Swap

pg. 29

$$\dfrac{\dfrac{\dfrac{}{M : A \wedge B}\,M}{\text{snd } M : B}\wedge E_R \qquad \dfrac{\dfrac{}{M : A \wedge B}\,M}{\text{fst } M : A}\wedge E_L}{\langle \text{snd } M, \text{fst } M \rangle : B \wedge A}\wedge I}{(\lambda M.\langle \text{snd } M, \text{fst } M\rangle) : (A \wedge B) \supset (B \wedge A)}\supset I^M$$

Given a function that maps A to (B, C), program gives two functions, A→B and A→C.

pg. 31

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{M : A \supset (B \wedge C)}\,M \quad \dfrac{}{W : A}\,W}{M\ W : B \wedge C}\supset E}{\text{fst } (M\ W) : B}\wedge E_L}{\lambda W.\ \text{fst}(M\ W) : A \supset B}\supset I^W \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{M : A \supset (B \wedge C)}\,M \quad \dfrac{}{V : A}\,V}{M\ V : B \wedge C}\supset E}{\text{snd } (M\ V) : C}\wedge E_R}{\lambda V.\ \text{snd}(M\ V) : A \supset C}\supset I^V}{\langle (\lambda W.\ \text{fst}(M\ W)), (\lambda V.\ \text{snd}(M\ V))\rangle : (A \supset B) \wedge (A \supset C)}\wedge I}{\lambda M.\langle (\lambda W.\ \text{fst}(M\ W)), (\lambda V.\ \text{snd}(M\ V))\rangle : (A \supset (B \wedge C)) \supset ((A \supset B) \wedge (A \supset C))}\supset I^M$$

pg. 13

Not sure what inl and case — of inl means yet in program extraction rules, so tree not filled out.

$$\dfrac{\dfrac{}{A \vee B}\,M \qquad \dfrac{\dfrac{}{A}\,V}{B \vee A}\vee I_R \qquad \dfrac{\dfrac{}{B}\,W}{B \vee A}\vee I_L}{B \vee A}\vee E^{V,W}}{(A \vee B) \supset (B \vee A)}\supset I^M$$

Comprehension of functions

pg. 33

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{M : (A \to B) \wedge (B \to C)}\,M}{snd\ M : B \to C}\,\wedge E_R \qquad \dfrac{\dfrac{\dfrac{}{M : (A \to B) \wedge (B \to C)}\,M}{fst\ M : A \to B}\,\wedge E_L \qquad \dfrac{}{w : A}\,w}{(fst\ M)\ w : B}\,\to E}{(snd\ M)((fst\ M)\ w) : C}\,\to E}{\lambda w.\ (snd\ M)((fst\ M)\ w) : A \to C}\,\to I^{w}}{(\lambda M.\ \lambda w.\ (snd\ M)((fst\ M)\ w)) : ((A \to B) \wedge (B \to C)) \to (A \to C)}\,\to I^{M}$$

My own example

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{M : (A \to B) \to ((C \to D) \wedge C)}\,M \qquad \dfrac{}{V : (A \to B)}\,V}{M\ V : (C \to D) \wedge C}\,\to E}{fst\ (M\ V) : C \to D}\,\wedge E_L \qquad \dfrac{\dfrac{\dfrac{}{M : (A \to B) \to ((C \to D) \wedge C)}\,M \qquad \dfrac{}{V : (A \to B)}\,V}{M\ V : (C \to D) \wedge C}\,\to E}{snd\ (M\ V) : C}\,\wedge E_R}{fst\ (M\ V)\ snd\ (M\ V) : D}\,\to E}{\lambda V.\ fst\ (M\ V)\ snd\ (M\ V) : (A \to B) \to D}\,\to I^{V}}{\lambda M.\ \lambda V.\ fst\ (M\ V)\ snd\ (M\ V) : ((A \to B) \to ((C \to D) \wedge C)) \to ((A \to B) \to D)}\,\to I^{M}$$