2019 MATH3201 Assignment 2

Please provide all your workings. Please attach a print out of your codes to your assignment. Each code should be clearly labelled and should contain comments showing what the different variables are and what a part of the code is doing.
If no code printouts are attached then points will be taken off.

Question 1 [6 marks]

This is a pen and paper question. Please show all your workings. Use Euler's method to approximate the solution for the following initial value problem:

$$y' = 1 + y/t, \quad 1 \leq t \leq 2,$$
$$y(1) = 2,$$

with $h = 0.25$.

Question 2 [12 marks]

This is a pen and paper question. Please show all your workings.

(a) [8 marks] Derive the Taylor's method of order two for general initial value problems:

$$y' = f(t, y), \qquad a \leq t \leq b,$$
$$y(a) = \alpha,$$

(b) [4 marks] Derive the general update formula, $y_{n+1} = y_n +$ other terms, for the initial value problem using the method derived in (a):

$$y' = 1 + (t - y)^2,$$
$$y(2) = 1,$$

with $h = 0.5$ (use $t_n = 2 + 0.5n$). There is no need to calculate individual y_n.

Question 3 [10 marks]

Write a code to implement the trapezoidal rule and the Simpson's rule for the integral:

$$\int_0^5 \exp(x)dx$$

for N intervals. Consider $N = 2, 4, 8, 16, 32, 64, 128$ and tabulate the error/h^n for these values. Comment on how they compare to theory.

Question 4 [12 marks]

Write a code to use the Euler and the fourth order Runge Kutta methods to approximate the solution to the initial value problem:

$$y' = 1 + (t - y)^2, \qquad 2 \le t \le 10,$$

$$y(2) = 1,$$

with stepsize $h = 0.1$ and $h = 0.5$. The exact solution is $y(t) = t + \frac{1}{1-t}$. Tabulate the values of the approximate solutions and the exact solution and compare the results.