2019 MATH3201 Assignment 4

Due: 5pm 27-May-2019

Instructions for assignment preparation:

Assignment should include all your workings: formulations, steps, and explanations/justifications taken to obtain the formulations, tables, figures, and Matlab codes.

Notes:

- Figures: your plots should have clear label for every axis. Include a clear caption which includes: the main idea drawn from the figure, brief introduction of all symbols used in the figure, and addressing what settings the colors correspond to. Every figure should be references in the text where appropriate.
- Tables: include a clear caption which includes: a general title for the table, and brief introduction of all symbols used in the table. Every table should be references in the text where appropriate.
- Matlab code: your code should be clearly labelled containing comments showing what the different variables are and what a part of the code is doing.

Note:

Marks will be deducted if your assignment does not meet the instructions outlined above.

Question 4.1 [70 marks]

Consider the differential equation $y" = y$ subject to the Neumann boundary conditions $y'(0) = 0$ and $y(1) = \cosh(1)$.

(a) Formulate a numerical solution over N intervals using the simple approximation $y'(x) = \big(y(x+h) - y(x)\big)/h$, where h is the step size [5 marks].

(b) Implement your formulation in part (a) in Matlab [15 marks].

(c) For every value of N (i.e., 4, 8, and 16), calculate and tabulate the maximum absolute error $\varepsilon = \max(|y_i - Y_i|)$, where y_i and Y_i are the estimated and real values of solution, respectively, at grid points x_i with $i = 0, \ldots, N$ [10 marks].

(d) Plot the exact and approximated solutions (for N = 16 only) [5 marks].

(e) Use the three-point forward approximation formula to estimate for the first derivative. Implement this approximation of the boundary condition $y'(0)$ in your Matlab code, and repeat the parts (a) to (c) [20 marks].

(f) Plot the exact and approximated solutions (for N = 16 only) [5 marks].

(g) Provide a short paragraph describing the differences in your plots derived from parts (d) and (f) [10 marks].

Question 4.2 [30 marks]

Kinesins are molecular motors that carry cargoes within cells by walking along cellular tracks [1]. Suppose that the velocity $v(F)$ of a molecular motor has been recorded at five forces F in table 1.

Table 1. Motor force-velocity data $v(F)$.

$\boldsymbol{F}$ **(pN)**	$\boldsymbol{v(F)}$ **(nm/s)**
0	800
2	700
4	550
5	500
7	50

(a) Use Newton's divide differences and find a polynomial that interpolates (i.e., fits) the force-velocity data in Table 1 [10 marks].

(b) Implement your fit equation derived in part (a) in Matlab [5 marks].

(c) Plot your fit equation (curve) and data points (open circles) together [5 marks].

(d) Add data points $(F, v(F)) = \{(6, 150), (7.3, 15)\}$ to Table 1. Repeat steps (a-c) and explain how your fit changes as you add more data [10 marks].

Reference:

[1] J. Howard, Mechanics of Motor Proteins and the Cytoskeleton (Sinauer, Sunderland, MA, USA, 2001).