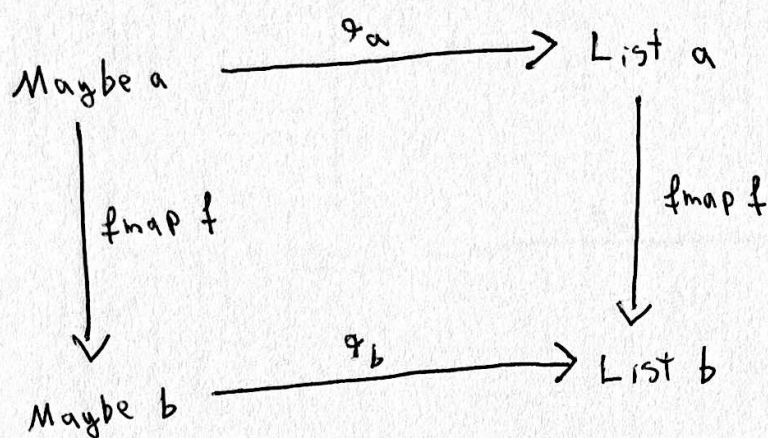


①

Define a natural transformation from the ~~Maybe~~ functor to the List functor.

Solution

I need to find an $\eta_a :: \text{Maybe } a \rightarrow \text{List } a$ s.t. the following diagram commutes $\forall f$:



Let

$$\eta_a \text{ Nothing} = []$$

$$\eta_a (\text{Just } x) = [x]$$

Then

$$fmap f \circ \eta_a \text{ Nothing} = [] = \eta_b \circ fmap f \text{ Nothing} \checkmark$$

$$fmap f \circ \eta_a (\text{Just } x) = fmap f [x] = [fx]$$

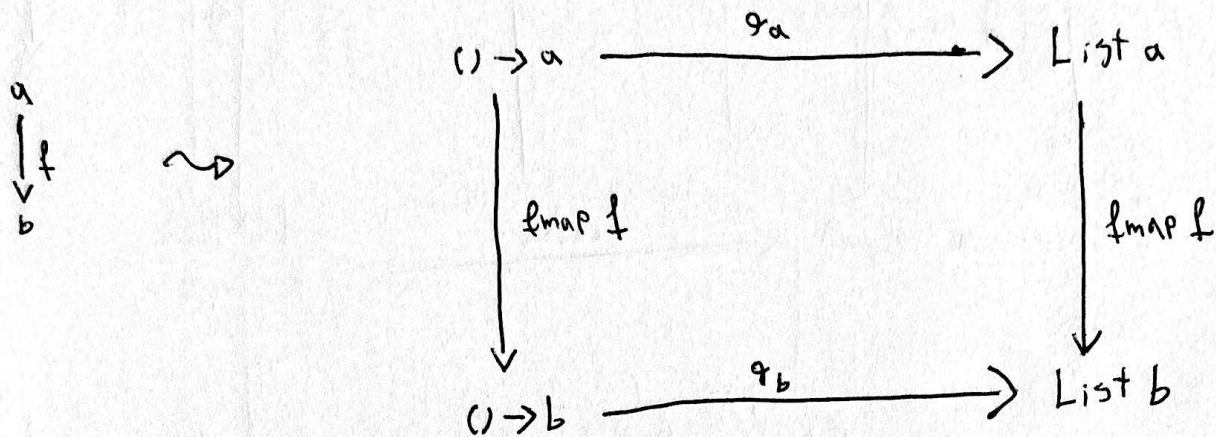
$$= \eta_b (\text{Just } (fx))$$

$$= \eta_b \circ fmap f (\text{Just } x) \checkmark$$

② Define 2 natural transformations between $\text{Reader}()$ and List .

Solution

We want $\eta_a, \beta_a :: () \rightarrow a \rightarrow \text{List } a$ s.t. the following diagram commutes $\forall f$:



Let

$$\eta_a \ x = [x()],$$

$$\beta_a \ x = []$$

then

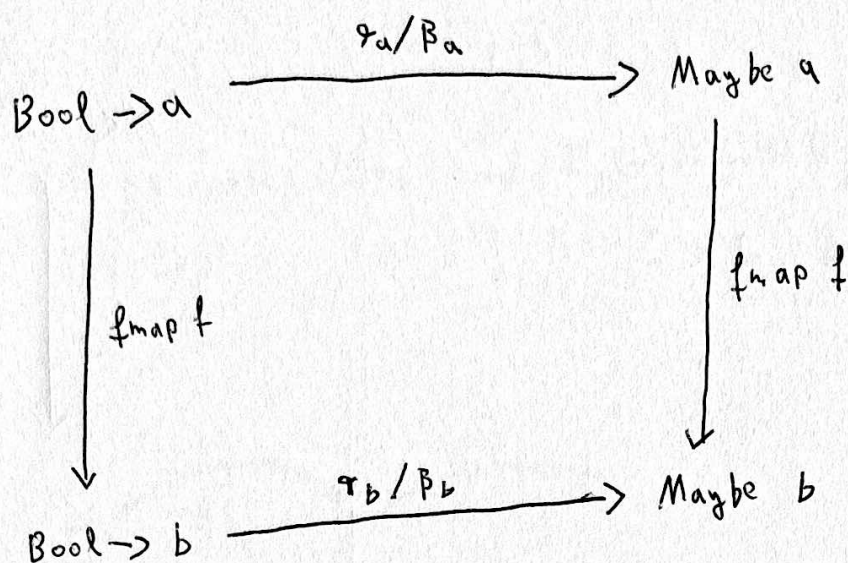
$$\text{fmap } f \circ \eta_a \ x = [f(x())] = \eta_b \circ \text{fmap } f \ x \quad \checkmark$$

$$\text{fmap } f \circ \beta_a \ x = [] = \beta_b \circ \text{fmap } f \ x \quad \checkmark$$

There are ∞ many lists of $()$, namely $[], [()], [(), ()], \dots$

③ Define 2 natural transformations between Reader Bool and Maybe.

Solution:



Let $\eta_a \ g = \text{Just } (g \ \text{True}) \ \forall \ f \ g,$

$\beta_a \ g = \text{Sust } (g \ \text{False})$

Then

$$\begin{aligned}
 \eta_a \circ \text{fmap } f \ g &= \text{fmap } f \ (\text{Just } (g \ \text{True})) \\
 &= \text{Just } (f \circ g \ \text{True})
 \end{aligned}$$

and

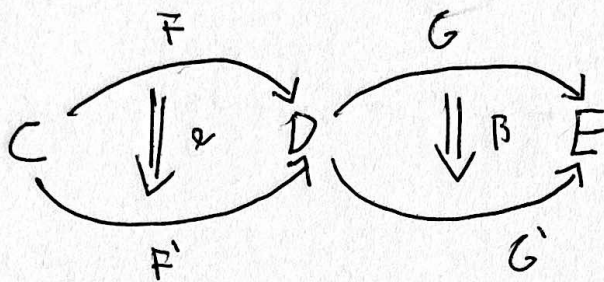
$$\begin{aligned}
 \eta_b \circ \text{fmap } f \ g &= \eta_b \ f \circ g \\
 &= \text{Just } (f \circ g \ \text{True})
 \end{aligned}$$

Same goes for β .

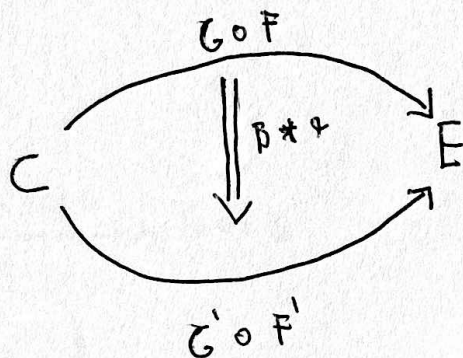
④ Show that horizontal composition of natural transformations satisfies the naturality condition.

Solution:

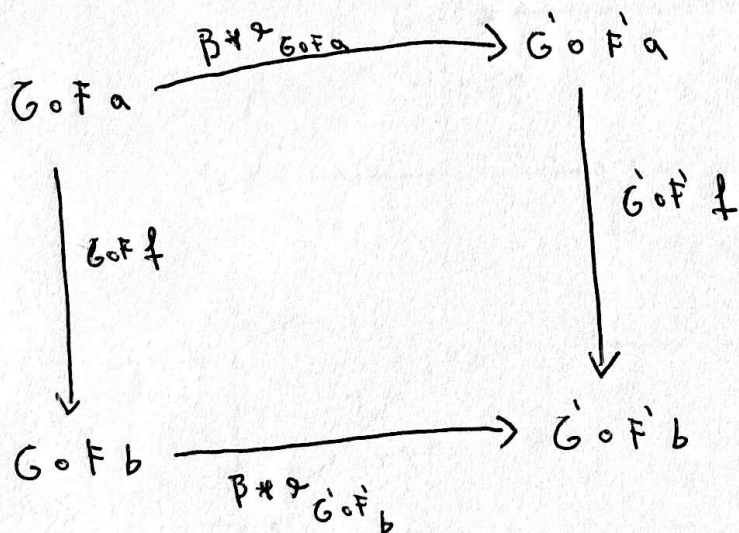
Let C, D, E be categories and $F, F' :: C \rightarrow D$, $G, G' :: D \rightarrow E$ be functors. Also, let $\eta :: F \rightarrow F'$, $\beta :: G \rightarrow G'$ be natural transformations. I.e.



We will show that \exists a natural transformation of the form

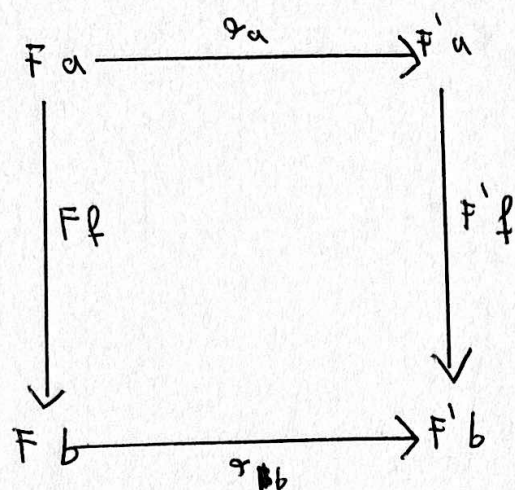


by diagram chasing. We will do this by defining $\beta * \eta$ and then showing that the following diagram commutes:

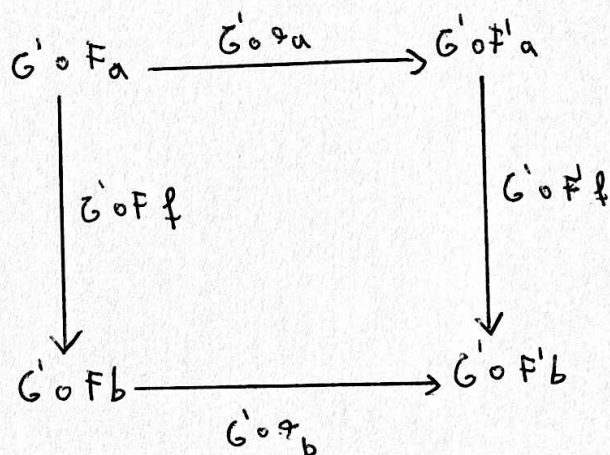
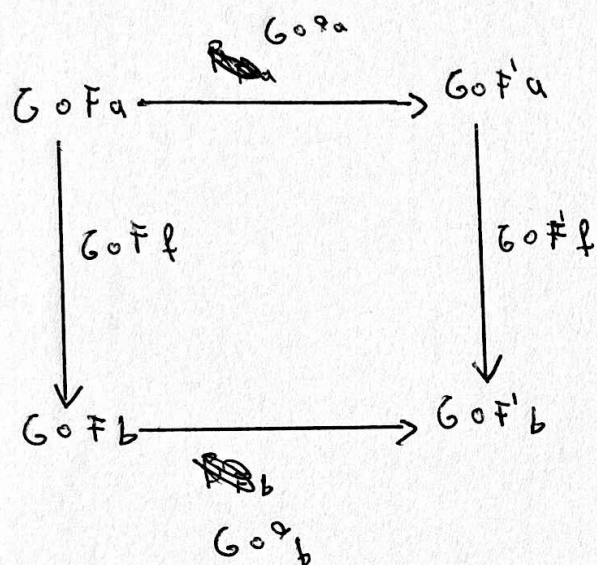


$\forall a :: C, b :: C,$
 $f :: a \rightarrow b$

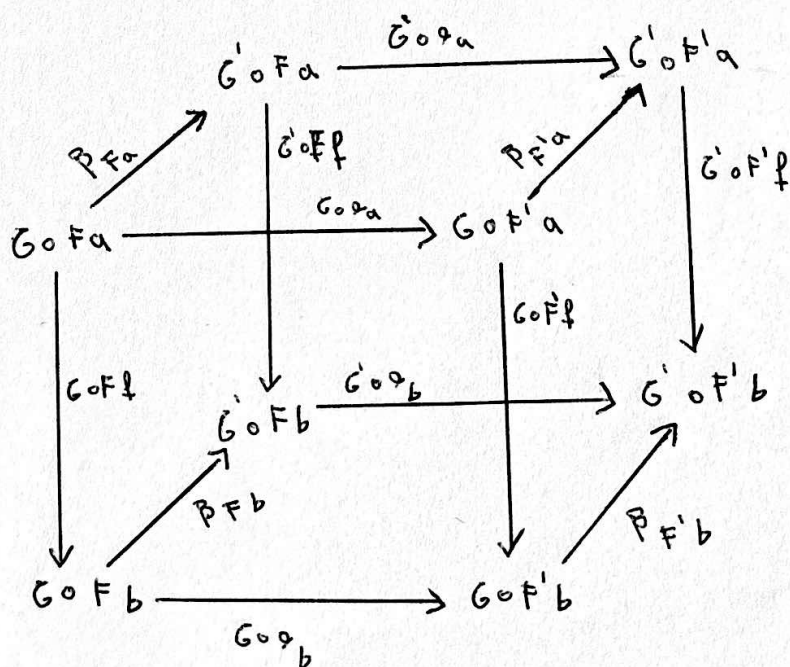
By the naturality of τ , the following commutes:



which in turn gives rise to 2 other commuting diagrams, by applying G and G' :



But these can be linked with β :



By chasing the diagram, we find 2 equal definitions for $\beta * \gamma$:

$$\beta * \gamma_{G \circ F \alpha} = \beta_{F \alpha} \circ G \circ \gamma_{\alpha}$$

$$= G' \circ \gamma_{\alpha} \circ \beta_{F \alpha} \quad \square$$