MATH3201 Practical Session
Week 10

Question 1
Use an update rule to calculate solutions of the forward difference approximation of the heat equation:

$$u_t = Du_{xx}$$

with boundary conditions

$$u(x,0) = \sin(\pi x/L),\ u(0,t) = 0, \text{ and } u(L,t) = 0$$

with parameters D = 0.5, L = 2.0 (i.e., $0<x<L$) for $0<t<1.0$. Set $\sigma = D(dt)/(dx)^2$. Consider M = 11 spatial points, and study the behaviour of the solutions for different values of Δt (i.e. consider σ < > = 0.5). The exact solution is $\exp(-D\pi^2 t/L^2)\sin(\pi x/L)$.
Plot your numerical solution against the exact solution. Show the numerical and exact solutions as solid curve and open circles, respectively. Label the solutions using Matlab legend.
(Note, you do not need to write the equation in matrix form to perform the update rule).

Question 2
Use a backward difference scheme to solve the heat equation

$$u_t = Du_{xx}$$

with boundary conditions

$$u(x,0) = \sin(\pi x/L),\ u(0,t) = 0, \text{ and } u(L,t) = 0$$

with parameters D = 0.5, L = 2.0 for $0<t<1.0$. Set $\sigma = D(dt)/(dx)^2$. Consider M = 11 spatial points, and study the behaviour of the solutions for different values of Δt (i.e. consider σ < > = 0.5). The exact solution is $\exp(-D\pi^2 t/L^2)\sin(\pi x/L)$.
Plot your numerical solution against the exact solution. Show the numerical and exact solutions as solid curve and open circles, respectively. Label the solutions using Matlab legend.