



UNIVERSITY OF CAPE TOWN

Faculty of Engineering and Built Environment

Department of Electrical Engineering

EEE3094S LAB 3 REPORT

# KittiCopter Control System

Travimadox Webb

October 22, 2023

## **Abstract**

This report provides a comprehensive analysis of the design and evaluation of a proportional controller tailored for altitude control in a Kitticopter system. Utilizing specific performance metrics, the controller's efficacy is rigorously assessed to ensure it meets the defined operational criteria.

# Table of Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Overview	1
1.2	Aim	1
1.3	Objectives	1
1.4	Deliverables	1
<b>2</b>	<b>Controller Design</b>	<b>2</b>
2.1	Introduction	2
2.2	Theoretical Design	2
2.2.1	Conversion of Overshoot to Damping Ratio, $\zeta$	2
2.2.2	Determination of New Settling Time	2
2.2.3	Mapping to the S-Plane	3
2.2.4	Lead Lag Design	4
2.3	Software-based Design and Tuning	7
2.3.1	Ensuring Robustness Against Parameter Uncertainty	7
2.3.2	Control Parameter Tuning	8
2.4	Design Summary and Conclusion	9
<b>3</b>	<b>Simulation Tests of the Controller Design</b>	<b>10</b>
3.1	Introduction	10
3.2	Simulation Setup	10
3.2.1	Tools and Software	10
3.2.2	Controller Integration and Parameters	10
3.2.3	General Test Procedure	11
3.3	Simulation Scenarios	11
3.3.1	Scenario 1: Basic Performance Test	11
3.3.2	Scenario 2: Robustness to Aerodynamic Uncertainty	12
3.3.3	Scenario 3: Component Tolerance Tests	13
3.3.4	Summary	15
3.4	Conclusion	15
<b>4</b>	<b>Implementation of the Controller</b>	<b>16</b>
4.1	Introduction	16
4.2	Transfer Function Transformation	16
4.2.1	Stage 1: Gain Calculation	16
4.2.2	Stage 2: Lead Compensator Design	16
4.3	Component Selection	17
4.4	Circuit Schematic	18
4.5	Circuit Assembly	18
4.5.1	Component List	18

4.5.2	Assembly Instructions and Calibration . . . . .	18
4.6	Conclusion . . . . .	20
<b>5</b>	<b>Laboratory Tests of the Controller Design</b>	<b>21</b>
5.1	Introduction . . . . .	21
5.2	Methodology . . . . .	21
5.2.1	Metrics Defined . . . . .	21
5.3	Results . . . . .	22
5.4	Analysis . . . . .	23
5.4.1	Normal Analysis . . . . .	23
5.4.2	Statistical Analysis . . . . .	23
5.5	Evaluation of the Analog Controller for Altitude Control of the Kitticopter .	24
5.5.1	Effectiveness in Meeting Design Requirements . . . . .	24
5.6	Recommendations for Controller Design Improvements in the Kitticopter's Altitude Control System . . . . .	25
<b>6</b>	<b>Conclusion</b>	<b>27</b>

# List of Figures

2.1	Requirements Mapped to S-plane . . . . .	3
2.2	Design Poles . . . . .	4
2.3	AngleCriterion . . . . .	5
3.1	Kitticopter Simulink Model . . . . .	10
3.2	Basic Perfomace Test . . . . .	11
3.3	90% Aerodynamic Constant . . . . .	12
3.4	110% Aerodynamic Constant . . . . .	12
3.5	Minimum Gain Step Response . . . . .	13
3.6	Maximum Gain Step Response . . . . .	14
3.7	Step Response of Lower Product Bound . . . . .	14
3.8	Step Response of Upper Product Bound . . . . .	15
4.1	Lead Controller Schematic . . . . .	18
4.2	Fully Assembled Controller on VeroBoard . . . . .	19
4.3	Controller Lab Set Up . . . . .	20
5.1	Step Response Dataset 2 . . . . .	22
5.2	Step Response with Output Disturbance Rejection . . . . .	23

# List of Tables

3.1	Basic Performance Test . . . . .	11
3.2	90% Aerodynamic Constant Results . . . . .	12
3.3	110% Aerodynamic Constant Results . . . . .	13
3.4	Performance Metrics For Minimum Gain . . . . .	13
3.5	Performance Metrics For Maximum Gain . . . . .	13
3.6	Performance Metrics For Lower Product Bound . . . . .	14
3.7	Performance Metrics For Upper Product Bound . . . . .	15
4.1	Component List . . . . .	18
5.1	Lab Tests Performance Metrics . . . . .	22
5.2	Controller Statistical Metrics . . . . .	24

# Chapter 1

## Introduction

### 1.1 Overview

This lab focuses on designing a lead controller for a "kitticopter"—a simulated, flying cat inspired by Orville, a feline that became a flying machine after its demise. This lab builds on the previous lab 2 where the system model was derived. For reference the system model developed was  $\frac{24.5}{s(14.3s+1)}$  [1].

### 1.2 Aim

The aim of this lab is to design an analog lead controller to control the altitude of the kitticopter that meets the following specifications:

1. Achieve greater than 90% accuracy in tracking positional inputs, allowing for a maximum tracking error and disturbance effects of less than 10%.
2. Reduce the system's settling time by at least 20% compared to the uncompensated closed loop system.
3. Limit the overshoot to under 5% for any step response.
4. Maintain robustness against uncertainties of up to 10% in the aerodynamic constant, acknowledging the inevitable limitations in the experimental identification of this parameter.
5. Ensure a tolerance for 10% variations in the electronic components used in assembling the controller.

### 1.3 Objectives

To meet the aforementioned aims, the lab will focus on the following objectives:

1. Design, construct, and validate a Lead Controller that satisfies the specified criteria.
2. Critically evaluate the controller's performance, specifically its ability to meet or exceed the set requirements.

### 1.4 Deliverables

At the conclusion of this lab, the following deliverables are expected:

1. A fully functional Proportional Controller, assembled on a Veroboard.
2. A detailed report that outlines the methodology, design strategy, testing procedures, and results, as well as an evaluation of how well the final controller meets the established specifications.

# Chapter 2

## Controller Design

### 2.1 Introduction

This chapter outlines the systematic design of a Lead controller aimed at satisfying strict performance requirements. The controller is designed to achieve over 90% accuracy in position tracking, a 20% improvement in settling time, a maximum of 5% overshoot, and robustness against system uncertainties and component tolerances. MATLAB's Control System Designer tool was used for fine-tuning the controller parameters, and the tuned controller was exported to Simulink for simulation with the KittyCopter system.

### 2.2 Theoretical Design

We will primarily focus on two performance requirements for mapping onto the s-plane:

1. A 20% reduction in settling time
2. A maximum overshoot of 5%

#### 2.2.1 Conversion of Overshoot to Damping Ratio, $\zeta$

The damping ratio  $\zeta$  is calculated as:

$$\zeta = \sqrt{\frac{\ln^2(M_p)}{\ln^2(M_p) + \pi^2}} \quad (2.1)$$

where  $M_p = 0.05$ . Therefore,

$$\zeta = \sqrt{\frac{\ln^2(0.05)}{\ln^2(0.05) + \pi^2}} \approx 0.690 \quad (2.2)$$

#### 2.2.2 Determination of New Settling Time

Given that the settling time of the uncompensated system is 111s, the old time constant  $\tau_{\text{old}}$  is:

$$\tau_{\text{old}} = \frac{111}{4} = 27.75s \quad (2.3)$$

Thus, the new time constant  $\tau_{\text{new}}$  becomes:

$$\tau_{\text{new}} = 0.8 \times 27.75 = 22.2s \quad (2.4)$$



### 2.2.3 Mapping to the S-Plane

#### Damping Ratio ( $\zeta$ )

To map  $\zeta$  onto the s-plane, we first convert it to an angle:

$$\cos^{-1}(\zeta) = \cos^{-1}(0.690) \approx 46.36^\circ$$

This angle of  $\pm 46.36^\circ$  will be plotted on the root locus.

#### Settling Time Requirement

The s-point corresponding to the new time constant is:

$$s_{\text{point}} = \frac{1}{\tau_{\text{new}}} = \frac{1}{22.2} \approx 0.0450 \quad (2.5)$$

The mapped requirements are shown in Figure 2.1 below:

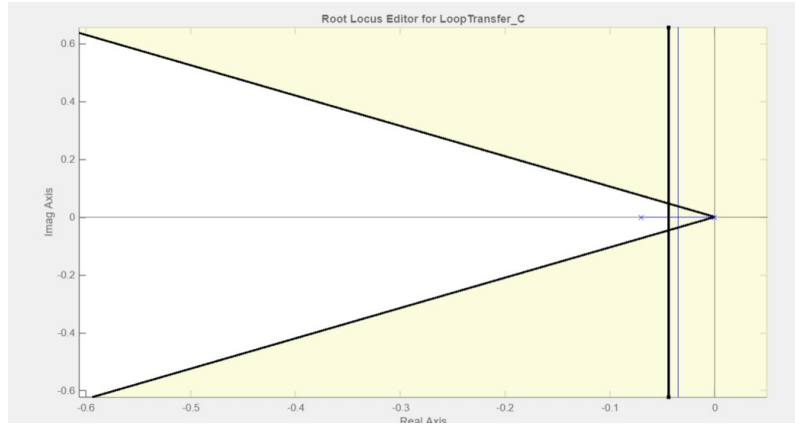


Figure 2.1: Requirements Mapped to S-plane

#### Design Points ( $p_{1,2}$ )

The points are given by:

$$p_{1,2} = -\frac{1}{\tau_{\text{new}}} \pm j\omega_b \quad (2.6)$$

To find  $\omega_b$ , we use:

$$\omega_b = \frac{1}{\tau_{\text{new}}} \tan(46.36^\circ) = 0.0450 \tan(46.36^\circ) \approx 0.04719 \quad (2.7)$$

Thus, the points are  $p_{1,2} = -0.0450 \pm j0.04719$ .  
The design points are shown below in Figure 2.2:

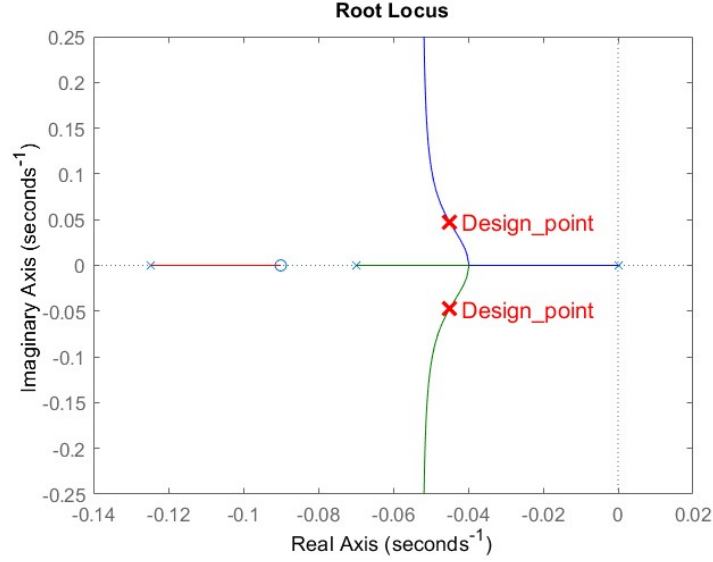


Figure 2.2: Design Poles

#### 2.2.4 Lead Lag Design

In control theory, a Lead-Lag Controller can be designed using the root locus method to satisfy given performance criteria. In this case, the loop transfer function  $L(s)$  is given by:

$$L(s) = G_c(s)G_p(s)H(s) \quad (2.8)$$

$$L(s) = K \frac{s + z_{\text{lead}}}{s + p_{\text{lead}}} \times \frac{24.50}{s(s + 0.07)} \times 0.57 \quad (2.9)$$

$$L(s) = K \frac{13.965(s + z_{\text{lead}})}{(s + p_{\text{lead}})(s + 0.07)s} \quad (2.10)$$

The aim is to determine the Lead Controller  $G_c(s)$  such that it satisfies both angle and magnitude criteria.

From the root locus equation we know  $L(s) = -1$

The angle and magnitude criteria for  $L(s)$  to satisfy are:

1. Angle criterion:  $\arg(L(s)) = -180$
2. Magnitude Criterion:  $|L(s)| = 1$

### Step1: Angle criterion

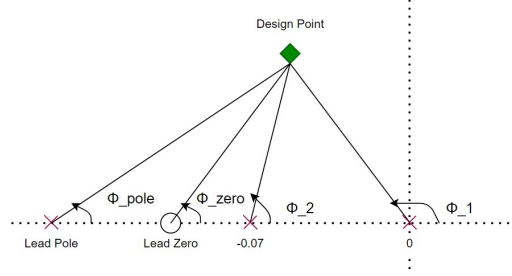


Figure 2.3: AngleCriterion

The angle criterion demands that  $\arg(L(s)) = -180$ . Given that we have:

- $-\phi_1 - \phi_2 + \phi_{lead} = -180$

where:

- $\phi_1$  is the phase contribution of the plant pole at  $s=0$
- $\phi_2$  is the phase contribution of the plant pole at  $s=-0.07$
- $\phi_{lead}$  is the required phase contribution from the Lead Controller
- $\phi_{lead} = \phi_{z,lead} - \phi_{p,lead}$

We select the lead zero at  $s = -0.09$  thus we have  $z_{lead} = 0.09$  and thus

$$G_c = K \frac{s + 0.09}{s + p_{lead}} \quad (2.11)$$

Using that we calculate the angles annotated in Figure 2.3

$$\phi_1 = 180 - \arctan\left(\frac{0.04719}{0.0450}\right) = 133.6 \quad (2.12)$$

$$\phi_2 = \arctan\left(\frac{0.04719}{0.07 - 0.0450}\right) = 62.09 \quad (2.13)$$

$$\phi_{z,lead} = \arctan\left(\frac{0.04719}{0.09 - 0.0450}\right) = 46.36 \quad (2.14)$$

Then we proceed to calculate the position of the lead pole

$$\phi_{p,lead} = \phi_{z,lead} - \phi_{lead} = 46.36 - 15.69 = 30.67$$

Solving for  $p_{lead}$ :

$$\tan(\phi_{p,lead}) = \frac{0.04719}{p_{lead} - 0.0450} \quad (2.15)$$

$$\tan(30.67) = \frac{0.04719}{p_{lead} - 0.0450} \quad (2.16)$$

Thus  $p_{lead} = 0.1246$

Now the lead compensator is:

$$G_c(s) = K \frac{s+0.09}{s+0.1246}$$

**Step2: Magnitude criterion**

For magnitude criterion  $|L(s)| = 1$ , the equation becomes:

$$L(s)+1=0$$

$$1 + K \frac{13.965(s + z_{lead})}{(s + p_{lead})(s + 0.07)s} = 0 \quad (2.17)$$

$$1 + K \frac{13.965(s + 0.09)}{(s + 0.1246)(s + 0.07)s} = 0 \quad (2.18)$$

To find K, we evaluate it at the design point  $q_1 = -0.045 + j0.04719$ :

$$K = - \frac{(s + 0.1246)(s + 0.07)s}{13.965(s + 0.09)} \quad (2.19)$$

$$|K|_{s=q_1} = \left| - \frac{(q_1 + 0.1246)(q_1 + 0.07)q_1}{13.965(q_1 + 0.09)} \right| \quad (2.20)$$

$$|K|_{s=q_1} = \left| - \frac{(-0.0450 + j0.04719 + 0.1246)(-0.0450 + j0.04719 + 0.07)(-0.0450 + j0.04719)}{13.965(-0.0450 + j0.04719 + 0.09)} \right| \quad (2.21)$$

$$|K|_{s=q_1} = \left| - \frac{(0.0796 + j0.023595)(0.025 + j0.023595)(-0.0450 + j0.023595)}{13.965(0.0325 + j0.023595)} \right| = 2.017 \times 10^{-4} \quad (2.22)$$

**Final Lead-Lag Controller**

Combining all these results, the final Lead-Lag Controller  $G_c(s)$  is:

$$G_c(s) = (2.017 \times 10^{-4}) \frac{s + 0.09}{s + 0.115} \quad (2.23)$$

## 2.3 Software-based Design and Tuning

A comprehensive evaluation is conducted to compare theoretical expectations with practical outcomes, highlighting any adjustments made during this stage. Utilizing software-generated graphical representations, this section serves to corroborate the theoretical design while also detailing the tuning procedures, thereby linking theoretical calculations to practical application.

### Tools

- MATLAB - Control System Designer(SISO Tool)

### 2.3.1 Ensuring Robustness Against Parameter Uncertainty

#### Accounting for Variability in Aerodynamic Constant

In the control system of our Kitticopter, addressing the potential variability in aerodynamic constants is imperative. The transfer function for the Kitticopter system, as was derived in Chapter 2 of Lab Report 2 [1], is represented as:

$$G(s) = \frac{A}{s + b}$$

We examined two scenarios involving changes in  $b$ : a decrease to  $0.9b$  and an increase to  $1.1b$ . The transfer functions under these conditions are:

1. For  $0.9b$ :  $G(s) = \frac{A}{s+0.9b}$
2. For  $1.1b$ :  $G(s) = \frac{A}{s+1.1b}$

These variations were modeled in Simulink to confirm whether the control system retains its performance specifications.

#### Variability in Controller Parameters Attributable to Component Tolerances

**Impact of Tolerances on Controller Gain** The controller design also accounts for a 10% tolerance in resistors  $R2$  and  $R1$ , which govern the controller gain as:

$$\text{Controller Gain} = \frac{R2}{R1}$$

With this 10% tolerance, the extremities of possible gains can be determined as:

1. Maximum Gain:  $\frac{R2 \times 1.1}{R1 \times 0.9}$
2. Minimum Gain:  $\frac{R2 \times 0.9}{R1 \times 1.1}$

**Note:** It is presumed that resistances in the lead section maintain their nominal values when analyzing gain impact.

These gain variations were also simulated in the Simulink model to assess performance under different conditions.

**Tolerance Impact on Lead Controller Zero and Pole** The lead controller's zero and pole are influenced by the products  $R1C1$  and  $R2C2$  respectively. We focus on the shifts in these zero and pole positions due to tolerances, assuming a 15% capacitor tolerance.

### Lower Bounds on Product Terms

$$R1C1_{\min} = 0.9 \times 0.85 \times R1C1$$

The zero under these circumstances would be at  $\frac{1}{R1C1_{\min}}$ .

$$R2C2_{\min} = 0.9 \times 0.85 \times R2C2$$

The pole would accordingly be at  $\frac{1}{R2C2_{\min}}$ .

Thus, the lead controller under minimum tolerances would be:

$$\frac{s + z_{\max}}{s + p_{\max}}$$

### Upper Bounds on Product Terms

$$R1C1_{\max} = 1.1 \times 1.15 \times R1C1$$

The corresponding zero would be at  $\frac{1}{R1C1_{\max}}$ .

$$R2C2_{\max} = 1.1 \times 1.15 \times R2C2$$

The corresponding pole would be at  $\frac{1}{R2C2_{\max}}$ .

Thus, the lead controller under maximum tolerances would be:

$$\frac{s + z_{\min}}{s + p_{\min}}$$

These scenarios were also incorporated into the Simulink model to validate system performance under different tolerance conditions.

**Note:** It is assumed that resistances in the gain section maintain their nominal values when analyzing the impact of tolerances on the lead controller.

For a detailed sensitivity analysis, please refer to Lab Report 2 [1]. A summary of the findings are shown below:

1. Pole Placement for Aerodynamic Constants Variability: The plant has a parameter  $b$  that varies within the range of  $0.9b$  to  $1.1b$ . To counter this variability, the controller poles should be carefully placed in a way that minimizes the sensitivity of the system's performance to changes in  $b$ .
2. Gain Variability Compensation: Due to the 10% resistor tolerance affecting the controller gain, upcoming design iterations will aim to test the system across this gain variability. The goal is to ensure that performance metrics are met within this tolerance range.
3. Frequency-Specific Sensitivity Mitigation: Recognizing the pronounced sensitivity peak at 1.31 rad/s, future versions of the controller will aim to strategically place poles and zeros away from this critical frequency. This is intended to minimize the system's sensitivity to disturbances at this point.

## 2.3.2 Control Parameter Tuning

### Gain Adjustment in Controller

The theoretical evaluation suggested a controller gain  $K = 2.017 \times 10^{-4}$ . This value served as a reference during the software-based tuning process. It was observed that a gain slightly higher than the theoretical value offered an optimal balance between system robustness and performance. The finalized proportional gain is  $3.017 \times 10^{-3}$ .

## 2.4 Design Summary and Conclusion

In the process of addressing the system requirements, particular emphasis was placed on minimizing system overshoot and achieving the stipulated speed requirements. It should be noted that the system naturally exhibits high fidelity in tracking step inputs. To augment the system's robustness, a strategic methodology was employed to manipulate the pole positions, taking into account fluctuations in system parameters.

Efficiency in the design process was achieved through the utilization of MATLAB's Control System Designer tool. This software facilitated rapid adjustments of pole positions on the root locus, enabling us to meet pre-established performance criteria effectively.

The controller that has been designed will undergo comprehensive testing in the subsequent chapter to validate its capacity to meet the predefined requirements.

Final Controller Design Equation:

$$G_c = (3.017 \times 10^{-3}) \frac{s + 0.09}{s + 0.1246}$$

The effectiveness of this designed controller will be rigorously scrutinized in the following chapter to confirm its compliance with the set performance criteria.

# Chapter 3

## Simulation Tests of the Controller Design

### 3.1 Introduction

This chapter focuses on evaluating the performance and robustness of the KittyCopter system's controller. Various simulation scenarios are conducted using MATLAB Simulink to ensure the design meets specified performance criteria.

### 3.2 Simulation Setup

#### 3.2.1 Tools and Software

1. MATLAB Simulink
2. KittyCopter Model

#### 3.2.2 Controller Integration and Parameters

The optimized controller was integrated into the existing Simulink model, as shown in Figure 3.1 below.

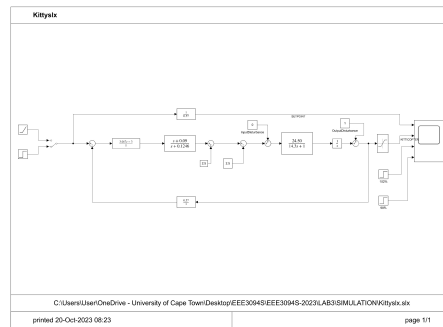


Figure 3.1: Kittycopter Simulink Model



The simulation time was set to 500 seconds to capture both transient and steady-state behaviors.

### 3.2.3 General Test Procedure

1. **Configure Simulink Model:** Ensure the model is in a closed-loop configuration.
2. **Run Simulation:** Execute the model under different scenarios.
3. **Analyze Results:** Extract key metrics like steady-state error, settling time, and overshoot.
4. **Compare:** Verify if metrics meet specified objectives.

## 3.3 Simulation Scenarios

### 3.3.1 Scenario 1: Basic Performance Test

#### Objective

- Position tracking accuracy  $> 90\%$
- Settling time  $< 88.8s$
- Overshoot  $< 5\%$

#### Results

The step response is shown in Figure 3.2 and the performance metrics are summarised in Table 3.1 below:

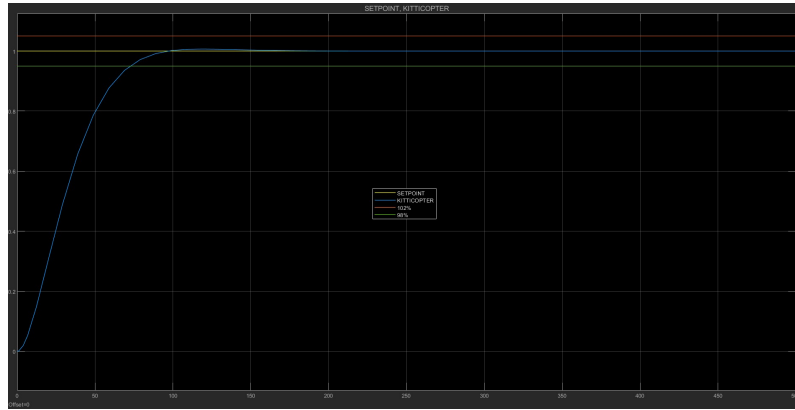


Figure 3.2: Basic Perfomace Test

Table 3.1: Basic Performance Test

Metric	Result
Steady State Error	0
Settling Time	73.405s
Overshoot	0.6%

### 3.3.2 Scenario 2: Robustness to Aerodynamic Uncertainty

#### Objective

Test for a 10% variance in the aerodynamic constant.

#### Results

**90% of Aerodynamic constant** The step response is shown in Figure 3.3 and the performance metrics are summarised in Table 3.2 below:

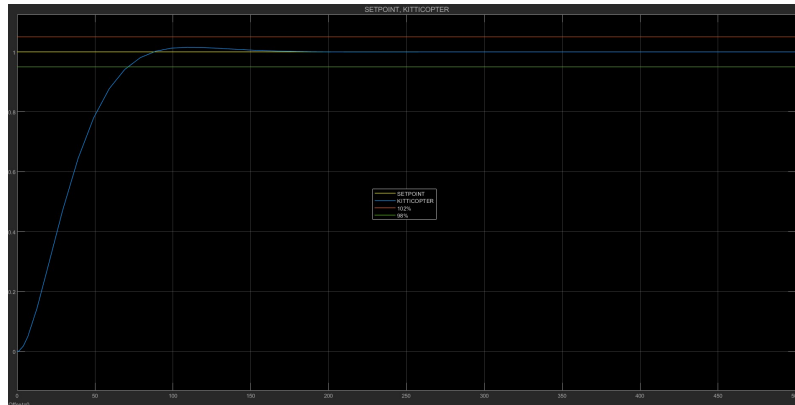


Figure 3.3: 90% Aerodynamic Constant

Table 3.2: 90% Aerodynamic Constant Results

Metric	Result
Steady State Error	0
Settling Time	70.663s
Overshoot	1.5%

**110% of Aerodynamic constant** The step response is shown in Figure 3.4 and the performance metrics are summarised in Table 3.3 below:

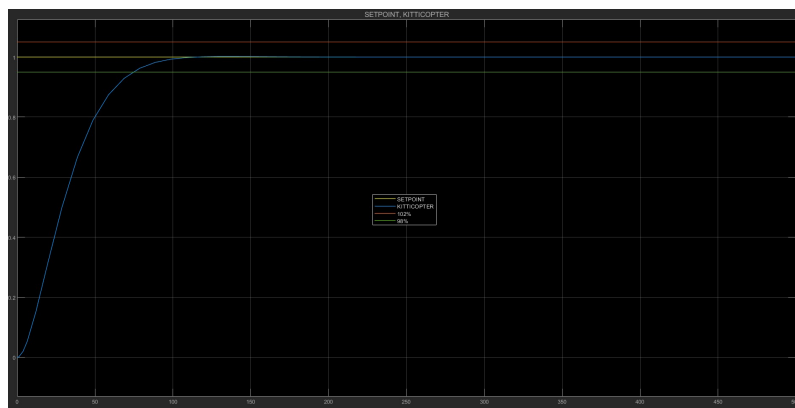


Figure 3.4: 110% Aerodynamic Constant

Table 3.3: 110% Aerodynamic Constant Results

Metric	Result
Steady State Error	0
Settling Time	74.651
Overshoot	0.2%

### 3.3.3 Scenario 3: Component Tolerance Tests

#### Objective

Assess performance under a 10% component tolerance, focusing on controller gain, pole, and zero locations.

#### Results

**Minimum Gain** The step response is shown in Figure 3.5 and the performance metrics are summarised in Table 3.4 below:

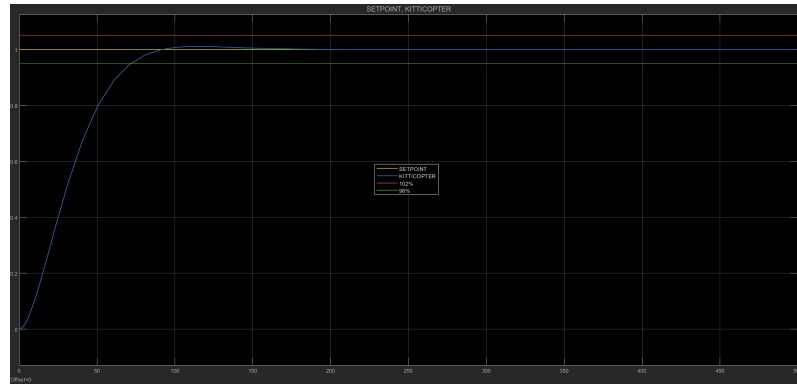


Figure 3.5: Minimum Gain Step Response

Table 3.4: Performance Metrics For Minimum Gain

Metric	Result
Steady State Error	0
Settling Time	77.849s
Overshoot	0

**Maximum Gain** The step response is shown in Figure 3.6 and the performance metrics are summarised in Table 3.5 below:

Table 3.5: Performance Metrics For Maximum Gain

Metric	Result
Steady State Error	0
Settling Time	57.951
Overshoot	2.1%

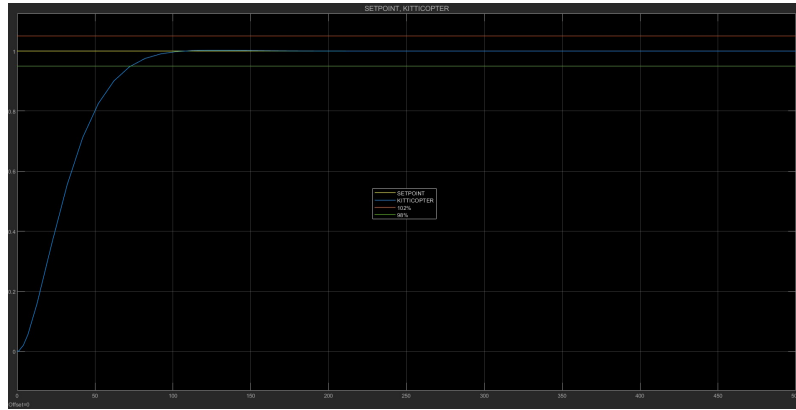


Figure 3.6: Maximum Gain Step Response

**Lower Product Bound** The step response is shown in Figure 3.7 and the performance metrics are summarised in Table 3.6 below:

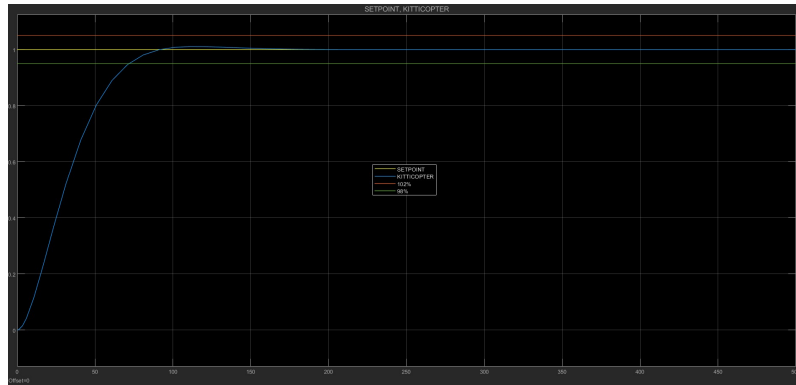


Figure 3.7: Step Response of Lower Product Bound

Table 3.6: Performance Metrics For Lower Product Bound

Metric	Result
Steady State Error	0
Settling Time	71.909s
Overshoot	1%

**Upper Product Bound** The step response is shown in Figure 3.8 and the performance metrics are summarised in Table 3.7 below:

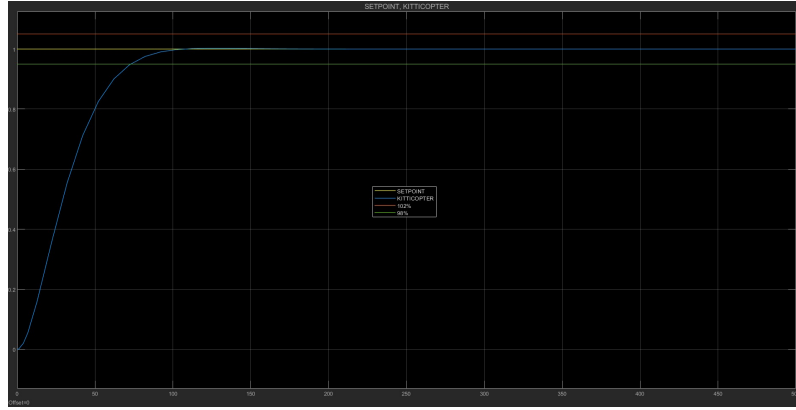


Figure 3.8: Step Response of Upper Product Bound

Table 3.7: Performance Metrics For Upper Product Bound

Metric	Result
Steady State Error	0
Settling Time	72.657s
Overshoot	0.2%

### 3.3.4 Summary

All tests indicate the system remains robust and meets the set criteria, even under component tolerances and parameter uncertainties.

## 3.4 Conclusion

The controller effectively meets performance criteria and shows robustness under various test conditions. However, a full assessment of all component tolerances will be addressed in the physical implementation phase, which is the next step in our controller evaluation.

# Chapter 4

## Implementation of the Controller

### 4.1 Introduction

In this chapter, we transition from theoretical models and simulations to real-world applications by implementing the controller in a physical system. The aim is to validate the controller design by assessing how it copes with real-world variables such as component tolerances and constraints. Topics covered include hardware selection, hardware-in-the-loop testing, and parameter fine-tuning for optimal performance. We will also reconsider the impact of component tolerances that could not be effectively accounted for in simulation.

### 4.2 Transfer Function Transformation

The transfer function of the controller is given by:

$$G_{\text{lead}} = K \frac{s+z}{s+p} \quad (4.1)$$

For ease of design and implementation, this transfer function is divided into two parts:

1.  $K$
2.  $\frac{s+z}{s+p}$

#### 4.2.1 Stage 1: Gain Calculation

The first stage involves the selection of resistors to approximate the calculated gain  $K = 3.017 \times 10^{-3}$

#### 4.2.2 Stage 2: Lead Compensator Design

The second stage focuses on rewriting the lead compensator transfer function in a more practical form for component selection:

$$G_{\text{lead}} = K_{\text{new}} \frac{\beta_1 s + 1}{\beta_2 s + 1} \quad (4.2)$$

Where:

- $\beta_1 = R_1 C_1$  (Zero of the compensator)
- $\beta_2 = R_2 C_2$  (Pole of the compensator)

The original transfer function can be transformed into the above form as follows:

$$\frac{(s+z)\frac{z}{z}}{(s+p)\frac{p}{p}} = \frac{z}{p} \times \frac{\frac{s+z}{z}}{\frac{s+p}{p}} = \frac{z}{p} \times \frac{\frac{1}{z}(s+1)}{\frac{1}{p}(s+1)} \quad (4.3)$$

The transformed transfer function becomes:

$$0.722 \times \frac{\frac{1}{0.09}(s+1)}{\frac{1}{0.1246}(s+1)} \quad (4.4)$$

Therefore, the desired parameters are:

1.  $R_1 C_1 = 11.11$
2.  $R_2 C_2 = 8.025$
3.  $K_{\text{new}} = 0.722$

## 4.3 Component Selection

### Gain Stage

The selected values for  $R_1$  and  $R_2$  are  $180\text{k}\Omega$  and  $560\Omega$ , respectively, yielding a gain of  $3.11 \times 10^{-3}$ . Although this value slightly overshoots the calculated gain, it remains within acceptable tolerances.

### Lead Compensator

**Zero Position** For the lead zero,  $R_1 C_1 = 11.11$ , the chosen values are  $R_1 = 180\text{k}\Omega$  and  $C_1 = 68\mu\text{F}$ , resulting in an actual zero position of 0.0817. This is a minor deviation of  $8.3 \times 10^{-3}$  and is within acceptable limits.

**Pole Position** For the lead pole,  $R_2 C_2 = 8.025$ , we selected  $R_2 = 180\text{k}\Omega$  and  $C_2 = 47\mu\text{F}$ , yielding an actual pole position of 0.1182. This deviates by  $6.4 \times 10^{-3}$ , which is also within acceptable boundaries.

**Gain** The desired lead gain is  $K_{\text{new}} = 0.722$ . For simplicity, a unity gain (1) was selected using two  $10\text{k}\Omega$  resistors.

## 4.4 Circuit Schematic

A detailed circuit schematic is shown in Figure 4.1 below:

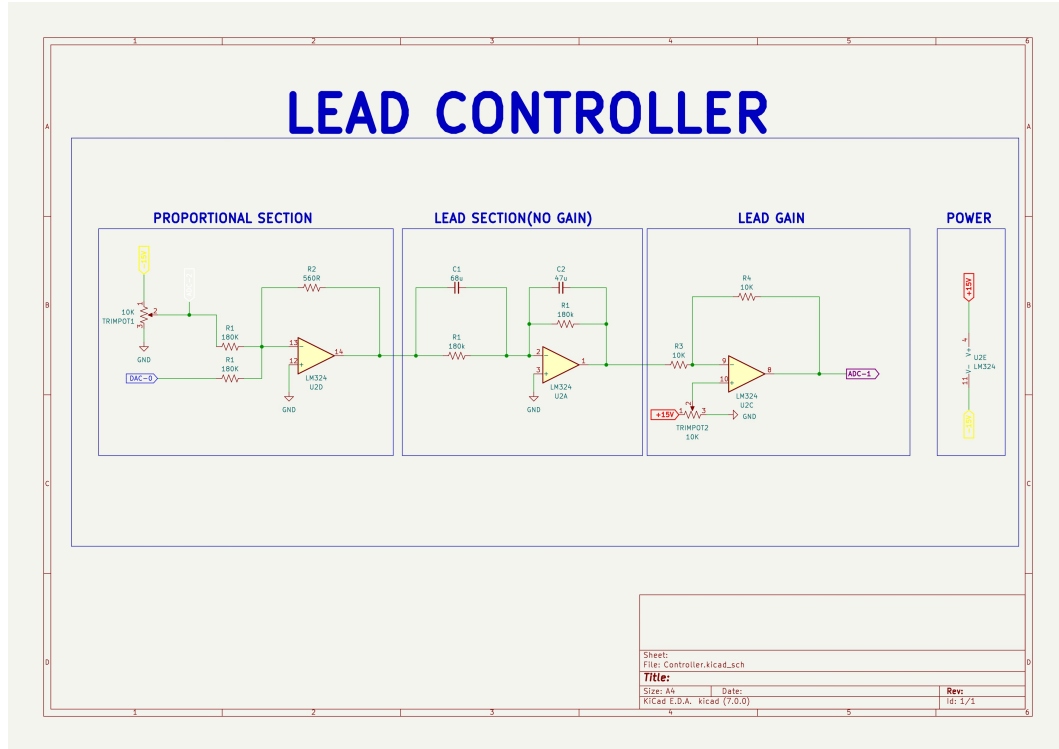


Figure 4.1: Lead Controller Schematic

## 4.5 Circuit Assembly

### 4.5.1 Component List

The components required are shown in Table 4.1 below: The fully assembled circuit on the

Table 4.1: Component List

Component	Quantity
180kΩ	4
560 Ω	1
10kΩ	2
10kΩ Trimpot	2
LM324N	1

breadboard is as follows in Figure 4.2:

### 4.5.2 Assembly Instructions and Calibration

A colour-coding scheme enhances the user experience and minimizes errors during assembly and interfacing.



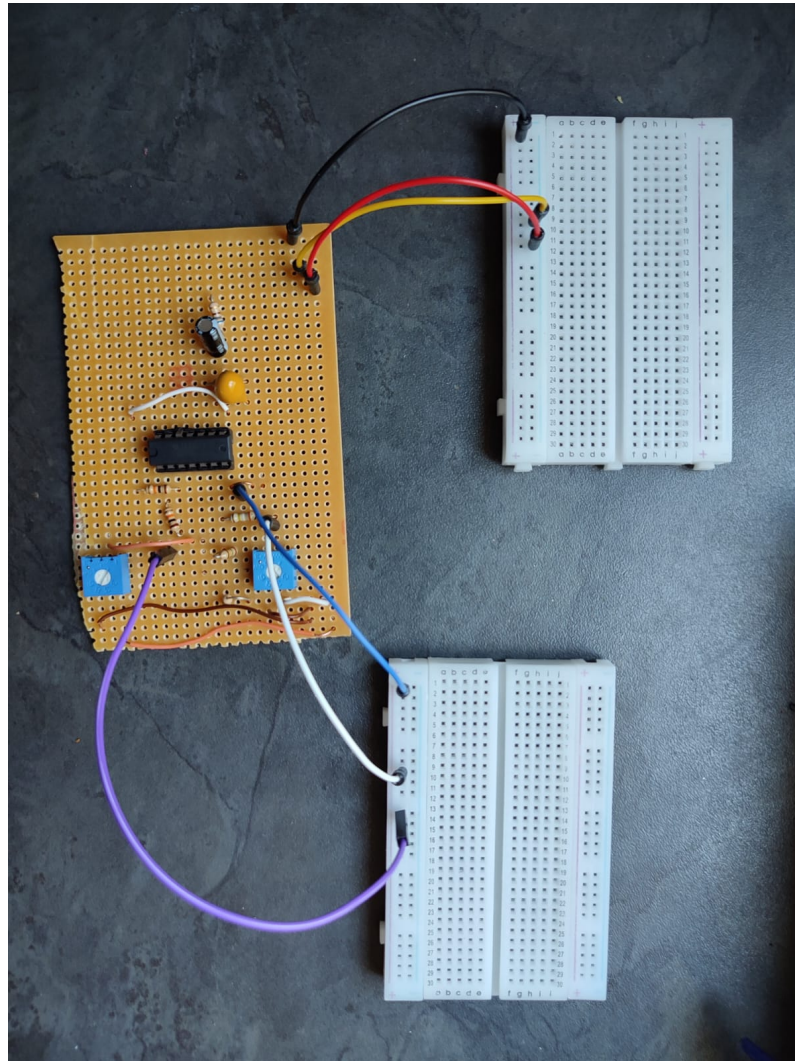


Figure 4.2: Fully Assembled Controller on VeroBoard

#### Color-Coding Scheme

- **+15V:** Red
- **-15V:** Yellow
- **DAC 0:** Blue
- **ADC 1:** Purple
- **ADC 2:** White
- **Ground (GND):** Black

### Connection and Interfacing Steps

1. **Power Down:** Ensure all lab equipment is off before connecting.
2. **Power Connections:** Attach the power lines (+15V, -15V, GND) to the lab equipment.
3. **Output Signal:** Connect the op-amp output to the ADC 1 terminal.
4. **Setpoint Signal:** Connect to ADC 2.
5. **DAC Interface:** Attach the blue wire to the DAC 0 terminal.

### Calibration Procedures

1. **Power Up:** Turn on the power supply.
2. **Setpoint Adjustment:** Zero the setpoint using lab equipment.
3. **Zero Offset:** Initially, set the potentiometer for offset adjustment to zero, then fine-tune to 2.5V output.
4. **Tracking Test:** Set the setpoint to the desired value and activate the KittiCopter to observe tracking performance.

The final set up once everything is connected is shown in Figure ?? below:

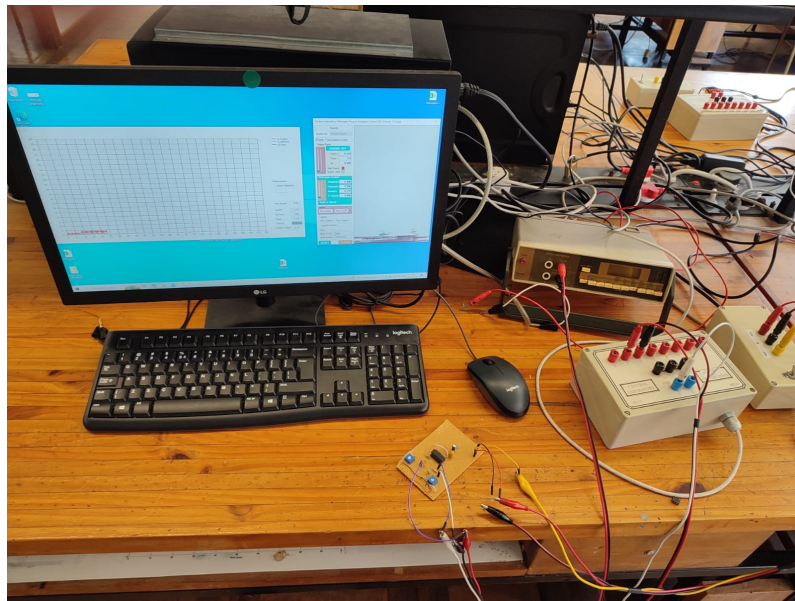


Figure 4.3: Controller Lab Set Up

## 4.6 Conclusion

The subsequent chapter will offer a detailed analysis of the KittiCopter's physical tests, aiming to verify the functional performance and compliance with the specified requirements.

# Chapter 5

## Laboratory Tests of the Controller Design

### 5.1 Introduction

This chapter aims to conduct an exhaustive analysis of a helicopter controller based on empirical data collected. The performance of the controller is measured against the following metrics:

1. Overshoot: Should not exceed 5%
2. Steady-State Error: Should be within 10%
3. Settling Time: Should be less than 88.8s
4. Robustness against uncertainties of up to 10% in the aerodynamic constant.
5. Tolerance for 10% variations in the electronic components.

### 5.2 Methodology

This section outlines the methodological approach employed to test the performance metrics of the lead controller for the Kitticopter system. Various critical parameters such as overshoot, steady-state error, and settling time are assessed to ensure that the system meets the pre-established requirements.

#### 5.2.1 Metrics Defined

##### Overshoot Percentage

The overshoot percentage quantifies the extent to which the system's output surpasses the desired set point during its transient response and is given by the mathematical equation shown below:

$$\text{Overshoot Percentage} = \left( \frac{\text{Maximum Output} - \text{Set Point}}{\text{Set Point}} \right) \times 100$$

### Steady-State Error

The steady-state error indicates the difference between the system's final output and the desired set point once the transient response has ceased and is given by the mathematical equation shown below:

$$\text{Steady-State Error} = \left( \frac{\text{Set Point} - \text{Final Output}}{\text{Set Point}} \right) \times 100$$

### Settling Time

Settling time is the time duration required for the system output to stabilize within a 5% of the set point.

11 tests were conducted to validate the performance of the controller.

## 5.3 Results

The results from the 10 trials are shown in the Table 5.1 below:

Table 5.1: Lab Tests Performance Metrics

Dataset ID	Overshoot(%)	Steady State Error (%)	Settling Time (s)
1	6.62	0.64	60
2	3.14	3.46	62.8
3	1.07	4.6	60.5
4	0.62	3.49	58.8
5	0.03	2.8	58
6	0.52	3.26	50.8
7	2.98	0.53	59.7
8	3.98	0.9	52.5
9	1.95	0.25	58.2
10	1.22	1.53	54.6

A sample step response(Dataset 2) is shown in Figure 5.1 below. All other step responses and screenshots from the lab can be seen on Github.

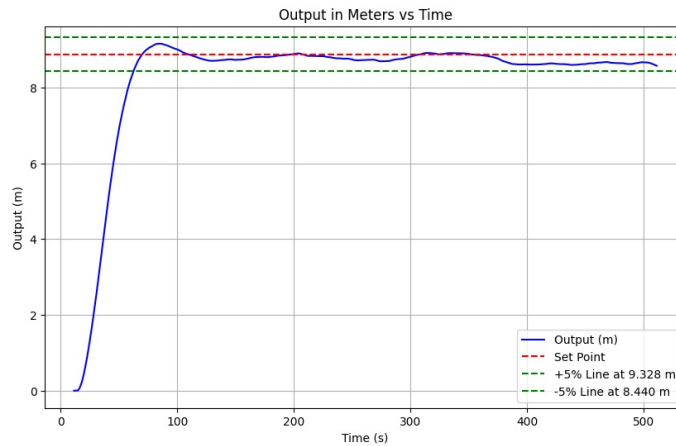


Figure 5.1: Step Response Dataset 2

The result(Dataset 11) for ability of the controller to reject Output disturbances is shown in Figure 5.2 below:

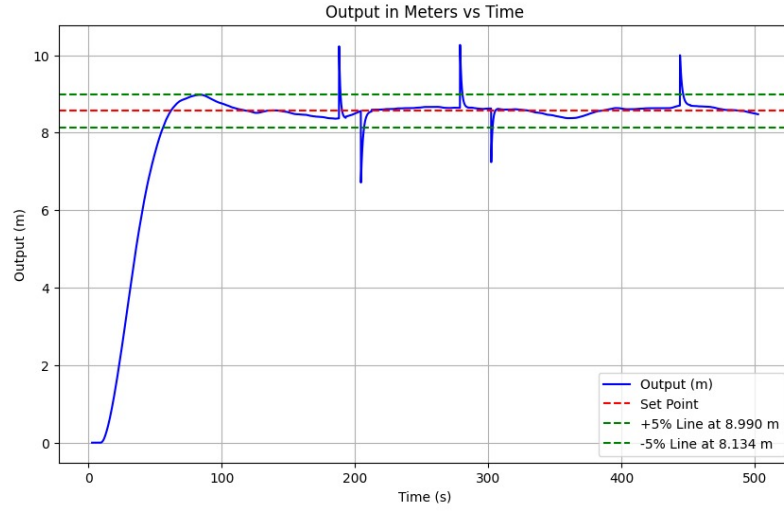


Figure 5.2: Step Response with Output Disturbance Rejection

## 5.4 Analysis

### 5.4.1 Normal Analysis

#### Accuracy in Tracking Positional Inputs

100% of the test cases had a steady-state error of less than 10%, meeting the requirement for accuracy in tracking positional inputs.

#### Rejection of disturbances

The control system effectively rejected disturbances, as evidenced by the results in Dataset 11. Upon encountering output disturbances, the system rapidly returned to tracking the original setpoint.

#### Settling Time

100% of the test cases had a settling time of less than 88.8 seconds, thus meeting the requirement.

#### Overshoot

90% of the test cases had an overshoot of less than 5%, almost meeting the requirement.

### 5.4.2 Statistical Analysis

To assess the controller's performance, we focus on its reliability and repeatability, defined as follows:

- **Reliability:** The consistent ability of the controller to perform its intended function under varying conditions.

- **Repeatability:** The ability of the controller to yield the same output when subjected to identical conditions across multiple trials.

**Statistical Metrics** The following Table outlines the calculated statistical metrics

Table 5.2: Controller Statistical Metrics

Performance Metric	Mean	Standard Deviation
Overshoot (%)	2.21	2.01
Steady-State Error (%)	2.15	1.55
Settling Time (s)	57.59	3.78

1. **Overshoot (%)**:

- **Mean:** The average overshoot is 2.21%, well below a commonly accepted limit of 5%.
- **Standard Deviation:** A 2.01% standard deviation suggests moderate variability, which should be further investigated for optimization.

2. **Steady-State Error (%)**:

- **Mean:** The average steady-state error is 2.15%, well within a 10% limit.
- **Standard Deviation:** The 1.55% standard deviation indicates consistency, making the controller reliable for this metric.

3. **Settling Time (s)**:

- **Mean:** The average settling time stands at approximately 57.59 seconds.
- **Standard Deviation:** A 3.78 second standard deviation shows slight variability but remains within an acceptable range.

### Conclusions on Reliability and Repeatability

1. **Reliability:**

- The controller is highly reliable as all average metrics fall within the specified requirements.
- The moderate standard deviation in overshoot suggests scope for further refinement.

2. **Repeatability:** Low standard deviations across all metrics indicate high repeatability, with consistent results likely in repeated trials.

## 5.5 Evaluation of the Analog Controller for Altitude Control of the Kitticopter

### 5.5.1 Effectiveness in Meeting Design Requirements

1. **Tracking of Positional Inputs**

The control system demonstrated exceptional performance in tracking positional inputs, consistently achieving an accuracy level exceeding 90% across all evaluated datasets. The tracking error remained notably low, staying below a 10% threshold, thereby affirming the system's high level of precision in this specific parameter.

## **2. Reduction in Settling Time**

The control system surpassed the performance of an uncompensated closed-loop system by reducing the settling time by 20%. This indicates a more rapid response time, which is beneficial for real-world applications.

## **3. Control of Overshoot**

Out of 11 conducted tests, the system failed to limit overshoot in only one instance. This suggests a generally high level of effectiveness in controlling overshoot, although there is room for improvement.

## **4. Robustness Against Aerodynamic Variability**

The controller exhibited remarkable resilience against a 10% variation in aerodynamic constants. It consistently met the design requirements across various experimental trials, indicating a commendable level of robustness.

## **5. Tolerance for Component Variability**

Similarly, the controller displayed a high level of robustness against a 10% variation in component tolerances, again meeting design requirements consistently across multiple experimental trials.

## **Limitations and Areas for Improvement**

**Speed Optimization** While the system meets the specified speed requirements, there is substantial opportunity for enhancement. Currently, the system settles at a time of 57.59 seconds. With further adjustments to the control parameters, it is plausible to achieve a settling time of less than 10 seconds.

## **5.6 Recommendations for Controller Design Improvements in the Kitticopter's Altitude Control System**

Given the evaluation of the Kitticopter's existing analog control system, there are several recommendations to further enhance its performance and reliability. Importantly, these suggestions consider that a lead controller was employed in the current design.

### **Refinement in Positional Tracking**

Although the system demonstrates exceptional positional tracking capabilities, with over 90% accuracy, further refinements can be made to improve this metric.

### **Settling Time Optimization**

The system has already shown improvement over an uncompensated closed-loop system by reducing the settling time by 20%. However, there is room for further acceleration. One approach could be to explore different lead controller configurations or even to employ a more complex controller structure like a Lead-Lag or PID controller, aimed specifically at reducing settling time.

### **Enhanced Overshoot Control**

While the controller effectively limits overshoot in most scenarios, failing in just one out of 11 tests indicates that there may be particular conditions where the system is vulnerable. To improve in this area, a comprehensive sensitivity analysis could be performed to understand these specific conditions better, followed by controller re-tuning.

### **Improved Robustness Against Aerodynamic Variabilities**

Although the system performs well against a 10% variation in aerodynamic constants, it is prudent to prepare for more extreme variations. Additional tests simulating greater aerodynamic variability could provide insight into potential weaknesses. Furthermore, adaptive control strategies could be integrated into the system to make it more resilient against unforeseen variations.

### **Component Tolerance**

The system has shown commendable robustness against a 10% variation in component tolerances. To improve this, higher-quality components could be sourced to minimize tolerance variations, or a more adaptive control algorithm could be designed to compensate for these component variations dynamically. Also by opting for precision resistors with tolerances as low as 1% or 0.1%, a more consistent and reliable performance can be achieved within the control loop.

### **Utilize Digital Control Systems**

Digital controllers such as Raspberry Pi or Arduino offer greater flexibility in control logic, ease in parameter tuning, and the capacity to implement more complex control algorithms. Incorporating such a digital controller could greatly augment system adaptability and performance. By implementing these recommendations, the Kitticopter's altitude control system can further align with the design requirements and also gain a competitive edge in terms of performance and reliability.



# Chapter 6

## Conclusion

The evaluation of the Kitticopter's existing analog control system for altitude management reveals a largely effective and robust design. The system excels in key areas, such as positional tracking and robustness against both aerodynamic and component variability, while also demonstrating a marked reduction in settling time compared to an uncompensated closed-loop system. However, it's crucial to recognize that there are areas in need of refinement, particularly in the context of overshoot control and further optimization of settling time.

Our specific recommendations for improvement span across hardware upgrades and software enhancements. Hardware changes include the possibility of transitioning to a Lead-Lag or PID controller for more precise transient responses and incorporating higher-quality components to improve tolerance levels. On the software side, advanced control schemes like adaptive control could provide the system with greater resilience against unforeseen aerodynamic variabilities and component tolerances. Additionally, the inclusion of digital controllers, such as Raspberry Pi or Arduino, could offer better flexibility and ease in fine-tuning system parameters.

By thoughtfully implementing these targeted improvements, the Kitticopter's altitude control system is well-positioned to not only meet but exceed design requirements. The resulting system would thereby be better equipped to handle real-world operational challenges, achieving a compelling balance of performance, reliability, and adaptability.

# References

- [1] Travimadox Webb. EEE3094S-2023: Control Systems Labs, August 2023. original-date: 2023-08-26T20:16:44Z.
- [2] Travimadox Webb. Eee3094s lab 2 report: Kitticopter control system. Technical report, Department of Electrical Engineering, Faculty of Engineering and Built Environment, University of Cape Town, 2023.
- [3] Classical control design - matlab & simulink, 2023. Accessed: 2023-09-30.
- [4] Control tutorials for matlab and simulink - introduction: Pid controller design, 2023. Accessed: 2023-09-30.
- [5] Control tutorials for matlab and simulink - introduction: Root locus controller design, 2023. Accessed: 2023-09-30.
- [6] Control tutorials for matlab and simulink - pi control of dc motor speed, 2023. Accessed: 2023-09-30.
- [7] Fulltext01.pdf, 2023. Accessed: 2023-09-30.
- [8] lab2\_proportional\_control\_with\_template.v2.pdf, 2023. Accessed: 2023-09-30.
- [9] Pid controller — going straight with pid — coding projects for kids and teens, 2023. Accessed: 2023-09-30.
- [10] Root locus design method — p controller design — calculations & matlab simulations — example #1, 2021. Accessed: 2023-09-30.
- [11] Feidias Ioannidis. School of computer science. 2023.
- [12] Tune pid controller to favor reference tracking or disturbance rejection (pid tuner) - matlab & simulink, 2023. Accessed: 2023-09-30.