Recursion, Linked Lists, and ADTs

Overview

- Recursion
- Singly-Linked Lists
- Abstract Data Types

References

- Bruno R. Preiss: Data Structures and Algorithms with Object-Oriented Design Patterns in C++. John Wiley & Sons, Inc. (1999)
- Richard F. Gilberg and Behrouz A. Forouzan: Data Structures - A Pseudocode Approach with C. 2nd Edition. Thomson (2005)
- Russ Miller and Laurence Boxer: Algorithms Sequential & Parallel. 2nd Edition. Charles River Media Inc. (2005)
- Stanley B. Lippman, Josée Lajoie, and Barbara E. Moo: C++ Primer. 5th Edition. Addison-Wesley (2013)

Recursion

- If a procedure contains within its body calls to itself, then this procedure is said to be recursively defined.
- This approach of program specification is called recursion and is found not only in programming.
- If we the define a procedure recursively, then there must exist at least one sub-problem that can be solved directly, that is without calling the procedure again.
- A recursively defined procedure must always contain a directly solvable sub-problem. Otherwise, this procedure does not terminate.

Problem-Solving with Recursion

- Recursion is an important problem-solving technique in which a given problem is reduced to smaller instances of the same problem.

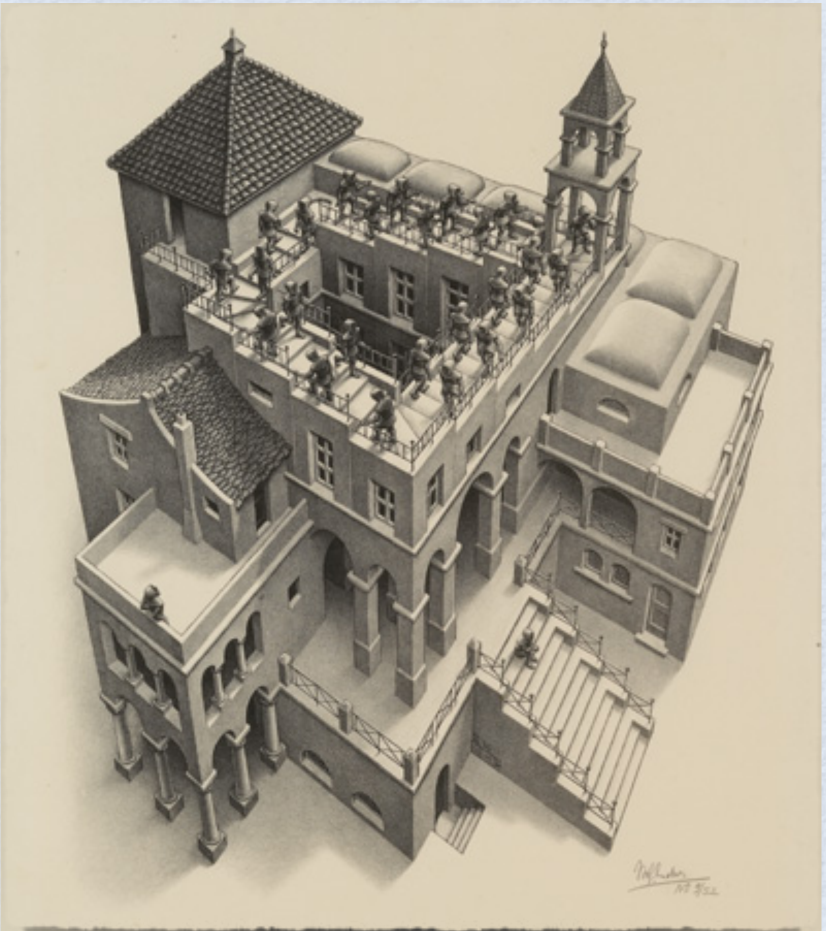
- The general structure of a recursive definition is

$$X = \dots X \dots$$

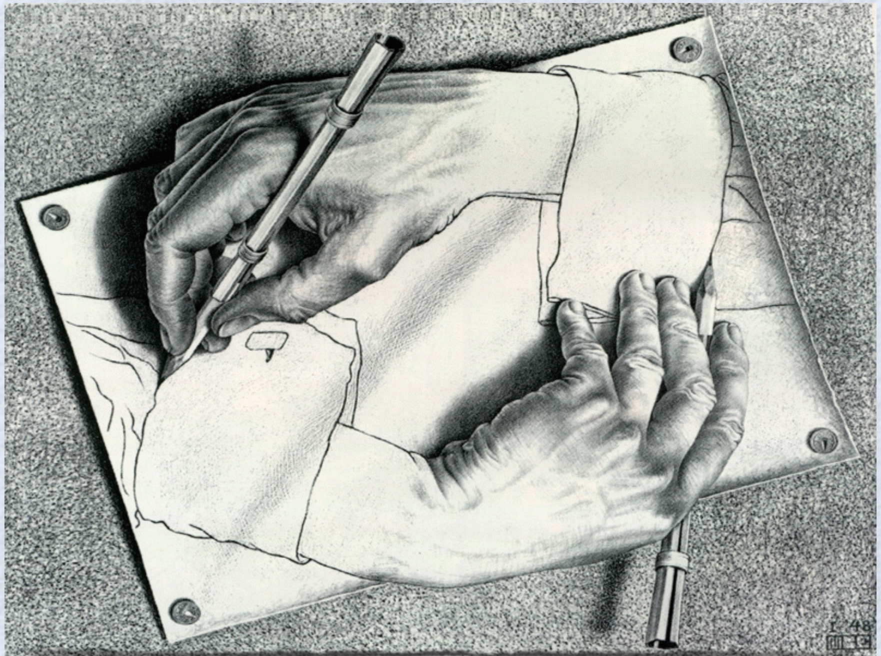
Left-hand side

Right-hand side

Impossible Recursive Structures



http://www.mcescher.com/



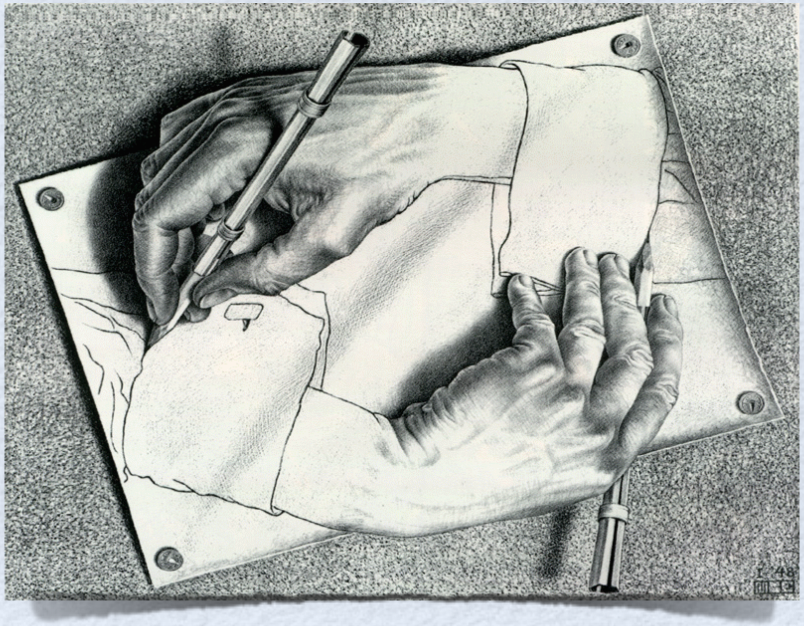
Impossible Structures in C++:

```
#include "ClassB.h"

class ClassA
{
    use ClassB
};
```

```
#include "ClassA.h"

class ClassB
{
    use ClassA
};
```



Forward Declaration

- Unlike in Java or C#, we cannot define mutual recursive classes in C++.
- We must resolve the dependency manually and use forward declarations in specifications.
- In the implementations, we must include all specifications, so that the compiler can resolve the dependencies.

```
class ClassB;

class ClassA
{
    use ClassB
};
```

```
class ClassA;

class ClassB
{
    use ClassA
};
```

```
#include "ClassA.h"
#include "ClassB.h"

implement ClassA
implement ClassB
```

Basic Recursive Problems

Fibonacci

- The Fibonacci numbers are the following sequence of numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …
- In mathematical terms, the sequence F(n) of Fibonacci numbers is defined recursively as follows:

$$F(1) = 1$$

$$F(2) = 1$$

$$F(n) = F(n-1) + F(n-2)$$

fib.cpp

```cpp
unsigned long fib( unsigned long n )
{
    if ( n <= 2)
        return 1;
    else
        return fib( n-1 ) + fib( n-2 );
}
```

Line: 1 Column: 2 C++ Tab Size: 4

Recursive Problem-Solving: Factorials

- The factorial for positive integers is

$$n! = n * (n - 1) * \ldots * 1$$

- The recursive definition:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n * (n-1)! & \text{if } n > 0 \end{cases}$$

Calculating Factorials

- The recursive definition tells us exactly how to calculate a factorial:

4!	= 4 * 3!	Recursive step: n=4
	= 4 * (3 * 2!)	Recursive step: n=3
	= 4 * (3 * (2 * 1!))	Recursive step: n=2
	= 4 * (3 * (2 * (1 * 0!)))	Recursive step: n=1
	= 4 * (3 * (2 * (1 * 1)))	Stop condition: n=0
	= 4 * (3 * (2 * 1))	
	= 4 * (3 * 2)	
	= 4 * 6	
	= 24	

Recursive Factorial

factorial.cpp

```cpp
#include <iostream>

using namespace std;

long factorial( unsigned long n )
{
  if (n == 0)
    return 1;
  else
    return n * factorial( n-1 );
}

int main()
{
    cout << "6! = " << factorial( 6 ) << endl;

    return 0;
}
```

Line: 19 Column: 1 C++ Tab Size: 4 main

Tail-Recursion

- A function is called tail-recursive if it ends in a recursive call that does not build-up any deferred operations.

gcd.cpp

```cpp
long gcd( long x, long y )
{
  if (y == 0)
    return x;
  else
    return gcd( y, x % y );
}
```

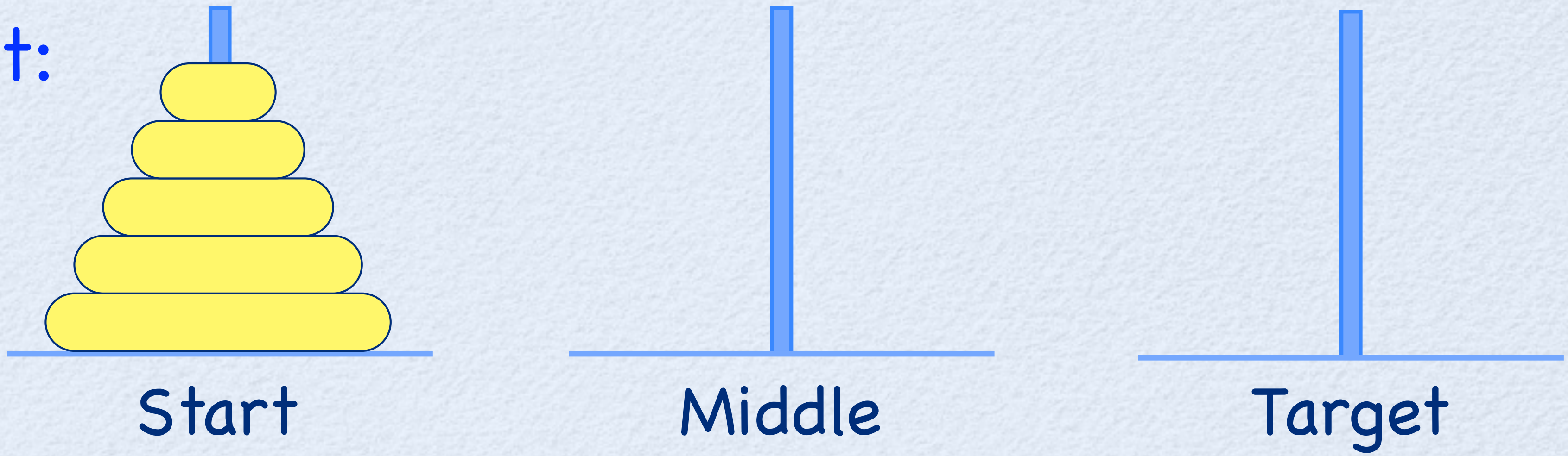
gcd(1246, 234):
➥ gcd(234, 76)
➥ gcd(76, 6)
➥ gcd(6, 4)
➥ gcd(4, 2)
➥ gcd(2, 0)
➥ 2

Towers of Hanoi

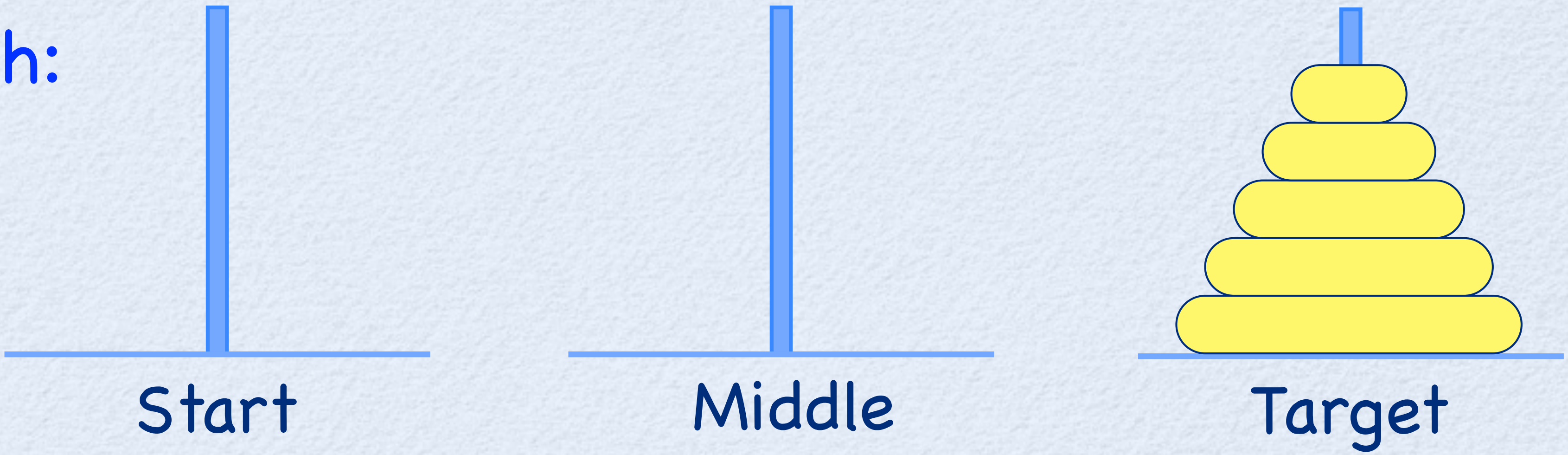
- Problem:
  - Move disks from a start peg to a target peg using a middle peg.
- Challenge:
  - All disks have a unique size and at no time must a bigger disk be placed on top of a smaller one.

Towers of Hanoi: Configuration

Start:



Finish:



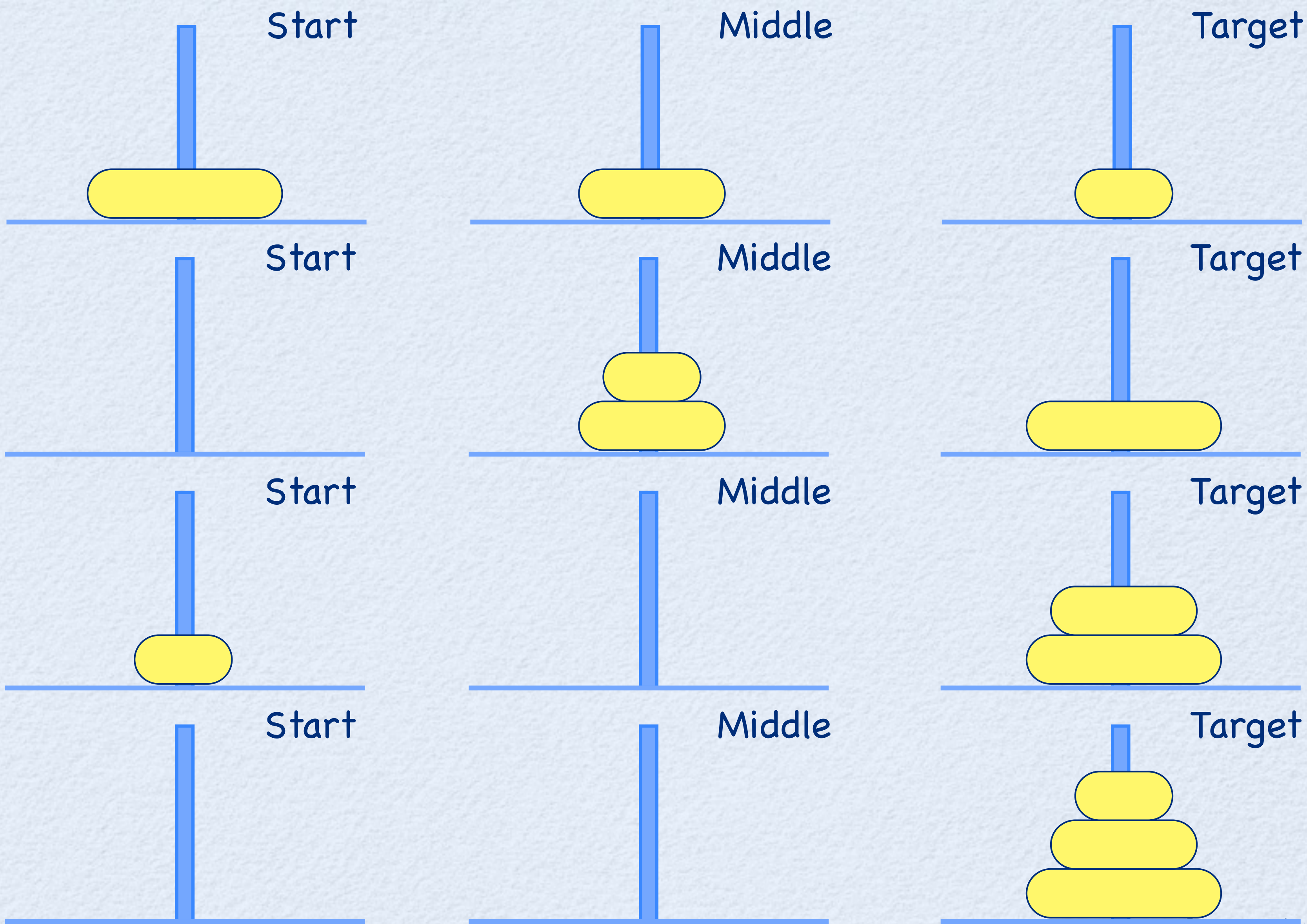
A Recursive Solution

1. Move n-1 disks from Start to Middle:

2. Move 1 disk from Start to Target:

3. Move n-1 disks from Middle to Target:

A Recursive Solution: Intermediate



The Recursive Procedure

hanoi.cpp

```cpp
#include <iostream>

using namespace std;

void move( int n, string start, string target, string middle )
{
    if ( n > 0 )
    {
        move( n-1, start, middle, target );
        cout << "Move disk "  << n << " from " << start
             << " to "<< target << "." << endl;
        move( n-1, middle, target, start );
    }
}

int main()
{
    move( 3, "Start", "Target", "Middle" );

    return 0;
}
```

Line: 22 Column: 1 C++ Tab Size: 4 main

COS30008

```
Kamala:COS30008 Markus$ ./hanoi
Move disk 1 from Start to Target.
Move disk 2 from Start to Middle.
Move disk 1 from Target to Middle.
Move disk 3 from Start to Target.
Move disk 1 from Middle to Start.
Move disk 2 from Middle to Target.
Move disk 1 from Start to Target.
Kamala:COS30008 Markus$ _
```

Recursion is a prerequisite for linked lists!