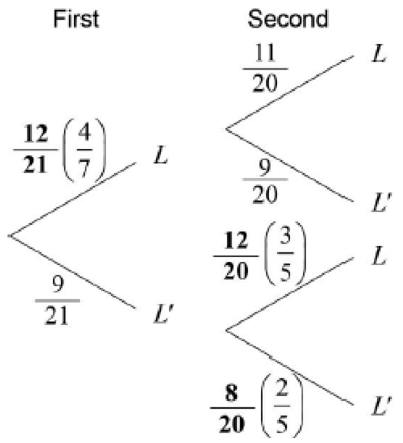
b. (i) $\frac{12}{21}\ \left(= \frac{4}{7}\right)$ ***A2 N2***

(ii) valid approach ***(M1)***

eg $8 + 6,\ r + s$

$\frac{14}{21}\ \left(= \frac{2}{3}\right)$ ***A1 N2*** c.

(i)



A1A1A1 N3

(ii) $\frac{11}{20}$ ***A1 N1***

[4 marks]

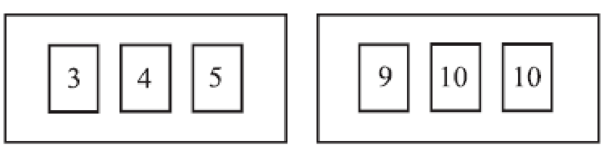
Examiners report

a. On the whole, candidates were very successful on this question, with the majority of candidates earning most of the available marks.

b. On the whole, candidates were very successful on this question, with the majority of candidates earning most of the available marks.

c. On the whole, candidates were very successful on this question, with the majority of candidates earning most of the available marks. The most common error was seen in part (c)(ii), where many candidates did not earn the mark. It is also interesting to note that many of the candidates who answered this part correctly did so by using the formula for conditional probability, rather than recognizing that the required probability is given to them in the second branch of the tree diagram.

Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

a. Copy and complete the table below to show all nine equally likely outcomes. [2]

3 , 9		
3 , 10		
3 , 10		

b. Let S be the sum of the numbers on the two cards. [2]

Find the probability of each value of S.

c. Find the expected value of S. [3]

d. Anna plays a game where she wins \$50 if S is even and loses \$30 if S is odd. [3]

Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.

Markscheme

a.

3, 9	4, 9	5, 9
3, 10	4, 10	5, 10
3, 10	4, 10	5, 10

A2 N2

[2 marks]

b. $\mathrm{P}(12) = \frac{1}{9}$, $\mathrm{P}(13) = \frac{3}{9}$, $\mathrm{P}(14) = \frac{3}{9}$, $\mathrm{P}(15) = \frac{2}{9}$ ***A2 N2***

[2 marks]

c. correct substitution into formula for $\mathrm{E}(X)$ ***A1***

e.g. $\mathrm{E}(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}$

$\mathrm{E}(S) = \frac{123}{9}$ ***A2 N2***

[3 marks]

d. **METHOD 1**

correct expression for expected gain $\mathrm{E}(A)$ for 1 game ***(A1)***

e.g. $\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$\mathrm{E}(A) = \frac{50}{9}$

amount at end = expected gain for 1 game $\times 36$ ***(M1)***

= 200 (dollars) ***A1 N2***

METHOD 2

attempt to find expected number of wins and losses ***(M1)***

e.g. $\frac{4}{9} \times 36$, $\frac{5}{9} \times 36$

attempt to find expected gain $\mathrm{E}(G)$ ***(M1)***

e.g. $16 \times 50 - 30 \times 20$

$\mathrm{E}(G) = 200$ (dollars) ***A1 N2***

[3 marks]

Examiners report

a. Most candidates completed parts (a), (b) and (c) successfully.

b. Most candidates completed part (b) successfully.

c. Many found the expected value correctly, while some showed difficulty with the arithmetic.

d. This was often left blank or only superficially attempted. Some found the expected value $\frac{50}{9}$ but did not answer the question about the amount of money.

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

a.i. Write down the independent variable. [1]

a.ii. Write down the boiling temperature of the liquid. [1]

b. Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient. [2]

$$0.992 \qquad 0.251 \qquad 0 \qquad -0.251 \qquad -0.992$$

c. Jim's model is $d = -2.24t + 105$, for $0 \leqslant t \leqslant 20$. Use his model to predict the decrease in temperature for any 2 minute interval. [2]

Markscheme

a.i. t ***A1 N1***

[1 mark]

a.ii. 105 ***A1 N1***

[1 mark]

b. -0.992 ***A2 N2***

[2 marks]

c. valid approach ***(M1)***

eg $\frac{\mathrm{d}d}{\mathrm{d}t} = -2.24;\ 2 \times 2.24,\ 2 \times -2.24,\ d(2) = -2 \times 2.24 \times 105,$

finding $d(t_2) - d(t_1)$ where $t_2 = t_1 + 2$

4.48 (degrees) ***A1 N2***

Notes: Award no marks for answers that **directly** use the table to find the decrease in temperature for 2 minutes *eg* $\frac{105-98.4}{2} = 3.3$.

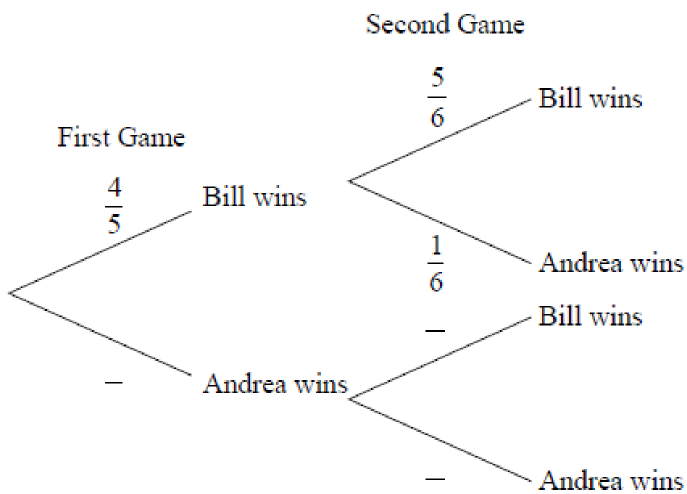
[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.
If Bill wins the first game, the probability that he wins the second game is $\frac{5}{6}$.
If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

a. **Copy** and complete the following tree diagram. *(Do **not** write on this page.)* [3]



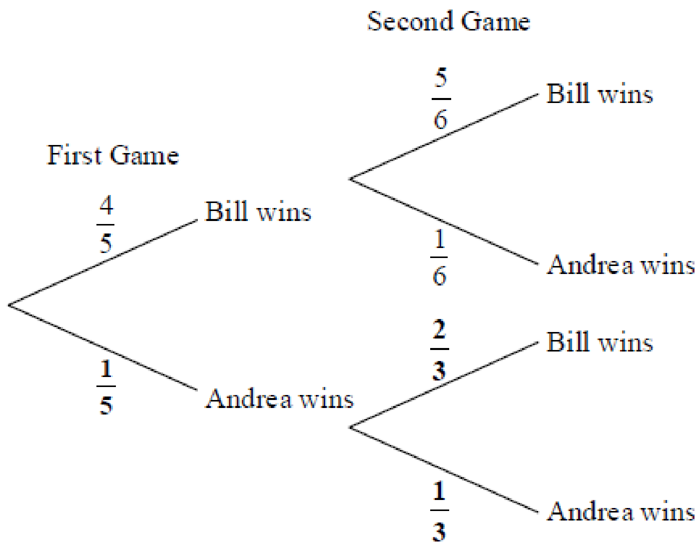
b. Find the probability that Bill wins the first game and Andrea wins the second game. [2]

c. Find the probability that Bill wins at least one game. [4]

d. Given that Bill wins at least one game, find the probability that he wins both games. [5]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** probability.

[3 marks]

b. multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5} \times \frac{1}{6}$

$\frac{4}{30} \left(\frac{2}{15}\right)$ ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5} \times \frac{5}{6},\ \frac{4}{5} \times \frac{1}{6},\ \frac{1}{5} \times \frac{2}{3}$

adding their probabilities of three mutually exclusive paths ***(M1)***

eg $\frac{4}{5} \times \frac{5}{6} + \frac{4}{5} \times \frac{1}{6} + \frac{1}{5} \times \frac{2}{3},\ \frac{4}{5} + \frac{1}{5} \times \frac{2}{3}$

correct simplification ***(A1)***

eg $\frac{20}{30} + \frac{4}{30} + \frac{2}{15},\ \frac{2}{3} + \frac{2}{15} + \frac{2}{15}$

$\frac{28}{30} \left(= \frac{14}{15}\right)$ ***A1*** ***N3***

METHOD 2

recognizing "Bill wins at least one" is complement of "Andrea wins 2" ***(R1)***

eg finding P (Andrea wins 2)

P (Andrea wins both) $= \frac{1}{5} \times \frac{1}{3}$ ***(A1)***

evidence of complement ***(M1)***

eg $1 - p,\ 1 - \frac{1}{15}$

$\frac{14}{15}$ ***A1*** ***N3***

[4 marks]

d. P (B wins both) $\frac{4}{5} \times \frac{5}{6} \left(= \frac{2}{3}\right)$ ***A1***

evidence of recognizing conditional probability ***(R1)***

eg $\mathrm{P}(A\,|B)$, P (Bill wins both |Bill wins at least one), tree diagram

correct substitution ***(A2)***

eg $\frac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$

$\frac{20}{28} \left(= \frac{5}{7}\right)$ ***A1*** ***N3***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

score on second die

score on first die	1	2	3	4	5	6
1	•	•	•	•	•	•
2	•	•	•	•	•	•
3	•	•	•	•	•	•
4	•	•	•	•	•	•
5	•	•	•	•	•	•
6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

a(i), (ii) and (iii). (i) Find $\mathrm{P}(X = 6)$. [6]

(ii) Find $\mathrm{P}(X > 6)$.

(iii) Find $\mathrm{P}(X = 7 | X > 6)$.

b. Elena plays a game where she tosses two dice. [8]

If the sum is 6, she **wins** 3 points.

If the sum is greater than 6, she **wins** 1 point.

If the sum is less than 6, she **loses** *k* points.

Find the value of *k* for which the game is fair.

Markscheme

a(i), (ii) and (iii). (i) number of ways of getting $X = 6$ is 5 ***A1***

$\mathrm{P}(X = 6) = \frac{5}{36}$ ***A1*** ***N2***

(ii) number of ways of getting $X > 6$ is 21 ***A1***

$\mathrm{P}(X > 6) = \frac{21}{36}\left(= \frac{7}{12}\right)$ ***A1*** ***N2***

(iii) $\mathrm{P}(X = 7 | X > 6) = \frac{6}{21}\left(= \frac{2}{7}\right)$ ***A2*** ***N2***

[6 marks]

b. attempt to find $\mathrm{P}(X < 6)$ ***M1***

e.g. $1 - \frac{5}{36} - \frac{21}{36}$

$\mathrm{P}(X < 6) = \frac{10}{36}$ ***A1***

fair game if $\mathrm{E}(W) = 0$ (may be seen anywhere) ***R1***

attempt to substitute into $\mathrm{E}(X)$ formula ***M1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right)$

correct substitution into $\mathrm{E}(W) = 0$ ***A1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$

work towards solving ***M1***

e.g. $15 + 21 - 10k = 0$

$36 = 10k$ ***A1***

$k = \frac{36}{10}(= 3.6)$ ***A1*** ***N4***

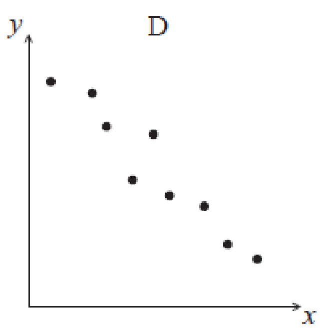
[8 marks]

Examiners report

a(i), (ii) and (iii). [N/A]

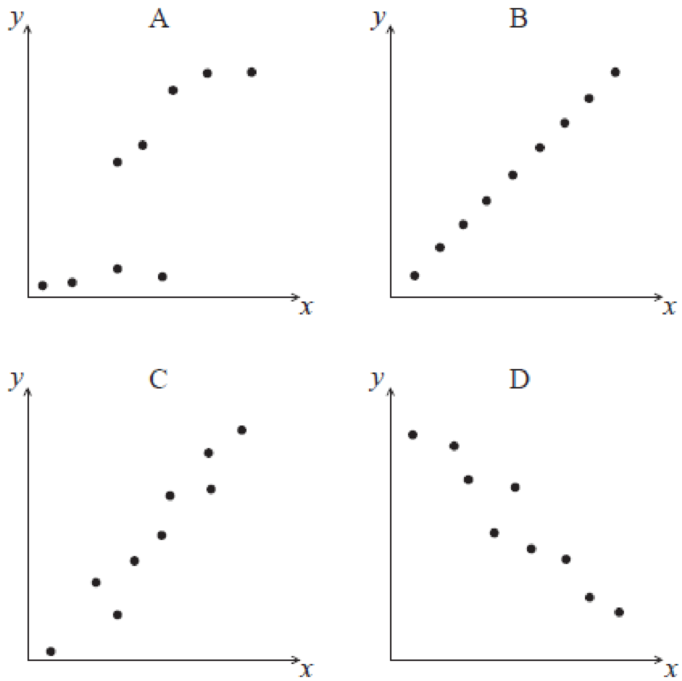
b. [N/A]

There are nine books on a shelf. For each book, x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.



a. Write down the possible minimum and maximum values of r . [2]

b. Given that $r = 0.95$, which of the following diagrams best represents the data. [1]



c. For the data in diagram D , which **two** of the following expressions describe the correlation between x and y ? [2]

perfect, zero, linear, strong positive, strong negative, weak positive, weak negative

Markscheme

a. min value of r is -1, max value of r is 1 ***A1A1 N2***

[2 marks]

b. C ***A1 N1***

[1 mark]

c. linear, strong negative ***A1A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A box contains 100 cards. Each card has a number between one and six written on it. The following table shows the frequencies for each number.

Number	1	2	3	4	5	6
Frequency	26	10	20	k	29	11

a. Calculate the value of k. [2]

b(i) and (ii). Find [5]

(i) the median;

(ii) the interquartile range.

Markscheme

a. evidence of using $\sum f_i = 100$ ***(M1)***

$k = 4$ ***A1 N2***

[2 marks]

b(i) and (ii). (i) evidence of median position ***(M1)***

e.g. 50th item, $26 + 10 + 20 = 56$

$\text{median} = 3$ ***A1 N2***

(ii) $Q_1 = 1$ and $Q_3 = 5$ ***(A1)(A1)***

$\text{interquartile range} = 4$ (accept 1 to 5 or $5 - 1$, etc.) ***A1 N3***

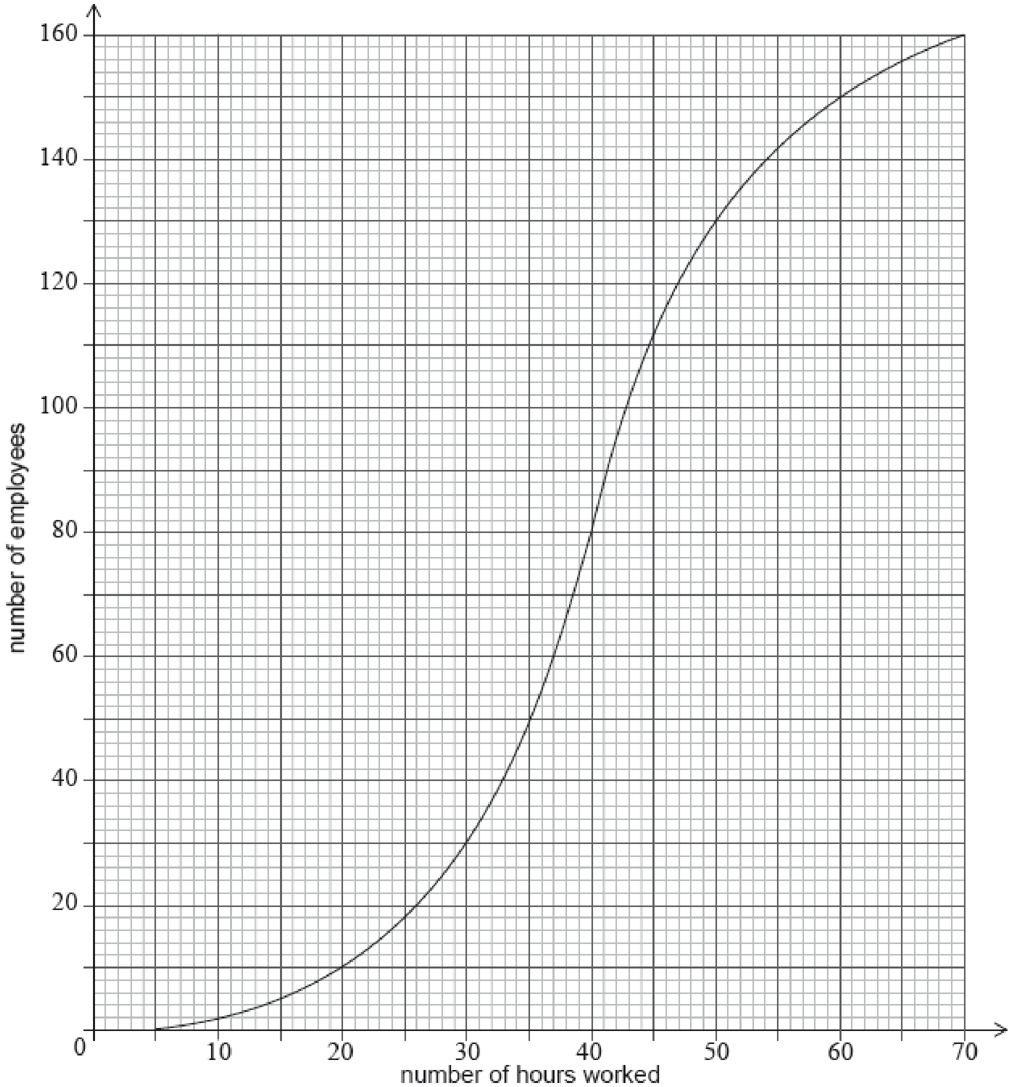
[5 marks]

Examiners report

a. Frequencies and median seemed well understood, but quartiles and inter-quartile range less so.

b(i) and (ii). Frequencies and median seemed well-understood, but quartiles and interquartile range less so. A few students, probably based on past papers, drew cumulative frequency diagrams, generating slightly different answers for median and quartiles.

A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.



The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours.

a.i. Find the median number of hours worked by the employees. [2]

a.ii. Write down the number of employees who worked 50 hours or less. [1]

b.i. Find the amount of money an employee earned for working 40 hours; [1]

b.ii Find the amount of money an employee earned for working 43 hours. [3]

c. Find the number of employees who earned £200 or less. [3]

d. Only 10 employees earned more than £k. Find the value of k. [4]

Markscheme

a.i. evidence of median position ***(M1)***

eg 80th employee

40 hours ***A1 N2***

[2 marks]

a.ii. 130 employees ***A1*** ***N1***

[1 mark]

b.i. £320 ***A1*** ***N1***

[1 mark]

b.ii splitting into 40 and 3 ***(M1)***

eg 3 hours more, 3×10

correct working ***(A1)***

eg $320 + 3 \times 10$

£350 ***A1*** ***N3***

[3 marks]

c. valid approach ***(M1)***

eg 200 is less than 320 so 8 pounds/hour, $200 \div 8,\ 25,\ \frac{200}{320} = \frac{x}{40},$

18 employees ***A2*** ***N3***

[3 marks]

d. valid approach ***(M1)***

eg $160 - 10$

60 hours worked ***(A1)***

correct working ***(A1)***

eg $40(8) + 20(10),\ 320 + 200$

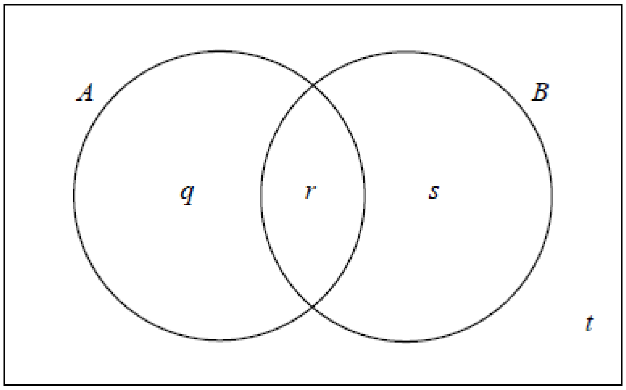
$k = 520$ ***A1*** ***N3***

[4 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]

Events A and B are such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.



The values q , r , s and t represent probabilities.

a. Write down the value of t . [1]

b(i) and (ii). (i) Show that $r = 0.2$. [3]

(ii) Write down the value of q and of s .

c(i) and (ii). (i) Write down $\mathrm{P}(B')$. [3]

(ii) Find $\mathrm{P}(A|B')$.

Markscheme

a. $t = 0.3$ ***A1 N1***

[1 mark]

b(i) and (ii). (i) correct values ***A1***

e.g. $0.3 + 0.6 - 0.7$, $0.9 - 0.7$

$r = 0.2$ ***AG N0***

(ii) $q = 0.1$, $s = 0.4$ ***A1A1 N2***

[3 marks]

c(i) and (ii). (i) 0.4 ***A1 N1***

(ii) $\mathrm{P}(A|B') = \frac{1}{4}$ ***A2 N2***

[3 marks]

Examiners report

a. Parts (a), (b), and (c)(i) of this Venn diagram probability question were answered quite well with candidates consistently earning full marks.

b(i) and (ii). Parts (a), (b), and (c)(i) of this Venn diagram probability question were answered quite well with candidates consistently earning full marks.

Only a few candidates worked backwards from the $r = 0.2$ given in the "show that" portion of part (b).

c(i) and (ii). Many candidates struggled on part (c)(ii), either not recognizing conditional probability or multiplying probabilities to find the numerator as if the events were independent. A number of candidates who successfully found the probability in part (c)(ii) left their incomplete answer of $\frac{0.1}{0.4}$.

In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

a. (i) Find the number of boys who play both sports. [3]

(ii) Write down the number of boys who play only rugby.

b. One boy is selected at random. [4]

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

c. Let A be the event that a boy plays football and B be the event that a boy plays rugby. [2]

Explain why A and B are **not** mutually exclusive.

d. Show that A and B are **not** independent. [3]

Markscheme

a. (i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ***(M1)***

e.g. $75 + 55 - 100$, Venn diagram

30 ***A1 N2***

(ii) 45 ***A1 N1***

[3 marks]

b. (i) **METHOD 1**

evidence of using complement, Venn diagram ***(M1)***

e.g. $1 - p$, $100 - 30$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ ***A1 N2***

METHOD 2

attempt to find P(only one sport) , Venn diagram ***(M1)***

e.g. $\frac{25}{100} + \frac{45}{100}$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ ***A1 N2***

(ii) $\frac{45}{70} \left(= \frac{9}{14}\right)$ ***A2 N2***

[4 marks]

c. valid reason in words or symbols ***(R1)***

e.g. $\mathrm{P}(A \cap B) = 0$ if mutually exclusive, $\mathrm{P}(A \cap B) \neq 0$ if not mutually exclusive

correct statement in words or symbols ***A1 N2***

e.g. $\mathrm{P}(A \cap B) = 0.3$, $\mathrm{P}(A \cup B) \neq \mathrm{P}(A) + \mathrm{P}(B)$, $\mathrm{P}(A) + \mathrm{P}(B) > 1$, some students play both sports, sets intersect

[2 marks]

d. valid reason for independence ***(R1)***

e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, $\mathrm{P}(B|A) = \mathrm{P}(B)$

correct substitution ***A1A1*** ***N3***

e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}$, $\frac{30}{55} \neq \frac{75}{100}$

[3 marks]

Examiners report

a. Overall, this question was very well done. There were some problems with the calculation of conditional probability, where a considerable amount of candidates tried to use a formula instead of using its concept and analysing the problem. It is the kind of question where it can be seen if the concept is not clear to candidates.

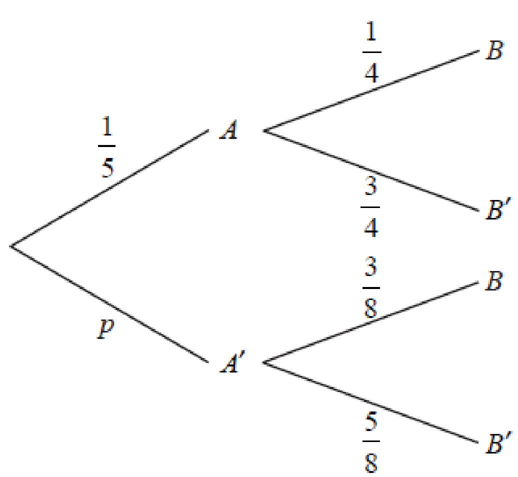
b. Overall, this question was very well done. There were some problems with the calculation of conditional probability, where a considerable amount of candidates tried to use a formula instead of using its concept and analysing the problem. It is the kind of question where it can be seen if the concept is not clear to candidates.

c. In part (c), candidates were generally able to explain in words why events were mutually exclusive, though many gave the wrong values for $\mathrm{P}(A)$ and $\mathrm{P}(B)$.

d. There was a great amount of confusion between the concepts of independent and mutually exclusive events. In part (d), the explanations often referred to mutually exclusive events.

It was evident that candidates need more practice with questions like (c) and (d).
Some students equated probabilities and number of elements, giving probabilities greater than 1.

The diagram below shows the probabilities for events A and B , with $\mathrm{P}(A') = p$.



a. Write down the value of p . [1]

b. Find $\mathrm{P}(B)$. [3]

c. Find $\mathrm{P}(A'|B)$. [3]

Markscheme

a. $p = \frac{4}{5}$ ***A1*** ***N1***

[1 mark]

b. multiplying along the branches ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4}$, $\frac{12}{40}$

adding products of probabilities of two mutually exclusive paths ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}$, $\frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40} \left(= \frac{7}{20}\right)$ ***A1*** ***N2***

[3 marks]

c. appropriate approach which must include A' (may be seen on diagram) ***(M1)***

e.g. $\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (do not accept $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$)

$\mathrm{P}(A'|B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$ ***(A1)***

$\mathrm{P}(A'|B) = \frac{12}{14} \left(= \frac{6}{7}\right)$ ***A1*** ***N2***

[3 marks]

Examiners report

a. While nearly every candidate answered part (a) correctly, many had trouble with the other parts of this question.

b. In part (b), many candidates did not multiply along the branches of the tree diagram to find the required values, and many did not realize that there were two paths for P(B). There were also many candidates who understood what the question required, but then did not know how to multiply fractions correctly, and these calculation errors led to an incorrect answer.

c. In part (c), most candidates attempted to use a formula for conditional probability found in the information booklet, but very few substituted the correct values.

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs.

The taxicabs are randomly assigned by the company.

The probability that a taxicab is yellow is 0.4.

The probability that a taxicab is a Fiat is 0.3.

The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

Markscheme

recognize need for intersection of Y and F ***(R1)***

eg $\mathrm{P}(Y \cap F)$, 0.3×0.4

valid approach to find $\mathrm{P}(Y \cap F)$ ***(M1)***

eg $\mathrm{P}(Y) + \mathrm{P}(F) - \mathrm{P}(Y \cup F)$, Venn diagram

correct working (may be seen in Venn diagram) ***(A1)***

eg $0.4 + 0.3 - 0.6$

$\mathrm{P}(Y \cap F) = 0.1$ ***A1***

recognize need for complement of $Y \cap F$ ***(M1)***

eg $1 - \mathrm{P}(Y \cap F), 1 - 0.1$

$\mathrm{P}\left((Y \cap F)'\right) = 0.9$ ***A1 N3***

[6 marks]

Examiners report

[N/A]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Markscheme

a. evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

eg $\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2(2\cos^2\theta - 1) = 1,\ 4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1 \pm \sqrt{1 - 4 \times 4 \times (-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1),\ \frac{-1 \pm 7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \frac{3}{4}$ ***AG N0***

b. valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working

(A1)

eg missing side $= \sqrt{7}$, $\frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1 N2***

[3 marks]

c. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_\theta^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_\theta^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ ***(A1)***

eg $1 - \tan\theta$

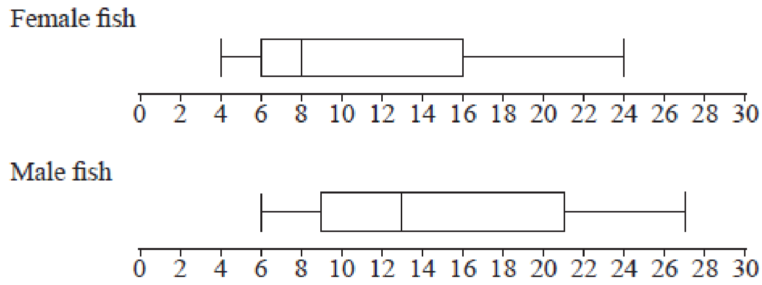
$V = \pi - \frac{\pi\sqrt{7}}{3}$ ***A1 N3***

[6 marks]

Examiners report

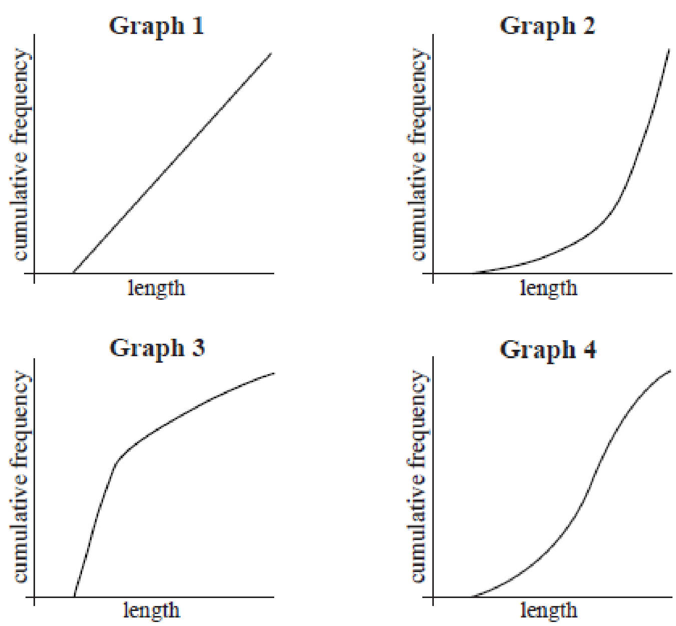
a. [N/A]
b. [N/A]
c. [N/A]

A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.



a. Find the range of the lengths of **all** 200 fish. [3]

b. Four cumulative frequency graphs are shown below. [2]



Which graph is the best representation of the lengths of the **female** fish?

Markscheme

a. correct end points ***(A1)(A1)***

max = 27 , min = 4

range = 23 ***A1*** ***N3***

[3 marks]

b. Graph 3 ***A2*** ***N2***

[2 marks]

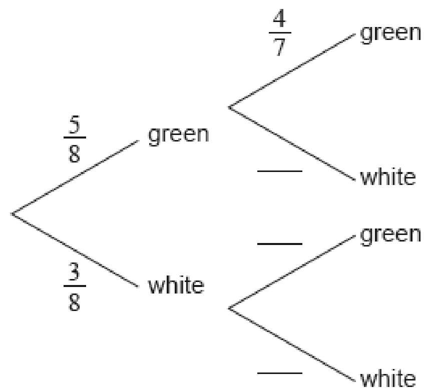
Examiners report

a. While there were a large number of candidates who answered both parts of this question correctly, a surprising number did not know how to find the range of all 200 fish in part (a). Common errors included finding the ranges of the male and female fish separately, or averaging the separate ranges of the male and female fish.

b. Some candidates did not interpret the cumulative frequency graphs correctly, or just seemed to guess which graph was correct. The most common incorrect "guess" was graph 4, likely because this graph had a more familiar cumulative shape.

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

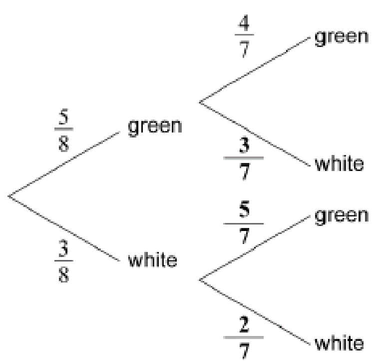
a. Complete the following tree diagram. [3]



b. Find the probability that exactly one of the selected balls is green. [3]

Markscheme

a. correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

b. multiplying along branches ***(M1)***

eg $\frac{5}{8}\times\frac{3}{7},\ \frac{3}{8}\times\frac{5}{7},\ \frac{15}{56}$

adding probabilities of correct mutually exclusive paths ***(A1)***

eg $\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7},\ \frac{15}{56}+\frac{15}{56}$

$\frac{30}{56}\ \left(=\frac{15}{28}\right)$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The random variable X has the following probability distribution, with $\mathrm{P}(X > 1) = 0.5$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	0.2

a. Find the value of r . [2]

b. Given that $\mathrm{E}(X) = 1.4$, find the value of p and of q . [6]

Markscheme

a. attempt to substitute $\mathrm{P}(X > 1) = 0.5$ ***(M1)***

e.g. $r + 0.2 = 0.5$

$r = 0.3$ ***A1*** ***N2***

[2 marks]

b. correct substitution into $\mathrm{E}(X)$ (seen anywhere) ***(A1)***

e.g. $0 \times p + 1 \times q + 2 \times r + 3 \times 0.2$

correct equation ***A1***

e.g. $q + 2 \times 0.3 + 3 \times 0.2 = 1.4$, $q + 1.2 = 1.4$

$q = 0.2$ ***A1*** ***N1***

evidence of choosing $\sum p_i = 1$ ***M1***

e.g. $p + 0.2 + 0.3 + 0.2 = 1$, $p + q = 0.5$

correct working ***(A1)***

$p + 0.7 = 1$, $1 - 0.2 - 0.3 - 0.2$, $p + 0.2 = 0.5$

$p = 0.3$ ***A1 N2***

Note: Exception to the ***FT*** rule. Award ***FT*** marks on an incorrect value of q, even if q is an inappropriate value. Do not award the final ***A*** mark for an inappropriate value of p.

[6 marks]

Examiners report

a. The majority of candidates were successful in earning full marks on this question.

b. In part (b), a small number of candidates did not use the correct formula for $\mathrm{E}(X)$, even though this formula is given in the formula booklet.

There were also a few candidates who incorrectly assumed that $p = 0$, forgetting that the sum of the probabilities must equal 1. There were a few candidates who left this question blank, which raises concerns about whether they had been exposed to probability distributions during the course.

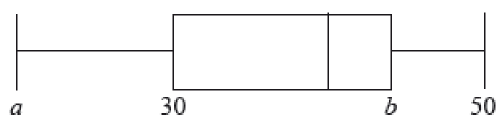
A school collects cans for recycling to raise money. Sam’s class has 20 students.

The number of cans collected by each student in Sam’s class is shown in the following stem and leaf diagram.

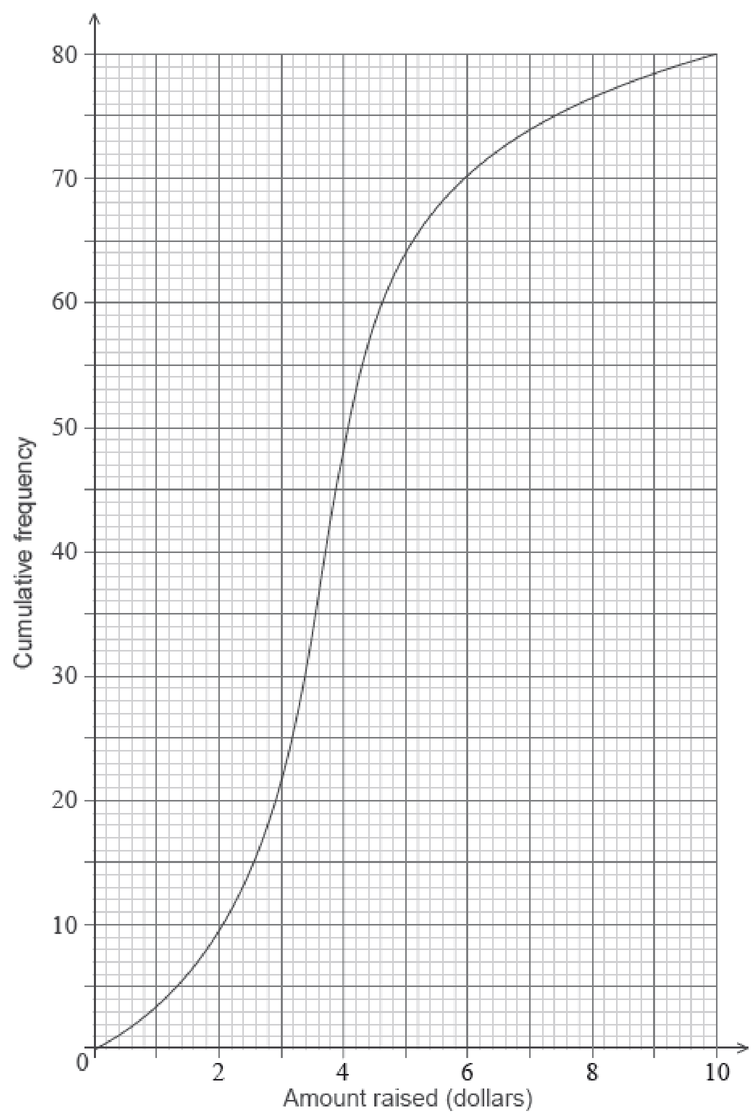
Stem	Leaf
2	0, 1, 4, 9, 9
3	1, 7, 7, 7, 8, 8
4	1, 2, 2, 3, 5, 6, 7, 8
5	0

Key: 3|1 represents 31 cans

The following box-and-whisker plot also displays the number of cans collected by students in Sam’s class.



There are 80 students in the school.



The mean number of cans collected is 39.4. The standard deviation is 18.5.

Each student then collects 2 more cans.

a. Find the median number of cans collected. [2]

b. (i) Write down the value of a. [3]

(ii) The interquartile range is 14. Find the value of b.

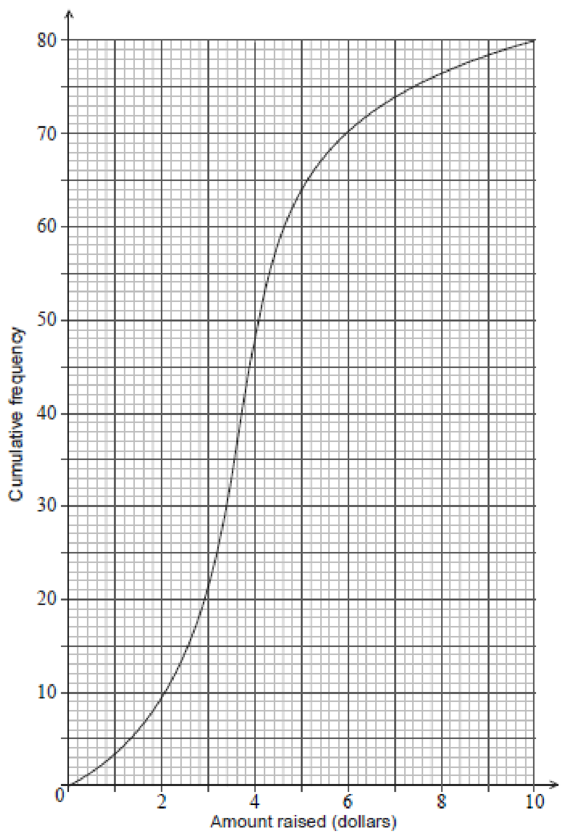
c. Sam’s class collected 745 cans. They want an average of 40 cans per student. [3]

How many more cans need to be collected to achieve this target?

d. The students raise \$0.10 for each recycled can. [5]

(i) Find the largest amount raised by a student in Sam’s class.

(ii) The following cumulative frequency curve shows the amounts in dollars raised by all the students in the school. Find the percentage of students in the school who raised more money than anyone in Sam’s class.



e. (i) Write down the new mean. [2]

(ii) Write down the new standard deviation.

Markscheme

a. valid approach ***(M1)***

eg between 10th and 11th, $\frac{8+8}{2}$

median $= 38$ ***A1 N2***

[2 marks]

b. (i) $a = 20$ ***A1 N1***

(ii) valid approach ***(M1)***

eg $Q_3 - Q_1,\ Q_1 + 14,\ b - 30 = 14$

$b = 44$ ***A1*** ***N2***

[3 marks]

c. valid approach ***(M1)***

*eg*x $40 \times 20,\ \frac{x+745}{20},\ 40 - \frac{745}{20}$

correct working ***(A1)***

eg $800 - 745,\ 20 \times 2.75$

55 (more cans) ***A1*** ***N2***

[3 marks]

d. (i) most cans in Sam's class $= 50$ ***(A1)***

5 ($) ***A1*** ***N2***

(ii) correct value of 64 or 16 ***A1***

valid approach ***(M1)***

eg $\frac{64}{80},\ 80\%,\ 80 - 64,\ \frac{16}{80}$

20% ***A1*** ***N2***

[5 marks]

e. (i) 41.4 (exact) ***A1 N1***

(ii) 18.5 ***A1*** ***N1***

[2 marks]

Examiners report

a. Generally, candidates were very successful with this question, appearing to move easily between the three different representations of data. The main conceptual errors appeared in part d) where a percentage of 100 was found (instead of 80) and in part e) where the new standard deviation was often given as 20.5. Arithmetic errors seemed to be the other factor, with a surprising number of candidates finding in part c) that $800 - 745 = 15$.

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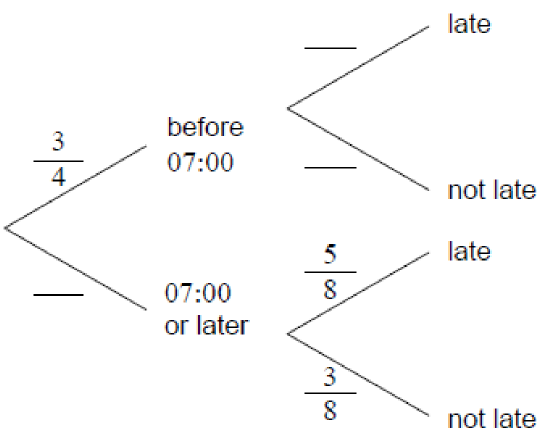
$800 - 745 = 15.$

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$.

If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

a. **Copy** and complete the following tree diagram. [3]



b. Find the probability that Pablo leaves home before 07:00 and is late for work. [2]

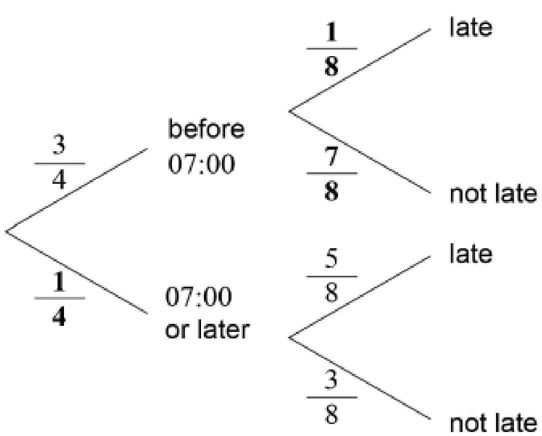
c. Find the probability that Pablo is late for work. [3]

d. Given that Pablo is late for work, find the probability that he left home before 07:00. [3]

e. Two days next week Pablo will drive to work. Find the probability that he will be late at least once. [3]

Markscheme

a.



A1A1A1 N3

Note: Award ***A1*** for each bold fraction.

[3 marks]

b. multiplying along correct branches ***(A1)***

eg $\frac{3}{4} \times \frac{1}{8}$

P(leaves before 07:00 ∩ late) = $\frac{3}{32}$ ***A1 N2***

[2 marks]

c.

multiplying along other "late" branch ***(M1)***
eg $\frac{1}{4} \times \frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***
eg $\left(\frac{3}{4} \times \frac{1}{8}\right) + \left(\frac{1}{4} \times \frac{5}{8}\right),\ \frac{3}{32} + \frac{5}{32}$

$\mathrm{P}\left(L\right) = \frac{8}{32}\ \left(= \frac{1}{4}\right)$ ***A1 N2***

[3 marks]

d. recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}\left(A|B\right),\ \mathrm{P}\left(\text{before } 7|\text{late}\right)$

correct substitution of **their** values into formula ***(A1)***
eg $\frac{\frac{3}{32}}{\frac{1}{4}}$

$\mathrm{P}\left(\text{left before } 07{:}00|\text{late}\right) = \frac{3}{8}$ ***A1 N2***

[3 marks]

e. valid approach ***(M1)***

eg 1 − P(not late twice), P(late once) + P(late twice)

correct working ***(A1)***
eg $1 - \left(\frac{3}{4} \times \frac{3}{4}\right),\ 2 \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}$

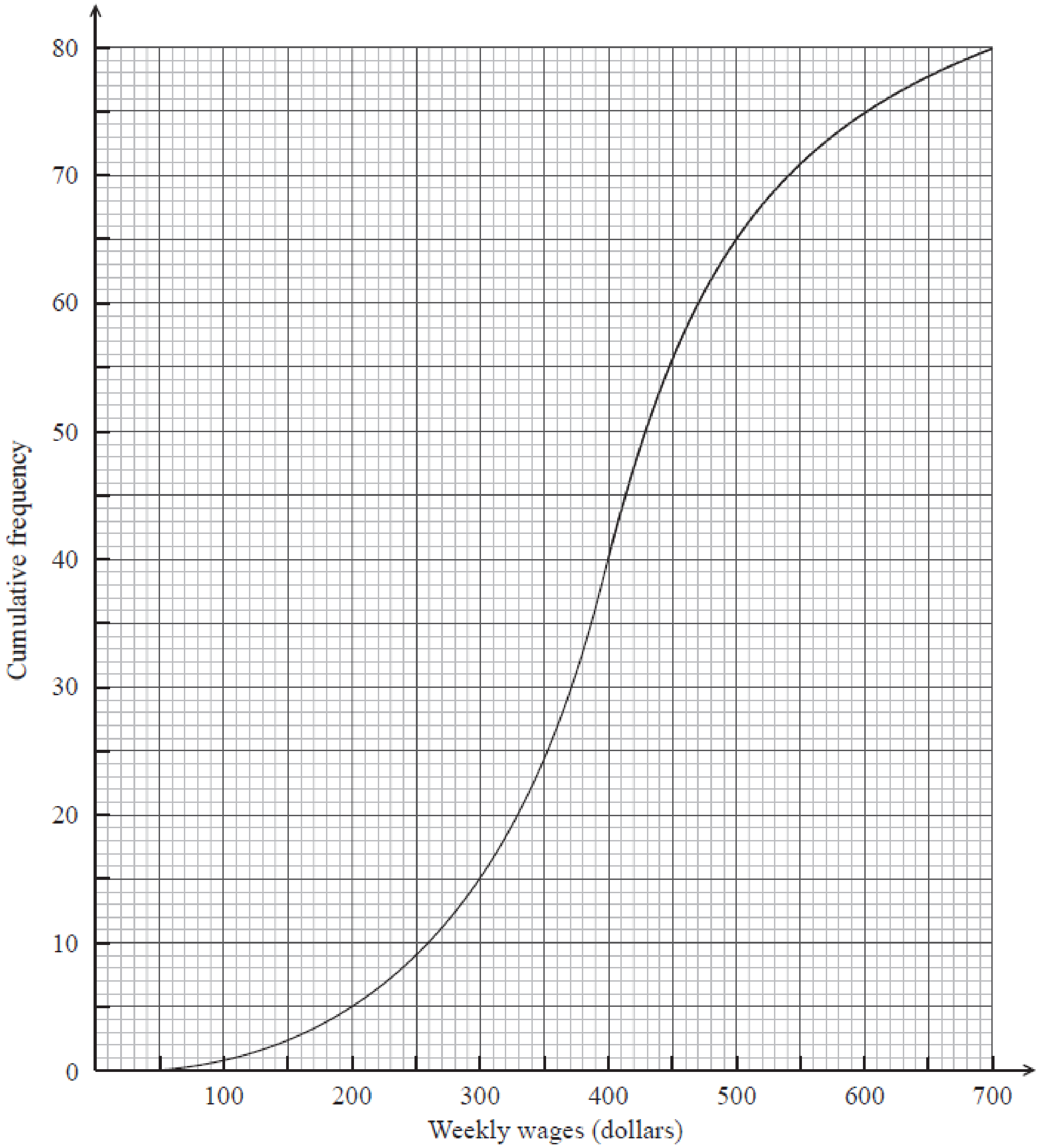
$\frac{7}{16}$ ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

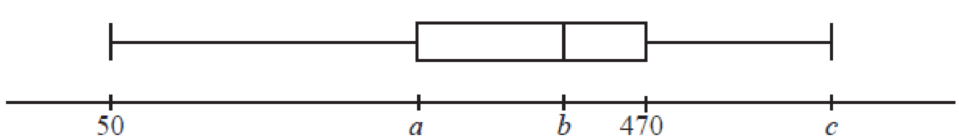
The weekly wages (in dollars) of 80 employees are displayed in the cumulative frequency curve below.



a(i) and (ii) (i) Write down the median weekly wage. [4]

(ii) Find the interquartile range of the weekly wages.

b(i), (ii), (iii). The box-and-whisker plot below displays the weekly wages of the employees. [3]



Write down the value of

(i) a ;

(ii) b ;

(iii) c .

c. Employees are paid $ 20 per hour. [3]

Find the median number of **hours** worked per week.

d. Employees are paid $20 per hour. [5]

Find the number of employees who work more than 25 hours per week.

Markscheme

a(i) and (ii). (i) median weekly wage $=400$ (dollars) ***A1*** ***N1***

(ii) lower quartile $=330$, upper quartile $=470$ ***(A1)(A1)***

IQR $=140$ (dollars) (accept any notation suggesting interval 330 to 470) ***A1*** ***N3***

Note: Exception to the ***FT*** rule. Award ***A1(FT)*** for an incorrect IQR **only** if both quartiles are explicitly noted.

[4 marks]

b(i), (ii), (iii). (i) 300 (dollars) ***A1*** ***N1***

(ii) 400 (dollars) ***A1*** ***N1***

(iii) 700 (dollars) ***A1*** ***N1***

[3 marks]

c. valid approach ***(M1)***

e.g. hours $=\frac{\text{wages}}{\text{rate}}$

correct substitution ***(A1)***

e.g. $\frac{400}{20}$

median hours per week $=20$ ***A1*** ***N2***

[3 marks]

d. attempt to find wages for 25 hours per week ***(M1)***

e.g. wages $=$ hours $\times$ rate

correct substitution ***(A1)***

e.g. 25×20

finding wages $=500$ ***(A1)***

65 people (earn $500\leqslant$) ***(A1)***

15 people (work more than 25 hours) ***A1*** ***N3***

[5 marks]

Examiners report

a(i) and (ii). Many candidates answered this question completely correctly, earning full marks in all parts of the question. In parts (a) and (b), there were some who gave the frequency values on the *y*-axis, rather than the wages on the *x*-axis, as their quartiles and interquartile range.

b(i), (ii), (iii). Many candidates answered this question completely correctly, earning full marks in all parts of the question. In parts (a) and (b), there were some who gave the frequency values on the *y*-axis, rather than the wages on the *x*-axis, as their quartiles and interquartile range.

c. For part (c), the majority of candidates seemed to understand what was required, though there were a few who used an extreme value such as 700, rather than the median value.

d. In part (d), some candidates simply answered 65, which is the number of workers earning \$500 or less, rather than finding the number of workers who earned more than \$500. It was interesting to note that quite a few candidates gave their final answer as 14, rather than 15.